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MEASUREMENTS ON THE EFFECT OF TRANSDUCER SIZE ON THE RESOLUTION OF BOUNDARY-LAYER PRESSURE FLUCTUATIONS

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER
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F.E. Geib, Jr.

DEPARTMENT OF ACOUSTICS AND VIBRATION
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Measurements on the Effect of Transducer Size on the Resolution of Boundary-Layer Pressure Fluctuations

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The response of a flush-mounted transducer to the pressure field in a turbulent boundary layer is known to depend on the spatial and temporal characteristics of the transducer. This paper presents an experimental study of this dependence. The reduced data are presented in a manner similar to that used by Corcos to present his estimation of the response of transducers to a corresponding pressure field.

INTRODUCTION

The response of a flush-mounted transducer to the pressure field in a turbulent boundary layer is known to depend on the spatial and temporal characteristics of the transducer. This paper presents the results of an experimental study of this dependence. The frequency spectral density of the pressure fluctuations on the boundary beneath a turbulent boundary layer was measured with transducers of various radii. These measurements were taken in air at several flow speeds and analyzed by passing the signal through a narrow-band frequency analyzer. Similar experimental work was performed by Gilchrist and Strawderman.[1] The experimental work reported in the present paper constitutes, therefore, additional experimental data. The data reported by Gilchrist and Strawderman and in the present paper were reduced in a form that corresponds to Corcos' presentation of his semiempirical predictions of this phenomenon.[1,2] The method of data reduction is contingent on a number of assumptions and idealizations of the pressure field in a turbulent boundary layer and the response characteristics of the flush-mounted transducer. Therefore, to make the presentation of the reduced data meaningful, the nature of the assumptions and idealizations that are involved must be made explicit. For this purpose, Sec. I is devoted to the discussion of the salient elements of the data-reduction procedure adopted in the present paper. However, no attempt is made to estimate the scatter in the reduced data that occurs as different assumptions and idealizations are violated in the experiments; this attempt awaits further research. In Sec. II, the measurement techniques are briefly considered, and in Sec. III the instruments and the transducers used in the experiment are described. Section IV is devoted to the discussion of the results obtained in the experiment. Additional remarks and conclusions are presented in Sec. V.

I. DATA-REDUCTION PROCEDURE

The data obtained in the present experiment are presented in a manner similar to that used by Corcos[2] to present his semiempirical estimation of the response of flush-mounted transducers to the pressure field in a turbulent boundary layer. Since the method employed to reduce the data to this form is contingent on a number of assumptions and idealizations, it is pertinent to consider the salient features of the method in order to make the presentation of the results meaningful.

If the turbulent pressure field is assumed to be statistically stationary and homogeneous, an expression describing the frequency spectral response of a flush-mounted transducer can be derived in the form[2,3]

$$\Phi_m(\omega,\nu)=\int_\infty d\omega' D(\omega',\omega,\Delta) \times\int\int_\infty dA(\boldsymbol{\varepsilon})\Psi(\boldsymbol{\varepsilon},\omega')\Gamma(\boldsymbol{\varepsilon},\omega',\nu). \quad (1)$$

[1] R. B. Gilchrist and W. A. Strawderman, "Experimenta Hydrophone-Size Correction Factor for Boundary-Layer Pressure Fluctuations," J. Acoust. Soc. Amer. 38, 298–302 (1965).

[2] G. M. Corcos, "Resolution of Pressure in Turbulence," J. Acoust. Soc. Amer. 35, 192–199 (1963).

[3] G. Maidanik and D. W. Jorgensen, "Boundary Wave-Vector Filters for the Study of the Pressure Field in a Turbulent Boundary Layer," J. Acoust. Soc. Amer. 42, 494–501 (1967).

In Eq. 1, $\Phi_m(\omega,\nu)$ is the frequency spectral density measured by the flush-mounted transducer; $D(\omega',\omega,\Delta)$ describes the normalized filtering action of an incorporated frequency-band analyzer with center frequency ω and half-bandwidth Δ; $\Psi(\boldsymbol{\varepsilon},\omega')$ is the function that describes the spatial- and frequency-response characteristics of the flush-mounted transducer; $\Gamma(\boldsymbol{\varepsilon},\omega',\nu)$ is the cross frequency spectral density of the pressure field in the turbulent boundary layer; $dA(\boldsymbol{\varepsilon})$ is an incremental area on the plane boundary in which the transducer is flush-mounted, and $\boldsymbol{\varepsilon}$ is the spatial separation vector between two points in that plane; ν is the kinematic viscosity of the fluid; and ω' is the frequency variable. In stating Eq. 1, it is assumed that the flush-mounted transducer does not modify the pressure field as it would have existed if the transducer were removed and the boundary extended to replace it.[4]

The integration over ω', in Eq. 1, can be readily performed if the bandwidth 2Δ is kept small so that, for values of $|\boldsymbol{\varepsilon}|\lesssim L$, $\Psi(\boldsymbol{\varepsilon},\omega')|\Gamma(\boldsymbol{\varepsilon},\omega',\nu)|$ remains substantially constant as ω' varies within the band, and if, in addition, the restriction

$$L\Delta/U\ll 1 \tag{2}$$

is imposed in order to ensure that the phase of $\Gamma(\boldsymbol{\varepsilon},\omega',\nu)$ remains substantially constant as ω' varies within the band.[4,5] The parameter L is a typical linear spatial dimension of the flush-mounted transducer, and the parameter U is the group velocity associated with the convection of the pressure field in the turbulent boundary layer.[5] (The group velocity is in reference to those frequency spectral components of the pressure field that lie within the frequency bandwidth under consideration. This velocity U is expected to be of the order of U_∞, the free-stream velocity.) Upon integration over ω', Eq. 1 reduces to the form

$$\Phi_m(\omega,\nu)=\int\int_\infty dA(\boldsymbol{\varepsilon})\Psi(\boldsymbol{\varepsilon},\omega)\Gamma(\boldsymbol{\varepsilon},\omega,\nu). \tag{3}$$

It is convenient to express Eq. 3 in a dimensionless form. For this purpose, the cross frequency spectral density is written in the form

$$\frac{\Gamma(\boldsymbol{\varepsilon},\omega,\nu)}{\rho^2U_\infty{}^3\delta^*}=\gamma\left(\frac{\omega\boldsymbol{\varepsilon}}{U_\infty},\frac{\omega\delta^*}{U_\infty},\frac{\delta^*U_\infty}{\nu}\right), \tag{4}$$

where ρ is the density of the fluid and δ^* is the displacement thickness of the turbulent boundary layer. The response function can also be expressed in dimensionless form:

$$dA(\boldsymbol{\varepsilon})\Psi(\boldsymbol{\varepsilon},\omega)=dA(\boldsymbol{\xi})\psi(\boldsymbol{\xi},\Omega), \tag{5}$$

where

$$\boldsymbol{\xi}=\boldsymbol{\varepsilon}/L,\quad \Omega=\omega/\omega_L, \tag{6}$$

and ω_L is a typical relaxation frequency of the flush-mounted transducer. The insertion of Eqs. 4 and 5 into Eq. 3 yields a dimensionless form for the measured frequency spectral density:

$$\frac{\Phi_m(\omega,\nu)}{\rho^2U_\infty{}^3\delta^*}=\varphi_m\left(\frac{\omega L}{U_\infty},\Omega,\frac{\omega\delta^*}{U_\infty},\frac{\delta^*U_\infty}{\nu}\right). \tag{7}$$

Equation 7 is of limited utility as it stands, except to indicate formally the dependence of the normalized response on the dimensionless parameters characterizing the pressure field and the transducer. To expand the utility of Eq. 7, more intimate knowledge of this dependence is needed.

A suggestion by Corcos,[2] backed by some experimental evidence, makes it possible to cast Eq. 7 in a form suited to making predictions concerning the frequency spectral response of a class of flush-mounted transducers. Corcos has shown that, for a range of pressure fields in turbulent boundary layers, the experimental data obtained with two small flush-mounted transducers for a range of separation distances, frequencies, and convection velocities are consistent with a cross-frequency spectral density, Eq. 4, that is separable in the form[2,6,7]

$$\gamma\left(\frac{\omega\boldsymbol{\varepsilon}}{U_\infty},\frac{\omega\delta^*}{U_\infty},\frac{\delta^*U_\infty}{\nu}\right)=\varphi\left(\frac{\omega\delta^*}{U_\infty}\right)\lambda\left(\frac{\omega\boldsymbol{\varepsilon}}{U_c}\right), \tag{8}$$

where U_c is the convection velocity of the pressure field. The convection velocity is related to the free-stream velocity through a weak dependence on frequency,[2]

$$\frac{U_c}{U_\infty}=\frac{U_c}{U_\infty}\left(\frac{\omega\delta^*}{U_\infty}\right). \tag{9}$$

The functional dependence of φ and λ on the Reynolds number δ^*U_∞/ν is assumed to be weak and the dependence on this parameter is therefore dropped in Eq. 8.

The response operator, Eq. 5, can be simplified somewhat by assuming that it is separable in the form

$$dA(\boldsymbol{\xi})\psi(\boldsymbol{\xi},\Omega)=dA(\boldsymbol{\xi})\theta(\boldsymbol{\xi})|F(\Omega)|^2, \tag{10}$$

where $\theta(\boldsymbol{\xi})$ is a spatial-response function and $|F(\Omega)|^2$ is the frequency-response function. Not all flush-mounted transducers possess a separable response function, at least not in all of the frequency range. Therefore, if Eq. 10 is utilized, it is to be understood that the analysis is limited to those flush-mounted

[4] G. Maidanik, "System of Small Size Transducers as Elemental Unit in Sonar System," J. Acoust. Soc. Amer. 44, 488–496 (1968).

[5] H. Cox and M. Strasberg, "Bandwidth Limitations in Measurements of Cross-Spectral Density," J. Acoust. Soc. Amer. 42, 1217(A) (1967).

[6] G. M. Corcos, "Pressure Fluctuations in Shear Flows," Univ. California Inst. Eng. Res. Rep. Ser. 183, No. 2 (July 1962).

[7] W. W. Willmarth and C. E. Wooldridge, "Measurements of the Fluctuating Pressure at the Wall beneath a Thick Turbulent Boundary Layer," Univ. Mich. ORA Rep. 02920-1-T (1962).

transducers and those frequency ranges where this separation is a valid approximation. (In his analysis, Corcos has essentially assumed that the transducers that he considered are separable and that the frequency-response operators are identically unity over the entire frequency range.[2])

When Eqs. 8 and 10 are substituted into Eq. 3, the measured frequency spectral density is obtained in the form

$$\Phi_m(\omega)/\rho^2U_\infty^3\delta^* = \varphi(\omega\delta^*/U_\infty)\,|F(\Omega)|^2\sigma(\omega L/U_c), \quad (11)$$

with

$$\sigma\left(\frac{\omega L}{U_c}\right) = \int\int_\infty dA(\boldsymbol{\xi})\theta(\boldsymbol{\xi})\lambda\left(\frac{\omega\boldsymbol{\xi}}{U_c}\right). \quad (12)$$

In Eq. 11, the factor $\varphi(\omega\delta^*/U_\infty)$ pertains only to the characteristics of the pressure field; the factor $|F(\Omega)|^2$ pertains only to the frequency behavior of the flush-mounted transducer; and the factor $\sigma(\omega L/U_c)$ pertains to the spatial characteristics of the flush-mounted transducer. Because of the assumptions of separability and the universality of the function $\lambda(\omega\boldsymbol{\xi}/U_c)$ employed by Corcos, the characteristics of the pressure field enter into the functional dependence of σ only through the value of U_c. The functional form of $\sigma(\omega L/U_c)$, Eq. 12, is obviously dependent on the form of the spatial-response function. However, it is possible to define a class of similar flush-mounted transducers such that the spatial-response function of any member of the class is derivable from that of any other member of the class by a simple scaling of its typical linear dimension L. For such a class, $\sigma(\omega L/U_c)$ is a universal function. In general, different classes of flush-mounted transducers will possess different functional forms for σ. Indeed, the computations performed by Corcos were directed towards the determination of $\sigma(\omega L/U_c)$ as a function of $\omega L/U_c$ for the uniform circular class and the uniform square class of flush-mounted transducers.[2] The square class was oriented with one side parallel to the direction of flow. Corcos derived the functional forms for the spatial-response operators and utilized his previously mentioned empirical determination of $\lambda(\omega\boldsymbol{\xi}/U_c)$. Recently, White extended these computations to other classes of flush-mounted transducers.[8] White, however, employed a slightly different form for $\lambda(\omega\boldsymbol{\xi}/U_c)$ than that used by Corcos.

In the present experiment, an attempt is made to determine experimentally the universal function σ for two nominal classes of circular flush-mounted transducers. Since experimental procedures for determining $|F(\Omega)|^2$ are well known, it is assumed that its functional form can be obtained, and Eq. 11 is rewritten in the form

$$\Phi_m'(\omega)/\rho^2U_\infty^3\delta^* = \varphi(\omega\delta^*/U_\infty)\sigma(\omega L/U_c), \quad (13)$$

where

$$\Phi_m'(\omega) = \Phi_m(\omega)/|F(\Omega)|^2. \quad (14)$$

It is considered that the terms on the left-hand side of Eq. 13 are experimentally determined quantities. It remains to determine $\varphi(\omega\delta^*/U_\infty)$ in order to obtain $\sigma(\omega L/U_c)$. A method of determining $\varphi(\omega\delta^*/U_\infty)$ is to employ a transducer of vanishing size, where the spatial-response function can be represented by a delta function:

$$dA(\boldsymbol{\xi})\theta_0(\boldsymbol{\xi}) = dA(\boldsymbol{\xi})\delta(\xi_x)\delta(\xi_y), \quad (15)$$

where $\boldsymbol{\xi} = \{\xi_x, \xi_y\}$. When Eq. 15 is utilized, one obtains

$$\Phi_m'(\omega)/\rho^2U_\infty^3\delta^* = \varphi(\omega\delta^*/U_\infty)\sigma(0), \quad \sigma(0) = \lambda(0). \quad (16)$$

Normalizing $\sigma(0)$ to be unity,[2] $\varphi(\omega\delta^*/U_\infty)$ can be determined experimentally. However, this method cannot be implemented in practice because transducers of vanishing size cannot be constructed. Thus, a procedure is required that circumvents the necessity for prior knowledge of $\varphi(\omega\delta^*/U_\infty)$ in order to obtain $\sigma(\omega L/U_c)$.

The factor $\varphi(\omega\delta^*/U_\infty)$, in Eq. 13, can be eliminated by designing an appropriate data-reduction procedure. This procedure utilizes in combination the frequency spectral responses of a pair of transducers of differing typical linear spatial dimension. This pair must belong to the same class of flush-mounted transducers as defined previously. It is assumed that the pressure fields to which the pair of transducers is subjected are nominally identical so that the frequency spectral responses of the two transducers can be expressed in the form

$$\Phi_{m\alpha}'(\omega)/\rho^2U_\infty^3\delta^* = \varphi(\omega\delta^*/U_\infty)\sigma(\omega L_\alpha/U_c), \quad (17)$$

and

$$\Phi_{m\beta}'(\omega)/\rho^2U_\infty^3\delta^* = \varphi(\omega\delta^*/U_\infty)\sigma(\omega L_\beta/U_c), \quad (18)$$

respectively. The subscript α designates quantities associated with the transducer whose typical linear spatial dimension is L_α and the subscript β designates quantities associated with the transducer whose typical linear spatial dimension is L_β.

By choosing a sequence of center frequencies such that $\omega_1 < \omega_2 < \cdots < \omega_i < \cdots < \omega_n$, it can be readily derived from Eqs. 17 and 18 that

$$\prod_{i=1}^{n}\frac{\Phi_{m\alpha}'(\omega_i)}{\Phi_{m\beta}'(\omega_i)} = \prod_{i=1}^{n}\frac{\sigma(\omega_iL_\alpha/U_c)}{\sigma(\omega_iL_\beta/U_c)}, \quad (19)$$

where for an arbitrary function $f(\omega)$,

$$\prod_{i=1}^{n} f(\omega_i) = f(\omega_1)f(\omega_2)\cdots f(\omega_n).$$

Equation 19 is seen to be independent of $\varphi(\omega\delta^*/U_\infty)$. A considerable simplification can be achieved by choosing the center frequencies such that $\omega_{(i+1)}L_\beta/U_c$

[8] P. H. White, "Effects of Transducer Size, Shape, and Surface Sensitivity on the Measurement of Boundary-Layer Pressures," J. Acoust. Soc. Amer. 41, 1358–1363 (1967).

$=\omega_i L_\alpha/U_c$. When this is done, Eq. 19 reduces to

$$\prod_{i=1}^{n}\frac{\Phi_{m\alpha}'(\omega_i)}{\Phi_{m\beta}'(\omega_i)}=\frac{\sigma(\omega_n L_\alpha/U_c)}{\sigma(\omega_1 L_\beta/U_c)}. \tag{20}$$

By starting the center-frequency sequence at a frequency that is low enough so that

$$\sigma(\omega_1 L_\beta/U_c)\simeq\sigma(0)=1, \tag{21}$$

Eq. 20 becomes

$$\sigma\left(\frac{\omega_n L_\alpha}{U_c}\right)\simeq\prod_{i=1}^{n}\frac{\Phi_{m\alpha}'(\omega_i)}{\Phi_{m\beta}'(\omega_i)}. \tag{22}$$

The above choice for the center-frequency sequence implies that $L_\alpha > L_\beta$.

Since the values of $\Phi_{m\alpha}'(\omega_i)$ and $\Phi_{m\beta}'(\omega_i)$ are obtained from the experimental data, the repeated use of Eq. 22 leads to an experimental determination of $\sigma(\omega L/U_c)$. This procedure is used to reduce the data obtained in the experiment reported here. (See Appendix A.)

The validity of the procedure that has been outlined is contingent upon a number of assumptions and idealizations. These have been made explicit in this Section. Some violations of these assumptions and idealizations are expected and, therefore, some of the scatter in the reduced data may have to be attributed to this source. No attempt is made in this paper to estimate the degree to which each of the assumptions and the idealizations is violated and the effect that it may have had on the reduced data. It should be further noted that knowledge of the frequency-response function $|F(\Omega)|^2$ for the transducers being used is an obvious requirement when Eq. 22 is utilized, since $\Phi_m(\omega)$, rather than $\Phi_m'(\omega)$, is the term that is actually measured. The examination of the functional form of $\sigma(\omega L/U_c)$ is therefore meaningful for a given transducer only in the frequency range where $|F(\Omega)|^2$ is known. In the present experiment, difficulties were encountered in obtaining a frequency-response calibration for certain transducers at the higher frequencies. Data were limited in these cases to the lower-frequency ranges.

II. MEASUREMENTS

The experimental results reported in this paper were obtained by taking frequency spectral density measurements of the pressure fluctuations beneath a turbulent boundary layer. These measurements were made in air with transducers that were mounted flush with the wall of a subsonic wind tunnel. Data were taken at a single location for a range of frequencies and free-stream velocities. Two nominal classes of circular transducers were employed, with several radii available within each class. The amplified output signal from each transducer was passed through a frequency-band analyzer and recorded with a graphic level recorder. These data were then treated by the reduction procedure described in Sec. I (see Eq. 22), which provided an experimental determination of the functional form of $\sigma(\omega L/U_c)$. Since the transducers employed were circular, the linear dimension L was chosen to be the physical radius r of the sensitive area.

A constant 3-Hz bandwidth was used for the frequency-band analyzer to ensure that the bandwidth criterion imposed in Eq. 2 was well satisfied. This narrow a bandwidth required taking data for several minutes at each center frequency so that a reasonably good average of the recorded signal was obtained. The boundary-layer displacement thickness δ^* was determined from velocity-profile measurements. Values for δ^* along with those of other parameters are shown in Table I. The convection velocity U_c was not determined experimentally; rather, U_c was calculated by using the same functional dependence on frequency that was assumed by Corcos.[2]

Acoustic background-noise measurements were taken at the centerline of the wind tunnel at a point opposite the point on the wall where the frequency spectral density measurements were made. A $\frac{1}{2}$-in. condenser microphone with a nose cone was used for these measurements. Total electrical-noise measurements were made with a $\frac{1}{4}$-in. condenser microphone mounted flush in the wind-tunnel wall. The diaphragm of this microphone was covered so that pressure fluctuations would not induce a response. Both types of background-noise measurements were made for a range of frequencies and free-stream velocities. In general, the data reported in this paper are believed to be 10 dB or more above background noise.

III. INSTRUMENTATION

The wind tunnel employed in this experiment is a closed-circuit, subsonic tunnel with a 15×18-in. cross section in the area where measurements were taken. This tunnel has been used and discussed by previous experimenters.[9,10] The remaining instrumentation, with the exception of the transducers, is indicated in the schematic diagram, Fig. 1.

A total of seven transducers was employed; these transducers formed nominally two classes of transducers. The three larger transducers were 1-, $\frac{1}{2}$-, and $\frac{1}{4}$-in. (cartridge diameters) condenser microphones with their open diaphragm mounted flush with the wind-tunnel wall. These microphones constitute one nominal class of transducers. The frequency-response calibration of the manufacturer was used for these microphones after a single-frequency calibration of each gave values that were in close agreement with the calibration curve provided by the manufacturer.

[9] D. W. Jorgensen, "Measurements of Fluctuating Pressures on a Wall Adjacent to a Turbulent Boundary Layer," David Taylor Model Basin Rep. 1744 (July 1963).

[10] M. Harrison, "Pressure Fluctuations on the Wall Adjacent to a Turbulent Boundary Layer," David Taylor Model Basin Rep. 1260 (Dec. 1958).

TABLE I. Experimental range.

Transducer radii		Boundary layer			Experiment		
					Range of some parameters		
Type of transducer	r (in.)	U_∞ (ft/sec)	δ^* (in.)	$R_{\delta^*}=U_\infty\delta^*/\nu$	Parameter	Lower	Upper
Pinhole microphones	0.008	50	0.083	2100	$\omega\delta^*/U_\infty$	0.12	7
	0.016	75	0.089	3400	r/δ^*	0.08	4.3
	0.031	100	0.096	4700	$\omega/2\pi$	200 Hz	$\approx$12 000 Hz
	0.062	150	0.106	7600			Function of
Condenser microphones	0.085	200	0.101	9300			U_∞, r, and $\|F(\omega)\|^2$
	0.19						
	0.36						

The remaining four transducers were an adaptation of the $\frac{1}{4}$-in. condenser microphone referred to here as "pinhole" microphones. These microphones constituted the second nominal class of transducers. The pinhole microphone is made by enclosing the head of the $\frac{1}{4}$-in. condenser microphone so that a small cavity remains in front of the diaphragm. A small pinhole in the outer wall of the cavity provides the sensor for the pressure fluctuations. The pinhole microphones, with pinhole diameters of $\frac{1}{64}$, $\frac{1}{32}$, $\frac{1}{16}$, and $\frac{1}{8}$ in., were thus constructed to obtain transducers with small sensing areas. However, while decreasing the size of a transducer usually results in an increased frequency response, an inherent disadvantage of the pinhole adaptation is a decreased frequency response. This decrease occurs because the cavity/hole combination acts in the manner of a Helmholtz resonator. The resonant frequency of such a system is proportional to $(S/hV)^{\frac{1}{2}}$, where S is the area of the pinhole, h is the length of the hole leading to the cavity, and V is the volume of the cavity.[11] The resonant frequency of the system is directly proportional to the radius of the pinhole, once h and V have been set in the process of construction.

Several attempts were made to obtain reproducible frequency calibrations for the pinhole microphones. The results were deemed untrustworthy in the high-frequency regions. The Helmholtz resonances were detectable in the data by a change in slope *before* the resonances and by a sharp decrease in signal *after* the resonances. The usable frequency range for a particular pinhole microphone was limited to frequencies below the point where the preresonance change in slope occurred. In the usable frequency region, the frequency response for the pinhole microphones was assumed to be the same as that of the $\frac{1}{4}$-in. condenser microphone that was used in the adaptation.

The spatial-response functions of the transducers employed in the present experiment were not measured. However, measurements of this type have been reported for condenser microphones like those that were employed.[12] The spatial-response functions reported by Ref. 12 indicate that, for the frequency range covered in this experiment, the condenser microphones can be considered to belong to the same nominal class. An investigation by Fitzpatrick[13] indicates that the spatial-response function of a pinhole microphone is inherently different from that of a condenser microphone. Fitzpatrick argued that the pinhole microphone is most sensitive near the edges of the hole forming its sensing area. The condenser microphone, on the other hand, is least sensitive near the edges of the sensing area formed by its diaphragm. The pinhole microphones are, therefore, assumed to comprise a second nominal class of flush-mounted transducers.

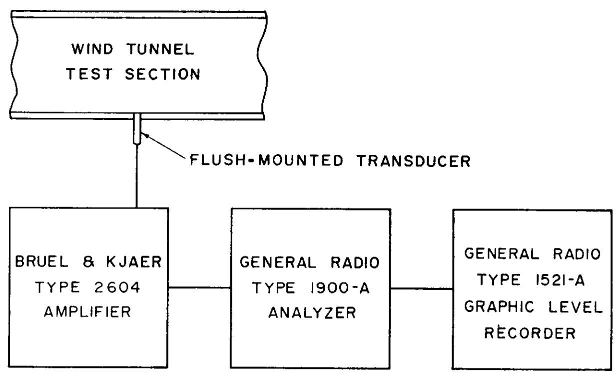

FIG. 1. Schematic of instrumentation.

IV. RESULTS

The results obtained from the present measurements are summarized in Figs. 2–8. Figure 2 presents typical frequency spectral density data in nondimensional form. The data presented in Fig. 2 were obtained at a flow speed of 50 ft/sec and clearly show the effect of size on the ability of a transducer to respond to particular pressure-field components. Marked decreases in the values obtained for $\Phi_m'(\omega)$ are evident as the radius of the transducer is increased.

The results obtained when the data-reduction procedure of Eq. 22 was applied to the frequency-spectral-density data are presented in Figs. 3–8. Figure 3 presents data obtained at a flow speed of 50 ft/sec

[11] P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Co., New York, 1948), 2nd ed., p. 235.
[12] Brüel and Kjær Tech. Rev. No. 1 (1959); No. 2 (1959); No. 1 (1962).

[13] H. M. Fitzpatrick, "Spatial Resolution Effected by a Recessed Microphone," J. Acoust. Soc. Amer. **40**, 1247(A) (1966).

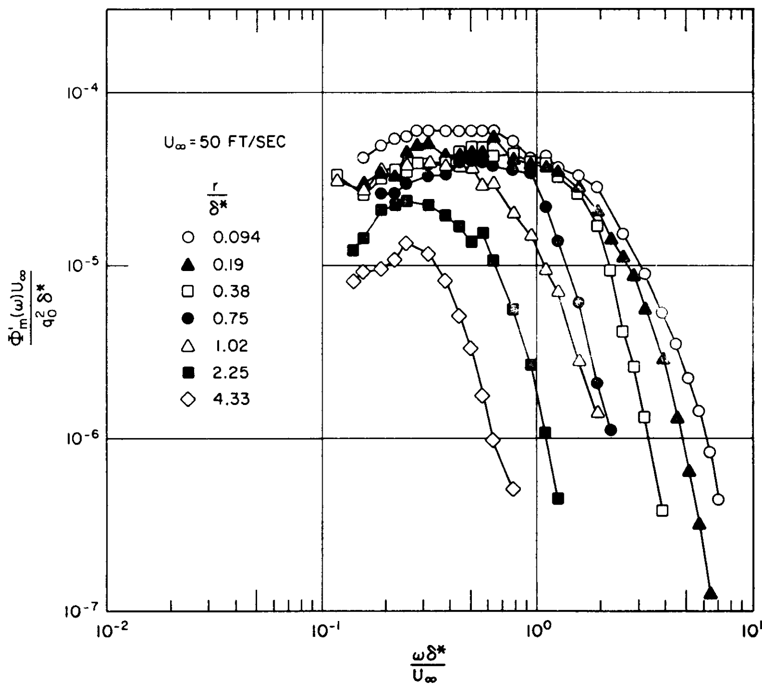

FIG. 2. Typical frequency-spectral-density data obtained with seven transducers of different radii. In the ordinate, $q_0=\frac{1}{2}\rho U_\infty^2$.

with the pinhole microphones. The six pinhole microphone pair combinations that were available for the data-reduction procedure are shown in Fig. 3. Figures 4 and 5 present two of these combinations, showing data obtained at the five flow speeds that were employed. Three pair combinations were available for the condenser microphones. Reduced data for these combinations are presented in Figs. 6 and 7, which show data obtained at flow speeds of 50 and 100 ft/sec. Figure 8 presents reduced data obtained at five flow speeds for one of the condenser microphone pair combinations.

At the lower three flow speeds, the reduced data collapsed quite well. The data obtained at the two higher flow speeds, however, showed some scatter. This was true for both nominal classes of transducers. There is some evidence that extraneous noise may have contributed to the scatter at the two higher flow speeds, but this could not be ascertained as fact.

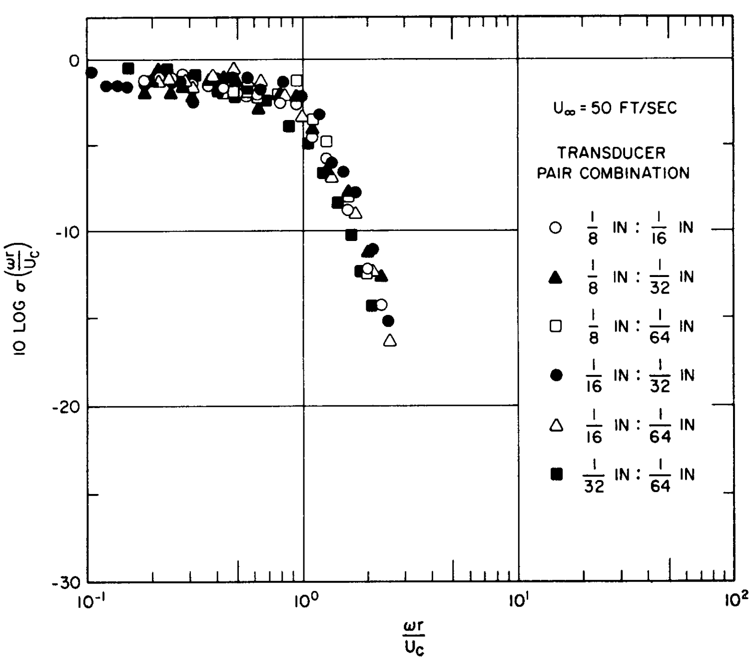

FIG. 3. Results obtained when the data-reduction procedure of Eq. 22 was applied to frequency-spectral-density data obtained with pinhole microphones at a flow speed of 50 ft/sec.

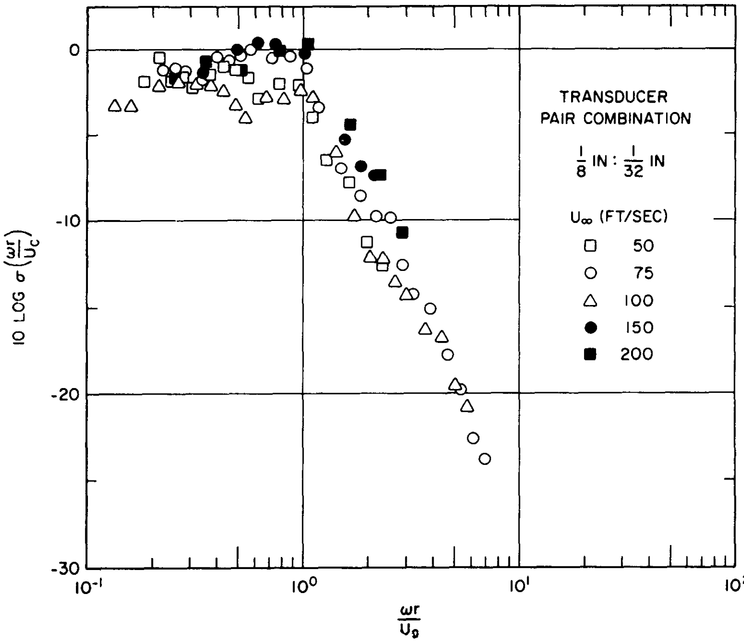

FIG. 4. Results obtained when the data-reduction procedure of Eq. 22 was applied to frequency-spectral-density data obtained with two pinhole microphones at five flow speeds.

V. ADDITIONAL REMARKS AND CONCLUSIONS

As was shown in developing Eq. 11, when the cross-frequency spectral density of the pressure field in a turbulent boundary layer is assumed separable in the manner stated in Eq. 8, and the function $\lambda(\omega\varepsilon/U_c)$ in that equation is universal, there exist universal functions $\sigma(\omega L/U_c)$ for each class of transducers. The form of the function $\sigma(\omega L/U_c)$ provides a description of the spatial-response characteristics of each transducer that falls within the class to which this function belongs. However, the definition of a class of transducers does not specify how the typical length L is to be chosen. The only requirement imposed on L is that it be chosen similarly for all transducers within a given class. The choice for L is, therefore, quite arbitrary, and several legitimate choices may exist within the given class. When comparisons between the response characteristics of different classes of transducers are to be made, care is necessary in defining the typical length since the form of $\sigma(\omega L/U_c)$ is strongly dependent on L. For

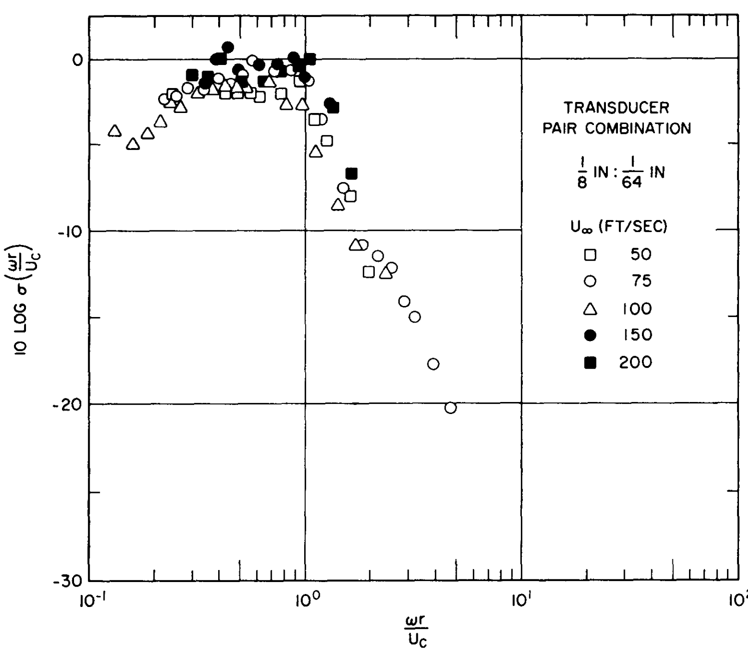

FIG. 5. Results obtained when the data-reduction procedure of Eq. 22 was applied to frequency-spectral-density data obtained with two pinhole microphones at five flow speeds.

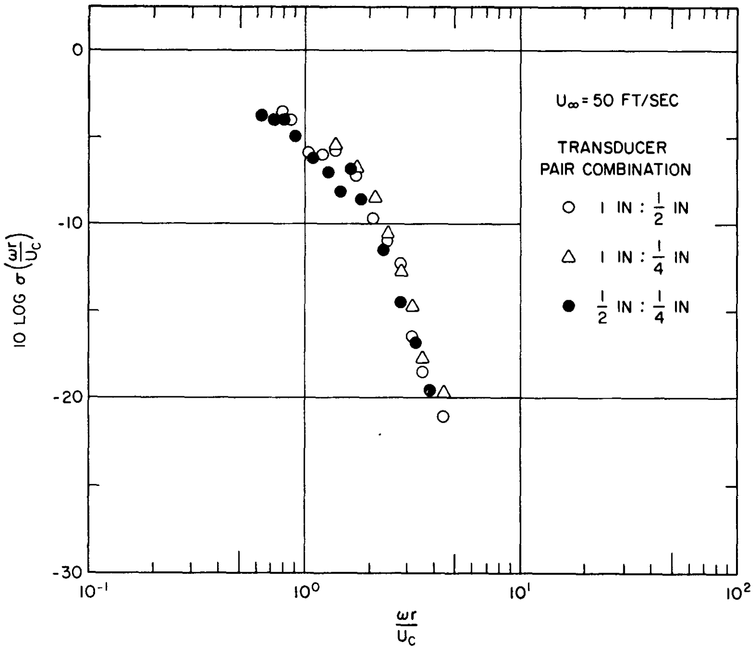

FIG. 6. Results obtained when the data-reduction procedure of Eq. 22 was applied to frequency-spectral-density data obtained with condenser microphones at a flow speed of 50 ft/sec.

transducers with a nonuniform spatial sensitivity, the appropriate choice of L may not be easily discernible.

The results obtained in the present experiment are not considered a validation of the predictions of either Corcos[2] or White.[8] Corcos and White treated transducers that belonged to uniform classes, whereas nonuniform transducers were used in the present experiment. Further, it should be emphasized that the function $\lambda(\omega\varepsilon/U_c)$ was derived, both by Corcos and White, for limited ranges of the parameters ω, ε, and U_c. Thus, strictly one may base predictions on their results only for uniform classes of transducers and within the limited ranges where both separability of the cross-frequency spectral density and the form of $\lambda(\omega\varepsilon/U_c)$ have been fairly established; outside these limits, one must proceed with caution.

A practical point of consideration concerns the limits of variability that one may allow in the form of the spatial sensitivity of transducers that nominally belong to the same class. This problem can be estimated analytically within the framework of the formalism presented in this paper. Limited variability in other

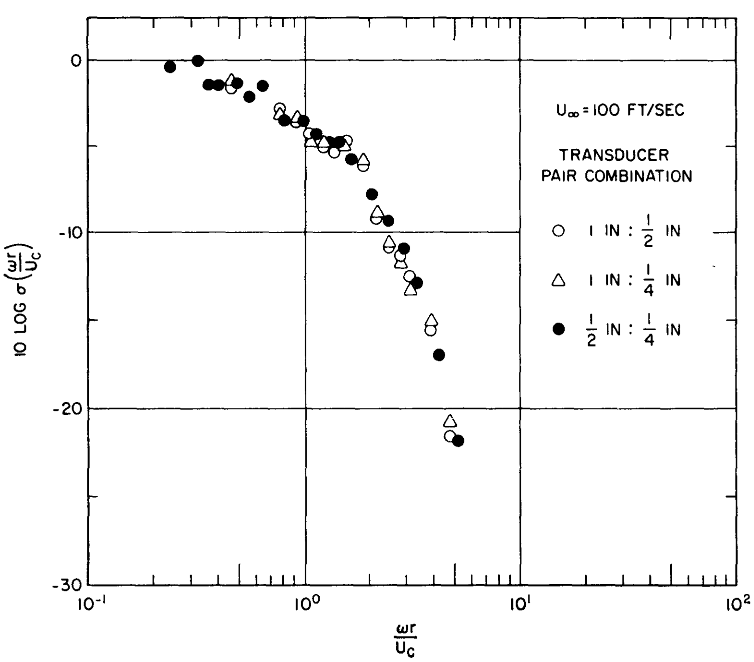

FIG. 7. Results obtained when the data-reduction procedure of Eq. 22 was applied to frequency-spectral-density data obtained with condenser microphones at a flow speed of 100 ft/sec.

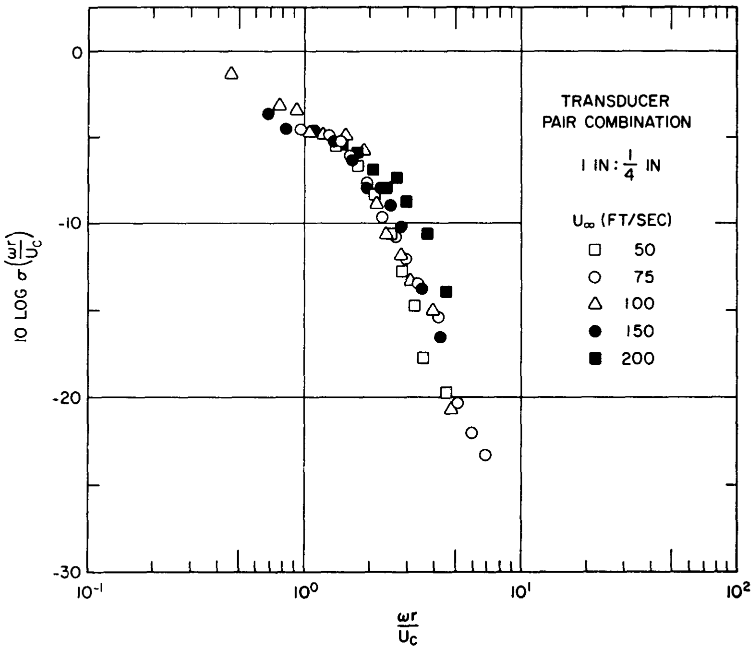

FIG. 8. Results obtained when the data-reduction procedure of Eq. 22 was applied to frequency-spectral-density data obtained with two condenser microphones at five flow speeds.

contingencies on which the formalism is based must also be examined to estimate the practical limits on the analysis. These items call for further investigations.

Finally, it is stressed that it is a dangerous procedure to apply the data-reduction procedure developed in this paper to a pair of transducers that obviously do not belong to the same class. It is apparent from Eqs. 19 and A4 that a minor difference in the universal functions to which each transducer pertains can cause a major difference in the final values owing to accumulations of errors. Moreover, it is not clear to which class one assigns this composite universal function.

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Appendix A

The data-reduction procedure that is described in Sec. I of this paper is developed to provide the functional form of $\sigma(\omega L/U_c)$ in a manner that obviates the need to know the frequency spectral density $\varphi(\omega\delta^*/U_\infty)$ of the turbulent-boundary-layer pressure field. A similar procedure is now described that provides the functional form of $\varphi(\omega\delta^*/U_\infty)$ rather than $\sigma(\omega L/U_c)$. This procedure is subject to the assumptions made previously with respect to Eq. 22. A sequence of center frequencies identical to the previously described sequence is constructed for this procedure; that is, $\omega_1<\omega_2<\cdots<\omega_i<\cdots<\omega_n$. Again, a pair of transducers belonging to the same class is utilized, but their responses are set in the form

$$\Phi_{m\alpha}'(\omega_i)/\rho^2U_\infty^3\delta^*=\varphi(\omega_i\delta^*/U_\infty)\sigma(\omega_iL_\alpha/U_c), \tag{A1}$$

and

$$\Phi_{m\beta}'(\omega_{i+1})/\rho^2U_\infty^3\delta^*=\varphi(\omega_{i+1}\delta^*/U_\infty)\sigma(\omega_{i+1}L_\beta/U_c). \tag{A2}$$

Imposing the condition that

$$\omega_iL_\alpha/U_c=\omega_{i+1}L_\beta/U_c,\quad L_\alpha>L_\beta, \tag{A3}$$

leads to the relation

$$\frac{\varphi(\omega_n\delta^*/U_\infty)}{\varphi(\omega_1\delta^*/U_\infty)}=\frac{\Phi_{m\beta}'(\omega_n)}{\Phi_{m\alpha}'(\omega_1)}\prod_{i=2}^{n-1}\frac{\Phi_{m\beta}'(\omega_i)}{\Phi_{m\alpha}'(\omega_i)}. \tag{A4}$$

When ω_1 is chosen low enough, and fixed for all pairs of transducers, Eq. (A4) reduces to

$$\varphi\left(\frac{\omega_n\delta^*}{U_\infty}\right)\simeq\frac{\Phi_{m\beta}'(\omega_n)}{\rho^2U_\infty^3\delta^*}\prod_{i=2}^{n-1}\frac{\Phi_{m\beta}'(\omega_i)}{\Phi_{m\alpha}'(\omega_i)}, \tag{A5}$$

and the functional form of $\varphi(\omega\delta^*/U_\infty)$ can be obtained from measurements of $\Phi_m'(\omega)$ for transducers belonging to the same class but having various typical linear spatial dimensions.

Equations A4 and 19 are seen to be compatible in the sense that, under the prevailing assumptions, one equation can be obtained from the other equation by using the response equations (e.g., Eqs. A1 and A2).

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13 ABSTRACT

The response of a flush-mounted transducer to the pressure field in a turbulent boundary layer is known to depend on the spatial and temporal characteristics of the transducer. This paper presents an experimental study of this dependence. The reduced data are presented in a manner similar to that used by Corcos to present his estimation of the response of transducers to a corresponding pressure field.

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