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ITERATIVE METHOD FOR THE LAMINAR BOUNDARY LAYER WITH PRESSURE GRADIENT AND SUCTION OR BLOWING

Paul S. Granville,

Department of the Navy
Washington, D C.

November 1968

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DEPARTMENT OF THE NAVY
NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER
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ITERATIVE METHOD FOR THE LAMINAR BOUNDARY LAYER WITH PRESSURE GRADIENT AND SUCTION OR BLOWING

by

Paul S. Granville

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c_f	Drag coefficient
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$\mathcal{D}$	Drag
E	Function, Equation [38]
F	Function, Equation [39]
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L	Arbitrary length
R_x	Reynolds number
R_{δ^*}	Displacement thickness Reynolds number
r	Radius of body of revolution
U	Velocity outside boundary layer
u	Velocity in x-direction
v	Velocity in y-direction
v_w	Velocity of suction or blowing normal to wall
w	Subscript referring to wall conditions
x	Distance on body in streamwise direction
y	Distance normal to wall
α	Constant, Equation [54]
δ	Boundary layer thickness

δ^*	Displacement thickness, $\delta^* \equiv \int_0^\delta (1 - \frac{u}{U})\, dy$
$\tilde{\eta}$	Function, Equation [53]
θ	Momentum thickness, $\theta \equiv \int_0^\delta \frac{u}{U} (1 - \frac{u}{U})\, dy$
ν	Kinematic viscosity of fluid
τ_w	Shearing stress at wall

ABSTRACT

The laminar boundary layer equations with pressure gradient and with suction or blowing are approximately solved analytically by an iteration method. The resulting velocity profile provides criteria for separation and stability as a function of pressure gradient and suction or blowing. This method is simpler and more versatile as well as more accurate than the classical Kármán-Pohlhausen method.

ADMINISTRATIVE INFORMATION

This report represents a dissertation accepted at the Catholic University of America in partial fulfillment of the degree, Master of Mechanical Engineering.

INTRODUCTION

An iterative method is developed for obtaining approximate analytical solutions to the laminar boundary layer equations with pressure gradient and suction or normal blowing. Iterative methods are common in solving nonlinear ordinary differential equations in simple vibration systems such as the Duffing equation.[1] Shvets[2] to a limited degree applied iterative methods to laminar boundary layers especially in the case of pressure gradients. This paper extends Shvets' analysis and furthermore includes the case of suction or blowing.

Iterative methods are related to the integral methods of Kármán-Pohlhausen[3] and Whitehead.[4] The Kármán-Pohlhausen method uses a single integral and the Whitehead method uses a double integral of the equation of motion for pressure gradients but without suction or blowing. The iterative method also uses a double integral with the difference that the assumed velocity profile is introduced before the integration and results in an

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[1]References are listed on page 17.

improved regenerated velocity profile. On the other hand the Kármán-Pohlhausen and Whitehead methods start with the assumed velocity profile and use the integral relations to evaluate the boundary layer characteristics and no improved velocity profile results. The iterative method seems to give an improved velocity profile together with more accurate properties of the boundary layer. It appears to be a simple and rational one for approximately solving the laminar boundary layer equations.

A variety of approximate methods are in existence based on different degrees of analytical complexity and/or empiricism which are summarized in various texts.[5-10]

The iterated procedure is developed in detail for a linear initial velocity profile. An iterated velocity profile results which has the same form as the Pohlhausen velocity profile. The equation for the variation of momentum thickness with distance is almost identical to that of Thwaites which was empirically obtained for the case of the solid boundary. As an example the drag coefficient is derived for the flat plate with uniform suction or blowing. The effect of suction or blowing on separation and stability is shown for the iterated linear velocity profile.

A general expression is derived for arbitrary initial velocity profiles such as the quadratic, sinusoidal, and Blasius flat plate solution. The results for predicting separation are compared for the various initial velocity profiles.

In addition the case of asymptotic solutions for suction is treated by an initial exponential velocity profile.

The extension to axisymmetric flows by Mangler's transformations is also included.

ITERATED SOLUTION OF LAMINAR BOUNDARY LAYER EQUATIONS USING INITIAL LINEAR VELOCITY PROFILE

LAMINAR BOUNDARY LAYER EQUATIONS

For the case of two-dimensional, incompressible flow with pressure gradient, the equation of motion or momentum equation[11] is

$$\nu \frac{\partial^2 u}{\partial y^2} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - U \frac{dU}{dx} \qquad [1]$$

and the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad [2]$$

The boundary conditions to be used are

$$y = 0, \quad u = 0, \; v = v_w \qquad (v_w < 0: \text{ suction;}$$
$$y = \delta, \quad u = U \qquad v_w > 0: \text{ normal blowing;}^{*}$$
$$v_w = 0: \text{ solid boundary)}$$

A finite limit to the boundary layer thickness δ is specified where $U - u$ is considered negligible. Here u and v are the velocity components in the boundary layer in the x- and y-directions respectively. x is the coordinate along the body contour and y is the coordinate normal to the body contour. ν is the kinematic viscosity of the fluid and δ is the thickness of the boundary layer. $U[x]$** is the velocity outside the boundary layer and is considered as a given quantity.

ITERATED SOLUTION

The iterative method consists in inserting a suitable velocity profile u[y] in the right hand side of Equation [1] and performing the indicated operations. The simplest velocity profile u[y] is the linear relation

$$\frac{u}{U} = \frac{y}{\delta}, \quad y \leq \delta \qquad [3]$$

which satisfies boundary conditions $u = 0$ at $y = 0$ and $u = U$ at $y = \delta$. The linear velocity profile is used by Shvets[2] for the condition $v_w = 0$. The equation of continuity gives

$$v - v_w = - \int_0^y \frac{\partial u}{\partial x} \, dy \qquad [4]$$

---

*Hereafter blowing means normal blowing.

**Note: functional notation is indicated by brackets instead of parentheses to prevent confusion with multiplication in equations.

for the general case of suction or blowing and for the linear velocity profile

$$v = v_w + \left(U \frac{d\delta}{dx} - \delta \frac{dU}{dx}\right) \frac{1}{2} \left(\frac{y}{\delta}\right)^2 \qquad [5]$$

This expression represents the normal velocity profile v[y]. Then

$$\nu \frac{\partial^2 u}{\partial y^2} = \left(U \frac{dU}{dx} - \frac{U^2}{\delta} \frac{d\delta}{dx}\right) \frac{1}{2} \left(\frac{y}{\delta}\right)^2 + \frac{v_w U}{\delta} - U \frac{dU}{dx} \qquad [6]$$

Integrating produces

$$\nu \frac{\partial u}{\partial y} - \nu \left(\frac{\partial u}{\partial y}\right)_w = \left(\delta U \frac{dU}{dx} - U^2 \frac{d\delta}{dx}\right) \frac{1}{6} \left(\frac{y}{\delta}\right)^3 + v_w U \left(\frac{y}{\delta}\right) - \delta U \frac{dU}{dx} \left(\frac{y}{\delta}\right) \qquad [7]$$

Using the boundary condition that $\frac{\partial u}{\partial y} = 0$ at $y = \delta$ results in

$$\nu \left(\frac{\partial u}{\partial y}\right)_w = - \frac{1}{6}\left(\delta U \frac{dU}{dx} - U^2 \frac{d\delta}{dx}\right) - v_w U + \delta U \frac{dU}{dx} \qquad [8]$$

Then

$$\nu \frac{\partial u}{\partial y} = \left(\delta U \frac{dU}{dx} - U^2 \frac{d\delta}{dx}\right)\left[\frac{1}{6}\left(\frac{y}{\delta}\right)^3 - \frac{1}{6}\right] + v_w U \left(\frac{y}{\delta} - 1\right) - \delta U \frac{dU}{dx}\left(\frac{y}{\delta} - 1\right) \qquad [9]$$

Integrating again produces

$$\nu u = \left(\delta^2 U \frac{dU}{dx} - U^2 \frac{\delta d\delta}{dx}\right)\left[\frac{1}{24}\left(\frac{y}{\delta}\right)^4 - \frac{1}{6}\left(\frac{y}{\delta}\right)\right] + v_w U\delta \left[\frac{1}{2}\left(\frac{y}{\delta}\right)^2 - \frac{y}{\delta}\right]$$

$$- \delta^2 U \frac{dU}{dx}\left[\frac{1}{2}\left(\frac{y}{\delta}\right)^2 - \frac{y}{\delta}\right] \qquad [10]$$

Using the boundary condition that u = U at y = δ gives

$$\nu U = - \frac{1}{8}\left(\delta^2 U \frac{dU}{dx} - U^2 \frac{\delta d\delta}{dx}\right) - \frac{1}{2} v_w U\delta + \frac{1}{2} \delta^2 U \frac{dU}{dx} \qquad [11]$$

or

$$U^2 \delta \frac{d\delta}{dx} = - 3 \delta^2 U \frac{dU}{dx} + 8 \nu U + 4 v_w U\delta \qquad [12]$$

or

$$\frac{d\delta}{dx} + 3 \frac{\delta}{U} \frac{dU}{dx} = 8 \frac{\nu}{U\delta} + 4 \frac{v_w}{U} \qquad [13]$$

This result was obtained by Shvets[2] for the solid boundary, $v_w = 0$. Now substituting $U^2\ \delta\ \frac{d\delta}{dx}$, Equation [12], into Equation [10] produces the iterated expression for the velocity profile

$$\frac{u}{U} = \frac{1}{3}\left[4\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^4\right] + \left(\frac{\delta^2}{\nu}\frac{dU}{dx} - \frac{v_w\delta}{\nu}\right)\left[\frac{1}{3}\ \frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2 + \frac{1}{6}\left(\frac{y}{\delta}\right)^6\right] \qquad [14]$$

The form of this expression is like that of Pohlhausen for $v_w = 0$.

NORMAL VELOCITY PROFILE

The variation of normal velocity v across the boundary layer is given by

$$\frac{v\delta}{\nu} = 4\left(\frac{y}{\delta}\right)^2 - 2\,\frac{\delta^2}{\nu}\,\frac{dU}{dx}\left(\frac{y}{\delta}\right)^2 + \frac{v_w\delta}{\nu}\left[1 + 2\left(\frac{y}{\delta}\right)^2\right] \qquad [15]$$

after Equation [13] is substituted into Equation [5].

This result may be compared to the exact Blasius solution for a flat plate $\left(\frac{dU}{dx} = 0,\ v_w = 0\right)$ as follows. At the outer edge of the boundary layer, $y = \delta$, the iterated solution reduces to

$$\frac{y\delta}{\nu} = 4 \qquad [16]$$

For a valid comparison momentum thickness θ is to be substituted for the boundary layer thickness δ. The iterated solution for an impervious flat plate, Equation [14] yields

$$\frac{\theta}{\delta} = \frac{58}{405} \qquad [17]$$

Then the limiting value, Equation [16], becomes

$$\frac{v\theta}{\nu} = 0.573 \qquad [18]$$

The Blasius solution[11] for the limiting case, $y \to \infty$, is

$$v = 0.865 \sqrt{\frac{\nu U}{x}} \qquad [19]$$

Since the Blasius solution has

$$\theta = 0.664 \sqrt{\frac{\nu x}{U}} \qquad [20]$$

the limiting case becomes for the Blasius solution

$$\frac{v\theta}{\nu} = 0.574 \qquad [21]$$

This compares remarkably close to the iterated solution of 0.573.

VARIATION OF MOMENTUM THICKNESS

The variation of boundary layer thickness δ with x is given by Equation [13]. Since δ is not too precise a quantity for laminar boundary layers, it is preferable to consider the variation of momentum thickness θ. As a first approximation $\frac{\theta}{\delta} = \frac{1}{6}$ obtained from the linear velocity profile Equation [3] is used in Equation [13] with the result

$$\frac{U}{\nu} \frac{d\theta^2}{dx} = \frac{4}{9} - 6 \frac{\theta^2}{\nu} \frac{dU}{dx} + \frac{4}{3} \frac{v_w \theta}{\nu} \qquad [22]$$

This result is almost identical in values of constants to the result empirically obtained by Thwaites[12] for the solid boundary ($v_w = 0$) as the best fit to known calculated solution of the laminar boundary layer.

For $v_w = 0$, Equation [22] integrates as a linear differential equation to

$$\theta^2 = \frac{4}{9} \frac{\nu}{U^6} \int_0^x U^5 \, dx \qquad [23]$$

as shown by Thwaites[12] ($\theta = 0$ at $x = 0$).

FLAT PLATE

For the case of zero pressure gradient, $\frac{dU}{dx} = 0$, and uniform suction or blowing, v_w = constant, Equation [22] integrates to

$$\theta - \frac{1}{3}\frac{\nu}{v_w}\ln\left(1 + 3\frac{v_w\theta}{\nu}\right) = \frac{2}{3}\frac{v_w}{U}x \qquad [24]$$

($\theta = 0$ at $x = 0$).

A similar result was obtained by Gandin and Soloveichik[13] in terms of boundary layer thickness δ.

For $\frac{v_w\theta}{\nu} \ll 1$ the logarithim may be expanded into a series and if the first three terms are retained, there results

$$\theta = \frac{2}{3}\sqrt{\frac{\nu x}{U}} + \frac{4}{9}\frac{v_w}{U}x \qquad [25]$$

In terms of overall drag coefficient $c_f = \frac{\mathcal{D}}{1/2\rho U^2 x}$ ($\mathcal{D}$ = drag of whole plate) $\left(\frac{\tau_w}{\rho U^2} = \frac{d\theta}{dx} - \frac{v_w}{U}\right)^{11}$

$$c_f = \frac{2\theta}{x} - 2\frac{v_w}{U} \qquad [26]$$

and hence

$$c_f = \frac{4}{3}R_x^{-1/2} - \frac{10}{9}\frac{v_w}{U} \qquad [27]$$

where $R_x \equiv \frac{Ux}{\nu}$.

Note that the exact Blasius solution[11] for $v_w = 0$ gives

$$c_f = 1.328\ R_x^{-1/2} \qquad [28]$$

which is remarkably close to the solution in Equation [27]

$$c_f = 1.333\ R_x^{-1/2} \qquad [29]$$

SEPARATION

The position of separation is indicated by the criterion that $\tau_w = 0$ or $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$. The iterated velocity profile, Equation [14], gives

$$\frac{\delta^2}{\nu}\frac{dU}{dx} - \frac{v_w \delta}{\nu} = -4 \qquad [30]$$

and with $\frac{\theta}{\delta} = \frac{1}{6}$

$$\frac{\theta^2}{\nu}\frac{dU}{dx} = -\frac{1}{9} + \frac{1}{6}\frac{v_w \theta}{\nu} \qquad [31]$$

For $v_w = 0$, $\frac{\theta^2}{\nu}\frac{dU}{dx} = -\frac{1}{9} = -0.111$. Compare with the Pohlhausen value of -0.157 and with the empriical result of Curle and Skan[14] of -0.09.

A $\frac{\theta}{\delta} = 0.15$ will give the empirical result of Curle and Skan of $\frac{\theta^2}{\nu}\frac{dU}{dx} = -0.09$. Then Equation [31] may be written

$$\frac{\theta^2}{\nu}\frac{dU}{dx} = -0.09 + 0.15\frac{v_w \theta}{\nu} \qquad [32]$$

for greater accuracy in the case of suction or blowing.

The effect of pressure gradient and suction or blowing is shown in Figure 1.

STABILITY

The shape of the iterated linear velocity profile, Equation [14], provides an indication of the effect of pressure gradient and/or suction on the stability of the laminar flow with respect to disturbances leading to transition to turbulent flow.

The small disturbance theory predicts a neutral stability point downstream of which velocity fluctuations start to amplify and upstream of which velocity fluctuations decay. Since extensive calculations are required to solve the resulting Orr-Sommerfeld equation and since an approximate method has been used to obtain the velocity profile, it is appropriate to employ an approximate method of Lin[15] to determine the neutral stability point as a function of pressure gradient and/or suction.

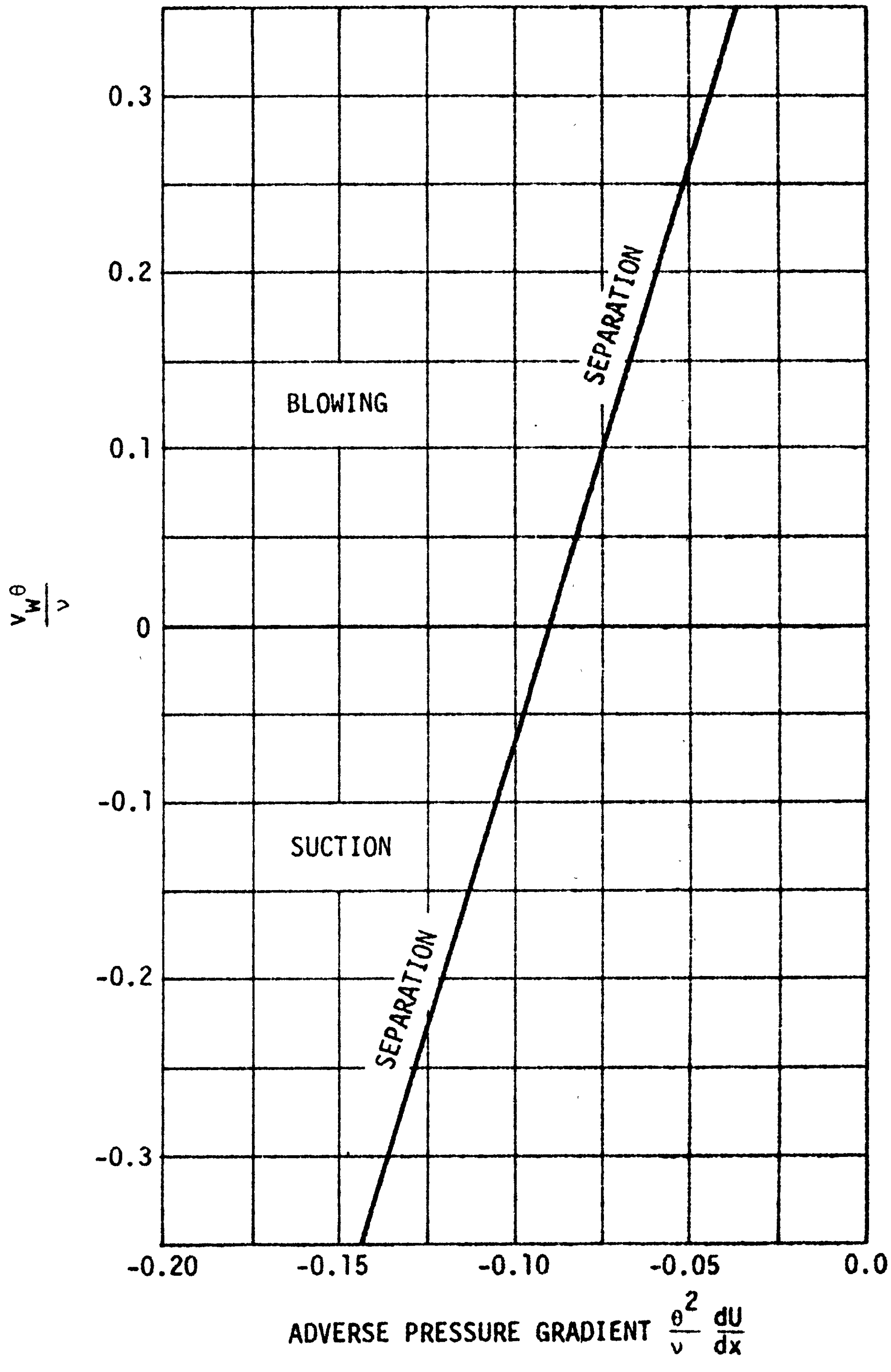

Figure 1 - Iterated Linear Velocity Profile: Separation

In Lin's method the Reynolds number of the neutral stability point is given by

$$R_\delta{}^* = 25\left[\frac{d(u/U)}{d(y/\delta)}\right]_w \frac{\delta^*}{\delta}\Big/\left(\frac{u}{U}\right)_{cr}^4 \qquad [33]$$

The critical value of $\frac{u}{U}$, $\left(\frac{u}{U}\right)_{cr}$, is a function of the critical value of $\frac{y}{\delta}$, $\left(\frac{y}{\delta}\right)_{cr}$, which is implicitly determined from the condition that

$$-\pi\left[\frac{d(u/U)}{d(y/\delta)}\right]_w \left(\frac{u}{U}\right)_{cr}\left[\frac{d^2(u/U)}{d(y/\delta)^2}\right]_{cr}\Big/\left[\frac{d(u/U)}{d(y/\delta)}\right]_{cr}^3 = 0.58 \qquad [34]$$

Here $R_\delta{}^* = \frac{U\delta^*}{\nu}$ and the displacement thickness $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$. Also

$$\left[\frac{d(u/U)}{d(y/\delta)}\right]_w = \frac{d(u/U)}{d(y/\delta)} \text{ at } y/\delta = 0$$

The neutral stability point for the iterated linear velocity profile is shown in Figure 2 where $R_\delta{}^*$ is plotted against $\frac{\delta^2}{\nu}\frac{dU}{dx} - \frac{v_w\delta}{\nu}$. It is to be observed that both a favorable pressure gradient $\left(\frac{\delta^2}{\nu}\frac{dU}{dx} > 0\right)$ and suction $\left(\frac{v_w\delta}{\nu} < 0\right)$ give a higher value of $R_\delta{}^*$ and hence greater stability.

ITERATED SOLUTION USING INITIAL GENERAL VELOCITY PROFILE

The iterated solution may be performed with any initial function of form

$$\frac{u}{U} = g\left[\frac{y}{\delta}\right] \qquad [35]$$

The result is

$$\frac{u}{U} = D\left[\frac{y}{\delta}\right] + \frac{\delta^2}{\nu}\frac{dU}{dx}E\left[\frac{y}{\delta}\right] + \frac{v_w\delta}{\nu}F\left[\frac{y}{\delta}\right] \qquad [36]$$

where

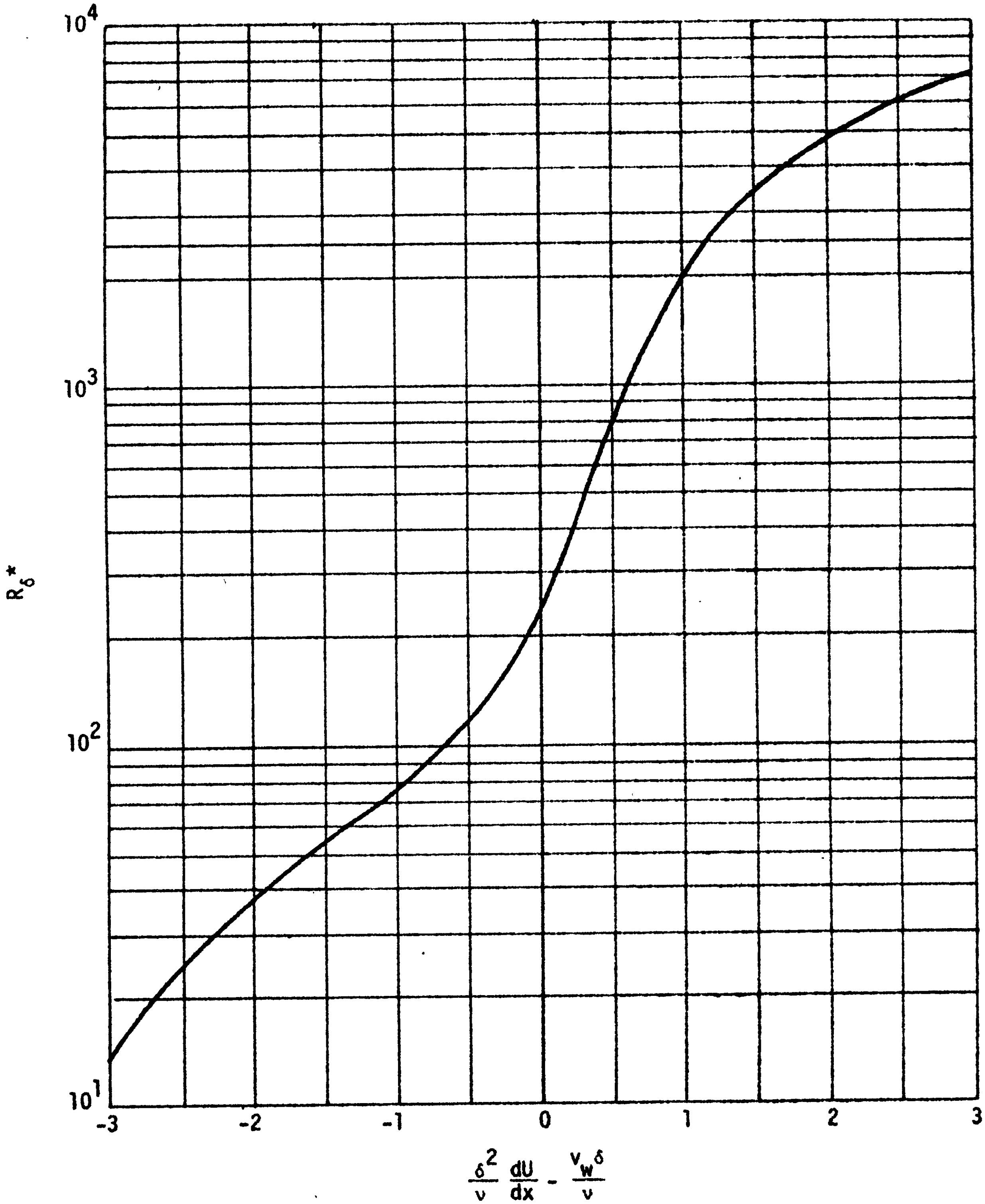

Figure 2 - Neutral Stability Point for Iterated Linear Velocity Profile

$$D = A\left[a\left(\frac{y}{\delta}\right) - b - \frac{1}{2}c^2 + (c_1 - a_1)\left(\frac{y}{\delta}\right)\right] \quad [37]$$

$$E = (2 + B)\left[a\left(\frac{y}{\delta}\right) - b\right] - \frac{(1 + B)}{2}c^2 - \frac{1}{2}\left(\frac{y}{\delta}\right)^2$$

$$- \left[(2 + B)a_1 - (1 + B)c_1 - 1\right]\left(\frac{y}{\delta}\right) \quad [38]$$

$$F = c + C\left[a\left(\frac{y}{\delta}\right) - b - \frac{1}{2}c^2\right] - \left[C(a_1 - c_1) + 1\right]\left(\frac{y}{\delta}\right) \quad [39]$$

$$A = -1/(b_1 + \frac{1}{2}c_1{}^2 - c_1) \quad [40]$$

$$B = -(2b_1 + \frac{1}{2}c_1{}^2 - c_1 - \frac{1}{2}) \quad [41]$$

$$C = -(1 - c_1)/(b_1 + \frac{1}{2}c_1{}^2 - c_1) \quad [42]$$

$$a = \int_o^{y/\delta} g^2 d\left[\frac{y}{\delta}\right] \quad [43]$$

$$a_1 = \int_o^1 g^2 d\left[\frac{y}{\delta}\right] \quad [44]$$

$$b = \int_o^{y/\delta} g^2\left(\frac{y}{\delta}\right) d\left[\frac{y}{\delta}\right] \quad [45]$$

$$b_1 = \int_o^{y/\delta} g^2 d\left[\frac{y}{\delta}\right] \quad [46]$$

$$c = \int_o^{y/\delta} g\ d\left[\frac{y}{\delta}\right] \quad [47]$$

$$c_1 = \int_o^1 g\ d\left[\frac{y}{\delta}\right] \quad [48]$$

Suitable functions are

quadratic: $$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \quad [49]$$

cubic:- $$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \qquad [50]$$

sinusoidal: $$\frac{u}{U} = \sin\left[\frac{\pi}{2}\ \frac{y}{\delta}\right] \qquad [51]$$

Blasius flat plate solution:[11] $$\frac{u}{U} = f^1 = \frac{df}{d\tilde{\eta}} \qquad [52]$$

where $$\tilde{\eta} = \alpha\left(\frac{y}{\delta}\right) \qquad [53]$$

$$\alpha = \tilde{\eta} \text{ at } \frac{u}{U} \sim 1 \qquad [54]$$

The condition for separation is derived for these various initial velocity profiles with the exception of the cubic (which is close to the sinusoidal) and the results are shown in Figure 3. The $\frac{\theta}{\delta}$ is obtained from the initial velocity profile. It is seen that the linear initial velocity profile provides the best answer to the first approximation.

ITERATED SOLUTION USING INITIAL EXPONENTIAL VELOCITY PROFILE

The asymptotic solution[11] for suction for $\frac{dU}{dx} = 0$ is

$$\frac{u}{U} = 1 - e^{\frac{-|v_w|y}{\nu}} \qquad [55]$$

with $u = 0$ at $y = 0$
$u = U$ at $y \to \infty$

For other conditions it is interesting to consider the use of such an exponentional form as an initial velocity profile. Then

$$\frac{u}{U} = 1 - e^{-y/\delta} \qquad [56]$$

with $u = 0$ at $y = 0$
$u = U$ at $y \to \infty$

where δ is the nondimensionalizing length parameter. Owing to the boundary condition at infinity, the solution to the general form, Equation [36], is

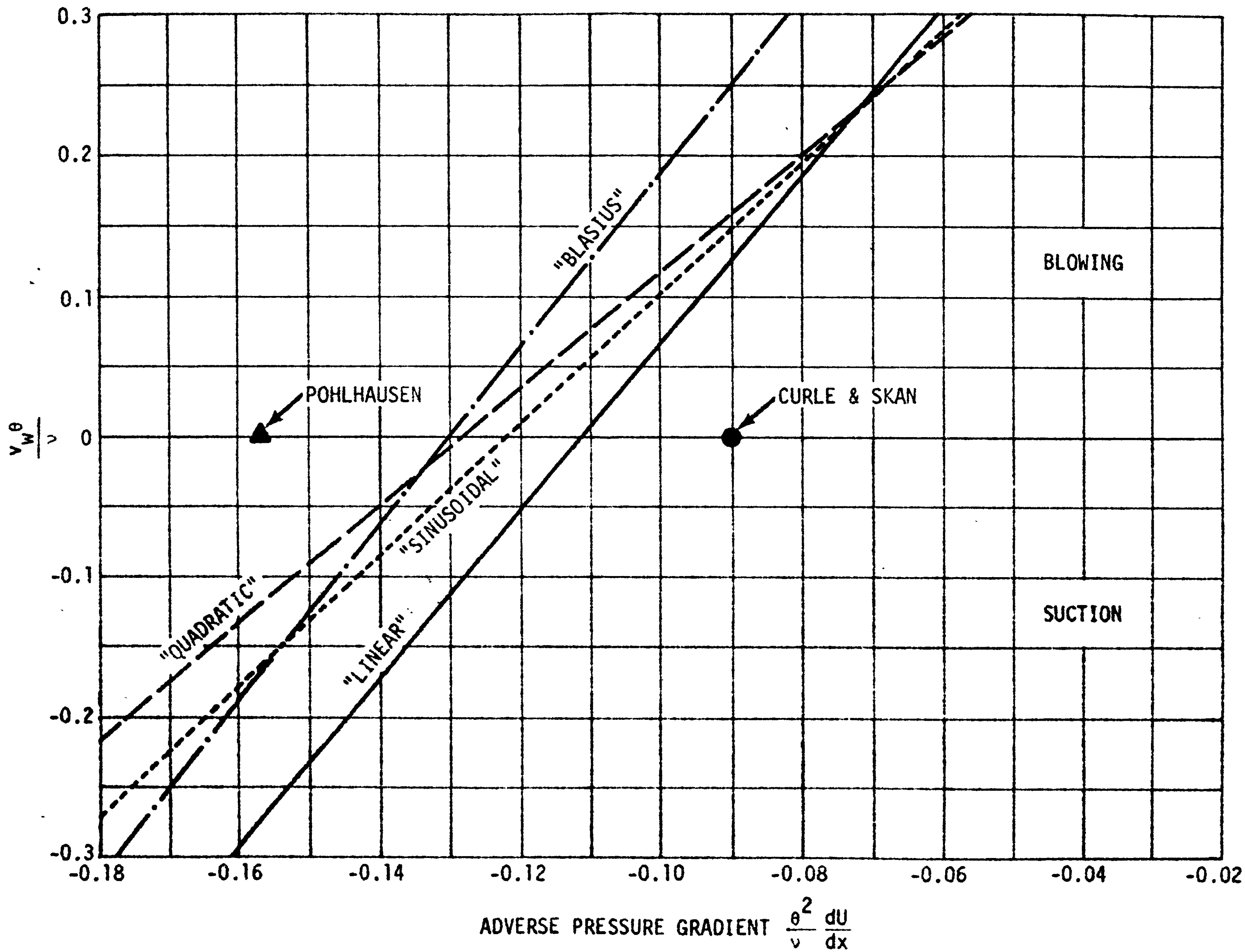

Figure 3 - Prediction of Separation by Various Velocity Profiles

not applicable. However a solution may be obtained by the straight forward iteration of the laminar boundary layer equations, Equations [1] and [2]. The iterated velocity profile is

$$\frac{u}{U} = \frac{1}{5}\left[-4\left(\frac{y}{\delta}\right)e^{-y/\delta} - 4e^{-y/\delta} - e^{-2y/\delta} + 5\right]$$
$$+ \frac{\delta^2}{\nu}\frac{dU}{dx}\frac{1}{5}\left[-3e^{-y/\delta} + 7\left(\frac{y}{\delta}\right)e^{-y/\delta} + 3e^{-2y/\delta}\right]$$
$$+ \frac{v_w\delta}{\nu}\frac{1}{5}\left[e^{-y/\delta} - 4\left(\frac{y}{\delta}\right)e^{-y/\delta} - e^{-2y/\delta}\right] \qquad [57]$$

Also

$$\frac{d\delta}{dx} + \frac{12}{5}\frac{\delta}{U}\frac{dU}{dx} = \frac{4}{5}\frac{\nu}{U\delta} + \frac{4}{5}\frac{v_w}{U} \qquad [58]$$

With $\frac{\theta}{\delta} = \frac{1}{2}$ from the initial velocity profile

$$\frac{U}{\nu}\frac{d\theta^2}{dx} = \frac{2}{5} - \frac{24}{5}\frac{\theta^2}{\nu}\frac{dU}{dx} + \frac{4}{5}\frac{v_w}{U} \qquad [59]$$

The iterated velocity profile reduces to the asymptotic solution $\left(\frac{dU}{dx} = 0\right)$ for $\delta = \frac{\nu}{|v_w|}$, $v_w = -|v_w|$.

Since the asymptotic solution represents a condition not varying with x for the case of zero pressure gradient, an analogous condition for pressure gradient is $\frac{d\delta}{dx} = \frac{d\theta}{dx} = 0$. Then from Equation [59]

$$\frac{\theta^2}{\nu}\frac{dU}{dx} = \frac{1}{12} + \frac{1}{6}\frac{v_w\theta}{\nu} \qquad [60]$$

which is plotted in Figure 4.

For zero pressure gradient

$$\frac{v_w\theta}{\nu} = \frac{1}{2} \qquad [61]$$

which is also the asymptotic solution.

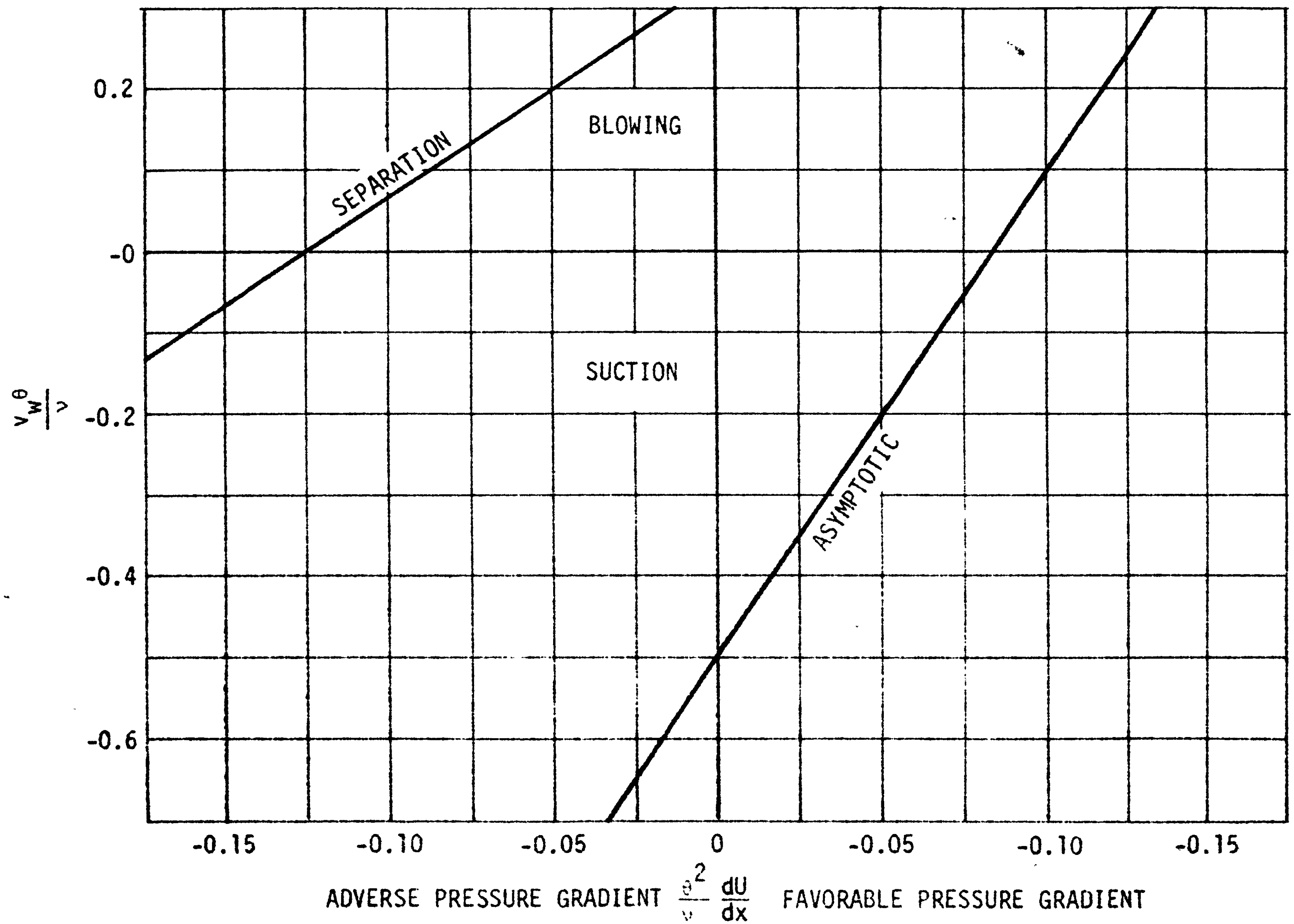

Figure 4 - Properties of Initial Exponential Velocity Profile

AXISYMMETRIC FLOW

By Mangler's transformations the results for two-dimensional flow are readily transferred to axisymmetric flow.[11] If the tildes represent two-dimensional flow, then

$$\tilde{\theta} = \frac{r}{L}\theta, \quad \tilde{U}\,[\tilde{x}] = U(x) \tag{62}$$

$$\tilde{x} = \frac{r^2}{L^2}x, \quad \tilde{v}_w = \frac{L}{r}v_w$$

where r = radius of axisymmetric body and L = arbitrary length. Equation [15] for two-dimensional flow

$$\frac{\tilde{U}}{\nu}\frac{d\tilde{\theta}^2}{d\tilde{x}} = \frac{4}{9} - 6\,\frac{\tilde{\theta}^2}{\nu}\frac{d\tilde{U}}{d\tilde{x}} + \frac{4}{3}\frac{\tilde{v}_w\tilde{\theta}}{\nu} \tag{63}$$

transforms into

$$\frac{U}{\nu}\frac{d(r^2\theta^2)}{r^2dx} = \frac{4}{9} - 6\,\frac{\theta^2}{\nu}\frac{dU}{dx} + \frac{4}{3}\frac{v_w\theta}{\nu} \tag{64}$$

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13. ABSTRACT

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