NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Washington, D.C. 20007

BOUNDARY WAVE-VECTOR FILTERS FOR THE STUDY OF THE PRESSURE FIELD IN A TURBULENT BOUNDARY LAYER

~~This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made ONLY with prior approval of Commanding Officer and Director, Naval Ship Research and Development Center.~~

Distribution Unlimited

23 Apr. 1968

ACOUSTICS AND VIBRATION LABORATORY
RESEARCH AND DEVELOPMENT REPORT

December 1967 | Report 2638

The Naval Ship Research and Development Center is a U.S. Navy center for laboratory effort directed at achieving improved sea and air vehicles. It was formed in March 1967 by merging the David Taylor Model Basin at Carderock, Maryland and the Marine Engineering Laboratory at Annapolis, Maryland.

Naval Ship Research and Development Center
Washington, D. C. 20007

BOUNDARY WAVE-VECTOR FILTERS FOR THE STUDY OF THE PRESSURE FIELD IN A TURBULENT BOUNDARY LAYER

G. Maidanik and D. W. Jorgensen

ABSTRACT

A proposal for constructing a boundary wave-vector filter is made. The wave-vector filter consists of a flush-mounted pressure transducer system. The elements and characteristics of the system are discussed. The manner in which the system can be employed to study the nature of the boundary pressure field in a turbulent boundary layer is discussed.

Reprinted from THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA, Vol. 42, No. 2, 494–501, August 1967

Boundary Wave-Vector Filters for the Study of the Pressure Field in a Turbulent Boundary Layer

G. MAIDANIK AND D. W. JORGENSEN

Naval Ship Research and Development Center, Washington, D. C. 20007

A proposal for constructing a boundary wave-vector filter is made. The wave-vector filter consists of a flush-mounted pressure transducer system. The elements and characteristics of the system are discussed. The manner in which the system can be employed to study the nature of the boundary pressure field in a turbulent boundary layer is discussed.

INTRODUCTION

THE study of the boundary pressure field in a turbulent boundary layer is of considerable import to those who design vehicles that are destined to move through fluid media. Among problems associated with the pressure field in the turbulent boundary layer are the interference of this pressure field with the proper operation of flush-mounted transducers in an array system and the excitation of panellike structures of the vehicle, which results in undesirable acoustic fields both internally and externally. Before means can be devised for suppressing the spurious effects caused by this pressure field, its nature must be deciphered. Indeed, a considerable effort has been devoted to this study.[1–9] Hitherto, the theoretical approach has not been able to account for the major properties of the pressure field. The nonlinear nature of the pressure field renders the theoretical tools most difficult to manipulate.[6] On the experimental front the situation is somewhat better. With the use of narrow-band frequency filters and small-size transducers (usually two) spaced appropriately from each other, a large volume of experimental data has accrued.[1,4,5,10,11] Further experimental data have been obtained employing a single transducer.[10–12] These latter data have been compared in terms of a formalism based on similarity con cepts proposed by Corcos.[3] However, most of the analyses of the data have been limited to the frequency domain, and only rough inferences of the wave-vector-spectral nature of the pressure field have been made. It is not surprising that to date only inferences could be made with respect to the wave-vector spectra of the pressure field since the frequency and the wavenumber in this case are not simply related. An analysis of the wave-vector nature of a single transducer and the two transducer systems would show that these systems are

[1] M. K. Bull, "Properties of the Fluctuating Wall-Pressure Field of a Turbulent Boundary Layer," Univ. of Southampton A. A. S. U. Rept. 234 (Mar. 1963).

[2] K. L. Chandiramani, "Interpretation of Wall Pressure Measurements under a Turbulent Boundary Layer," Bolt Beranek and Newman Inc. Rept. No. 1310, Contract No. Nonr 2321(00) (Aug. 1965).

[3] G. M. Corcos, "Resolution of Pressure in Turbulence," J. Acoust. Soc. Am. 35, 192–199 (1963).

[4] M. Harrison, "Pressure Fluctuations on the Wall Adjacent to a Turbulent Boundary Layer," David Taylor Model Basin Rept. 1260 (Dec. 1958).

[5] D. W. Jorgensen, "Measurements of Fluctuating Pressures on a Wall Adjacent to a Turbulent Boundary Layer," David Taylor Model Basin Rept. 1744 (July 1963).

[6] R. H. Kraichnan, "Decay of Isotropic Turbulence in the Direct-Interaction Approximation," Phys. Fluids 7, 1030–1048 (1964).

[7] P. H. White, "Cross-Spectral Density of Pressure Field Behind an Infinite Plate Excited by Boundary Layer Turbulence," Measurements Analysis Corp. Rept. 602-01, Contract N167-275(X) (May 1966).

[8] W. W. Willmarth and F. W. Roos, "Resolution and Structure of the Wall Pressure Field Beneath a Turbulent Boundary Layer," J. Fluid Mech. 22, 81–94 (1965).

[9] Refs. 1–8 are not meant to be exhaustive, but rather, illustrative.

[10] W. W. Willmarth, "Space-Time Correlations and Spectra of Wall Pressure in a Turbulent Boundary Layer," NASA Memorandum 3-17-59W (1959).

[11] W. W. Willmarth and C. E. Wooldridge, "Measurements of the Fluctuating Pressure at the Wall Beneath a Thick Turbulent Boundary Layer," J. Fluid Mech. 14, 187–210 (1962).

[12] F. E. Geib, Jr., "Measurements on the Effect of Transducer Size on the Resolution of Boundary-Layer Pressure Fluctuations." Naval Ship Research and Development Center Rept. 2503 (Aug. 1967).

very crude wave-vector filters.[13] Since a pressure field is completely specified only when its spectral nature is known, the lack of proper wave-vector filters has hampered the quest for better data concerning the pressure field in a turbulent boundary layer.

With the considerable literature that is available on the transducer array system, it is surprising that scientists concerned with the measurements of the pressure field in a turbulent boundary layer have not designed and employed more refined wave-vector filters to enhance the effectiveness of their research. In this paper, the elements of a boundary wave-vector filter that seem to hold considerable promise for detailed measurements of the pressure field in a turbulent boundary layer are described. It is not intended that this paper be inclusive of all possibilities of such a device. Rather, the purpose is to suggest a more sophisticated device for the study of the nature of this pressure field.

I. RELATIONSHIP BETWEEN THE TRUE PRESSURE FIELD AND THE MEASURED PRESSURE FIELD ON A BOUNDARY

The pressure field on a flat boundary is considered to be stationary, both spatially and temporally. The normalized pressure-field cross correlation can then be written[2]

$$\phi_{\mathrm{p}}(\mathbf{x},t)=\langle p^2\rangle^{-1}\langle p(\mathbf{x}',t')p(\mathbf{x}'+\mathbf{x},\,t'+t)\rangle, \tag{1}$$

where $\langle p^2\rangle$ is the mean square of the pressure field, $\mathbf{x}$ is the spatial position vector variable on the boundary, t is the temporal variable, and $\langle\ \rangle$ indicates the appropriate statistical averaging.

The cross-frequency spectral density $\tilde{\phi}_{\mathrm{p}}(\mathbf{x},\omega)$ of the pressure field is defined

$$\tilde{\phi}_{\mathrm{p}}(\mathbf{x},\omega)=\frac{1}{(2\pi)^{\frac{1}{2}}}\int_{-\infty}^{\infty}dt\phi_{\mathrm{p}}(\mathbf{x},t)\exp(i\omega t), \tag{2}$$

and the spectral density $\Phi_{\mathrm{p}}(\mathbf{k},\omega)$ of the pressure field is defined

$$\Phi_{\mathrm{p}}(\mathbf{k},\omega)=\frac{1}{2\pi}\iint_{-\infty}^{\infty}d\mathbf{x}\tilde{\phi}_{\mathrm{p}}(\mathbf{x},\omega)\exp(-i\mathbf{k}\cdot\mathbf{x}), \tag{3}$$

where ω is the Fourier conjugate variable of the temporal variable t, and $\mathbf{k}$ is the Fourier conjugate variable of the spatial variable $\mathbf{x}$.

A flush-mounted transducer system, consisting of several individual transducers, is placed in the boundary and the response of this system to the pressure field on the boundary is measured. The transducer system is assumed to be a linear device whose impulse response function is defined as follows:

$$G(\mathbf{y},\tau|\mathbf{x},t)=\mathbf{h}(\mathbf{y}|\mathbf{x})\cdot\mathbf{f}(\tau|t), \tag{4}$$

where

$$\mathbf{h}(\mathbf{y}|\mathbf{x})=\{h_i(\mathbf{y}+\mathbf{y}_i-\mathbf{x})\}, \tag{5}$$

$$\mathbf{f}(\tau|t)=\{f_i(\tau+\tau_i+t)\}, \tag{6}$$

h_i is the spatial "sensitivity function" of the ith transducer, f_i is the temporal sensitivity function of the ith transducer, $\mathbf{y}$ is the spatial position vector defining the "center" of the transducer system, τ is the "time delay" associated with the transducer system, $\mathbf{y}_i$ is the spatial position vector defining the center of the ith transducer relative to the transducer system center $\mathbf{y}$, and τ_i is the time delay associated with the ith transducer relative to the transducer system time delay τ.

In Eq. 4, it is assumed that the spatial and temporal dependence of the impulse-response function of each of the transducers in the transducer system are separable. Most practical transducers would, for the frequency range of interest, conform with such a description. In any case, the generalization of the formalism to a non-separable spatial and temporal response function is straightforward. In terms of the impulse-response function G, the measured response of the transducer system is

$$p_{\mathrm{m}}(\mathbf{y}',\tau')=\iint_{-\infty}^{\infty}d\mathbf{x}'\int_{-\infty}^{\infty}dt'p(\mathbf{x}',t')G(\mathbf{y}',\tau'|\mathbf{x}',t'). \tag{7}$$

From Eqs. 2, 4, and 7 one readily obtains

$$\langle p_{\mathrm{m}}{}^2\rangle\tilde{\phi}_{\mathrm{m}}(\mathbf{y},\omega)$$
$$=2\pi\langle p^2\rangle\iint_{-\infty}^{\infty}d\mathbf{x}'\iint_{-\infty}^{\infty}d\mathbf{x}\tilde{\phi}_{\mathrm{p}}(\mathbf{x},\omega)[\mathbf{h}(\mathbf{y}'|\mathbf{x}')\cdot\mathbf{F}(\omega)]$$
$$\times[\mathbf{h}(\mathbf{y}'+\mathbf{y}|\mathbf{x}'+\mathbf{x})\cdot\mathbf{F}^*(\omega)], \tag{8}$$

where

$$\mathbf{F}(\omega)=\{F_i(\omega)\exp(-i\omega\tau_i)\}, \tag{9}$$

$$F_i(\omega)=\frac{1}{(2\pi)^{\frac{1}{2}}}\int_{-\infty}^{\infty}dtf_i(t)\exp(i\omega t), \tag{10}$$

and

$$\langle p_{\mathrm{m}}{}^2\rangle\Phi_{\mathrm{m}}(\mathbf{k},\omega)$$
$$=8\pi^3\bar{p}^2\Phi_{\mathrm{p}}(\mathbf{k},\omega)[\mathbf{H}(\mathbf{k})\cdot\mathbf{F}(\omega)][\mathbf{H}(\mathbf{k})\cdot\mathbf{F}(\omega)]^*, \tag{11}$$

where

$$\mathbf{H}(\mathbf{k})=\{H_i(\mathbf{k})\exp(-i\mathbf{k}\cdot\mathbf{y}_i)\}, \tag{12}$$

$$H_i(\mathbf{k})=\frac{1}{2\pi}\iint_{-\infty}^{\infty}d\mathbf{x}h_i(\mathbf{x})\exp(-i\mathbf{k}\cdot\mathbf{x}). \tag{13}$$

The cross correlation of the response $\phi_{\mathrm{m}}(\mathbf{y},\tau)$ is normalized by the mean-square response $\langle p_{\mathrm{m}}{}^2\rangle$ and not by $\langle p^2\rangle$. This normalization is made explicit in Eqs. 8 and 11.[2]

[13] G. Maidanik and D. W. Jorgensen, "The Response of a Transducer System to Turbulent Boundary Layer Pressure Field" (to be published).

II. NATURE OF THE SPECTRAL DENSITY OF THE PRESSURE IN A TURBULENT BOUNDARY LAYER

In order to be specific, as well as to establish criteria for a boundary "wave-vector filter" for analyzing the pressure field in a turbulent boundary layer, it is necessary to have some idea of the nature of this pressure field. Since the pressure field is convective the spectral density of the pressure field in a turbulent boundary layer has the approximate form[2,7]

$$\Phi_p(\mathbf{k},\omega)=\Phi_t(\mathbf{k},\omega-\mathbf{k}\cdot\mathbf{U}), \tag{14}$$

where $\mathbf{U}$ is the convection velocity.

The Fourier conjugate of the temporal variable enters in the form $(\omega-\mathbf{k}\cdot\mathbf{U})$; this form accounts for the convective nature of the pressure field. It is of interest to note that should Taylor's hypothesis hold,[14] the functional form of Φ_t would contain a Dirac delta function factor in the variable $(\omega-\mathbf{k}\cdot\mathbf{U})$. However, the pressure field in a turbulent boundary layer does not strictly obey Taylor's hypothesis.[14] Nevertheless, the function Φ_t is such that it peaks substantially in the range where $\omega\simeq\mathbf{k}\cdot\mathbf{U}$. The precise form of the peak is in part controlled by the dependence of $\mathbf{U}$ on ω and $\mathbf{k}$. Previous studies[2,7] indicate that Φ_t may be separable in the form

$$\Phi_t=\Phi_{t1}(k_1,\omega)\Phi_{t3}(k_3,\omega)\Phi_{t0}(\omega-k_1U), \tag{15}$$

where k_1 and k_3 are defined so that

$$\mathbf{k}\cdot\mathbf{U}=\{k_1,k_3\}\cdot\mathbf{U}=(k_1U,O) \quad\text{and}\quad |\mathbf{U}|=U.$$

In Eq. 15, $\Phi_{t1}(k_1,\omega)$ and $\Phi_{t3}(k_3,\omega)$ are assumed to be symmetric, and reasonably smooth functions of ω, k_1, and k_3. The peak in Φ_t is accounted for primarily by the function Φ_{t0}, and occurs, as previously observed, when $\omega\rightarrow k_1U$. Equation 15 is verified, to a degree, by experimental data pertaining to those spectral components that lie in the vicinity of the spectral region defined by $\omega\simeq k_1U$. It is noted that in this region ω and k_1U are, to a first order of approximation, interchangeable. In this paper, a device is described that may yield better and more detailed experimental data of the pressure field in a turbulent boundary layer than has been achieved hitherto. The spectral region of interest is that where $\omega\simeq k_1U$. For the purpose of illustration, the form of the spectral density is assumed to closely resemble the form stated in Eq. 15. It becomes apparent, however, that the device could provide data concerning the pressure field that extend beyond merely a test of the appropriateness of Eq. 15 as a description of the spectral density of the pressure field.

III. ELEMENTS OF A DIFFRACTION GRATING; A BOUNDARY WAVE-VECTOR FILTER

Return to Eq. 11. It is assumed that the transducer system is constructed of nominally identical transducers. Denote by s_i the sensitivity of the ith transducer. Equation 11 may then be written in the series form

$$\langle p_m^2\rangle\Phi_m(\mathbf{k},\omega)=8\pi^3\langle p^2\rangle\Phi_p(\mathbf{k},\omega)|H(\mathbf{k})|^2|F(\omega)|^2 \times\sum_i\sum_j s_is_j\exp\{-i[\mathbf{k}\cdot(\mathbf{y}_i-\mathbf{y}_j)+\omega(\tau_i-\tau_j)]\}, \tag{16}$$

where

$$H(\mathbf{k})=H_i(\mathbf{k}) \quad\text{for all } i\text{'s},$$
$$F(\omega)=F_i(\omega) \quad\text{for all } i\text{'s}.$$

In this paper, special consideration is given to cases where $\tau_i=\tau_j=0$, and to transducers that are aligned so that the distances between the centers of successive transducers are equal. Then Eq. 16 becomes

$$\langle p_m^2\rangle\Phi_m(\mathbf{k},\omega)=\{8\pi^3\langle p^2\rangle\Phi_p(\mathbf{k},\omega)|H(\mathbf{k})|^2|F(\omega)|^2\} \times\{|\sum_{n=0}^{N-1}s_n\exp(-in\mathbf{k}\cdot\mathbf{d})|^2\}, \tag{17}$$

where $\mathbf{d}$ is the vector formed by a line joining the centers of two successive transducers, and N is the number of transducers in the system.

The first term within braces on the right-hand side of Eq. 17 describes the spectral density as measured by a single transducer of the system; the multiplicity of transducers is accounted for completely by the second term within braces. An examination of the nature of the latter term is now considered in some detail. There is no practical difficulty in adjusting the magnitude of the s_n's; this can be accomplished by simple electrical means. Further, the sign of the s_n's can also be adjusted by simple electrical means, e.g., reversing the electrical terminals. However, consideration in this paper is limited specifically to cases where $|s_n|=1$, particularly where $s_n=(-1)^{2n}$ and $s_n=(-1)^n$. The following relationships are then readily obtained:

$$|\sum_{n=0}^{N-1}s_n\exp(-in\mathbf{k}\cdot\mathbf{d})|^2=\begin{cases}\sin^2(\frac{1}{2}Nk_dd)/\sin^2(\frac{1}{2}k_dd), & s_n=(-1)^{2n} \quad (18a)\\ \sin^2(\frac{1}{2}Nk_dd)/\cos^2(\frac{1}{2}k_dd), & s_n=(-1)^n \text{ for } N \text{ even} \quad (18b)\\ \cos^2(\frac{1}{2}Nk_dd)/\cos^2(\frac{1}{2}k_dd), & s_n=(-1)^n \text{ for } N \text{ odd} \quad (18c)\end{cases}$$

where $\mathbf{k}_d/k_d=\mathbf{d}/d$, $[|\mathbf{k}_d|=k_d,\ |\mathbf{d}|=d]$.

The equivalence between Eqs. 17 and 18a and the corresponding equations that describe an optical diffraction grating is apparent.[15] It is clear that because of

[14] M. J. Fisher and P. O. A. L. Davies, "Correlation Measurements in a Non-Frozen Pattern of Turbulence," J. Fluid Mech. 18, 97–116 (1964).

[15] M. Born and E. Wolf, *Principles of Optics* (Pergamon Press Ltd., New York, 1959), Chap. 8.

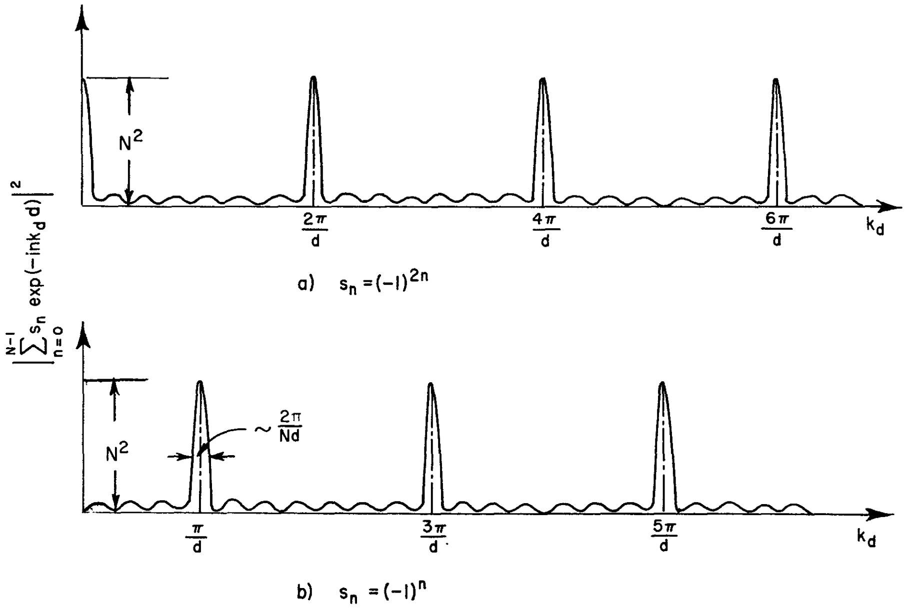

FIG. 1. Schematic plot of Eq. 18 as function of k_d.

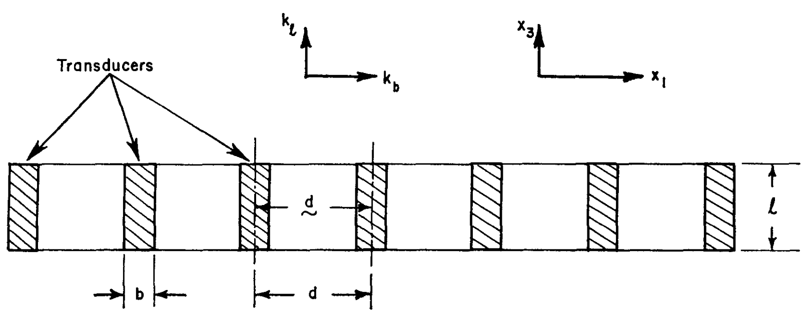

FIG. 2. Flush-mounted rectangular pressure-transducer system.

the ease with which the values and signs of the s_n's can be adjusted, that the pressure diffraction grating is a more versatile instrument.

For $s_n=(-1)^{2n}$, the major maxima appear whenever $k_d=(2\pi/d)q$, where q is an integer inclusive of zero. For $s_n=(-1)^n$, the major maxima appear whenever $k_d=(2\pi/d)(q+\frac{1}{2})$. It is clear that in the latter case no major maximum appears at $k_d=0$. This feature may be taken advantage of to reduce the response to noise that may be present in the pressure field, provided that the noise field is convected with velocities that substantially exceed the flow velocity of the fluid. A schematic plot of the functions described in Eqs. 18 is shown in Figs. 1(a) and 1(b).

IV. BOUNDARY WAVE-VECTOR FILTER EMPLOYING RECTANGULAR TRANSDUCERS

The wave-vector nature of the transducer system is, from Eq. 17, given by

$$W(\mathbf{k})=|H(\mathbf{k})|^2\left|\sum_{n=0}^{N-1} s_n \exp(-in\mathbf{k}\cdot\mathbf{d})\right|^2. \tag{19}$$

Equation 19 describes the wave-vector filtering action of the transducer system.[16] In discussing the nature of this wave-vector filter, a particular case is considered where the system of transducers consists of rectangular transducers. In this case

$$|H(\mathbf{k})|^2=\left[\frac{\sin(k_b b/2)}{(k_b b/2)}\right]^2\left[\frac{\sin(k_l l/2)}{(k_l l/2)}\right]^2, \tag{20}$$

where b is the width of the transducer and l is its length. The wavenumbers k_b and k_l are measured along the width and the length of the transducers, as shown in Fig. 2.

For simplicity, $\mathbf{d}$ is assumed to lie along the width of the transducers (see Fig. 2). The characteristics of the wave-vector filter as described in Eqs. 19 and 20, for a few special cases of interest, are shown in Fig. 3. In

[16] The function $W(\mathbf{k})$ is essentially the spatial "joint acceptance" of the transducer system as defined by A. Powell. Indeed, the nature of $W(\mathbf{k})$ is closely related to the spatial joint-acceptance function of a responding flat plate (or a beam) [see for example: A. Powell, Aero-Acoustic Excitation of Structures," David Taylor Model Basin Rept. 2232 (Sept. 1966)].

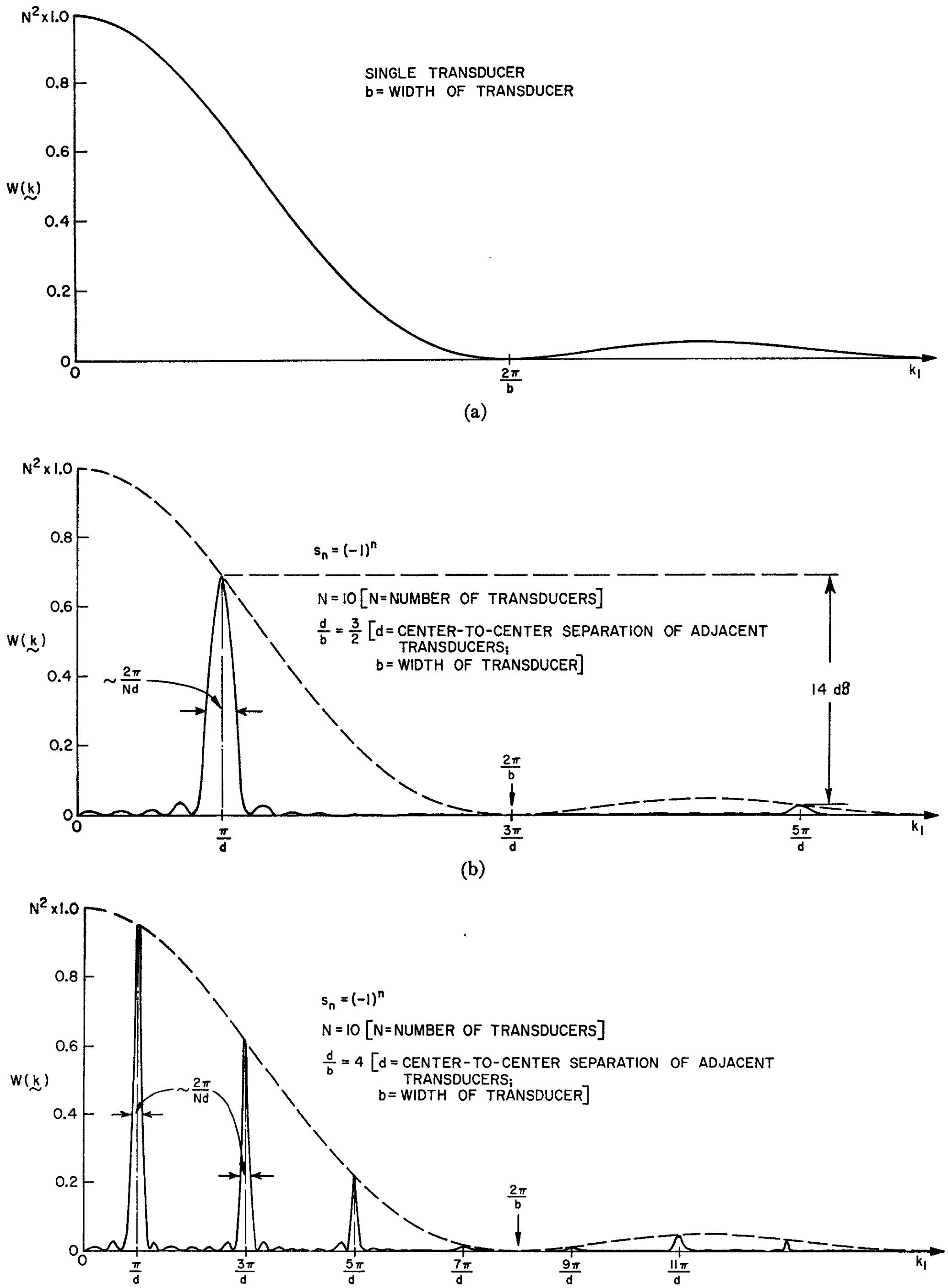

FIG. 3. Characteristics of various boundary wave-vector filters (Eq. 19). (a) Low-pass. (b) Single-band. (c) Multiband.

Fig. 3(a), a case of a low-pass boundary wave-vector filter, is shown. This type of filter has been analyzed and used extensively by several workers.[3,17] In Fig. 3(b) is shown a single-bandpass boundary wave-vector filter. The most important features of this type of wave-vector filter are indicated in Fig. 3(b). Finally, in Fig. 3(c) a multibandpass boundary wave-vector filter is shown; its most important features are indicated in this figure.

Some variations on the theme may be introduced by

[17] K. L. Chandiramani, private communication.

using different transducer geometries and different transducer alignments. Further variations can also be introduced by variations in the s_n's (in magnitude and phase). In fact, many of the techniques of shading and phasing of planar array systems are directly applicable to "building in" the desired features in the boundary wave-vector filter.

The wavenumber range of interest in a pressure field in a turbulent boundary layer may involve wavenumbers as high as $10^2\sim 10^3$ (in.)$^{-1}$. This requires transducers of a width of the order of 10^{-2} in. and separations of the same order of magnitude between successive transducers. A bank of transducers of such dimensionalities and separations can be readily achieved by present-day state of the art concerning ceramic slabs and ultrasonic cutting.

V. USE OF A BOUNDARY WAVE-VECTOR FILTER FOR THE STUDY OF THE PRESSURE FIELD IN A TURBULENT BOUNDARY LAYER

Return to Eq. 17. The frequency spectral response of the system is given by

$$\langle p_{\mathrm{m}}^2\rangle\tilde{\phi}_{\mathrm{m}}(0,\omega)=\frac{1}{2\pi}\langle p^2\rangle\int\int_{-\infty}^{\infty}\Phi_{\mathrm{m}}(\mathbf{k},\omega)d\mathbf{k}. \tag{21}$$

For simplicity, and also because this case is the most relevant to the specific problem of interest in this paper, k_d and k_b are set equal to k_1, and k_l is set equal to k_3. From Eqs. 15 and 18–21, one obtains

$$\langle p_{\mathrm{m}}^2\rangle\tilde{\phi}_{\mathrm{m}}(0,\omega)\simeq 8\pi^3\frac{N^2\bar{p}^2}{Nd}\{|F(\omega)|^2\}$$
$$\times\left\{\int_{-\infty}^{\infty}dk_3\Phi_{t3}(k_3,\omega)\left[\frac{\sin(k_3l/2)}{(k_3l/2)}\right]^2\right\}$$
$$\times\left\{\sum_q[\Phi_{t1}(k_{1q},\omega)\{\Phi_{t0}(\omega-k_{1q}U)+\Phi_{t0}(\omega+k_{1q}U)\}]\times\left[\frac{\sin(k_{1q}b/2)}{k_{1q}b/2}\right]^2\right\}, \tag{22}$$

where the wavenumber $k_{1q}=(2\pi/d)q$ for the case where the $s_n=(-1)^{2n}$, and $k_{1q}=(2\pi/d)(q+\frac{1}{2})$ for the case where the $s_n=(-1)^n$. The approximation implied in Eq. 22 is better for higher values of N, the number of transducers in the transducer system.

In practice, one measures the response of the transducer system in a chosen frequency band. We denote by ω_0 the center frequency of the band, and by 2Δ its width. It can be shown that if the frequency filter has sufficiently sharp skirts, the measured response of the system is given approximately by

$$\langle p_{\mathrm{m}}^2\rangle\phi_{\mathrm{m}}(0,\omega_0,\Delta)\simeq 2\Delta\langle p_{\mathrm{m}}^2\rangle\phi_{\mathrm{m}}(0,\omega_0), \tag{23}$$

provided that $(\Delta Nd)/(2\pi U)\ll 1$. This inequality simply ensures that within the incremental time associated with the incremental bandwidth Δ, a sample of the pressure field has interacted properly with all the transducers of the system.[13]

The combined filtering action of the frequency filter and the boundary wave-vector filter in the $k_1-\omega$ plane is illustrated in Fig. 4. A rough representation of the pressure field in a turbulent boundary layer in this plane is also illustrated in Fig. 4.

If l is chosen large enough so that $l\gg\gamma$, where γ is a typical correlation length of the pressure field, in the direction normal to the flow, Eqs. 22 and 23 can be further approximated

$$\langle p_{\mathrm{m}}^2\rangle\phi_{\mathrm{m}}(0,\omega_0,\Delta)\simeq 16\pi^4\Delta\frac{N^2\bar{p}^2}{A}|F(\omega_0)|^2\Phi_{t3}(0,\omega_0)$$
$$\times\left\{\sum[\Phi_{t1}(k_{1q},\omega_0)\{\Phi_{t0}(\omega_0-k_{1q}U)+\Phi_{t0}(\omega_0+k_{1q}U)\}]\times\left[\frac{\sin(k_{1q}b/2)}{(k_{1q}b/2)}\right]^2\right\}, \tag{24}$$

where $A=Ndl$, essentially the total area of the transducer system, and U in Eq. 24 is a function of k_{1q} and ω_0.

It was argued previously that $\Phi_{t0}(\omega-k_1U)$ peaks substantially whenever $\omega\rightarrow k_1U$. Thus, by measuring $\langle p_{\mathrm{m}}^2\rangle\phi_{\mathrm{m}}(0,\omega_0,\Delta)$ as a function of ω_0 (with the k_{1q}'s fixed) the peaks in the measured curves will furnish knowledge of the values of $U(k_{1q},\omega_0)$. This can be achieved provided the center wavenumbers k_{1q}'s are sufficiently separated. Since, at the present time the criteria for the required separation between the k_{1q}'s is not established, the single-band wave-vector filter should be employed. In this case only the first term in the summations in Eqs. 22 and 24 need be retained. The measurement of $\langle p_{\mathrm{m}}^2\rangle\phi(0,\omega_0,\Delta)$ as a function of ω_0 can be used further to explore the functional form of $\langle p_{\mathrm{m}}^2\rangle\phi_{\mathrm{m}}$ in the vicinity of the condition $\omega_0=k_{1q}U$ (see Fig. 4). It is thus clear that the boundary wave vector filter can profitably be used to study details of the boundary pressure field in a turbulent boundary layer.

VI. ADDITIONAL COMMENTS

This Section is devoted to comments and brief discussions of some special aspects of the design and the use of the boundary wave-vector filter for the study of the pressure field in a turbulent boundary layer.

In the final paragraph of Sec. V, it is suggested that the output of the transducer system $\langle p_{\mathrm{m}}^2\rangle\phi_{\mathrm{m}}(0,\omega_0,\Delta)$ be measured as a function of the center frequency ω_0. This suggestion is made in view of the ease with which the center frequency ω_0 can be varied to any predetermined value; variable single-band frequency filters possessing bandwidths of the order of a few cycles per second, with sharp skirts, are in common use. On the

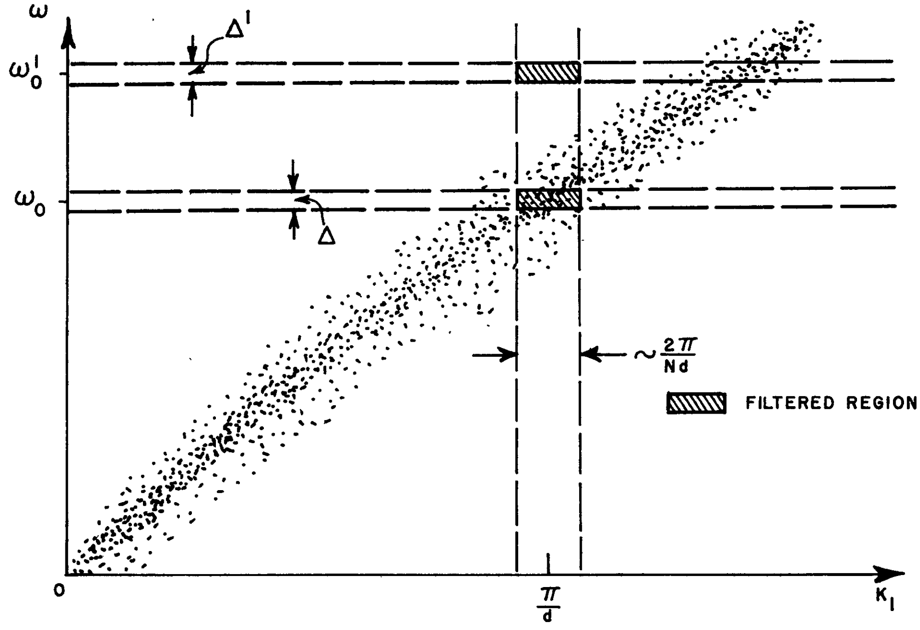

FIG. 4. Combined filtering action of frequency filters and boundary Wave-vector filter. Also illustrated is the spectral density of the pressure field in a turbulent boundary layer (Spectral density is proportional to density of dots.)

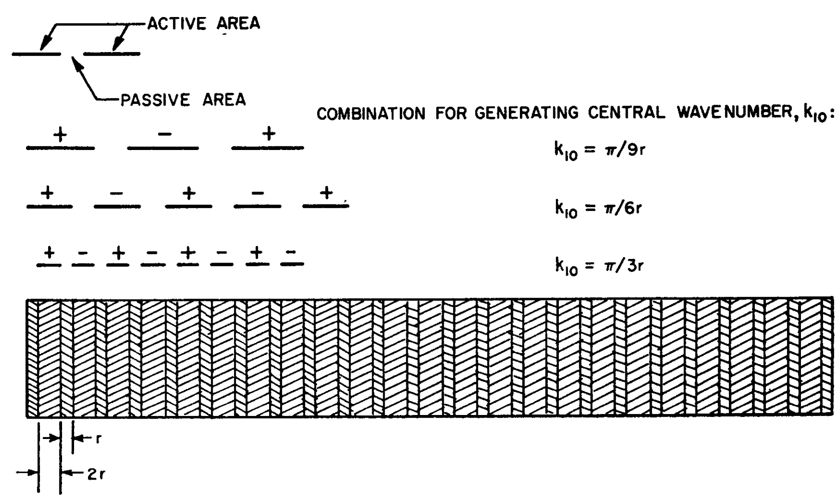

FIG. 5. A single rectangular pressure-transducer system capable of generating several single-band wavevector filters.

other hand, varying the center wave vector of a single-band wave-vector filter is usually not readily accomplished. However, with a proper design, a single transducer system can be made to accommodate several distinct center wave vectors by the appropriate combination of the transducer's electrical leads. In Fig. 5, an example of such a single transducer system is illustrated. In this transducer system, the center wavenumbers k_{10} can be made to take on the values $k_{10} = \pi/3r$, $\pi/6r$, $\pi/9r$, etc., while k_3 remains unchanged. This transducer system is made, for example, of a laminated stack of transducers, separated just sufficiently so as to isolate adjacent transducers from each other. The transducers are alternately of width r and $2r$. Each of the transducers has its own separate electrical leads. If the number of elements is such that for the centerwave number $\pi/3r$ the filter is an $\alpha\%$ filter, it is essentially a $2\alpha\%$ and a $3\alpha\%$ filter with respect to the center wavenumbers $\pi/6r$ and $\pi/9r$, respectively.

One may inquire as to whether a multiband wave-vector filter can be successfully employed to study the spectral nature of the pressure field in a turbulent boundary layer. Unfortunately, at the present time, the distribution of the wave-vector spectrum of the pressure field, as a function of frequency, is not sufficiently well known to answer this query definitively. Should, however, the measurements performed with the single-band wave-vector filter show this spectrum to be substantially localized, multiband wave-vector filters may be appropriately designed so as to resolve adequately the spectrum at the various center wave vectors of the transducer system with the use of a narrow enough single-band frequency filter. It is observed that the transducer system illustrated in Fig. 5 can be made to generate multiband wave-vector filters of considerable variety. Further, should the spectrum be of a form such that multiband wave-vector filters (possessing more than two center wave vectors) can be employed, the transducer system depicted in Fig. 6 may be used to advantage. With this proposed transducer system, the nature of the pressure field spectrum can be examined readily in a direction other than the direction of flow. However, the largest center wavenumber that can be used in this system to examine the pressure field, with some degree of accuracy, is of the order of $\pi/6R$, where R is the radius of the transducer.

Finally, it is worth mentioning that should a method be devised to obtain the "simultaneous" signals of the transducers in the transducer system, a computer program can be instituted to generate the required wave-vector filtering. Such a program would substitute mathematical procedures for the many actual experiments that would otherwise be required to utilize the full range of a given transducer system.

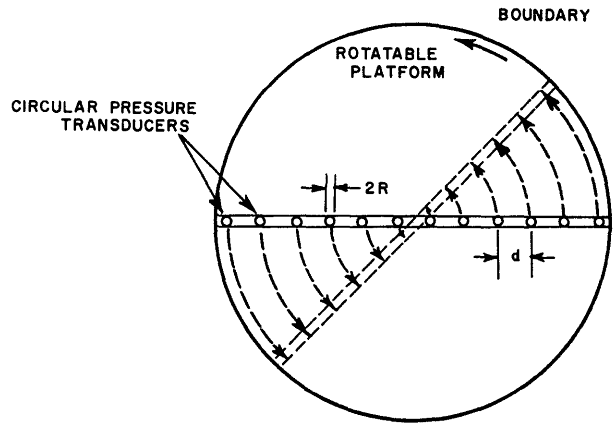

FIG. 6. Circular pressure-transducer system on a rotatable platform.

INITIAL DISTRIBUTION

Copies	
4	CHNAVMAT 1 DIR, Laboratory Programs (MAT 03L) 1 Laboratory Infor Office (MAT 033A) 1 Operations Planning & Analysis Br (MAT 0331) 1 Public Affairs Officer (MAT 09D)
8	NAVSHIPSYSCOM 1 Office of the Commander (SHIPS 00) 1 Office of Public Affairs & Congressional Relations (SHIPS 00D) 1 Technical Lib (SHIPS 2052) 1 Chief Scientist for R&D (SHIPS 031) 1 RDT&E Planning Div (SHIPS 033) 1 Applied Research Div (SHIPS 034) 1 Warfare Systems Div (SHIPS 035) 1 Sonar Systems Office (SHIPS 1610)
7	NAVSEC 1 Commander (SEC 6000) 1 Rewson Project Support Office (SEC 6050) 1 Ship Systems Engineering & Des Dept (SEC 6100) 1 Ship Concept Des Div (SEC 6110) 1 Hull Systems & Weapons Support Div (SEC 6120) 1 Propulsion, Power & Aux Systems Div (SEC 6140) 1 Command & Surveillance Div (SEC 6170)
1	NAVSEC PHILA

Copies	
5	NAVAIRSYSCOM 1 Plans & Program Div (AIR 302) 1 Adv Systems Concepts Div (AIR 303) 1 Aero & Structures Adm (AIR 320) 1 Air Frame Div (AIR 530) 1 Avionics Div (AIR 533)
1	NAVORDSYSCOM Weapons Dynamics Div (ORD 035)
1	NAVSUPSYSCOM
1	NAVFACENGCOM
1	ASTSECNAV (R&D)
1	ASTSECNAV (INSLOG)
2	ONR
1	CO & DIR, USNASL
1	CO & DIR, USNUSL
1	CO & DIR, NMDL
1	CO & DIR, NRDL
1	DIR, NRL
1	CDR, NWL
1	CDR, NOL
1	CO & DIR, NAVCIVENGRLAB
1	CO, NAVMISCEN
1	CO, NAVAIRENGCEN
1	CO, NAVAIRDEVCEN
1	CO, NAVUWRES
1	CDR, NUWC
1	CDR, NWC CHINA LAKE
1	CDR, NWC Corona Laboratories
1	CDR, NCCCLC
1	ONR, LONDON
1	CO, USNROTC & NAVADMINUMIT
20	DDC

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY *(Corporate author)* Naval Ship Research and Development Center Washington, D. C. 20007	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP ---

3 REPORT TITLE

BOUNDARY WAVE-VECTOR FILTERS FOR THE STUDY OF THE PRESSURE FIELD IN A TURBULENT BOUNDARY LAYER

4 DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Reprint of article published in the Journal of the Acoustical Society of America

5. AUTHOR(S) *(First name, middle initial, last name)*

G. Maidanik and D. W. Jorgensen

6 REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO OF REFS
December 1967	10	17

8a. CONTRACT OR GRANT NO	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. Sub-Project ZR-011-0101	Report 2638
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	AD 667 274

10. DISTRIBUTION STATEMENT

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of CO & DIR Naval Ship Research and Development Center.

11 SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Ship Systems Command

13 ABSTRACT

A proposal for constructing a boundary wave-vector filter is made. The wave-vector filter consists of a flush-mounted pressure transducer system. The elements and characteristics of the system are discussed. The manner in which the system can be employed to study the nature of the boundary pressure field in a turbulent boundary layer is discussed.

DD FORM 1473 1 NOV 65 (PAGE 1)

S/N 0101-807-6801

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Wave-Vector Filters						
Turbulent Boundary Layer						
Planar Array System						