Report 2507

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Washington, D.C. 20007

A COMPUTER PROGRAM FOR USE IN DESIGNING DUCTED PROPELLERS

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The Naval Ship Research and Development Center is a U.S. Navy center for laboratory effort directed at achieving improved sea and air vehicles. It was formed in March 1967 by merging the David Taylor Model Basin at Carderock, Maryland and the Marine Engineering Laboratory at Annapolis, Maryland.

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A COMPUTER PROGRAM FOR USE IN DESIGNING DUCTED PROPELLERS

by

E. B. Caster

ABSTRACT

This report presents a computer program that can be used to design and predict the performance of ducted propellers if the geometric shape of the duct is specified. The duct is represented mathematically by a distribution of ring vortices and ring sources along a cylinder of constant diameter which implies that the boundary conditions are linearized. The propeller design characteristics and performance are computed from lifting-line theory, as developed by Lerbs for moderately loaded finite-bladed propellers. Only the steady velocity components induced on the duct by the finite-bladed propeller are considered in determining the effect of the propeller on the duct.

The computer program contains an option for the input of velocities induced on the duct by sources other than the propeller. Another option allows calculation of the ideal angle of attack of the duct section in the presence of a propeller. A duct operating at this angle will have a favorable pressure distribution and flow should not separate. Results of calculations are presented and compared to experimental results.

ADMINISTRATIVE INFORMATION

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NOTATION

a	Duct-chord length
C_{P+}, C_{P-}	Pressure coefficients on outside and inside of duct $\dfrac{[p(x_d, x_l) - p_0]}{(1/2)\rho V^2}$
C_{Ps}	Propeller viscous power coefficient $\dfrac{550 P_s}{\frac{\rho}{2} \pi R^2 V^3}$
C_{Th}	Thrust coefficient $\left(\dfrac{T}{\frac{\rho}{2} \pi R^2 V^2}\right)$
C_{THP}	Propeller viscous thrust-horsepower coefficient $\dfrac{T V_a}{\frac{\rho}{2} \pi R^2 V^3}$
D	Propeller diameter
$G(x)$	Propeller nondimensionalized circulation distribution $\dfrac{\Gamma}{2\pi R V}$
h	Chord-diameter ratio of duct
J	Propeller speed coefficient V_a/nD
M_{x0}, M_{y0}	Bending moments on blade sections
n	Propeller rps
P_s	Shaft horsepower
$p(x_d, x_l)$	Local pressure on duct
p_0	Pressure infinitely ahead of duct
R	Propeller radius
R_d	Duct radius
T	Thrust
V	Ship speed
V_a	Speed of advance $(1 - w_0) V$
w_a	Axial velocity induced by propeller

w_{rp}	Radial velocity induced by propeller
w_{tp}	Tangential velocity induced by propeller
x	Nondimensional radius
x_d	Ratio of duct radius to the propeller radius
x_h	Nondimensional propeller hub radius
x_l	Axial distance from leading edge of duct nondimensionalized by the duct chord
x_0, y_0	Nondimensional radius of section being analyzed
x_p	Axial distance from the propeller plane nondimensionalized by the propeller radius
Z	Number of blades
β	Propeller advance angle
β_i	Propeller hydrodynamic pitch angle
Γ.	Propeller circulation distribution
ρ	Mass density
ϕ	Propeller pitch angle
$(1 - w_x)$	Radial distribution of propeller inflow velocity
$(1 - w_0)$	Propeller effective inflow velocity
Subscripts	
d	Duct
p	Propeller
sv	Stator vanes

INTRODUCTION

There is at present considerable interest in the application of ducted propeller systems to naval vessels since the velocity at the propeller can be increased or decreased by choosing the proper type of duct. Kort nozzles, which increase the flow at the propeller, have been used for many years[1] for increasing the efficiency of heavily loaded propellers. Pumpjets, a more recent innovation which decrease the flow at the propeller, are used to delay propeller cavitation. These applications have, therefore, led to various experimental and theoretical investigations of ducted propellers.

A number of systematic experimental studies have been made on the performance of ducted propellers.[2–6] Because of the large number of ducted propeller design parameters that must be investigated, results obtained from theoretical as well as experimental studies are important in determining the performance characteristics of ducted propellers. This is expecially true since the complex theoretical solutions required can now be accomplished on high-speed computers.

In most theoretical investigations,[7–11] the duct is represented by a linearized theory and the propeller by lifting-line theory. The linearized theory of the duct implies that the ring-vortex and ring-source distributions lie on a cylinder with a diameter representative of the duct and a length equal to the duct length. A nonlinear theory for the duct representing the static and free-flight conditions has also been developed by Chaplin[12] who represents the duct by a system of ring vortices lying on the duct surfaces.

The duct forces, circulation and pressure distributions, and the effect of the duct on the propeller is computed here using the linearized theory of the duct as discussed by Morgan.[7] An investigation of the linearized theory of the duct[13] showed satisfactory agreement between theory and experiments except when separation occurred on the duct. A computer program was also developed and results presented for the inverse problem of the duct and ducted propeller,[14] where the duct shape is computed once the duct pressure distribution has been specified.

In the design procedure presented here, the design and predicted performance of the propeller is computed from lifting-line theory as derived by Lerbs[15] for moderately loaded,

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[1] References are listed on page 61.

finite-bladed propellers. The effect of the propeller on the duct, considered when computing the duct thrust and pressure distribution, is determined by computing only the average axial and radial velocity components induced on the duct by the propeller from lifting-line theory as derived by Hough and Ordway.[16]

A procedure for designing ducted propellers has also been developed by Therm.[17] The design calculations presented here were performed on high-speed computers whereas Therm's calculations were done by hand. The advantage of Therm's approach is that a high-speed computer is not required, but this severely limits the calculations which can be performed. Consequently, features which are not included in the Therm approach but are presented here include the following:

1. Any shape can be considered for the duct.
2. The ducted propeller can be designed for a given thrust or horespower.
3. The design and predicted performances of the propeller can be obtained.
4. An option in the computer program allows calculations of the ideal angle of attack of the duct section in the presence of a propeller.

DUCT LINEARIZED THEORY

As previously stated, the computer program includes the linearized theory of the duct as presented in Reference 13, the Lerbs moderately loaded propeller theory,[15] and the theory used to compute the steady velocities induced on the duct by a finite-bladed propeller.

Since the linearized theory of the duct used to develop the computer program is identical to the one presented in Reference 13, the assumptions made will be reviewed only briefly here:

1. The fluid is inviscid and incompressible and no separation occurs on the duct.
2. The free-stream flow must be axisymmetric.
3. The duct is axisymmetric and of finite length.
4. The duct can be represented mathematically by a distribution of ring vortices and ring sources along a cylinder of constant diameter. This implies that the boundary conditions are linearized.

PROPELLER DESIGN THEORY

The lifting-line theory, as developed by Lerbs[15] for moderately loaded, finite-bladed propellers, is used to design and predict the performance of the propeller. Since this theory is discussed in detail in Reference 18, only the assumptions made in developing the theory and the equations used in computing the thrust and power coefficients, efficiency, and blade bending moments will be given here. In this theory, Lerbs considers the influence of the induced velocities on the shape of the helical vortex sheet at the lifting line, but neglects the effect of centrifugal forces and of the contraction of the slipstream. In addition, the change in shape of the vortex lines is neglected in the axial direction, i.e., each vortex line is assumed to be of constant pitch. However, the vortex sheets are not necessarily true helical surfaces since the pitch may vary along the radius. These same assumptions are made in the theory used here, and it is further assumed that the influence of the duct on the change in the shape of the helical vortex sheet in the axial direction can be neglected.

The propeller thrust and power coefficients, propeller efficiency, and blade bending moments parallel and perpendicular to the noise-tail line of the blade section are some of the more important parameters required in designing a propeller. The equations used in the computer program to calculate these parameters are as follows:

Propeller thrust coefficient:

$$C_{Ts} = \int_{x_h}^{1} (1 - \epsilon \tan \beta_i) \frac{dC_{Tsi}}{dx} dx \qquad [1]$$

Propeller thrust-horsepower coefficient:

$$C_{THP} = \int_{x_h}^{1} (1 - w_x)(1 - \epsilon \tan \beta_i) \frac{dC_{Tsi}}{dx} dx \qquad [2]$$

Propeller power coefficient:

$$C_{Ps} = \int_{x_h}^{1} \left(1 + \frac{\epsilon}{\tan \beta_i}\right) \frac{dC_{Psi}}{dx} dx \qquad [3]$$

Propeller efficiency:

$$\eta = \frac{C_{THP}}{C_{Ps}} \qquad [4]$$

Bending moments:

$$M_{x0} = M_{Tb} \cos \phi + M_{Qb} \sin \phi \qquad [5]$$

$$M_{y0} = M_{Tb} \sin \phi - M_{Qb} \cos \phi \qquad [6]$$

where the bending moments due to thrust M_{Tb} and torque M_{Qb}, are given as:

$$M_{Tb} = \frac{\rho}{2} \frac{R^3 V^2}{Z} \int_{x_h}^{1} (x - x_0)(1 - \epsilon \tan \beta_i) \frac{dC_{Tsi}}{dx} dx$$

$$M_{Qb} = \frac{\rho}{2} \frac{R^3 V^3}{Z} \int_{x_h}^{1} (x - x_0) \left(1 + \frac{\epsilon}{\tan \beta_i}\right) \frac{dC_{Psi}}{dx} dx$$

where

$\frac{dC_{Tsi}}{dx}$	is the radial distribution of the nonviscous thrust coefficient,
$\frac{dC_{Psi}}{dx}$	is the radial distribution of the nonviscous power coefficient,
ϵ	is the section drag-lift ratio,
$\tan \beta_i$	is the tangent of the propeller hydrodynamic pitch angle,
$(1 - w_x)$	is the radial distribution of the inflow velocity to the propeller and includes the velocity induced by the duct,
x_h	is the nondimensional hub radius,
x	is the nondimensional propeller radius,
R	is the propeller radius in feet,
V	is the ship speed,
Z	is the number of blades,
x_0, y_0	is the nondimensional radius of section being analyzed, and
ϕ	is the propeller pitch angle.

In addition, the velocities induced on the duct by the propeller must also be obtained. A brief discussion of the equations used is presented in the next section.

PROPELLER STEADY INDUCED VELOCITIES

As stated previously, the computer program calculates the steady axial and radial velocities induced by a finite-bladed propeller using the method presented in Reference 16. These average propeller-induced velocity components, derived from finite-bladed propeller theory, are essentially the same as those induced by infinitely bladed propellers. Equations for computing the steady and unsteady propeller-induced velocities have also been derived in terms of elliptic integrals by Morgan[7,10] for the lifting-line case. The numerical procedure presented in Reference 16 was used in computing the average propeller-induced axial and radial velocities because the computer programming of Legendre functions were preferred over the elliptic integrals presented in References 7 and 10.

Axial velocity:

$$\frac{w_{ap}}{V} = \frac{Z x_p}{2\pi x_d^{3/2}} \int_{x_h}^{1} \frac{G(x)}{x^{3/2} \tan \beta_i} Q'_{-1/2}(\omega_1)\, dx, \text{ for } x_d > 1 \qquad [7]$$

Radial velocity:

$$\frac{w_{rp}}{V} = \frac{Z}{2\pi x_d^{3/2}} \int_{x_h}^{1} \frac{G'(x)}{x^{1/2} \tan \beta_i} Q_{1/2}(\omega_1)\, dx, \text{ for } x_d > 1 \qquad [8]$$

where the Legendre function of the second kind and half-order $Q_{1/2}(\omega_1)$ and its first derivative $Q'_{-1/2}(\omega_1)$ are given as

$$Q_{1/2}(\omega_1) = \int_{-\pi/2}^{\pi/2} \frac{\cos 2\alpha}{[2(\omega_1 - 1) + 4 \sin^2 \alpha]^{1/2}}\, d\alpha$$

$$Q'_{-1/2}(\omega_1) = -\int_{-\pi/2}^{\pi/2} \frac{1}{[2(\omega_1 - 1) + 4 \sin^2 \alpha\ ]^{3/2}}\, d\alpha$$

$$\omega_1 = 1 + \frac{x_p^2\, (x_d - x)^2}{2\, x_d x}$$

where

$G(x)$ is the propeller nondimensional circulation distribution,

Z is the number of propeller blades,

x_d is the ratio of the duct radius to the propeller radius at the propeller plane, and

x_p is the axial distance from the propeller plane nondimensionalized on the propeller radius.

DESIGN PROCEDURE

The computer program presented calculates the duct thrust, duct pressure distribution, velocities induced at the propeller plane by the duct. and the design characteristics and predicted performance of the propeller in the following steps:

1. For an inflow velocity, the propeller calculations are made using the Lerbs moderately loaded propeller theory.[15]

2. The steady axial and radial velocity components induced on the duct by the propeller are computed using Equations [7] and [8].

3. The linearized theory of the duct,[13] which takes these propeller-induced velocities into account plus other induced velocities if given as input, is used to compute the duct thrust, pressure distribution, and velocities induced at the propeller plane by the duct. The velocity induced at the propeller by the duct is then combined with the inflow velocity used in Step 1 to design the propeller.

With this new inflow velocity, Steps 1, 2, and 3 are repeated until the inflow velocity at the propeller converges to four significant figures. Investigations to date indicate that this condition of convergence is usually obtained in less than six iterations.

The propeller can be designed on the basis of thrust or shaft horsepower and for a prescribed blade circulation distribution or pitch distribution. The viscous effects of the propeller are taken into account by giving as input the blade-section drag coefficient and the propeller blade outline. The viscous drag on the duct, which includes both the skin-friction and pressure drag, can also be calculated on the computer. The frictional drag on the duct is computed by giving as input the frictional drag as presented by Gertler[19] where the Reynolds number is based on the duct length. The computer program calculates the pressure drag on the duct according to the method developed by Granville.[20]

COMPUTER PROGRAM

The calculations based on the aforementioned theory of ducted propellers have been programmed for the IBM-7090 high-speed computer. The input for the duct consists mainly of the section camber and thickness ordinates, the section angle of attack, and the chord-diameter ratio of the duct. There are options whereby the ideal angle of attack of the duct section can be determined in the presence of the propeller. The propeller input consists of the propeller diameter, propeller speed in revolutions per second, design thrust (or propeller shaft horsepower), ship speed, number of blades, inflow velocity, and circulation (or pitch distribution). If the propeller is to be designed using the Lerbs optimum pitch distribution, only an estimate of the propeller ideal efficiency is given as input instead of the circulation or pitch distribution.

The output consists of the propeller design characteristics and performance as well as the duct thrust and pressure distribution. It takes approximately 15 minutes to complete the first iteration and 2 minutes for each additional iteration. Results from this investigation indicate the duct circulation distribution usually converges in less than six iterations or approximately 27 minutes of computer time. A FORTRAN listing of the computer program developed for computation on the IBM-7090 high-speed computer is presented in the appendices along with the input and output obtained for one ducted propeller design.

INPUT FORMAT

The input data (see the example in Appendix A) for the ducted propeller design computer program are punched on IBM cards using Format F8.6, where up to nine parameters are punched on a card; each parameter has a field width of eight columns. The duct and propeller identifications are key-punched on IBM cards using Format 12A6 where Columns 2 through 72 are used. The input formats are described in Reference 21. The input parameters are punched on each IBM card as follows:

1st Card (Format 2F8.6)

1) The duct identification where any alphanumeric characters may appear in Columns 2 through 72.

2nd Card (Format 2F8.6)

1) This quantity indicates three options for making ducted propeller design calculations. If the complete design of a ducted propeller is desired, the value should be −1.0. If calculations for the duct and propeller are desired after only one iteration, the value should be zero. If calculations for the duct alone are desired, the value should be 1.0.

2) If the design is based on a propeller design thrust or shaft horsepower, the quantity should be 1.0. If the design thrust is to be the duct plus propeller thrust or if no propeller is present, the value should be zero.

3rd Card (Format 5F8.6)

1) The number of chordwise camber ordinates used in describing the duct. This number may range from 18 to 39.

2) The number of chordwise thickness ordinates used in describing the duct. This number may range from 18 to 39.

3) The chord-diameter ratio of the duct.

4) The duct section angle of attack in degrees. If the duct section ideal angle of attack is to be computed, the value should be zero.

5) This quantity indicates the duct section ideal angle of attack option. If the ideal angle of attack is to be computed, the value should be 1.0. If the ideal angle of attack is not to be computed, the value should be zero.

6) This quantity indicates the option for inputting radial velocities induced on the duct. If these velocities are not given as input, the value should be zero. If these velocities are given as input, the value is the number of ordinates used in describing the velocities which may range from 18 to 39.

7) This quantity indicates the option for inputting axial velocities induced on the duct. If these induced velocities are not given as input, the value should be zero. If these velocities are given as input, the value is the number of ordinates used in describing these velocities which may range from 18 to 39.

4th Card (Format 3F8.6)

1) The ratio of the duct radius to that of the propeller at the centerline of the propeller.

2) The axial distance from the leading edge of the duct to the propeller centerline nondimensionalized on the duct chord length.

3) The duct frictional drag coefficient such as given in Reference 19.

5th Set of Cards (Format 9F8.6)

The stations along the duct chord nondimensionalized by the duct chord where the camber ordinates are given as input.

6th Set of Cards (Format 9F8.6)

The camber ordinates nondimensionalized by the duct chord that corresponds to stations just given.

7th Set of Cards (Format 9F8.6)

The stations along the duct chord nondimensionalized by the duct chord where the thickness ordinates are given as input.

8th Set of Cards (Format 9F8.6)

The half-thickness ordinates nondimensionalized by the duct chord that corresponds to stations just given.

9th Set of Cards (Format 9F8.6)

The stations along the duct chord nondimensionalized by the duct chord were radial velocities induced on the duct are given as input. If these velocities are not given, the cards are not included as input.

10th Set of Cards (Format 9F8.6)

The radial velocities nondimensionalized on ship speed induced on the duct that corresponds to stations just given. If these velocities are not given, the cards are not included as input.

11th Set of Cards (Format 9F8.6)

The stations along the duct chord nondimensionalized on the duct chord where axial velocities induced on the duct are given as input. If these velocities are not given, these cards are not included as input.

12th Set of Cards (Format 9F8.6)

The axial velocities nondimensionalized on ship speed on the duct that corresponds to stations just given. If these velocities are not given, the cards are not included as input. No additional input cards are required unless the propeller design calculations are to be included in the calculations.

13th Set of Cards (Format 12A6)

The propeller identification whereby any alphanumeric characters may appear in Columns 2 through 72.

14th Card (Format 4F8.6)

1) This quantity indicates the option for designing the propeller for a given pitch or circulation distribution. If the propeller is to be designed for a given circulation distribution, the value should be 1.0. If the propeller is to be designed for a specified pitch distribution, the value should be zero.

2) The propeller section drag coefficient is punched on this card if its radial distribution is constant. If the drag distribution is not constant, the value should be zero.

3) This quantity indicates the option for designing the propeller for a given advance angle tan β distribution. This option is valuable when a propeller with a specified advance angle distribution is to operate in any axisymmetric radially varying inflow velocity. If the propeller is to be designed for a given tan β distribution, the value should be 1.0; otherwise, the value should be zero.

4) This quantity is an estimate of the ideal efficiency of the propeller and is given only if Lerbs optimum pitch distribution is desired.

15th Card (Format 9F8.6)

1) This quantity is 1.0 if the propeller design is based on thrust and zero if based on shaft horsepower.

2) The ratio of the nonviscous to viscous propeller thrust or power coefficients, depending on whether the design is based on thrust or shaft horsepower. This quantity is used only because it reduces the number of iterations required in the computer computations.

3) Propeller diameter in feet.

4) The absolute pressure at the shaft centerline minus the vapor pressure in feet of water.

5) The propeller revolutions per second.

6) The design thrust or shaft horsepower.

7) The ship speed in feet per second.

8) The density of the fluid.

9) The number of propeller blades.

16th Card (Format 9F8.6)

Nine stations along the propeller radius nondimensionalized on the propeller radius.

17th Card (Format 9F8.6)

Nine free-stream inflow velocities that correspond to the radial locations just given. This allows the propeller to be designed for operation in axisymmetric radially varying inflow.

18th Card (Format 9F8.6)

Nine values of the tangent of the hydrodynamic pitch angle distribution. If Lerbs optimum pitch distribution is desired, no values are punched on this card. If the design is to be based on a given blade circulation distribution, nine values describing this distribution are punched on this card.

19th Card (Format 9F8.6)

The nine values describing the propeller blade outline are punched on this card if the propeller viscous effects are to be taken into account; otherwise no values should be punched on this card.

20th Card (Format 9F8.6)

Nine values of the propeller section drag distribution are punched on this card only if the distribution is not constant. Otherwise this card is not included as input.

21st Card (Format 9F8.6)

Nine values of the tangent of the advance angle distribution. If this input option is not desired, this input card is not included as input.

An example of the input data for a ducted propeller design is shown in Appendix A.

OUTPUT FORMAT

An example of the output data is also given in Appendix A. The first page of output contains the input data. The design and predicted performance of the propeller computed on the basis of lifting-line theory[15,18] are printed on the second page of the output. The symbols used to identify the parameters printed on this page are defined at the bottom of the page of output.

The steady axial and radial velocity components induced on the duct by the finite-bladed propeller using Equations [7] and [8] are printed on the third page of output. These velocities are combined with those given as input if the option for inputting such velocities is used.

The fourth page of output contains the induced duct thrust coefficient $C_{T_{di}}$ computed using the method derived in References 7 and 10, and the viscous duct thrust coefficients due to the duct frictional[19] and pressure drag,[20] $C_{T_{df}}$ and $C_{T_{dp}}$, respectively. If calculations are made for the duct alone, the viscous duct thrust calculations are based on a duct diameter of 1.0. The propeller inflow velocity given as input, the axial velocity induced at the propeller by the duct,[13] and the sum of these velocities are also printed on the fourth page of output.

The last page of output gives the duct circulation distribution, nonlinear corrections, and nonlinear and linear pressure distributions. The method used to compute these parameters is described in detail in Reference 13. If the ideal angle of attack input option is used, the computed angle in degrees is printed on the fourth page of output, and the other parameters just cited are printed on succeeding page.

FORTRAN LISTING

The FORTRAN listing of the computer program is given in Appendix B. In addition to the subroutines furnished automatically by the FORTRAN IV monitor system on the IBM-7090 computer, the bindary coding for YSEL, ELLIP, CBRT, SIMPUN, GAUSS, MATINV, DISCOT, and LEGEND subroutines, must be added to the FORTRAN listing of this program; these are all available from SHARE listings and the NSRDC Applied Mathematics Laboratory.

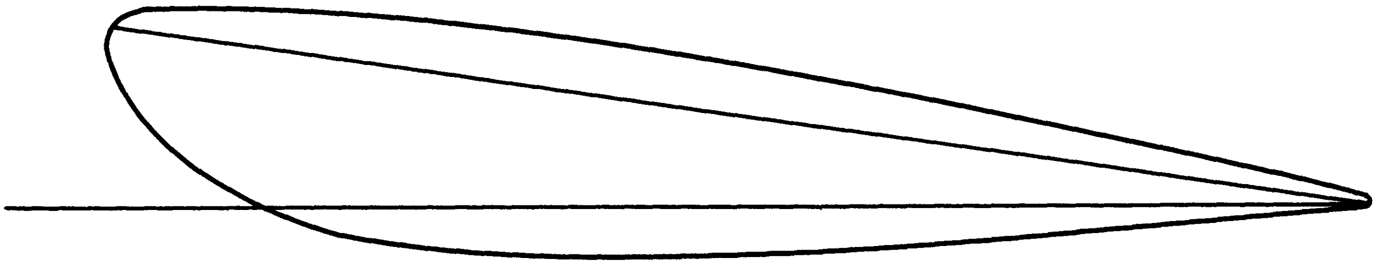
EXPERIMENTAL AND THEORETICAL RESULTS

This section presents the experimental duct plus stator thrust and pressure distributions of the ERG ducted propeller system[22,23] at two propeller operating conditions and compares them to theoretical results obtained using the computer program presented here.

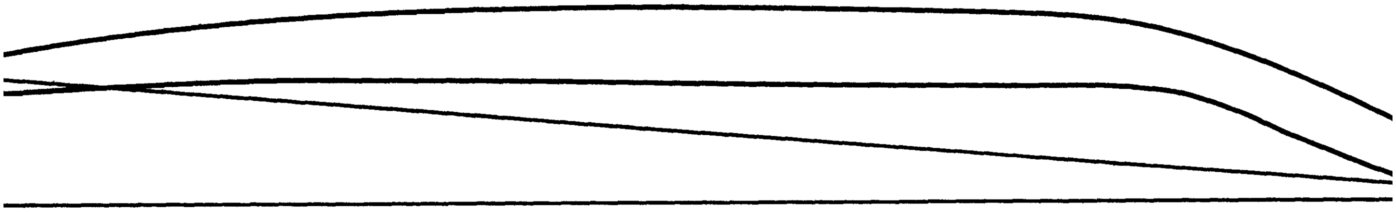
The geometric properties of the duct and the design condition of the propeller were obtained from References 22 and 23. The ERG duct had a maximum camber ratio of 0.067 (which occurred at 67.1 percent of the duct chord from the leading edge) and a maximum half-thickness ratio of 0.0225 (which occurred at 62.9 percent of the duct chord length). Each duct section operated at an angle of attack of 3.88 degrees and the chord diameter ratio of the duct (h) was 1.57. The duct shape is shown in Figure 1. A five-bladed propeller with a hub radius (x_h) of 0.4 was located at 28.1 percent of the duct-chord length from the leading edge. Six stator vanes were located aft of the propeller at 53.6 percent of the duct-chord length.

The measured pressure distributions on the inside and outside of the duct[22] are presented in Figure 2 for the propeller operating at a J of 2.26 and in Figure 3 for the propeller operating at a J of 1.27. The propeller thrust C_{Tp} and the duct C_{Td} plus stator thrust C_{Tsv} coefficients[23] were 0.28 and −0.17, respectively, when the propeller operated at a J of 2.26. The comparable values were 1.33 and −0.28, respectively, when the propeller operated at a J of 1.27. These measured results, including the total measured thrust coefficients C_{Th}, are shown in Table 1 and the pressure measurements on the duct are listed in Table 2.

In the theoretical calculations of the duct thrust and pressure distributions, difficulty was encountered in determining accurate input data for the computer program because no measured results were available for the performance of the stator vanes. The effect of the stator vanes can be taken into account in the theoretical calculations if the axial and radial velocities induced on the duct by the stator vanes are given as input to the computer program. These velocities are calculated by assuming that the velocities induced on the duct by the



KORT NOZZLE



ERG DUCT

Figure 1 – Duct Cross Sections

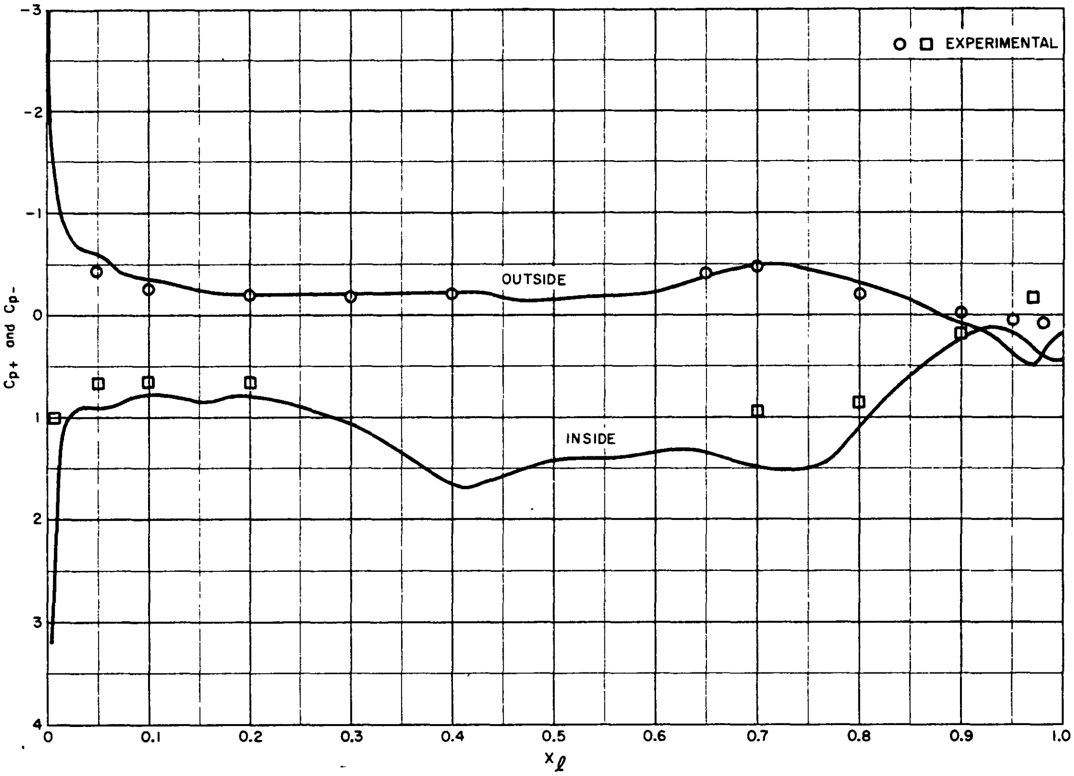


Figure 2 – Pressure Distribution on ERG Duct Surface for $J = 2.26$

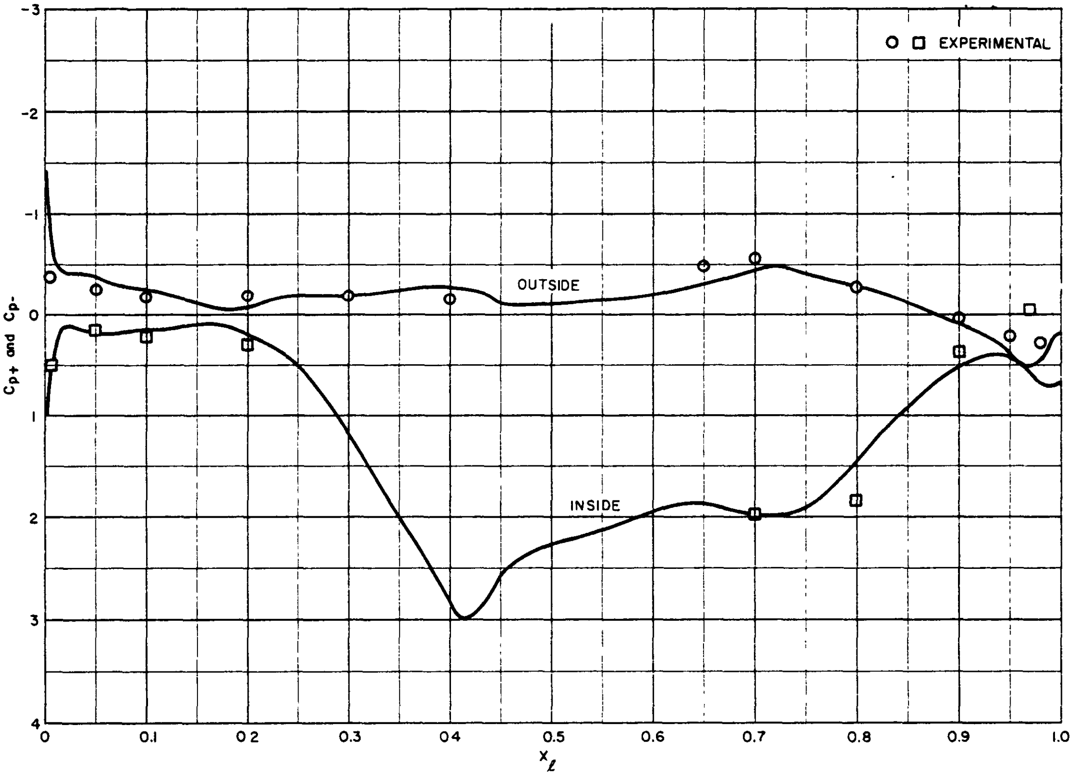


Figure 3 – Pressure Distribution on ERG Duct Surface for $J = 1.27$

TABLE 1

Theoretical and Measured Thrust Coefficients for the ERG and a Kort Nozzle Type Ducted Propeller

Ducted Propeller	J	C_{Tp}	C_{Td}	C_{Tsv}	$C_{Td} + C_{Tsv}$	C_{Th}
Theory - ERG	2.260	0.270*	-0.309	0.142*	-0.167	0.103
Measured - ERG	2.260	0.270	----	-----	-0.170	0.100
Theory - ERG	1.270	1.330*	-0.638	0.354*	-0.284	1.046
Measured - ERG	1.270	1.330	----	-----	-0.280	1.050
Theory - Kort Nozzle	0.40	2.75*	0.425	-----	-----	3.175
Measured - Kort Nozzle	0.40	2.75	0.40	-----	-----	3.150
*Input						

TABLE 2

Measured Pressure Distribution on ERG Duct for $J = 2.26$ and $J = 1.27$

x_ℓ	J = 2.26		J = 1.271	
	C_{P+}	C_{P-}	C_{P+}	C_{P-}
Leading Edge	1.000	1.000	1.000	1.000
0.005		0.993	-0.387	0.498
0.050	-0.441	0.658	-0.263	0.134
0.100	-0.271	0.642	-0.192	0.199
0.200	-0.210	0.660	-0.193	0.284
0.300	-0.197		-0.193	
0.400	-0.214		-0.171	
0.650	-0.416		-0.472	
0.700	-0.473	0.931	-0.553	1.980
0.800	-0.210	0.861	-0.265	1.838
0.900	-0.009	0.190	0.035	0.362
0.950	0.060		0.208	
0.970		-0.165		-0.063
0.980	0.097		0.277	
1.000	1.000	1.000	1.000	1.000

TABLE 3

Theoretical Pressure Distributions on ERG Duct for $J = 2.26$ and $J = 1.27$

	J = 2.26		J = 1.27	
x_ℓ	C_{P+}	C_{P-}	C_{P+}	C_{P-}
Leading Edge	1.000	1.000	1.000	1.000
0.0019	-3.087	3.191	-1.412	1.031
0.0076	-1.435	1.647	-0.710	0.404
0.0170	-0.886	1.046	-0.450	0.122
0.0302	-0.694	0.903	-0.414	0.115
0.0469	-0.619	0.894	-0.407	0.180
0.0670	-0.465	0.863	-0.315	0.178
0.0904	-0.380	0.789	-0.262	0.144
0.1170	-0.349	0.795	-0.228	0.132
0.1464	-0.255	0.842	-0.131	0.090
0.1786	-0.226	0.786	-0.058	0.106
0.2132	-0.228	0.809	-0.131	0.259
0.2500	-0.222	0.904	-0.202	0.483
0.2887	-0.234	0.991	-0.190	0.870
0.3290	-0.236	1.240	-0.236	1.661
0.3706	-0.227	1.488	-0.290	2.330
0.4132	-0.233	1.670	-0.272	2.950
0.4564	-0.153	1.543	-0.138	2.577
0.5000	-0.156	1.414	-0.110	2.259
0.5436	-0.177	1.390	-0.145	2.126
0.5868	-0.212	1.357	-0.184	1.997
0.6294	-0.318	1.303	-0.277	1.851
0.6710	-0.416	1.413	-0.383	1.900
0.7113	-0.514	1.520	-0.491	1.965
0.7500	-0.445	1.514	-0.417	1.909
0.7868	-0.352	1.240	-0.322	1.597
0.8214	-0.260	0.865	-0.240	1.203
0.8536	-0.125	0.565	-0.108	0.885
0.8830	0.017	0.352	0.039	0.641
0.9096	0.111	0.187	0.128	0.466
0.9330	0.225	0.115	0.234	0.387
0.9532	0.402	0.182	0.416	0.439
0.9698	0.502	0.322	0.516	0.570
0.9830	0.398	0.421	0.403	0.672
0.9924	0.220	0.446	0.225	0.693
0.9981	0.185	0.444	0.195	0.684
1.0	1.000	1.000	1.000	1.000

stator vanes are essentially the same as those induced by an infinitely bladed propeller at the same loading.

Since no measured results were available for the stator vanes, an estimate of their thrust was calculated by combining (1) the theoretical duct thrust obtained from the computer program when the stator vanes were not taken into account and (2) the measured propeller and duct plus stator thrust presented in Reference 23. That is to say, the stator thrust was calculated to be equal to the total measured thrust minus the measured propeller thrust and the theoretical duct thrust. Consequently the procedure used in making the calculations was indirect in that the method requires use of measured data obtained for the total thrust of the ducted propeller system being investigated.

When the propeller operated at a J of 2.26, the duct and stator vanes thrust coefficients C_{Td} and C_{Tsv} were computed to be -0.303 and 0.142, respectively. Comparable values were -0.636 and 0.354, respectively, when the propeller operated at a J of 1.27.

The theoretical and measured duct plus stator thrust coefficients were -0.167 and -0.170, respectively, when the propeller operated at a J of 2.26. Comparable values were -0.284 and -0.280, respectively, when the propeller operated at a J of 1.27. These results, which show very good agreement between measured and theoretical results, are also given in Table 1.

The theoretical pressure distributions computed for the ERG ducted propeller includes an approximate nonlinear correction to the linearized Bernoulli equation as discussed in Reference 13 since this equation calculates the pressure on the duct to be infinite at the leading edge. The pressure distributions computed for the inside and outside of the duct are shown as solid lines in Figures 2 and 3 for propeller operation at $J = 2.26$ and $J = 1.27$, respectively. These theoretical pressure distributions are listed in Table 3. It is apparent from Figures 2 and 3 that the predicted pressure distributions on the outside of the duct were slightly better than those predicted for the inside of the duct especially when the propeller operated at a J of 2.26. This may be due to the inaccuracy involved in preparing input data for the effect of the stator vanes since the propeller loading for this case is small and the change in velocities induced inside the duct has a greater effect on the inside duct distribution than on the outside duct pressure distribution.

In order to determine whether reasonable predictions of the duct thrust coefficient can be obtained for a ducted propeller, the computer results were compared with the duct thrust coefficients from open-water tests for a Kort-nozzle type ducted propeller presented by Nakonechny.[5] This duct had a maximum camber ratio of -0.05 (which occurred at 45 percent of the duct chord from the leading edge of the duct) and a maximum half-thickness ratio of 0.0764 (which occurred at 40 percent of the duct-chord length from the leading edge). Each duct section operated at an angle of attack of 12.7 degrees and the chord diameter ratio of the duct (h) was 0.5. The duct shape is also shown in Figure 1. A three-bladed propeller with a hub radius (x_h) of 0.2 was located at 50 percent of the duct-chord length. When the propeller was operating at a J of 0.40, which corresponds to a propeller thrust coefficient C_{T_P} of 2.75, the measured and computed duct thrust coefficients C_{Td} were 0.40 and 0.425, respectively. The theoretical and measured thrust coefficients are presented in Table 1. The theoretical pressure distributions on the duct are not presented here since no measured results were available for comparisons.

CONCLUSIONS

1. The linearized theory used in the computer program gives better predictions of the duct thrust and pressure distribution than expected for the duct shapes investigated.

2. Better agreement would have been obtained probably between the measured and computed duct pressure distributions if measured results were available on the performance of the stator vanes.

3. The computer program is quite versatile in that it can be used to design and predict ~~the~~ performance of the propeller as well as to compute the ideal angle of attack of the duct section in the presence of the propeller if desired.

4. The computed duct thrust and pressure distributions presented here are based only on the steady velocities induced on the duct by a finite-bladed propeller. It is believed, however, that these predictions will improve when the unsteady velocities associated with finite-bladed propellers are taken into account.

ACKNOWLEDGMENTS

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APPENDIX A

INPUT AND OUTPUT FOR A DUCTED PROPELLER DESIGN

1	9	17	25	33	41	49	57	65 72
KORT NOZZLE								
−1.0	1.0							
18.0	18.0	0.5	12.7					
1.01	0.5	0.0047						
0.0	0.0125	0.025	0.05	0.075	0.10	0.15	0.20	0.25
0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	1.0
0.0	− 0.0079	−0.0106	−0.0156	−0.0208	−0.0254	−0.0341	−0.0402	−0.0445
− 0.0478	− 0.0505	−0.0479	−0.0416	−0.0333	−0.0240	−0.0186	−0.0072	0.0
0.0	0.0125	0.025	0.05	0.075	0.10	0.15	0.20	0.25
0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	1.0
0.0	0.0259	0.0355	0.0469	0.0551	0.0610	0.0705	0.0751	0.0775
0.0783	0.0764	0.07	0.0596	0.0472	0.0337	0.0186	0.0105	0.0
PROPELLER FOR KNORT NOZZLE								
0.0	0.0085							
1.0	1.0251	0.667	33.0	45.0	77.0	9.0	1.9905	3.0
0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.273	0.849	0.637	0.509	0.424	0.364	0.318	0.283	0.225
0.333	0.389	0.444	0.472	0.500	0.472	0.444	0.356	0.0

```
KORT NOZZLE
    -1.0000     1.0000
   18.0000     18.0000      0.5000     12.7000      0.         -0.         -0.         -0.
    1.0100      0.5000      0.0047     -0.
    0.          0.0125      0.0250      0.0500      0.0750      0.1000      0.1500      0.2000      0.2500
    0.3000      0.4000      0.5000      0.6000      0.7000      0.8000      0.9000      0.9500      1.0000
    0.         -0.0079     -0.0106     -0.0156     -0.0208     -0.0254     -0.0341     -0.0402     -0.0445
   -0.0478     -0.0505     -0.0479     -0.0416     -0.0333     -0.0240     -0.0186     -0.0072      0.
    0.          0.0125      0.0250      0.0500      0.0750      0.1000      0.1500      0.2000      0.2500
    0.3000      0.4000      0.5000      0.6000      0.7000      0.8000      0.9000      0.9500      1.0000
    0.          0.0259      0.0355      0.0469      0.0551      0.0610      0.0705      0.0751      0.0775
    0.0783      0.0764      0.0700      0.0596      0.0472      0.0337      0.0186      0.0105      0.
PROPELLER FOR KORT NOZZLE
    0.          0.0085     -0.         -0.         -0.         -0.         -0.         -0.         -0.
    1.0000      1.0251      0.6670     33.0000     45.0000     77.0000      9.0000      1.9905      3.0000
    0.2000      0.3000      0.4000      0.5000      0.6000      0.7000      0.8000      0.9000      1.0000
    1.0000      1.0000      1.0000      1.0000      1.0000      1.0000      1.0000      1.0000      1.0000
    1.2730      0.8490      0.6370      0.5090      0.4240      0.3640      0.3180      0.2830      0.2550
    0.3330      0.3890      0.4440      0.4720      0.5000      0.4720      0.4440      0.3560      0.
    0.0085      0.0085      0.0085      0.0085      0.0085      0.0085      0.0085      0.0085      0.0085
```

```
    K=9.5817E-01   CTSI=2.8020E 00   CPSI=5.6114E 00

   XI          TAN BI       TAN B          GS          UT/2VS        UA/2VS       DCTSI        DCPSI        CLL/D        SIGMA X

 2.0000E-01  9.5572E-01  6.4852E-01  0.          3.1326E-01  3.4432E-01 -0.          -0.          0.           4.3427E 00
 3.0000E-01  6.3740E-01  4.3572E-01  4.9931E-02  2.1730E-01  4.9541E-01  1.7531E 00  3.5122E 00  9.0419E-02  2.2775E 00
 4.0000E-01  4.7823E-01  3.3051E-01  5.9773E-02  2.0544E-01  5.2083E-01  2.8587E 00  5.7294E 00  8.5013E-02  1.3612E 00
 5.0000E-01  3.8214E-01  2.6851E-01  6.2602E-02  1.8980E-01  5.2274E-01  3.7928E 00  7.5927E 00  7.2775E-02  8.9543E-01
 6.0000E-01  3.1832E-01  2.2824E-01  6.1764E-02  1.6350E-01  5.1422E-01  4.5381E 00  9.0810E 00  6.0396E-02  6.3077E-01
 7.0000E-01  2.7328E-01  2.0053E-01  5.8223E-02  1.2858E-01  4.9841E-01  5.0343E 00  1.0090E 01  4.8975E-02  4.6717E-01
 8.0000E-01  2.3874E-01  1.8071E-01  5.1281E-02  8.6377E-02  4.6581E-01  5.1047E 00  1.0215E 01  3.7780E-02  3.5933E-01
 9.0000E-01  2.1247E-01  1.6614E-01  3.8851E-02  4.7923E-02  4.2665E-01  4.3738E 00  8.7625E 00  2.5452E-02  2.8471E-01
 1.0000E 00  1.9144E-01  1.4933E-01  0.          8.1118E-02  4.2366E-01 -0.          -0.          0.           2.3148E-01


   CPT=4.0093E 00      CPS=7.2567E 00      EP=5.5250E-01      CTS=2.7378E 00


      XI          EPSILON      MTB          MQB          MXO          MYO            M        G(M)


 2.0000E-01  0.           3.8513E 00  1.4577E 00  4.0244E 00  8.7255E-01        1       6.4096E-02
 3.0000E-01  3.6569E-02  0.           0.          0.          0.                 2       5.1905E-03
 4.0000E-01  4.4393E-02  2.2096E 00  7.9650E-01  2.3479E 00 -6.2850E-02        3       2.2460E-03
 5.0000E-01  5.3128E-02  0.           0.          0.          0.                 4       1.5596E-04
 6.0000E-01  7.0369E-02  9.1258E-01  3.1672E-01  9.6017E-01 -1.0572E-01        5      -1.2372E-04
 7.0000E-01  8.1319E-02  0.           0.          0.          0.                 6      -3.1004E-04
 8.0000E-01  9.9894E-02  1.7800E-01  6.0509E-02  1.8592E-01 -2.7892E-02        7      -1.0227E-04
 9.0000E-01  1.1889E-01  0.           0.          0.          0.                 8      -1.3151E-04
 1.0000E 00  0.           0.           0.          0.          0.                 9      -6.3895E-05

K=RATIO OF ESTIMATED AND FINAL HYDRODYNAMIC PITCH ANGLE
CTSI=PROPELLER NONVISCOUS THRUST COEFFICIENT
CPSI=PROPELLER NONVISCOUS POWER COEFFICIENT
XI=NONDIMENSIONAL PROPELLER RADIUS
TANBI=TANGENT OF HYDRODYNAMIC PITCH ANGLE
TANB=TANGENT OF PROPELLER ADVANCE ANGLE
GS=PROPELLER NONDIMENSIONAL CIRCULATION DISTRIBUTION
UT/2VS=TANGENTIAL VELOCITY INDUCED AT THE LIFTING LINE
UA/2VS=AXIAL VELOCITY INDUCED AT THE LIFTING LINE
DCTSI=LOCAL NONVISCOUS PROPELLER THRUST COEFFICIENT
DCPSI=LOCAL NONVISCOUS PROPELLER POWER COEFFICIENT
CLL/D=NONDIMENSIONAL PROPELLER LIFT COEFFICIENT
SIGMA X=CAVITATION NO. BASED ON INFLOW VELOCITY
CPT=PROPELLER VISCOUS THRUST-HORSE POWER
CPS=PROPELLER VISCOUS POWER COEFFICIENT
EP=PROPELLER EFFICIENCY (CPT/CPS)
CTS=PROPELLER VISCOUS THRUST  COEFFICIENT
EPSILON=PROPELLER SECTION DRAG-LIFT RATIO
MTB=BENDING MOMENT DUE TO THRUST
MQB=BENDING MOMENT DUE TO TORQUE
MXO=BENDING MOMENT PARALLEL TO SECTION NOSE-TAIL LINE
MYO=BENDING MOMENT PERPENDICULAR TO NOSE-TAIL LINE
M,G(M) NO. OF HARMONICS AND  FOURIER SINE COEFF. OF GS
```

```
VELOCITIES INDUCED BY PROPELLER ALONG THE DUCT
       X           AXIAL          RADIAL
  0.             -0.9758E-01   -0.1308E 00
  0.1903E-02     -0.9772E-01   -0.1312E 00
  0.7596E-02     -0.9816E-01   -0.1326E 00
  0.1704E-01     -0.9888E-01   -0.1350E 00
  0.3015E-01     -0.9988E-01   -0.1384E 00
  0.4685E-01     -0.1011E 00   -0.1429E 00
  0.6699E-01     -0.1026E 00   -0.1486E 00
  0.9042E-01     -0.1044E 00   -0.2306E 00
  0.1170E 00     -0.1077E 00   -0.3719E 00
  0.1464E 00     -0.1099E 00   -0.3833E 00
  0.1786E 00     -0.1121E 00   -0.3967E 00
  0.2132E 00     -0.8885E-01   -0.4975E 00
  0.2500E 00     -0.8787E-01   -0.5162E 00
  0.2887E 00     -0.8414E-01   -0.5378E 00
  0.3290E 00     -0.5845E-01   -0.6104E 00
  0.3706E 00     -0.5244E-01   -0.6395E 00
  0.4132E 00     -0.3978E-01   -0.6663E 00
  0.4564E 00     -0.2092E-01   -0.6814E 00
  0.5000E 00     -0.9096E-08   -0.6877E 00
  0.5436E 00      0.2092E-01   -0.6814E 00
  0.5868E 00      0.3978E-01   -0.6663E 00
  0.6294E 00      0.5244E-01   -0.6395E 00
  0.6710E 00      0.5845E-01   -0.6104E 00
  0.7113E 00      0.8414E-01   -0.5378E 00
  0.7500E 00      0.8787E-01   -0.5162E 00
  0.7868E 00      0.8885E-01   -0.4975E 00
  0.8214E 00      0.1121E 00   -0.3967E 00
  0.8536E 00      0.1099E 00   -0.3833E 00
  0.8830E 00      0.1077E 00   -0.3719E 00
  0.9096E 00      0.1044E 00   -0.2306E 00
  0.9330E 00      0.1026E 00   -0.1486E 00
  0.9532E 00      0.1011E 00   -0.1429E 00
  0.9698E 00      0.9988E-01   -0.1384E 00
  0.9830E 00      0.9888E-01   -0.1350E 00
  0.9924E 00      0.9816E-01   -0.1326E 00
  0.9981E 00      0.9772E-01   -0.1312E 00
  0.1000E 01      0.9758E-01   -0.1308E 00
```

```
CHORD DIAMETER RATIO                                          0.5000E 00

DUCT THRUST COEFF. DUE TO PROPELLER                           0.4269E 00

EFFECT OF DUCT PROFILE DRAG ON CT                            -0.1884E-01

EFFECT OF DUCT PRESSURE ON CT                                -0.7736E-01




VELOCITIES INDUCED BY THE DUCT AT THE PROPELLER PLANE
     X            1-WP           WD             1-W
  0.2000E 00   0.1000E 01   0.3784E 00   0.1378E 01
  0.3000E 00   0.1000E 01   0.3894E 00   0.1389E 01
  0.4000E 00   0.1000E 01   0.4055E 00   0.1406E 01
  0.5000E 00   0.1000E 01   0.4276E 00   0.1428E 01
  0.6000E 00   0.1000E 01   0.4566E 00   0.1457E 01
  0.7000E 00   0.1000E 01   0.4934E 00   0.1493E 01
  0.8000E 00   0.1000E 01   0.5381E 00   0.1538E 01
  0.9000E 00   0.1000E 01   0.5906E 00   0.1591E 01
  0.1000E 01   0.1000E 01   0.5906E 00   0.1591E 01
```

```
GEOMETRIC ANGLE OF ATTACK IN DEGREES                          0.
POSITION OF SECTION AT ANGLE (PA) IN DEGREES                  0.
     T          X           GSTAR         NCO           NCI             PO            PI            POP           PIP
     0   -0.              -0.2698E 00   0.            0.             -0.0000E-19   -0.0000E-19   -0.           -0.
     5     0.1903E-02   -0.2824E 00   0.4647E 00    0.3128E 00     0.5261E 01   -0.7689E 01    0.1132E 02   -0.2458E 02
    10     0.7596E-02   -0.3189E 00   0.5500E 00    0.4220E 00     0.2413E 01   -0.4904E 01    0.4388E 01   -0.1162E 02
    15     0.1704E-01   -0.3749E 00   0.7675E 00    0.7443E 00     0.1615E 01   -0.4130E 01    0.2104E 01   -0.5549E 01
    20     0.3015E-01   -0.4444E 00   0.8759E 00    0.9024E 00     0.1295E 01   -0.3824E 01    0.1478E 01   -0.4238E 01
    25     0.4685E-01   -0.5201E 00   0.9231E 00    0.9321E 00     0.1133E 01   -0.3673E 01    0.1228E 01   -0.3940E 01
    30     0.6699E-01   -0.5950E 00   0.9496E 00    0.9645E 00     0.1029E 01   -0.3569E 01    0.1083E 01   -0.3700E 01
    35     0.9042E-01   -0.6633E 00   0.9650E 00    0.9809E 00     0.9391E 00   -0.3472E 01    0.9732E 00   -0.3540E 01
    40     0.1170E 00   -0.7208E 00   0.9698E 00    0.9865E 00     0.8456E 00   -0.3369E 01    0.8720E 00   -0.3415E 01
    45     0.1464E 00   -0.7655E 00   0.9781E 00    0.9980E 00     0.7498E 00   -0.3251E 01    0.7666E 00   -0.3257E 01
    50     0.1786E 00   -0.7970E 00   0.9821E 00    0.9997E 00     0.6546E 00   -0.3117E 01    0.6665E 00   -0.3118E 01
    55     0.2132E 00   -0.8163E 00   0.9816E 00    0.9973E 00     0.6090E 00   -0.2927E 01    0.6204E 00   -0.2935E 01
    60     0.2500E 00   -0.8247E 00   0.9844E 00    0.9927E 00     0.5231E 00   -0.2776E 01    0.5314E 00   -0.2796E 01
    65     0.2887E 00   -0.8229E 00   0.9855E 00    0.9883E 00     0.4488E 00   -0.2614E 01    0.4554E 00   -0.2645E 01
    70     0.3290E 00   -0.8109E 00   0.9843E 00    0.9806E 00     0.4212E 00   -0.2406E 01    0.4279E 00   -0.2454E 01
    75     0.3706E 00   -0.7876E 00   0.9842E 00    0.9734E 00     0.3543E 00   -0.2233E 01    0.3600E 00   -0.2294E 01
    80     0.4132E 00   -0.7511E 00   0.9828E 00    0.9611E 00     0.3004E 00   -0.2037E 01    0.3056E 00   -0.2119E 01
    85     0.4564E 00   -0.6990E 00   0.9831E 00    0.9535E 00     0.2504E 00   -0.1819E 01    0.2547E 00   -0.1908E 01
    90     0.5000E 00   -0.6298E 00   0.9831E 00    0.9402E 00     0.1972E 00   -0.1584E 01    0.2006E 00   -0.1685E 01
    95     0.5436E 00   -0.5433E 00   0.9838E 00    0.9315E 00     0.1359E 00   -0.1338E 01    0.1381E 00   -0.1436E 01
   100     0.5868E 00   -0.4413E 00   0.9830E 00    0.9253E 00     0.6495E-01   -0.1087E 01    0.6608E-01   -0.1175E 01
   105     0.6294E 00   -0.3280E 00   0.9840E 00    0.9195E 00    -0.2063E-01   -0.8476E 00   -0.2096E-01   -0.9218E 00
   110     0.6710E 00   -0.2098E 00   0.9824E 00    0.9151E 00    -0.1134E 00   -0.6256E 00   -0.1154E 00   -0.6836E 00
   115     0.7113E 00   -0.9417E-01   0.9823E 00    0.9117E 00    -0.1543E 00   -0.3776E 00   -0.1571E 00   -0.4142E 00
   120     0.7500E 00    0.1082E-01   0.9807E 00    0.9064E 00    -0.2195E 00   -0.1945E 00   -0.2239E 00   -0.2146E 00
   125     0.7868E 00    0.9809E-01   0.9919E 00    0.9246E 00    -0.2653E 00   -0.4414E-01   -0.2675E 00   -0.4774E-01
   130     0.8214E 00    0.1625E 00   0.9975E 00    0.9397E 00    -0.2387E 00    0.1198E 00   -0.2394E 00    0.1275E 00
   135     0.8536E 00    0.2016E 00   0.9910E 00    0.9149E 00    -0.2352E 00    0.2013E 00   -0.2373E 00    0.2200E 00
   140     0.8830E 00    0.2161E 00   0.9786E 00    0.8888E 00    -0.2070E 00    0.2530E 00   -0.2116E 00    0.2846E 00
   145     0.9096E 00    0.2094E 00   0.9554E 00    0.8526E 00    -0.1617E 00    0.2774E 00   -0.1692E 00    0.3254E 00
   150     0.9330E 00    0.1867E 00   0.9646E 00    0.8543E 00    -0.1008E 00    0.2859E 00   -0.1045E 00    0.3346E 00
   155     0.9532E 00    0.1545E 00   0.9770E 00    0.8581E 00    -0.3499E-01    0.2815E 00   -0.3581E-01    0.3280E 00
   160     0.9698E 00    0.1186E 00   0.9860E 00    0.8659E 00     0.2855E-01    0.2694E 00    0.2896E-01    0.3111E 00
   165     0.9830E 00    0.8368E-01   0.9899E 00    0.8651E 00     0.8389E-01    0.2527E 00    0.8474E-01    0.2921E 00
   170     0.9924E 00    0.5233E-01   0.9947E 00    0.8729E 00     0.1257E 00    0.2308E 00    0.1264E 00    0.2644E 00
   175     0.9981E 00    0.2492E-01   0.9929E 00    0.8701E 00     0.1425E 00    0.1924E 00    0.1435E 00    0.2211E 00
   180     0.1000E 01    0.1578E-07   0.            0.            -0.6230E 00   -0.6230E 00   -0.6230E 00   -0.6230E 00
T AND X=STATIONS ALONG CHORD WHERE X=.5(1-COS(T))
GSTAR =CIRCULATION DISTRIBUTION
NCO AND NCI=NONLINEAR CORRECTION OUTSIDE AND INSIDE    DUCT RESPECTIVELY
PO AND PI=NONLINEAR PRESSURE DISTRIBUTION OUTSIDE AND  INSIDE THE DUCT RESPECTIVELY FOR PA=0 DEGREES
POP AND PIP =   LINEAR PRESSURE DISTRIBUTION OUTSIDE   AND INSIDE THE DUCT RESPECTIVELY FOR PA=0
```

APPENDIX B

FORTRAN LISTING OF COMPUTER PROGRAM

```
CX14
C     CONTROL PROGRAM
      DIMENSION SSM(181),SCM(181),CT(181),ST(181),CTH(181),STH(181),G(40
     1,20),SM(182),APM(20,20),F(20),CB(40,20),B(60,20),C(20,20),FF(181),
     2A(20
     2),G1(181),W(181),SU(181),VG(200),VQ(200),BVQ(181),FII(20),DIMM(20)
     3,FIMM(20),AX(100),BX(100),RY(200),GX(200),Y(181),S(181),TITLE(12),
     4X(181),XX(181),XC(40,2),XT(40,2),XSC(40,2),AX1(40),BX1(40),AX2(40)
     5,BX2(40),GG(181),GP(181),SMZ(20),XI(20),T1(181),T2(181),Z1(20),Z2(
     620),WR6(20),POL(181),PIL(181),PORL(181),PIRL(181),POO(181),PII(181
     7),RV9(9),TB9(9),GM9(9),G9(9),GP9(9),SM9(10),F9(9),F9(9),UE9(37),RE
     89(37),WE9(37),UP9(37),SP9(37),SE9(37),X9(37),T9(181),AAT(50),XXT(5
     90),ZZT(50),V1(37),V2(37),SCD(181),CXT(181),P(181),R(181)
      DIMENSION IA(6),CA(9),CE(9),CO(9),RC(9),RD(9),GA(9),GC(9),BI(9),
     1        PA(9),PB(9),PTB(9),PQB(9),PXF(9),PYF(9),PTBW(81),PQBW(81),PQ
     2(9),CP(12),DA(9),DB(3),DC(3),DD(3,3),GB(9),GD(9),BBB(9,7),EA(9),EB
     3(9),EC(9),ED(9)            ,H1(81),Q(9),O(81),AH(10,9),AJ(9,9),AL(9),A7(
     49,9),AQ(9,9),AR(9,9),AS(9,9),AT1(9,9),AU(9,9),AV(9,9),AW(9,9),BA(9
     5,9),BB(9,9),BC(9,9),BD(9,9),BE(9,9),BF1(9,9),BH(9,9),CD(10,9),CF(9
     6,1),CG(9,1),CH(9,9),A8(9,9),AZ(9,9),CCC(9,7),BBF(361),BBS(361),CC(
     720,20)
      DIMENSION B2(9),B3(9),B4(72),B5(100),B6(10)
      DIMENSION CCB(40,6),AAB(40,6),AAF(6),ADIM(6,6),AFIM(6,6)
      COMMON   SSM     , SCM     , CT      , ST      , CTH     , STH
      COMMON   G       , SM      , APM     , F       , CB      , B
      COMMON   C       , FF      , A       , G1      , W       , NPP
      COMMON   SU      , VG      , VQ      , BVQ     , FII     , DIMM
      COMMON   FIMM    , AX      , BX      , RY      , GX      , H
      COMMON   I1      , I2      , I3      , DX      , X1      , X3
      COMMON   X4      , NP      , NH      , Y       , S       , TITLE
      COMMON   X       , XX      , XC      , XT      , XSC     , BF
      COMMON   BS      , AK      , A01     , A02     , ALFA    , MP1
      COMMON   GG      , GP      , SMZ     , XI      , T1      , Z1
      COMMON   Z2      , T2      , WK      , AT      , WR6     , CDI
      COMMON   POL     , PIL     , PORL    , PIRL    , POO     , PII
      COMMON   CTD     , ALFAR   , CL      , CM      , CM1     , CM2
      COMMON   PHI     , RV9     , TB9     , GM9     , G9      , GP9
      COMMON   SM9     , F9      , E9      , UE9     , RE9     , WE9
      COMMON   UP9     , SP9     , SE9     , X9      , T9      , RDI
      COMMON   R9      , AST9    , V1      , V2      , SCD     , CXT
      COMMON   P       , R       , AAT     , XXT     , ZZT     , I7
      COMMON   AX1     , AX2     , BX1     , BX2     , IA      , CA
      COMMON   CE      , CO      , RC      , RD      , GA      , GC
      COMMON   BI      , AQ      , PA      , PB      , PTB     , PQB
      COMMON   PXF     , PYF     , PTBW    , PQBW    , PQ      , CP
      COMMON   DA      , DB      , DC      , DD      , GB      , GD
      COMMON   BBB     , EA      , EB      , EC      , ED      , H1
      COMMON   G       , O       , AH      , AJ      , AL      , A7
      COMMON   AR      , AS      , AT1     , AU      , AV      , AW
      COMMON   BA      , BB      , BC      , BD      , BE      , BF1
      COMMON   BH      , CD      , CF      , CG      , CH      , A8
      COMMON   AZ      , CCC     , CN      , BBF     , BBS     , CC
      COMMON   ID      , WW      , C8
      COMMON B2,B3,B4,B5,B6
      COMMON CCB,AAB,AAF,ADIM,AFIM
```

```
      I1=1
      I3=181
      I2=5
      NH=36
      NPP=5
      MP1=5
      IN=0
      IM1=0
      DO 9999 I=I1,I3,I2
      IN=IN+1
      IF(I-1) 9992,9993,9992
 9992 IM1=IM1+I2
 9993 T=FLOAT(IM1)/57.29578
      TH=.5*T
      CTH(I)=COS(TH)
      STH(I)=SIN(TH)
      CT(I)=COS(T)
9999  ST(I)=SIN(T)
  103 FORMAT(1H1)
223   IN=0
      DO 2901 I=1,181,5
      T1(I)=.0
      T2(I)=.0
      IN=IN+1
      AX1(IN)=.0
      AX2(IN)=.0
      BX2(IN)=.0
      BBF(IN)=.0
      BBS(IN)=.0
      XX(IN)=.5*(1.0-CT(I))
 2901 X(IN)=FLOAT(I-1)
      READ (5,21993)TITLE
      WRITE (6,21990)TITLE
      READ (5,21998)AIT,BIT
      WRITE(6,206)AIT,BIT
      NIT=AIT
      READ(5,21998)C5,TT,H,ALFA,AT,WK,WW,YY
      WRITE(6,206   )C5,TT,H,ALFA,AT,WK,WW,YY
 96   READ (5,21998)R9,AST9,DF,GTR9
      WRITE (6,206 )R9,AST9,DF,GTR9
      VG(200)=-8.*H*DF
      NC=C5
      NT=TT
      M=NC/2
21993 FORMAT(12A6)
21990 FORMAT(1H1,12A6)
21998 FORMAT(9F8.6)
      IF(C5) 2601,2602,2601
 2601 READ (5,21998)(X(I),I=1,NC)
      READ (5,21998)(Y(I),I=1,NC)
      WRITE (6,206 )(X(I),I=1,NC)
      WRITE (6,206 )(Y(I),I=1,NC)
      DO 2599 I=1,37
      S1=XX(I)
      CALL DISCOT(S1,S1,X,Y,Y,-120,NC,0,S2)
      V1(I)=S2
```

```
 2599 T1(I)=S2
 2602 READ (5,21998)(X(I),I=1,NT)
      READ (5,21998)(Y(I),I=1,NT)
      WRITE (6,206 )(X(I),I=1,NT)
      WRITE (6,206 )(Y(I),I=1,NT)
      DO 2598 I=1,37
      S1=XX(I)
      CALL DISCOT(S1,S1,X,Y,Y,-120,NT,0,S2)
      V2(I)=S2
      UP9(I)=V1(I)+V2(I)
      SP9(I)=V1(I)-V2(I)
 2598 T2(I)=S2
      MV=WK
      IF(WK)2604,2604,2603
2603  READ(5,21998)(X(I),I=1,MV)
      READ(5,21998)(Y(I),I=1,MV)
      WRITE(6,206  )(X(I),I=1,MV)
      WRITE(6,206  )(Y(I),I=1,MV)
      DO 2605 I=1,37
      S1=XX(I)
      CALL DISCOT(S1,S1,X,Y,Y,-120,MV,0,S2)
2605  BX2(I)=S2
2604  MV=WW
      IF(WW)2610,2610,2606
2606  READ(5,21998)(X(I),I=1,MV)
      READ(5,21998)(Y(I),I=1,MV)
      WRITE(6,206  )(X(I),I=1,MV)
      WRITE(6,206  )(Y(I),I=1,MV)
      DO 2607 I=1,37
      S1=XX(I)
      CALL DISCOT(S1,S1,X,Y,Y,-120,MV,0,S2)
2607  AX2(I)=S2
2610  MV=YY
      IF(YY)2611,2611,2608
2608  READ(5,21998)(X(I),I=1,MV)
      READ(5,21998)(Y(I),I=1,MV)
      WRITE(6,206  )(X(I),I=1,MV)
      WRITE(6,206  )(Y(I),I=1,MV)
      DO 2609 I=1,37
      S1=XX(I)
      CALL DISCOT(S1,S1,X,Y,Y,-120,MV,0,S2)
2609  BX2(I)=S2
2611  CONTINUE
      WK=1.0
      WW=1.0
 3000 FORMAT(3E13.4)
21005 FORMAT(I10,4E15.6)
21000 FORMAT(I8,4F8.6)
      F9=.0
      CDI=.0
  802 READ (5,3)
      READ (5,4)((BBB(I,J),I=1,9),J=1,6)
      IF(BBB(2,1))402,8021,8022
 8021 READ (5,4)(BBB(I,7),I=1,9)
      GO TO 401
 8022 DO 8023 I=1,9
```

```
8023  BBB(I,7)=BBB(2,1)
      WRITE (6,3)
      WRITE (6,206)((BBB(I,J),I=1,9),J=1,7)
402   BX1(40)=BBB(1,1)
      BX1(39)=BBB(2,1)
      BX1(38)=BBB(3,1)
      BX1(37)=BBB(4,1)
      IF(BX1(40))1000,1000,1005
1005  GO TO 1004
1000  IF (BX1(37))1004,1004,1001
1001  DO 1002 I=1,9
1002  P(I)=(2./(1.-BBB(1,3)**2))*(1.-BBB(I,4))*BBB(I,3)
      RSL=BBB(7,2)/(3.1416*BBB(5,2)*BBB(3,2))
      VAL=SIMPUN(BBB(I,3),P(I),9)
      WO=1.-VAL
      DO 1003 I=1,9
1003  BBB(I,5)=RSL*SQRT(WO)*SQRT(BBB(I,4))/(BBB(I,3)*BBB(3,1))
1004  BBB(4,1)=BX1(39)
      BBB(1,1)=.95
      BBB(2,1)=1.0
      BBB(3,1)=1.05
      BBB(5,1)=BX1(38)
8020  DO 842 I=1,9
842   AX1(I)=BBB(I,4)
      DO 841 I7=1,NIT
      I1=1
      I3=181
      I2=5
      NH=36
      NPP=5
      MP1=5
      IN=0
      IM1=0
      IN=0
      DO 22222 I=1,181,5
      W(I)=.0
      IN=IN+1
      BBF(IN)=.0
      X(IN)=FLOAT(I-1)
22222 XX(IN)=.5*(1.0-CT(I))
224   IN=0
      DO 750 I=1,181,5
      IN=IN+1
 750  X9(IN)=2.0*H*(XX(IN)-AST9)
      DO 500 I=1,9
      RV9(I)=BBB(I,3)
      DO 500 J=1,9
 500  CCC(I,J)=BBB(I,J)
      B9=CCC(9,2)
      XH9=CCC(1,3)
      CDI=(1.0-BIT)*CDI
      NN=1
    3 FORMAT(72H
     1                             )
    4 FORMAT(9F8.4)
  401 DO 200 I=1,3
```

```
  200 B(I+11,1)=BBB(I,1)
      IF(BBB(5,1))2001,2001,2000
 2000 READ (5,4)(CO(I),I=1,9)
 2001 B(5,1)=BBB(10,1)
      DO 201 I=1,4
      B(I,1)=BBB(I+10,1)
  201 CONTINUE
      DO 202 I=1,3
      B(I+5,1)=BBB(I+14,1)
  202 CONTINUE
      B(10,1)=BBB(18,1)
      DO 203 I=1,18
  203 B(I+14,1)=BBB(I+18,1)
      DO 204 I=1,9
      B(I+32,1)=BBB(I,6)
      B(I+41,1)=BBB(I,7)
  204 B(I+50,1)=BBB(I+36,1)
      B(9,1)=BBB(5,1)
      DO 2041 I=1,9
      B2(I)=B(I+14,1)
2041  B3(I)=B(I+50,1)
      WRITE(6,103)
  650 FORMAT(1H1,26X,69HAML PROBLEM 106H MODERATELY LOADED PROPELLERS US
     1ING INDUCTION FACTORS,12X,5HPAGE I2)
  205 FORMAT(4H    )
 2050 FORMAT(54X,10HINPUT DATA)
  206 FORMAT(9F12.4)
    9 ECC=B(10,1)
      BBB(4,1)=-1.0
      BBB(5,1)=0.0
      CCC(4,1)=-1.
      CCC(5,1)=.0
      PM=B(12,1)
      KK=0
      BI(1)=3.0
      BJ=0.0
      BJ1=0.0
      BJ2=0.0
      J1=0
      ZZ=B(51,1)
   12 KK=KK+1
      IF(BX1(40))1200,1200,120
120   IN=0
      DO 121 I=1,181,5
      IN=IN+1
121   FF(IN)=.5*(1.+B(15,1))-.5*(1.-B(15,1))*COS((FLOAT(I-1)/57.2957))
      DO 122  I=1,37
      S1=FF(I)
      CALL DISCOT(S1,S1,B2,B3,B3,-120,9,0,S2)
122   B4(I)=S2
      B4(1)=0.0
      B4(37)=0.0
      DO 123I=1,35
      N1=37+I
      N2=37-I
123   B4(N1)=-B4(N2)
```

```
      CALL GMHAS(72,10,B4,B5)
      DO 124 I=1,10
      J=I*5
124   B6(I)=B5(J+2)
1200  CALL SUB
      IF(BX1(40))1201,1201,1240
1240  DO 1241 I=1,9
1241  BBB(I+36,1)=B(I+50,1)
      BX1(40)=.0
      GO TO 8020
1201  CP(1)=B(12,1)
      CALL SUB
      CP(1)=B(12,1)
      B(12,1)=B(13,1)
      CP(5)=CM
      CP(9)=CN
      CALL SUB
      CP(2)=B(12,1)
      CP(6)=CM
      CP(10)=CN
      B(12,1)=B(14,1)
      CALL SUB
      CP(3)=B(12,1)
      CP(7)=CM
      CP(11)=CN
      DA(1)=1.0
      DA(4)=1.0
      DA(7)=1.0
      DA(2)=CP(1)
      DA(5)=CP(2)
      DA(8)=CP(3)
      DA(3)=CP(1)**2
      DA(6)=CP(2)**2
      DA(9)=CP(3)**2
      IF(B(60,1)-B(5,1))700,710,710
  700 DB(1)=CP(5)
      DB(2)=CP(6)
      DB(3)=CP(7)
      DE=B(6,1)*B(1,1)/((B(8,1)*B(2,1)**2*3.1415927*B(7,1)**2)/8.0)
      DE=DE-CDI
      GO TO 701
  710 DB(1)=CP(9)
      DB(2)=CP(10)
      DB(3)=CP(11)
      DE=550.0*B(6,1)*B(1,1)/((B(8,1)*B(2,1)**2*3.1415927*B(7,1)**3)/8.0
     1)
  701 DO711I=1,3
      DO711J=1,3
      K=3*(J-1)+I
      DD(J,I)=DA(K)
  711 CONTINUE
      D=1.0
      DO 715 I=1,3
      DO 715 J=1,3
      C(I,J)=DD(I,J)
715   CC(I,1)=DB(I)
```

```
      CALL MATINV(C,3,CC,1,Q,ID)
      DO 714 J=1,3
714   DD(J,1)=CC(J,1)
      IF(ID-1)713,712,713
713   WRITE (6,716)
  716 FORMAT(11H MATRIX SIN)
      STOP
712   XYZ=DD(2,1)**2-4.*DD(3,1)*(DD(1,1)-DE)
      IF(XYZ)813,812,812
813   XYZ=-XYZ
812   CP(4)=(-DD(2,1)+SQRT(XYZ))/(2.*DD(3,1))
      B(12,1)=CP(4)
      CALL SUB
      DO725I=1,9
      GD(I)=ATAN(F(I))
  725 CONTINUE
      DO726I=1,9
      GD(I)=COS(GD(I))
  726 CONTINUE
      DO727I=1,9
      GA(I)=2.0*3.1415927*CE(I)*GD(I)/(1.0/CO(I)-CB(I,1))
  727 CONTINUE
      DO728I=1,9
      GB(I)=B(3,1)-(B(I+14,1)*B(2,1))/2.0
  728 CONTINUE
      DO729I=1,9
      GD(I)=ARCOS(GD(I))
  729 CONTINUE
      DO730I=1,9
      BBB(I+36,1)=CO(I)
      CO(I)=ATAN(CO(I))
      PA(I)=COS(CO(I))
  730 PB(I)=SIN(CO(I))
      DO731I=1,9
      GC(I)=64.31*GB(I)*(SIN(CO(I))/(B(I+23,1)*B(7,1)*COS(GD(I)-CO(I))))
     1**2
  731 CO(I)=BBB(I+36,1)
      WRITE (6,501)
  501 FORMAT(1H )
      WRITE (6,501)
      WRITE (6,502)B(12,1),CM,CN
  502 FORMAT(8H      K=1PE10.4,8H   CTSI=1PF10.4,8H   CPSI=1PE10.4)
      WRITE (6,501)
      WRITE (6,503)
  503 FORMAT(118H    XI          TAN BI         TAN B          GS         UT/2V
     1S        UA/2VS       DCTSI        DCPSI        CLL/D        SIGMA X )
      DO 504I=1,9
      CE(I)=CE(I)*B(I+23,1)
      CB(I,1)=CB(I,1)*B(I+23,1)
  504 CA(I)=CA(I)*B(I+23,1)
      WRITE (6,501)
      DO 505 I=1,9
  505 WRITE (6,506)B(I+14,1),F(I),CO(I),CE(I),CB(I,1),CA(I),RC(I),RD(I),
     1GA(I), GC(I)
  506 FORMAT(1P10E12.4)
      DO 7310 I=1,9
```

```
      EA(I)=GA(I)/BBB(I,6)
      EB(I)=BBB(I,7)/EA(I)
      EC(I)=((1.0-EB(I)*F(I))*B(I+23,1))*RC(I)
 7310 ED(I)= (1.0+EB(I)/F(I))*RD(I)
      VALUE=SIMPUN(BI,EC,9)
      CTT=VALUE
      VALUE=SIMPUN(BI,ED,9)
      CCPT=VALUE
      EPT=CTT/CCPT
      DO 900 I=1,9
  900 EC(I)=(1.0-EB(I)*F(I))*RC(I)
      VALUE=SIMPUN(BI,EC,9)
      PPT=VALUE
      WRITE (6,7311)
      WRITE (6,7311)
      WRITE (6,7311)
 7311 FORMAT(4H    )
      WRITE (6,7312)CTT,CCPT,EPT,PPT
 7312 FORMAT(9H      CPT=1PE10.4,9H      CPS=1PE10.4,8H      EP=1PE10.4,9H
     1     CTS=1PE10.4)
      WRITE (6,7311)
      WRITE (6,7311)
      WRITE (6,7313)
 7313 FORMAT(8X,2HXI,8X,7HEPSILON,5X,3HMTB,8X,3HMQB,8X,3HMXO,10X,3HMYO,1
     11X,1HM,10X,4HG(M))
      WRITE (6,7311)
      WRITE (6,7311)
      DO 8030 I=1,9
      DO 8030 J=1,9
      K=9*(I-1)+J
      PTBW(K)=(B(J+14,1)-B(I+14,1))*(1.0-EB(J)*F(J))*RC(J)
 8030 PQBW(K)=((B(J+14,1)-B(I+14,1))/B(J+14,1))*(1.0+EB(J)/F(J))*RD(J)
      PTB(1)=SIMPUN(BI(1),PTBW(1),9)
      PQB(1)=SIMPUN(BI(1),PQBW(1),9)
      PTB(2)=0.0
      PQB(2)=0.0
      PTB(3)=SIMPUN(BI(3),PTBW(21),7)
      PQB(3)=SIMPUN(BI(3),PQBW(21),7)
      PTB(4)=0.0
      PQB(4)=0.0
      PTB(5)=SIMPUN(BI(5),PTBW(41),5)
      PQB(5)=SIMPUN(BI(5),PQBW(41),5)
      PTB(6)=0.0
      PQB(6)=0.0
      PTB(7)=SIMPUN(BI(7),PTBW(61),3)
      PQB(7)=SIMPUN(BI(7),PQBW(61),3)
      PTB(8)=0.0
      PQB(8)=0.0
      PTB(9)=0.0
      PQB(9)=0.0
      PTBC=(B(8,1)*B(2,1)**3*3.14159265*B(7,1)**2)/(16.0*B(10,1))
      PQBC=B(8,1)*B(2,1)**2*B(7,1)**3/(16.0*B(4,1)*B(10,1))
      DO 8033 I=1,9
      PQ(I)=FLOAT(I)
      PTB(I)=PTB(I)*PTBC
 8033 PQB(I)=PQB(I)*PQBC
```

```
      DO 8034 I=1,9
      PXF(I)=PTB(I)*PA(I)+PQB(I)*PB(I)
 8034 PYF(I)=PTB(I)*PB(I)-PQB(I)*PA(I)
      DO 7314 I= 1,9
 7314 WRITE (6,7315)B(I+14,1),EB(I),PTB(I),PQB(I),PXF(I),PYF(I),I,AQ(I,1
     1)
 7315 FORMAT(1P6E12.4,6X,I1,5X,1PE12.4)
      CALL SUB F
      DO 601 I=1,9
      F9(9)=F9(8)
      TB9(I)=F(I)
      CCC(I,5)=TB9(I)
 601  GM9(I)=AQ(I,1)
      WRITE (6,103)
      CALL SUB HA
      CALL EL
      CALL SUB WR
 841  CONTINUE
      GO TO 1603
26009 FORMAT(7E15.6)
 2102 FORMAT(I10,5E15.6)
  101 FORMAT(8F9.6)
  100 FORMAT(5I4,4F10.6/18I4)
 1752 FORMAT(8F10.6)
 1758 FORMAT(55H  INDUCED VELOCITY INSIDE THE DUCT AT A DUCT RADIUS OF
     1E11.4//55H      1-WP              WD               1-W                 )
  801 FORMAT(8E15.6)
1603  CONTINUE
803   CONTINUE
1668  STOP
      END
```

```
  SUBROUTINE EL
 DIMENSION SSM(181),SCM(181),CT(181),ST(181),CTH(181),STH(181),G(40
1,20),SM(182),APM(20,20),F(20),CB(40,20),B(60,20),C(20,20),FF(181),
2A(20
2),G1(181),W(181),SU(181),VG(200),VQ(200),BVQ(181),FII(20),DIMM(20)
3,FIMM(20),AX(100),BX(100),RY(200),GX(200),Y(181),S(181),TITLE(12),
4X(181),XX(181),XC(40,2),XT(40,2),XSC(40,2),AX1(40),BX1(40),AX2(40)
5,BX2(40),GG(181),GP(181),SMZ(20),XI(20),T1(181),T2(181),Z1(20),Z2(
620),WR6(20),POL(181),PIL(181),PORL(181),PIRL(181),POO(181),PII(181
7),RV9(9),TB9(9),GM9(9),G9(9),GP9(9),SM9(10),F9(9),E9(9),UE9(37),RE
89(37),WE9(37),UP9(37),SP9(37),SE9(37),X9(37),T9(181),AAT(50),XXT(5
90),ZZT(50),V1(37),V2(37),SCD(181),CXT(181),P(181),R(181)
 DIMENSION IA(6),CA(9),CE(9),CO(9),RC(9),RD(9),GA(9),GC(9),BI(9),
1       PA(9),PB(9),PTB(9),PQB(9),PXF(9),PYF(9),PTBW(81),PQBW(81),PQ
2(9),CP(12),DA(9),DB(3),DC(3),DD(3,3),GB(9),GD(9),BBB(9,7),EA(9),EB
3(9),EC(9),ED(9)          ,H1(81),Q(9),O(81),AH(10,9),AJ(9,9),AL(9),A7(
49,9),AQ(9,9),AR(9,9),AS(9,9),AT1(9,9),AU(9,9),AV(9,9),AW(9,9),BA(9
5,9),BB(9,9),BC(9,9),BD(9,9),BE(9,9),BF1(9,9),BH(9,9),CD(10,9),CF(9
6,1),CG(9,1),CH(9,9),A8(9,9),AZ(9,9),CCC(9,7),BBF(361),BBS(361),CC(
720,20)
 DIMENSION B2(9),B3(9),B4(72),B5(100),B6(10)
 DIMENSION CCB(40,6),AAB(40,6),AAF(6),ADIM(6,6),AFIM(6,6)
 COMMON  SSM    , SCM   , CT    , ST    , CTH   , STH
 COMMON  G      , SM    , APM   , F     , CB    , B
 COMMON  C      , FF    , A     , G1    , W     , NPP
 COMMON  SU     , VG    , VQ    , BVQ   , FII   , DIMM
 COMMON  FIMM   , AX    , BX    , RY    , GX    , H
 COMMON  I1     , I2    , I3    , DX    , X1    , X3
 COMMON  X4     , NP    , NH    , Y     , S     , TITLE
 COMMON  X      , XX    , XC    , XT    , XSC   , BF
 COMMON  BS     , AK    , AO1   , AO2   , ALFA  , MP1
 COMMON  GG     , GP    , SMZ   , XI    , T1    , Z1
 COMMON  Z2     , T2    , WK    , AT    , WR6   , CDI
 COMMON  POL    , PIL   , PORL  , PIRL  , POO   , PII
 COMMON  CTD    , ALFAR , CL    , CM    , CM1   , CM2
 COMMON  PHI    , RV9   , TB9   , GM9   , G9    , GP9
 COMMON  SM9    , F9    , E9    , UE9   , RE9   , WE9
 COMMON  UP9    , SP9   , SE9   , X9    , T9    , RDI
 COMMON  R9     , AST9  , V1    , V2    , SCD   , CXT
 COMMON  P      , R     , AAT   , XXT   , ZZT   , I7
 COMMON  AX1    , AX2   , BX1   , BX2   , IA    , CA
 COMMON  CE     , CO    , RC    , RD    , GA    , GC
 COMMON  BI     , AQ    , PA    , PB    , PTB   , PQB
 COMMON  PXF    , PYF   , PTBW  , PQBW  , PQ    , CP
 COMMON  DA     , DB    , DC    , DD    , GB    , GD
 COMMON  BBB    , EA    , EB    , EC    , ED    , H1
 COMMON  Q      , O     , AH    , AJ    , AL    , A7
 COMMON  AR     , AS    , AT1   , AU    , AV    , AW
 COMMON  BA     , BB    , BC    , BD    , BE    , BF1
 COMMON  BH     , CD    , CF    , CG    , CH    , A8
 COMMON  AZ     , CCC   , CN    , BBF   , BBS   , CC
 COMMON  ID     , WW    , C8
 COMMON B2,B3,B4,B5,B6
 COMMON CCB,AAB,AAF,ADIM,AFIM
```

```
      IF(I7-1)8000,8000,8003
8000  X1=.0
      J=0
      IN=0
      DO 1 I=I1,I3,I2
      IN=IN+1
      IF(I-1)92,93,92
   92 J=J+I2
   93 T=FLOAT(J)/57.29578
      DO 90 K=1,MP1
      M=K
      NP=NPP+M
      SLT=-CBRT(3.1416-T)
      ULT=CBRT(T)
      PR=GAUSS(SLT,ULT,NP,DT)
      AK=SQRT(4.0/(H**2*(CT(I)-COS(T-DT**3))**2+4.0))
      IF(AK-1.0) 2,10,10
2     IF(AK-.99)8,3104,3104
    8 CALL ELLIP(X1,AK,X3,X4,BF,BS)
      GO TO 7
3104  AK2=AK**2
      CALL YSELL(AK2,BS,BF)
      GO TO 7
10    BF=.0
      BS=.0
    7 PR=DT**2*AK*(BF-BS)*COS(FLOAT(M)*(T-DT**3))
   90 G(IN,K)=PR
1     NP=NPP
      DX=FLOAT(I2)/171.88734
      DO 1801 I=2,I3,2
      SM(I)=4.0
 1801 SM(I+1)=2.0
      SM(1)=1.0
      SM(I3)=1.0
      DO  3102 M=1,MP1
      AP=.0
      IN=0
      DO 300 I=I1,I3,I2
      IN=IN+1
  300 AP=AP+G(IN,M)*SM(I)
      APM(1,M)=AP*DX/3.1416
      F(M)=6.0*H*FLOAT(M)*APM(1,M)/3.1416
3102  AAF(M)=F(M)
      DO 400 IP=2,MP1
      DO 400 M=1,MP1
      AP=.0
      IN=0
      DO 11500 I=I1,I3,I2
      IN=IN+1
11500 AP=AP+G(IN,M)*SM(I)*COS(FLOAT((IP-1)*(I-1))/57.29578)
  400 APM(IP,M)=.636618*AP*DX
      IN=0
      DO 3100 I=I1,I3,I2
      IN=IN+1
      DO 3100 M=1,MP1
      SUM=.0
```

```
      DO 3101 IP=2,MP1
3101  SUM=SUM+APM(IP,M)*SIN(FLOAT((I-1)*(IP-1))/57.29578)
      CB(IN,M)=-6.0*H*FLOAT(M)*SUM/3.1416
3100  CCB(IN,M)=CB(IN,M)
      IN=0
      DO 600 I=I1,I3,I2
      IN=IN+1
      DO 611 K=1,MP1
      M=K-1
      NP=NPP+M
      PR=GAUSS(0.0,3.1416,NP,DT)
      CMC=COS(DT)-CT(I)
      IF(CMC) 699,610,699
  699 AK=SQRT(4.0/(H**2*CMC**2+4.0))
      IF(AK-1.0) 602,610,610
  602 IF(AK-.99) 608,609,609
  608 CALL ELLIP(X1,AK,X3,X4,BF,BS)
      GO TO 607
  609 AK2=AK**2
      CALL YSELL (AK2,BS,BF)
      GO TO 607
  610 FX=0.0
      GO TO 660
  607 FX=(1.0/CMC)*(2.0+AK*(H**2*CMC**2*(BF-BS)-2.0*BS))
  660 IF(M) 666,666,555
  555 FX=FX*COS(FLOAT(M)*DT)
  666 PR=FX
      IF(M) 777,777,888
  777 B(IN,K)=.319308*PR
      AAB(IN,K)=B(IN,K)
      GO TO 611
  888 B(IN,K)=.636618*PR
      AAB(IN,K)=B(IN,K)
  611 CONTINUE
600   NP=NPP
      DO 1301 K=1,MP1
      FI=.0
      IN=0
      DO 1201 I=I1,I3,I2
      IN=IN+                                       1
 1201 FI=FI+B(IN,K)*CTH(I)**2*SM(I)*DX
 1301 FII(K)=FI
      DO 8004 K=1,MP1
      DO 1500 M=1,MP1
      DIM=.0
      FIM=.0
      IN=0
      DO 1400 I=I1,I3,I2
      IN=IN+1
      DIM=DIM+B(IN,K)*ST(I)*SM(I)*DX*SIN(FLOAT((I-1)*M)/57.29578)
 1400 FIM=FIM+B(IN,K)*CB(IN,M)*ST(I)*SM(I)*DX
      DIMM(M)=DIM
      ADIM(K,M)=DIM
      AFIM(K,M)=FIM
 1500 FIMM(M)=FIM
8004  CONTINUE
```

```
8001  DO 8002  I=1,40
      DO 8002 K=1,MP1
8002  B(I,K)=AAB(I,K)
      DX=FLOAT(I2)/171.88734
      C11=.0
      C12=.0
      IN=0
      DO 910 I=I1,I3,I2
           IN    =   IN+1
      C11=C11+B(IN ,1)*(1.0+CT(I))*SM(I)
  910 C12=C12+B(IN
     1              ,1)*((ST(I))**2)*SM(I)
      C(1,1)=-.159154*C11*DX
      C(1,2)=.159154*C12*DX
      DO 911 J=3,MP1
      XC1=.0
      IN=0
      DO 912 I=I1,I3,I2
      IN=IN+1
912   XC1=XC1+B(IN,1)*ST(I)*SM(I)*SIN(FLOAT((I-1)*(J-1))/57.29578)
  911 C(1,J)=.159154*XC1*DX
      DO 920 K=2,MP1
      XCK1=.0
      IN=0
      DO 913 I=I1,I3,I2
      IN=IN+1
  913 XCK1=XCK1+B(IN,K)*(1.0+CT(I))*SM(I)
  920 C(K,1)=-.159154*XCK1*DX
      DO 921 K=2,MP1
      DO 921 J=2,MP1
      IN=0
      XC(1,1)=.0
      DO 922 I=I1,I3,I2
      IN=IN+1
922   XC(1,1)=XC(1,1)+B(IN,K)*ST(I)*SM(I)*SIN(FLOAT((I-1)*(J-1))/57.2957
     18)
  921 C(K,J)=.159154*XC(1,1)*DX
      IF(AT) 1312,1313,1312
 1312 C(1,1)=1.0
      DO 1314 K=2,MP1
 1314 C(K,1)=0.0
 1313 DO 1100 K=1,MP1
      DO 1100 J=1,MP1
      IF(K-J) 1102,1101,1102
 1101 C(K,J)=1.0-C(K,J)
      AAB(K,J)=C(K,J)
      GO TO 1100
 1102 C(K,J)=-C(K,J)
      AAB(K,J)=C(K,J)
 1100 CONTINUE
8003  ALFA=ALFA/57.29578
      DO 8010 I=1,MP1
      DO 8010 J=1,MP1
8010  C(I,J)=AAB(I,J)
      DO 1405 K=1,MP1
      S4=2.0*(SIN(ALFA)/COS(ALFA)+SCM(1))*FII(K)
```

```
      S1=.0
      S2=.0
      S3=.0
      DO 1404 M=1,MP1
      DIMM(M)=ADIM(K,M)
      FIMM(M)=AFIM(K,M)
      F(M)=AAF(M)
      S1=S1+SSM(M)*F(M)*FII(K)
      S2=S2+SCM(M+1)*DIMM(M)
 1404 S3=S3+SSM(M)*FIMM(M)
      IF(AT)1318,1333,1318
 1318 A(K)=-2.0*S2+S3
      A(1)=0.0
      GO TO 1405
 1333 A(K)=S1-S2+.5*S3+S4
 1405 CONTINUE
      IF(AT)2200,3103,2200
 2200 MP1=MP1-1
      DO 3333 K=1,MP1
      DO 3333 J=1,MP1
      C(K,J)=C(K+1,J+1)
 3333 A(K)=A(K+1)
      MP1=MP1+1
3103  DO 718 I=1,MP1
718   CC(I,1)=A(I)
      CALL MATINV(C,MP1,CC,1,Q,ID)
      IN=0
      DO 1300 I=I1,I3,I2
      IN=IN+1
      SUM1=.0
      SUM2=.0
      SUM3=.0
      DO 1200 M=1,MP1
      F(M)=AAF(M)
      CB(IN,M)=CCB(IN,M)
      SUM1=SUM1+SSM(M)*F(M)
      SUM2=SUM2+SCM(M+1)*SIN(FLOAT((I-1)*M)/57.29578)
 1200 SUM3=SUM3+SSM(M)*CB(IN,M)
      IF(AT) 1316,1323,1316
 1316 FF(I)=-2.0*SUM2+SUM3
      GO TO 1300
 1323 FF(I)=((2.0*SIN(ALFA)/COS(ALFA)+2.0*SCM(1)+SUM1)*CTH(I)+STH(I)*(-2
     1.0*SUM2+SUM3))
 1300 CONTINUE
      DO 717 I=1,MP1
717   A(I)=CC(I,1)
      IF(ID-1) 1601,1600,1601
 1601 WRITE (6,88)
   88 FORMAT(14H C IS SINGULAR)
      GO TO 1603
 1600 IF(AT)1660,1670,1660
 1670 G1(1)=FF(1)-.318309*A(1)
      IN=1
      I9=I1+I2
      DO 1700I=I9,I3,I2
      IN=IN+1
```

```
      SUM=.0
      DO 1701 J=2,MP1
1701  SUM=SUM+.318309*STH(I)*A(J)*SIN(FLOAT((I-1)*(J-1))/57.29578)
 1700 G1(I)=FF(I)-.318309*CTH(I)*A(1)+SUM
29993 NH=36
      NX=0
      DO 20001 I=I1,I3,I2
      NX=NX+1
20001 GX(NX)=G1(I)
      NN=2*NX-2
      KX=NX-2
      DO 20002 KK=1,KX
      NX1=NX+KK
      NX2=NX-KK
20002 GX(NX1)=+GX(NX2)
      CALL GMHAS(NN,NH,GX,RY)
      A0=RY(1)
      B0=RY(2)
      DO 20003 J=1,NH
      JFIX=5*J+1
      AX(J)=RY(JFIX)
20003 BX(J)=RY(JFIX+1)
      IN=0
      J=0
      DO 1900 I=I1,I3,I2
      XU=.0
      IN=IN+1
      DO 1901 M=1,MP1
1901  XU=XU+FLOAT(M)*SSM(M)*COS(FLOAT((I-1)*M)/57.29578)
 1900 SU(I)=XU
      IN=0
      DO 20007 IB=I1,I3,I2
      IN=IN+1
      IF(IB-1)20009,20008,20009
20009 J=J+I2
20008 T=FLOAT(J)/57.29578
      SLT=-CBRT(T)
      ULT=CBRT(3.1416-T)
      PR=GAUSS(SLT,ULT,NP,DT)
      AK=SQRT(4.0/(H**2*(CT(IB)-COS(DT**3+T))**2+4.0))
      IF(AK-1.0) 20011,20012,20012
20011 IF(AK-.99) 20014,20013,20013
20014 CALL ELLIP(X1,AK,X3,X4,BF,BS)
      GO TO 20015
20013 AK2=AK**2
      CALL YSELL(AK2,BS,BF)
      GO TO          2         0        015
20012 BS=.0
      BF=.0
20015 Y1=.0
      Y2=.0
      DO 20016 K=1,NH
      ZK=FLOAT(K)*(DT**3+T)
      Y1=Y1+AX(K)*COS(ZK)
20016 Y2=Y2+BX(K)*SIN(ZK)
      Y3=A0+Y1+B0+Y2
```

```
      PR=Y3*AK*(BF-BS)*COS(.5*(DT**3+T))*3.0*DT**2
       VG(IB)=.159154*H*PR
      DO 1807 I=I1,I3,I2
      AK=SQRT(4.0/(H**2*(CT(IB)-CT(I))**2+4.0))
      IF(AK-1.0)1802,1810,1810
 1802 IF(AK-.99)1808,1809,1809
 1808 CALL ELLIP(X1,AK,X3,X4,BF,BS)
      GO TO 1807
 1809 AK2=AK**2
      CALL YSELL(AK2,BS,BF)
      GO TO 1807
 1810 BS=1.0
 1807 BVQ(I)=SU(I)*AK*BS
      GXO=.0
      DO 6666 I=I1,I3,I2
 6666 GXO=GXO+BVQ(I)
     1             *SM(I)*DX/3.1416
      DO 6662 M=1,MP1
      GXX=.0
      INN=0
      DO 6661 I=I1,I3,I2
      INN=INN+1
6661  GXX=GXX+2.0*BVQ(I)*SM(I)*DX/3.1416*COS(FLOAT((I-1)*M)/57.29578)
 6662 GX(M)=GXX
      SUM10=.0
      SUM9=.0
      SUM8=.0
      DO 11905 M=1,MP1
      SUM10=SUM10+GX(M)*SIN(FLOAT((IB-1)*M)/57.29578)
      SUM9=SUM9-FLOAT(M)*GX(M)*2.0
11905 SUM8=SUM8+2.0*(-1.0**M)*FLOAT(M)*GX(M)
      IF(IB-1) 11912,11908,11912
11912 IF(IB-181) 11907,11906,11907
11907 WQ=(-2.0/ST(IB))*(GXO+SUM10)
      GO TO 20007
11908 WQ=SUM9
      GO TO 20007
11906 WQ=SUM8
20007 VQ(IB)=WQ
      IN=0
20000 DO 20020 IB=I1,I3,I2
      IN=IN+1
      IF(IB-1)20021,20020,20021
20021 HV6=VG(IB)+T2(IB)
                            G1STH=.5*G1(IB)/STH(IB)
      WGO=HV6-G1STH
      WGI=HV6+G1STH
      WQQ=         VQ(IB)
      WGG=VG(IB)
      PO=2.0*(WGO+WQQ)
      PI=2.0*(WGI+WQQ)
      POL(IB)=PO
      PIL(IB)=PI
      PON=S(IN)*PO
      PIN=Y(IN)*PI
      X(IN)=.5*PO
```

```
      XX(IN)=.5*PI
20020 CONTINUE
20023 FORMAT(I10,7E15.6)
      IN=0
      DO 28001 I=1,181,5
      IN=IN+1
      GG(IN)=FLOAT(I-1)/57.29578
28001 GX(IN)=-8.0*H*G1(I)*W(I)*CTH(I) *R9**2/2.0
      CDI=SIMPUN(GG,GX,37)
      CDI=-.5*CDI
997   IN=0
      DO 9998 I=1,181,5
      IN=IN+1
      GG(IN)=FLOAT(I-1)/57.29578
      GX(IN)=6.2832*UP9(IN)*POL(I)*SF9(IN)
9998  GP(IN)=6.2832*SP9(IN)*PIL(I)*SE9(IN)
      CDP=SIMPUN(GG,GX,37)+SIMPUN(GG,GP,37)
      CDP=CDP/(4.*R9**4)
      DF=VG(200)
      WRITE (6,1754)H,CDI,DF,CDP
      WRITE (6,205)
      WRITE (6,205)
      WRITE (6,205)
      WRITE(6,205)
      WRITE(6,205)
      WRITE(6,205)
      WRITE(6,205)
      WRITE(6,205)
205   FORMAT(4H      )
1754  FORMAT(55H1CHORD DIAMETER RATIO
     1E13.4//55H DUCT THRUST COEFF. DUE TO PROPELLER
     2E13.4//55H EFFECT OF DUCT PROFILE DRAG ON CT
     3E13.4//55H EFFECT OF DUCT PRESSURE ON CT
     4E13.4)
      CDI=CDI+DF+CDP
1768  ANT=1.0
      ANX=9.0
      ANZ=1.0
      NNX=ANX
      NNZ=ANZ
      NNT=1
      AAT(1)=1.0
26021 FORMAT(3I8)
      DO 26022 JA=1,NNT
      DO 26022 JX=1,NNX
      XXT(JX)=RV9(JX)
      DO 26024 JZ=1,NNZ
      ZZT(JZ)=1.0-AST9
      TST=ARCOS(2.0*ZZT(JZ)-1.0)
      VAG=.0
      VRG=.0
       VAQ=.0
       VRQ=.0
      DO 26023 I=1,181,5
      TSP=FLOAT(I-1)/57.29578
      TD=COS(TST)-COS(TSP)
```

```
      B1=TD
      B2=H**2*TD**2+(XXT(JX)-1.0)**2
      B5=H**2*TD**2+(XXT(JX)+1.0)**2
      B3=SQRT(XXT(JX))
      B4=B3**3
      AK=SQRT(4.0*XXT(JX)/B5)
      IF(AK-1.0) 26032,26030,26030
26032 IF(AK-.99) 26038,26039,26039
26038 CALL ELLIP(X1,AK,X3,X4,BF, BS)
      GO TO 26037
26039 AK2=AK**2
      CALL YSELL(AK2,BS,BF)
      GO TO 26037
26030 BS=1.0
26037 VAG=VAG+(-H/(6.2832*B3))*G1(I)*AK*(BF-BS-2.0*(XXT(JX)-1.0)*BS/B2)*
     1CTH(I)*SM(I)*DX *.7854
      VRG=VRG+(H/(6.2832*B4))*G1(I)*AK*H*B1*(BF-BS-2.0*XXT(JX)*BS/B2)*CT
     1H(I)*SM(I)*DX *1.5708
      VAQ=VAQ+(-H/(3.1416*B3))*SU(I)*AK*2.0*H*B1*BS*SM(I)*DX/B2
26023 VRQ=VRQ+(-H/(3.1416*B4))*SU(I)*AK*(BF-BS+2.0*XXT(JX)*(XXT(JX)-1.0)
     1*BS/B2)*SM(I)*DX
      VA=VAG+VAQ
      VR=VRG+VRQ
26024 F9(JX)=VA
26022 CONTINUE
1858  FORMAT(115H VELOCITIES INDUCED BY THE DUCT AT THE PROPELLER PLANE
     1                                                                  )
      WRITE(6,1858)
1859  FORMAT(115H          X          1-WP          WD              1-W
     1                                                                  )
      WRITE(6,1859)
1860  FORMAT(8E13.4)
      F9(9)=F9(8)
      DO 902 I=1,9
      CCC(I,4)=AX1(I)
      WX=AX1(I)+F9(I)
      WRITE(6,1860) RV9(I),CCC(I,4),F9(I),WX
      CCC(I,4)=WX
      CCC(I,5)=TB9(I)
      DO 902 J=1,7
 902  BBB(I,J)=CCC(I,J)
      ALFA=ALFA*57.29578
      RETURN
1660  IN=0
      A(1)=0.0
      DO 1669 I=I1,I3,I2
      IN=IN+1
      SUM=.0
      DO 1661 J=2,MP1
1661  SUM=SUM+A(J)*SIN(FLOAT((J-1)*(I-1))/57.29578)
      G1(I)=FF(I)+.159154*SUM
      GG(I)=G1(I)
      IX=I-1
 1669 CONTINUE
      IN=0
      DO 1234 I=I1,I3,I2
```

```
      IN=IN+1
      GG(IN)=FLOAT(I-1)/57.29578
1234  GX(IN)=B(IN,1)*GI(I)*ST(I)
      T=SIMPUN(GG,GX,37)
      ANT=ATAN(.079577*T-.5*SUM1-SCM(1))*57.29578
 1753 FORMAT(55H1IDEAL ANGLE OF ATTACK IN DEGREES
     1F9.6)
      WRITE (6,1753)ANT
      AT=.0
      ALFA=ANT
      EBC=ALFA
      ALFA=SIN(ALFA/57.29578)/COS(ALFA/57.29578)
29001 DO 22001 I=1,37
      S(I)=1.0/SQRT(1.0+(XSC(I,1)+ALFA+XSC(I,2))**2)
22001 Y(I)=1.0/SQRT(1.0+(XSC(I,1)+ALFA-XSC(I,2))**2)
      ALFA=ANT
      IN=0
      GO TO 8000
26009 FORMAT(7E15.6)
  103 FORMAT(1H1)
 1758 FORMAT(55H  INDUCED VELOCITY INSIDE THE DUCT AT A DUCT RADIUS OF
     1E11.4//55H      1-WP                WD                1-W                  )
1603  STOP
      END
```

```
CSUB
      SUBROUTINE SUB
      DIMENSION SSM(181),SCM(181),CT(181),ST(181),CTH(181),STH(181),G(40
     1,20),SM(182),APM(20,20),F(20),CB(40,20),B(60,20),C(20,20),FF(181),
     2A(20
     2),G1(181),W(181),SU(181),VG(200),VQ(200),BVQ(181),FII(20),DIMM(20)
     3,FIMM(20),AX(100),BX(100),RY(200),GX(200),Y(181),S(181),TITLE(12),
     4X(181),XX(181),XC(40,2),XT(40,2),XSC(40,2),AX1(40),BX1(40),AX2(40)
     5,BX2(40),GG(181),GP(181),SMZ(20),XI(20),T1(181),T2(181),Z1(20),Z2(
     620),WR6(20),POL(181),PIL(181),PORL(181),PIRL(181),POO(181),PII(181
     7),RV9(9),TB9(9),GM9(9),G9(9),GP9(9),SM9(10),F9(9),E9(9),UE9(37),RE
     89(37),WE9(37),UP9(37),SP9(37),SE9(37),X9(37),T9(181),AAT(50),XXT(5
     90),ZZT(50),V1(37),V2(37),SCD(181),CXT(181),P(181),R(181)
      DIMENSION IA(6),CA(9),CE(9),CO(9),RC(9),RD(9),GA(9),GC(9),BI(9),
     1      PA(9),PB(9),PTB(9),PQB(9),PXF(9),PYF(9),PTBW(81),PQBW(81),PQ
     2(9),CP(12),DA(9),DB(3),DC(3),DD(3,3),GB(9),GD(9),BBB(9,7),EA(9),EB
     3(9),EC(9),ED(9)       ,H1(81),Q(9),O(81),AH(10,9),AJ(9,9),AL(9),A7(
     49,9),AQ(9,9),AR(9,9),AS(9,9),AT1(9,9),AU(9,9),AV(9,9),AW(9,9),BA(9
     5,9),BB(9,9),BC(9,9),BD(9,9),BE(9,9),BF1(9,9),BH(9,9),CD(10,9),CF(9
     6,1),CG(9,1),CH(9,9),A8(9,9),AZ(9,9),CCC(9,7),BBF(361),BBS(361),CC(
     720,20)
      DIMENSION B2(9),B3(9),B4(72),B5(100),B6(10)
      DIMENSION CCB(40,6),AAB(40,6),AAF(6),ADIM(6,6),AFIM(6,6)
      COMMON  SSM    , SCM    , CT     , ST     , CTH    , STH
      COMMON  G      , SM     , APM    , F      , CB     , B
      COMMON  C      , FF     , A      , G1     , W      , NPP
      COMMON  SU     , VG     , VQ     , BVQ    , FII    , DIMM
      COMMON  FIMM   , AX     , BX     , RY     , GX     , H
      COMMON  I1     , I2     , I3     , DX     , X1     , X3
      COMMON  X4     , NP     , NH     , Y      , S      , TITLE
      COMMON  X      , XX     , XC     , XT     , XSC    , BF
      COMMON  BS     , AK     , AO1    , AO2    , ALFA   , MP1
      COMMON  GG     , GP     , SMZ    , XI     , T1     , Z1
      COMMON  Z2     , T2     , WK     , AT     , WR6    , CDI
      COMMON  POL    , PIL    , PORL   , PIRL   , POO    , PII
      COMMON  CTD    , ALFAR  , CL     , CM     , CM1    , CM2
      COMMON  PHI    , RV9    , TB9    , GM9    , G9     , GP9
      COMMON  SM9    , F9     , E9     , UE9    , RE9    , WE9
      COMMON  UP9    , SP9    , SE9    , X9     , T9     , RDI
      COMMON  R9     , AST9   , V1     , V2     , SCD    , CXT
      COMMON  P      , R      , AAT    , XXT    , ZZT    , I7
      COMMON  AX1    , AX2    , BX1    , BX2    , IA     , CA
      COMMON  CE     , CO     , RC     , RD     , GA     , GC
      COMMON  BI     , AQ     , PA     , PB     , PTB    , PQB
      COMMON  PXF    , PYF    , PTBW   , PQBW   , PQ     , CP
      COMMON  DA     , DB     , DC     , DD     , GB     , GD
      COMMON  BBB    , EA     , EB     , EC     , ED     , H1
      COMMON  Q      , O      , AH     , AJ     , AL     , A7
      COMMON  AR     , AS     , AT1    , AU     , AV     , AW
      COMMON  BA     , BB     , BC     , BD     , BE     , BF1
      COMMON  BH     , CD     , CF     , CG     , CH     , A8
      COMMON  AZ     , CCC    , CN     , BBF    , BBS    , CC
      COMMON  ID     , WW     , C8
      COMMON B2,B3,B4,B5,B6
      COMMON CCB,AAB,AAF,ADIM,AFIM
```

```
      IF(BX1(40))8210,8210,820
8210  CA(1)=4.0
      DO 5 I=1,9
      C(I,1)=((1.0+B(15,1))-2.0*B(I+14,1))/(1.0-B(15,1))
      AH(2,I)=C(I,1)
      CE(1)=C(I,1)**2
      IF(1.-CE(1))200,200,201
200   CD(2,I)=.0
      GO TO 202
201   CD(2,I)=SQRT((1.0-CE(1)))
202   AH(1,I)=1.0
      CD(1,I)=0.0
    5 CONTINUE
      AH(2,1)=1.0
      AH(2,9)=-1.0
      CD(2,1)=0.0
      CD(2,9)=0.0
      DO 6 J=3,10
      L=J-1
      DO 6 K=1,9
      AH(J,K)=(AH(L,K)*AH(2,K))-(CD(L,K)*CD(2,K))
      CD(J,K)=(CD(L,K)*AH(2,K))+(AH(L,K)*CD(2,K))
    6 CONTINUE
      DO 7 I=2,8
      A(I)=3.1415927/CD(2,I)
    7 CONTINUE
    8 RSL=B(7,1)/(3.1415927*B(4,1)*B(2,1))
820   IF(BX1(40))222,222,8820
8820  DO 821 I=1,9
      P1=B(I+14,1)**2*B(I+23,1)**2
      P2=B(10,1)*B(I+50,1)*(2.*B(I+14,1)**2*B(10,1)*B(I+50,1)/RSL)
      P3=SQRT(P1+P2)
      P4=B(I+14,1)*B(I+23,1)
      P5=(2.*B(I+14,1)**2/RSL-B(10,1)*B(I+50,1)   )
821   F(I)=(P4+P3)/P5
      GO TO 160
222   DO 10 J=1,9
      F(J)=B(12,1)*B(J+50,1)
      TB9(J)=F(J)
   10 CONTINUE
  160 DO 11 J=1,9
      G(J,1)=1.0/F(J)
   11 CONTINUE
      DO 12 I=1,9
      JNO=I-1
      DO 12 J=1,9
      K=9*JNO+J
      H1(K)=B(I+14,1)/B(J+14,1)*G(J,1)
   12 CONTINUE
      DO 13 J=1,9
      Q(J)=ATAN(F(J))
   13 CONTINUE
      DO 14 N=1,9
      KNO=N-1
      LNO=N-1
      MNO=N-1
```

```
      DO 14 I=1,9
      J=9*MNO+I
      IF(H1(J)-G(I,1)) 15,18,15
   18 K=9*KNO+I
      L=9*LNO+I
      O(K)=COS(Q(I))
      P(L)=SIN(Q(I))
      GO TO 14
 5    S8=1.0+H1(J)**2
      T8=SQRT(S8)
      V8=1.0+G(I,1)**2
      W8=SQRT(V8)
      AE=T8-W8
      U=EXP(AE)
      R8=(((T8-1.0)/H1(J)*(G(I,1)/(W8-1.0)))*U)**B(10,1)
      AC=1.5
      AD=.25
      X8=(1.0/(2.0*B(10,1)*G(I,1)))*((V8/S8)**AD)
      Y8=((9.0*G(I,1)**2)+2.0)/(V8**AC)+((3.0*H1(J)**2-2.0)/(S8**AC))
      Z8=1.0/(24.0*B(10,1))*Y8
      IF(H1(J)-G(I,1)) 17,17,16
 6    AF=1.0+1.0/(R8-1.0)
      AA=X8*(1.0/(R8-1.0)-Z8*ALOG(AF))
      K=9*KNO+I
      L=9*LNO+I
      O(K)=2.0*B(10,1)**2*G(I,1)*H1(J)*(1.0-G(I,1)/H1(J))*AA
      P(L)=B(10,1)*(1.0-G(I,1)/H1(J))*(1.0+2.0*B(10,1)*G(I,1)*AA)
      GO TO 14
 7    AG=1.0+1.0/(1.0/R8-1.0)
      AB=-X8*(1.0/(1.0/R8-1.0)+Z8*ALOG(AG))
      K=9*KNO+I
      L=9*LNO+I
      O(K)=B(10,1)*G(I,1)*(1.0-H1(J)/G(I,1))* (1.0-2.0*B(10,1)*G(I,1)*AB
     1)
      P(L)=2.0*B(10,1)**2*G(I,1)*(1.0-G(I,1)/H1(J))*AB
   14 CONTINUE
      DO 40 I=1,9
      IMO=I-1
      DO 40 L=1,9
      K=9*IMO+L
      A8(L,I)=O(K)
   40 CONTINUE
      DO 41 I=1,9
      JMO=I-1
      DO 41 L=1,9
      K=9*JMO+L
      AQ(L,I)=P(K)
   41 CONTINUE
      DO 42 I=1,9
      DO 42 L=1,9
      AJ(L,I)=AH(I,L)
   42 CONTINUE
      DO 43 I=1,9
      DO 43 L=1,9
      A7(L,I)=AH(I,L)
   43 CONTINUE
```

```
      AK=1.0
      DO 27 I=1,9
      DO 27 J=1,9
      C(I,J)=AJ(I,J)
27    CC(I,J)=A8(I,J)
      CALL MATINV(C,9,CC,9,Q,ID)
      DO 26 I=1,9
      DO 26 J=1,9
26    AJ(I,J)=CC(I,J)
      IF(ID-1)25,24,25
25    WRITE (6,29)
      STOP
   29 FORMAT (26H I(A) IS SINGULAR FOR Z(F))
   24 AK=1.0
      DO 32 I=1,9
      DO 32 J=1,9
      C(I,J)=A7(I,J)
32    CC(I,J)=AQ(I,J)
      CALL MATINV(C,9,CC,9,Q,ID)
      DO 33 I=1,9
      DO 33 J=1,9
33    A7(I,J)=CC(I,J)
      IF(ID-1)34,31,34
34    WRITE (6,36)
      STOP
   36 FORMAT (25H I(T) IS SINGULAR CF Z(F))
   31 AS(1,1)=AJ(1,1)+AJ(2,1)
      DO 44 J=2,8
      AS(J,1)=AS(J-1,1)+AJ(J+1,1)
   44 CONTINUE
      AS(9,1)=AS(8,1)
      DO 45 L=1,9
      AW(L,1)=FLOAT(L)*AS(L,1)
   45 CONTINUE
      DO 46 L=2,9
      K=L-1
      AU(K,1)=AJ(L,1)*FLOAT(K)
   46 CONTINUE
      AU(9,1)=0.0
      AV(9,1)=0.0
      DO 47 L=2,9
      J=10-L
      AV(J,1)=AV(J+1,1)+AU(J+1,1)
   47 CONTINUE
      DO 48 L=1,9
      AZ(L,1)=(AW(L,1)+AV(L,1))*3.1415927
   48 CONTINUE
      DO 49 I=2,8
      DO 49 L=2,9
      K=L-1
      AR(K,I)=AJ(L,I)*AH(L,I)
   49 CONTINUE
      DO 51 I=2,8
      AS(1,I)=AJ(1,I)*AH(1,I)+AR(1,I)
      DO 50 J=2,9
      K=J-1
```

```
    AS(J,I)=AS(K,I)+AR(J,I)
 50 CONTINUE
    AS(9,I)=AS(8,I)
 51 CONTINUE
    DO 52 I=2,8
    DO 52 L=1,9
    J=L+1
    AW(L,I)=CD(J,I)*AS(L,I)
 52 CONTINUE
    DO 54 I=2,8
    DO 53 L=2,9
    K=L-1
    AU(K,I)=AJ(L,I)*CD(L,I)
 53 CONTINUE
    AU(9,I)=0.0
 54 CONTINUE
    DO 55 I=2,8
    AV(9,I)=0.0
    DO 55 L=2,9
    J=10-L
    AV(J,I)=AV(J+1,I)+AU(J+1,I)
 55 CONTINUE
    DO 56 I=2,8
    DO 56 L=1,9
    J=L+1
    C(L,I)=AH(J,I)*AV(L,I)
 56 CONTINUE
    DO 57 I=2,8
    DO 57 L=1,9
    AZ(L,I)=(AW(L,I)+C(L,I))*A(I)
 57 CONTINUE
    DO 58 L=1,4
    J=2*L
    AJ(J,9)=-1.0*AJ(J,9)
 58 CONTINUE
    AR(1,9)=AJ(1,9)+AJ(2,9)
    DO 59 L=2,8
    AR(L,9)=AR(L-1,9)+AJ(L+1,9)
 59 CONTINUE
    AR(9,9)=AR(8,9)
    DO 130 L=1,9
    AS(L,9)=FLOAT(L)*AR(L,9)
130 CONTINUE
    DO 131 L=2,9
    K=L-1
    AU(K,9)=FLOAT(K)*AJ(L,9)
131 CONTINUE
    AU(9,9)=0.0
    AV(9,9)=0.0
    DO 132 L=2,9
    J=10-L
    AV(J,9)=AV(J+1,9)+AU(J+1,9)
132 CONTINUE
    DO 133 L=1,9
    AZ(L,9)=(AS(L,9)+AV(L,9))*3.1415927
133 CONTINUE
```

```
      DO 176 L=1,4
      J=2*L
      AZ(J,9)=-1.0*AZ(J,9)
  176 CONTINUE
      BA(1,1)=A7(1,1)+A7(2,1)
      DO 134 J=2,8
      BA(J,1)=BA(J-1,1)+A7(J+1,1)
  134 CONTINUE
      BA(9,1)=BA(8,1)
      DO 135 L=1,9
      BB(L,1)=FLOAT(L)*BA(L,1)
  135 CONTINUE
      DO 136 L=2,9
      K=L-1
      BC(K,1)=A7(L,1)*FLOAT(K)
  136 CONTINUE
      BC(9,1)=0.0
      BD(9,1)=0.0
      DO 137 L=2,9
      J=10-L
      BD(J,1)=BD(J+1,1)+BC(J+1,1)
  137 CONTINUE
      DO 138 L=1,9
      BH(L,1)=(BB(L,1)+BD(L,1))*3.1415927
  138 CONTINUE
      DO 139 I=2,8
      DO 139 L=2,9
      K=L-1
      BA(K,I)=A7(L,I)*AH(L,I)
  139 CONTINUE
      DO 141 I=2,8
      BB(1,I)=A7(1,I)*AH(1,I)+BA(1,I)
      DO 140 J=2,9
      K=J-1
      BB(J,I)=BB(K,I)+BA(J,I)
  140 CONTINUE
      BB(9,I)=BB(8,I)
  141 CONTINUE
      DO 142 I=2,8
      DO 142 L=1,9
      J=L+1
      BC(L,I)=CD(J,I)*BB(L,I)
  142 CONTINUE
      DO 144 I=2,8
      DO 143 L=2,9
      K=L-1
  143 CONTINUE
      BD(K,I)=A7(L,I)*CD(L,I)
      BD(9,I)=0.0
  144 CONTINUE
      DO 145 I=2,8
      BE(9,I)=0.0
      DO 145 L=2,9
      J=10-L
      BE(J,I)=BE(J+1,I)+BD(J+1,I)
  145 CONTINUE
```

```
      DO 146 I=2,8
      DO 146 L=1,9
      J=L+1
      BF1(L,I)=AH(J,I)*BE(L,I)
  146 CONTINUE
      DO 147 I=2,8
      DO 147 L=1,9
      BH(L,I)=(BC(L,I)+BF1(L,I))*A(I)
  147 CONTINUE
      DO 148 L=1,4
      J=2*L
      A7(J,9)=-1.0*A7(J,9)
  148 CONTINUE
      BA(1,9)=A7(1,9)+A7(2,9)
      DO 149 L=2,8
      BA(L,9)=BA(L-1,9)+A7(L+1,9)
  149 CONTINUE
      BA(9,9)=BA(8,9)
      DO 150 L=1,9
      BB(L,9)=FLOAT(L)*BA(L,9)
  150 CONTINUE
      DO 151 L=2,9
      K=L-1
      BC(K,9)=FLOAT(K)*A7(L,9)
  151 CONTINUE
      BC(9,9)=0.0
      BD(9,9)=0.0
      DO 152 L=2,9
      J=10-L
      BD(J,9)=BD(J+1,9)+BC(J+1,9)
  152 CONTINUE
      DO 153 L=1,9
      BH(L,9)=(BB(L,9)+BD(L,9))*3.1415927
  153 CONTINUE
      DO 175 L=1,4
      J=2*L
      BH(J,9)=-1.0*BH(J,9)
  175 CONTINUE
      IF(B(9,1))1750,1750,1751
 1750 DO 602 I=1,9
  602 CO(I)=RSL/B(I+14,1)*B(I+23,1)
 1751 T=6.0
      DO 60 J=1,9
      CF(J,1)=(1.0-B(15,1))*(F(J)/CO(J)-1.0)*B(J+23,1)
   60 CONTINUE
      DO 125 J=1,9
      CG(J,1)=CF(J,1)
  125 CONTINUE
      DO 75 I=1,9
      DO 75 J=1,9
      AQ(I,J)=FLOAT(J)*(AZ(J,I)+F(I)*BH(J,I))
   75 CONTINUE
      DO 121 J=1,9
      DO 121 K=1,9
      CH(K,J)=AQ(K,J)
  121 CONTINUE
```

```
      AK=1.0
      DO 67 I=1,9
      DO 67 J=1,9
      C(I,J)=AQ(I,J)
67    CC(I,1)=CG(I,1)
      CALL MATINV(C,9,CC,1,Q,ID)
      DO 68 J=1,9
68    AQ(J,1)=CC(J,1)
      IF(ID-1)69,85,69
69    WRITE (6,70)
      STOP
   70 FORMAT (27H G(MX) IS SINGULAR FOR Z(F))
   85 CG(1,1)=1.0/(1.0-B(15,1))
      DO601I=1,9
      CA(I)=0.0
      CB(I,1)=0.0
      CE(I)=0.0
      DO601J=1,9
      IF(BX1(40))87,87,88
87    CA(I)=CA(I)+FLOAT(J)*AQ(J,1)*AZ(J,I)/B(I+23,1)
      CB(I,1)=CB(I,1)+FLOAT(J)*AQ(J,1)*BH(J,I)/B(I+23,1)
      CE(I)=AQ(J,1)*CD(J+1,I)/B(I+23,1)+CE(I)
      GO TO 601
88    CA(I)=CA(I)+FLOAT(J)*B6(J)*AZ(J,I)/B(I+23,1)
      CB(I,1)=CB(I,1)+FLOAT(J)*B6(J)*BH(J,I)/B(I+23,1)
      CE(I)=CE(I)+B6(J)*CD(J+1,I)/B(I+23,1)
  601 CONTINUE
      DO 660I=1,9
      CA(I)=CG(1,1)*CA(I)
      CB(I,1)=CG(1,1)*CB(I,1)
      RC(I)=(CE(I)*B(I+23,1)*(B(I+23,1)/CO(I)-CB(I,1)*B(I+23,1)))*4.0*B(
     110,1)
      RD(I)=(B(I+23,1)/CO(I)*CE(I)*B(I+23,1)*(B(I+23,1)+CA(I)*B(I+23,1))
     1)*4.0*B(10,1)
  660 CONTINUE
      CE(1)=0.0
      CE(9)=0.0
      CM=0.0
      CN=0.0
      DO603I=1,9
  603 BI(I)=B(I+14,1)
      VALUE=SIMPUN(BI,RC,9)
      CM=VALUE
      VALUE=SIMPUN(BI,RD,9)
      CN=VALUE
      IF(BX1(40))604,604,6030
6030  DO 851 I=1,9
      F(I)=(B(I+23,1)+CA(I))/((B(I+14,1)/RSL)-CB(I,1))
      F(1)=((1.9932/1.5045)*RSL/B(15,1))
      F(9)=((.33456/.30087)*RSL)
851   B(I+50,1)=F(I)
604   RETURN
      END
```

```
,ORIGIN        XC13A
      SUBROUTINE SUB HA
      DIMENSION SSM(181),SCM(181),CT(181),ST(181),CTH(181),STH(181),G(40
     1,20),SM(182),APM(20,20),F(20),CB(40,20),B(60,20),C(20,20),FF(181),
     2A(20
     2),G1(181),W(181),SU(181),VG(200),VQ(200),BVQ(181),FII(20),DIMM(20)
     3,FIMM(20),AX(100),BX(100),RY(200),GX(200),Y(181),S(181),TITLE(12),
     4X(181),XX(181),XC(40,2),XT(40,2),XSC(40,2),AX1(40),BX1(40),AX2(40)
     5,BX2(40),GG(181),GP(181),SMZ(20),XI(20),T1(181),T2(181),Z1(20),Z2(
     620),WR6(20),POL(181),PIL(181),PORL(181),PIRL(181),POO(181),PII(181
     7),RV9(9),TB9(9),GM9(9),G9(9),GP9(9),SM9(10),F9(9),E9(9),UE9(37),RE
     89(37),WE9(37),UP9(37),SP9(37),SE9(37),X9(37),T9(181),AAT(50),XXT(5
     90),ZZT(50),V1(37),V2(37),SCD(181),CXT(181),P(181),R(181)
      DIMENSION IA(6),CA(9),CE(9),CO(9),RC(9),RD(9),GA(9),GC(9),BI(9),
     1      PA(9),PB(9),PTB(9),PQB(9),PXF(9),PYF(9),PTBW(81),PQBW(81),PQ
     2(9),CP(12),DA(9),DB(3),DC(3),DD(3,3),GB(9),GD(9),BBB(9,7),EA(9),EB
     3(9),EC(9),ED(9)        ,H1(81),Q(9),O(81),AH(10,9),AJ(9,9),AL(9),A7(
     49,9),AQ(9,9),AR(9,9),AS(9,9),AT1(9,9),AU(9,9),AV(9,9),AW(9,9),BA(9
     5,9),BB(9,9),BC(9,9),BD(9,9),BE(9,9),BF1(9,9),BH(9,9),CD(10,9),CF(9
     6,1),CG(9,1),CH(9,9),A8(9,9),AZ(9,9),CCC(9,7),BBF(361),BBS(361),CC(
     720,20)
      DIMENSION B2(9),B3(9),B4(72),B5(100),B6(10)
      DIMENSION CCB(40,6),AAB(40,6),AAF(6),ADIM(6,6),AFIM(6,6)
      COMMON  SSM    , SCM    , CT     , ST     , CTH    , STH
      COMMON  G      , SM     , APM    , F      , CB     , B
      COMMON  C      , FF     , A      , G1     , W      , NPP
      COMMON  SU     , VG     , VQ     , BVQ    , FII    , DIMM
      COMMON  FIMM   , AX     , BX     , RY     , GX     , H
      COMMON  I1     , I2     , I3     , DX     , X1     , X3
      COMMON  X4     , NP     , NH     , Y      , S      , TITLE
      COMMON  X      , XX     , XC     , XT     , XSC    , BF
      COMMON  BS     , AK     , AO1    , AO2    , ALFA   , MP1
      COMMON  GG     , GP     , SMZ    , XI     , T1     , Z1
      COMMON  Z2     , T2     , WK     , AT     , WR6    , CDI
      COMMON  POL    , PIL    , PORL   , PIRL   , POO    , PII
      COMMON  CTD    , ALFAR  , CL     , CM     , CM1    , CM2
      COMMON  PHI    , RV9    , TB9    , GM9    , G9     , GP9
      COMMON  SM9    , F9     , E9     , UE9    , RE9    , WE9
      COMMON  UP9    , SP9    , SE9    , X9     , T9     , RDI
      COMMON  R9     , AST9   , V1     , V2     , SCD    , CXT
      COMMON  P      , R      , AAT    , XXT    , ZZT    , I7
      COMMON  AX1    , AX2    , BX1    , BX2    , IA     , CA
      COMMON  CE     , CO     , RC     , RD     , GA     , GC
      COMMON  BI     , AQ     , PA     , PB     , PTB    , PQB
      COMMON  PXF    , PYF    , PTBW   , PQBW   , PQ     , CP
      COMMON  DA     , DB     , DC     , DD     , GB     , GD
      COMMON  BBB    , EA     , EB     , EC     , ED     , H1
      COMMON  Q      , O      , AH     , AJ     , AL     , A7
      COMMON  AR     , AS     , AT1    , AU     , AV     , AW
      COMMON  BA     , BB     , BC     , BD     , BE     , BF1
      COMMON  BH     , CD     , CF     , CG     , CH     , A8
      COMMON  AZ     , CCC    , CN     , BBF    , BBS    , CC
      COMMON  ID     , WW     , C8
      COMMON B2,B3,B4,B5,B6
      COMMON CCB,AAB,AAF,ADIM,AFIM
```

```
      B9=CCC(9,2)
      XH9=CCC(1,3)
      GTR9=.0
      DO 11 K=1,9
      PHI9=ARCOS((1.+RV9(1)-2.*RV9(K))/(1.-RV9(1)))
      SUM1=.0
      SUM2=.0
      IF(PHI9) 1383,1382,1383
 1383 IF(ABS(PHI9)-3.1416) 1381,1380,1381
 1382 TOS=2.0/SIN(1.0/57.29578)
      GO TO 1300
 1380 TOS=2.0/SIN(179.0/57.29578)
      GC TO 1300
 1381 TOS=2.0/SIN(PHI9)
 1300 DO 10 M=1,9
      SUM1=SUM1+GM9(M)*SIN(FLOAT(M)*PHI9)
   10 SUM2=SUM2+FLOAT(M)*GM9(M)*COS(FLOAT(M)*PHI9)
      G9(K)=SUM1
   11 GP9(K)=SUM2
      Q1=.0
      SMA=-1.5708
      SMB=-SMA
      DO 21 K=1,37
      SUM1=.0
      SUM2=.0
      DO 2 J=1,9
      XW=1.0+(X9(K)**2+(R9-RV9(J))**2)/(2.0*R9*RV9(J))
      IF(XW-1.0001)80,80,90
80    XW=1.00012
90    IF(XW-1000.0)20,100,100
100   XW=999.9998
20    NW=0
      CALL LEGEND(NW,XW,QNX,DQNX)
      IF(ABS(DQNX)-10.0)201,200,200
200   DQNX=-10.0
201   CONTINUE
      F9(J)=DQNX*G9(J)/(TB9(J)*SQRT(RV9(J))**3)*B9*X9(K)/(6.2832*SQRT(R9
     1)**3)
      NW=1
      CALL LEGEND(NW,XW,QNX,DQNX)
      IF(QNX-4.33826)301,300,300
300   QNX=4.33826
301   CONTINUE
2     E9(J)=QNX*GP9(J)/(TB9(J)*SQRT(RV9(J)))*B9/(6.2832*SQRT(R9))
      UE9(K)=SIMPUN(RV9,F9,9)
      RE9(K)=SIMPUN(RV9,E9,9)
21    WE9(K)=.0
      IN=0
      DO 998 I=1,181,5
      IN=IN+1
      T1(I)=.5*(1.0-CT(I))
      T2(I)=-UE9(IN)-AX2(IN)
      W(I)=RE9(IN)*6.2832-BX2(IN)
998   T9(I)=-WE9(IN)-BX2(IN)
1858  FORMAT(115H VELOCITIES INDUCED BY PROPELLER ALONG THE DUCT
     1                                                              )
```

```
      WRITE(6,1858)
      WRITE(6,1859)
1860  FORMAT(3E13.4)
      WRITE(6,1860)(T1(I),T2(I),W(I),        I=1,181,5)
1859  FORMAT(115H           X        AXIAL        RADIAL
     1
      DO 2597 I=1,37
 2597 Y(I)=V1(I)
   99 DO 21888 IC=1,2
      Y(1)=.0
      Y(37)=.0
 3001 FORMAT(9E13.4)
 2498 FORMAT(3E15.6)
      DO 21001 KK=1,35
      NX1=37+KK
      NX2=37-KK
21001 Y(NX1)=-Y(NX2)
      IF(IC-2) 2408,2409,2408
 2408 IG=2
      GO TO 2410
 2409 IG=1
 2410 DO 21002 IS=1,IG
      IF(V1(5)) 21977,21978,21977
21978 DO 702 I=1,200
702   RY(I)=.0
      GO TO 21976
21977 CALL GMHAS(72,36,Y,RY)
21976 AO=RY(1)
      XC(1,IS)=RY(1)
      BO=RY(2)
      DO 21997 J=1,36
      JFIX=5*J+1
      AX(J)=RY(JFIX)
      XC(J+1,IS)=AX(J)
      BX(J)=RY(JFIX+1)
21997 XT(J,IC)=BX(J)
      IH=0
21996 FORMAT(I10,2E15.6)
      IN=0
      DO 29000 I=1,181,5
      IN=IN+1
      Y1=.0
      Y2=.0
      SS=.0
      L=I-1
      DO 21995 K=1,36
      IF(IS-2)24000,23000,24000
23000 DO 703 I5=1,181,5
703   S(I5)=.0
      GO TO 21984
24000 IF(I-1)21983,21982,21983
21983 IF(I-181)21981,21980,21981
21982 TOS=2.0/SIN(1.0/57.29578)
      GO TO 21995
21980 TOS=2.0/SIN(179.0/57.29578)
      GO TO 21995
```

```
21981 TOS=2.0/ST(I)
21995 SS=SS-FLOAT(K)*BX(K)*TOS*COS(FLOAT((I-1)*K)/57.29578)
      Y(IN)=-SS-W(I)
      XSC(IN,IC)=-SS
      WE9(IN)=XSC(IN,1)
      S(I)=-SS
21984 CONTINUE
29000 CONTINUE
      DO 21006 KK=1,35
      NX1=37+KK
      NX2=37-KK
21006 Y(NX1)=+Y(NX2)
      Y(37)=Y(36)
21002 Y(38)=Y(36)
      DO 2596 I=1,37
2596  Y(I)=V2(I)
21888 CONTINUE
21994 FORMAT(I10,2E15.6,I10,3E15.6)
      ANT=ALFA
      ALFA=SIN(ALFA/57.29578)/COS(ALFA/57.29578)
29001 DO 22001 I=1,37
      S(I)=1.0/SQRT(1.0+(XSC(I,1)+ALFA+XSC(I,2))**2)
22001 Y(I)=1.0/SQRT(1.0+(XSC(I,1)+ALFA-XSC(I,2))**2)
      ALFA=ANT
      DO 1 I=1,37
      RE9(I)=ATAN(WE9(I))
      UE9(I)=COS(RE9(I))
    1 SE9(I)=SIN(RE9(I))
      DO 22889 J=1,36
      SCM(J)=-XC(J,2)
22889 SSM(J)=XT(J,2)
      S(1)=.0
      Y(1)=.0
      S(37)=.0
      Y(37)=.0
 1100 FORMAT(9F8.6)
 1200 FORMAT(9F8.6)
 1752 FORMAT(9F10.6)
  103 FORMAT(1H1)
   95 RETURN
      END
```

```
CSUBW
      SUBROUTINE SUBWR
      DIMENSION SSM(181),SCM(181),CT(181),ST(181),CTH(181),STH(181),G(40
     1,20),SM(182),APM(20,20),F(20),CB(40,20),B(60,20),C(20,20),FF(181),
     2A(20
     2),G1(181),W(181),SU(181),VG(200),VQ(200),BVQ(181),FII(20),DIMM(20)
     3,FIMM(20),AX(100),BX(100),RY(200),GX(200),Y(181),S(181),TITLE(12),
     4X(181),XX(181),XC(40,2),XT(40,2),XSC(40,2),AX1(40),BX1(40),AX2(40)
     5,BX2(40),GG(181),GP(181),SMZ(20),XI(20),T1(181),T2(181),Z1(20),Z2(
     620),WR6(20),POL(181),PIL(181),PORL(181),PIRL(181),POO(181),PII(181
     7),RV9(9),TB9(9),GM9(9),G9(9),GP9(9),SM9(10),F9(9),E9(9),UE9(37),RE
     89(37),WE9(37),UP9(37),SP9(37),SE9(37),X9(37),T9(181),AAT(50),XXT(5
     90),ZZT(50),V1(37),V2(37),SCD(181),CXT(181),P(181),R(181)
      DIMENSION IA(6),CA(9),CE(9),CO(9),RC(9),RD(9),GA(9),GC(9),BI(9),
     1      PA(9),PB(9),PTB(9),PQB(9),PXF(9),PYF(9),PTBW(81),PQBW(81),PQ
     2(9),CP(12),DA(9),DB(3),DC(3),DD(3,3),GB(9),GD(9),BBB(9,7),EA(9),EB
     3(9),EC(9),ED(9)       ,H1(81),Q(9),O(81),AH(10,9),AJ(9,9),AL(9),A7(
     49,9),AQ(9,9),AR(9,9),AS(9,9),AT1(9,9),AU(9,9),AV(9,9),AW(9,9),BA(9
     5,9),BB(9,9),BC(9,9),BD(9,9),BE(9,9),BF1(9,9),BH(9,9),CD(10,9),CF(9
     6,1),CG(9,1),CH(9,9),A8(9,9),AZ(9,9),CCC(9,7),BBF(361),BBS(361),CC(
     720,20)
      DIMENSION B2(9),B3(9),B4(72),B5(100),B6(10)
      DIMENSION CCB(40,6),AAB(40,6),AAF(6),ADIM(6,6),AFIM(6,6)
      COMMON  SSM     , SCM     , CT      , ST      , CTH     , STH
      COMMON  G       , SM      , APM     , F       , CB      , B
      COMMON  C       , FF      , A       , G1      , W       , NPP
      COMMON  SU      , VG      , VQ      , BVQ     , FII     , DIMM
      COMMON  FIMM    , AX      , BX      , RY      , GX      , H
      COMMON  I1      , I2      , I3      , DX      , X1      , X3
      COMMON  X4      , NP      , NH      , Y       , S       , TITLE
      COMMON  X       , XX      , XC      , XT      , XSC     , BF
      COMMON  BS      , AK      , A01     , A02     , ALFA    , MP1
      COMMON  GG      , GP      , SMZ     , XI      , T1      , Z1
      COMMON  Z2      , T2      , WK      , AT      , WR6     , CDI
      COMMON  POL     , PIL     , PORL    , PIRL    , POO     , PII
      COMMON  CTD     , ALFAR   , CL      , CM      , CM1     , CM2
      COMMON  PHI     , RV9     , TB9     , GM9     , G9      , GP9
      COMMON  SM9     , F9      , E9      , UE9     , RE9     , WE9
      COMMON  UP9     , SP9     , SE9     , X9      , T9      , RDI
      COMMON  R9      , AST9    , V1      , V2      , SCD     , CXT
      COMMON  P       , R       , AAT     , XXT     , ZZT     , I7
      COMMON  AX1     , AX2     , BX1     , BX2     , IA      , CA
      COMMON  CE      , CO      , RC      , RD      , GA      , GC
      COMMON  BI      , AQ      , PA      , PB      , PTB     , PQB
      COMMON  PXF     , PYF     , PTBW    , PQBW    , PQ      , CP
      COMMON  DA      , DB      , DC      , DD      , GB      , GD
      COMMON  BBB     , EA      , EB      , EC      , ED      , H1
      COMMON  Q       , O       , AH      , AJ      , AL      , A7
      COMMON  AR      , AS      , AT1     , AU      , AV      , AW
      COMMON  BA      , BB      , BC      , BD      , BE      , BF1
      COMMON  BH      , CD      , CF      , CG      , CH      , A8
      COMMON  AZ      , CCC     , CN      , BBF     , BBS     , CC
      COMMON  ID      , WW      , C8
      COMMON B2,B3,B4,B5,B6
      COMMON CCB,AAB,AAF,ADIM,AFIM
```

```
 1753 FORMAT(55H1CHORD DIAMETER RATIO
     1F9.4///55H LIFT COEFFICIENT PER DEGREE (EQ3.3-2)
     2F9.4///55H INDUCED DRAG COEFFICIENT (EQ3.3-5)
     3F9.4///55H MOMENT COEFFICIENT FROM VERTICAL FORCES (EQ3.3-7)
     4F9.4///55H MOMENT COEFFICIENT FROM HORIZONTAL FORCES (EQ3.3-8)
     5F9.4///55H TOTAL MOMENT COEFFICIENT ABOUT L. E. (EQ3.3-6)
     6F9.4///55H DUCT THRUST COEFF. DUE TO PROPELLER
     7F9.4///55H EFFECT OF DUCT PROFILE DRAG ON CT
     8F9.4)
 1755 FORMAT(55H1GEOMETRIC ANGLE OF ATTACK IN DEGREES
     1E13.4 /55H POSITION OF SECTION AT ANGLE (PA) IN DEGREES
     2E13.4/)
 1756 FORMAT(115H       T          X            GSTAR          NCO              NC
     1I           PO          PI           POP          PIP        )
 1757 FORMAT(55H T AND X=STATIONS ALONG CHORD WHERE X=.5(1-COS(T))
     1      /55H GSTAR =CIRCULATION DISTRIBUTION
     2      /55H NCO AND NCI=NONLINEAR CORRECTION OUTSIDE AND INSIDE
     3     55H DUCT RESPECTIVELY
     4      /55H PO AND PI=NONLINEAR PRESSURE DISTRIBUTION OUTSIDE AND
     5      55H INSIDE THE DUCT RESPECTIVELY FOR PA=0 DEGREES
     6      /55H POP AND PIP =   LINEAR PRESSURE DISTRIBUTION OUTSIDE
     7      55H AND INSIDE THE DUCT RESPECTIVELY FOR PA=0
     8)
      A4=.0
      A5=.0
      WRITE (6,1755)A4,A5
      WRITE (6,1756)
      IN=0
      DO 7777 I=I1,I3,I2
      IN=IN+1
      S(1)=1.0
      Y(1)=1.0
      S(37)=1.0
      Y(37)=1.0
      PCT=POL(I)
      PIT=PIL(I)
      PORT=POL(I)/S(IN)
      PIRT=PIL(I)/Y(IN)
      IF(I-176) 14001,14002,14001
14002 AY=POT
      AZ(1,1)=PIT
14001 IX=I-1
      Z=-(.5*(1.0+CT(I)))+1.0
      S(1)=.0
      Y(1)=.0
      S(37)=.0
      Y(37)=.0
 7777 WRITE (6,20023)IX,Z,G1(I),S(IN),Y(IN),POT,PIT,PORT,PIRT
      WRITE (6,1757)
26022 CONTINUE
20023 FORMAT(I8,8E13.4)
26000 FORMAT(9F8.6)
26009 FORMAT(7E15.6)
20024 FORMAT(9I8)
20025 FORMAT(9F8.6)
  801 FORMAT(//8E15.6)
```

```
103 FORMAT(1H1)
    RETURN
    END
```

```
SUBF
      SUBROUTINE  SUBF
      DIMENSION SSM(181),SCM(181),CT(181),ST(181),CTH(181),STH(181),G(40
     1,20),SM(182),APM(20,20),F(20),CB(40,20),B(60,20),C(20,20),FF(181),
     2A(20
     2),G1(181),W(181),SU(181),VG(200),VQ(200),BVQ(181),FII(20),DIMM(20)
     3,FIMM(20),AX(100),BX(100),RY(200),GX(200),Y(181),S(181),TITLE(12),
     4X(181),XX(181),XC(40,2),XT(40,2),XSC(40,2),AX1(40),BX1(40),AX2(40)
     5,BX2(40),GG(181),GP(181),SMZ(20),XI(20),T1(181),T2(181),Z1(20),Z2(
     620),WR6(20),POL(181),PIL(181),PORL(181),PIRL(181),POO(181),PII(181
     7),RV9(9),TB9(9),GM9(9),G9(9),GP9(9),SM9(10),F9(9),E9(9),UE9(37),RE
     89(37),WE9(37),UP9(37),SP9(37),SE9(37),X9(37),T9(181),AAT(50),XXT(5
     90),ZZT(50),V1(37),V2(37),SCD(181),CXT(181),P(181),R(181)
      DIMENSION IA(6),CA(9),CE(9),CO(9),RC(9),RD(9),GA(9),GC(9),BI(9),
     1       PA(9),PB(9),PTB(9),PQB(9),PXF(9),PYF(9),PTBW(81),PQBW(81),PQ
     2(9),CP(12),DA(9),DB(3),DC(3),DD(3,3),GB(9),GD(9),BBB(9,7),EA(9),EB
     3(9),EC(9),ED(9)       ,H1(81),Q(9),O(81),AH(10,9),AJ(9,9),AL(9),A7(
     49,9),AQ(9,9),AR(9,9),AS(9,9),AT1(9,9),AU(9,9),AV(9,9),AW(9,9),BA(9
     5,9),BB(9,9),BC(9,9),BD(9,9),BE(9,9),BF1(9,9),BH(9,9),CD(10,9),CF(9
     6,1),CG(9,1),CH(9,9),A8(9,9),AZ(9,9),CCC(9,7),BBF(361),BBS(361),CC(
     720,20)
      DIMENSION B2(9),B3(9),B4(72),B5(100),B6(10)
      DIMENSION CCB(40,6),AAB(40,6),AAF(6),ADIM(6,6),AFIM(6,6)
      COMMON  SSM     , SCM     , CT      , ST      , CTH     , STH
      COMMON  G       , SM      , APM     , F       , CB      , B
      COMMON  C       , FF      , A       , G1      , W       , NPP
      COMMON  SU      , VG      , VQ      , BVQ     , FII     , DIMM
      COMMON  FIMM    , AX      , BX      , RY      , GX      , H
      COMMON  I1      , I2      , I3      , DX      , X1      , X3
      COMMON  X4      , NP      , NH      , Y       , S       , TITLE
      COMMON  X       , XX      , XC      , XT      , XSC     , BF
      COMMON  BS      , AK      , AO1     , AO2     , ALFA    , MP1
      COMMON  GG      , GP      , SMZ     , XI      , T1      , Z1
      COMMON  Z2      , T2      , WK      , AT      , WR6     , CDI
      COMMON  POL     , PIL     , PORL    , PIRL    , POO     , PII
      COMMON  CTD     , ALFAR   , CL      , CM      , CM1     , CM2
      COMMON  PHI     , RV9     , TB9     , GM9     , G9      , GP9
      COMMON  SM9     , F9      , E9      , UE9     , RE9     , WE9
      COMMON  UP9     , SP9     , SE9     , X9      , T9      , RDI
      COMMON  R9      , AST9    , V1      , V2      , SCD     , CXT
      COMMON  P       , R       , AAT     , XXT     , ZZT     , I7
      COMMON  AX1     , AX2     , BX1     , BX2     , IA      , CA
      COMMON  CE      , CO      , RC      , RD      , GA      , GC
      COMMON  BI      , AQ      , PA      , PB      , PTB     , PQB
      COMMON  PXF     , PYF     , PTBW    , PQBW    , PQ      , CP
      COMMON  DA      , DB      , DC      , DD      , GB      , GD
      COMMON  BBB     , EA      , EB      , EC      , ED      , H1
      COMMON  Q       , O       , AH      , AJ      , AL      , A7
      COMMON  AR      , AS      , AT1     , AU      , AV      , AW
      COMMON  BA      , BB      , BC      , BD      , BE      , BF1
      COMMON  BH      , CD      , CF      , CG      , CH      , A8
      COMMON  AZ      , CCC     , CN      , BBF     , BBS     , CC
      COMMON  ID      , WW      , C8
      COMMON B2,B3,B4,B5,B6
      COMMON CCB,AAB,AAF,ADIM,AFIM
```

```
      WRITE(6,205)
      WRITE(6,205)
205   FORMAT(4H    )
      WRITE(6,7525)
      WRITE(6,7526)
      WRITE(6,7527)
7525  FORMAT(56H K=RATIO OF ESTIMATED AND FINAL HYDRODYNAMIC PITCH ANGLE
     1        /55H CTSI=PROPELLER NONVISCOUS THRUST COEFFICIENT
     2        /55H CPSI=PROPELLER NONVISCOUS POWER COEFFICIENT
     3        /55H XI=NONDIMENSIONAL PROPELLER RADIUS
     4        /55H TANBI=TANGENT OF HYDRODYNAMIC PITCH ANGLE
     5        /55H TANB=TANGENT OF PROPELLER ADVANCE ANGLE
     6        /55H GS=PROPELLER NONDIMENSIONAL CIRCULATION DISTRIBUTION
     7        /55H UT/2VS=TANGENTIAL VELOCITY INDUCED AT THE LIFTING LINE
     8        /55H UA/2VS=AXIAL VELOCITY INDUCED AT THE LIFTING LINE
     9        )
7526  FORMAT(55H DCTSI=LOCAL NONVISCOUS PROPELLER THRUST COEFFICIENT
     1        /55H DCPSI=LOCAL NONVISCOUS PROPELLER POWER COEFFICIENT
     2        /55H CLL/D=NONDIMENSIONAL PROPELLER LIFT COEFFICIENT
     3        /55H SIGMA X=CAVITATION NO. BASED ON INFLOW VELOCITY
     4        /55H CPT=PROPELLER VISCOUS THRUST-HORSE POWER
     5        /55H CPS=PROPELLER VISCOUS POWER COEFFICIENT
     6        /55H EP=PROPELLER EFFICIENCY (CPT/CPS)
     7        /55H CTS=PROPELLER VISCOUS THRUST  COEFFICIENT
     8        /55H EPSILON=PROPELLER SECTION DRAG-LIFT RATIO
     9        )
7527  FORMAT(55H MTB=BENDING MOMENT DUE TO THRUST
     1        /55H MQB=BENDING MOMENT DUE TO TORQUE
     2        /55H MXO=BENDING MOMENT PARALLEL TO SECTION NOSE-TAIL LINE
     3        /55H MYO=BENDING MOMENT PERPENDICULAR TO NOSE-TAIL LINE
     4        /55H M,G(M) NO. OF HARMONICS AND  FOURIER SINE COEFF. OF GS
     5        )
      RETURN
      END
```

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13. ABSTRACT

This report presents a computer program that can be used to design and predict the performance of ducted propellers if the geometric shape of the duct is specified. The duct is represented mathematically by a distribution of ring vortices and ring sources along a cylinder of constant diameter which implies that the boundary conditions are linearized. The propeller design characteristics and performance are computed from lifting-line theory, as developed by Lerbs for moderately loaded finite-bladed propellers. Only the steady velocity components induced on the duct by the finite-bladed propeller are considered in determining the effect of the propeller on the duct.

The computer program contains an option for the input of velocities induced on the duct by sources other than the propeller. Another option allows calculation of the ideal angle of attack of the duct section in the presence of a propeller. A duct operating at this angle will have a favorable pressure distribution and flow should not separate. Results of calculations are presented and compared to experimental results.

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3. Ducted propellers--Pressure distribution--Theory
4. Digital computers--IBM 7090--Applications

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