MODEL BASIN
WASHINGTON, D.C.

DEPARTMENT OF THE NAVY

HYDROMECHANICS

AERODYNAMICS

STRUCTURAL MECHANICS

APPLIED MATHEMATICS

ACOUSTICS AND VIBRATION

A FORTRAN PROGRAM FOR CALCULATING THE SOUND RADIATION FROM A VIBRATING SURFACE OF REVOLUTION

by

George Chertock, Ph.D.

[redacted]

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ACCOUSTICS AND VIBRATION LABORATORY
RESEARCH AND DEVELOPMENT REPORT

December 1965

Report 2083

ERRATA SHEET

for

David Taylor Model Basin 2083, December 1965

The following changes are to be made to Report 2083.

Page 17 – Equation [75] reads:

$$Z = \frac{2\pi pc}{\epsilon_m} \int_{-a}^{a} F(x) \cdot G(x)\, dx \qquad [75]$$

Equation [75] should read:

$$Z = \frac{2\pi pca}{\epsilon_m} \int_{-a}^{a} F(x) \cdot G(x)\, dx \qquad [75]$$

Page 20 – Equation [89] reads:

$$J_{n+1}'(x) = \left[1 - \frac{n(n+1)}{x^2}\right] J_m(x) + \left[\frac{n(n+1)}{x}\right] J_m'(x), \qquad [89]$$

Equation [89] should read:

$$J_{n+1}'(x) = \left[1 - \frac{n(n+1)}{x^2}\right] J_n(x) + \left[\frac{n(n+1)}{x}\right] J_n'(x), \qquad [89]$$

Insert attached sheet 38a between pages 38 and 39. This page was inadvertently omitted from the Appendix listing the FORTRAN program.

Corrections made 10/3/67
Jed

```
C       MODIFIED GAUSS - SEIDEL ITERATION FOR SURFACE PRESSURE
        SUBROUTINE SURFPR(N,M,A,W,VEL,I4,HALF,EPS2,TZR,TZI,NST)
        COMMON X,Y,YDY,D,E,F1,F2,G,T1,T2,U1,U2                         UC07A040
       1,F7,F8,G8,G9,Z1,Z2                                              UC070041
        DIMENSION X(73),Y(73),YDY(73),D(73),E(73),F1(73),F2(73),       UC07A060
       1G(73),T1(73,73),T2(73,73),U1(73,73),U2(73,73),                  UC07A080
       2G8(73),G9(73),F7(73),F8(73),Z1(73),Z2(73)                       UC070062
        IF(EPS2) 51,51,77                                               UC074790
   51 EPS2=1.E-6                                                        UC074795
   77 S1=0.                                                             UC074800
        L=1                                                             UC074820
        DO211 J=1,N                                                     UC074830
        DO218 K=1,N                                                     UC074840
        Z1(K)=T2(J,K)*G(K)                                              UC074860
  218 Z2(K)=-T1(J,K)*G(K)                                               UC074880
        F7(J)=SM3PT(X,Z1,N,J)*W                                         UC074902
        F8(J)=SM3PT(X,Z2,N,J)*W                                         UC074922
        F1(J) = F7(J)                                                   UC074960
  211 F2(J) = F8(J)                                                     UC074965
  215 DO 40 K=1,N                                                       UC074980
        Z1(K)=F1(K)*G(K)                                                UC075000
   40 Z2(K)=F2(K)*G(K)                                                  UC075020
        TZR=SM3PT(X,Z1,N,2)                                             UC075042
        TZI=SM3PT(X,Z2,N,2)                                             UC075062
  181 PRINT 182,L,TZR ,TZI                                              UC075100
  182 FORMAT(//2X,8HSURFACE ,13HITERATION NO.I3,10X,5HTZR =E14.6,10X    UC075120
       1,5HTZI =E14.6)                                                      5122
        IF (I4-1) 80,281,281                                                5130
  281 PRINT 381                                                             5132
  381 FORMAT (94X,9HF1 EQUALS)
        PRINT313,(F1(J),J=1,N)                                          UC075140
        PRINT161                                                        UC075160
        PRINT313,(F2(J),J=1,N)                                          UC075180
   80 L=L+1                                                             UC075200
        IF(L-50)83,83,79                                                UC075220
   83 S2=TZR**2+TZI**2                                                  UC075240
        IF(((S2-S1)/S2)**2-EPS2**2)79,79,85                             UC075260
   85 S1=S2                                                             UC075280
  161 FORMAT (94X,9HF2 EQUALS)                                          UC075300
  313 FORMAT (7E17.8)                                                   UC075320
        DO 184 J=1,N                                                    UC075370
        DO119 K=1,N                                                     UC075380
        Z1(K)=F1(K)*U1(J,K)-F2(K)*U2(J,K)                               UC075400
  119 Z2(K)=F1(K)*U2(J,K)+F2(K)*U1(J,K)                                 UC075420
        G8(J)=F7(J)+SM3PT(X,Z1,N,J)/A                                   UC075442
   84 G9(J)=F8(J)+SM3PT(X,Z2,N,J)/A                                     UC075462
        F1(J)=G8(J)+(F1(J)-G8(J))*HALF                                  UC075480
  184 F2(J)=G9(J)+(F2(J)-G9(J))*HALF                                    UC075500
        IF(I4-2)215,86,88                                               UC075522
   86 F1(1)=F1(2)*G(1)/G(2)                                             UC075542
        F2(1)=F2(2)*G(1)/G(2)                                           UC075562
   88 F1(N)=F1(NM1)*G(N)/G(NM1)                                         UC075602
        F2(N)=F2(NM1)*G(N)/G(NM1)                                       UC075622
        GO TO 215                                                       UC075640
   79 NST=2                                                                 5660
        RETURN                                                          UC07A100
```

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ABSTRACT

A FORTRAN program is presented which computes the sound pressures, at the surface and in the near or far field, radiated by an arbitrary surface of revolution which is vibrating in an almost arbitrary pattern with arbitrary frequency and phase. The method of solution, the input data, and the output data are all described in detail.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

This report describes and explains detailed procedures for the numerical calculation of the sound pressure field outside a surface of revolution vibrating with a specified distribution of velocity. More generally, these procedures provide for the numerical solution of the scalar wave equation when the normal gradient is specified on a surface of revolution. The method of calculation, which is based on the theory of Reference 1*, is presented in a FORTRAN program that can be used to solve specific problems with an IBM 7090 computer. In addition, the procedures are explained in detail so that they may be more easily modified and adapted to solve special problems.

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*References are listed on page 40.

The surface boundary must be idealized as a closed surface of revolution without discontinuities. The normal velocity of the surface must be of the form

$$v(x,y,\varphi,t) = v_o\, \psi(x) \cos(m\varphi) \cos(\omega t + \delta) \qquad [1]$$

where x, y, and φ are the cylindrical coordinates of a surface point with the x-axis being the axis of symmetry; v_o is a velocity amplitude; $\psi(x)$ is an arbitrary function of longitudinal position; and m and ω are likewise arbitrary. If the phase angle δ is not constant over the surface, the velocity distribution must be written as the sum of two component distributions, 90 degrees apart in the time phase, and a separate numerical solution must be made for each component.

In the method of solution described here the sound pressure at any point in the field is computed by simple quadratures from the Helmholtz integral in terms of velocity and sound pressure at the surface of the vibrating body. But first, the sound pressure at the surface must be computed by solution of an integral equation of the second kind, and it is this solution of the integral equation that is the salient feature of this method.

The program and procedures are described in five parts: (a) specifying the surface shape, (b) computing the influence coefficients for the specific surface shape and vibration frequency, (c) specifying the velocity distribution on the surface, (d) computing the sound pressure at the surface, and (e) computing the sound pressure in the field. Each part may be accomplished by one or more alternative procedures as explained below, and in addition any new procedure may be introduced by a suitable subroutine. The FORTRAN program itself is listed in the Appendix.

SURFACE SHAPE

The minimum data required to specify the surface shape are the values for the section radii $y(x)$ at an odd number, $n \leq 73$, of stations along the x-axis. The spacing between stations need not be uniform; in fact, the accuracy of the calculation is increased if stations are concentrated where the profile curvature is high or where the velocity distribution changes fast. The first and last stations must be points at which $y = 0$. The function $y dy/dx$ is used often in many of the subsequent calculations. It is therefore computed or specified at each station and stored. There are three alternative methods of specifying the station data.

(a) The numerical values for x_i, y_i, $(y dy/dx)_i$; $i = 1, 2, \ldots n$, are specified in the input data. If the $y dy/dx$ data are not specified, the program will then compute this quantity (cards 380 to 560 in the FORTRAN program). At stations 1 and 2 it is assumed that a circular profile passes through these points, whence

$$\left(y \frac{dy}{dx} \right)_1 = \frac{y_2^2 + (x_2 - x_1)^2}{2(x_2 - x_1)} \qquad [2]$$

and

$$\left(y \frac{dy}{dx} \right)_2 = \left(y \frac{dy}{dx} \right)_1 + x_1 - x_2 \qquad [3]$$

Similar relations are assumed for stations n and n-1. At the remaining n-4 stations it is assumed that the profile is parabolic through each point and its two neighbors. Hence

$$\left(y \frac{dy}{dx} \right)_i = y_i \left[\frac{y_{i+1} - y_i}{x_{i+1} - x_i} + \frac{y_i - y_{i-1}}{x_i - x_{i-1}} - \frac{y_{i+1} - y_{i-1}}{y_{i+1} - y_{i-1}} \right] \qquad [4]$$

(b) If the vibrating surface is a spheroid, either prolate or oblate, and of any eccentricity, it is sufficient to input the semilongitudinal axis a and the midsection radius b. The program (subroutine SHAPE) will then select n-values for x which are either equally spaced,

$$x_i = \frac{a}{n-1} \ (2i - 1 - n) \qquad [5]$$

or, optionally, distributed as

$$x_i = a \cos\left(\frac{n-i}{n-1} \ \pi\right). \qquad [6]$$

This last distribution concentrates most stations at the two ends where the curvature is highest for a prolate body.

Then, y_i and $(ydy/dx)_i$ are computed from the equations

$$y_i = b(1 - x_i^2 / a^2)^{\frac{1}{2}}, \qquad [7]$$

and

$$\left(y \frac{dy}{dx}\right)_i = x_i \, b^2/a^2 \qquad [8]$$

(c) If the profile of the vibrating surface is some special analytical shape, or if a special station spacing is desired, then a special program may be introduced as an alternative subroutine SHAPE.

VELOCITY DISTRIBUTION ON SURFACE

The vibration velocity must be specified in the form of Equation [1]. If the time phase of the velocity is variable over the surface, the velocitydistribution is specified as the sum of two single-phased distributions in quadrature with each other. The entire calculation, including the components of the far-field pressures, is made separately for each single-phase distribution, and the far-field pressures are then added, with proper regard for phase, to give the net field pressure.

In general, the quantity used to characterize a single-phase velocity distribution is a nondimensional surface velocity amplitude,

$$G(x) = \psi y/(a \sin\beta); \text{ where } \cot\beta = -dy/dx, \qquad [9]$$

which must be computed and stored for the n stations x_1 to x_n. The $\psi(x)$ data for the calculation may be specified in three alternative ways.

(a) If the motion is predominantly a longitudinal vibration, with an associated radial motion due to a Poisson ratio effect, the relative velocity normal to the surface is (see cards 800 to 960 in the FORTRAN program)

$$\psi(x) = \psi_\ell(x)\cos\beta + \psi_r(x)\sin\beta \qquad [10]$$

where $v_0\,\psi_\ell \cos(m\varphi)\cos(\omega t + \delta)$ is the longitudinal velocity at x,

$$\psi_r(x) = -\sigma\, y\, d\psi_\ell/dx \qquad [11]$$

and σ is the effective Poisson ratio for the material and structure (e.g., $\sigma \simeq 0.30$ for steel). If the values of $\psi_\ell(x)$ are specified at the n stations, the derivative is computed from the 3-point parabolic formula

$$\left(\frac{d\psi_\ell}{dx}\right)_i = \frac{(\psi_\ell)_i - (\psi_\ell)_{i-1}}{x_i - x_{i-1}} + \frac{(\psi_\ell)_{i+1} - (\psi_\ell)_i}{x_{i+1} - x_i} - \frac{(\psi_\ell)_{i+1} - (\psi_\ell)_{i-1}}{x_{i+1} - x_{i-1}} \qquad [12]$$

(b) If the motion is as specified by Equations [10] and [11], except that $\psi_\ell(x)$

is approximated by a simple trigonometric formula, then

$$\psi_{\ell}(x) = \cos\left[\frac{L\pi}{2a}(x - x_1)\right], \qquad [13]$$

and

$$\psi_r(x) = \frac{\sigma\pi Ly}{2a} \sin\left[\frac{L\pi}{2a}(x - x_1)\right]. \qquad [14]$$

Thus $L = 0$ means a rigid body vibration in the longitudinal direction, and $L > 0$ means an accordion vibration mode with L longitudinal nodal sections.

(c) If the velocity distribution has some simple analytical form, or if G(x) is specified by a table of experimental data, the velocity may be specified by a special subroutine which replaces SURVEL.

INFLUENCE FUNCTIONS

The purpose here is to compute and store numerical values for the four functions:

$$T_1(x', x) = \frac{a}{\pi} \int_0^{\pi} \frac{\cos(kr)}{r} \cos(m\varphi)\, d\varphi, \qquad [15]$$

$$T_2(x', x) = \frac{a}{\pi} \int_0^{\pi} \frac{\sin(kr)}{r} \cos(m\varphi)\, d\varphi, \qquad [16]$$

$$U_1(x', x) = \frac{a}{\pi} \int_0^{\pi} \left[\frac{\cos(kr)}{r} + k\sin(kr)\right]$$

$$\times \left[y\frac{dy}{dx}(x - x') - y^2 + yy'\cos\varphi\right] \frac{\cos(m\varphi)}{r^2}\, d\varphi, \qquad [17]$$

$$U_2(x', x) = \frac{a}{\pi} \int_0^{\pi} \left[\frac{\sin(kr)}{r} - k\cos(kr)\right]$$

$$\times \left[y\frac{dy}{dx}(x - x') - y^2 + yy'\cos\varphi\right] \frac{\cos(m\varphi)}{r^2}\, d\varphi. \qquad [18]$$

where

$$r^2 = (x - x')^2 + (y - y')^2 + 2yy'(1 - \cos\varphi). \qquad [19]$$

These must be calculated for each ordered pair of values of (x', x) where $x' = x_i$, $x = x_j$ and $i, j = 1, 2, \ldots n$. Note that these functions depend not only on the surface shape, but also on the wave number k, and on m which specifies the number of circumferential variations in the vibration pattern. The procedure for computing these functions depends on whether $x = x'$, and whether x or x' is an end station.

Case I. If $x \neq x'$ and $y, y' \neq 0$ (cards 1140 to 2060 in the FORTRAN program), then the eight values for T_1, T_2, U_1, and U_2 are evaluated in terms of six different integrals S_1 to S_6 defined over the range $0 \leq \varphi \leq \pi$,

$$T_1(x',x) = T_1(x, x') = aS_1/\pi \qquad [20]$$

$$T_2(x',x) = T_2(x,x') = aS_2/\pi \qquad [21]$$

$$U_1(x',x) = \frac{a}{\pi}\left[y\frac{dy}{dx}(x - x') - y^2\right] S_3 + \frac{a}{\pi} yy' S_5 \qquad [22]$$

$$U_1(x, x') = \frac{a}{\pi}\left[y'\frac{dy'}{dx'}(x' - x) - y'^2\right] S_3 + \frac{a}{\pi} yy' S_5 \qquad [23]$$

$$U_2(x',x) = \frac{a}{\pi}\left[y\frac{dy}{dx}(x - x') - y^2\right] S_4 + \frac{a}{\pi} yy' S_6 \qquad [24]$$

$$U_2(x, x') = \frac{a}{\pi}\left[y'\frac{dy'}{dx'}(x' - x) - y'^2\right] S_4 + \frac{a}{\pi} yy' S_6 \qquad [25]$$

where

$$S_1 = \int_0^\pi \frac{\cos(kr)\cos(m\varphi)}{r}\, d\varphi \qquad [26]$$

$$S_2 = \int_0^\pi \frac{\sin(kr)\cos(m\varphi)}{r}\, d\varphi \qquad [27]$$

$$S_3 = \int_0^\pi \left[\frac{\cos(kr)}{r^3} + \frac{k\sin(kr)}{r^2}\right] \cos(m\varphi)\, d\varphi \qquad [28]$$

$$S_4 = \int_0^{\pi} \left[\frac{\sin (kr)}{r^3} - \frac{k \cos (kr)}{r^2} \right] \cos (m\varphi)\, d\varphi \qquad [29]$$

$$S_5 = \int_0^{\pi} \left[\frac{\cos (kr)}{r^3} + \frac{k \sin (kr)}{r^2} \right] \cos (m\varphi) \cos \varphi \, d\varphi \qquad [30]$$

$$S_6 = \int_0^{\pi} \left[\frac{\sin (kr)}{r^3} - \frac{k \cos (kr)}{r^2} \right] \cos (m\varphi) \cos \varphi \, d\varphi \qquad [31]$$

The six integrals are evaluated simultaneously by dividing the range o to π into b/2 equal subintervals and by using a two-point Guassian quadrature formula for each subinterval, i.e., by using a b-point formula for the range.

Thus for S_1 ,

$$S_1 = \sum_{i=1}^{b} [\cos (m\varphi_i) \cos (kr_i)]/r_i, \qquad [32]$$

where

$$r_i^2 = (x - x')^2 + (y - y')^2 + 2yy'(1 - \cos \varphi_i), \qquad [33]$$

$$\varphi_i = (i - 0.7887)\, 2\pi/b, \text{ if } i \text{ is odd}, \qquad [34]$$

$$\varphi_i = (i - 0.2113)\, 2\pi/b, \text{ if } i \text{ is even}, \qquad [35]$$

$$x = x_j \,;\; x' = x_k \,;\; y = y_j \,;\; y' = y_k \,; \qquad [36]$$

$$j\,,\; k = 2,3,\ldots,n - 1;\; j \neq k,$$

and similar equations are obtained for S_2 to S_6. The number of subintervals is at least 10 and increases with k and with m, and with the disparity between r for $\varphi = 0$ and r for $\varphi = \pi$, in Equation [33]. Specifically, b/2 is taken as the largest integer within

$$b/2 = \max \; [\, (92. + 8.4\, r_0^2/r_\pi^2)^{\frac{1}{2}},\; 2k\,(r_\pi - r_0),\; 20m \qquad [37]$$

This prescription for b is intended to ensure that the integrals are calculated with at least 3-figure accuracy. It is based on some intuitive arguments plus a few check calculations. But the prescription does require further investigation and possible modification. It is clear that S_2, for example, can be evaluated with adequate precision at far fewer stations than S_1, but it is advantageous to evaluate all six integrals simultaneously at every value of φ.

Case II. When y or $y' = 0$, i.e., one station is an end station (cards 2100 to 2680 in the FORTRAN program). If the velocity distribution is not uniform in φ, i.e., if $m > 0$, all four functions T_1, T_2, U_1, and U_2 must be zero. If $m = 0$, and $y = 0$ i.e., $x = x_1$ or x_n, the four functions may be calculated without quadrature.

$$T_1\ (x', x) = T_1\ (x, x') = \frac{a}{s}\ \cos\ (ks), \qquad [38]$$

$$T_2\ (x', x) = T_2\ (x, x') = \frac{a}{s}\ \sin\ (ks), \qquad [39]$$

$$U_1\ (x', x) = ay\frac{dy}{dx}\ (x - x')\ \left[\frac{\cos\ (ks)}{s^3} + \frac{k\sin\ (ks)}{s^2}\right], \qquad [40]$$

$$U_1\ (x,x') = a\left[y'\frac{dy'}{dx'}\ (x' - x) - y'^2\right]\left[\frac{\cos\ (ks)}{s^3} + \frac{k\ \sin\ (ks)}{s^2}\right], \qquad [41]$$

$$U_2\ (x', x) = ay\ \frac{dy}{dx}\ (x - x')\left[\frac{\sin\ (ks)}{s^3} - \frac{k\cos\ (ks)}{s^2}\right], \qquad [42]$$

$$U_2\ (x,x') = a\left[y'\ \frac{dy'}{dx'}\ (x' - x) - y'^2\right]\left[\frac{\sin\ (ks)}{s^3} - \frac{k\cos\ (ks)}{s^2}\right], \qquad [43]$$

where

$$s^2 = (x - x')^2 + y'^2\ . \qquad [44]$$

Note that ydy/dx always is finite and not zero, even though y may be zero.

Case III. $x = x'$; $y = y' \neq 0$. These conditions give the dominant terms in the T_1 and U_1 matrices, but they are the most difficult to compute with accuracy because T_1 and U_1 are singular at $x = x'$.

These terms will be used later (see Equation [66]) in a modified version of a 3-point Simpson's rule quadrature formula over the range $x_{j-1} \leq x \leq x_{j+1}$, $x' = x_j$. Hence

$T_1(x_j, x_j)$; $j = 2,3,\ldots,n-1$, is computed (cards 2700 to 3780) as that value which used in conjunction with $T_1(x_j, x_{j-1})$ and $T_1(x_j, x_{j+1})$ in this quadrature formula gives an accurate value to $\int_{x_{j-1}}^{x_{j+1}} T_1(x_j, x)\,dx$. That is,

$$\int_{x_{j-1}}^{x_j} T_1(x_j, x)\,dx + \int_{x_j}^{x_{j+1}} T_1(x_j, x)\,dx$$

$$= \frac{x_{j+1} - x_{j-1}}{6} \left[\frac{1+3p}{1+p} T_1(x_j, x_{j-1}) + \frac{4}{1-p^2} T_1(x_j, x_j) + \frac{1-3p}{1-p} T_1(x_j, x_{j-1}) \right] \qquad [45]$$

where

$$p = (2x_j - x_{j-1} - x_{j+1}) / (x_{j+1} - x_{j-1}). \qquad [46]$$

If x_j is midway between x_{j-1} and x_{j+1}, then $p = 0$ and Equation [45] reduces to the common 3-point Simpson's rule. But if the stations are not equally spaced, Equation [45] is equivalent to first using a 3-point Lagrange interpolation formula to obtain T_1 at the midpoint and then applying the conventional Simpson's rule.

Each integral on the left-hand side of Equation [45] is evaluated by a 4-point approximation which avoids the point $x = x_j$. Thus, for the first integral,

$$\int_{x_{j-1}}^{x_j} T_1\,dx = \sum_{i=1}^{4} h_i (x_j - x_{j-1})\, T_1(x_j, x_i) \qquad [47]$$

where

$$\frac{x_i - x_j}{x_{j-1} - x_j} = .0736,\ .2,\ .6,\ .2 \qquad [48]$$

and

$$h_i = \frac{1}{5}, \frac{4}{30}, \frac{16}{30}, \frac{4}{30} \qquad [49]$$

for $i = 1,2,3,4$ respectively.

This quadrature formula, Equation [47], results from fitting a special 1-point logarithmic quadrature approximation (see Reference 1, Equation B2) to the range $x_j - (x_j - x_{j-1}) / 5 < x < x_j$, which is one-fifth of the total range, and a 3-point Simpson's rule formula to the remaining four-fifths of the range. The second integral in Equation [45] is evaluated by the same formulas, but replacing $j-1$ by $j+1$ in Equations [47] to [49].

The net result of Equations [45] to [49] is that $T_1 (x_j, x_j)$ is replaced by a particular mean of eight values of $T_1 (x_j, x)$ which are computed at six new stations for x in the neighborhood of x_j, in addition to the two stations already computed at $x = x_{j-1}$ and $x = x_{j+1}$.

First, y(x) is computed for the three x-points between x_{j-1} and x_j from the equation

$$y^2 = y_k^2 + 2 (x-x_k)\left(y \frac{dy}{dx}\right)_k + \frac{(x-x_k)^2}{x_{k-1}-x_k} \left[\left(y \frac{dy}{dx}\right)_{k-1} - \left(y \frac{dy}{dx}\right)_k \right] \qquad [50]$$

For the three points between x_k and x_{k+1}, $k+1$ is substituted for $k-1$ in Equation [50]. This equation may be interpreted as a Taylor expansion for y^2 about $x = x_k$, with the last factor in square brackets being an approximation for $d^2 (y^2)/dx^2$ at $x = x_k$. The equation is possibly the simplest form using y and ydy/dx at $x = x_k$ and ydy/dx at x_{k-1}, and would be an exact expression if the profile through x_k and x_{k-1} was an ellipse centered on the x axis. If an elliptic section were a grossly inadequate representation of the local profile, then Equation [50] may give impossible results, i.e., y^2 may be negative.

Each of the six values of $T_1 (x_j, x)$ is now computed from Equation [15] by a new quadrature program (see cards 3040 to 3640 in the FORTRAN program) designed to decrease the station spacing in the range where the integrand changes rapidly. We divide the interval $0 \leq \varphi \leq \pi$ into five equal subintervals of width $\pi/5$ and further subdivide each subinterval into 2,4,8,16, etc. segments, in each case halving the segment until the resulting value for T_1 converges within a specified limit ϵ.

For example, suppose the integration over the first subinterval, $0 \leq \varphi \leq \pi/5$, has converged to S_1, and suppose that the integration over the second subinterval, $\pi/5 \leq \varphi \leq 2\pi/5$, results in S_2 (m) when the second subinterval is divided into m segments, then m is increased to 2m , and the integration results in S_2 (2m). If

$$\left| \frac{S_2\,(2m) - S_2\,(m)}{S_2\,(2m) + S_1} \right| \leq \epsilon \tag{51}$$

Then S_2 (2m) is accepted as S_2, and the integration is now continued to the third interval $2\pi/5 \leq \varphi \leq 3\pi/5$, starting with m = 2. The program is arranged so that every time the interval is halved, all of the previous station computations are used again and need not be recomputed.

These special procedures, Equations [45] to [51], are used only to compute $T_1\,(x_j, x_j)$; $j = 2,3,\ldots,n-1$.

The function $T_2\,(x', x)$ is finite at $x = x' = x_j$ and is computed from (see cards 3800 to 4140 in the FORTRAN program)

$$T_2\,(x', x) = \frac{a}{\pi} \int_0^{\pi/2} \frac{\sin\,(kr)}{r} \cos\,(2m\theta)\, d\theta \tag{52}$$

where

$$r = 2y \sin\,\theta \tag{53}$$

The integral is evaluated by dividing the range $0 \leq \theta \leq \pi/2$ into b subintervals and using a two-point Gaussian quadrature formula for each subinterval. The number of subintervals is taken as

$$b = \text{Max} \quad [10,\ 3\,ky,\ 5\,m]\ , \tag{54}$$

which again is an arbitrary prescription intended to give an accuracy of at least three figures.

The functions $U_1\ (x', x)$ and $U_2\,(x', x)$, for $x \to x' = x_j$, are both computed on the partial assumption that the profile section near $x \to x_j$ can be approximated by a circle, whence (see Reference 1, Equation [A4]).

$$U_1\ (x_j, x_j) = -\frac{1}{2}\ T_1\ (x_j, x_j) - \frac{ka}{\pi} \int_0^{\pi/2} \sin\,(kr)\cos(2m\theta)d\theta, \tag{55}$$

and

$$U_2\ (x_j, x_j) = -\frac{1}{2}\ T_2\ (x_j, x_j) + \frac{ka}{\pi} \int_0^{\pi/2} \cos(kr)\cos(2m\theta)d\theta. \tag{56}$$

The two integrals are evaluated, simultaneously with Equation [52], by the method described for $T_2\ (x_j, x_j)$.

<u>Case IV.</u> If $x = x' = x_1$ or x_n; $y = y' = 0$. Then T_1, T_2, U_1, and U_2 are all zero unless $m = 0$. In the latter event, it is assumed that stations 1 and 2 are on the same spherical cap for which simple exact expressions for T and U are available (see Reference 1, Equation [A5] and see cards 4160 to 4460 of the FORTRAN program). For $x' = x_1$,

$$T_1\ (x_1,\ x_1) = \frac{4a}{ks^2}\ \sin\,(ks) - T_1\ (x_1,\ x_2) \tag{57}$$

$$T_2\ (x_1,\ x_1) = ka \tag{58}$$

$$U_1\ (x_1,\ x_1) = -T_1\ (x_1,\ x_1)/2 \tag{59}$$

$$U_2\ (x_1,\ x_1) = 0 \tag{60}$$

where

$$s^2 = (x_1 - x_2)^2 + y_2^2 \qquad [61]$$

For $x' = x_n$, similar forms are used, by replacing x_1 by x_n, and x_2 by x_{n-1}.

PRESSURE AT VIBRATING SURFACE

The components of the nondimensional surface pressure,

$$F_1(x) + iF_2(x) = \frac{P_1(x) + iP_2(x)}{\rho c v_0 \cos(m\phi)} \qquad [62]$$

are computed for each station x_1 to x_n. This requires a solution for the set of 2n simultaneous integral equations

$$F_1(x') = k\int_{-a}^{a} G(x)\, T_2(x',x)\, dx + \frac{1}{a}\int_{-a}^{a} [F_1(x)\, U_1(x',x) - F_2(x) U_2(x',x)]dx \qquad [63]$$

$$F_2(x') = -k\int_{-a}^{a} G(x)\, T_1(x',x) dx + \frac{1}{a}\int_{-a}^{a} [F_1(x)\, U_2(x',x) + F_2(x)\, U_1(x',x)]\, dx \qquad [64]$$

where

$$x' = x_j;\ j = 1,2,3\ldots n. \qquad [65]$$

Each integral is approximated by the modified Simpson's rule formula previously described under Equation [45]. If station j is an even-numbered station, this quadrature formula is

$$\int_{x_1}^{x_n} f dx = \sum_{i=2,4}^{n-1} \frac{x_{i+1} - x_{i-1}}{6} \left[\frac{1+3p}{1+p} f_{i-1} \frac{+4}{1-p^2} f_i + \frac{1-3p}{1+p} f_{i+1}\right] \qquad [66]$$

where

$$p = (2x_i - x_{i-1} - y_{i+1})/(x_{i+1} - y_{i-1}). \qquad [67]$$

And if station j is an odd-numbered station,

$$\int_{x_1}^{x_n} f dx = (x_2 - x_1)\frac{f_1 + f_2}{2} + (x_n - x_{n-1})\ \frac{f_n + f_{n-1}}{2}$$

$$+ \sum_{i=3,5}^{n-2} \frac{x_{i+1} - x_i}{6} \left[\frac{1+3p}{1+p} f_{i-1} + \frac{4}{1-p^2} f_i + \frac{1-3p}{1+p} f_{i+1}\right]. \qquad [68]$$

There are now $2n \leq 146$ linear algebraic equations to solve simultaneously. The method normally used (see cards 4800 to 5500) is the Gauss-Seidel process of simple iteration. We start with the initial trial solution obtained by taking F_1 as only the first term on the right-hand side of Equation [63], and F_2 as only the first term on the right of Equation [64]. These initial values are then substituted into Equations [63] and [64] for $j = 1$, and a new pair of values for $F_1(x_1)$ and $F_2(x_1)$ are determined. Then, using these new values at station 1, a new pair of values is calculated for station 2; the process being continued until the second pair of values has been calculated for every station. Then the entire cycle for the n-stations is repeated to get a third approximation to the vectors $F_1(x_i)$ and $F_2(y_i)$; etc., etc.

The iteration calculation (cards 5220 to 5260) is normally terminated after 50 cycles of calculation, or before, if a particular weighted mean square value of the surface pressure has converged within a specified limit ϵ_2. That is, if S_2 is the current value of this mean square surface pressure and S_1 is the value after the previous iteration cycle, then the criterion for terminating the iterations is

$$(1 - S_1/S_2)^2 \leq \epsilon_2^2 \qquad [69]$$

where ϵ_2 is ordinarily taken as 0.001, but may be prescribed otherwise in the input data.

The mean square surface pressure for this purpose is defined by

$$S_2 = \left[\int_0^{\ell} F_1 \ G dx\right]^2 + \left[\int_0^{\ell} F_2 \, G dx\right]^2 \qquad [70]$$

and is calculated after every iteration cycle by the quadrature formula of Equation [66].

The simple iteration method may not converge properly at high frequencies, e. g., if ka is greater than some critical eigenfrequency. In such cases an alternative procedure in the calculating program (cards 5480, 5500) is to use as the trial vector for the (k+1) iteration, not the solution vector of the kth iteration, but the mean between that and the trial vector for the kth iteration.

Also, at high frequencies, the computed values for F_1 and F_2 at stations 1 and N tend to be particularly inaccurate unless the station spacing at the ends is very small. An alternative procedure (see cards 5520 to 5620), used when kydy/dx > 4, is to compute the values for stations 1 and n by extrapolation from the values for stations 2 and n-1, according to the formulas

$$F_1(x_1) = F_1(x_2)\, G(x_1)/G(x_2) \qquad [71]$$

$$F_1(x_n) = F_1(x_{n-1})\, G(x_n)/G(x_{n-1}) \qquad [72]$$

and similarly for $F_2(x_1)$ and $F_2(x_n)$. This is valid because the ratio $F(x)/G(x)$ depends on the local surface impedance and the local radius of curvature, and at high frequencies these are the same for station 1 as for station 2 (see Reference 1, Equation [3]).

A special situation occurs when the shape of the vibrating surface is similar to a prolate ellipsoid, and the pattern of the nondimensional velocity distribution G(x) is proportional to the angular-spheroidal wave function $S_{m\ell}(x)$ at that frequency. Then the surface pressure may be calculated from the simple equation

$$F_1(x) + i\, F_2(x) = (c_1 + i\, c_2)\, G(x) \qquad [73]$$

where (see Reference 1, Equation [6])

$$c_1 + ic_2 = \frac{ika}{\xi} \; \frac{a^2}{b^2} \; \frac{R_{m\ell}(3)}{R_{m\ell}(3)'} \tag{74}$$

and where ξ is the reciprocal eccentricity of the elliptic section, $R_{m\ell}^{(3)}$ is the radial-spheroidal wave function of the third kind with arguments ka/ξ and ξ, and $R_{m\ell}^{(3)'}$ is the derivative with respect to ξ. In this spheroidal approximation it is not necessary to calculate the influence functions, Equations [15] to [18], and it is not necessary to solve the set of simultaneous equations, [63] and [64], but merely to supply the constants c_1 and c_2 as part of the input data (see cards 4620 to 4780).

A completely different procedure for calculating the surface pressures can most easily be introduced as a new subroutine SURFPR to replace the iteration process.

RADIATION IMPEDANCE AND POWER OUTPUT

The modal radiation impedance is defined in Reference 1, Equation [23], as

$$Z = \frac{2\pi\rho c a}{\epsilon_m} \int_{-a}^{a} F(x) \cdot G(x)\, dx \tag{75}$$

where $\epsilon_m = 1$ for $m = 0$, and $\epsilon_m = 2$ for $m > 0$. It is convenient to compute the modal impedance coefficient defined by

$$\bar{z}_1 + i\,\bar{z}_2 = Z/(4\pi\rho c a^2) \tag{76}$$

These coefficients depend on the shape of the surface, the vibration pattern, and the reduced frequency ka, but are independent of the absolute size of the surface, the vibration amplitude, and the characteristic impedance of the medium. The modal impedance coefficients are first calculated from Equation [76] by the quadrature formula of Equation [66], (see cards 5000 to 5060 and 5660 to 5760).

The time average of the radiated power is then

$$\Pi = 4\pi \rho c a^2 \frac{\bar{z}_1}{2} v_0^2 , \qquad [77]$$

and the source power level of the vibrating surface is

$$\bar{L} = 10 \log_{10} (\Pi/\Pi_{ref}) \text{ db}, \qquad [78]$$

where the reference power Π_{ref} is that of a spherical source which generates a sound pressure of 0.0002 μ bars rms at 1 yd from the center. Thus, if a is in feet, v_0 is the velocity amplitude (defined in Equation [1]) in feet per second, and the fluid is sea water, then

$$\bar{L} = 10 \log_{10} [\bar{z}_1 (5.49 \cdot 10^9 a v_0)^2]. \qquad [79]$$

Note that v_0 is here defined as a peak velocity, not an rms velocity.

FAR-FIELD SOUND PRESSURES

The sound pressure in the far field at distance R', longitude angle φ', azimuth angle θ', relative to the longitudinal axis of the vibrating surface, is given by (see Reference 1, Equation [28])

$$\begin{aligned} p_1(R',\theta',\varphi') &= \rho c v_0 \cos(m\varphi')(-i)^{m+1} e^{ikR'}/R' \\ &\times \int_{-a}^{a} \frac{k dx}{2} \left\{ \frac{\psi y}{\sin\beta} J_m(ky \sin\theta') \cos(kx \cos\theta') \right. \\ &- \frac{p_1(x) \cos\theta'}{\rho c v_0 \cos(m\varphi)} y \frac{dy}{dx} J_m(ky \sin\theta') \cos(kx \cos\theta') \\ &- \frac{p_2(x) \cos\theta'}{\rho c v_0 \cos(m\varphi)} y \frac{dy}{dx} J_m(ky \sin\theta') \sin(kx \cos\theta') \\ &+ \frac{p_1(x) \sin\theta'}{\rho c v_0 \cos(m\varphi)} y J_m'(ky \sin\theta') \sin(kx \cos\theta') \\ &\left. - \frac{p_2(x) \sin\theta'}{\rho c v_0 \cos(m\varphi)} y J_m'(ky \sin\theta') \sin(kx \cos\theta') \right\}, \end{aligned} \qquad [80]$$

$$P_2(R',\theta',\varphi') = \rho c v_0 \cos(m\varphi')\,(-i)^{m+1} e^{ikR'}/R'$$

$$\times \int_{-a}^{a} \frac{k\,dx}{2} \left\{ \frac{\psi y}{\sin\beta} J_m(ky\sin\theta')\cos(kx\cos\theta') \right.$$

$$+ \frac{p_1(x)\cos\theta'}{\rho c v_0 \cos(m\varphi)}\, y\, \frac{dy}{dx}\, J_m(ky\sin\theta')\sin(kx\cos\theta')$$

$$- \frac{p_2(x)\cos\theta'}{\rho c v_0 \cos(m\varphi)}\, y\, \frac{dy}{dx}\, J_m(ky\sin\theta')\cos(kx\cos\theta')$$

$$+ \frac{p_1(x)\sin\theta'}{\rho c v_0 \cos(m\varphi)}\, y\, J_m'(ky\sin\theta')\cos(kx\cos\theta')$$

$$\left. - \frac{p_2(x)\sin\theta'}{\rho c v_0 \cos(m\varphi)}\, y\, J_m'(ky\sin\theta')\sin(kx\cos\theta') \right\} . \qquad [81]$$

The two integrals in Equations [80] and [81] (divided by a) may be interpreted as pressure coefficients, $q_1(\theta')$ and $q_2(\theta')$ respectively, in terms of which the amplitude of the far-field sound pressure at R', θ', φ' is

$$|p(R',\theta',\varphi')| = \rho c v_0 \cos(m\varphi')\, aq/R' \qquad [82]$$

where

$$[q(\theta')]^2 = q_1{}^2 + q_2{}^2 \quad . \qquad [83]$$

Hence the time average of the radiated power is

$$\Pi = \frac{\pi \rho c v_0{}^2 a^2}{\epsilon_m} \int_0^{\pi} q^2 \sin\theta'\, d\theta', \qquad [84]$$

which should give the same value as Equation [77]. Comparing Equations [79] and [77], the directivity factor at azimuth θ' and longitude φ' is

$$D(\theta',\varphi') = q^2 \cos^2(m\varphi')/\bar{z}_1 \qquad [85]$$

And, when averaged over φ', the directivity factor is

$$\bar{D}(\theta') = q^2/(\bar{z}_1\, \epsilon_m) \qquad [86]$$

The sound pressure level in the far field at direction θ', and averaged over φ', and relative to 0.0002 μ bar at 1 yd from a spherical source in sea water, is given by

$$L(\theta') = \bar{L} + 10 \log_{10} \bar{D} \qquad [87]$$

where $\bar{L}$ is given by Equation [78].

These equations are used to compute the far-field pressure level and directivity factors for every angular direction that is specified in the input data (see cards 6040 to 7040). The integrals of Equations [80] and [81] are evaluated by the 3-point quadrature formula of Equation [66]. The two Bessel functions J_m and J_m' are evaluated (see cards 6180 to 6460) by the recurrence relations,

$$J_{n+1}(x) = \frac{n}{x} J_n(x) - J_n'(x), \qquad [88]$$

$$J_{n+1}'(x) = \left[1 - \frac{n(n+1)}{x^2}\right] J_{\cancel{m}\,n}(x) + \left[\frac{n(n+1)}{x}\right] J_{\cancel{m}\,n}'(x), \qquad [89]$$

which can easily be obtained from the more common recurrence formula. $J_0(x)$ and $J_0'(x)$ are first computed by a library subroutine.

NEAR-FIELD SOUND PRESSURES

The sound pressure in the near field, at a point whose cylindrical coordinates are x', y', φ', is expressed in nondimensional units in almost the same form as Equations [63] and [64] for the surface pressure. Thus

$$F_1(x') = \int_{-a}^{a} \frac{dx}{2} \left[kG(x)\, T_2(x', x) + a^{-1} F_1(x)\, U_1(x', x) - a^{-1} F_2(x) U_2(x', x)\right] \qquad [90]$$

$$F_2(x') = \int_{-a}^{a} \frac{dx}{2} \left[-kG(x) T_1(x', x) + a^{-1} F_1(x)\, U_2(x', x) + a^{-1} F_2(x)\, U_1(x', x)\right] \qquad [91]$$

See Correction sheet.
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Note the factor $\frac{1}{2}$, which is not present in the surface pressure equations.

In this equation $T_1(x',x)$, $T_2(x',x)$, $U_1(x',x)$ and $U_2(x',x)$ are defined as in Equations (15) to (18), and are computed for $x = x_k$; k=1, n, by the methods described in Equations (20) to (37). Values for $F_1(x)$, $F_2(x)$, and $G(x)$ are available for these values of x. Hence $F_1(x')$ and $F_2(x')$ are obtained by the modified Simpson's rule formula described under Equation (45).

The computation procedures for the near field are specified by a special subroutine FIELD which can be called by the main program.

FORTRAN PROGRAM

The FORTRAN programs for the main computing routine and some subroutines are listed in the Appendix. The FORTRAN names for the more common variables in the preceding equations are listed here. Some additional names are explained under Input Data.

X(K) $\equiv x_k$, the longitudinal coordinate of station K

Y(K) $\equiv y_k$, the section radius at x = X(K)

YDY(K) $\equiv$ ydy/dx at x = X(K)

F1(K) $\equiv F_1(x)$, the nondimensional surface pressure of Equation [62]

F2(K) $\equiv F_2(x)$, of Equation [62]

G(K) $\equiv G(x)$, the nondimensional velocity function of Equation [9]

PSX(K) $\equiv \psi_\ell(x)$, the longitudinal velocity distribution of Equation [10]

T1(J, K) $\equiv T_1(x',x)$ at x' = X(J) and x = X(K)

T2(J, K) $\equiv T_2(x',x)$

$U1(J, K) \equiv U_1\ (x',x)$

$U2(J, K) \equiv U_2\ (x',x)$

INPUT DATA

For any particular problem, the shape and dimensions of the vibrating surface, the amplitude and distribution of the vibration velocity, and the location of the field points, must all be specified. In addition, certain alternative procedures in the computation program must be selected by input control data. Each variation in the surface shape, or in the velocity distribution, must be preceded by these control data. However, if the velocity distribution consists of two distributions in time quadrature, the control data should precede the data for the first velocity distribution only.

The units used in the input data may be any self-consistent set except only in the calculation of the sound pressure levels where it is assumed that c = 9740 slugs $ft^{-2}\ sec^{-1}$ and it is necessary that all distances be in feet and that the velocity v_o be in feet/second.

The input data should be supplied on at least two punched cards in the following order. Note that in most cases it is both usual and proper to have a zero entry for any particular data.

1. On two punched cards enter I1, I2, I3, I4, I5, M, N, AL, HALF, EPS, EPS2, W, FREQ, VEL, PHASE, AGL, DA, A, B, ZR, ZI, SIGMA in FORMAT (7I3, F3.0, F4.0, 2E5.2, 3F7.3, F5.1, 2F4.0/2F7.5, 2E10.6, F4.2).

I1 is used to select the method for specifying the surface shape. I1 $\leq$ 4 means that the surface shape data will be supplied on subsequent input cards. I1 = 5 means bypass the surface shape calculation and go to the calculation of the velocity distribution. I1 $\geq$ 6 means call the special subroutine SHAPE for specifying the surface shape (e.g., if the surface shape is a speroid). I1 = 6 means, in addition to calling SHAPE, that the station values for $x = X(K)$ will be selected according to Equation [6], which concentrates stations near the ends.

I2 is used to select the method for specifying the surface velocity. I2 $\leq$ 4 means that the surface velocity will be computed from values for $\psi_{\ell}(x)$ in Equation [10] which are to be read from input data cards. I2 = 5 means bypass the specification of the surface velocity. I2 $\geq$ 6 means call the special subroutine SURVEL to compute the surface velocity, e.g., if the velocity distribution is specified by an analytical function.

I3 is used to select the method for specifying the calculation of the influence functions. I3 $\leq$ 4 means that the influence functions will be calculated by the standard method of Equations [20] to [26]. I3 = 5 means bypass the calculation of the influence functions. I3 $\geq$ 6 means call the special subroutine INFLU to compute the influence functions.

I4 may be used to control the calculation and print-out of the surface pressures. I4 = 0 means that TZR and TZI (related to surface impedance) will be printed out after every iteration cycle. I4 = 1 means, in addition, that F1(K) and F2(K) will be printed out after each iteration cycle. I4 = 2 means, in addition to the foregoing, that the high frequency approximation of Equations [71] and [72] will be used at station N. I4 = 3 or 4 means, in addition to the foregoing, that this approximation will also be used at station 1.

I5 is used to control the calculation and print-out of the field data. I5 = 0 or 1 means that far-field pressures will be calculated by the standard method and printed out. I5 = 2 means that the far-field pressure data is not printed out but is stored for later operations. I5 = 3 or 4 means that the far-field will be computed by the standard method, and printed out, and also that the mean far-field pressure level is computed from Equation [84]. I5 = 5 means bypass the computation of the far-field pressures. I5 $\geq$ 6 means call the special subroutine FIELD, which may, for example, compute the near field.

M $\equiv$ m in Equation [1] and is equal to the number of circumferential variations in the velocity distribution.

N $\equiv$ n, the number of stations along the x-axis. N must be an odd number not greater than 73. If N + I1 = 0, the program will END JOB.

AL $\equiv$ L in Equation [13] and is equal to the number of nodal sections in the velocity distribution. Thus, AL = 0 is a rigid body translation in the axial direction. AL = 1 is a longitudinal "accordion" vibration mode with a single nodal section at midsection.

HALF = 0 is standard and means that in the iteration calculation of the surface pressure the direct results of the kth iterative calculation are used in the equations for the (k + 1) iteration. HALF = 0.5 means that the mean of the input for the kth iteration and the results of the kth iteration are used as the input for the (k + 1) iteration.

EPS $\equiv \epsilon$, the convergence limit in Equation [51]. If EPS is left blank, it will be taken as 0.001.

EPS2 = ϵ_2 in Equation [69]. If EPS2 is left blank, it will be taken as 1×10^{-6}.

W $\equiv$ k, the wave number times 2π.

FREQ = vibration frequency in cycles per second. Either W or FREQ must be specified in the input data. If the frequency is specified, then W is calculated from W = FREQ/795, which implies that the frequency is expressed in cycles per second, and the sound velocity = 2π (795) = 4995 ft/sec.

VEL = v_0 in Equation [1], in feet/second.

PHASE = 0 is standard. PHASE = 2 means that the subsequent velocity distribution data will consist of two distributions which are in time quadrature and which act concurrently. The far field of each distribution is calculated separately, but only the far field of the combination is printed out.

AGL is the initial value of θ', measured in degrees, i.e., AGL is the initial angular direction in the far field for which the far-field pressure will be calculated.

DA is the increment, $d\theta'$ in degrees, in the angular direction. That is, the far-field pressure is calculated for AGL, AGL + DA, AGL + 2 (DA), AGL + 3 (DA), etc.; to 180 deg.

A = semilongitudinal axis of a spheroidal surface.

B = semitransverse axis of a spheroidal surface.

ZR $\equiv$ c_1 in Equation [74] .

ZI $\equiv$ c_2 in Equation [74] . If ZR + ZI = 0, the surface pressures will be computed by the SURFPR subroutine.

SIGMA $\equiv \sigma$ in Equation [11] . Note σ is defined as a positive number.

2. If I1 $\leq$ 4, then at this point there must follow N cards with X(K), Y(K), YDY(K), D(K), and E(K) on each card in FORMAT (5E14.8). Only X(K) and Y(K) must be listed; the cards must be arranged in order of increasing or decreasing X, and the first and last cards must have Y = 0. D(K) and E(K) reserve space in COMMON storage for variables which may be necessary in future subroutines.

3. If I1 $\geq$ 5, the SHAPE subroutine may require input data at this point.

4. If I2 $\leq$ 4, then at this point there must follow input data cards with PSX(K), (K = 1, N), in FORMAT (6E12.4).

5. If I2 $\geq$ 6, the SURVEL subroutine may require input data at this point.

6. If I4 $\geq$ 5, and/or I5 $\geq$ 6, then the special subroutine for surface pressures or far-field pressures may require input data at this point.

7. If PHASE = 2.0, there must follow data cards to specify the second velocity distribution.

8. The control data (as in item 1) for the next problem follows. If there is no other problem to be calculated, add two blank cards.

PRINT-OUT DATA

The print-out data include the following, all appropriately labeled, and in the following order:

1. Data of two control cards.

2. Print-out, if any, of SHAPE subroutine.

3. Print-out, if any, of SURVEL subroutine.

4. Surface shape and velocity distribution, specified by table of X(K), Y(K), YDY(K), PSX(K), and G(K) versus station number K. Also velocity phase, ka, and velocity angle number M.

5. Print-out, if any, of INFLU subroutine.

6. Print-out, if any, of SURFPR subroutine.

7. If the surface pressures are computed by iteration, there is a print-out of $\int F_1\, Gdx$ (labeled TZR) and $\int F_2\, Gdx$ (labeled TZI) for every iteration cycle. If $I4 \geq 1$, there is also a print-out of F1(K) and F2(K); K = 1, N, for every iteration cycle.

8. Surface pressures, specified by table of F1(K) and F2(K) versus station K. Also modal resistance coefficient $\bar{z}_1$ and modal reactance coefficient $\bar{z}_2$, Equation [76], and source power level $\bar{L}$, Equation [79].

9. Print-out, if any, of FIELD subroutine.

10. Far-field pressures, specified by a table of far-field pressure coefficients q of Equation [83], phase angles defined by $\tan^{-1}(q_1/q_2)$, directivity factor $\bar{D}$ of

Equation [82], and sound pressure level of Equation [86], all tabulated versus angular position θ'. If the surface velocity is specified in two phases, the far-field pressure data includes the combined effect of both phases.

REMARKS AND NOTES

1. This FORTRAN program could be used to calculate the incompressible flow about a vibrating or moving surface of revolution, or--more generally--to obtain solutions to Laplace's equation, given Neumann boundary conditions on a surface of revolution. The procedure is simply to specify both W and FREQ, in the control data, as zero.

However, an abbreviated and more efficient version of this program is available for this problem. It is designated UC08 and can accommodate up to 100 stations as written and undoubtedly more than 100 stations with slight modifications.

2. The present program, UC07, and the necessary set of subroutines are written for a maximum of 73 stations and require about 27,700 storage locations. Thus, on the IBM 7090 computer, about 5000 storage locations remain and are available for "overhead" operations.

The storage requirements may be reduced in several minor ways without materially increasing the calculation time. For example, since the arrays D, E, F1, F2, F7, F8, G8, and G9 are normally used after T1 and T2, the former arrays may share storage locations with the latter arrays. Also, since T1 and T2 are symmetric arrays, it is only necessary to store about half of each.

In this way, the maximum number of stations can be increased to perhaps 77 without increasing calculation time. If tape storage is used, the number of stations can be increased indefinitely but at the expense of substantially increasing the calculation time.

3. If the vibrating surface has a flat section normal to the longitudinal axis, the general method of solution remains valid and applicable. But some particular details of the FORTRAN program cannot be used without modification. In particular, the variables $G(x)$, defined in Equation [9], and $U(x', x)$, defined in Equations [17] to [25], become infinite at values of x for which $\sin\beta = 0$.

One solution is to replace these surface sections by sections of very high but finite slope. An alternative technique is to modify the FORTRAN program by replacing $G(x)$ and $U(x',x)$ by $G'(x)$ and $U'(x',x)$ where $G' = G \sin\beta$ and $U' = U \sin\beta(x)$ and by replacing integrations with respect to x by integrations (at these points only) with respect to the slant distance s where $ds = dx/\sin\beta$.

4. The program has been written so as to be applicable, without modification, to a wide range of conditions. This means that it is not necessarily the most efficient program for any particular problem. Certainly much more efficient programs can be written for the special and common case where $m = 0$ in Equation [1] and also for the special cases where the vibrating surface has some special symmetry.

5. A final word of caution--the program is sufficiently complex that it is difficult to avoid a mistake either in input data or in interpretation of the limitations of the solution. A new solution should be accepted only after comparison with some relevant independent analysis.

APPENDIX

LISTING OF THE FORTRAN PROGRAM

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C     UC07 PROGRAM OF G. CHERTOCK
C     SOUND RADIATION BY VIBRATING SURFACE OF REVOLUTION
      COMMON X,Y,YDY,D,E,F1,F2,G,T1,T2,U1,U2
     1,F7,F8,G8,G9,Z1,Z2
      DIMENSION X(73),Y(73),YDY(73),D(73),E(73),F1(73),F2(73),
     1G(73),T1(73,73),T2(73,73),U1(73,73),U2(73,73)
     2,SM1(73),SM2(73),G8(73),G9(73),Q(4),QH(4),PSX(73)
     3,Z1(73),Z2(73),F8(73),F7(73)
   95 READ666,I1,I2,I3,I4,I5,M,N,AL,HALF,EPS,EPS2,W,
     1FREQ,VEL,PHASE,AGL,DA,A,B,ZR,ZI,SIGMA
  666 FORMAT (7I3,F3.0,F4.0,2E5.2,3F7.3,F5.1,2F4.0/
     12F7.5,2E10.6,F4.2)
      PRINT 667,I1,I2,I3,I4,I5,M,N,AL,HALF,EPS,EPS2,W,
     1FREQ,VEL,PHASE,AGL,DA,A,B,ZR,ZI,SIGMA
  667 FORMAT (1H1,1X,12HCONTROL DATA,5X,3HI1=I3,5X,
     13HI2=I3,5X,3HI3=I3,5X,3HI4=I3,5X,3HI5=I3,
     2          5X,2HM=I3,5X,2HN=I3,5X,3HAL=F3.0,5X,5HHALF=F5.1/
     35X,4HEPS=E7.2,5X,5HEPS2=E7.2,5X,2HW=F7.3,5X,5HFREQ=F7.3,5X,
     44HVEL=F8.3,5X,6HPHASE=F5.1,5X,4HAGL=F4.0/5X,3HDA=F4.0,5X,2HA=
     5F7.3,5X,2HB=F7.3,5X,3HZR=E10.6,5X,3HZI=E10.6,5X,6HSIGMA=F4.2)
      PI=3.1415927
      NM1=N-1
      NM2=N-2
      IF(N+I1)38,99,38
   99 CALL END JOB
   38 IF(I1-5)52,59,53
   53 CALL SHAPE (N,A,B,I1)
      GO TO 59
   52 READ 54,(X(K),Y(K),YDY(K),D(K),E(K),K=1,N)
   54 FORMAT(5E14.8)
      A=(X(N)-X(1))/2.
      IF(YDY(2)+YDY(1))59,55,59
   55 YDY(1)=(Y(2)**2+(X(2)-X(1))**2)/2./(X(2)-X(1))
      YDY(2)=YDY(1)+X(1)-X(2)
      YDY(N)=(Y(NM1)**2+(X(N)-X(NM1))**2)/2./(X(NM1)-X(N))
      YDY(NM1)=YDY(N)+X(N)-X(NM1)
      DO113 K=3,NM2
      KP1=K+1
      KM1=K-1
  113 YDY(K)=Y(K)*((Y(KP1)-Y(K))/(X(KP1)-X(K))+(Y(K)
     1-Y(KM1))/(X(K)-X(KM1))-(Y(KP1)-Y(KM1))/(X(KP1)-X(KM1)))
   59 IF(EPS)71,72,71
   72 EPS=.001
   71 IF(W)57,58,57
   58 W=FREQ/795.
   57 IF(I2-5)60,62,61
   61 CALL SURVEL(N,M,A,W,SIGMA,VEL,PHASE,I2,AL,PSX)
      GO TO 62
   60 READ 65,(PSX(K),K=1,N)
   65 FORMAT(6E12.4)
      DO111K=2,NM1
      KP1=K+1
      KM1=K-1
      PSR=SIGMA*Y(K)*((PSX(KP1)-PSX(K))/(X(KP1)-X(K))
     1+(PSX(K)-PSX(KM1))/(X(K)-X(KM1))-(PSX(KP1)-PSX(KM1))
     2/(X(KP1)-X(KM1)))*(-1.)
  111 G(K)=(Y(K)*PSR-YDY(K)*PSX(K))/A
      G(1)=-YDY(1)*PSX(1)/A
      G(N)=-YDY(N)*PSX(N)/A
  213 FORMAT(I19,5E20.8)
```

```
 62 H1 = W*A
    IF (PHASE-2.) 162,162,262
162 VPHASE = 0.
    GO TO 362
262 VPHASE = PHASE
362 PRINT 112,M,H1,VPHASE
    PRINT213,(K,X(K),Y(K),YDY(K),PSX(K),G(K),K=1,N)
112 FORMAT (// , 5X,39HSURFACE SHAPE AND VELOCITY DISTRIBUTION,5X,
   115HANGLE NUMBER M=I3,5X,3HKA=F6.3,5X,15HVELOCITY PHASE=F5.1,
   2//11X,8H STATION,8X,12HAXIAL COORD.,6X,14HSECTION RADIUS,
   36X,14HRADIUS X SLOPE,6X,14HAXIAL VELOCITY,
   46X,14HVELOCITY  G(K))
 67 IF (PHASE-2.) 296,296,169
169 RAR1=RAR
    GO TO 69
296 RAR1=0.
196 IF(I3-5)96,69,68
 68 CALL INFLU(N,M,A,W,I3,EPS)
    GO TO 69
 96 AM=M
    DO 11 K=2,NM2
    KP1=K+1
    DO 11 J=KP1,NM1
    RS=(X(J)-X(K))
   X               **2+(Y(J)-Y(K))**2
    RT=RS+4.*Y(J)*Y(K)
    NN=XMAX1F(SQRTF(92.+8.4*RT/RS),2.*W*(SQRTF(RT)
   1-SQRTF(RS)),AM*20.)
    ANN=NN+NN
    DT=PI/ANN*2.
    PHI=-DT*.2113249
    DPHI=-2.*PHI
    SUM1=0.
    SUM2=0.
    SUM11=0.
    SUM21=0.
    SUM13=0.
    SUM23=0.
 31 PHI=PHI+DPHI
    DPHI=DT-DPHI
    CA=COSF(PHI)
    S1=SQRTF(RS+2.*Y(J)*Y(K)*(1.-CA))
    S2=COSF(W*S1)/S1*COSF(AM*PHI)
    S3=SINF(W*S1)/S1*COSF(AM*PHI)
    S4=(S2/S1+S3*W)/S1
    S5=(S3/S1-S2*W)/S1
    SUM1=SUM1+S2
    SUM2=SUM2+S3
    SUM11=SUM11+S4
    SUM21=SUM21+S5
    SUM13=SUM13+S4*CA
    SUM23=SUM23+S5*CA
    IF(PHI-PI+DPHI)31,32,32
 32 T1(J,K)=A/ANN*SUM1
    T2(J,K)=A/ANN*SUM2
    T1(K,J)=T1(J,K)
    T2(K,J)=T2(J,K)
    S11JK=(X(J)-X(K))*SUM11
    S21JK=(X(J)-X(K))*SUM21
    S12JK=Y(J)*SUM13-Y(K)*SUM11
```

```
      S12KJ=Y(K)*SUM13-Y(J)*SUM11
      S22JK=Y(J)*SUM23-Y(K)*SUM21
      S22KJ=Y(K)*SUM23-Y(J)*SUM21
      U1(J,K)=(Y(K)*S12JK-YDY(K)*S11JK)*A/ANN
      U1(K,J)=(Y(J)*S12KJ+YDY(J)*S11JK)*A/ANN
      U2(J,K)=(Y(K)*S22JK-YDY(K)*S21JK)*A/ANN
   11 U2(K,J)=(Y(J)*S22KJ+YDY(J)*S21JK)*A/ANN
      K=1
      JJ=N
   17 DO 14 J=2,JJ
      IF(M)12,13,12
   12 T1(J,K)=0.
      T2(J,K)=0.
      U1(J,K)=0.
      U2(J,K)=0.
      T1(K,J)=0.
      T2(K,J)=0.
      U1(K,J)=0.
      U2(K,J)=0.
      GO TO 14
   13 S=SQRTF((
     X          X(J)-X(K))**2+Y(J)**2)
      T1(J,K)=COSF(W*S)/S*A
      T2(J,K)=SINF(W*S)/S*A
      T1(K,J)=T1(J,K)
      T2(K,J)=T2(J,K)
      T11=(T1(J,K)/S+T2(J,K)*W)/S*(X(J)-X(K))
      T21=(T2(J,K)/S-T1(J,K)*W)/S*(X(J)-X(K))
      U1(J,K)=-T11*YDY(K)
      U1(K,J)=T11*(YDY(J)-Y(J)**2/(X(J)-X(K)))
      U2(J,K)=-T21*YDY(K)
      U2(K,J)=T21*(YDY(J)-Y(J)**2/(X(J)-X(K)))
   14 CONTINUE
      IF(K-N)15,16,15
   15 K=N
      JJ=NM1
      GO TO 17
   16 Q(1)=.07358
      Q(2)=.2
      Q(3)=.6
      QH(1)=.2
      QH(2)=4./30.
      QH(3)=16./30.
      QH(4)=4./30.
      IF (I3-2) 80,83,80
   83 NM=(N+1)/2
      GO TO 85
   80 NM=2
   85 DO 18 K=NM,NM1
      KM1=K-1
      KP1=K+1
      JJ=KM1
   19 S4=T1(JJ,K)*QH(4)
      DO 20 KX=1,3
      DX=Q(KX)*(X(JJ)-X(K))
      YSQ=Y(K)**2+DX*(2.*YDY(K)+Q(KX)*(YDY(JJ)-YDY(K)))
      YY=SQRTF(YSQ)
      RS=DX**2+(YY-Y(K))**2
      TS=0.
      PHI1=0.
```

```
      PHI2=PI/5.
      H=PI/5.
      S3=0.
      PHI=PHI1-H
   34 S2=0.
   23 PHI=PHI+H
      CA=COSF(PHI)
      S1=SQRTF(RS+2.*Y(K)*YY*(1.-CA))
      DS2=COSF(W*S1)/S1*COSF(AM*PHI)
      S2=S2+DS2
      IF(PHI+.6*H-PHI2)23,24,24
   24 IF(PHI+.1*H-PHI2)25,26,26
   26 SUM1=S2
      DT=DS2
      SUM2=SUM1*H/2.
   29 PHI=PHI1-H/2.
      GO TO 34
   25 SUM3=H/6.*(SUM1+4.*S2+2.*S3)
      IF(((SUM3-SUM2)/(SUM3+TS))**2-EPS**2)27,28,28
   28 SUM2=SUM3
      S3=S3+S2
      H=H/2.
      GO TO 29
   27 TS=TS+SUM3
      IF(PHI2-PI+.1*H)30,133,133
   30 PHI1=PHI2
      PHI2=PHI2+PI/5.
      S2=DT
      S3=0.
      H=PI/5.
      PHI=PHI1
      GO TO 23
  133 S4=S4+QH(KX)*TS*A/PI
   20 CONTINUE
      IF(JJ-KM1)21,22,21
   22 SL4=S4
      JJ=KP1
      GO TO 19
   21 P=(2.*X(K)-X(KP1)-X(KM1))/(X(KP1)-X(KM1))
      T1(K,K)=SL4*.75*(1.+P)*(1.-P**2)+S4*.75*(1.-P)*
     1(1.-P**2)-T1(K,KM1)/4.*(1.-P)*(1.+3.*P)-T1(K,KP1)/
     24.*(1.+P)*(1.-3.*P)
      SUM2=0.
      NN=XMAX1F(10.,3.*W*Y(K),AM*5.)
      ANN=NN+NN
      DT=PI/ANN
      PHI=-DT*.2113249
      DPHI=-2.*PHI
      SUM3=0.
      SUM1=0.
   43 PHI=PHI+DPHI
      DPHI=DT-DPHI
      S=2.*Y(K)*SINF(PHI)
      S3=SINF(W*S)*COSF(AM*PHI*2.)
      SUM1=SUM1+S3/S
      SUM2=SUM2+COSF(W*S)*COSF(2.*AM*PHI)
      SUM3=SUM3+S3
      IF(PHI-PI/2.+DPHI)43,44,44
   44 T2(K,K)=SUM1*A/ANN
      U1(K,K)=-T1(K,K)/2.-W*A*SUM3*.5/ANN
```

```
 18 U2(K,K)=-T2(K,K)/2.+W*A*SUM2*.5/ANN
    IF(M) 145,47,145
145 T1(1,1)=0.
    T2(1,1)=0.
    T1(N,N)=0.
    T2(N,N)=0.
    GO TO 146
 47 S=SQRTF((X(1)-X(2))**2+Y(2)**2)
    T1(1,1)=(4.*A*SINF(W*S)/S**2/W-T1(1,2))
    T2(1,1)=  W*A
    S=SQRTF((X(N)-X(NM1))**2+Y(NM1)**2)
    T1(N,N)=(4.*A*SINF(W*S)/S**2/W-T1(N,NM1))
    T2(N,N)=  W*A
146 U1(1,1)=-T1(1,1)*.5
    U2(1,1)=0.
    U1(N,N)=-T1(N,N)*.5
    U2(N,N)=0.
 69 CONTINUE
 73 IF(ZR+ZI)75,74,75
 74 CALL SURFPR(N,M,A,W,VEL,I4,HALF,EPS2,TZR,TZI,NST)
    GO TO (75,79,37,284),NST
 75 DO 78 J=1,N
    F1(J)=ZR*G(J)
    F2(J)=ZI*G(J)
    Z1(J)=ZR*G(J)**2
 78 Z2(J)=ZI*G(J)**2
    TZR=SM3PT(X,Z1,N,2)
    TZI=SM3PT(X,Z2,N,2)
 79 IF(M)89,90,89
 90 ETA=1.
    GO TO 35
 89 ETA=2.
 35 RAR=TZR/ETA/A/2.+1.E-37
    RAI=TZI/ETA/A/2.
    ASPL=10.*LOG10F(RAR*(VEL*A*5.49E9)**2)
 37 PRINT 91
 91 FORMAT(1H1,10X,17HSURFACE PRESSURES,
  2                                      //13X,7HSTATION,11X,9HF1 EQUALS,1
  31X,9HF2 EQUALS,13X,7HSTATION,11X,9HF1 EQUALS,11X,9HF2 EQUALS)
    PRINT92,(K,F1(K),F2(K),K=1,N)
    PRINT 97,RAR,RAI,ASPL
 97 FORMAT(//13X,17HMODAL RES. COEF.=E15.8,/13X,18HMODAL REAC. COEF.
  1             E15.8/13X,37HSOURCE POWER LEVEL(DB RE .0002/1YD.)=
  2        E15.8)
284 CONTINUE
 92 FORMAT(I20,2E20.8,I20,2E20.8)
    IF(I5-5)122,95,94
 94 CALL FIELD (N,M,A,W,VEL,I5,AGL,DA)
    GO TO  95
118 FORMAT(1H0,50X,19HFAR FIELD PRESSURES//10X,9HDIRECTION,6X,23HREL.
  1TRANSFER IMPEDANCE,10X,14HRELATIVE PHASE,2X,18HDIRECTIVITY FACTOR,
  35X,20HSPL(DB RE .0002/1YD))
318 FORMAT(F17.1,E31.8,F24.3,2E20.8)
122 L=1
    ANG=AGL
 33 CA=COSF(ANG*.017453)
    SA=SQRTF(1.-CA**2)
    DO131 K=1,N
    H8=W*Y(K)*SA
    H9=W*X(K)*CA
```

```
      G1=BEJOF(H8)
      G2=-BEJ1F(H8)
      MM=0
   48 IF(M-MM)46,46,147
  147 AMM=MM
      IF(H8)81,82,81
   82 G1=0.
      G2=0.
      GOTO 46
   81 CONTINUE
      TG=G1*AMM/H8-G2
      G2=G1*(1.-AMM*(1.+AMM)/H8**2)+G2/H8*(1.+AMM)
      G1=TG
      MM=MM+1
      GO TO 48
   46 G3=W*G1*(-G(K)*A+YDY(K)*F1(K)*CA)+G2*F2(K)*H8
      G4=W*G1*YDY(K)*F2(K)*CA-G2*F1(K)*H8
      G5=COSF(H9)
      G6=SINF(H9)
      G8(K)=G5*G3+G6*G4
  131 G9(K)=G5*G4-G6*G3
      SUM1=SM3PT(X,G8,N,2)
      SUM2=SM3PT(X,G9,N,2)
      IF (PHASE - 2.) 143,143,144
  143 SM1(L)=SUM1
      SM2(L)=SUM2
      GO TO 45
  144 TEM=SM1(L)-SUM2
      SUM2=SM2(L)+SUM1
      SUM1=TEM
   45 L=L+1
      IF (PHASE - 2.) 121,120,121
  121 SUM=SQRTF(SUM1**2+SUM2**2)/A/2.  +1.E-37
      FASANG=ATN1F(SUM1,SUM2)
      DIRFAC=SUM**2/(RAR+RAR1)/ETA
      SPL=20.*LOG10F(VEL*SUM*A*5.49E9/ETA)
      LM=L-1
      Z1(LM)=ANG
      Z2(LM)=DIRFAC*SA
      IF(L-2)36,136,36
  136 IF(PHASE)336,236,336
  336 PRINT 337
  337 FORMAT(1H0,13X,63HFAR FIELD BELOW IS COMBINED FIELD OF TWO VELOCIT
     1Y DISTRIBUTIONS)
  236 PRINT 118
   36 PRINT 318,ANG, SUM, FASANG, DIRFAC, SPL
  120 ANG=ANG+DA
      IF(ANG-180.1) 33,33,150
  150 IF(I5-2)159,159,359
  359 IF(PHASE-2.)76,259,76
   76 AVDF=SM3PT(Z1,Z2,LM,2)/(ANG-DA-AGL)*PI/2.
  511 SPL=SPL+LOG10F(AVDF/DIRFAC)*10.
      PRINT 513,SPL
  513 FORMAT (13X,32HTOTAL RADIATED POWER-ALL PHASES=E20.8)
  512 PRINT 98,AVDF
   98 FORMAT(13X,25HMEAN DIRECTIVITY FACTOR =E20.8)
  159 IF (PHASE - 2.) 95,259,95
  259 PHASE = 90.
      GO TO 57
      END
```

```
C       SPHEROID SURFACE SHAPE WITH UNIFORM OR COSINE
C       SPACING OF STATIONS
        SUBROUTINE SHAPE(N,A,B,I1)
        COMMON X,Y,YDY,D,E,F1,F2,G,T1,T2,U1,U2
        DIMENSION X(73),Y(73),YDY(73),D(73),E(73),F1(73),F2(73),
       1G(73),T1(73,73),T2(73,73),U1(73,73),U2(73,73)
        NM=(N+1)/2
        AN=N
        DO 11 K=NM,N
        AK=K
        IF(I1-6)12,13,12
     12 X(K)=A*(2.*AK-AN-1.)/(AN-1.)
        GO TO 14
     13 X(K)=A*COSF(3.14159*(AN-AK)/(AN-1.))
     14 Y(K) = B*SQRTF(1. - (X(K)/A)**2)
        YDY(K)=-X(K)*(B/A)**2
        NMK=N-K+1
        X(NMK)=-X(K)
        Y(NMK)=Y(K)
     11 YDY(NMK)=-YDY(K)
        RETURN
        END
```

```
C       ACCORDION VIBRATION MODE WITH L NODES
        SUBROUTINE SURVEL (N,M,A,W,SIGMA,VEL,PHASE,I2,AL,PSX)
        COMMON X,Y,YDY,D,E,F1,F2,G,T1,T2,U1,U2
        DIMENSION X(73),Y(73),YDY(73),D(73),E(73),F1(73),F2(73),
       1G(73),T1(73,73),T2(73,73),U1(73,73),U2(73,73),PSX(73)
        PI=3.1415927
        DO 11 K=1,N
        ANG=AL*PI/A*(X(K)-X(1))/2.
        PSX(K)=COSF(ANG)
        PSR=AL*SIGMA*Y(K)/A*SINF(ANG)*PI/2.
     11 G(K)=(Y(K)*PSR-YDY(K)*PSX(K))/A
        RETURN
        END
```

```
C       DUMMY SUBROUTINE INFLU
        SUBROUTINE INFLU(N,M,A,W,I3,EPS)
        COMMON X,Y,YDY,D,E,F1,F2,G,T1,T2,U1,U2
        DIMENSION X(73),Y(73),YDY(73),D(73),E(73),F1(73),F2(73),
       1G(73),T1(73,73),T2(73,73),U1(73,73),U2(73,73)
        RETURN
        END
```

```
C       DUMMY SUBROUTINE FIELD
        SUBROUTINE FIELD(N,M,A,W,VEL,I5,AGL,DA)
        COMMON X,Y,YDY,D,E,F1,F2,G,T1,T2,U1,U2
        DIMENSION X(73),Y(73),YDY(73),D(73),E(73),F1(73),F2(73),
       1G(73),T1(73,73),T2(73,73),U1(73,73),U2(73,73)
        RETURN
        END
```

```
C       INTEGRAL OF YDX BY 3-POINT SIMPSONS RULE
C       MODIFIED FOR UNEQUAL INTERVALS AND ODD OR EVEN
C       NUMBER OF STATIONS
        FUNCTION SM3PT(X,Y,N,J)
        DIMENSION X(100),Y(100)
        NM1=N-1
        JP=(-1)**J
        IF(JP)12,11,11
     11 Z=0.
        MM=2
        MMM=N-1
        GO TO 14
     12 Z=(X(2)-X(1))/2.*(Y(1)+Y(2))+(X(N)-X(NM1))/2.
       1*(Y(N)+Y(NM1))
        MM=3
        MMM=N-2
     14 DO 13 M=MM,MMM,2
        MM1=M-1
        MP1=M+1
        P=(2.*X(M)-X(MM1)-X(MP1))/(X(MP1)-X(MM1))
     13 Z=Z+(X(MP1)-X(MM1))/6.*(Y(MM1)*(1.+3.*P)/
       1(1.+P)+Y(M)*4./(1.-P**2)+Y(MP1)*(1.-3.*P)/
       2(1.-P))
        SM3PT=Z
        RETURN
        END
```

REFERENCES

1. Chertock, George, "Sound Radiation from Vibrating Surfaces," J. Acoust. Soc. Am., Vol. 36, No. 7 (July 1964). Also David Taylor Model Basin Report 1873 (Jan 1965).

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	Bureau of Ships Washington D.C.

13. ABSTRACT

A FORTRAN program is presented which computes the sound pressures, at the surface and in the near or far field, radiated by an arbitrary surface of revolution which is vibrating in an almost arbitrary pattern with arbitrary frequency and phase. The method of solution, the input data, and the output data are all described in detail.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Sound Radiation						
Vibrating Surfaces						
FORTRAN Program						
Numerical Solutions						
Integral Equations						

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