A FORTRAN PROGRAM FOR CALCULATING THE SOUND RADIATION FROM A VIBRATING SURFACE OF REVOLUTION

by

George Chertock, Ph.D.

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ACOUSTICS AND VIBRATION LABORATORY
RESEARCH AND DEVELOPMENT REPORT

December 1965

Report 2083
ERRATA SHEET
for
David Taylor Model Basin 2083, December 1965

The following changes are to be made to Report 2083.

Page 17 – Equation [75] reads:
\[ Z = \frac{2\pi pc}{\epsilon_m} \int_{-a}^{a} F(x) \cdot G(x) \, dx \]  
Equation [75] should read:
\[ Z = \frac{2\pi pc}{\epsilon_m} \int_{-a}^{a} F(x) \cdot G(x) \, dx \]

Page 20 – Equation [89] reads:
\[ J_{n+1}'(x) = \left[ 1 - \frac{n(n+1)}{x^2} \right] J_n(x) + \left[ \frac{n}{x} \right] J'_n(x), \]
Equation [89] should read:
\[ J_{n+1}'(x) = \left[ 1 - \frac{n(n+1)}{x^2} \right] J_n(x) + \left[ \frac{n(n+1)}{x} \right] J'_n(x), \]

Insert attached sheet 38a between pages 38 and 39. This page was inadvertently omitted from the Appendix listing the FORTRAN program.

Corrections made 10/3/67

[Signature]
C MODIFIED GAUSS - SEIDEL ITERATION FOR SURFACE PRESSURE
SUBROUTINE SURFPR(NoMoAWVEL,14,HALF,EPS2,TZRTZI,NST)
COMMON X,Y,YDY,VEL,HALF,EPS2,TZRTZI,NST

DIMENSION X(73),Y(73),YDY(73),VEL(73),F1(73),F2(73),

188 GT1(73),GT2(73),ST1(73),ST2(73),U1(73),U2(73),
1(LF(73),G8(73),G9(73),F7(73),F8(73),Z1(73),Z2(73)

51 EPS2=1.E-6
77 S1=0.0
L=1
D021 J=1,N
D0218 K=1,N
Z1(K)=T1(J+K)*G(K)
Z2(K)=-T1(J-K)*G(K)
F7(J)=SM3PT(X*Z1(NJ)*F1(J)
F8(J)=SM3PT(X*Z2(NJ)*F2(J)
F1(J)=F7(J)
F2(J)=F8(J)
DO 40 K=1,N
40 Z1(K)=F1(K)*G(K)
TZR=SM3PT(XZ1.N.2)
TZI=SM3PT!X.Z2#N.2)
182 PRINT 182,L,TZR,TZI
182 FORMAT(/5X,8HSURFACE ITERATION NO.I3910X,TZR =E14.6,10X,
10HTZI =E14.6)
IF (14-1) 809281,281 5130
281 PRINT 381
381 FORMAT (94X,9HF1 EQUALS)
PRINT313*(F1(J),J=1,N)
PRINT161
PRINT313*(F2(J),J=1,N)
80 L=L+1
IF(L>50)83,84
83 S2=TZR**2+TZI**2
83 S2=S2
161 FORMAT (94X,9HF2 EQUALS)
1313 FORMAT (7E17.8)
DO 313 J=1,N
D0119 K=1,N
Z1(K)=F1(K)*U1(J*K)-F2(K)*U2(J*K)
Z2(K)=F1(K)*U2(J,K)+F2(K)*U1(J,K)
G8(J)=F7(J)*SM3PT(X+Z1*N,J)/A
G9(J)=F8(J)*SM3PT(X+Z2*N,J)/A
F1(J)=G8(J)+F1(J)-GB(J))*HALF
F2(J)=G9(J)+F2(J)-GB(J))*HALF
184 F1(J)=F1(J)*G1(J)/G2(J)
F2(J)=F2(J)*G1(J)/G2(J)
313 FORMAT (TE17.8)
DO 184 J=1,N
D0119 K=1,N
Z1(K)=F1(K)*U1(J+K)-F2(K)*U2(J+K)
Z2(K)=F1(K)*U2(J+K)+F2(K)*U1(J+K)
084 G9(J)=F8(J)*SM3PT(X+Z2*N+J)/A
F1(J)=G8(J)+F1(J)-GB(J))*HALF
184 F1(J)=F1(J)*G1(J)/G2(J)
F2(J)=F2(J)*G1(J)/G2(J)
GO TO 215
79 NST=2
RETURN
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ABSTRACT

A FORTRAN program is presented which computes the sound pressures, at the surface and in the near or far field, radiated by an arbitrary surface of revolution which is vibrating in an almost arbitrary pattern with arbitrary frequency and phase. The method of solution, the input data, and the output data are all described in detail.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

This report describes and explains detailed procedures for the numerical calculation of the sound pressure field outside a surface of revolution vibrating with a specified distribution of velocity. More generally, these procedures provide for the numerical solution of the scalar wave equation when the normal gradient is specified on a surface of revolution. The method of calculation, which is based on the theory of Reference 1*, is presented in a FORTRAN program that can be used to solve specific problems with an IBM 7090 computer. In addition, the procedures are explained in detail so that they may be more easily modified and adapted to solve special problems.

*References are listed on page 40.
The surface boundary must be idealized as a closed surface of revolution without discontinuities. The normal velocity of the surface must be of the form

\[ v(x, y, \varphi, t) = v_0 \psi(x) \cos(m\varphi) \cos(\omega t + \delta) \]  

where \( x, y, \) and \( \varphi \) are the cylindrical coordinates of a surface point with the \( x \)-axis being the axis of symmetry; \( v_0 \) is a velocity amplitude; \( \psi(x) \) is an arbitrary function of longitudinal position; and \( m \) and \( \omega \) are likewise arbitrary. If the phase angle \( \delta \) is not constant over the surface, the velocity distribution must be written as the sum of two component distributions, 90 degrees apart in the time phase, and a separate numerical solution must be made for each component.

In the method of solution described here the sound pressure at any point in the field is computed by simple quadratures from the Helmholtz integral in terms of velocity and sound pressure at the surface of the vibrating body. But first, the sound pressure at the surface must be computed by solution of an integral equation of the second kind, and it is this solution of the integral equation that is the salient feature of this method.

The program and procedures are described in five parts: (a) specifying the surface shape, (b) computing the influence coefficients for the specific surface shape and vibration frequency, (c) specifying the velocity distribution on the surface, (d) computing the sound pressure at the surface, and (e) computing the sound pressure in the field. Each part may be accomplished by one or more alternative procedures as explained below, and in addition any new procedure may be introduced by a suitable subroutine. The FORTRAN program itself is listed in the Appendix.
SURFACE SHAPE

The minimum data required to specify the surface shape are the values for the section radii \( y(x) \) at an odd number, \( n \leq 73 \), of stations along the \( x \)-axis. The spacing between stations need not be uniform; in fact, the accuracy of the calculation is increased if stations are concentrated where the profile curvature is high or where the velocity distribution changes fast. The first and last stations must be points at which \( y = 0 \). The function \( ydy/dx \) is used often in many of the subsequent calculations. It is therefore computed or specified at each station and stored. There are three alternative methods of specifying the station data.

(a) The numerical values for \( x_i, y_i, (ydy/dx)_i; i = 1, 2, \ldots n, \) are specified in the input data. If the \( ydy/dx \) data are not specified, the program will then compute this quantity (cards 380 to 560 in the FORTRAN program). At stations 1 and 2 it is assumed that a circular profile passes through these points, whence

\[
\left( y \frac{dy}{dx} \right)_1 = \frac{y_2^2 + (x_2 - x_1)^2}{2(x_2 - x_1)}
\]  

[2]

and

\[
\left( y \frac{dy}{dx} \right)_2 = \left( y \frac{dy}{dx} \right)_1 + x_2 - x_1
\]  

[3]

Similar relations are assumed for stations \( n \) and \( n-1 \). At the remaining \( n-4 \) stations it is assumed that the profile is parabolic through each point and its two neighbors. Hence

\[
\left( y \frac{dy}{dx} \right)_i = y_i \left[ \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \right] - \left[ \frac{y_{i-1} - y_i}{x_i - x_{i-1}} \right] = \frac{y_{i+1} - y_{i-1}}{y_{i+1} - y_{i-1}}
\]

[4]
(b) If the vibrating surface is a spheroid, either prolate or oblate, and of any eccentricity, it is sufficient to input the semilongitudinal axis a and the midsection radius b. The program (subroutine SHAPE) will then select n-values for x which are either equally spaced,

\[ x_i = \frac{a}{n-1} \left(2i - 1 - n\right) \]  

[5]

or, optionally, distributed as

\[ x_i = a \cos\left( \frac{n - i}{n - 1} \pi \right) \]  

[6]

This last distribution concentrates most stations at the two ends where the curvature is highest for a prolate body.

Then, \( y_i \) and \( (dy/dx)_i \) are computed from the equations

\[ y_i = b\left(1 - x_i^2 / a^2\right)^{\frac{1}{2}} \]  

[7]

and

\[ \left(\frac{dy}{dx_i}\right) = x_i \frac{b^2}{a^2} \]  

[8]

(c) If the profile of the vibrating surface is some special analytical shape, or if a special station spacing is desired, then a special program may be introduced as an alternative subroutine SHAPE.

VELOCITY DISTRIBUTION ON SURFACE

The vibration velocity must be specified in the form of Equation [1]. If the time phase of the velocity is variable over the surface, the velocity distribution is speci-
fied as the sum of two single-phased distributions in quadrature with each other. The entire calculation, including the components of the far-field pressures, is made separately for each single-phase distribution, and the far-field pressures are then added, with proper regard for phase, to give the net field pressure.

In general, the quantity used to characterize a single-phase velocity distribution is a nondimensional surface velocity amplitude,

\[ G(x) = \frac{\psi y}{(a \sin \beta)}; \text{ where } \cot \beta = -\frac{dy}{dx}, \]  

which must be computed and stored for the \( n \) stations \( x_1 \) to \( x_n \). The \( \psi(x) \) data for the calculation may be specified in three alternative ways.

(a) If the motion is predominantly a longitudinal vibration, with an associated radial motion due to a Poisson ratio effect, the relative velocity normal to the surface is (see cards 800 to 960 in the FORTRAN program)

\[ \psi(x) = \psi_L(x) \cos \beta + \psi_r(x) \sin \beta \]  

where \( v_0 \psi_L \cos (m \varphi) \cos (\omega t + \delta) \) is the longitudinal velocity at \( x \),

\[ \psi_r(x) = -\sigma \frac{yd\psi_L}{dx} \]  

and \( \sigma \) is the effective Poisson ratio for the material and structure (e.g., \( \sigma \approx 0.30 \) for steel). If the values of \( \psi_L(x) \) are specified at the \( n \) stations, the derivative is computed from the 3-point parabolic formula

\[ \left( \frac{d\psi_L}{dx} \right)_x \approx \frac{(\psi_L)_i - (\psi_L)_{i-1}}{x_i - x_{i-1}} + \frac{(\psi_L)_{i+1} - (\psi_L)_i}{x_{i+1} - x_i} - \frac{(\psi_L)_{i+1} - (\psi_L)_{i-1}}{x_{i+1} - x_{i-1}} \]

(b) If the motion is as specified by Equations [10] and [11], except that \( \psi_L(x) \)
is approximated by a simple trigonometric formula, then
\[ \psi_\ell(x) = \cos \left( \frac{L\pi}{2a} (x - x_\ell) \right), \]
and
\[ \psi_\ell(x) = \frac{\alpha\pi Ly}{2a} \sin \left( \frac{L\pi}{2a} (x - x_\ell) \right). \]

Thus \( L = 0 \) means a rigid body vibration in the longitudinal direction, and \( L > 0 \) means an accordion vibration mode with \( L \) longitudinal nodal sections.

(c) If the velocity distribution has some simple analytical form, or if \( G(x) \) is specified by a table of experimental data, the velocity may be specified by a special subroutine which replaces SURVEL.

**INFLUENCE FUNCTIONS**

The purpose here is to compute and store numerical values for the four functions:

\[ T_1(x',x) = \frac{a}{\pi} \int_0^{\pi} \frac{\cos (kr)}{r} \cos (m\varphi) d\varphi, \]  \[ 15 \]

\[ T_2(x',x) = \frac{a}{\pi} \int_0^{\pi} \frac{\sin (kr)}{r} \cos (m\varphi) d\varphi, \]  \[ 16 \]

\[ U_1(x',x) = \frac{a}{\pi} \int_0^{\pi} \left[ \frac{\cos (kr)}{r} + k \sin (kr) \right] \]
\[ \times \left[ \gamma \frac{dy}{dx} (x - x') - y^2 + yy' \cos \varphi \right] \frac{\cos (m\varphi)}{r^2} d\varphi, \]  \[ 17 \]

\[ U_2(x',x) = \frac{a}{\pi} \int_0^{\pi} \left[ \frac{\sin (kr)}{r} - k \cos (kr) \right] \]
\[ \times \left[ \gamma \frac{dy}{dx} (x - x') - y^2 + yy' \cos \varphi \right] \frac{\cos (m\varphi)}{r^3} d\varphi. \]  \[ 18 \]

where
\[ r^2 = (x - x')^2 + (y - y')^2 + 2yy' (1 - \cos \varphi). \]  \[ 19 \]
These must be calculated for each ordered pair of values of \((x', x)\) where \(x' = x_i, x = x_j\) and \(i, j = 1, 2, \ldots n\). Note that these functions depend not only on the surface shape, but also on the wave number \(k\), and on \(m\) which specifies the number of circumferential variations in the vibration pattern. The procedure for computing these functions depends on whether \(x = x'\), and whether \(x\) or \(x'\) is an end station.

**Case I.** If \(x \neq x'\) and \(y, y' \neq 0\) (cards 1140 to 2060 in the FORTRAN program), then the eight values for \(T_1, T_2, U_1,\) and \(U_2\) are evaluated in terms of six different integrals \(S_1\) to \(S_6\) defined over the range \(0 \leq \varphi \leq \pi\),

\[
T_1 (x', x) = T_1 (x, x') = \frac{aS_1}{\pi} \quad [20]
\]

\[
T_2 (x', x) = T_2 (x, x') = \frac{aS_2}{\pi} \quad [21]
\]

\[
U_1 (x', x) = \frac{a}{\pi} \left[ y \frac{dy'}{dx'} (x' - x) - y'^2 \right] S_3 + \frac{a}{\pi} yy' S_6 \quad [22]
\]

\[
U_1 (x, x') = \frac{a}{\pi} \left[ y' \frac{dy}{dx} (x' - x) - y'^2 \right] S_3 + \frac{a}{\pi} yy' S_6 \quad [23]
\]

\[
U_2 (x', x) = \frac{a}{\pi} \left[ y \frac{dy'}{dx'} (x - x') - y'^2 \right] S_4 + \frac{a}{\pi} yy' S_6 \quad [24]
\]

\[
U_2 (x, x') = \frac{a}{\pi} \left[ y' \frac{dy}{dx} (x' - x) - y'^2 \right] S_4 + \frac{a}{\pi} yy' S_6 \quad [25]
\]

where

\[
S_1 = \int_0^\pi \frac{\cos (kr) \cos (m \varphi)}{r} d\varphi \quad [26]
\]

\[
S_2 = \int_0^\pi \frac{\sin (kr) \cos (m \varphi)}{r} d\varphi \quad [27]
\]

\[
S_3 = \int_0^\pi \left[ \frac{\cos (kr) + k \sin (kr)}{r^3} \right] \cos (m \varphi) d\varphi \quad [28]
\]
\[ S_4 = \int_0^\pi \left[ \frac{\sin (kr)}{r^3} - \frac{k \cos (kr)}{r^3} \right] \cos (m\varphi) \, d\varphi \]  
[29]

\[ S_6 = \int_0^\pi \left[ \frac{\cos (kr)}{r^3} + \frac{k \sin (kr)}{r^3} \right] \cos (m\varphi) \cos \varphi \, d\varphi \]  
[30]

\[ S_8 = \int_0^\pi \left[ \frac{\sin (kr)}{r^3} - \frac{k \cos (kr)}{r^3} \right] \cos (m\varphi) \cos \varphi \, d\varphi \]  
[31]

The six integrals are evaluated simultaneously by dividing the range 0 to \( \pi \) into \( b/2 \) equal subintervals and by using a two-point Gaussian quadrature formula for each subinterval, i.e., by using a \( b \)-point formula for the range.

Thus for \( S_1 \),
\[ S_1 = \sum_{i=1}^{b} \cos (m\varphi_i) \cos (kr_i) / r_i, \]  
[32]

where
\[ r_i^2 = (x - x')^2 + (y - y')^2 + 2yy' (1 - \cos \varphi_i), \]  
[33]

\[ \varphi_i = (i - 0.7887) 2\pi/b, \text{ if } i \text{ is odd,} \]  
[34]

\[ \varphi_i = (i - 0.2113) 2\pi/b, \text{ if } i \text{ is even,} \]  
[35]

\[ x = x_j; \ x' = x_k; \ y = y_j; \ y' = y_k. \]  
[36]

and similar equations are obtained for \( S_2 \) to \( S_6 \). The number of subintervals is at least 10 and increases with \( k \) and with \( m \), and with the disparity between \( r \) for \( \varphi = 0 \) and \( r \) for \( \varphi = \pi \), in Equation [33]. Specifically, \( b/2 \) is taken as the largest integer within
\[ b/2 = \max \left( \left( 92. + 8.4 \frac{r_0^2}{r^2} \right)^{1/2}, 2k (r_\pi - r_0) \right) 20m \]  
[37]

This prescription for \( b \) is intended to ensure that the integrals are calculated with at least 3-figure accuracy. It is based on some intuitive arguments plus a
few check calculations. But the prescription does require further investigation and possible modification. It is clear that $S_2$, for example, can be evaluated with adequate precision at far fewer stations than $S_1$, but it is advantageous to evaluate all six integrals simultaneously at every value of $\varphi$.

**Case II.** When $y$ or $y' = 0$, i.e., one station is an end station (cards 2100 to 2680 in the FORTRAN program). If the velocity distribution is not uniform in $\varphi$, i.e., if $m > 0$, all four functions $T_1$, $T_2$, $U_1$, and $U_2$ must be zero. If $m = 0$, and $y = 0$ i.e., $x = x_1$ or $x_n$, the four functions may be calculated without quadrature.

$$T_1 (x', x) = T_2 (x, x') = \frac{a}{s} \cos (ks),$$
$$T_2 (x', x) = T_2 (x, x') = \frac{a}{s} \sin (ks),$$
$$U_1 (x', x) = ay \frac{dy}{dx} (x - x') \left[ \frac{\cos (ks)}{s^3} + \frac{k \sin (ks)}{s^2} \right],$$
$$U_1 (x, x') = ay \frac{dy'}{dx'} (x' - x) - y \left[ \frac{\cos (ks)}{s^3} + \frac{k \sin (ks)}{s^2} \right],$$
$$U_2 (x', x) = ay \frac{dy}{dx} (x - x') \left[ \frac{\sin (ks)}{s^3} - \frac{k \cos (ks)}{s^2} \right],$$
$$U_2 (x, x') = ay \frac{dy'}{dx'} (x' - x) - y \left[ \frac{\sin (ks)}{s^3} - \frac{k \cos (ks)}{s^2} \right],$$

where

$$s^2 = (x - x')^2 + y'^2.$$

Note that $y dy/dx$ always is finite and not zero, even though $y$ may be zero.

**Case III.** $x = x'$; $y = y' \neq 0$. These conditions give the dominant terms in the $T_1$ and $U_1$ matrices, but they are the most difficult to compute with accuracy because $T_1$ and $U_1$ are singular at $x = x'$.

These terms will be used later (see Equation [66]) in a modified version of a 3-point Simpson's rule quadrature formula over the range $x_{j-1} \leq x \leq x_{j+1}$, $x' = x_j$. Hence
T_1 (x, x); i = 2, 3, ..., n - 1, is computed (cards 2700 to 3780) as that value which
used in conjunction with T_1 (x, x) and T_1 (x, x) in this quadrature
formula gives an accurate value to \( \int_{x_{j-1}}^{x_j+1} T_1 (x, x) \, dx \). That is,

\[
\int_{x_{j-1}}^{x_j} T_1 (x, x) \, dx + \int_{x_j}^{x_{j+1}} T_1 (x, x) \, dx
\]

\[
= \frac{x_{j+1} - x_{j-1}}{6} \left[ \frac{1 + 3p}{1 + p} T_1 (x, x_{j-1}) + \frac{4}{1 - p^2} T_1 (x, x_j)
\right. \\
\left. + \frac{1 - 3p}{1 - p} T_1 (x, x_{j+1}) \right] \quad [45]
\]

where

\[
p = \frac{(2 x_j - x_{j-1} - x_{j+1}) / (x_{j+1} - x_{j-1})}. \quad [46]
\]

If \( x_j \) is midway between \( x_{j-1} \) and \( x_{j+1} \), then \( p = 0 \) and Equation [45] reduces
to the common 3-point Simpson's rule. But if the stations are not equally spaced,
Equation [45] is equivalent to first using a 3-point Lagrange interpolation formula
to obtain \( T_1 \) at the midpoint and then applying the conventional Simpson's rule.

Each integral on the left-hand side of Equation [45] is evaluated by a 4-point
approximation which avoids the point \( x = x_j \). Thus, for the first integral,

\[
\int_{x_{j-1}}^{x_j} T_1 (x, x) \, dx = \frac{4}{\sum_{i=1}^{4} h_i (x_j - x_{j-1})} T_1 (x_j, x_i) \quad [47]
\]

where

\[
\frac{x_i - x_j}{x_{j-1} - x_j} = .0736, .2, .6, .2 \quad [48]
\]

and

\[
h_i = \frac{1}{5}, \frac{4}{30}, \frac{16}{30}, \frac{4}{30} \quad [49]
\]

for \( i = 1, 2, 3, 4 \) respectively.
This quadrature formula, Equation [47], results from fitting a special 1-point logarithmic quadrature approximation (see Reference 1, Equation B2) to the range 
\[ x_j - (x_j - x_{j-1}) / 5 < x < x_j \] which is one-fifth of the total range, and a 3-point Simpson's rule formula to the remaining four-fifths of the range. The second integral in Equation [45] is evaluated by the same formulas, but replacing \( j-1 \) by \( j+1 \) in Equations [47] to [49].

The net result of Equations [45] to [49] is that \( T_1 (x_j, x_l) \) is replaced by a particular mean of eight values of \( T_2 (x_j, x) \) which are computed at six new stations for \( x \) in the neighborhood of \( x_j \), in addition to the two stations already computed at \( x = x_{j-1} \) and \( x = x_{j+1} \).

First, \( y(x) \) is computed for the three \( x \)-points between \( x_{j-1} \) and \( x_j \) from the equation
\[ y^2 = y_k^2 + 2 (x-x_k) \left( \frac{dy}{dx} \right)_k + \frac{(x-x_k)^2}{x_{k-1}-x_k} \left[ \left( \frac{dy}{dx} \right)_{k-1} - \left( \frac{dy}{dx} \right)_k \right] \] [50]

For the three points between \( x_k \) and \( x_{k+1} \), \( k+1 \) is substituted for \( k-1 \) in Equation [50]. This equation may be interpreted as a Taylor expansion for \( y^2 \) about \( x = x_k \), with the last factor in square brackets being an approximation for \( d^2 (y^2)/dx^2 \) at \( x = x_k \).

The equation is possibly the simplest form using \( y \) and \( ydy/dx \) at \( x = x_k \) and \( ydy/dx \) at \( x_{k-1} \), and would be an exact expression if the profile through \( x_k \) and \( x_{k-1} \) was an ellipse centered on the \( x \) axis. If an elliptic section were a grossly inadequate representation of the local profile, then Equation [50] may give impossible results, i.e., \( y^2 \) may be negative.

Each of the six values of \( T_1 (x_j, x) \) is now computed from Equation [15] by a new quadrature program (see cards 3040 to 3640 in the FORTRAN program) designed...
to decrease the station spacing in the range where the integrand changes rapidly.
We divide the interval $0 \leq \varphi \leq \pi$ into five equal subintervals of width $\pi/5$ and further subdivide each subinterval into $2, 4, 8, 16,$ etc. segments, in each case halving the segment until the resulting value for $T_1$ converges within a specified limit $\epsilon$.

For example, suppose the integration over the first subinterval, $0 \leq \varphi \leq \pi/5$, has converged to $S_1$, and suppose that the integration over the second subinterval, $\pi/5 \leq \varphi \leq 2\pi/5$, results in $S_2 (m)$ when the second subinterval is divided into $m$ segments, then $m$ is increased to $2m$, and the integration results in $S_2 (2m)$. If

$$\left| \frac{S_2 (2m) - S_2 (m)}{S_2 (2m) + S_1} \right| \leq \epsilon \quad [51]$$

Then $S_2 (2m)$ is accepted as $S_2$, and the integration is now continued to the third interval $2\pi/5 \leq \varphi \leq 3\pi/5$, starting with $m = 2$. The program is arranged so that every time the interval is halved, all of the previous station computations are used again and need not be recomputed.

These special procedures, Equations [45] to [51], are used only to compute $T_1 (x_j, x_j)$; $j = 2, 3, \ldots, n-1$.

The function $T_2 (x', x)$ is finite at $x = x' = x_j$ and is computed from (see cards 3800 to 4140 in the FORTRAN program)

$$T_2 (x', x) = \frac{a}{\pi} \int_0^{\pi/2} \frac{\sin (kr)}{r} \cos (2m\theta) d\theta \quad [52]$$

where

$$r = 2y \sin \theta \quad [53]$$
The integral is evaluated by dividing the range $0 \leq \theta \leq \pi/2$ into $b$ subintervals and using a two-point Gaussian quadrature formula for each subinterval. The number of subintervals is taken as

$$b = \text{Max} \ [10, 3 \ k, 5 \ m],$$

which again is an arbitrary prescription intended to give an accuracy of at least three figures.

The functions $U_1(x', x)$ and $U_2(x', x)$, for $x \to x' = x_j$, are both computed on the partial assumption that the profile section near $x \to x_j$ can be approximated by a circle, whence (see Reference 1, Equation [A4]).

$$U_1(x_j, x_j) = -\frac{1}{2} T_1(x_j, x_j) = \frac{ka}{\pi} \int_0^{\pi/2} \sin (kr) \cos(2m\theta) d\theta,$$

and

$$U_2(x_j, x_j) = -\frac{1}{2} T_2(x_j, x_j) + \frac{ka}{\pi} \int_0^{\pi/2} \cos (kr) \cos(2m\theta) d\theta.$$

The two integrals are evaluated, simultaneously with Equation [52], by the method described for $T_2(x_j, x_j)$.

**Case IV.** If $x = x' = x_1$ or $x_n'; y = y' = 0$. Then $T_1$, $T_2$, $U_1$, and $U_2$ are all zero unless $m = 0$. In the latter event, it is assumed that stations 1 and 2 are on the same spherical cap for which simple exact expressions for $T$ and $U$ are available (see Reference 1, Equation [A5] and see cards 4160 to 4460 of the FORTRAN program). For $x' = x_1$,

$$T_1(x_1, x_1) = \frac{4a}{ks^2} \sin (ks) - T_1(x_1, x_2),$$

$$T_2(x_1, x_1) = ka,$$

$$U_1(x_1, x_1) = -T_1(x_1, x_1)/2,$$

$$U_2(x_1, x_1) = 0.$$
where

\[ s^2 = (x_1 - x_2)^2 + y_2^2 \]  \[61\]

For \( x' = x_n \), similar forms are used, by replacing \( x_1 \) by \( x_n \), and \( x_2 \) by \( x_{n-1} \).

### PRESSURE AT VIBRATING SURFACE

The components of the nondimensional surface pressure,

\[
F_1(x) + iF_2(x) = \frac{P_1(x) + iP_2(x)}{\rho c v_0 \cos(m\phi)}
\]  \[62\]

are computed for each station \( x_1 \) to \( x_n \). This requires a solution for the set of \( 2n \) simultaneous integral equations

\[
F_1(x') = k \int_{-a}^{a} G(x) T_1(x',x) \, dx + \frac{1}{a} \int_{-a}^{a} \left[ F_1(x) U_1(x',x) - F_2(x) U_2(x',x) \right] \, dx
\]  \[63\]

\[
F_2(x') = -k \int_{-a}^{a} G(x) T_1(x',x) \, dx + \frac{1}{a} \int_{-a}^{a} \left[ F_1(x) U_2(x',x) + F_2(x) U_1(x',x) \right] \, dx
\]  \[64\]

where

\[
x' = x_j; \quad j = 1, 2, 3 \ldots n.
\]  \[65\]

Each integral is approximated by the modified Simpson's rule formula previously described under Equation [45]. If station \( j \) is an even-numbered station, this quadrature formula is

\[
\int_{x_1}^{x_n} f(x) \, dx = \sum_{i=2,4}^{n-1} \frac{x_{i+1} - x_{i-1}}{6} \left[ \frac{1 + 3p}{1 + p} f_{i-1} + \frac{4}{1 - p^2} f_i + \frac{1 - 3p}{1 + p} f_{i+1} \right]
\]  \[66\]

where

\[
p = \left( 2x_{i-1} - x_{i-1} - y_{i+1} \right) / (x_{i+1} - y_{i-1}).
\]  \[67\]

And if station \( j \) is an odd-numbered station,

\[
\int_{x_1}^{x_n} f(x) \, dx = \left( x_2 - x_1 \right) \frac{f_1 + f_2}{2} + \left( x_n - x_{n-1} \right) \frac{f_n + f_{n-1}}{2}
\]

\[
+ \sum_{i=3,5}^{n-2} \frac{x_{i+1} - x_i}{6} \left[ \frac{1 + 3p}{1 + p} f_{i-1} + \frac{4}{1 - p^2} f_i + \frac{1 - 3p}{1 + p} f_{i+1} \right].
\]  \[68\]

There are now \( 2n \leq 146 \) linear algebraic equations to solve simultaneously. The
method normally used (see cards 4800 to 5500) is the Gauss-Seidel process of simple iteration. We start with the initial trial solution obtained by taking \( F_1 \) as only the first term on the right-hand side of Equation [63], and \( F_2 \) as only the first term on the right of Equation [64]. These initial values are then substituted into Equations [63] and [64] for \( j = 1 \), and a new pair of values for \( F_1(x_1) \) and \( F_2(x_1) \) are determined. Then, using these new values at station 1, a new pair of values is calculated for station 2; the process being continued until the second pair of values has been calculated for every station. Then the entire cycle for the n-stations is repeated to get a third approximation to the vectors \( F_1(x_i) \) and \( F_2(y_i) \); etc., etc.

The iteration calculation (cards 5220 to 5260) is normally terminated after 50 cycles of calculation, or before, if a particular weighted mean square value of the surface pressure has converged within a specified limit \( \epsilon^2 \). That is, if \( S_2 \) is the current value of this mean square surface pressure and \( S_1 \) is the value after the previous iteration cycle, then the criterion for terminating the iterations is

\[
(1 - S_1/S_2)^2 \leq \epsilon^2 \]  

[69]

where \( \epsilon^2 \) is ordinarily taken as 0.001, but may be prescribed otherwise in the input data.

The mean square surface pressure for this purpose is defined by

\[
S_2 = \left[ \int_{-\ell}^{\ell} F_1 G dx \right]^2 + \left[ \int_{-\ell}^{\ell} F_2 G dx \right]^2
\]  

[70]

and is calculated after every iteration cycle by the quadrature formula of Equation [66].
The simple iteration method may not converge properly at high frequencies, e. g., if \( ka \) is greater than some critical eigenfrequency. In such cases an alternative procedure in the calculating program (cards 5480, 5500) is to use as the trial vector for the \((k+1)\) iteration, not the solution vector of the \(k\)th iteration, but the mean between that and the trial vector for the \(k\)th iteration.

Also, at high frequencies, the computed values for \( F_1 \) and \( F_2 \) at stations 1 and \( N \) tend to be particularly inaccurate unless the station spacing at the ends is very small. An alternative procedure (see cards 5520 to 5620), used when \( kydy/dx > 4 \), is to compute the values for stations 1 and \( n \) by extrapolation from the values for stations 2 and \( n-1 \), according to the formulas

\[
F_1(x_1) = F_1(x_2) \frac{G(x_1)}{G(x_2)} \quad [71]
\]

\[
F_1(x_n) = F_1(x_{n-1}) \frac{G(x_n)}{G(x_{n-1})} \quad [72]
\]

and similarly for \( F_2(x_1) \) and \( F_2(x_n) \). This is valid because the ratio \( F(x)/G(x) \) depends on the local surface impedance and the local radius of curvature, and at high frequencies these are the same for station 1 as for station 2 (see Reference 1, Equation [3]).

A special situation occurs when the shape of the vibrating surface is similar to a prolate ellipsoid, and the pattern of the nondimensional velocity distribution \( G(x) \) is proportional to the angular-spheroidal wave function \( S_{mt}(x) \) at that frequency. Then the surface pressure may be calculated from the simple equation

\[
F_1(x) + i F_2(x) = (c_1 + i c_2) G(x) \quad [73]
\]
where (see Reference 1, Equation [6])

\[ c_1 + ic_2 = \frac{ika}{\xi} \frac{a^2}{b^2} \frac{R_{m\ell}}{R_{m\ell}(3)} \]  

[74]

and where \( \xi \) is the reciprocal eccentricity of the elliptic section, \( R_{m\ell}(3) \) is the radial-spheroidal wave function of the third kind with arguments \( ka/\xi \) and \( \xi \), and \( R_{m\ell}(3)' \) is the derivative with respect to \( \xi \). In this spheroidal approximation it is not necessary to calculate the influence functions, Equations [15] to [18], and it is not necessary to solve the set of simultaneous equations, [63] and [64], but merely to supply the constants \( c_1 \) and \( c_2 \) as part of the input data (see cards 4620 to 4780).

A completely different procedure for calculating the surface pressures can most easily be introduced as a new subroutine SURFPR to replace the iteration process.

**RADIATION IMPEDANCE AND POWER OUTPUT**

The modal radiation impedance is defined in Reference 1, Equation [23], as

\[ Z = \frac{2\pi\rho c a}{\epsilon_m} \int_{-a}^{a} F(x) \cdot G(x) \, dx \]  

[75]

where \( \epsilon_m = 1 \) for \( m = 0 \), and \( \epsilon_m = 2 \) for \( m > 0 \). It is convenient to compute the modal impedance coefficient defined by

\[ \bar{z}_1 + i\bar{z}_2 = Z/(4\pi\rho ca^2) \]  

[76]

These coefficients depend on the shape of the surface, the vibration pattern, and the reduced frequency \( ka \), but are independent of the absolute size of the surface, the vibration amplitude, and the characteristic impedance of the medium. The modal impedance coefficients are first calculated from Equation [76] by the quadrature formula of Equation [66], (see cards 5000 to 5060 and 5660 to 5760).
The time average of the radiated power is then
\[ \Pi = 4\pi \rho c a^2 \frac{v_o^2}{2}, \quad [77] \]
and the source power level of the vibrating surface is
\[ L = 10 \log_{10} \left( \frac{\Pi}{\Pi_{ref}} \right) \text{ dB}, \quad [78] \]
where the reference power \( \Pi_{ref} \) is that of a spherical source which generates a sound pressure of 0.0002 \( \mu \) bars rms at 1 yd from the center. Thus, if \( a \) is in feet, \( v_o \) is the velocity amplitude (defined in Equation [1]) in feet per second, and the fluid is sea water, then
\[ L = 10 \log_{10} \left[ \frac{\Pi}{5.49 \cdot 10^9 a v_o^2} \right]. \quad [79] \]
Note that \( v_o \) is here defined as a peak velocity, not an rms velocity.

FAR-FIELD SOUND PRESSURES

The sound pressure in the far field at distance \( R' \), longitude angle \( \varphi' \), azimuth angle \( \theta' \), relative to the longitudinal axis of the vibrating surface, is given by (see Reference 1, Equation [28])
\[
P_1(R', \theta', \varphi') = \rho c v_o \cos (m \varphi') (\imath)^{m+1} e^{\imath k R'/R'}
\]
\[
\times \int_a^a \frac{k dx}{2} \left\{ \psi_y \sin \theta' \right\} J_m (k y \sin \theta') \cos (k x \cos \theta')
\]
\[
- \frac{p_1(x) \cos \theta'}{\rho c v_o \cos (m \varphi')} y \frac{dy}{dx} J_m (k y \sin \theta') \cos (k x \cos \theta')
\]
\[
- \frac{p_2(x) \sin \theta'}{\rho c v_o \cos (m \varphi')} y \frac{dy}{dx} J_m (k y \sin \theta') \sin (k x \cos \theta')
\]
\[
+ \frac{p_1(x) \sin \theta'}{\rho c v_o \cos (m \varphi')} y J_m' (k y \sin \theta') \sin (k x \cos \theta')
\]
\[
- \frac{p_2(x) \sin \theta'}{\rho c v_o \cos (m \varphi')} y J_m' (k y \sin \theta') \sin (k x \cos \theta') \right\}, \quad [80] \]
\[ P'_e(R', \theta', \varphi') = \rho c v_0 \cos (m \varphi') (-i)^{m+1} e^{i k R'/R'} \]

\[
\times \int_{-\alpha}^{\alpha} \frac{k dx}{2} \left\{ \frac{\psi y}{\sin \beta} J_m(\nu y) \cos (k x \sin \theta') \right. \\
+ \frac{p_1(x) \cos \theta'}{\rho c v_0 \cos (m \varphi')} \frac{dy}{dx} J_m(\nu y) \sin (k x \cos \theta') \\
- \frac{p_2(x) \cos \theta'}{\rho c v_0 \cos (m \varphi')} \frac{dy}{dx} J_m(\nu y) \cos (k x \cos \theta') \\
+ \frac{p_1(x) \sin \theta'}{\rho c v_0 \cos (m \varphi')} \frac{dy}{dx} J_{m'}(k y) \sin (k x \cos \theta') \\
- \frac{p_2(x) \sin \theta'}{\rho c v_0 \cos (m \varphi')} \frac{dy}{dx} J_{m'}(k y) \cos (k x \cos \theta') \left\} . \tag{81} \]

The two integrals in Equations [80] and [81] (divided by \(a\)) may be interpreted as pressure coefficients, \(q_1(\theta')\) and \(q_2(\theta')\) respectively, in terms of which the amplitude of the far-field sound pressure at \(R', \theta', \varphi'\) is

\[ |p(R', \theta', \varphi')| = \rho c v_0 \cos (m \varphi') a q / R' \tag{82} \]

where

\[ [q(\theta')]^2 = q_1^2 + q_2^2 . \tag{83} \]

Hence the time average of the radiated power is

\[ \Pi = \frac{\pi \rho c v_0^2 a^2}{\epsilon_m} \int_0^{\pi} q^2 \sin \theta' d \theta', \tag{84} \]

which should give the same value as Equation [77]. Comparing Equations [79] and [77], the directivity factor at azimuth \(\theta'\) and longitude \(\varphi'\) is

\[ D(\theta', \varphi') = q^2 \cos^2 (m \varphi') / z_1 \tag{85} \]

And, when averaged over \(\varphi'\), the directivity factor is

\[ \bar{D}(\theta') = q^2 / (z_1 \epsilon_m) \tag{86} \]
The sound pressure level in the far field at direction $\theta'$, and averaged over $\varphi'$, and relative to 0.0002 $\mu$ bar at 1 yd from a spherical source in sea water, is given by

$$L(\theta') = \bar{L} + 10 \log_{10} D$$ \[87\]

where $\bar{L}$ is given by Equation [78].

These equations are used to compute the far-field pressure level and directivity factors for every angular direction that is specified in the input data (see cards 6040 to 7040). The integrals of Equations [80] and [81] are evaluated by the 3-point quadrature formula of Equation [66]. The two Bessel functions $J_m$ and $J_m'$ are evaluated (see cards 6180 to 6460) by the recurrence relations,

$$J_{n+1}(x) = \frac{n}{x} J_n(x) - J'_n(x),$$ \[88\]

$$J'_n(x) = \left[1 - \frac{n(n+1)}{x^2}\right] J_n(x) + \left[\frac{n(n+1)}{x}\right] J'_n(x),$$ \[89\]

which can easily be obtained from the more common recurrence formula. $J_0(x)$ and $J_0'(x)$ are first computed by a library subroutine.

NEAR-FIELD SOUND PRESSURES

The sound pressure in the near field, at a point whose cylindrical coordinates are $x'$, $y'$, $\varphi'$, is expressed in nondimensional units in almost the same form as Equations [63] and [64] for the surface pressure. Thus

$$F_1(x') = \int_a^x \frac{dx}{2} \left[-kG(x) T_2(x', x) + \alpha^{-1} F_1(x) U_1(x', x) - \alpha^{-1} F_2(x) U_2(x', x)\right]$$ \[90\]

$$F_2(x') = \int_a^x \frac{dx}{2} \left[-kG(x) T_1(x', x) + \alpha^{-1} F_1(x) U_2(x', x) + \alpha^{-1} F_2(x) U_2(x', x)\right]$$ \[91\]

See Correction sheet.
Note the factor $\frac{1}{2}$, which is not present in the surface pressure equations.

In this equation $T_1(x',x)$, $T_2(x',x)$, $U_1(x',x)$ and $U_2(x',x)$ are defined as in Equations (15) to (18), and are computed for $x = x_k$; $k=1, n$, by the methods described in Equations (20) to (37). Values for $F_1(x)$, $F_2(x)$, and $G(x)$ are available for these values of $x$. Hence $F_1(x')$ and $F_2(x')$ are obtained by the modified Simpson's rule formula described under Equation (45).

The computation procedures for the near field are specified by a special subroutine FIELD which can be called by the main program.

FORTRAN PROGRAM

The FORTRAN programs for the main computing routine and some subroutines are listed in the Appendix. The FORTRAN names for the more common variables in the preceding equations are listed here. Some additional names are explained under Input Data.

- $X(K) \equiv x_k$, the longitudinal coordinate of station $K$
- $Y(K) \equiv y_k$, the section radius at $x = X(K)$
- $YDY(K) \equiv y dy/dx$ at $x = X(K)$
- $F_1(K) \equiv F_1(x)$, the nondimensional surface pressure of Equation [62]
- $F_2(K) \equiv F_2(x)$, of Equation [62]
- $G(K) \equiv G(x)$, the nondimensional velocity function of Equation [9]
- $PSX(K) \equiv \psi(x)$, the longitudinal velocity distribution of Equation [10]
- $T_1(J, K) \equiv T_1(x',x)$ at $x' = X(J)$ and $x = X(K)$
- $T_2(J, K) \equiv T_2(x',x)$
U1(J, K) \equiv U_1 (x', x)
U2(J, K) \equiv U_2 (x', x)

INPUT DATA

For any particular problem, the shape and dimensions of the vibrating surface, the amplitude and distribution of the vibration velocity, and the location of the field points, must all be specified. In addition, certain alternative procedures in the computation program must be selected by input control data. Each variation in the surface shape, or in the velocity distribution, must be preceded by these control data. However, if the velocity distribution consists of two distributions in time quadrature, the control data should precede the data for the first velocity distribution only.

The units used in the input data may be any self-consistent set except only in the calculation of the sound pressure levels where it is assumed that \( c = 9740 \) slugs \( \text{ft}^2 \text{s}^{-1} \) and it is necessary that all distances be in feet and that the velocity \( v_0 \) be in feet/second.

The input data should be supplied on at least two punched cards in the following order. Note that in most cases it is both usual and proper to have a zero entry for any particular data.

1. On two punched cards enter 11, 12, 13, 14, 15, M, N, AL, HALF, EPS, EPS2, W, FREQ, VEL, PHASE, AGL, DA, A, B, ZR, ZI, SIGMA in FORMAT (713, F3.0, F4.0, 2E5.2, 3F7.3, F5.1, 2F4.0/2F7.5, 2E10.6, F4.2).
11 is used to select the method for specifying the surface shape. \( 11 \leq 4 \) means that the surface shape data will be supplied on subsequent input cards. \( 11 = 5 \) means bypass the surface shape calculation and go to the calculation of the velocity distribution. \( 11 \geq 6 \) means call the special subroutine SHAPE for specifying the surface shape (e.g., if the surface shape is a speroid). \( 11 = 6 \) means, in addition to calling SHAPE, that the station values for \( x = X(K) \) will be selected according to Equation \([6]\), which concentrates stations near the ends.

12 is used to select the method for specifying the surface velocity. \( 12 \leq 4 \) means that the surface velocity will be computed from values for \( \psi_t(x) \) in Equation \([10]\) which are to be read from input data cards. \( 12 = 5 \) means bypass the specification of the surface velocity. \( 12 \geq 6 \) means call the special subroutine SURVEL to compute the surface velocity, e.g., if the velocity distribution is specified by an analytical function.

13 is used to select the method for specifying the calculation of the influence functions. \( 13 \leq 4 \) means that the influence functions will be calculated by the standard method of Equations \([20]\) to \([26]\). \( 13 = 5 \) means bypass the calculation of the influence functions. \( 13 \geq 6 \) means call the special subroutine INFLU to compute the influence functions.

14 may be used to control the calculation and print-out of the surface pressures. \( 14 = 0 \) means that TZR and TZI (related to surface impedance) will be printed out after every iteration cycle. \( 14 = 1 \) means, in addition, that F1(K) and F2(K) will be printed.
out after each iteration cycle. \( I_4 = 2 \) means, in addition to the foregoing, that the high frequency approximation of Equations [71] and [72] will be used at station \( N \).

\( I_4 = 3 \) or 4 means, in addition to the foregoing, that this approximation will also be used at station 1.

\( I_5 \) is used to control the calculation and print-out of the field data. \( I_5 = 0 \) or 1 means that far-field pressures will be calculated by the standard method and printed out. \( I_5 = 2 \) means that the far-field pressure data is not printed out but is stored for later operations. \( I_5 = 3 \) or 4 means that the far-field will be computed by the standard method, and printed out, and also that the mean far-field pressure level is computed from Equation [84]. \( I_5 = 5 \) means bypass the computation of the far-field pressures. \( I_5 \geq 6 \) means call the special subroutine FIELD, which may, for example, compute the near field.

\( M = m \) in Equation [1] and is equal to the number of circumferential variations in the velocity distribution.

\( N = n \), the number of stations along the x-axis. \( N \) must be an odd number not greater than 73. If \( N + 11 = 0 \), the program will END JOB.

\( AL = L \) in Equation [13] and is equal to the number of nodal sections in the velocity distribution. Thus, \( AL = 0 \) is a rigid body translation in the axial direction. \( AL = 1 \) is a longitudinal "accordion" vibration mode with a single nodal section at midsection.

\( \text{HALF} = 0 \) is standard and means that in the iteration calculation of the surface
pressure the direct results of the kth iterative calculation are used in the equations for
the \((k + 1)\) iteration. \(\text{HALF} = 0.5\) means that the mean of the input for the kth
iteration and the results of the kth iteration are used as the input for the \((k + 1)\)
iteration.

\(\text{EPS} \equiv \epsilon, \) the convergence limit in Equation [51]. If EPS is left blank, it will
be taken as 0.001.

\(\text{EPS2} = \epsilon_2\) in Equation [69]. If EPS2 is left blank, it will be taken as
\(1 \times 10^{-8}\).

\(W \equiv k, \) the wave number times \(2\pi\).

\(\text{FREQ} = \) vibration frequency in cycles per second. Either \(W\) or \(\text{FREQ}\) must
be specified in the input data. If the frequency is specified, then \(W\) is calculated
from \(W = \text{FREQ}/795,\) which implies that the frequency is expressed in cycles per
second, and the sound velocity = \(2\pi (795) = 4995 \text{ ft/sec}\).

\(\text{VEL} = v_0\) in Equation [1], in feet/second.

\(\text{PHASE} = 0\) is standard. \(\text{PHASE} = 2\) means that the subsequent velocity
distribution data will consist of two distributions which are in time quadrature and
which act concurrently. The far field of each distribution is calculated separately,
but only the far field of the combination is printed out.

\(\text{AGL}\) is the initial value of \(\theta',\) measured in degrees, i.e., \(\text{AGL}\) is the initial
angular direction in the far field for which the far-field pressure will be calculated.

\(\text{DA}\) is the increment, \(d\theta'\) in degrees, in the angular direction. That is, the
far-field pressure is calculated for \(\text{AGL}, \text{AGL} + \text{DA}, \text{AGL} + 2(\text{DA}), \text{AGL} + 3(\text{DA}),\)
etc.; to 180 deg.

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A = semilongitudinal axis of a spheroidal surface.

B = semitransverse axis of a spheroidal surface.

ZR \equiv c_1 \text{ in Equation } [74].

ZI \equiv c_2 \text{ in Equation } [74]. \text{ If } ZR + ZI = 0, \text{ the surface pressures will be computed by the SURFPR subroutine.}

\sigma \equiv \sigma \text{ in Equation } [11]. \text{ Note } \sigma \text{ is defined as a positive number.}

2. If \text{11} \leq 4, \text{ then at this point there must follow } N \text{ cards with } X(K), Y(K), YDY(K), D(K), \text{ and } E(K) \text{ on each card in FORMAT } (5E14.8). \text{ Only } X(K) \text{ and } Y(K) \text{ must be listed; the cards must be arranged in order of increasing or decreasing } X, \text{ and the first and last cards must have } Y = 0. \text{ D(K) and E(K) reserve space in COMMON storage for variables which may be necessary in future subroutines.}

3. If \text{11} \geq 5, \text{ the SHAPE subroutine may require input data at this point.}

4. If \text{12} \leq 4, \text{ then at this point there must follow input data cards with } PSX(K), \text{ (}K = 1, N\text{), in FORMAT } (6E12.4).

5. If \text{12} \geq 6, \text{ the SURVEL subroutine may require input data at this point.}

6. If \text{14} \geq 5, \text{ and/or } \text{15} \geq 6, \text{ then the special subroutine for surface pressures or far-field pressures may require input data at this point.}

7. If PHASE = 2.0, \text{ there must follow data cards to specify the second velocity distribution.}

8. The control data (as in item 1) for the next problem follows. \text{ If there is no other problem to be calculated, add two blank cards.}
PRINT-OUT DATA

The print-out data include the following, all appropriately labeled, and in
the following order:

1. Data of two control cards.

2. Print-out, if any, of SHAPE subroutine.

3. Print-out, if any, of SURVEL subroutine.

4. Surface shape and velocity distribution, specified by table of X(K),
   Y(K), YDY(K), PSX(K), and G(K) versus station number K. Also velocity phase,k_a,
   and velocity angle number M.

5. Print-out, if any, of INFLU subroutine.

6. Print-out, if any, of SURFPR subroutine.

7. If the surface pressures are computed by iteration, there is a print-out of
   \[ \int F_1 \, G dx \] (labeled TZR) and \[ \int F_2 \, G dx \] (labeled TZI) for every iteration cycle. If
   \( l_4 > 1 \), there is also a print-out of \( F_1(K) \) and \( F_2(K) \); \( K = 1, N \), for every iteration
   cycle.

8. Surface pressures, specified by table of \( F_1(K) \) and \( F_2(K) \) versus station \( K \).
   Also modal resistance coefficient \( \tilde{z}_1 \) and modal reactance coefficient \( \tilde{z}_2 \), Equation
   [76], and source power level \( \bar{L} \), Equation [79].

9. Print-out, if any, of FIELD subroutine.

10. Far-field pressures, specified by a table of far-field pressure coefficients
    \( q \) of Equation [83], phase angles defined by \( \tan^{-1} \left( q_{12} / q_a \right) \), directivity factor \( \tilde{D} \) of
Equation [82], and sound pressure level of Equation [86], all tabulated versus angular position $\theta$'. If the surface velocity is specified in two phases, the far-field pressure data includes the combined effect of both phases.

REMARKS AND NOTES

1. This FORTRAN program could be used to calculate the incompressible flow about a vibrating or moving surface of revolution, or--more generally--to obtain solutions to Laplace's equation, given Neumann boundary conditions on a surface of revolution. The procedure is simply to specify both $W$ and $FREQ$, in the control data, as zero.

However, an abbreviated and more efficient version of this program is available for this problem. It is designated UC08 and can accommodate up to 100 stations as written and undoubtedly more than 100 stations with slight modifications.

2. The present program, UC07, and the necessary set of subroutines are written for a maximum of 73 stations and require about 27,700 storage locations. Thus, on the IBM 7090 computer, about 5000 storage locations remain and are available for "overhead" operations.

The storage requirements may be reduced in several minor ways without materially increasing the calculation time. For example, since the arrays $D$, $E$, $F1$, $F2$, $F7$, $F8$, $G8$, and $G9$ are normally used after $T1$ and $T2$, the former arrays may share storage locations with the latter arrays. Also, since $T1$ and $T2$ are symmetric arrays, it is only necessary to store about half of each.
In this way, the maximum number of stations can be increased to perhaps 77 without increasing calculation time. If tape storage is used, the number of stations can be increased indefinitely but at the expense of substantially increasing the calculation time.

3. If the vibrating surface has a flat section normal to the longitudinal axis, the general method of solution remains valid and applicable. But some particular details of the FORTRAN program cannot be used without modification. In particular, the variables $G(x)$, defined in Equation [9], and $U(x', x)$, defined in Equations [17] to [25], become infinite at values of $x$ for which $\sin \beta = 0$.

One solution is to replace these surface sections by sections of very high but finite slope. An alternative technique is to modify the FORTRAN program by replacing $G(x)$ and $U(x', x)$ by $G'(x)$ and $U'(x', x)$ where $G' = G \sin \beta$ and $U' = U \sin \beta(x)$ and by replacing integrations with respect to $x$ by integrations (at these points only) with respect to the slant distance $s$ where $ds = dx/\sin \beta$.

4. The program has been written so as to be applicable, without modification, to a wide range of conditions. This means that it is not necessarily the most efficient program for any particular problem. Certainly much more efficient programs can be written for the special and common case where $m = 0$ in Equation [1] and also for the special cases where the vibrating surface has some special symmetry.

5. A final word of caution—the program is sufficiently complex that it is difficult to avoid a mistake either in input data or in interpretation of the limitations of the solution. A new solution should be accepted only after comparison with some relevant independent analysis.
APPENDIX

LISTING OF THE FORTRAN PROGRAM
PROGRAM OF G. CHERTOCK

COMMON XY, YD, XD, F2, G1, T2, U1, U2
1 F7 = F8 * GB * G9 * Z1 * Z2
DIMENSION X(73), YD(73), YDY(73), D(73), E(73), F1(73), F2(73)
1 G(73), T1(73), T2(73), U1(73), U2(73)
2 SM1(73), SM2(73), GB(73), G9(73), G4(4), QH(4), PSX(73)
3 Z1(73), Z2(73), F8(73), F7(73)

READ 666, 11, 12, 13, 14, 15, MN, AL, EPS, EPS2, W
1 FREQ, VEL, PHASE, AGL, DA, A, ZR, ZI, SIGMA
1 FORMAT (113F3.0, 4E5.2, 9F7.3, 9F5.1, 2F4.0/
1 2 F7.5, 2E10.6, 5F4.2)
PRINT 667, 11, 12, 13, 14, 15, MN, AL, EPS, EPS2, W
1 FORMAT (1H191X, 12HCONTROL DATA, 5X, 3HI1=I3, 5X, 3HI2=I3,
1 13HI3=I3, 5X, 3HI4=I3, 5X, 3HI5=I3, 5X, 3HI6=I3,
1 5X, 2HM=I3, 5X, 2HN=I3, 5X, 3HALF=F3.0, 5X, 5HALF=F5.1/
1 5X, 4HEPS=E7.2, 5X, 5HEPS2=E7.2, 5X, 2HFW=F7.3, 5X, 5HREQ=F7.3, 5X,
1 4HVEL=F8.3, 5X, 6PHASE=F5.1, 5X, 4HAGL=F4.0, 5X, 3HDA=F4.0, 5X, 2HMA=
1 5F7.3, 5X, 2HBA=F7.3, 5X, 3HZR=E10.6, 5X, 3HZI=E10.6, 5X, 6HSIGMA=F4.2)
PI = 3.1415927
NM1 = N - 1
NM2 = N - 2
IF (NM1) 38, 999, 59
99 CALL END JOB
38 IF (N1 - 5) 52, 59, 53
53 CALL SHAPE (N, A, B, I1)
GO TO 59
52 READ 54, (X(K), Y(K), YDY(K), D(K), E(K), K=1,N)
54 FORMAT (5E14.8)
A = (X(N) - X(1))/2.
IF (YDY(1) + YDY(I1)) 59, 55, 59
55 YDY(1) = (Y(2)**2 + (X(2) - X(1))**2)/2 ./ (X(2) - X(1))
YDY(I1) = YDY(1) + (X(1) - X(2))
YDY(N) = (Y(NM1)**2 + (X(N) - X(NM1))**2)/2 ./ (X(NM1) - X(N))
YDY(NM1) = YDY(N) + X(N) - X(NM1)
DO113 K = 3, NM2
KP1 = K + 1
113 YDY(K) = (Y(K) - Y(K1) - Y(K1)) / (X(KP1) - X(K1)) + (Y(K)
1 - Y(KM1))/ (X(K1) - X(KM1)) - (Y(K1) - Y(K1))/ (X(KP1) - X(KM1))
59 IF (EPS) 71, 72, 71
72 EPS = .001
71 IF (W1) 57, 58, 57
58 W = FREQ / 795.
57 IF (I2 - 5) 60, 62, 61
61 CALL SURVEL (N, A, W, SIGMA, VEL, PHASE, 12, AL, PSX)
GO TO 62
60 READ 65, (PSX(K), K=1, N)
65 FORMAT (6E12.4)
D0111 K = 2, NM1
KP1 = K + 1
111 PSR = SIGMA * Y(K) * (PSX(KP1) - PSX(K)) / (X(KP1) - X(K))
1 + (PSX(K) - PSX(KM1)) / (X(K1) - X(KM1)) - (PSX(KP1) - PSX(KM1))
2 / (X(KP1) - X(KM1)) * (4.)
111 G(K) = (Y(K) * PSR - YDY(K) * PSX(K))/A
G(K) = (Y(K) * PSX(K))/A
G(N) = (YDY(N) * PSX(N))/A
213 FORMAT (I19, 5E20.8)
62 H1 = W*A
   IF (PHASE=2*) 162,162,262
162 VPHASE = 0*
   GO TO 362
262 VPHASE = PHASE
362 PRINT 112,112,VPHASE
   PRINT13*,(K*,X(K)*,Y(K)*)*,YDY(K)*,PSX(K)*,G(K)*)K=1,N)
112 FORMAT (//,5X,39HSURFACE SHAPE AND VELOCITY DISTRIBUTION,5X,
   115HANDLE NUMBER M=13,5X,3HKA=F6.3,5X,15VELOCITY PHASE=F5.1*,
   2//11X8H STATION=8X,12HAXIAL COORD.=6X,14HSECTION RADIUS,
   36X14HRADIUS X SLOPE,6X,14HAXIAL VELOCITY,
   46X14HVELOCITY G(K))
67 IF (PHASE=2*) 296,296,169
169 RAR1=RAR
   GO TO 69
296 RAR1=0*
196 IF((I3-5)96,69,68
68 CALL INFLU(N,M,A,W,3,EPS)
   GO TO 69
96 AM=M
   DO 11 K=2*NM2
   KP1=K+1
   DO 11 J=KP1*NM1
   RS=(X(J)-X(K))**2+(Y(J)-Y(K))**2
1200 RT=RS+4.*Y(J)*Y(K)
1220 NN=XMAX1F(SQRTF(92.+8.4*RT/RS)*2.*W*(SQRTF(RT)
   1-SQRTF(RS)))*AM*20.*
   ANN=NN+NN
   DT=PI/ANN*2.
   PHI=-DT*.2113249
   DPHI=-2.*PHI
   SUM1=0.
   SUM2=0.
   SUM11=0.
   SUM21=0.
   SUM13=0.
   SUM23=0.
   31 PHI=PHI+DPHI
   DPHI=DT-DPHI
   CA=COSF(PHI)
   S1=SQRTF(RS+2.*Y(J)*Y(K)*)*(1*-CA)
   S2=COSF(W*S1/S1)*COSF(AM*PHI)
   S3=SQRTF(W*S1/S1)*COSF(AM*PHI)
   S4=(S2/S1+S3*W)/S1
   S5=(S5/S1-S2*W)/S1
   SUM1=SUM1+S2
   SUM2=SUM2+S3
   SUM11=SUM11+S4
   SUM12=SUM12+S5
   SUM13=SUM13+S4*CA
   SUM14=SUM14+S5*CA
   IF(PHI-P1+DPHI131,32,32
   32 T1(J,K)=A/ANN*SUM1
   T2(J,K)=A/ANN*SUM2
   T1(K,J)=T1(J,K)
   T2(K,J)=T2(J,K)
   S11JK=(X(J)-X(K))*SUM1
   S21JK=(X(J)-X(K))*SUM2
   S12JK=Y(J)*SUM13-Y(K)*SUM11
   IF(PHI-P1+DPHI131,32,32
S12KJ=Y(K)*SUM13-Y(J)*SUM11
S22JK=Y(J)*SUM23-Y(K)*SUM21
U1(K+J)=(Y(Y)*S12JK+YDY(J)*S11JK)*A/ANN
U2(K+J)=(Y(K)*S22JK+YDY(K)*S21JK)*A/ANN
U1(J+K)=(Y(Y)*S12JK+YDY(J)*S11JK)*A/ANN
U2(J+K)=(Y(K)*S22JK+YDY(K)*S21JK)*A/ANN

K=1

11 U2(K+J)=(Y(Y)*S22KJ+YDY(J)*S21KJ)*A/ANN

JJ=N
DO 14 J=2,N
IF(M)12,13,12
T1(J+K)=0.*
T2(J+K)=0.*
U1(J+K)=0.*
U2(J+K)=0.*
GO TO 14
12 Q(1)=QH(1)
Q(2)=QH(2)
Q(3)=QH(3)
Q(4)=QH(4)
IF (13-2) 80,83,80
80 NM=(N+1)/2
GO TO 85
83 NM=2
DO 18 K=NM,NM1
KM1=K-1
KP1=K+1
JJ=KM1
19 S4=T1(J*J)*QH(4)
DO 20 XX=1,3
DX=Q(K)*X(J+K)
YSQ=Y(K)**2+DX*(2.*YDY(K)+Q(K)*(YDY(J+K)-YDY(K))
YY=SQRTF(YSQ)
RS=DX*(YY-Y(K))**2
TS=0.*
PHI1=0.*

DO 10 J=1,3
DX=Q(K)*X(J+K)
YSQ=Y(K)**2+DX*(2.*YDY(K)+Q(K)*(YDY(J+K)-YDY(K))
YY=SQRTF(YSQ)
RS=DX*(YY-Y(K))**2
TS=0.*
PHI1=0.*

DO 10 J=1,3
DX=Q(K)*X(J+K)
YSQ=Y(K)**2+DX*(2.*YDY(K)+Q(K)*(YDY(J+K)-YDY(K))
YY=SQRTF(YSQ)
RS=DX*(YY-Y(K))**2
TS=0.*
PHI1=0.*

DO 10 J=1,3
DX=Q(K)*X(J+K)
YSQ=Y(K)**2+DX*(2.*YDY(K)+Q(K)*(YDY(J+K)-YDY(K))
YY=SQRTF(YSQ)
RS=DX*(YY-Y(K))**2
TS=0.*
PHI1=0.*

DO 10 J=1,3
DX=Q(K)*X(J+K)
YSQ=Y(K)**2+DX*(2.*YDY(K)+Q(K)*(YDY(J+K)-YDY(K))
YY=SQRTF(YSQ)
RS=DX*(YY-Y(K))**2
TS=0.*
PHI1=0.*

DO 10 J=1,3
DX=Q(K)*X(J+K)
YSQ=Y(K)**2+DX*(2.*YDY(K)+Q(K)*(YDY(J+K)-YDY(K))
YY=SQRTF(YSQ)
RS=DX*(YY-Y(K))**2
TS=0.*
PHI1=0.*

DO 10 J=1,3
DX=Q(K)*X(J+K)
YSQ=Y(K)**2+DX*(2.*YDY(K)+Q(K)*(YDY(J+K)-YDY(K))
YY=SQRTF(YSQ)
RS=DX*(YY-Y(K))**2
TS=0.*
PHI1=0.*

DO 10 J=1,3
DX=Q(K)*X(J+K)
YSQ=Y(K)**2+DX*(2.*YDY(K)+Q(K)*(YDY(J+K)-YDY(K))
YY=SQRTF(YSQ)
RS=DX*(YY-Y(K))**2
TS=0.*
PHI1=0.*
PHI2=PI/5.
H=PI/5.
S3=0.
PHI=PHI1-H
S2=0.

23 PHI=PHI+H
CA=COSF(PHI)
S1=SQRTF(RS+2.*Y(K)*YY*(1.-CA))
DS2=COSF(W*S1)/S1*COSF(AM*PHI)
S2=S2+DS2
IF(PHI+.6*H-PHI2)23,24
24 IF(PHI+.1*H-PHI2)25
26 SUM1=S2
27 DT=DS2
SUM2=SUM1*H/2.
29 PHI=PHI1-H/2.
GO TO 34
25 SUM3=H/6.*(SUM1+4.*S2+2.*S3)
IF((SUM3-SUM2)/(SUM3+TS))**2-EPS**2)27,28
28 SUM2=SUM3
S3=S3+S2
H=H/2.
GO TO 29
27 TS=TS+SUM3
25 IF(PHI2-PI+H)30
30 PHI1=PHI2
PHI2=PHI2+PI/5.
S2=DT
S3=0.
H=PI/5.
PHI=PHI1
GO TO 23
133 S4=S4+QH(KK)*TS*A/PI
20 CONTINUE
IF(JJ-KM1)21
22 SL4=S4
JJ=KP1
GO TO 19
21 P=(2.*X(K)-X(KP1)-X(KM1))/(X(KP1)-X(KM1))
T1(K)=SL4*.75*(1.+P)*(1.-P**2)+S4*.75*(1.-P)*
(1.-P**2)-T1(KK)/4.*(1.+3.*P)-T1(K(KP1))/
24.*(1.+3.*P)
SUM2=0.
NN=XMAX1F(10.*3.*W*Y(K)*AM*5.)
ANN=NN+NN
DT=PI/ANN
PHI=-DT**2113249
DPHI=-2.*PHI
SUM3=0.
SUM1=0.
34 PHI=PHI+DPHI
DPHI=DT-DPHI
S=2.*Y(K)*SINF(PHI)
S3=SINF(W*S)*COSF(AM*PHI+2.)
SUM1=SUM1+S3/S
SUM2=SUM2+COSF(W*S)*COSF(2.*AM*PHI)
SUM3=SUM3+S3
IF(PHI-P/2.+DPHI)43
44 T2(K)=SUM1*A/ANN
U1(K)=T1(K)/2.-W*A*SUM3**5/ANN
4080
4080
4100
4120
U2(K,K) = -T2(K,K)/2 + W*A*SUM2*.5/ANN

145 T1(1,1) = 0.
    T2(1,1) = 0.
    T1(N,N) = 0.*
    T2(N,N) = 0.*
    GO TO 146

47 S = SQRTF((X(1) - X(2))**2 + Y(2)**2)
    T1(1,1) = (4.*A*SINF(W*S)/S**2/W - T1(1,2))
    T2(1,1) = W*A
    S = SQRTF((X(N) - X(NM1))**2 + Y(NM1)**2)
    T1(N,N) = (4.*A*SINF(W*S)/S**2/W - T1(N,NM1))
    T2(N,N) = W*A

146 U1(1,1) = -T1(lo1)*.5
    U2(1,1') = 0.*
    U1(N,N) = -T1(N#N)**5
    U2(N,N) = 0.*

69 CONTINUE

73 IF(SQB(ZR*ZI) & GT & 75, 74, 97)
74 CALL SURFPR(NMAW, 14, HALFEPS2, TZR, TZI, NST)
    GO TO (75, 79, 284) NST
75 DO 78 J = 1, N
    F1(J) = ZR*G(J)
    F2(J) = ZI*G(J)
    ZI(J) = ZR*G(J)**2
78 Z2(J) = ZI*G(J)**2
    TZR = SM3PT(XtZ19N.2)
    TZI = SM3PT(X9Z29N92)
79 IF(15 - 5) 122, 284, 94
89 ETA = 1.
    RAR = TZR/ETA/A/2 + 1.E-37
    RAI = TZI/ETA/A/2.
    ASPL = 10.*LOG10F(RAR*(VEL*A*5.49E9)**2)
37 PRINT 91
91 FORMAT(1H1G10X, 7HSTATION, 11X, 9HF1 EQUALS, 11X, 9HF2 EQUALS, 1X, 18HMODAL REAC. COEF. =E15.8, 1X, 18HSOURCE POWER LEVEL(DB RE .0002/1YD.) = E15.8)
284 CONTINUE
92 FORMAT(120, 2E20.8, 120, 2E20.8)
94 CALL FIELD (NMAW, VEL, AGL, DA)
    GO TO 95
118 FORMAT(1H0, 50X, 19HFAR FIELD PRESSURES/10X, 9HDIRECTION, 6X, 23HREL. 1TRANSFER IMPEDANCE, 10X, 14HRELATIVE PHASE, 2X, 18HDIRECTIVITY FACTOR, 35X, 20HSPL(DB RE .0002/1YD.))
G1 = BEJOF(H8)
G2 = BEJ1F(H8)
MM = 0
48 IF(M-MM)46,46,147
147 AMM = MM
IF(H8)81,82,81
82 G1 = 0.
G2 = 0.
GOTO 46
81 CONTINUE
TG = G1 * AMM / H8 - G2
G2 = G1 * (1. - AMM * (1. + AMM) / H8 ** 2) + G2 / H8 * (1. + AMM)
G1 = TG
MM = MM + 1
GO TO 48
46 G3 = W * G1 * (-G(K) * DY(K) * F1(K) * CA) + G2 * F2(K) * H8
G4 = W * G1 * DY(K) * F2(K) * CA - G2 * F1(K) * H8
G5 = COSF(H9)
G6 = SINF(H9)
G8(K) = G5 * G3 + G6 * G4
131 G9(K) = G5 * G4 - G6 * G3
SUM1 = SM3PT(X * G8 + N * 2)
SUM2 = SM3PT(X * G9 + N * 2)
143 IF (PHASE - 2*) 143, 143, 144
144 TEM = SUM1 - SUM2
SUM2 = SM2 + SUM1 - TEM
SUM1 = TEM
45 L = L + 1
121 IF (PHASE - 2*) 121, 120, 121
120 SUM = SQRTF(SUM1 ** 2 + SUM2 ** 2) / A / 2 + 1E-37
FASANG = ATN1F(SUM1 / SUM2)
DIRFAC = SUM2 / (RAR + RAR1) / ETA
SPL = 20 * LOG10F(VEL * SUM * A * 5.49E9 / ETA)
LM = L - 1
Z1(LM) = ANG
Z2(LM) = DIRFAC * SA
136 IF (PHASE - 2*) 136, 135, 136
336 PRINT 337
337 PRINT(H0, 13X, 63HField below is combined field of two velocity distributions)
337 PRINT 336
36 PRINT 318, ANG, SUM, FASANG, DIRFAC, SPL
120 ANG = ANG + DA
150 IF (FL = 2) 150, 155, 150
155 IF (FL = 2) 155, 156, 155
511 SPL = SPL + LOG10F(AVDF / DIRFAC) * 10.
513 FORMAT (13X, 32HTotal Radiated Power = All Phases = E20.8)
512 PRINT 98, AVDF
513 FORMAT (13X, 25HMean Directivity Factor = E20.8)
159 IF (PHASE = 90) 95, 259, 95
259 PHASE = 90.
GO TO 57
END
C SPHEROID SURFACE SHAPE WITH UNIFORM OR COSINE SPACING OF STATIONS
SUBROUTINE SHAPE(N, A, B, 11)
COMMON X, Y, YDY, D, E, F1, F2, G, T1, T2, U1, U2
DIMENSION X(73, 7), Y(73, 7), YDY(73, 7), D(73, 7), E(73, 7), F1(73, 7), F2(73, 7)
1G(73, 7), T1(73, 7), T2(73, 7), U1(73, 7), U2(73, 7)
NM=(N+1)/2
AN=N
DO 11 K=NM, N
AK=K
IF(K-1)12, 13, 20
12 X(K)=A*(2*K-AN-1)/(AN-1.)
GO TO 14
13 X(K)=A*COSF(3.14159*(AN-AK)/(AN-1.))
14 Y(K)=B*SQRTF(1.0-(X(K)/A)**2)
YDY(K)=-X(K)*(B/A)**2
NMK=N-K+1
X(NMK)=-X(K)
Y(NMK)=Y(K)
11 YDY(NMK)=-YDY(K)
RETURN
END

C ACCORDION VIBRATION MODE WITH L NODES
SUBROUTINE SURVEL(N, M, A, W, SIGMA, VEL, PHASE, I2, AL, PSX)
COMMON X, Y, YDY, D, E, F1, F2, G, T1, T2, U1, U2
DIMENSION X(73, 7), Y(73, 7), YDY(73, 7), D(73, 7), E(73, 7), F1(73, 7), F2(73, 7)
1G(73, 7), T1(73, 7), T2(73, 7), U1(73, 7), U2(73, 7), PSX(73)
PI=3.1415927
DO 11 K=1, N
ANG=AL*PI/A*(X(K)-X(1))/2
PSX(K)=COSF(ANG)
PSR=AL*SIGMA*Y(K)/A*SINF(ANG)*PI/2
G(K)=(Y(K)*PSR-YDY(K)*PSX(K))/A
11 RETURN
END

C DUMMY SUBROUTINE INFLU
SUBROUTINE INFLU(N, M, A, W, EPS)
COMMON X, Y, YDY, D, E, F1, F2, G, T1, T2, U1, U2
DIMENSION X(73, 7), Y(73, 7), YDY(73, 7), D(73, 7), E(73, 7), F1(73, 7), F2(73, 7)
1G(73, 7), T1(73, 7), T2(73, 7), U1(73, 7), U2(73, 7)
RETURN
END

38
C DUMMY SUBROUTINE FIELD

SUBROUTINE FIELD(N+M+A+W+VEL+15+AGL+DA)
COMMON X+Y+YDY+D+E+F1+F2+G+T1+T2+U1+U2
DIMENSION X(73),Y(73),YDY(73),D(73),E(73),F1(73),F2(73),
G(73),T1(73),T2(73),U1(73),U2(73)
RETURN
END

C INTEGRAL OF YDX BY 3-POINT SIMPSONS RULE
C MODIFIED FOR UNEQUAL INTERVALS AND ODD OR EVEN
C NUMBER OF STATIONS
FUNCTION SM3PT(X+Y+J)
DIMENSION X(100),Y(100)
NM1=N-1
JP=(-1)**J
IF(JP)12911.11
11 Z=0.
MM=2
MMMM=N-1
GO TO 14
12 Z=(X(2)-X(1))/2*Y(1)+Y(2)+(X(N)-X(NM1))/2*
1*(Y(N)+Y(NM1))
MM=3
MMMM=N-2
14 DO 13 M=MM,MMMM
MM1=M-1
MP1=M+1
P=(2*X(M)-X(MM1)-X(MP1))/(X(MP1)-X(MM1))
13 Z=Z+(X(MP1)-X(MM1))/6*(Y(MM1)*(1+3*P)/
1*(1+P)+Y(M)+(1-3*P)/
2*(1-P))
SM3PT=Z
RETURN
END
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41
A FORTRAN program is presented which computes the sound pressures, at the surface and in the near or far field, radiated by an arbitrary surface of revolution which is vibrating in an almost arbitrary pattern with arbitrary frequency and phase. The method of solution, the input data, and the output data are all described in detail.
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