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AXISYMMETRIC ELASTIC STRESSES IN CIRCULAR CYLINDRICAL SHELLS STIFFENED BY INTERNAL CHANNEL SECTIONS AND SUBJECTED TO UNIFORM EXTERNAL PRESSURE LOADING

by

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STRUCTURAL MECHANICS LABORATORY
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S-F013 03 02

TABLE OF CONTENTS

LIST OF FIGURES

NOTATION

a_i, b_i, d_i, g_i, f_i', f_i'', s_i, t_i', t_i''	Coefficients representing edge rotation and displacement per unit edge or surface load for cylindrical shell Element i with symmetric loading
a_3^A, a_3^B, a_3^{AB}, a_3^{BA}, c_3^A, c_3^B, g_3^A, g_3^B, g_3^{AB}, g_3^{BA}, q_3^B, q_3^{BA}, $t_3'^B$	Coefficients representing edge rotation and displacements per unit edge load for circular annulus Element 3.
D_i	$= \dfrac{Eh_i^3}{12(1-\nu^2)}$, flexural rigidity of shells and plates
E	Young's modulus
H_i	Edge force normal to axis of symmetry
h_i	Thickness of Element i
l_i	Length of shell Element i
M_i	Edge bending moment in a meridional plane
P_i	Axial edge force
p	External uniform pressure
R_i	Radial distance from axis of symmetry
r	Variable radial distance from axis of symmetry
w_i	Radial displacement of Element i
x_i	Axial coordinate taken along shell Element i
β_i	$= \dfrac{\sqrt[4]{3(1-\nu^2)}}{\sqrt{R_i h_i}}$
ϵ	Strain
θ_i	Axial rotation of Element i
ν	Poisson's ratio
σ	Stress

ABSTRACT

A theoretical analysis of the axisymmetric elastic deformations and stresses in a circular cylindrical shell internally stiffened by inverted channel frames under uniform external pressure loading is presented. The solution is based on the use of edge coefficients for plate and shell elements to determine the edge forces and moments arising at the common juncture of these elements. Equations are given for computing numerically the stresses in the elements of the composite structure once the edge forces and moments are determined.

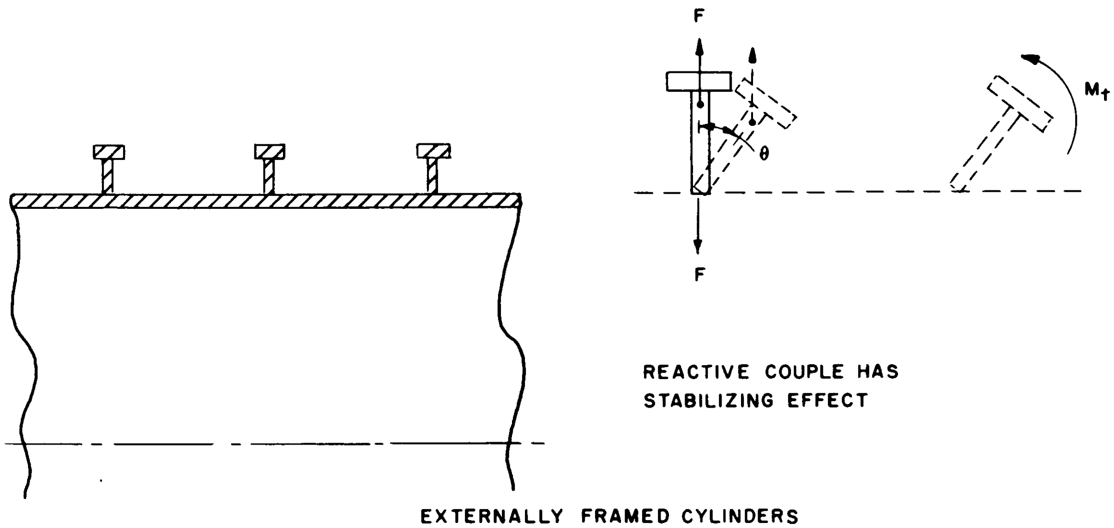
INTRODUCTION

The use of T-frames has become a common practice for transverse stiffening of circular cylindrical shells subjected to external pressure loading. Such frames offer a relatively large moment of inertia for a given weight or cross section of material to resist bending in their own plane. However, internal T-frames are unstable against twisting out of their plane, as can be seen in Figure 1, where any tilt deformation θ sets up a reactive couple which has a destabilizing effect on the frame. External frames are elastically stable against this twisting. Furthermore, the compressive radial forces F acting on an internal frame tend to buckle the web, whereas these forces are tensile for external frames. Hence, because internal T-frames are more susceptible to twisting, they will undergo less yielding through their cross section prior to collapse than will external frames, so that in such a case the full strength potential of the material will not be realized.

The relative torsional instability of internal T-frames suggests the possible use of other types of stiffeners for internally framed cylindrical shells. One such type would be frames of inverted channel cross section; see Figure 2a. This type of stiffener is not only less susceptible to ~~twisting~~ but also offers the advantage of effectively reducing the unsupported length of shell plating between adjacent stiffeners, thereby changing the deformation pattern and possibly increasing the strength against interbay collapse.

At the time this type of stiffening system was conceived for pressure vessel application, no rational analyses were available to determine the possible advantages, if any, of resorting to the use of such an arrangement over the conventional T-frames. An experimental program was initiated at the David Taylor Model Basin, where intuition and engineering considerations were utilized for proportioning the structural elements of the first few models. In conjunction with the experimental program, an analytical study was also initiated to develop an analysis for determining the axisymmetric elastic deformation and stresses occurring in a circular cylindrical shell stiffened by inverted channel frames.

In this report an analysis is developed for determining the deformations and stresses in a typical portion of a structure such as that shown in Figure 2a. The method is based on



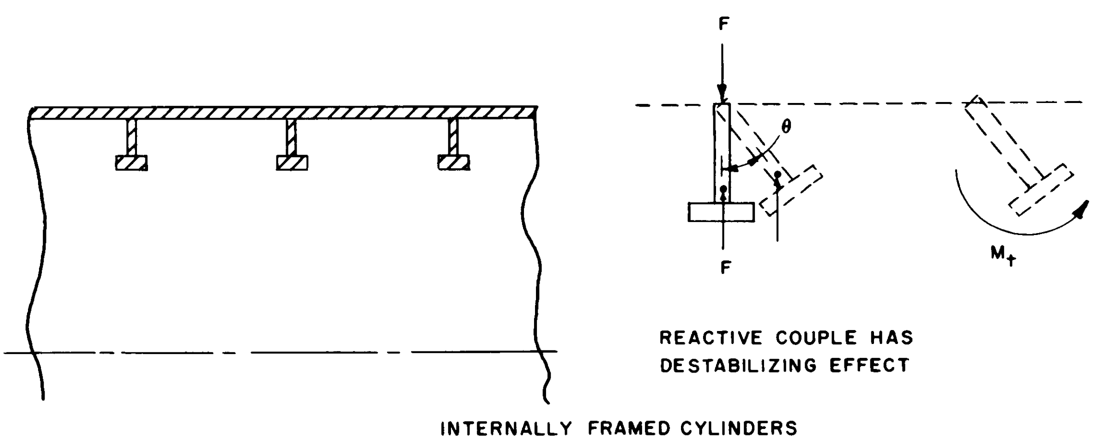


Figure 1 – Torsional Stability of External and Internal T-Frames

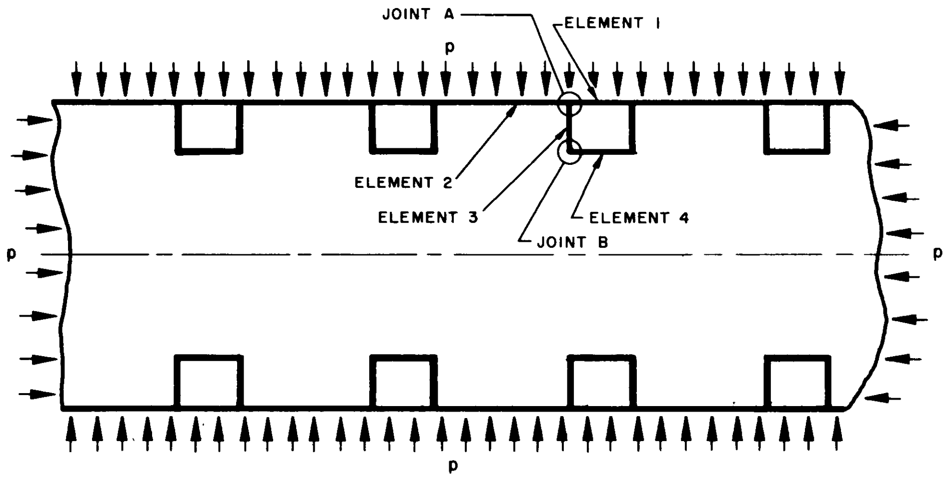


Figure 2a – Cylinder Stiffened by Inverted Channel Frames

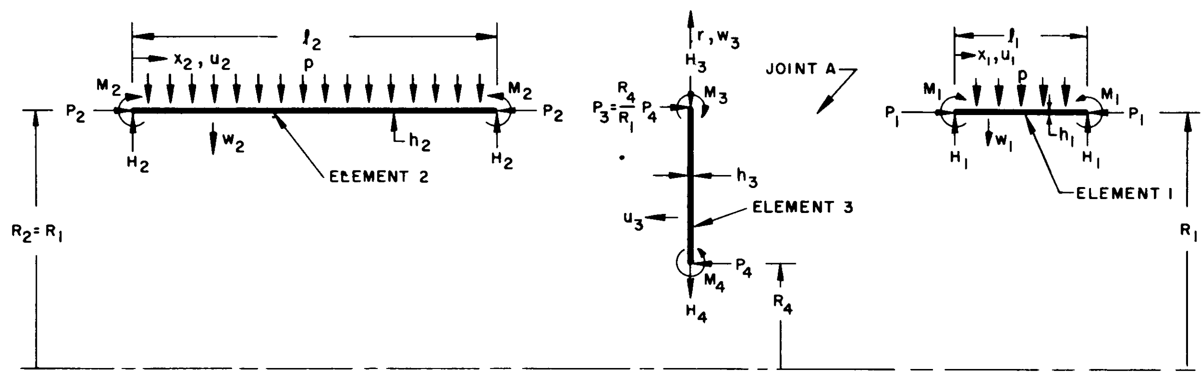


Figure 2b – Joint A

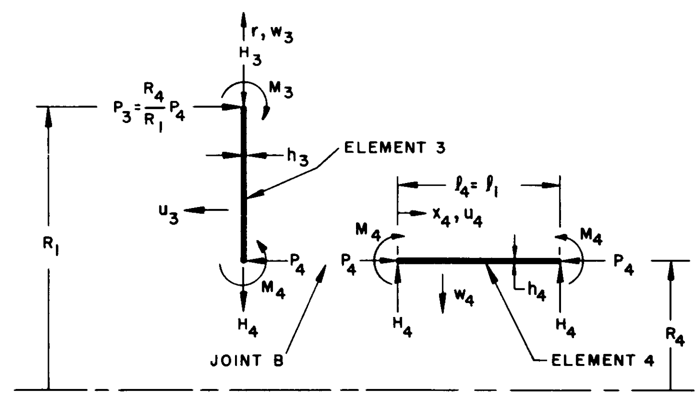


Figure 2c – Joint B

Figure 2 – Sign Convention and Free-Body Diagrams Showing Edge Forces and Moments Acting on Shell and Annulus Elements

the use of edge coefficients for plate and shell elements, equilibrium of forces and moments, and compatibility of deformations at the common junctures of the elements comprising the structure.

Edge coefficients are developed and defined in Appendixes A and B, and the input required to obtain numerical results from this theoretical analysis on the IBM-7090 computer at the Model Basin is given in Appendix C.

THEORETICAL ANALYSIS

The elastic axisymmetric deformations occurring in uniformly stiffened cylindrical shells subjected to external hydrostatic pressure can be determined from the analysis of Von Sanden and Günther[1] if it is assumed that the cross section of the stiffeners does not change shape under loading. In their analysis these investigators assumed that the frames were rigid in the sense that only radial translation of any point of the frame would occur. This leads to the fact that the shell at the toe of the frame will not rotate, a boundary condition used in the analysis to solve the differential equation of equilibrium. However, frames of channel cross section such as that shown in Figure 2a can bend; the webs can bend out of their plane, also the flange and shell material between the two webs can deform.

Therefore, additional considerations are required in order to develop an analysis for determining the elastic axisymmetric deformations in circular cylindrical shells stiffened by transverse frames of channel cross section. This is done by treating the entire structure to be composed of concentric cylindrical shell elements and circular annulii as shown in Figure 2a. Each element is considered to be loaded at its boundaries by unknown axisymmetric edge forces and moments and, as the case may be, by pressure loading on its outside surface. The redundant edge forces and moments, which arise from the interaction of the various elements with each other, are then determined by enforcing compatibility of deformations at the junctures and satisfying force and moment equilibrium. Once the edge forces and moments are determined, the axisymmetric deformations and stresses in each element are readily obtainable.

The method of analysis based on the use of edge coefficients has found wide application in studying stresses and deformations in complex structures composed of ring, plate, and shell elements; see References 2 through 6.

DETERMINATION OF EDGE MOMENTS M_i AND EDGE FORCES H_i, P_i

Proceeding as just outlined, Elements 1, 2, 3, and 4 which comprise the structure are isolated for study as shown in the "free-body" diagrams of Figure 2. The nomenclature and sign conventions used in the analysis are as defined in that figure. The quantities M_i, H_i and P_i are edge moments and edge forces which are developed along the edges of each element from the interaction of the elements with each other.

---

[1]References are listed on page 27.

The edge rotations and displacements of each element can be expressed in terms of edge coefficients. These edge coefficients are functions of the geometry and elasticity of their respective elements; they represent the amount of rotation or displacement per unit edge moment, unit edge vertical force, unit edge horizontal force, and unit surface pressure. The total rotation θ_i^A and displacements w_i^A , u_i^A at Joint A for Elements 1, 2, and 3 are:

$$
\begin{aligned}
\theta_1^A &= \left[\frac{dw_1}{dx_1}\right]_{x=0} \doteq a_1 M_1 + b_1 H_1 \\
w_1^A &= [w_1]_{x=0} = d_1 M_1 + g_1 H_1 + f_1' P_1 + f_1'' p \\
u_1^A &= [u_1]_{x=0} = s_1 H_1 + t_1' P_1 + t_1'' p \\
\theta_2^A &= \left[\frac{dw_2}{dx_2}\right]_{x=l_2} = -a_2 M_2 - b_2 H_2 \\
w_2^A &= [w_2]_{x=l_2} = d_2 M_2 + g_2 H_2 + f_2' P_2 \cdot f_2'' p \\
\theta_3^A &= \left[\frac{du_3}{dr}\right]_{r=R_1} = a_3^A M_3 + a_3^{AB} M_4 + c_3^A P_4 \\
w_3^A &= [w_3]_{r=R_1} = g_3^A H_3 + g_3^{AB} H_4 \\
u_3^A &= [u_3]_{r=R_1} = 0
\end{aligned}
\qquad [1]
$$

The quantities a_1, a_2, a_3^A, a_3^{AB}, b_1, b_2 . . . . are edge coefficients developed and defined in Appendixes A and B for a cylindrical shell and an annulus, respectively.

Likewise, Elements 3 and 4 are isolated for study in Figure 2c. The total rotation θ_i^B and displacement w_i^B , u_i^B at Joint B for Elements 3 and 4 are:

$$
\begin{aligned}
\theta_3^B &= \left[\frac{du_3}{dr}\right]_{r=R_4} = a_3^B M_4 + a_3^{BA} M_3 + c_3^B P_4 \\
w_3^B &= [w_3]_{r=R_4} = g_3^B H_4 + g_3^{BA} H_3 \\
u_3^B &= [u_3]_{r=R_4} = q_3^B M_4 + q_3^{BA} M_3 + t_3'^B P_4 \\
\theta_4^B &= \left[\frac{dw_4}{dx_4}\right]_{x=0} = a_4 M_4 + b_4 H_4 \\
w_4^B &= [w_4]_{x=0} = d_4 M_4 + g_4 H_4 + f_4' P_4 \\
u_4^B &= s_4 H_4 + t_4' P_4
\end{aligned}
\qquad [1]
$$

Compatibility of rotations and radial displacements at Joint A requires that:

$$
\begin{aligned}
\theta_1^A &= -\theta_3^A \\
\theta_2^A &= -\theta_3^A \\
w_1^A &= -w_3^A \\
w_2^A &= -w_3^A
\end{aligned}
\qquad [2]
$$

and at Joint B that:

$$
\begin{aligned}
\theta_4^B &= -\theta_3^B \\
w_4^B &= -w_3^B
\end{aligned}
\qquad [2]
$$

u_1^A and u_4^B are the relative axial displacements at the edge of the shell with respect to a datum located at midbay of Elements 1 and 4, respectively. From symmetry of the structure and from a consideration of only axisymmetric deformations, it can be concluded that there is no relative axial displacement of points located at midbay of the shell of Element 1 with respect to the same region in Element 4. Therefore,the difference between the axial displacements u_1^A and u_4^B represents the difference between the axial displacement of the inner and outer edges of the annulus (Element 3). Thus,

$$u_4^B - u_1^A = -(u_3^B - u_3^A) \qquad [3]$$

The following equations involving only the edge forces, edge moments, and applied external pressure are obtained when the appropriate expressions of Equations [1] are substituted into Equations [2] and [3]:

$$
\begin{aligned}
a_1 M_1 + a_3^A M_3 + a_3^{AB} M_4 + b_1 H_1 + c_3^A P_4 &= 0 \\
-a_2 M_2 + a_3^A M_3 + a_3^{AB} M_4 - b_2 H_2 + c_3^A P_4 &= 0 \\
d_1 M_1 + g_1 H_1 + g_3^A H_3 + g_3^{AB} H_4 + f_1' P_1 &= -f_1'' p \\
d_2 M_2 + g_2 H_2 + g_3^A H_3 + g_3^{AB} H_4 + f_2' P_2 &= -f_2'' p \\
a_3^{BA} M_3 + (a_3^B + a_4) M_4 + b_4 H_4 + c_3^B P_4 &= 0 \\
d_4 M_4 + g_3^{BA} H_3 + (g_3^B + g_4) H_4 + f_4' P_4 &= 0 \\
q_3^{BA} M_3 + q_3^B M_4 - s_1 H_1 + s_4 H_4 - t_1' P_1 + (t_3'^B + t_4') P_4 &= t_1'' p
\end{aligned}
\qquad [4]
$$

Equilibrium of the edge moments and edge forces at Joint A requires that:

$$
\begin{aligned}
&M_1 - M_2 + M_3 = 0 \\
&H_1 + H_2 - H_3 = 0 \\
&P_1 - P_2 + \frac{R_4}{R_1} P_4 = 0
\end{aligned}
\qquad [5]
$$

Equations [4] and [5] represent ten equations with eleven unknowns in the edge moments and edge forces. An additional relationship is thus required in order to solve these unknowns, and it can be obtained by considering the influence of the axial pressure acting on the ends of the structure. With reference to Figure 3, the force equilibrium in the longitudinal direction requires that:

$$2\pi R_1 P_1 + 2\pi R_4 P_4 = \pi R_1{}^2 p \qquad [6]$$

Rewriting Equation [6] gives:

$$P_1 + \frac{R_4}{R_1} P_4 = \frac{R_1}{2} p \qquad [7]$$

COMPUTATION OF STRESSES AND STRAINS IN CYLINDRICAL SHELL ELEMENT i

The method for determining the edge forces H_i and P_i and edge moments M_i shown in Figure 2 was described in the preceding section of this report. Once the edge forces and moments are known, the following formulas may be used for determining the longitudinal and circumferential stresses and strains which occur at any point on the surfaces of the cylindrical shell element:

Circumferential strain $$\epsilon_\phi = -\frac{w_i}{R_i} \qquad [8]$$

Longitudinal stress $$\sigma_x = -\frac{P_i}{h_i} \mp \frac{6M_x}{h_i^2} = -\frac{P_i}{h_i} \pm \frac{6D_i}{h_i^2} \frac{d^2 w_i}{dx_i^2} \qquad [9]$$

Circumferential stress $$\sigma_\phi = E\epsilon_\phi + \nu\sigma_x \qquad [10]$$

Longitudinal strain $$\epsilon_x = \frac{1-\nu^2}{E}\sigma_x - \nu\epsilon_\phi \qquad [11]$$

where, in the above equations, the upper sign is for the outer fiber and the lower sign for the inner fiber of each shell plating. The expressions for w_i and $\dfrac{d^2 w_i}{dx_i^2}$ can be obtained from Equations [A.4] and [A.6] of Appendix A, respectively.

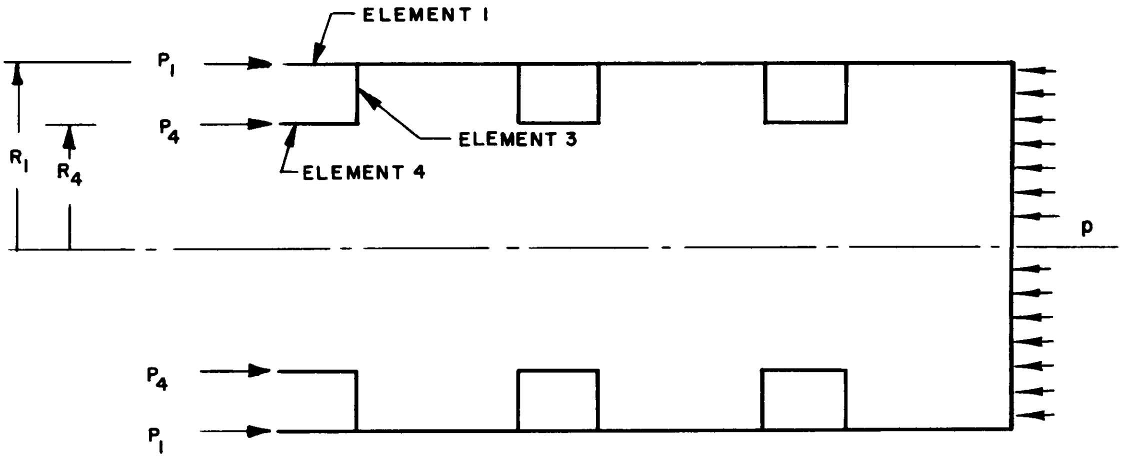


Figure 3 – Distribution of Axial-Pressure Load to the Cylindrical Shell Elements 1 and 4

COMPUTATION OF STRESSES IN CIRCULAR ANNULUS (ELEMENT 3)

Expressions for the direct and bending stresses in both the radial and tangential directions of annulus Element 3 are developed in Appendix B. The direct stresses in the annulus are influenced only by the radial edge forces H_3 and H_4, while the bending stresses are functions of both the edge moments M_3 and M_4 and the axial edge force P_4. The total stress in the radial and tangential directions is the sum of the direct stress and bending stress. Therefore,

$$\begin{aligned} \sigma_r &= (\sigma_r)_d \mp (\sigma_r)_b \\ \sigma_t &= (\sigma_t)_d \mp (\sigma_t)_b \end{aligned} \qquad [12]$$

Substituting into Equation [12] the expressions developed in Appendix B for the direct and bending stresses yields the following expressions for the radial and tangential stresses of any point on the lateral surfaces of the annulus:

$$\sigma_r = \frac{-r^2(H_4R_4^2 + H_3R_1^2) + (H_4 + H_3)\,R_4^2R_1^2}{r^2h_3(R_1^2 - R_4^2)} \mp \frac{6}{h_3^2}\left\{\frac{R_1^2}{R_1^2 - R_4^2}\left(1 - \frac{R_4^2}{r^2}\right)M_3 + \frac{R_4^2}{R_1^2 - R_4^2}\left(\frac{R_1^2}{r^2} - 1\right)M_4 + \frac{1+\nu}{2}R_4\left[\left(\frac{R_4}{r}\right)^2\left(\frac{R_1^2 - r^2}{R_1^2 - R_4^2}\right)\ln\frac{R_4}{R_1} - \ln\frac{r}{R_1}\right]P_4\right\} \qquad [13]$$

$$\sigma_t = \frac{-r^2(H_4R_4^2 + H_3R_1^2) - (H_4 + H_3)R_4^2R_1^2}{r^2h_3\,(R_1^2 - R_4^2)} \mp \frac{6}{h_3^2}\left\{\frac{R_1^2}{R_1^2 - R_4^2}\left(1 + \frac{R_4^2}{r^2}\right)M_3 - \frac{R_4^2}{R_1^2 - R_4^2}\left(\frac{R_1^2}{r^2} + 1\right)M_4 + \frac{1+\nu}{2}R_4\left[\frac{1-\nu}{1+\nu} - \left(\frac{R_4}{r}\right)^2\left(\frac{R_1^2 + r^2}{R_1^2 - R_4^2}\right)\ln\frac{R_4}{R_1} - \ln\frac{r}{R_1}\right]P_4\right\} \qquad [14]$$

where, in the above equations, the upper sign is for Surface X and the lower sign for Surface Y; see Figure 5 in Appendix B.

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APPENDIX A

DEVELOPMENT OF EDGE COEFFICIENTS FOR A CIRCULAR CYLINDRICAL SHELL WITH SYMMETRIC LOADING

If the beam-column effect[7] due to the axial force P_i were neglected, the differential equation of equilibrium governing the axisymmetric elastic deformations of a thin-walled circular cylinder based on small-deflection theory would be given by (see Appendix A of Reference 7):

$$D_i \frac{d^4 w_i}{dx_i^4} + \frac{E h_i w_i}{R_i^2} = p - \frac{\nu P_i}{R_i} \qquad [A.1]$$

When the following notation is used:

$$\beta_i^4 = \frac{E h_i}{4 R_i^2 D_i} = \frac{3(1-\nu^2)}{R_i^2 h_i^2} \qquad [A.2]$$

Equation [A.1] can be represented in the simplified form:

$$\frac{d^4 w_i}{dx_i^4} + 4\beta_i^4 w_i = \frac{1}{D_i}\left(p - \frac{\nu P_i}{R_i}\right) \qquad [A.3]$$

The general solution of this equation is:

$$\begin{aligned} w_i = {} & \frac{R_i^2}{E h_i} p - \frac{\nu R_i}{E h_i} P_i + C_1 \sin \beta_i x_i \sinh \beta_i x_i \\ & + C_2 \sin \beta_i x_i \cosh \beta_i x_i + C_3 \cos \beta_i x_i \sinh \beta_i x_i \\ & + C_4 \cos \beta_i x_i \cosh \beta_i x_i \end{aligned} \qquad [A.4]$$

and the first three derivatives of Equation [A.4] are:

$$\begin{aligned} \frac{1}{\beta_i} \frac{dw_i}{dx_i} = {} & (C_2 - C_3) \sin \beta_i x_i \sinh \beta_i x_i + (C_1 - C_4) \sin \beta_i x_i \cosh \beta_i x_i \\ & + (C_1 + C_4) \cos \beta_i x_i \sinh \beta_i x_i + (C_2 + C_3) \cos \beta_i x_i \cosh \beta_i x_i \end{aligned} \qquad [A.5]$$

$$\begin{aligned} \frac{1}{2\beta_i^2} \frac{d^2 w_i}{dx_i^2} = {} & -C_4 \sin \beta_i x_i \sinh \beta_i x_i - C_3 \sin \beta_i x_i \cosh \beta_i x_i \\ & + C_2 \cos \beta_i x_i \sinh \beta_i x_i + C_1 \cos \beta_i x_i \cosh \beta_i x_i \end{aligned} \qquad [A.6]$$

$$\frac{1}{2\beta_i^3}\frac{d^3 w_i}{dx_i^3} = -(C_2+C_3)\sin\beta_i x_i \sinh\beta_i x_i - (C_1+C_4)\sin\beta_i x_i\cosh\beta_i x_i$$
$$+(C_1-C_4)\cos\beta_i x_i\sinh\beta_i x_i + (C_2-C_3)\cos\beta_i x_i\cosh\beta_i x_i \qquad [A.7]$$

C_1, C_2, C_3, and C_4 are the constants of integration which must be determined from the conditions at the ends of the cylinder; see Figure 4. The longitudinal bending moment M_x and the transverse shearing force Q_x are related to the derivatives of w_i by the following equations:

$$M_x = -D_i\frac{d^2 w_i}{dx_i^2}$$
$$Q_x = \frac{dM_x}{dx} = -D_i\frac{d^3 w_i}{dx_i^3} \qquad [A.8]$$

With reference to Figure 4, the load boundary conditions are given by:

$$\text{At}\quad x_i = 0:\quad M_x = M_i;\quad Q_x = H_i$$
$$\text{At}\quad x_i = l_i:\quad M_x = M_i;\quad Q_x = -H_i \qquad [A.9]$$

When Equations [A.6], [A.7], and [A.8] are substituted into the boundary conditions, [A.9], and solved simultaneously, the following expressions are found for the four integration constants C_1, C_2, C_3, and C_4:

$$C_1 = -\frac{M_i}{2D_i\beta_i^2}$$

$$C_2 = \frac{M_i}{2D_i\beta_i^2}\cdot\frac{\cosh\beta_i l_i - \cos\beta_i l_i}{\sinh\beta_i l_i + \sin\beta_i l_i} - \frac{H_i}{2D_i\beta_i^3}\cdot\frac{\sin\beta_i l_i}{\sinh\beta_i l_i + \sin\beta_i l_i}$$

$$C_3 = \frac{M_i}{2D_i\beta_i^2}\cdot\frac{\cosh\beta_i l_i - \cos\beta_i l_i}{\sinh\beta_i l_i + \sin\beta_i l_i} + \frac{H_i}{2D_i\beta_i^3}\cdot\frac{\sinh\beta_i l_i}{\sinh\beta_i l_i + \sin\beta_i l_i} \qquad [A.10]$$

$$C_4 = -\frac{M_i}{2D_i\beta_i^2}\cdot\frac{\sinh\beta_i l_i - \sin\beta_i l_i}{\sinh\beta_i l_i + \sin\beta_i l_i} - \frac{H_i}{2D_i\beta_i^3}\cdot\frac{\cosh\beta_i l_i + \cos\beta_i l_i}{\sinh\beta_i l_i + \sin\beta_i l_i}$$

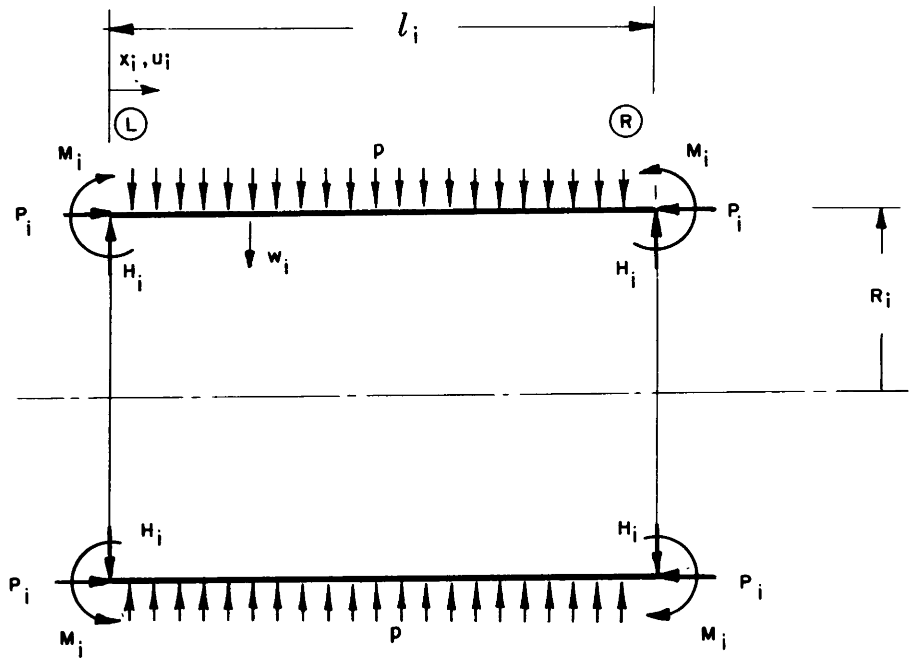

Figure 4 – Edge Forces and Moments for a Cylindrical Shell Element i

Substituting integration constants, Equations [A.10], into both the deflection function, [A.4], and the slope function, [A.5], and then evaluating the resulting expressions at the left and right edges of the shell element, that is, at $x = 0$ and $x = l_i$, respectively, results in the following equations:

$$[w_i]_{x=0,\ x=l_i} = -\frac{1}{2D_i\beta_i^2} \cdot \frac{\sinh \beta_i l_i - \sin \beta_i l_i}{\sinh \beta_i l_i + \sin \beta_i l_i} M_i - \frac{1}{2D_i\beta_i^3} \cdot \frac{\cosh \beta_i l_i + \cos \beta_i l_i}{\sinh \beta_i l_i + \sin \beta_i l_i} H_i - \frac{\nu R_i}{Eh_i} P_i + \frac{R_i^2}{Eh_i} p \qquad \text{[A.11]}$$

and

$$\left[\frac{dw_i}{dx_i}\right]_{\substack{x=0\\ x=l_i}} = \pm \frac{1}{D_i\beta_i} \cdot \frac{\cosh \beta_i l_i - \cos \beta_i l_i}{\sinh \beta_i l_i + \sin \beta_i l_i} M_i \pm \frac{1}{2D_i\beta_i^2} \cdot \frac{\sinh \beta_i l_i - \sin \beta_i l_i}{\sinh \beta_i l_i + \sin \beta_i l_i} H_i \qquad \text{[A.12]}$$

Equations [A.11] and [A.12] describe the deformation occurring at the edges of the cylindrical shell Element i, as shown in Figures 2 and 4, and are written in terms of the unknown edge forces and moments and known applied loading. These equations can be rewritten in simplified form of edge coefficients as follows:

$$
\begin{aligned}
\theta_i^L &= \left[\frac{dw_i}{dx_i}\right]_{x=0} = a_i M_i + b_i H_i \\
\theta_i^R &= \left[\frac{dw_i}{dx_i}\right]_{x=l_i} = -\,a_i M_i - b_i H_i \\
w_i^L = w_i^R &= [w_i]_{x=0,\ x=l_i} = d_i M_i + g_i H_i + f_i' P_i + f_i'' p
\end{aligned}
\qquad [A.13]
$$

When the terms of Equations [A.11] and [A.12] are compared with the corresponding terms of Equation [A.13], it can be readily seen that the appropriate edge coefficients are:

$$
\begin{aligned}
a_i &= \frac{1}{D_i\beta_i} \cdot \frac{\cosh \beta_i l_i - \cos \beta_i l_i}{\sinh \beta_i l_i + \sin \beta_i l_i} \\
b_i &= \frac{1}{2D_i\beta_i^2} \cdot \frac{\sinh \beta_i l_i - \sin \beta_i l_i}{\sinh \beta_i l_i + \sin \beta_i l_i} \\
d_i &= -\frac{1}{2D_i\beta_i^2} \cdot \frac{\sinh \beta_i l_i - \sin \beta_i l_i}{\sinh \beta_i l_i + \sin \beta_i l_i} = -\,b_i \\
g_i &= -\frac{1}{2D_i\beta_i^3} \cdot \frac{\cosh \beta_i l_i + \cos \beta_i l_i}{\sinh \beta_i l_i + \sin \beta_i l_i} \\
f_i' &= \frac{\nu R_i}{E h_i} \\
f_i'' &= \frac{R_i^2}{E h_i}
\end{aligned}
\qquad [A.14]
$$

The longitudinal membrane stress $(\sigma_{x_i})_m$ is related to the displacements u_i and w_i by the following equation:

$$
(\sigma_{x_i})_m = -\frac{P_i}{h_i} = \frac{E}{1-\nu^2}\left(\epsilon_{x_i} + \nu\epsilon_{\phi_i}\right) = \frac{E}{1-\nu^2}\left(\frac{du_i}{dx_i} - \nu\frac{w_i}{R_i}\right) \qquad [A.15]
$$

When Equation [A.15] is rewritten as

$$\frac{du_i}{dx_i} = \frac{\nu}{R_i} w_i - \frac{1-\nu^2}{Eh_i} P_i \qquad \text{[A.16]}$$

the total axial displacement u_i of any point in the cylinder is equal to the integral of Equation [A.16], thus

$$u_i = \frac{\nu}{R_i} \int w_i dx_i - \frac{1-\nu^2}{Eh_i} P_i \int dx_i \qquad \text{[A.17]}$$

The integral $\int w_i \, dx_i$ can be determined from consideration of the differential Equation [A.1]. Rewriting Equation [A.1] yields

$$w_i = \frac{R_i^2}{Eh_i} \left[- D_i \frac{d^4 w_i}{dx_i^4} + p - \frac{\nu}{R_i} P_i \right] \qquad \text{[A.18]}$$

and integrating with respect to x_i leads to the following expression for the integral $\int w_i \, dx_i$:

$$\int w_i \, dx_i = \frac{R_i^2}{Eh_i} \left[- D_i \frac{d^3 w_i}{dx_i^3} + px_i - \frac{\nu}{R_i} P_i x_i \right] + K_i \qquad \text{[A.19]}$$

where K_i is the constant of integration. Since the transverse shearing force $Q_x = - D_i \dfrac{d^3 w_i}{dx_i^3}$, Equation [A.19] can be expressed as follows:

$$\int w_i \, dx_i = \frac{R_i^2}{Eh_i} \left[Q_x + px_i - \frac{\nu}{R_i} P_i x_i \right] + K_i \qquad \text{[A.20]}$$

Substituting [A.20] into Equation [A.17] and then integrating results in the following equation for u_i:

$$u_i = \frac{\nu R_i}{Eh_i} Q_x - \frac{1}{Eh_i} P_i x_i + \frac{\nu R_i}{Eh_i} px_i + \overline{K}_i \qquad \text{[A.21]}$$

The integration constant $\overline{K}_i$ is determined from the conditions at the middle of the shell, i.e., $x_i = \dfrac{l_i}{2}$. For a cylindrical shell with equal loading on both ends of the cylinder, the transverse shearing force Q_x at midlength of the shell is equal to zero. This can readily be substantiated by evaluating the third derivative of the radial displacement w_i, Equation [A.7], at $x_i = \dfrac{l_i}{2}$ since

$$Q_x = - D_i \frac{d^3 w_i}{dx_i^3}$$

The longitudinal displacement u_i requires a datum from which the displacement is determined. For the analysis in this report, it is convenient to assume that the longitudinal displacement is zero at midlength of the shell. Thus, the displacement u_i of any other point would be the relative axial displacement with respect to the "midlength" location. Imposing these two conditions into Equation [A.21], we find that

$$\bar{K}_i = \frac{1}{Eh_i} P_i \frac{l_i}{2} - \frac{\nu R_i}{Eh_i} p \frac{l_i}{2} \qquad \text{[A.22]}$$

When the integration constant, Equation [A.22], is substituted into the axial displacement function, [A.21], and the resulting expression is evaluated at the left edge of the shell element, i.e., at $x = 0$, we obtain the following:

$$[u_i]_{x=0} = \frac{\nu R_i}{Eh_i} H_i + \frac{l_i}{2Eh_i} P_i - \frac{\nu R_i l_i}{2Eh_i} p \qquad \text{[A.23]}$$

Equation [A.23] can be rewritten in simplified form of edge coefficients, thus

$$u_i^L = s_i H_i + t_i' P_i + t_i'' p \qquad \text{[A.24]}$$

where

$$s_i = \frac{\nu R_i}{Eh_i}$$

$$t_i' = \frac{l_i}{2Eh_i} \qquad \text{[A.25]}$$

$$t_i'' = - \frac{\nu R_i l_i}{2Eh_i}$$

For Element 4, shown in Figure 2, where the inner cylindrical shell is not subjected to the radial pressure loading, the terms f_4'' and t_4'' appearing in Equations [A.13] and [A.24], respectively, which are multiplied by the pressure p, will drop out when considering the deformation of the inner shell.

APPENDIX B

DEVELOPMENT OF EDGE COEFFICIENTS AND EXPRESSIONS FOR THE RADIAL AND TANGENTIAL STRESSES OF A CIRCULAR ANNULUS

In the development of edge coefficients for a circular annulus (Element 3) we will assume, with reference to Equations [1] and Figure 5, that the edge moments (M_3, M_4), axial thrust (P_3, P_4), and surface pressure p do not affect the radial displacement w_3 in the plane of the circular annulus. Only the radial forces H_3 and H_4 acting on the outer and inner edges, respectively, of Element 3 will give rise to radial displacement w_3. We will also assume that the longitudinal displacement u_3 and rotation du_3/dr are affected only by the edge moments and axial thrusts.

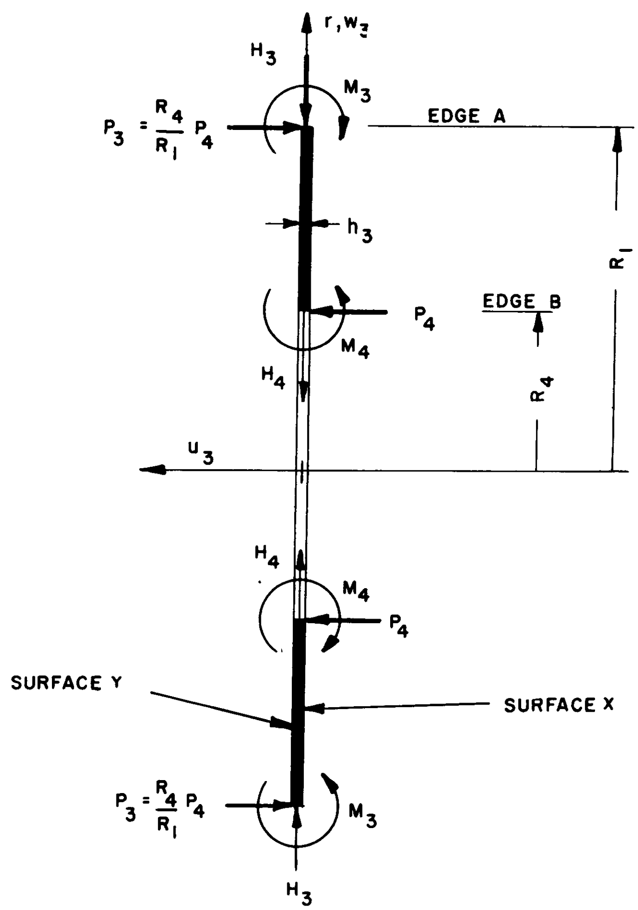

Figure 5 – Edge Forces and Moments for Circular Annulus (Element 3)

On page 418 of Reference 8, the following expression is given for a thick-walled tube subjected to internal pressure p_i and external pressure p_o:

$$w(r) = \frac{r^2(1-\nu)(p_i r_i^2 - p_o r_o^2) + (1+\nu)(p_i - p_o) r_i^2 r_o^2}{rE(r_o^2 - r_i^2)} \qquad [B.1]$$

When the solution [B.1] is adapted to the present problem of the circular annulus, we see that:

$$r_o = R_1; \quad r_i = R_4$$
$$p_o = \frac{H_3}{h_3}; \quad p_i = -\frac{H_4}{h_3} \qquad [B.2]$$

Substituting [B.2] into [B.1] gives the following results:

$$w_3(r) = -\frac{r^2(1-\nu)(H_4 R_4^2 + H_3 R_1^2) + (1+\nu)(H_4 + H_3) R_4^2 R_1^2}{rh_3 E(R_1^2 - R_4^2)} \qquad [B.3]$$

Evaluation of Equation [B.3] at the outer and inner edges of the annulus, that is, at $r = R_1$ and $r = R_4$, respectively, yields the following:

$$w_3^A = [w_3]_{r=R_1} = g_3^A H_3 + g_3^{AB} H_4$$
$$w_3^B = [w_3]_{r=R_4} = g_3^B H_4 + g_3^{BA} H_3 \qquad [B.4]$$

where the edge coefficients are:

$$g_3^A = -\frac{(1+\nu) R_1^2 R_4^2}{Eh_3 (R_1^2 - R_4^2)} \left[\frac{1}{R_1} + \left(\frac{1-\nu}{1+\nu}\right) \frac{R_1}{R_4^2} \right]$$

$$g_3^{AB} = -\frac{(1+\nu) R_1^2 R_4^2}{Eh_3 (R_1^2 - R_4^2)} \left[\left(\frac{2}{1+\nu}\right) \frac{1}{R_1} \right] \qquad [B.5]$$

$$g_3^B = -\frac{(1+\nu) R_1^2 R_4^2}{Eh_3 (R_1^2 - R_4^2)} \left[\frac{1}{R_4} + \left(\frac{1-\nu}{1+\nu}\right) \frac{R_4}{R_1^2} \right]$$

$$g_3^{BA} = -\frac{(1+\nu) R_1^2 R_4^2}{Eh_3 (R_1^2 - R_4^2)} \left[\left(\frac{2}{1+\nu}\right) \frac{1}{R_4} \right]$$

Also on page 418 of Reference 8, the following expressions are given for the radial and tangential stresses of a thick-walled tube:

$$\sigma_r = \frac{r^2 (p_i r_i^2 - p_o r_o^2) - (p_i - p_o) r_i^2 r_o^2}{r^2 (r_o^2 - r_i^2)}$$

$$\sigma_t = \frac{r^2 (p_i r_i^2 - p_o r_o^2) + (p_i - p_o) r_i^2 r_o^2}{r^2 (r_o^2 - r_i^2)} \qquad [B.6]$$

Substituting [B.2] into [B.6], gives, for the present problem of the circular annulus, the following results:

$$(\sigma_r)_d = \frac{-r^2 (H_4 R_4^2 + H_3 R_1^2) + (H_4 + H_3) R_4^2 R_1^2}{r^2 h_3 (R_1^2 - R_4^2)}$$

$$(\sigma_t)_d = \frac{-r^2 (H_4 R_4^2 + H_3 R_1^2) - (H_4 + H_3) R_4^2 R_1^2}{r^2 h_3 (R_1^2 - R_4^2)} \qquad [B.7]$$

Equations [B.7] are expressions for the direct stresses due to the radial forces H_3 and H_4. The bending stresses which arise from the edge moments and axial thrusts will be determined later in this appendix.

The differential equation governing symmetrical bending of circular plates is given by:[9]

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)\right] = \frac{Q}{D} \qquad [B.8]$$

where Q is the shearing force per unit length of cylindrical section of radius r. For the circular annulus shown in Figure 5:

$$Q = \frac{R_4 P_4}{r}^{*} \qquad [B.9]$$

---

*P_3 denotes the shearing force at the outer edge ($r = R_1$) of the circular annulus shown in Figure 5. However, from Equation [B.9] the shearing force on this same edge is given as follows:

$$[Q]_{r=R_1} = \frac{R_4 P_4}{R_1}$$

which means that

$$P_3 = \frac{R_4}{R_1} P_4$$

If Equation [B.9] is substituted into Equation [B.8], the differential equation for annulus Element 3 becomes

$$\frac{d}{dr}\left[\frac{1}{r}\ \frac{d}{dr}\left(r\ \frac{du_3}{dr}\right)\right] = \frac{R_4\ P_4}{D_3 r} \qquad \text{[B.10]}$$

By one integration of Equation [B.10] , we find

$$\frac{1}{r}\ \frac{d}{dr}\left(r\ \frac{du_3}{dr}\right) = \frac{R_4}{D_3}\ P_4\ \ln\ \frac{r}{R_1} - \overline{C}_1 \qquad \text{[B.11]}$$

where $\overline{C}_1$ is a constant of integration to be found later from the conditions at the edges of the annulus. Multiplying both sides of Equation [B.11] by r and making the second integration, we find

$$r\ \frac{du_3}{dr} = \frac{R_4 P_4}{D_3}\ r^2\left[\frac{1}{2}\ \ln\ \frac{r}{R_1} - \frac{1}{4}\right] - \frac{\overline{C}_1 r^2}{2} - \overline{C}_2 \qquad \text{[B.12]}$$

and

$$\frac{du_3}{dr} = \frac{R_4 P_4}{4 D_3}\ r\ \left[2\ \ln\ \frac{r}{R_1}\ - 1\right] - \frac{\overline{C}_1}{2}\ r - \frac{\overline{C}_2}{r} \qquad \text{[B.13]}$$

A new integration then gives:

$$u_3 = \frac{R_4 P_4}{4 D_3}\ r^2\ \left[\ln\ \frac{r}{R_1} - 1\right] - \frac{\overline{C}_1 r^2}{4} - \overline{C}_2 \ln\ \frac{r}{R_1} + \overline{C}_3 \qquad \text{[B.14]}$$

The second derivative of u_3 with respect to r is obtained by differentiating Equation [B.13], thus

$$\frac{d^2 u_3}{dr^2} = \frac{R_4 P_4}{4D_3}\left[2\ \ln\ \frac{r}{R_1} + 1\right] - \frac{\overline{C}_1}{2} + \frac{\overline{C}_2}{r^2} \qquad \text{[B.15]}$$

The bending moments of the circular annulus are given by:[9]

$$M_r = - D_3\left(\frac{d^2 u_3}{dr^2} + \frac{\nu}{r}\ \frac{du_3}{dr}\right) \qquad \text{[B.16]}$$

$$M_t = - D_3\left(\frac{1}{r}\ \frac{du_3}{dr} + \nu\ \frac{d^2 u_3}{dr^2}\right) \qquad \text{[B.17]}$$

where M_r and M_t denote the bending moments per unit length, that is, M_r along circumferential sections of the annulus and M_t along the diametral section of the annulus. Substituting Equations [B.13] and [B.15] into both [B.16] and [B.17] results in the following expressions for the bending moments:

$$M_r = -\frac{R_4 P_4}{4}\left[2(1+\nu)\ln\frac{r}{R_1} + (1-\nu)\right] + \frac{(1+\nu)D_3\overline{C}_1}{2} - (1-\nu)D_3\frac{\overline{C}_2}{r^2} \qquad [B.18]$$

$$M_t = -\frac{R_4 P_4}{4}\left[2(1+\nu)\ln\frac{r}{R_1} - (1-\nu)\right] + \frac{(1+\nu)D_3\overline{C}_1}{2} + (1-\nu)D_3\frac{\overline{C}_2}{r^2} \qquad [B.19]$$

With reference to Figure 5, the moment boundary conditions are given by:

$$\begin{aligned} &\text{At} \quad r = R_1 : \quad M_r = M_3 \\ &\text{At} \quad r = R_4 : \quad M_r = M_4 \end{aligned} \qquad [B.20]$$

Substituting Equation [B.18] into the boundary conditions, [B.20], and solving simultaneously yields the following expressions for the integration constants $\overline{C}_1$ and $\overline{C}_2$

$$\overline{C}_1 = \frac{2(R_1^2 M_3 - R_4^2 M_4)}{(1+\nu)D_3(R_1^2 - R_4^2)} + \frac{R_4 P_4}{2D_3}\left[\frac{(1-\nu)}{(1+\nu)} - \frac{2R_4^2}{R_1^2 - R_4^2}\ln\frac{R_4}{R_1}\right]$$

$$\overline{C}_2 = \frac{R_1^2 R_4^2 (M_3 - M_4)}{(1-\nu)D_3(R_1^2 - R_4^2)} - \frac{(1+\nu)R_4 P_4}{2(1-\nu)D_3} \cdot \frac{R_1^2 R_4^2}{R_1^2 - R_4^2}\ln\frac{R_4}{R_1} \qquad [B.21]$$

To determine the integration constant $\overline{C}_3$, the axial displacement u_3 at the edges of the annulus must be considered. u_3 is a relative displacement and requires a datum from which to measure the displacement. For the present problem of the circular annulus, where we are interested in the difference between the axial displacement of the outer and inner edges of the annulus, i.e., Edge A and Edge B, respectively (see Figure 5), it is convenient to assume

$$\text{At} \quad r = R_1 : \quad u_3 = 0 \qquad [B.22]$$

Substituting Equation [B.14] into the displacement boundary condition [B.22], we find the following expression for the integration constant $\overline{C}_3$:

$$\overline{C}_3 = \frac{R_1^2(R_1^2 M_3 - R_4^2 M_4)}{2(1+\nu)D_3(R_1^2 - R_4^2)} + \frac{R_1^2 R_4 P_4}{4D_3}\left[1 + \frac{1}{2}\cdot\frac{(1-\nu)}{(1+\nu)} - \frac{R_4^2}{R_1^2 - R_4^2}\ln\frac{R_4}{R_1}\right] \qquad [B.23]$$

When the integration constants, Equations [B.21] and [B.23], are substituted into the slope function, [B.13], and the deflection function, [B.14], and the resulting expressions are then evaluated at Edges A and B of annulus Element 3, that is, at $r = R_1$ and $r = R_4$, respectively, we obtain the following:

$$
\begin{aligned}
\theta_3^A &= \left[\frac{du_3}{dr}\right]_{r=R_1} = a_3^A M_3 + a_3^{AB} M_4 + c_3^A P_4 \\
\theta_3^B &= \left[\frac{du_3}{dr}\right]_{r=R_4} = a_3^B M_4 + a_3^{BA} M_3 + c_3^B P_4 \\
u_3^A &= [u_3]_{r=R_1} = 0 \\
u_3^B &= [u_3]_{r=R_4} = q_3^B M_4 + q_3^{BA} M_3 + t_3'^B P_4
\end{aligned}
\qquad [B.24]
$$

where the edge coefficients are:

$$
\begin{aligned}
a_3^A &= -\frac{R_1}{(1-\nu^2)\, D_3 (R_1^2 - R_4^2)} \cdot \left[(1-\nu)R_1^2 + (1+\nu)R_4^2\right] \\
a_3^{AB} &= \frac{2R_1 R_4^2}{(1-\nu^2)\, D_3 (R_1^2 - R_4^2)} \\
c_3^A &= \frac{R_1 R_4}{2(1-\nu^2)\, D_3 (R_1^2 - R_4^2)} \cdot \left[2(1+\nu) R_4^2 \ln \frac{R_4}{R_1} - (1-\nu)(R_1^2 - R_4^2)\right] \\
a_3^B &= \frac{R_4}{(1-\nu^2)\, D_3 (R_1^2 - R_4^2)} \cdot \left[(1-\nu)R_4^2 + (1+\nu)R_1^2\right] \\
a_3^{BA} &= -\frac{2 R_4 R_1^2}{(1-\nu^2)\, D_3 (R_1^2 - R_4^2)} \\
c_3^B &= \frac{R_4^2}{2(1-\nu^2)\, D_3 (R_1^2 - R_4^2)} \cdot \left[2(1+\nu) R_1^2 \ln \frac{R_4}{R_1} - (1-\nu)(R_1^2 - R_4^2)\right] \\
q_3^B &= \frac{R_4^2}{2(1-\nu^2)\, D_3 (R_1^2 - R_4^2)} \left[2(1+\nu) R_1^2 \ln \frac{R_4}{R_1} - (1-\nu)(R_1^2 - R_4^2)\right] = c_3^B \\
q_3^{BA} &= -\frac{R_1^2}{2(1-\nu^2)\, D_3 (R_1^2 - R_4^2)} \cdot \left[2(1+\nu) R_4^2 \ln \frac{R_4}{R_1} - (1-\nu)(R_1^2 - R_4^2)\right]
\end{aligned}
\qquad [B.25]
$$

$$t_3'^B = \frac{R_4}{4D_3}\left[\frac{(3+\nu)(R_1^2-R_4^2)}{2(1+\nu)} + \frac{2(1+\nu)\left(R_1R_2\ln\dfrac{R_4}{R_1}\right)^2}{(1-\nu)(R_1^2-R_4^2)}\right] \qquad [B.25]$$

To determine the bending stresses occurring at any point in the circular annulus, it is convenient to express the bending moments M_r and M_t in terms of the edge force P_4 and edge moments M_3 and M_4. Thus, when substituting Equations [B.21] into Equations [B.18] and [B.19], we find

$$\begin{aligned} M_r = {} & \frac{R_1^2}{R_1^2-R_4^2}\left(1-\frac{R_4^2}{r^2}\right)M_3 + \frac{R_4^2}{R_1^2-R_4^2}\left(\frac{R_1^2}{r^2}-1\right)M_4 \\ & + \frac{1+\nu}{2}\cdot R_4\left[\left(\frac{R_4}{r}\right)^2\left(\frac{R_1^2-r^2}{R_1^2-R_4^2}\right)\ln\frac{R_4}{R_1} - \ln\frac{r}{R_1}\right]P_4 \end{aligned} \qquad [B.26]$$

$$\begin{aligned} M_t = {} & \frac{R_1^2}{R_1^2-R_4^2}\left(1+\frac{R_4^2}{r^2}\right)M_3 - \frac{R_4^2}{R_1^2-R_4^2}\left(\frac{R_1^2}{r^2}+1\right)M_4 \\ & + \frac{1+\nu}{2}\cdot R_4\left[\frac{1-\nu}{1+\nu}\left(\frac{R_4}{r}\right)^2\left(\frac{R_1^2+r^2}{R_1^2-R_4^2}\right)\ln\frac{R_4}{R_1} - \ln\frac{r}{R_1}\right]P_4 \end{aligned} \qquad [B.27]$$

The radial and tangential bending stresses of any point on the lateral surfaces of the annulus are given by:

$$\begin{aligned} (\sigma_r)_b &= \frac{6}{h_3^2}M_r \\ (\sigma_t)_b &= \frac{6}{h_3^2}M_t \end{aligned} \qquad [B.28]$$

where the expressions for M_r and M_t are given by Equations [B.26] and [B.27], respectively.

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APPENDIX C

COMPUTER PROGRAM

The analysis presented in this report has been programmed for the IBM–7090 computer at the David Taylor Model Basin. This appendix gives the parameters required for the computation and the form in which the solution is obtained.

The identification number for this program is 4–958–XD12.

The following input, based on the shell elements identified in Figure 6, is required to perform the calculations:

FS = axial length of shell C, inches
FR = axial length of shell A, inches
RO = radius of shell C, inches
RI = radius of shell A, inches
HS = thickness of shell C, inches
HF = thickness of shell A, inches
HW = thickness of annulus, inches
U = Poisson's ratio
E = Young's modulus of the material, psi
NO = model identification where any combination of three numbers may be used.

The input is of the following format: 7F7.4, F4.3, E11.3, I3

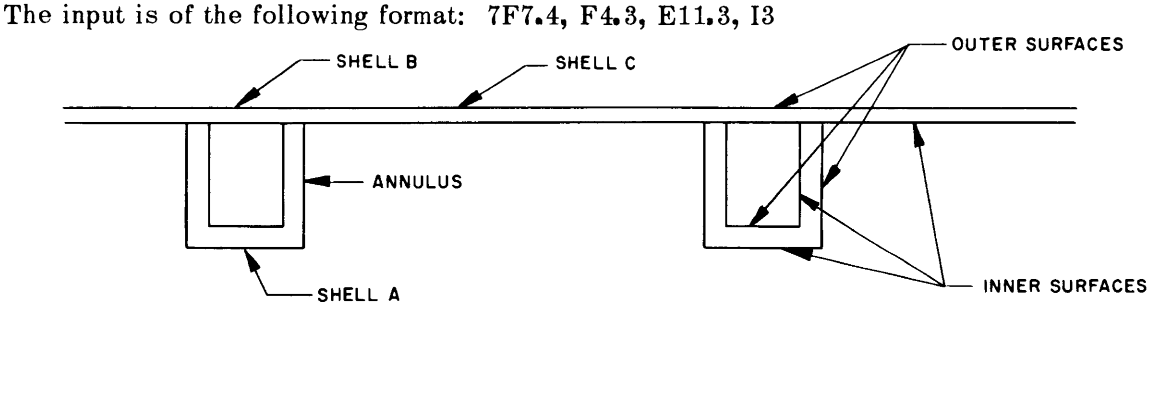

Figure 6 – Identification of Shell Elements for Computer Calculations

For each set of input parameters, stresses and strains are computed for a pressure of 1 psi and printed for points along the generator of each shell element. Likewise, the stresses of the annulus are printed. Figure 7 is an example of the printout.

STRESSES AND STRAINS IN CYLINDRICAL SHELL WITH HAT TYPE STIFFENERS

INPUT

LS= 2.0950 LR= 0.6240 RO=13.9170 RI=12.7370 HS=0.2610 HF=0.2100 HW=0.1000 NU=.300 E=0.3000E 08 MODEL NO 123

X/L	CIR STRESS OUT	CIR STRESS IN	LONG STRESS OUT	LONG STRESS IN	CIR STRAIN	LONG STRAIN OUT	LONG STRAIN IN
SHELL A							
.	-3.214542E 01	-3.160827E 01	-9.555933E-01	8.348894E-01	-1.061958E-06	2.896011E-07	3.439124E-07
.10000	-3.175065E 01	-3.196418E 01	2.955330E-01	-4.162369E-01	-1.061310E-06	3.273576E-07	3.057672E-07
.20000	-3.144024E 01	-3.223749E 01	1.268403E 00	-1.389107E 00	-1.060692E-06	3.566825E-07	2.760714E-07
.30000	-3.121672E 01	-3.243083E 01	1.963176E 00	-2.083880E 00	-1.060189E-06	3.776064E-07	2.548457E-07
.40000	-3.108188E 01	-3.254609E 01	2.379984E 00	-2.500688E 00	-1.059863E-06	3.901516E-07	2.421046E-07
.50000	-3.103682E 01	-3.258438E 01	2.518910E 00	-2.639614E 00	-1.059750E-06	3.943319E-07	2.378566E-07
SHELL B							
.	-3.448214E 01	-3.872171E 01	-1.955052E 01	-3.368244E 01	-9.538994E-07	-3.068625E-07	-7.355309E-07
.10000	-3.457484E 01	-3.852222E 01	-2.003751E 01	-3.319545E 01	-9.521195E-07	-3.221687E-07	-7.212927E-07
.20000	-3.464803E 01	-3.836793E 01	-2.041663E 01	-3.281633E 01	-9.507679E-07	-3.340741E-07	-7.101982E-07
.30000	-3.470089E 01	-3.825821E 01	-2.068761E 01	-3.254534E 01	-9.498201E-07	-3.425782E-07	-7.022627E-07
.40000	-3.473284E 01	-3.819256E 01	-2.085027E 01	-3.238268E 01	-9.492586E-07	-3.476807E-07	-6.974971E-07
.50000	-3.474353E 01	-3.817071E 01	-2.090451E 01	-3.232845E 01	-9.490725E-07	-3.493816E-07	-6.959079E-07
SHELL C							
.	-3.429254E 01	-3.893798E 01	-1.891851E 01	-3.440333E 01	-9.538994E-07	-2.876916E-07	-7.573978E-07
.10000	-3.612078E 01	-3.764337E 01	-2.412326E 01	-2.919857E 01	-9.627933E-07	-4.429010E-07	-5.968521E-07
.20000	-3.764955E 01	-3.676421E 01	-2.813649E 01	-2.518535E 01	-9.736202E-07	-5.613875E-07	-4.718695E-07
.30000	-3.879747E 01	-3.620530E 01	-3.098121E 01	-2.234063E 01	-9.834369E-07	-6.447322E-07	-3.826348E-07
.40000	-3.950821E 01	-3.589773E 01	-3.267838E 01	-2.064346E 01	-9.901566E-07	-6.941972E-07	-3.291379E-07
.50000	-3.974875E 01	-3.579986E 01	-3.324241E 01	-2.007943E 01	-9.925344E-07	-7.105927E-07	-3.113157E-07

ANNULUS

(R-RI)/(RO-RI)	RAD STRESS OUT	RAD STRESS IN	TAN STRESS OUT	TAN STRESS IN
0.	2.310387E 00	-5.585639E 00	-3.118071E 01	-3.351934E 01
0.20000	-6.024376E-02	-4.322262E 00	-3.112174E 01	-3.247106E 01
0.40000	-2.345506E 00	-3.085436E 00	-3.110647E 01	-3.143790E 01
0.60000	-4.550632E 00	-1.874024E 00	-3.113113E 01	-3.041952E 01
0.80000	-6.680422E 00	-6.869659E-01	-3.119232E 01	-2.941559E 01
1.00000	-8.739308E 00	4.767387E-01	-3.128695E 01	-2.842579E 01

Figure 7 – Sample of Computer Output

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1. Cylindrical shells (Stiffened)--Stresses--Mathematical analysis
2. Cylindrical shells (Stiffened)--Deformation-Mathematical analysis

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