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THE PATH ANGLES AND ANGLES OF ATTACK AND DRIFT FOR LARGE MOTIONS OF A MOVING BODY IN A MOVING FLUID

by

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NOTATION

In keeping with related work at the David Taylor Model Basin, the nomenclature in TMB Report 1319[1] was used as a guide in establishing the notation for this report. Departures from Reference 1, however, appeared desirable because of the special features of this study. Thus, angles defining the path of the body in space are identified by the subscript P. The use of a lowercase subscript here would create a possible risk of confusion with derivatives involving rolling velocity. Because the capital letter is selected for that subscript, capitals also are used for the corresponding subscripts R and F, which apply to motion of the body relative to the fluid and to the motion of the fluid in space, respectively, for consistency. The notation used is listed here and the items are also defined in the text at the point where first mentioned. Items that are used only once at some intermediate step in a development are omitted from the list.

O	Origin of the x, y, z body axes
t	Time
U	Magnitude of the absolute linear velocity vector $\bar{U}$
$\bar{U}$	Vector representing the absolute linear velocity of the body at the point O
U_F	Magnitude of the absolute linear velocity vector $\bar{U}_F$
$\bar{U}_F$	Vector representing the absolute linear velocity that the fluid would have at the point O in the absence of the body
U_R	Magnitude of the relative linear velocity vector $\bar{U}_R$
$\bar{U}_R$	Vector representing the relative linear velocity, with respect to the fluid, of the body at the point O
u_I, v_I, w_I	Components of $\bar{U}$ in the directions of the x_I, y_I, z_I axes
u_R, v_R, w_R	Components of $\bar{U}_R$ in the directions of the x, y, z axes
x, y, z	A set of moving, orthogonal, right-hand, Cartesian coordinate axes fixed in the body (Both the location of the origin O of the axes and the orientation of the axes with respect to the body are arbitrary.)
x_I, y_I, z_I	A set of orthogonal, right-hand, Cartesian coordinate axes fixed in space

[1]References are listed on page 33.

α	Angle of attack (To measure the angle, project the line of motion of the body relative to the fluid onto the x-positive portion of the xz plane. The angle of attack is the angle from this projection to the x axis, positive in the positive sense of rotation about the y axis.)
β	Angle of drift (The angle from $\overline{U}_R$ to the x-positive portion of the xz plane, positive in the positive sense of rotation about the z axis.)
ψ, θ, ϕ	Orientation (Euler) angles defining the attitude of the x, y, z axes with respect to the x_I, y_I, z_I axes
ψ_F, θ_F	Euler angles defining the orientation of $\overline{U}_F$ with respect to the x_I, y_I, z_I axes
ψ_P, θ_P	Path angles defining the instantaneous orientation of the path of the point O of the body in space (Equivalent definition: Euler angles fixing the direction of $\overline{U}$ with respect to the x_I, y_I, z_I axes.)
ψ_R, θ_R	Euler angles defining the orientation of $\overline{U}_R$ with respect to the x_I, y_I, z_I axes

ABSTRACT

The linear-velocity kinematics of a body moving through a moving or stationary fluid are discussed. Three distinct velocities are considered: the absolute velocity of the body, the absolute velocity of the fluid, and the velocity of the body relative to the fluid. Exact expressions are derived for obtaining information about any one of the the three velocities when certain data are known about the remaining two. Among the more utilitarian expressions offered are those for evaluating path angles and angles of attack and drift. Because exact solutions are given, there is no limitation on the severity of the maneuvers to which they may be applied. The application of the results to a few extreme maneuvers is illustrated by numerical examples.

INTRODUCTION

In typical treatments of the motion of a body in a fluid, references are made to various angles. Two such angles are the angles of attack and drift, which comprise a set showing the orientation of the body with respect to the relative motion of the body in the fluid. The pitch, roll and heading angles form another set that defines the orientation of the body in space. For motions in the vertical plane, the path angle is sometimes discussed. For completeness, this well-known path angle should be paired with another angle, corresponding to what is described as "course made good" in navigation, to form a set that defines the direction of motion of the body in space.

Students of the subject are aware that these various sets of angles are not independent. For example, in considering motions in the vertical plane, the path angle, pitch angle, and angle of attack are interrelated. These interrelationships make it impossible to treat the various angles as independent parameters in the equations of motion. It follows that the nature of these interrelationships must adequately be taken into account in writing equations of motion, where an intermixed use of the angles is often convenient.

Expressions that are approximate have customarily been used to define the relations among the angles. These approximations are based on three important assumptions: that the fluid is at rest; that the motion is restricted to one plane; and that the motion is small.

In many problems of current interest, one or more of these assumptions is not realistic. The fluid usually is not at rest but has some motion because of wind or current. In water, waves often are an important cause of fluid motion. If a craft is moving relatively slowly in such a disturbed fluid, the effect of the fluid motion on items such as the angle of attack must be considered. As a second consideration, the high degree of maneuverability of many modern naval craft results in distinctly nonplanar controlled motions of such large amplitude that serious doubts are raised regarding the adequacy of the approximate linearizing expressions customarily used.

The objective of this report is to provide exact expressions, free of any restrictive assumptions, for the relations among the kinematic variables under discussion. The expressions given can be applied, therefore, to any maneuver, however violent or complex, and for motions through either stationary or moving fluid.

Although the exact expressions are considered too complicated for practical routine use in studies of motion, it is considered desirable to present them in their complete form. The complete expressions then provide accurate bases for deriving simpler approximations in cases where a limited range of motion makes such simplification feasible. A few examples are given of how the complete expressions lead directly to suitable approximations when the motion is restricted in various ways.

Based on the assumption of small motions, the practice has been to treat motions in terms of variables that have an initial or constant value with a small deviation superimposed upon it. When the motion is nearly steady, such a treatment is adequate, but if violent maneuvers are involved, it is simpler not to separate the variable into constant and fluctuating parts but merely deal with instantaneous values of the total variable. The latter approach will be used in this report.

In selecting the material to be presented, consideration was given to the types of data that are likely to be available with respect to the motion of a body and, conversely, what deduced data may be sought. Because the angle of attack and the angle of drift depend solely on the relative motion of the body with respect to the fluid, kinematic relations dealing with angle of attack and angle of drift are given in the first major section of the paper; the second major section treats relations that are affected by the state of the linear velocity of the fluid.

Following the presentation of the kinematic relations, a summary of the principal equations in the report and some numerical examples, illustrating the generality and validity of the relations, appear. Finally, a few remarks are made on suggested applications. Derivation of the relations is discussed in Appendix A.

The work was carried out under the Fundamental Hydromechanics Research Program at the David Taylor Model Basin as part of a broader study of the general equations of motion of a body in a fluid. Earlier work on the same general subject has been reported in Reference 2, and the present paper may be considered as a companion work in a different area of the overall objective.

ANGLE OF ATTACK AND ANGLE OF DRIFT

The angles of attack and drift are well known as useful hydrodynamic parameters. Important hydrodynamic forces that act on a body that is moving through a fluid are usually defined in terms of these angles. If the body is moving through a fluid that is also in motion, the motion of the body relative to the fluid and the motion of the body in space become distinct. It is important to note that the angle of attack α and the angle of drift β depend on only the relative motion; hence they can be related to the relative motion whether the fluid is moving or stationary.

Physically, α and β describe the orientation of the body with respect to the apparent flight path of the body in the fluid. As an alternate description, they specify the direction of linear velocity through the fluid of a point of the body, as seen by an observer stationed at the point and moving with the body. Let the point be O and let $\overline{U}_R$ be a vector representing the relative linear velocity of O with respect to the fluid.

The same situation could be observed from a point fixed in space, whereupon α and β (because they describe the direction of $\overline{U}_R$ with respect to the body) are definable in terms of two Euler angles ψ_R, θ_R, used to fix the direction of the vector $\overline{U}_R$ in space, and three Euler angles ψ, θ, ϕ, which give the orientation of the body in space. The defining relationships are:

$$\alpha = \arctan \left[\frac{\tan\theta \cos(\psi - \psi_R) - \tan\theta_R}{\tan\theta \tan\theta_R + \cos(\psi - \psi_R)} \cos\phi + \frac{\sin(\psi - \psi_R)}{\sin\theta \tan\theta_R + \cos\theta \cos(\psi - \psi_R)} \sin\phi \right] \qquad [1]$$

$$\beta = \arcsin \{\sin(\psi - \psi_R) \cos\theta_R \cos\phi + [\cos\theta \sin\theta_R - \sin\theta \cos\theta_R \cos(\psi - \psi_R)] \sin\phi\} \qquad [2]$$

Derivation of Equations [1] and [2] is given in Appendix A. Some results calculated by means of these equations are presented in the section on Numerical Examples.

Because the unknown quantities in Equations [1] and [2], and in some of the following equations are evaluated by inverse trigonometric functions, the solutions are ambiguous. The ambiguity, however, can always be resolved from geometric considerations. Thus, for example, the ambiguity in evaluating α and β from Equations [1] and [2] can be removed as follows. Note that α must always lie in the range $-90° \leq \alpha \leq 90°$, and arctan always has one value that is in this range. The value of β can have the range $-180° \leq \beta \leq 180°$. The sign of arcsin β in Equation [2] will place β in one of two ranges: $-180° \leq \beta \leq 0°$ or $0° \leq \beta \leq 180°$.

For each range there are two angles that have the same sine; for example, β and $(180° - \beta)$. The proper value of β is the angle of smaller magnitude if the velocity vector $\bar{U}_R$ lies on the x-positive side of the yz plane; and conversely.

RELATIVE VELOCITY IN TERMS OF α AND β

The converse of the relations represented by Equations [1] and [2] is of considerable practical interest. Frequently the orientation of a body with respect to its relative motion in the fluid is known, i.e., α and β are known. If the orientation of the body in space is also known, it follows that the direction of the relative velocity vector in space can be determined. This determination becomes especially significant when the fluid is at rest, because the direction of the relative velocity vector then is also the path direction.

If, in addition to the foregoing, something is known about the magnitude of relative velocity, the relative velocity vector $\bar{U}_R$ is completely determinate. For example, assume that the component of $\bar{U}_R$ along the x body axis, denoted by u_R, is known and that α and β are known for the point where u_R is given. Data of this type frequently are obtained when measuring the speed of a body. Finally, assume that the angles ψ, θ, ϕ, which fix the orientation of the body in space, are also known.
Then

$$U_R = u_R \sec \alpha \sec \beta \tag{3}$$

$$\psi_R = \psi - \arctan\left(\frac{\sin \alpha \cos \beta \sin \phi + \sin \beta \cos \phi}{\cos \alpha \cos \beta \cos \theta + \sin \alpha \cos \beta \sin \theta \cos \phi - \sin \beta \sin \theta \sin \phi}\right) \tag{4}$$

$$\theta_R = \arcsin (\cos \alpha \cos \beta \sin \theta - \sin \alpha \cos \beta \cos \theta \cos \phi + \sin \beta \cos \theta \sin \phi) \tag{5}$$

where U_R is the magnitude of the vector $\bar{U}_R$.

For convenience, the inverse of Equation [3] is listed here with the remaining components of $\bar{U}_R$

$$u_R = U_R \cos \alpha \cos \beta \tag{6}$$

$$v_R = - U_R \sin \beta = - u_R \sec \alpha \tan \beta \tag{7}$$

$$w_R = U_R \sin \alpha \cos \beta = u_R \tan \alpha \tag{8}$$

where v_R and w_R are the components of $\bar{U}_R$ along the y and z axes.

Derivation of Equations [4] and [5] is discussed in Appendix A. Calculated results, based on these equations, are given in the section on Numerical Examples.

Equations [3], [4], and [5], as a group, give the magnitude and direction of the relative velocity vector $\bar{U}_R$. The ambiguity in defining θ_R and ψ_R can be resolved from geometric considerations. Note that U_R is always positive. If u_R is positive, then the relative velocity vector $\bar{U}_R$ and the x axis lie on the same side of the y_I, z_I plane; and conversely.

The mathematically oriented reader may note an incompleteness, at this point, in that a third set of equations, relating body orientation angles ψ, θ, ϕ to the parameters α , β, ψ_R, θ_R, is not given. The reason for the omission is that in practice the direct measurement of the body orientation angles is usually feasible, so development of expressions for their derivation was not attempted.

SIMPLIFIED EXPRESSIONS

If one or more degrees of freedom are eliminated from the motion, the exact expressions represented by Equations [4] and [5] are replaced by simpler forms. For example, if the roll angle is negligible during the motion, Equations [4] and [5] reduce to

$$\psi_R = \psi - \arctan \left[\frac{\tan \beta}{\cos (\theta - \alpha)} \right] \tag{9}$$

$$\theta_R = \arcsin [\cos \beta \sin (\theta - \alpha)] \tag{10}$$

If the drift angle is negligible, Equations [4] and [5] become

$$\psi_R = \psi - \arctan \left[\frac{\sin \alpha \sin \phi}{\cos \alpha \cos \theta + \sin \alpha \sin \theta \cos \phi} \right] \tag{11}$$

$$\theta_R = \arcsin (\cos \alpha \sin \theta - \sin \alpha \cos \theta \cos \phi) \tag{12}$$

Finally, if the drift and roll angles are both negligible (as in the case of a submarine maneuvering in a vertical plane) the equations take the forms

$$\psi_R = \psi \tag{13}$$

$$\theta_R = \theta - \alpha \tag{14}$$

If the fluid is stationary, θ_R is equivalent to θ_P, and Equation [14] reduces to the familar approximate expression for vertical path angle.

All of the Equations [1] through [14] are valid whether the fluid is moving or stationary. As in the case of Equation [14], just cited, some substitutions of nomenclature may make these equations more meaningful, however, when the fluid is motionless. With stationary fluid the relative motion of the body is also its absolute motion, so the following changes hold

Moving fluid	U_R	u_R	v_R	w_R	ψ_R	θ_R
Stationary fluid	U	u	v	w	ψ_P	θ_P

For instance, using the preceding substitutions, Equations [4] and [5] can be solved for flight path angles if the fluid is stationary, and Equations [1] and [2] give α and β in terms of path angles.

RELATIONS AMONG LINEAR VELOCITIES

The problem with which this section is concerned is basically one of particle kinematics. It is the problem of the interrelations that exist among three linear velocities when a body is moving in a fluid that is also in motion. The linear velocities of the body are dealt with at the point O, and the three velocities of interest are: (1) the absolute linear velocity of O, represented by the vector $\bar{U}$; (2) the relative linear velocity of O with respect to the fluid, represented by the vectoı $\bar{U}_R$; and (3) the absolute linear velocity of the fluid at the point O, represented by the vectoı $\bar{U}_F$.

The absolute velocity of the body, obviously, is its relative velocity with respect to the fluid plus the absolute velocity of the fluid. This fact is expressed by the vector equation

$$\bar{U} = \bar{U}_R + \bar{U}_F \tag{15}$$

Equation [15] is disarmingly simple. It demonstrates the basic interrelation that exists among the three velocities and establishes the fact that when any two of the velocities are known, the third can be derived. The equation is not in a form that is suited to the acquisition of data or to data processing, however, because of the vector addition that is involved.

Instead of dealing with the vectors of Equation [15], each of the vectors will be replaced by an equivalent set of three quantities. Two of these quantities are Euler angles that establish the direction of the vector in space, and the third quantity is the magnitude of the vector. The magnitude is indicated by omitting the bar from the vector symbol.

Euler angles used to give the direction of the vector $\bar{U}_R$ have already been defined as ψ_R, θ_R in the section on "Angle of Attack and Angle of Drift." Corresponding angles for the vector $\bar{U}_F$ are ψ_F, θ_F.

For the vector $\bar{U}$ the angles are ψ_P, θ_P. It should be noted that $\bar{U}$ gives the instantaneous direction of the path of the body in space (or over the ground when earth axes are considered fixed); hence, if the inertial axes x_I and y_I are chosen to lie in the horizontal plane (as is usually the case), then θ_P is what is customarily called the path angle. Note that θ_P only partly describes the direction of the path of the body in space, insofar as it gives the angular deviation of the path above or below the horizontal plane. To define the path direction completely, a second angle, ψ_P, is required so that the azimuth direction of the path can be specified. This latter angle corresponds to the angle descriptively termed "course made good" in navigation; see Reference 3.

ABSOLUTE VELOCITY OF BODY

The most direct measure of the absolute linear velocity of the point O is obtained if inertial guidance data are available. Inertial guidance systems usually develop information on the north-south and east-west components of absolute velocity of a moving body. This information can be combined with rate-of-change-of-depth data to provide three independent components of the absolute velocity. If a fixed reference frame of x_I, y_I, z_I axes is chosen in such a way that x_I is true north, y_I is due east, and z_I is vertically down, then the three components of velocity just described are along the respective axes. Let these components be u_I, v_I, w_I, in the order named. Then the magnitude of the absolute linear velocity of O is

$$U = \sqrt{u_I^2 + v_I^2 + w_I^2} \qquad [16]$$

(Note that U should always be given a positive sign.) The three independent components not only permit a determination of U, but also they are sufficient to determine ψ_P, θ_P, which define the path direction of the vector $\bar{U}$ (i.e., the direction of motion of the body in space). Thus

$$\psi_P = \arcsin\left(v_I/\sqrt{u_I^2 + v_I^2}\right) = \arccos\left(u_I/\sqrt{u_I^2 + v_I^2}\right) \qquad [17]$$

$$\theta_P = \arcsin\left(-w_I/U\right) = \arccos\left(\sqrt{u_I^2 + v_I^2}/U\right) \qquad [18]$$

The pair of inverse trigonometric expressions given for each angle remove any ambiguity as to value.

As an alternate approach, the magnitude of the absolute linear velocity of the body and its path direction can be derived if $\bar{U}_R$ and $\bar{U}_F$ are known. Information about $\bar{U}_R$ was discussed earlier, under the heading "Relative Velocity in Terms of α and β." Data on the absolute linear velocity of the fluid, $\bar{U}_F$, may be available through use of one, or a combination, of several techniques. As one example, currents may be predictable from oceanographic data. Assume that $\bar{U}_R$ is known in the form U_R, ψ_R, θ_R and that $\bar{U}_F$ is available in the form U_F, ψ_F, θ_F. Then

$$U = \sqrt{U_R^2 + U_F^2 + 2U_R U_F \cos \zeta} \qquad [19]$$

where $\cos \zeta = \sin \theta_R \sin \theta_F + \cos \theta_R \cos \theta_F \cos (\psi_R - \psi_F)$

$$\psi_P = \psi_R - \arctan \left[\frac{U_F \cos \theta_F \sin (\psi_R - \psi_F)}{U_R \cos \theta_R + U_F \cos \theta_F \cos (\psi_R - \psi_F)} \right] \qquad [20]$$

$$\theta_P = \arcsin \left(\frac{U_R}{U} \sin \theta_R + \frac{U_F}{U} \sin \theta_F \right) \qquad [21]$$

The derivation of Equations [19], [20], and [21] is given in Appendix A. Application of the equations to obtain some numerical results is discussed in the section on "Numerical Examples." Ambiguities in the equations can be removed by noting that U is always positive, and that $\bar{U}$ is the vector sum of $\bar{U}_R$ and $\bar{U}_F$. Geometrically, therefore, the direction of $\bar{U}$ must be in the plane containing $\bar{U}_R$ and $\bar{U}_F$; and, furthermore, the direction of $\bar{U}$ would lie in the sector of the lesser angle between $\bar{U}_R$ and $\bar{U}_F$.

In principle, if any two velocities are known in Equations [19], [20], and [21], the third can be found, but the trigonometric form of these equations does not encourage their direct transposition. Instead, in the following two sections, expressions of similar form are given for determining each of the two remaining velocities.

RELATIVE VELOCITY OF BODY

If information is available on the magnitudes and directions of $\bar{U}$ and $\bar{U}_F$, then the relative linear velocity of O with respect to the fluid is derivable from

$$U_R = \sqrt{U^2 + U_F^2 + 2UU_F \cos \lambda} \qquad [22]$$

where $\cos \lambda = - \sin \theta_P \sin \theta_F - \cos \theta_P \cos \theta_F \cos (\psi_P - \psi_F)$

$$\psi_R = \psi_F - \arctan \left[\frac{U \cos \theta_P \sin (\psi_P - \psi_F)}{U_F \cos \theta_F - U \cos \theta_P \cos (\psi_P - \psi_F)} \right] \qquad [23]$$

$$\theta_R = \arcsin \left(\frac{U}{U_R} \sin \theta_P - \frac{U_F}{U_R} \sin \theta_F \right) \qquad [24]$$

Remarks on derivation and ambiguity that follow Equation [21] also apply to Equations [22], [23], and [24].

ABSOLUTE VELOCITY OF FLUID

If sufficient information is available to define $\overline{U}$ and $\overline{U}_R$, then the absolute linear velocity of the fluid at the point O can be calculated from

$$U_F = \sqrt{U^2 + U_R^2 + 2UU_R \cos \xi} \qquad [25]$$

where $\cos \xi = - \sin \theta_P \sin \theta_R - \cos \theta_P \cos \theta_R \cos (\psi_P - \psi_R)$

$$\psi_F = \psi_R - \arctan \left[\frac{U \cos \theta_P \sin (\psi_P - \psi_R)}{U_R \cos \theta_R - U \cos \theta_P \cos (\psi_P - \psi_R)} \right] \qquad [26]$$

$$\theta_F = \arcsin \left(\frac{U}{U_F} \sin \theta_P - \frac{U_R}{U_F} \sin \theta_R \right) \qquad [27]$$

Comments on derivation and ambiguity, made following Equation [21], apply also to Equations [25], [26], and [27].

In closing this section on "Relations Among Linear Velocities," it is of interest to recall that the angles ψ, θ, ϕ define the orientation of the x, y, z axes with respect to the x_I, y_I, z_I axes. Hence, the following table of direction cosines provides the transformation from velocity components in either set of axes to corresponding components in the other set of axes (absolute velocity components are used for illustration).

	u	v	w
u_I	$\cos \psi \cos \theta$	$- \sin \psi \cos \phi + \cos \psi \sin \theta \sin \phi$	$\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi$
v_I	$\sin \psi \cos \theta$	$\cos \psi \cos \phi + \sin \psi \sin \theta \sin \phi$	$\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi$
w_I	$- \sin \theta$	$\cos \theta \sin \phi$	$\cos \theta \cos \phi$

For example, the result

$$w_I = \dot{z}_I = - u \sin \theta + v \cos \theta \sin \phi + w \cos \theta \cos \phi$$

is obtained from the table, and similar relations for relative velocities can be obtained.

SUMMARY OF PRINCIPAL EQUATIONS

The most important equations that have been presented are regrouped here to provide a convenient point of reference.

Given ψ, θ, ϕ and ψ_R, θ_R

$$\alpha = \arctan\left[\frac{\tan\theta\cos(\psi-\psi_R)-\tan\theta_R}{\tan\theta\tan\theta_R+\cos(\psi-\psi_R)}\cos\phi\right.$$

$$\left.+\frac{\sin(\psi-\psi_R)}{\sin\theta\tan\theta_R+\cos\theta\cos(\psi-\psi_R)}\sin\phi\right] \quad [1]$$

$$\beta = \arcsin\{\sin(\psi-\psi_R)\cos\theta_R\cos\phi$$

$$+[\cos\theta\sin\theta_R-\sin\theta\cos\theta_R\cos(\psi-\psi_R)]\sin\phi\} \quad [2]$$

Given u_R, α, β and ψ, θ, ϕ

$$U_R = u_R\sec\alpha\sec\beta \quad [3]$$

$$\psi_R = \psi - \arctan\left(\frac{\sin\alpha\cos\beta\sin\phi+\sin\beta\cos\phi}{\cos\alpha\cos\beta\cos\theta+\sin\alpha\cos\beta\sin\theta\cos\phi-\sin\beta\sin\theta\sin\phi}\right) \quad [4]$$

$$\theta_R = \arcsin(\cos\alpha\cos\beta\sin\theta-\sin\alpha\cos\beta\cos\theta\cos\phi+\sin\beta\cos\theta\sin\phi) \quad [5]$$

Given u_I, v_I, w_I

$$U = \sqrt{u_I^2+v_I^2+w_I^2} \quad [16]$$

$$\psi_P = \arcsin\left(v_I/\sqrt{u_I^2+v_I^2}\right) = \arccos\left(u_I/\sqrt{u_I^2+v_I^2}\right) \quad [17]$$

$$\theta_P = \arcsin(-w_I/U) = \arccos\left(\sqrt{u_I^2+v_I^2}/U\right) \quad [18]$$

Given U_R, ψ_R, θ_R and U_F, ψ_F, θ_F

$$U = \sqrt{U_R^2+U_F^2+2U_R\,U_F\cos\zeta} \quad [19]$$

where $\cos \zeta = \sin \theta_R \sin \theta_F + \cos \theta_R \cos \theta_F \cos (\psi_R - \psi_F)$

$$\psi_P = \psi_R - \arctan \left[\frac{U_F \cos \theta_F \sin (\psi_R - \psi_F)}{U_R \cos \theta_R + U_F \cos \theta_F \cos (\psi_R - \psi_F)} \right] \qquad [20]$$

$$\theta_P = \arcsin \left(\frac{U_R}{U} \sin \theta_R + \frac{U_F}{U} \sin \theta_F \right) \qquad [21]$$

Given U, ψ_P, θ_P and U_F, ψ_F, θ_F

$$U_R = \sqrt{U^2 + U_F^2 + 2UU_F \cos \lambda} \qquad [22]$$

where $\cos \lambda = - \sin \theta_P \sin \theta_F - \cos \theta_P \cos \theta_F \cos (\psi_P - \psi_F)$

$$\psi_R = \psi_F - \arctan \left[\frac{U \cos \theta_P \sin (\psi_P - \psi_F)}{U_F \cos \theta_F - U \cos \theta_P \cos (\psi_P - \psi_F)} \right] \qquad [23]$$

$$\theta_R = \arcsin \left(\frac{U}{U_R} \sin \theta_P - \frac{U_F}{U_R} \sin \theta_F \right) \qquad [24]$$

Given U, ψ_P, θ_P and U_R, ψ_R, θ_R

$$U_F = \sqrt{U^2 + U_R^2 + 2UU_R \cos \xi} \qquad [25]$$

where $\cos \xi = - \sin \theta_P \sin \theta_R - \cos \theta_P \cos \theta_R \cos (\psi_P - \psi_R)$

$$\psi_F = \psi_R - \arctan \left[\frac{U \cos \theta_P \sin (\psi_P - \psi_R)}{U_R \cos \theta_R - U \cos \theta_P \cos (\psi_P - \psi_R)} \right] \qquad [26]$$

$$\theta_F = \arcsin \left(\frac{U}{U_F} \sin \theta_P - \frac{U_R}{U_F} \sin \theta_R \right) \qquad [27]$$

NUMERICAL EXAMPLES

In addition to making the usual check of the mathematics involved in deriving the equations that form the essence of this paper, their validity and generality was also demonstrated by applying them to sets of conditions that might be described as extreme maneuvers. The sets of conditions selected were such that missing values of parameters could be deduced by other means in addition to direct calculation by the expressions being tested.

Results obtained by applying Equations [1] and [2] to six sets of given data are presented in Table 1. In this table the given data are the direction of the relative velocity vector, specified by values of ψ_R and θ_R, and the orientation of the body in space, indicated by the values of ψ, θ, ϕ. The directions and orientations assumed are such that corresponding values of α and β can be computed from the simple relations for the parts of right spherical triangles or, as in the case of Set 2, deduced directly from the geometry. These easily-computed or deduced values of α and β are labeled "Actual Results" in Table 1. Where the values were computed by spherical trigonometry, the high-resolution function tables of Reference 4 were used, with calculations performed on a 10-place desk calculator. Finally, the values of α and β were calculated by using Equations [1] and [2], and these values are labeled "Calculated Results" in Table 1. As a check on the suitability of Equations [1] and [2] for numerical calculation, all required calculations, including trigonometric operations required by Equations [1] and [2], were carried out on a 10-inch slide rule.

Comparison of the actual results and the calculated results in Table 1 shows that results accurate to within one-tenth degree are obtained readily from Equations [1] and [2], even when the maneuvers are extreme; and it also shows that the equations are not critical, i.e., slide-rule accuracy on intermediate steps will yield results accurate to three significant figures. The extremity of the assumed flight conditions is apparent from the values of α and β involved, both of which cover a range of approximately $\pm$ 50 degrees.

Table 2 provides an insight into the computational suitability of Equations [3], [4], and [5]. Giving a symbolic value to u_R, rather than a numerical one, in the table demonstrates that any consistent units for the linear velocities will be satisfactory in the equations.

The first eight sets of conditions in Table 2 portray forward motion, with the x axis directed in turn to the center of each of the four quadrants of the forward hemisphere and similar backing motion in each of the four possible quadrants. The negative sign given for the value of u_R in Sets 5 through 8 results from the assumed backing motion.

In Set 9, the combination of very large angles of attack and drift plus 180 degrees of roll results in a motion condition that is the same as Set 4. The very extreme conditions selected for this set illustrate that the equations presented, being exact, can be applied to any maneuver, however violent. For Set 10, the selected combination of 45-degree heading and 90-degree roll can result in the stipulated 45-degree angle of attack only if the direction of relative motion is in the horizontal plane and at zero heading; hence the actual values of ψ_R and θ_R are each zero. The 90-degree roll in Set 11 interchanges the usual influences of

TABLE 1

Calculation of α and β, Given ψ_R, θ_R and ψ, θ, ϕ

Set	Given Values deg					Actual Results deg		Calculated Results* deg	
	ψ_R	θ_R	ψ	θ	ϕ	α	β	α	β
1	20	45	−10	0	0	−49.11	−20.70	−49.1	− 2.07
2	20	45	−10	0	90	−30	45	−30	45
3	30	45	0	0	−63.43	0	−52.24	− 0.03	−52.2
4	45	−30	0	0	39.23	0	−52.24	0.06	−52.1
5	0	0	−45	30	26.57	0	−52.24	− 0.03	−52.2
6	0	0	−45	30	−63.43	52.24	0	52.2	0.00

*Based on Equations [1] and [2] and 10-inch slide rule accuracy.

TABLE 2

Calculation* of U_R, ψ_R and θ_R, Given u_R, α, β and ψ, θ, ϕ

Set	Given Values						Actual Results			Calculated Results**		
	u_R	α	β	ψ	θ	ϕ	U_R	ψ_R	θ_R	U_R	ψ_R	θ_R
1	u	45	45	45	45	0	$2u$	0	0	$2u$	0 180	0 180
2	u	−45	45	45	−45	0	$2u$	0	0	$2u$		
3	u	−45	− 45	− 45	−45	0	$2u$	0	0	$2u$		
4	u	45	− 45	− 45	45	0	$2u$	0	0	$2u$		
5	$-u$	45	135	135	45	0	$2u$	0	0	$2u$		
6	$-u$	−45	135	135	−45	0	$2u$	0	0	$2u$		
7	$-u$	−45	−135	−135	−45	0	$2u$	0	0	$2u$		
8	$-u$	45	−135	−135	45	0	$2u$	0	0	$2u$		
9	u	135	135	−45	45	180	$2u$	0	0	$2u$		
10	u	45	0	45	0	90	$\sqrt{2}u$	0	0	$\sqrt{2}u$	0 180	0 180
11	u	60	30	0	0	90	$\frac{4}{3}\sqrt{3}u$	−60	30	$\frac{4}{3}\sqrt{3}u$	−60 120	30.0 150.0
12	u	60	30	0	0	−33.69	$\frac{4}{3}\sqrt{3}u$	0	−64.34	$\frac{4}{3}\sqrt{3}u$	0 180	− 64.2 −115.8

*All values except velocities are in degrees.

**Based on Equations [3], [4], and [5] and 10-inch slide rule accuracy.

α on the value of θ_R, and β on the value of ψ_R. The orientation assumed for Set 12 is such that the relative velocity vector is rolled into the vertical plane, resulting in a large negative value for θ_R and a zero value for ψ_R.

The calculated results given in Table 2, obtained on a 10-inch slide rule, are in excellent agreement with the actual values. The rules for dealing with ambiguities, given on page 5 , were applied. For example, in Set 1, the pair $\psi_R = 0$, $\theta_R = 0$ defines the same direction for the vector $\bar{U}_R$ as does the pair ψ_R = 180 degrees, θ_R = 180 degrees. Hence, either pair of values may be discarded as being redundant. The ambiguity rules are used to reject the combinations $\psi_R = 0$, θ_R = 180 degrees or ψ_R = 180 degrees, $\theta_R = 0$.

Numerical examples of the use of Equations [19], [20], and [21] are summarized in Table 3. In the table, values of the magnitudes and directions of the vector velocities $\bar{U}_R$ and $\bar{U}_F$ are assumed known for eight sets of conditions, as represented by the given values for U_R, ψ_R, θ_R and U_F, ψ_F, θ_F. The magnitudes of the known velocities are treated symbolically as k, to demonstrate that any positive number is acceptable as a value. It is not necessary that each magnitude of the pair in a set have the same value. This is illustrated by the choices made in Sets 1 and 2. In each of the remaining Sets of Table 3, however, the magnitudes of $\bar{U}_R$ and $\bar{U}_F$ arbitrarily were made equal for convenience.

When the magnitude of a velocity is zero there can be no meaningful designation of a direction for the velocity. Dashes consequently appear in Table 3 for the values for the angles of direction whenever a velocity is zero.

TABLE 3

Calculation* of U, ψ_P and θ_P, Given U_R, ψ_R, θ_R and U_F, ψ_F, θ_F

Set	Given Values						Actual Results			Calculated Results**		
	U_R	ψ_R	θ_R	U_F	ψ_F	θ_F	U	ψ_P	θ_P	U	ψ_P	θ_P
1	k	30	45	0	–	–	k	30	45	k	30	45
2	0	–	–	k	30	45	k	30	45	k	30	45
3	k	−160	−40	k	20	40	0	–	–	0	–	–
4	k	20	40	k	20	40	$2k$	20	40	$2k$	20	40
5	k	− 90	0	k	0	0	$\sqrt{2}k$	−45	0	$\sqrt{2}k$	−45	0
6	k	45	0	k	−45	0	$\sqrt{2}k$	0	0	$\sqrt{2}k$	0	0
7	k	0	45	k	0	−45	$\sqrt{2}k$	0	0	$\sqrt{2}k$	0	0
8	k	0	0	k	90	90	$\sqrt{2}k$	0	45	$\sqrt{2}k$	0	45

*All values except velocities are in degrees.

**Based on Equations [19], [20], and [21] and 10-inch slide rule accuracy.

The first set of given values in Table 3 represents a body having a velocity of magnitude k relative to the fluid; the fluid being stationary. As a result, the direction of motion relative to the fluid is also the direction in space. It follows that the values for the path angles are the same as the values for the corresponding angles defining the direction of relative motion, as indicated by the tabulation under "Actual Results." The forms of Equations [19], [20], and [21] are such that the calculated results obtained from these equations are the same as the actual results for the first set of given values in Table 3.

For Set 2 in Table 3, the assumed given values represent a body drifting with a fluid (zero relative velocity), and the fluid has a velocity of magnitude k. Hence, the motion of the body in space is the same as the motion of the fluid. In Set 3, it is assumed that the fluid has a velocity of magnitude k, whereas the body has an equal and opposite velocity relative to the fluid; the body therefore has zero velocity in space. For Sets 5 through 8, the velocity of the body relative to the fluid is at right angles to the velocity of the fluid. In Sets 5 and 6, these two velocities lie in the horizontal plane; in Sets 7 and 8, the pair of velocities are in the vertical plane. Because the magnitudes of the velocities are equal in each set, it is easy to deduce the actual magnitude and direction of the resultant motion of the body in space. Calculated values in each case are the same as the actual values because of the forms of Equations [19], [20], and [21].

Because the derivation of the sets of Equations [22], [23], and [24]; and [25], [26], and [27] is based on the geometric analogies that relate them to the set of Equations [19], [20], and [21], it was considered unnecessary to test the remaining equations beyond Equation [21] by employing numerical examples.

SUGGESTED APPLICATIONS

The motivations for this work, mentioned in the Introduction, at once suggest two areas for application of the results. The first is for the determination of path angles or angles of attack and drift when the motion of a body involves excursions too large to be represented by the usual linear approximations. In such cases, it is recommended that approximations to the exact equations be made if the motion is sufficiently limited in one or more of the degrees of freedom to permit such action. Thus, the degree of complexity can be reduced by tailoring the equations to suit the type of motion being treated and the degree of accuracy desired for the resultant data.

The second, and possibly more noteworthy, area of application of the results is to problems where the fluid is moving. All of the equations presented have been derived to apply to this condition, but they are equally valid when the fluid is stationary. Two illustrations of problems where the motion of the fluid is a significant factor are those involving motions of a submarine hovering near the surface, or of a hydrofoil craft flying in waves.

A possible method of determining the local magnitude and direction of fluid velocity in ocean waves is suggested by Equations [25] through [27]. If a free body were instrumented to obtain inertial guidance data, the data would define the absolute linear velocity $\overline{U}$ of the body for use in the cited equations. If the same body were fitted with a suitable directional flow meter (for instance, a weather-vane-mounted pitot tube), the relative velocity $\overline{U}_R$ would also be known. The magnitude of the fluid velocity could then be obtained from Equation [25], and Equations [26] and [27] would give the direction of fluid flow.

A few remarks on the determination of α and β for a body, when the basic motion data are derived from inertial navigation type sensors, may be of interest. Via Equations [17] and [18], the path angles for the body can be derived from the inertial data. If the motion is through stationary fluid, these path angles can be used in conjunction with the body attitude angles ψ, θ, ϕ to obtain α and β from Equations [1] and [2] (by replacing ψ_R and θ_R in the equations with the corresponding path angles). If the motion occurs in water having a current of known characteristics, the path angles are obtained as before, and U can be obtained from Equation [16], in addition. The data on U and the path angles would then be combined with the information on the current, in Equations [23] and [24], to obtain the angles ψ_R and θ_R. The remainder of the data reduction would be by means of Equations [1] and [2], as before.

The data reduction procedures for obtaining absolute linear velocity and path direction from speed log data will be described briefly. Assume that a speed log measures u_R, α , and β. Also assume ψ, θ, ϕ are known. Then Equations [3] through [5] yield U_R, ψ_R, θ_R. If the fluid is stationary, these relative values are also the absolute values of velocity and path angle. If a current of known characteristics is present, correct for its effects by using Equations [19] through [21].

Finally, Equations [19] through [21] provide a general tool for calculating the resultant sum of two vectors when the two vectors are expressed in terms of their magnitudes and the Euler angles defining their directions. The vector sum involved is

$$\overline{U} = \overline{U}_R + \overline{U}_F \qquad [15]$$

Equations [22] through [24] perform a similar function for the difference of two vectors.

APPENDIX A

DERIVATIONS OF EQUATIONS

This section contains derivations of many of the equations given in the body of the paper. An inertial frame of reference, i.e., the fixed axes x_I, y_I, z_I, provides a logical starting point. It is an orthogonal right-hand Cartesian system and, in principle, can be fixed in space in any orientation. An orientation with the $x_I y_I$ plane horizontal and x_I directed to the true north is convenient and, consequently, is specified.

The inertial frame is useful in describing the directions of various vectors and axes, by means of Euler angles. In Figure 1, imagine a sphere of any convenient radius, located so that the horizontal $x_I y_I$ plane cuts the surface of the sphere at its equator. The z_I axis is vertically down and intersects the surface of the sphere at a pole. The orientation of any directed line, such as Ox in Figure 1, can be fixed by two Euler angles, shown as ψ and θ in Figure 1, and taken in the order just stated. Note that ψ is measured in the $x_I y_I$ plane (along the equator), and θ is measured along a meridian. Positive senses for the two angles are shown in Figure 1.

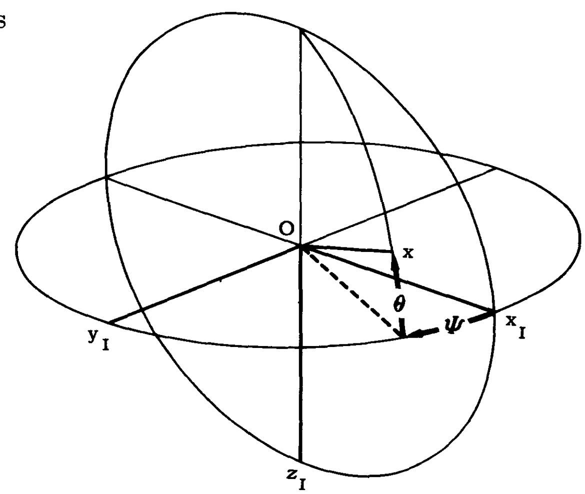

Figure 1 – Typical Euler Angles

The direction of any vector can be given by a pair of Euler angles of the type just described. Subscripts are used in the text to identify the various pairs associated with specified vectors. A pair of Euler angles without subscripts is used to show the direction of the Ox body axis. (This axis system will be described shortly.) All Euler angle pairs are measured from the same inertial reference frame, x_I, y_I, z_I.

Consider the axis Ox of Figure 1 as one member of an orthogonal right-hand Cartesian system x, y, z. Note that the orientation of x, y, z with respect to x_I, y_I, z_I can be specified completely if, in addition to the fixing of the direction of Ox by means of ψ and θ, a third angle ϕ is used to describe the rotation of the x, y, z system, out of the horizontal, about the Ox axis. The positive sense of ϕ is a clockwise rotation, as seen when looking in the direction Ox. The three Euler angles ψ, θ, ϕ, taken in the order stated, completely define the orientation of the x, y, z system relative to the x_I, y_I, z_I system.

The x, y, z reference frame is taken herein as a system fixed in a moving body, so that the origin O of the system is some point of the body. The whereabouts of O in the body, or the orientation of the x, y, z body axes with respect to the body configuration are of no significance for the present purpose. The motion of the body can be described completely by describing the linear and angular velocities of O. Only the linear velocity component of the motion is of interest in the present discussion. (If an angular velocity is present at O, its effect on the hydrodynamic forces experienced by the body customarily is treated separately and constitutes a problem outside the scope of this paper.)

Assume that the body is moving in a fluid and that in the region of O the fluid has a linear velocity, represented by the vector $\bar{U}_F$. Let the absolute linear velocity of the point O of the body in space be represented by the vector $\bar{U}$. Then

$$\bar{U} = \bar{U}_R + \bar{U}_F \qquad [15]$$

where $\bar{U}_R$ is a vector giving the linear velocity of the point O relative to the fluid.

The direction of $\bar{U}_R$ in space can be given by a pair of Euler angles ψ_R and θ_R. The direction of this same vector with respect to the moving body axes x, y, z can be given by another pair of angles β and α , as indicated in Figure 2, which is based on a sphere whose radius is the length of the vector $\bar{U}_R$ and whose center is at the point O. The x, y, z axes, with origin at O, pierce the surface of the sphere at the three points so labelled.

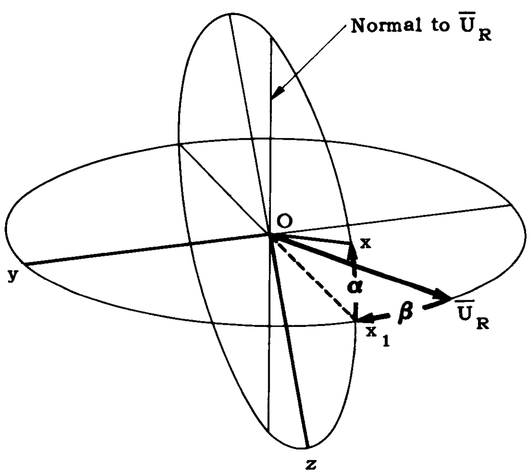

Figure 2 – Angles of Drift and Attack, and the Relative Velocity Vector $\bar{U}_R$

To find the drift of angle β, rotate $\bar{U}_R$ (about a line that lies in the xz plane and is normal to $\bar{U}_R$) until $\bar{U}_R$ lies in the x-positive portion of the xz plane. The resultant position of $\bar{U}_R$ is the dotted line Ox_1 in Figure 2. The rotation will be through the angle β, positive if $\bar{U}_R$ was initially on the y-negative side of the xz plane.

To find the angle of attack α , rotate the line Ox_1 (about the y axis) until the line Ox_1 coincides with the x axis. The rotation will be through the angle α , positive if Ox_1 was initially on the z-positive side of the xy plane.

Components of the vector $\bar{U}_R$ in the directions of the x, y, z axes are indicated by u_R, v_R, w_R, respectively. Then

$$U_R = \sqrt{u_R^2 + v_R^2 + w_R^2} \qquad [A1]$$

where U_R is the magnitude of the vector $\bar{U}_R$.

Let u_1 be the component of $\bar{U}_R$ in the xz plane (along the direction of the dotted line Ox_1 in Figure 2). Then, by noting the geometry involved, the value of u_1 can be expressed in four forms

$$\begin{aligned} u_1 &= u_R \sec \alpha \\ &= U_R \cos \beta \\ &= \sqrt{u_R^2 + w_R^2} \\ &= \sqrt{U_R^2 - v_R^2} \end{aligned}$$

It is likewise evident from Figure 2 that

$$\alpha = \arctan \frac{w_R}{u_R} = \arcsin \frac{w_R}{u_1} = \arccos \frac{u_R}{u_1} \qquad [A2]$$

$$\beta = \arctan \frac{-v_R}{u_1} = \arcsin \frac{-v_R}{U_R} = \arccos \frac{u_1}{U_R} \qquad [A3]$$

Combinations of certain of the preceding four forms for u_1 and some of the alternative expressions for α and β give

$$U_R = u_R \sec \alpha \sec \beta \qquad [3]$$

$$u_R = U_R \cos \alpha \cos \beta \qquad [6]$$

$$v_R = -U_R \sin \beta = -u_R \sec \alpha \tan \beta \qquad [7]$$

$$w_R = U_R \cos \beta \sin \alpha = u_R \tan \alpha \qquad [8]$$

Following a development analogous to that just described for $\bar{U}_R$, we note that the absolute velocity $\bar{U}$ of the body has components u_I, v_I, w_I in the directions of the fixed axes x_I, y_I, z_I. Hence

$$U = \sqrt{u_I^2 + v_I^2 + w_I^2} \qquad [16]$$

where U is the magnitude of the vector $\bar{U}$.

Figure 3 shows the relation of $\overline{U}$ to the fixed axes x_I, y_I, z_I. Let u_2 be the component of $\overline{U}$ in the $x_I z_I$ plane (along the direction of the dotted line Ox_2 in Figure 3). The value of this component can be obtained in several ways, as:

$$u_2 = U \cos \theta_P$$

$$= u_I \sec \psi_P$$

$$= \sqrt{u_I^2 + v_I^2}$$

$$= \sqrt{U^2 - w_I^2}$$

Also

$$\psi_P = \arctan \frac{v_I}{u_I} = \arcsin \frac{v_I}{u_2} = \arccos \frac{u_I}{u_2} \qquad [A4]$$

$$\theta_P = \arctan \frac{-w_I}{u_2} = \arcsin \frac{-w_I}{U} = \frac{u_2}{U} \qquad [A5]$$

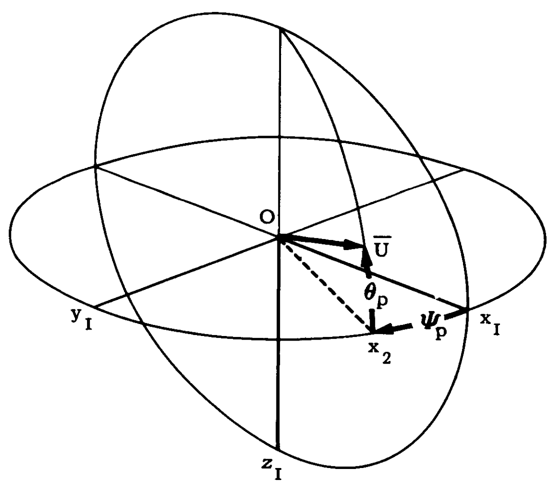

Figure 3 – Path Angles and the Absolute Velocity Vector $\overline{U}$

Substitution of the third of the expressions just given for u_2 into Equations [A4] and [A5] yields

$$\psi_P = \arcsin\left(v_I/\sqrt{u_I^2 + v_I^2}\right) = \arccos\left(u_I/\sqrt{u_I^2 + v_I^2}\right) \qquad [17]$$

$$\theta_P = \arcsin\,(-w_I/U) = \arccos\left(\sqrt{u_I^2 + v_I^2}/U\right) \qquad [18]$$

Derivations will next be given for Equations [1] and [2]. The sphere of Figure 1 is reportrayed in Figure 4, for use in the derivations. On the surface of the sphere, let the Ox body axis intersect at the point x. Assume that the relative velocity vector $\overline{U}_R$ intersects the surface at the point R. Arcs of great circles, representing the angles α and β are drawn, the two arcs meeting in a right angle at the point x_1 (compare with Figure 2). Also, pass a great circle through the points x and R. The resultant right spherical triangle x_1xR will be of interest.

If we recall that Euler angles can be used to define the direction of a line with respect to the $x_I y_I z_I$ axes, the Euler angles, measured along the equator and corresponding to the points x and R, are shown as ψ and ψ_R. We can visualize the related angles θ and θ_R, that would be measured (positive sense up) from the equator along the meridian lines shown passing through x and R. These meridian lines intersect at the pole C.

Note that the points x, C, and R form the vertices of a spherical triangle. Sides of spherical triangles will be designated by a pair of symbols corresponding to the terminal points. Vertex angles will be referred to by a triad of symbols, in customary fashion. For the triangle under discussion

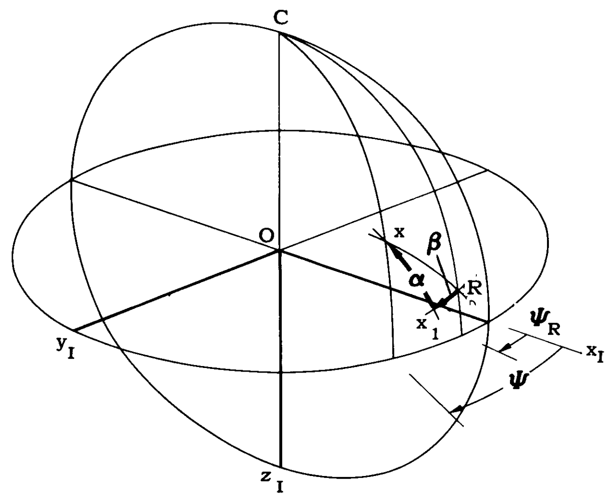

Figure 4 – Right Spherical Triangle Involving α and β

$$xC = \pi/2 - \theta \qquad [A6]$$

$$CR = \pi/2 - \theta_R \qquad [A7]$$

$$xCR = \psi - \psi_R \qquad [A8]$$

Using the Law of Cosines (see any representative text on spherical trigonometry) gives

$$\cos xR \equiv \cos xC \cos CR + \sin xC \sin CR \cos xCR$$

or

$$\cos xR = \sin \theta \sin \theta_R + \cos \theta \cos \theta_R \cos (\psi - \psi_R) \qquad [A9]$$

Using the Law of Sines supplies

$$\sin CxR \equiv \frac{\sin CR \sin xCR}{\sin xR} = \frac{\cos \theta_R \sin (\psi - \psi_R)}{\sin xR} \qquad [A10]$$

Also

$$\cos CxR \equiv \frac{\sin CxR}{\tan CxR} \qquad [A11]$$

In Appendix B the following identity is proved for spherical triangles

$$\tan CxR \equiv \frac{\sin xCR}{\cot CR \sin xC - \cos xCR \cos xC} \qquad [A12]$$

Substituting Equations [A 10] and [A 12] into [A 11] gives

$$\cos CxR \equiv \frac{\sin CR \sin xCR}{\sin xR} \left(\frac{\cot CR \sin xC - \cos xCR \cos xC}{\sin xCR} \right)$$

$$\equiv \frac{\cos CR \sin xC - \sin CR \cos xC \cos xCR}{\sin xR}$$

or

$$\cos CxR = \frac{\cos \theta \sin \theta_R - \sin \theta \cos \theta_R \cos (\psi - \psi_R)}{\sin xR} \qquad [A13]$$

Now consider the right spherical triangle $x_1 xR$, described previously. First note that arc $x_1 x$ makes an angle ϕ with the meridian passing through x. This follows because $x_1 x$ is contained in the xz body plane (see Figure 2), and that plane makes an angle ϕ with the vertical plane containing the x axis, i.e., the meridian plane passing through the point x in Figure 4. The angle $x_1 xR$, consequently, has the value

$$x_1 xR = \pi - \phi - CxR \qquad [A14]$$

Therefore

$$\sin x_1 xR = \sin (\phi + CxR) = \sin \phi \cos CxR + \cos \phi \sin CxR$$

$$\cos x_1 xR = - \cos (\phi + CxR) = - \cos \phi \cos CxR + \sin \phi \sin CxR$$

Substituting Equations [A10] and [A13] in the preceding pair, we obtain

$$\sin x_1 xR = \frac{\sin \phi}{\sin xR} [\cos \theta \sin \theta_R - \sin \theta \cos \theta_R \cos (\psi - \psi_R)]$$

$$+ \frac{\cos \phi}{\sin xR} [\cos \theta_R \sin (\psi - \psi_R)] \qquad [A15]$$

and

$$\cos x_1 xR = -\frac{\cos\phi}{\sin xR}\,[\cos\theta\sin\theta_R - \sin\theta\cos\theta_R\cos(\psi-\psi_R)]$$

$$+\frac{\sin\phi}{\sin xR}\,[\cos\theta_R\sin(\psi-\psi_R)] \qquad \text{[A16]}$$

Use will now be made of the identity for right spherical triangles

$$\tan x_1 x \equiv \tan xR \cos x_1 xR$$

or, noting that $x_1 x = \alpha$ and using Equation [A16], we get

$$\tan\alpha = \frac{1}{\cos xR}\,\{-\cos\phi\,[\cos\theta\sin\theta_R - \sin\theta\cos\theta_R\cos(\psi-\psi_R)]$$

$$+\sin\phi\,[\cos\theta_R\sin(\psi-\psi_R)]\}$$

Using Equation [A9] and simplifying gives

$$\alpha = \arctan\left[\frac{\tan\theta\cos(\psi-\psi_R)-\tan\theta_R}{\tan\theta\tan\theta_R+\cos(\psi-\psi_R)}\cos\phi\right.$$

$$\left.+\frac{\sin(\psi-\psi_R)}{\sin\theta\tan\theta_R+\cos\theta\cos(\psi-\psi_R)}\sin\phi\right] \qquad \text{[1]}$$

Equation [2] is derived by applying the identity

$$\sin x_1 R \equiv \sin x_1 xR \sin xR$$

to the right spherical triangle $x_1 xR$ of Figure 4. Making use of Equation [A15], this identity yields

$$\beta = \arcsin\{\sin(\psi-\psi_R)\cos\theta_R\cos\phi$$

$$+[\cos\theta\sin\theta_R - \sin\theta\cos\theta_R\cos(\psi-\psi_R)]\sin\phi\} \qquad \text{[2]}$$

A check on the calculation of α and β can be obtained, based on the identity

$$\cos xR \equiv \cos x_1 x \cos x_1 R$$

for the right spherical triangle of Figure 4. The identity is equivalent to

$$\cos xR = \cos\alpha\cos\beta \qquad \text{[A17]}$$

By using Equation [A6], the preceding expression can be written

$$\cos \alpha \cos \beta = \sin \theta \sin \theta_R + \cos \theta \cos \theta_R \cos (\psi - \psi_R) \qquad \text{[A18]}$$

The derivation of Equations [4] and [5] will be undertaken next. Derivation of these equations is based on the assumption that the body orientation angles ψ, θ, ϕ and the angles of attack and drift, α and β, are known. In Figure 4, consequently, xC is known from Equation [A 6].

From Equation [A 14]

$$CxR = (\pi - \phi) - x_1xR$$

so

$$\begin{aligned} \sin CxR &= \sin (\pi - \phi) \cos x_1xR - \cos (\pi - \phi) \sin x_1xR \\ &= \sin \phi \cos x_1xR + \cos \phi \sin x_1xR \end{aligned} \qquad \text{[A19]}$$

and, similarly,

$$\cos CxR = - \cos \phi \cos x_1xR + \sin \phi \sin x_1xR \qquad \text{[A20]}$$

Using the identities for right spherical triangles gives

$$\sin x_1xR \equiv \frac{\sin x_1R}{\sin xR} = \frac{\sin \beta}{\sin xR}$$

$$\cos x_1xR \equiv \frac{\cos x_1R \sin x_1x}{\sin xR} = \frac{\sin \alpha \cos \beta}{\sin xR}$$

Equations [A19] and [A20] become

$$\sin CxR = \frac{1}{\sin xR} (\sin \alpha \cos \beta \sin \phi + \sin \beta \cos \phi) \qquad \text{[A21]}$$

$$\cos CxR = \frac{1}{\sin xR} (-\sin \alpha \cos \beta \cos \phi + \sin \beta \sin \phi) \qquad \text{[A22]}$$

The identity developed in Appendix B leads to the relation for spherical triangle xCR in Figure 4

$$\tan xCR \equiv \frac{\sin CxR}{\cot xR \sin xC - \cos CxR \cos xC}$$

By using Equations [A6], [A21], and [A22], this becomes

$$\tan xCR = \frac{\sin\alpha\ \cos\beta \sin\phi + \sin\beta\cos\phi}{\cos xR \cos\theta - (-\sin\alpha\ \cos\beta\cos\phi + \sin\beta\sin\phi)\sin\theta}$$

Employing Equations [A8] and [A17] gives

$$\psi_R = \psi - \arctan\left(\frac{\sin\alpha\ \cos\beta\sin\phi + \sin\beta\cos\phi}{\cos\alpha\ \cos\beta\cos\theta + \sin\alpha\ \cos\beta\sin\theta\cos\phi - \sin\beta\sin\theta\sin\phi}\right) \quad [4]$$

In developing Equation [5], the identity

$$\cos CR \equiv \cos xR \cos xC + \sin xR \sin xC \cos CxR$$

is used for spherical triangle xCR of Figure 4. Then, on the basis of Equations [A6], [A17], and [A22]

$$\cos CR = \cos\alpha\ \cos\beta\sin\theta - \sin\alpha\ \cos\beta\cos\theta\cos\phi + \sin\beta\cos\theta\sin\phi$$

and, finally, by virtue of Equation [A7]

$$\theta_R = \arcsin(\cos\alpha\ \cos\beta\sin\theta - \sin\alpha\ \cos\beta\cos\theta\cos\phi + \sin\beta\cos\theta\sin\phi) \quad [5]$$

The derivation of Equations [19], [20], and [21] will now be described. These three equations define the resultant $\overline{U}$ that is the sum of two vectors $\overline{U}_R$ and $\overline{U}_F$; therefore, they may be considered as a general tool to find the sum of two given vectors.

In Figure 5, let OR and OF be the two given vectors and OP the required resultant vector. The three vectors are perforce coplanar and form two sides and a diagonal of a parallelogram. Hence, length FP is the same as the magnitude of the given vector OR, and this length is designated U_R. The length OF similarly is called U_F. The magnitude of the resultant vector is the length OP, to which the magnitude U is assigned. The Law of Cosines for plane triangles gives, for triangle OFP,

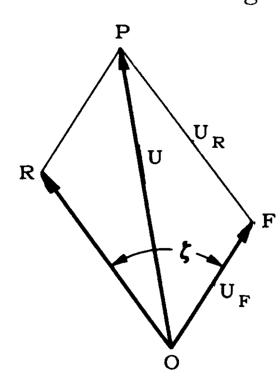

Figure 5 – Resultant of Sum of Two Vectors

$$(OP)^2 \equiv (FP)^2 + (OF)^2 - 2\,(FP)\,(OF)\cos OFP$$

Call the angle between OR and OF the angle ζ. Then

$$OFP = \pi - \zeta \qquad [A23]$$

and the preceding expression can be written

$$U = \sqrt{U_R^2 + U_F^2 + 2U_R U_F \cos \zeta} \qquad [19]$$

The value of cos ζ will now be determined. In Figure 5 visualize a sphere, centered at the point O, and of sufficiently small radius so that all three vectors pierce the surface of the sphere. The sphere described is shown in Figure 6, with the points of piercing labelled to correspond to the terminal points of the vectors. Because the vectors are coplanar, the points will lie on a common great circle. The angular measure of this great circle, between the points R and F, is the same as the angular separation of the vectors OR and OF in Figure 5, hence

$$\zeta = RF \qquad [A24]$$

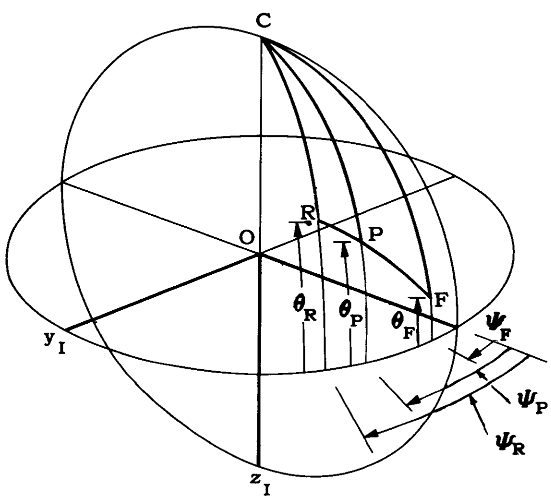

Figure 6 – Spherical Triangles Used in Derivation of Equations [20] and [21]

Because of the analogy between RF in Figure 6 and xR in Figure 4, Equation [A9] can be used as an analog to obtain

$$\cos \zeta = \sin \theta_R \sin \theta_F + \cos \theta_R \cos \theta_F \cos (\psi_R - \psi_F) \qquad [A25]$$

for use in Equation [19].

The development of Equation [20] will now be taken up. Using the identity given in Appendix B for the spherical triangle CRP in Figure 6 yields

$$\tan RCP \equiv \frac{\sin CRP}{\cot RP \sin CR - \cos CRP \cos CR}$$

Noting that angle CRP is the same as angle CRF in spherical triangle CRF, and that

$$CR = \pi/2 - \theta_R$$

leads to

$$\tan RCP = \frac{\sin CRF}{\cot RP \cos \theta_R - \cos CRF \sin \theta_R} \qquad [A26]$$

For the triangle CRF in Figure 6

$$RCF = \psi_R - \psi_F$$

$$CF = \pi/2 - \theta_F$$

Then, because of the similarity between triangle CRF in Figure 6 and triangle CxR in Figure 4, Equations [A10] and [A13] can be used as analogs to obtain

$$\sin CRF = \frac{\cos \theta_F \sin (\psi_R - \psi_F)}{\sin RF} \qquad [A27]$$

$$\cos CRF = \frac{\cos \theta_R \sin \theta_F - \sin \theta_R \cos \theta_F \cos (\psi_R - \psi_F)}{\sin RF} \qquad [A28]$$

The angular measure of the arc RP in Figure 6 is the same as the angle ROP in Figure 5 and, consequently, the same as angle OPF in Figure 5. The Law of Cosines for the plane triangle OPF in Figure 5 can be used to prove the identity

$$\cos OPF \equiv \frac{PF - OF \cos OFP}{OP}$$

Likewise the Law of Sines gives the identity

$$\sin OPF \equiv \frac{OF \sin OFP}{OP}$$

Hence, using Equation [A 23], we obtain

$$\cos RP = \frac{U_R + U_F \cos \zeta}{U} \qquad [A29]$$

$$\sin RP = \frac{U_F \sin \zeta}{U} \qquad [A30]$$

Therefore

$$\cot RP = \frac{U_R + U_F \cos \zeta}{U_F \sin \zeta} \qquad [A31]$$

Substituting Equations [A24], [A27], [A28], and [A31] into [A26] gives, upon simplification,

$$\tan RCP = \frac{U_F \cos \theta_F \sin (\psi_R - \psi_F)}{(U_R + U_F \cos \zeta) \cos \theta_R - U_F [\cos \theta_R \sin \theta_F - \sin \theta_R \cos \theta_F \cos (\psi_R - \psi_F)] \sin \theta_R} \qquad [A32]$$

Observing from Figure 6 that

$$RCP = \psi_R - \psi_P$$

using the value of $\cos \zeta$ from Equation [A25], the well-known trigonometric identity

$$\sin^2 \theta_R + \cos^2 \theta_R \equiv 1$$

and rearranging, Equation [A32] becomes

$$\psi_P = \psi_R - \arctan \left[\frac{U_F \cos \theta_F \sin (\psi_R - \psi_F)}{U_R \cos \theta_R + U_F \cos \theta_F \cos (\psi_R - \psi_F)} \right] \qquad [20]$$

The derivation of Equation [21] now follows. In Figure 6, the Law of Cosines gives, for triangle CRP,

$$\cos CRP \equiv \frac{\cos CP - \cos CR \cos RP}{\sin CR \sin RP} \qquad [A33]$$

and for triangle CRF

$$\cos CRF \equiv \frac{\cos CF - \cos CR \cos RF}{\sin CR \sin RF} \qquad [A34]$$

But

$$CRP = CRF$$

so the right-hand sides of Equations [A33] and [A34] are equal. This equality can be rearranged to give

$$\cos CP = \frac{\sin RP}{\sin RF} (\cos CF - \cos CR \cos RF) + \cos CR \cos RP \qquad \text{[A35]}$$

Equation [A35] can be simplified by utilizing Equations [A24], [A29] and [A30] to obtain

$$\cos CP = \frac{U_F}{U} (\cos CF - \cos CR \cos \zeta) + \cos CR \left(\frac{U_R}{U} + \frac{U_F}{U} \cos \zeta \right)$$

$$= \frac{U_F}{U} \cos CF + \frac{U_R}{U} \cos CR \qquad \text{[A36]}$$

However, Figure 6 shows

$$CR = \pi/2 - \theta_R$$

$$CP = \pi/2 - \theta_P$$

$$CF = \pi/2 - \theta_F$$

and, therefore, from Equation [A36], we obtain

$$\theta_P = \arcsin \left(\frac{U_R}{U} \sin \theta_R + \frac{U_F}{U} \sin \theta_F \right) \qquad \text{[21]}$$

Equations [22], [23], and [24] will now be derived. This set permits solution for $\overline{U}_R$ when $\overline{U}$ and $\overline{U}_F$ are known. The relation among these vectors has been given as

$$\overline{U} = \overline{U}_R + \overline{U}_F \qquad \text{[15]}$$

which can be reexpressed as

$$\overline{U}_R = \overline{U} - \overline{U}_F \qquad \text{[A37]}$$

The problem of finding $\overline{U}_R$, consequently, can be considered to be that of finding the sum of two vectors $\overline{U}$ and $-\overline{U}_F$. Hence it is equivalent to the problem just treated, and in which the diagram of Figure 5 was used. The present problem is portrayed in Figure 7. Comparison with Figure 5 shows the following correspondence of elements for the problems represented by the two figures:

Figure 5	U	U_R	U_F	ζ	ψ_P	θ_P	ψ_R	θ_R	ψ_F	θ_F
Figure 7	U_R	U_F	U	λ	ψ_R	θ_R	ψ_{-F}	θ_{-F}	ψ_P	θ_P

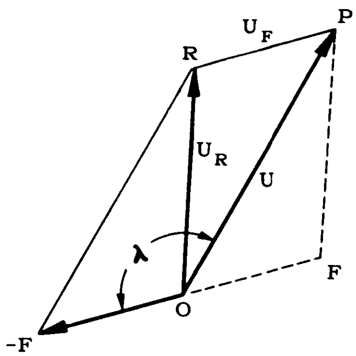

Figure 7 – Representation of the Vector Difference $\bar{U} - \bar{U}_F$

In the tabulation just given, the Euler angles for the vector $-\bar{U}_F$ are indicated by the symbols ψ_{-F} and θ_{-F}. Because $-\bar{U}_F$ is a vector opposite in direction to $\bar{U}_F$, these angles can be expressed in terms of similar angles for the vector $\bar{U}_F$, as:

$$\psi_{-F} = \psi_F + \pi$$

$$\theta_{-F} = -\theta_F$$

Note also that only the magnitudes of the vectors are given in the table, and the magnitude of $-\bar{U}_F$ is the same as that of $\bar{U}_F$.

The table can be used to convert Equations [19], [20], [21], and [A25], which apply to Figure 5, into new equations that apply to Figure 7. Thus, Equation [19] converts to

$$U_R = \sqrt{U^2 + U_F^2 + 2\,UU_F \cos\lambda} \qquad [22]$$

Equation [A25] supplies

$$\cos\lambda = -\sin\theta_P \sin\theta_F - \cos\theta_P \cos\theta_F \cos(\psi_P - \psi_F) \qquad [A38]$$

similarly, Equations [20] and [21] yield:

$$\psi_R = \psi_F - \arctan\left[\frac{U\cos\theta_P \sin(\psi_P - \psi_F)}{U_F\cos\theta_F - U\cos\theta_P\cos(\psi_P - \psi_F)}\right] \qquad [23]$$

$$\theta_R = \arcsin\left(\frac{U}{U_R}\sin\theta_P - \frac{U_F}{U_R}\sin\theta_F\right) \qquad [24]$$

The final set of equations to be derived pertains to the problem where $\bar{U}$ and $\bar{U}_R$ are known and $\bar{U}_F$ is sought. Again Equation [15] can be rewritten as

$$\bar{U}_F = \bar{U} - \bar{U}_R$$

Comparison with Equation [A37] shows that interchange of the subscripts R and F in Equations [22], [23], [24], and [A38] will suffice to obtain new equations that apply here. The angle λ of Figure 7 will be replaced by the angle ξ for the current problem. The interchange of subscripts is such a trivial operation that its result will not be carried out here. The resultant equations are available in the body of the paper as Equations [25], [26], and [27].

APPENDIX B

A LAW OF SPHERICAL TRIGONOMETRY

The identity

$$\tan A \equiv \frac{\sin C}{\cot a \sin b - \cos C \cos b}$$

for any spherical triangle ABC, shown in Figure 8, proved useful in the development contained in Appendix A. The author was unable to find this identity by referring to several standard textbooks on trigonometry that were readily available, so its proof will be given.

The following two identities for spherical triangles can be found in most textbooks:

$$\frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \equiv \frac{\tan \frac{1}{2}(A-B)}{\cot \frac{1}{2} C}$$

$$\frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \equiv \frac{\tan \frac{1}{2}(A+B)}{\cot \frac{1}{2} C}$$

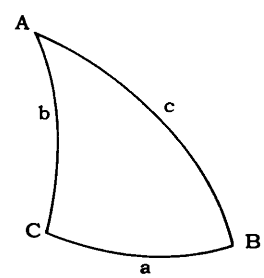

Figure 8 – Typical Spherical Triangle

The pair of identities can be rearranged to evaluate $\tan \frac{1}{2}(A-B)$ and $\tan \frac{1}{2}(A+B)$.

Then, because

$$\tan A = \tan \left[\frac{1}{2}(A-B) + \frac{1}{2}(A+B) \right]$$

and using the well-known identity

$$\tan (M+N) \equiv \frac{\tan M + \tan N}{1 - \tan M \tan N}$$

where M and N are any two angles, we obtain after some rearranging

$$\tan A \equiv \frac{\cot \frac{1}{2} C \left[\cos \frac{1}{2}(a+b) \sin \frac{1}{2}(a-b) + \sin \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b)\right]}{\sin \frac{1}{2}(a+b) \cos \frac{1}{2}(a+b) - \cot^2 \frac{1}{2} C \left[\sin \frac{1}{2}(a-b) \cos \frac{1}{2}(a-b)\right]}$$

Two other well-known identities:

$$\sin (M + N) \equiv \sin M \cos N + \cos M \sin N \qquad [B1]$$

$$\sin M \cos M \equiv \frac{1}{2} \sin 2 M \qquad [B2]$$

are used to simplify the preceding expression to

$$\tan A \equiv \frac{\cot \frac{1}{2} C \sin a}{\frac{1}{2} \sin (a + b) - \frac{1}{2} \cot^2 \frac{1}{2} C [\sin (a - b)]}$$

Applying Equation [B1] to the denominator, using

$$\cot M \equiv \frac{1}{\tan M}$$

and rearranging, gives

$$\tan A \equiv \frac{1}{\frac{1}{2} \tan \frac{1}{2} C [\cos b + \cot a \sin b] - \frac{1}{2} \cot \frac{1}{2} C [\cos b - \cot a \sin b]}$$

which, by regrouping and using the relation

$$\tan M \equiv \frac{\sin M}{\cos M}$$

becomes

$$\tan A \equiv \frac{1}{\frac{1}{2}\left[\frac{\sin^2 \frac{1}{2} C - \cos^2 \frac{1}{2} C}{\sin \frac{1}{2} C \cos \frac{1}{2} C}\right] \cos b + \frac{1}{2}\left[\frac{\sin^2 \frac{1}{2} C + \cos^2 \frac{1}{2} C}{\sin \frac{1}{2} C \cos \frac{1}{2} C}\right] \cot a \sin b}$$

By using the identities:

$$\cos^2 M - \sin^2 M \equiv \cos 2M$$

$$\sin^2 M + \cos^2 M \equiv 1$$

and identity [B2], the preceding expression reduces to

$$\tan A \equiv \frac{1}{\frac{1}{2}\left[\frac{-\cos C}{\frac{1}{2}\sin C}\right]\cos b + \frac{1}{2}\left[\frac{1}{\frac{1}{2}\sin C}\right]\cot a \sin b}$$

or, more simply arranged

$$\tan A \equiv \frac{\sin C}{\cot a \sin b - \cos C \cos b}$$

the identity sought.

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