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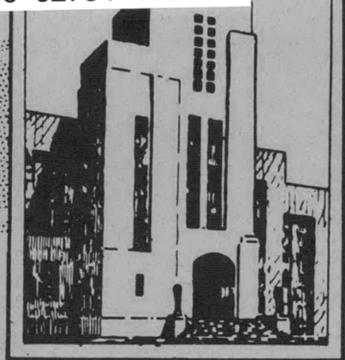

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THEORY OF STATIC AND DYNAMIC LOADS ON A RUDDER IN A STEADY TURN

by

Ralph C. Leibowitz
and
A.G. Strandhagen

[redacted]

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NOTATION

b	Span of sailplane; span of rudder
C_D, C_L, C_M or $C_{M_{shaft}}$ or $C_{M(\bar{c}/4)}$	Nondimensional lift, drag, and torque coefficients, respectively
$(C_{L\alpha F})_B$, $C_{L\alpha F}$	Total lift coefficient at sailplane and rudder, respectively
$\bar{C}_L$	Largest mean coefficient of rudder side force
$\overline{C_L^2}$	Mean square lift coefficient
C_y	Nondimensional mean lateral force
c	Distance from rudder stock to point of application of Y_R
$\bar{c}$	Mean geometric chord of rudder; mean chord of sailplane
D	Drag force; hull diameter at sailplane; hull diameter at rudder
F	Force
G	Shear modulus of elasticity
$\bar{i}$, $\bar{j}$	Unit vectors along x- and y-axes, respectively
J_e	Polar moment of inertia of cross section of rudder stock about a perpendicular through the centroid
k_M, k_y	Constants or transfer functions dependent on geometry of rudder-hull system and its moduli of elasticity
L	Lift force; scale of turbulence
l	**Horizontal distance between center of mass of hull and rudder stock**
l_T	Effective length of rudder stock for computing torsional flexibility
M_S	Torque (about rudder stock) exerted by fluid on rudder
p, q, r	Angular speeds of hull about x-, y-, and z-axes, respectively
R, $R_{\bar{c}}$	Reynolds number
S	Planform area of rudder

T	Torque; arbitrary time period
t	Time
U	Absolute velocity of center of mass; moment
u, v, w	Components of fluctuating turbulent flow velocity; v is also the induced velocity produced by horseshoe vortex in direction of negative y
X_R, Y_R	Components of lift and drag forces in direction of body x- and y-axes, respectively; Y_R is also a horizontal force positive in direction of negative y
x, y, z	Rectangular coordinates of a moving reference frame (i.e., body axes) with origin at ship center of mass, z-axes vertical and x- and y-axes in a horizontal plane along and perpendicular, respectively, to the ship's longitudinal axis; rectangular coordinates of rudder with x lying along the chord, y along the normal to chord and span, and z along span of rudder.
$y(t)$	Shear load at root of rudder stock
α	Local angle of attack of rudder relative to flow in neighborhood of rudder (time-dependent for turbulent flow)
α_0	Mean angle of attack of rudder under turbulent conditions
α_1	Angle between resultant velocity vector of rudder and absolute steady velocity U_0 of center of mass of hull
α_2	Angle of attack (angle between direction of motion or resultant velocity vector of rudder and chord line or centerplane of rudder)
α_e	Angle between resultant velocity vector of rudder and ship's median plane
α_R	Angle of attack of rudder in steady turning under turbulent conditions
β	Drift angle (angle between $\bar{U}$ and positive x-axis)
Γ	Mean distribution of vortex strength over hull and sailplane

$\Delta \alpha_R$	Change in angle of attack due to Y_R and/or M_S
δ_r	Rudder deflection from ship's median plane
η	Equals $\pi \bar{c}/L$, where $\bar{c}$ is the rudder chord and L is scale of turbulence
θ	Local angle of attack of rudder
ν	Kinematic viscosity
ξ	Distance from rudder stock to geometric center of sailplane
ρ	Fluid density
σ	Intensity of shear load
ϕ	Torsional angle of rotation about rudder stock axis
ω	Angular frequency
0	Subscript denoting steady (constant) values for particular steady turning maneuver
$-$	Bar over symbol denotes either vector quantity or mean value as designated in text

ABSTRACT

A rapid approximate procedure is given for predicting the static and dynamic loads on a rudder of a surface ship or submarine in a steady horizontal turn as a function of the rudder angle of attack.

INTRODUCTION

Sea trials of USS FORREST SHERMAN (DD 931) disclosed that severe vibrations were transmitted to the hull by the rudders during a steady horizontal turning maneuver.[1] Thus it appears that, as ship speeds increase, control-surface flutter may occur during a turn, within the operating speed range. This flutter is due to hydroelastic interactions which tend to feed hydrodynamic energy into the rudder system. This energy feedback is related to both the flow pattern at the rudder and the corresponding forces and moments on the rudder. Consequently, accurate *control-surface flutter prediction* and also ***rudder design for strength*** **require** that analytical methods be devised for determining the theoretical forces and moments on a ship's rudder during a turn.

The objective of this report is to represent the force and moment on the rudder of a submarine with sail during a steady turning maneuver in a horizontal plane;* the results will also be applicable to surface ships. Steady and unsteady lift and moment on the rudder are treated separately. The force and moment experienced by the rudder are dependent upon its angle of attack** and speed. For the *static* loads, the angle of attack is shown to be the algebraic sum of the drift angle, rudder angle, and angles related to the speed of steady turning, rudder-hull system flexibility, and the flow velocities in the neighborhood of the rudder caused by sailplane wake. For the *dynamic* or *unsteady* loads, the angle of attack is considered for a rudder in both a turbulent wake and an intermittently turbulent wake. The contribution of these loads to flutter can be mathematically formulated but this formulation will not be treated here.

The present analysis can be extended to include the effects of additional phenomena which have been omitted here; e.g., flow separation, ventilation along the rudder stock, cavitation, reduction of relative flow for a rudder lying in a sailplane wake, mutual hydrodynamic effects between skegs and rudders, rate of application of rudder angle[4,5] and propeller race,[5,6] angular changes about the steady angle of attack due to coupled flexural and torsional modal oscillations of a rudder-hull system,[2] and certain appendage interferences (e.g., diving planes).

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[1]References are listed on page 13.

*Force and moment equations used by the author in the flutter analysis for a *ship on course* are given in Reference 2.

**The angle of attack is defined as the angle between the direction of flow and rudder chord line. A discussion of the effective angle of attack during a turn is given in Appendix A of Reference 3.

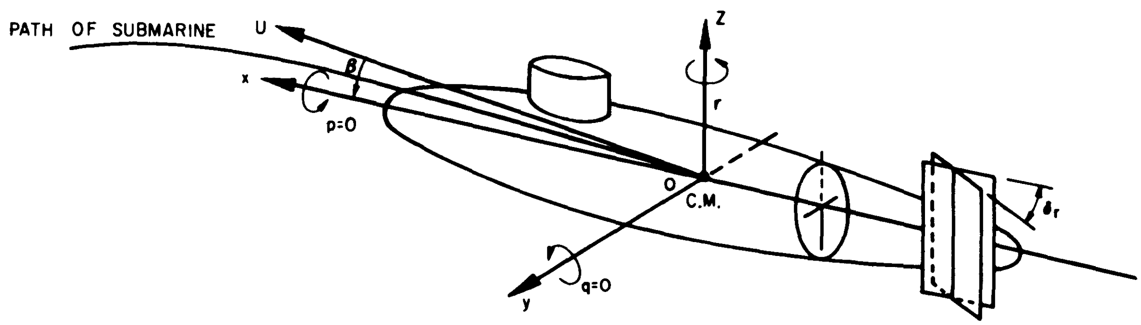

Figure 1a – Coordinate System

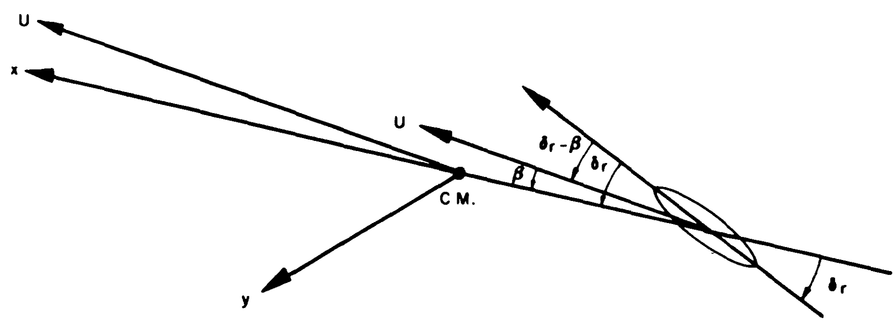

Figure 1b – Motion and Angle Relations

Figure 1 – Motion of Submarine in Turning

GENERAL APPROACH

Figure 1 illustrates the location of a moving reference frame $0xyz$ and the positive directions of linear and angular velocities. This frame is fixed to the moving submarine with origin at the submarine center of mass. The absolute velocity of the center of mass is denoted by the vector $\bar{U}$; the magnitude of $\bar{U}$ is represented by U.*

Steady turning in a horizontal plane[7] as defined here to mean $p = q = 0$ and therefore z is vertical, x and y along and perpendicular, respectively, to the ship's axis for all values of time, while $\bar{r}$, a constant vector perpendicular to the xy-plane, indicates angular velocity in turning. The angle between $\bar{U}$ and the positive x-axis which lies in a horizontal plane is called the drift angle β. The rudder deflection from the ship's median plane is denoted by δ_r. Figure 1 shows these quantities. The symbols U, r, etc., denote magnitudes only; the arrows indicate directions.

It is shown in Reference 8 that the following set of variables (U, β, δ_r, r), which are *mutually dependent*, completely define a steady turning maneuver. For example, if δ_r is given, then U, β, and r can be evaluated for a particular type of hull.

*A symbol with a bar over it denotes a vector quantity or a mean value as designated in the text.

Since rudder forces and moments depend also on rudder position relative to the *local* fluid flow, it is necessary to consider not only rudder orientation δ_r relative to an assumed *rigid* hull, but also changes in rudder orientation which arise from the rudder-hull system *flexibility*. For convenience, the treatment of this flexibility is subdivided under the headings of steady-state distortion and dynamic distortion. Steady-state distortion is associated with static loads. In dynamic distortion, the effects of flexibility are assumed to be time-dependent as in the cases of rudder buffeting,* rudder flutter, and whipping and torsional oscillation of the hull.

Evaluation of both the local angle of attack and the fluid speed relative to the rudder becomes even more complicated if we consider also: (1) proximity of rudder to propeller, (2) position of rudder in a possible separated flow region, or (3) a rudder in a region of turbulent flow, etc. A precise analytical evaluation of all these possibilities appears to be a difficult and perhaps impossible task. Therefore, we must consider simplifying assumptions, the use of experimental data, the evaluation of simpler cases, and, in some instances, statistical and stochastic concepts.** Since there is evidence of intermittent and nonharmonic hydrodynamic loads, we feel that improved engineering solutions can be achieved if statistical and stochastic concepts are employed in those cases where a rudder lies in a wake of turbulent flow or in a region of intermittent flow separation. In these cases, a buffeting phenomenon is more likely than a flutter phenomenon because a typical spectrum of rudder reduced-frequency responses suggests a positive hydrodynamic damping moment. By buffeting we mean a response problem, whereas we define flutter as a self-excited vibration.† On the other hand, interrelated buffeting–flutter phenomena can also occur.

Before we proceed with the evaluation of forces and moments, it will be useful to present a general approach to show how subsequent calculations fit into an overall pattern. Since we are interested in forces and moments experienced by rudders in a steady turn, we expect that forces and moments will depend upon the speed, angle of attack, and angular velocity of the rudder as it moves through a fluid medium. Hence, we put force $F = F(U, \theta, r)$ and moment $M = M(U, \theta, r)$, where θ is the local angle of attack of rudder and r is its angular speed.†† Suppose, also, that we are interested in this force and moment when a ship is in a steady turn, defined by the set $(U_0, \beta_0, \delta_{r0}, r_0)$ of mutually dependent variables. Here the subscript zero indicates steady or constant values for a particular steady turning maneuver. Clearly, the local angle of attack of rudder θ_0 is, to the first order, a linear combination of drift angle β_0, rudder deflection δ_{r0}, and induced angles of attack due to rigid-body angular

*Rudder buffeting is defined here as the transient or irregular vibrations of the rudder due to hydrodynamic impulses produced by the wake of the hull and its components, i.e., produced by turbulence.

**That is, the functions we are concerned with are functions of time and depend upon chance.

†Actually, hydrodynamic forces generate flutter.

††At this point, r is considered to be the angular speed of an *inflexible* rudder-hull system.

speed of the hull r_0 and other hydrodynamic effects from the hull, propeller, sailplane, and other appendage interference.

To continue with our general approach, we suppose that small changes or perturbations occur about the steady values U_0, θ_0, r_0. To mention only two possibilities, these perturbations may be induced either by rudder-hull system flexibility or, in many instances, by the energy arising from fluctuations in wakes of sailplane, propeller, and other appendages. Then we may write $F = F(U_0, \theta_0 + \Delta\theta, r_0 + \Delta r)$ and $M = M(U_0, \theta_0 + \Delta\theta, r_0 + \Delta r)$. A Taylor expansion about a steady state (U_0, θ_0, r_0) gives

$$F(U_0, \theta_0 + \Delta\theta, r_0 + \Delta r) = F(U_0, \theta_0, r_0) + a_1\Delta\theta + a_2\Delta r + \ldots$$

where $a_1 = \partial F(U_0, \theta_0, r_0)/\partial\theta$, $a_2 = \partial F(U_0, \theta_0, r_0)/\partial r$, etc. The coefficients $a_1, a_2, \ldots$ are constants evaluated at the steady-state condition. The force $F(U_0, \theta_0, r_0)$ and, similarly, the moment $M(U_0, \theta_0, r_0)$ represent the steady-state rudder forces and moments (or static load and static moment) which are capable of producing a *steady-state rudder-hull system distortion.* The terms $a_1\Delta\theta$, $a_2\Delta r$, and other similar terms, represent unsteady forces and moments, and they contribute to the *dynamic distortion* of the hull and appendages. As mentioned earlier, wake fluctuations distributed over a wide frequency range cause changes in flow near the rudder. Because this change is random, it is best specified by statistical parameters, such as the mean value and power spectrum. The response of a structure to buffeting is then a stochastic process.

The following analysis is devoted to the discussion of:

1. Steady-state lift and moment; i.e., terms of the *form* $F(U_0, \theta_0, r_0)$ and $M(U_0, \theta_0, r_0)$.
2. Unsteady lift and moment; i.e., terms of the *form* $a_1\Delta\theta$, $a_2\Delta r$, etc.

STEADY-STATE LIFT AND MOMENT ON THE RUDDER

In this section a rapid approximate procedure is outlined for predicting static and potential theory forces on rudders attached to submarine-shaped hull forms.

Here we are concerned with $F = F(U_0, \alpha, r_0)$ and $M = M(U_0, \alpha, r_0)$, where α is the local angle of attack of the rudder relative to the flow in the neighborhood of the rudder,* r_0 is the magnitude of the angular velocity of the hull about the z-axis through its center of mass, and U_0 is the magnitude of the velocity of the center of mass.

As a first approximation, we *neglect* the "effects" of the presence of the hull with its propeller, sailplane, etc. Figure 2 illustrates this case. The sign convention for β_0, δ_{r0}, and r_0 illustrated in Figure 1 is adopted in Figure 2a. In this figure the rudder has components of speed U_0 and $r_0 l/U_0$. The resultant velocity of rudder through the fluid is the vector $\overline{0C}$ approximately equal in magnitude to U_0. The distance l is measured from the center of mass of the hull to the rudder stock.

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*The more familiar symbol α for the angle of attack replaces θ_0.

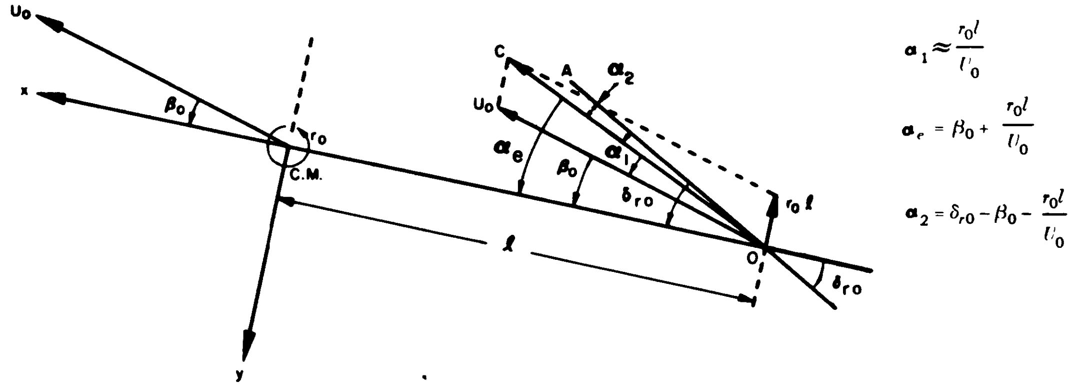

Figure 2a – Vector Relations for Determining Angle of Attack

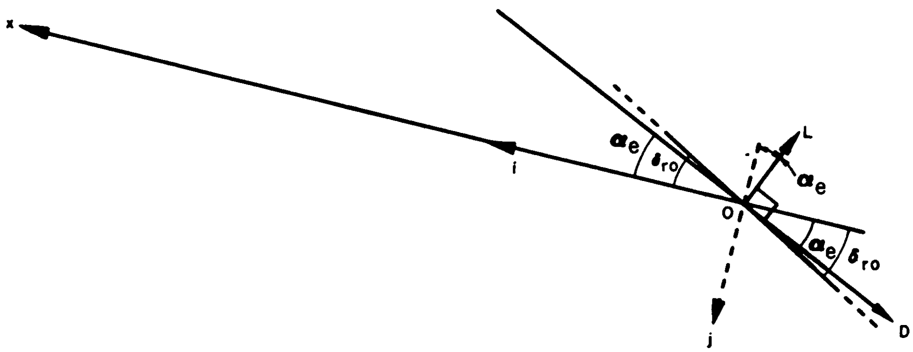

Figure 2b – Vector Relations for Determining Lift and Drag Forces

Figure 2 – Steady-State Force-Angle of Attack Relationship for Rudders

"Effects" of the presence of the hull with its propeller and sailplane, etc., are neglected here.

Angle

$$\alpha_1 \approx \tan^{-1} r_0 l/U_0 \approx r_0 l/U_0$$

and, since

$$\alpha_e = \beta_0 + \alpha_1 = \delta_{r0} - \alpha_2$$

then

$$\alpha_2 = \delta_{r0} - \alpha_e = \delta_{r0} - \beta_0 - \alpha_1 \qquad [1]$$

or

$$\alpha_2 = \delta_{r0} - \beta_0 - (r_0 l/U_0) \qquad [2]$$

Thus α_2 (angle AOC) is the angle of attack if we assume the *absence* of "effects" of the presence of the hull with its appendages and propeller, etc. It is easily seen from Figure 2b that the components of lift and drag forces in the direction of body axes x and y are

$$\bar{i}X_R = (-L \sin \alpha_e - D \cos \alpha_e)\bar{i} \qquad [3]$$

$$\bar{j}Y_R = (-L \cos \alpha_e + D \sin \alpha_e)\bar{j} \qquad [4]$$

L and D are the lift and drag components, respectively, of the resultant hydrodynamic force exerted by the fluid on the rudder. They are perpendicular and parallel, respectively, to the approaching *relative* stream. The lift, drag, and torque about the rudder stock (or shaft) are often expressed in terms of nondimensional force and moment coefficients $C_L(\alpha_2, R)$, $C_D(\alpha_2, R)$, $C_M(\alpha_2, R)$ as follows:[9,10]

$$L = C_L(\alpha_2, R)\rho\, SU_0^2/2;$$

$$D = C_D(\alpha_2, R)\rho\, SU_0^2/2; \text{ and}$$

$$M_S = M_{\text{shaft}} = C_{M_{\text{shaft}}}(\alpha_2, R)\rho\, SU_0^2\bar{c}/2$$

Here $\bar{c}$ is the rudder mean geometric chord, S is the rudder planform area and α_2 and R are, respectively, the angle of attack and Reynolds number based on chord $\bar{c}$ of the rudder; i.e., $R = U_0\bar{c}/\nu$, where ν is the kinematic viscosity. The torque (about the rudder stock) exerted by the fluid on the rudder is

$$M_S = C_{M_{\text{shaft}}}\,\rho\, SU_0^2\bar{c}/2 \qquad [5]$$

where $C_{M_{\text{shaft}}}$ is related to $C_{M(\bar{c}/4)}$, the coefficient about the quarter-chord point of the mean geometric chord for this explicit relationship. For additional pertinent details and information relative to the design of rudders with small aspect ratios, consult References 9 and 10.

Next we consider the influence of rudder-hull flexibility on the angle of attack. Assume that a horizontal force Y_R positive in the direction of negative y (normal to the body x-axis) bends the rudder-hull system so that the *rudder rotates* in the xy-plane through an angle $k_y Y_R$ causing a decrease $\Delta\alpha_R = -k_y Y_R$ in the *angle of attack*; see Figure 3a. Similarly, the positive moment M_S on the rudder bends the rudder-hull system so as to rotate the rudder in the xy-plane through an angle $k_M M_S$ (Figure 3b) and so *increase* the angle of attack by $\Delta\alpha R = +k_M M_S$. Both k_y and k_M are constants dependent upon the geometry of the rudder-hull system and its moduli of elasticity; both can be computed *analytically* or *experimentally*. The net angle of attack of the rudder will be

$$\alpha = \alpha_2 - k_y Y_R + k_M M_S \qquad [6]$$

where α_2 is given by Equation [2], Y_R by Equation [4], and M_S by Equation [5].

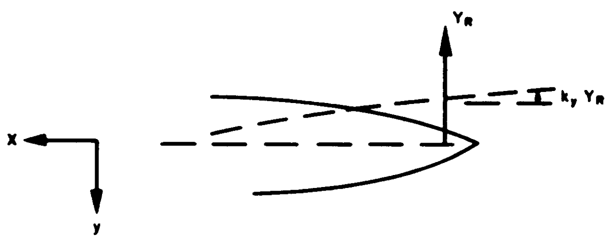

Figure 3a – Rudder Rotation Associated with Force Y_r

Figure 3b – Rudder Rotation Associated with Moment M_S

Figure 3 – Rudder Rotation Associated with Rudder-Hull System Flexibility

We now treat the analytic determination of k_y and k_M. Both of these quantities produce a $\Delta\alpha_R$ by virtue of the twisting of the stock. The effect of rudder-stock flexibility when the top of the stock is fixed is given by equations in Reference 2; the small effect of general ship flexibility can be found from equations describing main hull vibrations. More sophisticated analysis would permit inclusion of the effects of *local* flexibility in the rudder-stock parameters. Considering rudder-stock flexibility only, let Y_R act on the rudder at a distance c ahead of or behind the stock, thereby exerting a torque $T = \pm\, cY_R$ on the rudder end of the stock; Y_R could be caused by hydrodynamic forces on the rudder. Then from Equation [2] of Reference 2, the end of the stock and the rudder will rotate through an angle

$$\Delta\alpha_R = \pm\,\phi = \pm\,\frac{l_T T}{GJ_e} = \pm\,\frac{l_T c Y_R}{GJ_e}$$

so that

$$k_y = -\frac{\Delta\alpha_R}{Y_R} = \pm\,\frac{c l_T}{GJ_e}$$

where GJ_e is the torsional stiffness of the rudder stock and l_T is the effective length of the rudder stock for computing torsional flexibility. The sign depends upon whether c is ahead of or behind the stock. Similarly, M_S produces

$$\Delta\alpha_R = \pm\,\phi = \pm\,\frac{l_T M_S}{GJ_e}$$

so that

$$k_M = \frac{\Delta\alpha_R}{M_S} = \pm\,\frac{l_T}{GJ_e}$$

So far we have considered the effects of drift angle, rudder angle, angular velocity of steady-turning, and steady-state rudder-hull system distortions associated with static loads. Other effects on the angle of attack should be considered also; for example, interference between hull and rudder.* The lateral deflection of flow at the rudder caused by the sailplane is the most significant of these interference effects and the one usually predictable by theory. Obstruction of part of the rudder by the hull body is a second effect; the reduction of relative flow when the rudder lies in the wake of the sailplane is a third effect. The second effect is discussed in another part of this report where dynamic effects are considered; the third, for most types of maneuvers, is small compared to other effects and is usually neglected. The mutual hydrodynamic interference between skegs and rudders is mentioned and dismissed because of the difficulty in predicting it.

In the previously mentioned interference effect of flow between sailplane and hull, we assume a steady turn in a horizontal plane and note that both the hull and sailplane exert a lateral pressure on the surrounding fluid. This lateral pressure produces an incremental pressure on the fluid which deflects the flow past the rudder. In turn, this deflection produces a small change in the angle of attack of the rudder. The distribution of vortex strength over both the hull and sailplane is approximately elliptical,[11] as shown in Figure 4. For the purpose of rapid estimation, we assume that the elliptical distribution is replaced by an average distribution AB and denote this mean distribution by Γ. Since it is well known that the trailing sheet of free vortices is unstable and cannot persist, the sheet then rolls up into two cylindrical vortices within a chord length or less; see Reference 12. The cylindrical vortices are denoted by lines BC and AD which extend to infinity downstream.

Since distance ξ (the location of rudder relative to the geometric center of the sailplane) is large compared to distance b, it is shown in Reference 13 that the induced velocity produced by the above-mentioned horseshoe vortex, in the direction of negative y as drawn here, is $v = \Gamma/\pi b$. This velocity will increase the angle of attack on the rudder by $\Delta\alpha_R = (v/U_0) = (\Gamma/\pi b U_0)$.**

The mean distribution Γ may be estimated as follows: The lift on hull and sailplane is $L = 2\rho b U_0 \Gamma$ (Reference 13, Sec 11.201), in a direction opposite to the drift. The angle of attack on hull and sailplane equals the drift angle β; let it be expressed in degrees. Then $L = (C_{L\alpha F})_B \beta$ in terms of the total lift coefficient at the sailplane. The graphs in Figure 5, provided by the Stability and Control Division of the TMB Hydromechanics Laboratory, illustrate the dependency of the total lift coefficient† $(C_{L\alpha F})_B$ on two parameters D/b and $b/\bar{c}$. Here D is the hull diameter at the sailplane location, b is the span of the sailplane, and $\bar{c}$ is its mean chord. Eliminating L and Γ from these three equations gives

*This effect was not included in the results obtained for the coupled rudder-hull system considered heretofore.

**The direction of the arrows for Γ in Figure 4 indicates that the vortex lines cause water to flow in the direction of negative y, and a glance at Figure 2 shows that the angle of attack on the rudder must be increased.

†Γ includes a large contribution from the hull, and, of course, there is a lift on the hull too, but the lift on the hull relative to the lift on the sailplane is small and hence may be neglected. $(C_{L\alpha F})_B$ then refers to the the sailplane alone but in close proximity to the hull.

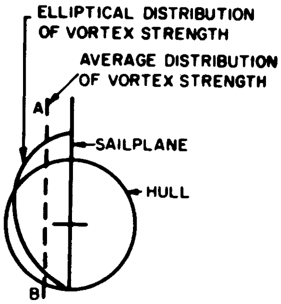

Figure 4a – Hull-Sailplane Vortex Distribution

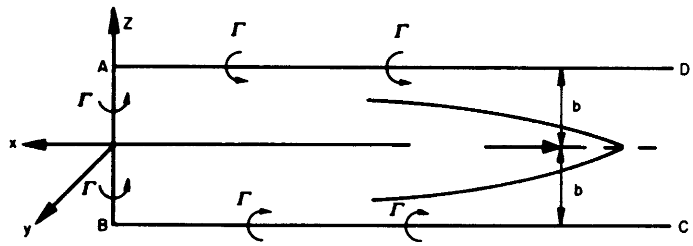

Figure 4b – Cylindrical Vortices along Rudder-Hull System

Figure 4 – Vortex Distribution for Submarine

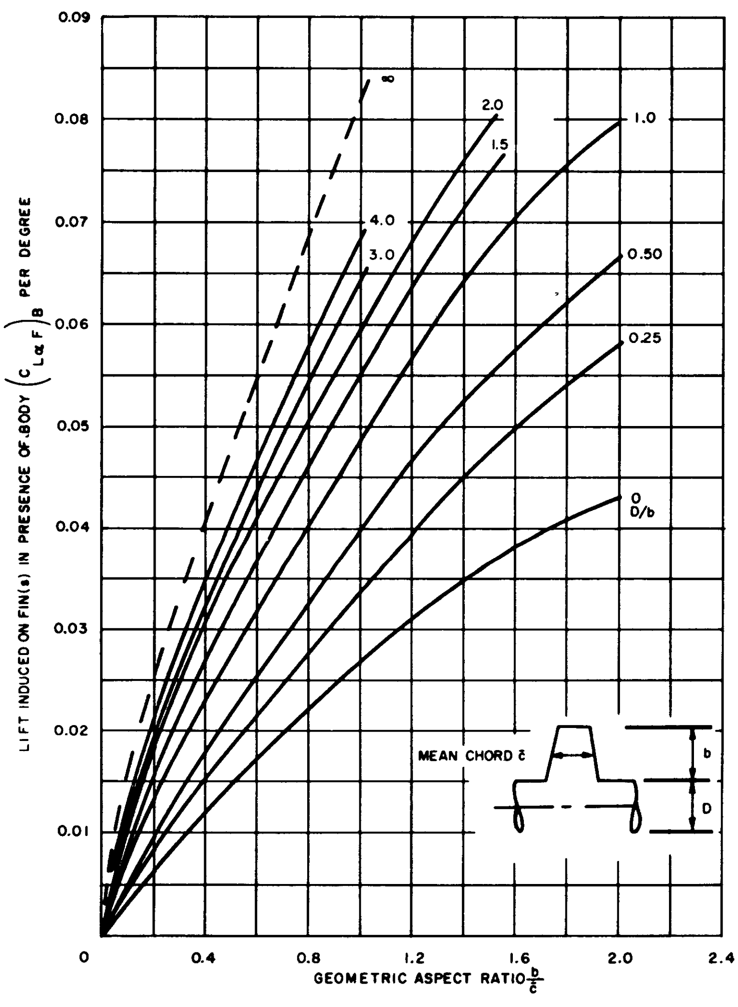

Figure 5 – Lift Induced at Sailplane versus Geometric Aspect Ratio $b/\bar{c}$ and Diameter Span Ratio D/b

$$D\alpha_R = \frac{(C_{L\alpha F})_B \beta}{2\pi\rho b^2 U_0^2}$$

The net angle of attack at the rudder from all effects considered so far is

$$\alpha = \alpha_2 + (k_M M_S - k_y Y_R) + (C_{L\alpha F})_B \frac{\beta}{2\pi\rho b^2 U_0^2} \qquad [7]$$

As already mentioned, Equation [7] is the net angle of attack of the rudder if the vessel is in a steady horizontal turn. The expression for the net angle of attack, as we have noted, includes changes in flow due to the sailplane, steady-state rudder-hull system distortion, angular speed in turning, rudder deflection, and drift angle.

To find the force and moment exerted by the fluid on the *rudder mounted to a hull,* we use again the expression $L = (C_{L\alpha F})\alpha$, where α is given by Equation [7]. To find $C_{L\alpha F}$, we use the graph (Figure 5) where the two parameters D/b and $b/\bar{c}$ correspond to rudder and hull; i.e., D is now the diameter of hull at the location of the rudder, b is now the span of rudder, and $\bar{c}$ is its mean chord.

Since we do not have *apriori* knowledge of Y_R and M_S in Equation [7], we obtain a solution for α by the method of successive approximations. First we assume that both Y_R and M_S are omitted in Equation [7] and obtain a first approximation to α. Then we obtain L, D, and $M_{(\bar{c}/4)}$ (using data from Reference 9) corresponding to the first approximation to α.

From this information we obtain a first approximation for M_S; from Equation [4] we find Y_R. A second approximation for α is obtained by using the derived Y_R and M_S and values of k_y and k_M computed analytically or experimentally, as mentioned previously. With this second approximation for α, we find a second approximation to L, D, $M_{(\bar{c}/4)}$, M_S, Y_R, and also X_R (Equation [3]). This process may be repeated again if more accurate values are desired.

DYNAMICAL CONSIDERATIONS

The discussion and analysis in this section are directed toward the consideration of dynamic forces.

It is known that the boundary layer is turbulent on the afterbody. Although the flow in this region is modified by the presence of the propeller, experimental evidence shows that the change in thickness and change in velocity in the boundary layer due to the presence of the rudder is negligible.[14] Since the rudder is located on the afterbody, the flow past the rudder is also turbulent.

Consider now a flat thin rudder of chord $\bar{c}$ moving with a speed U_0 through a turbulent region. Let x lie along the chord, z along the span, and y along the normal to the chord and span of the rudder. Assume that the components of fluctuating turbulent flow u, v, w, along

x, y, z, respectively, are small compared to U_0. Because of these turbulent fluctuations, a time-dependent apparent angle of attack α exists, and thus a fluctuating side-force in the y-direction is produced. The fluctuating angle of attack α is given by[15]

$$\alpha(t) = v(t)/U_0$$

Here $\alpha(t)$ plays the role of a forcing function and the response is the fluctuating side-force on the rudder or, in terms of a nondimensional coefficient, the lift can be expressed as $C_L(t)$. If it is assumed that $v(t)$ is sinusoidal with angular frequency ω and wave length $2\pi U_0/\omega$, then the mean square lift coefficient is (see Reference 15)

$$\overline{C_L^2} = 4\pi^2 \frac{\overline{V^2}}{U_0^2}\left[\frac{4\eta - \pi}{2\pi(\eta^2 + 1)} + \frac{\eta^2 + 3}{2\pi(\eta^2 + 1)}\left(\eta \log \eta^2 + \pi\right)\right]$$

with $\eta = \pi\bar{c}/L$, where L denotes the "scale of turbulence."* It is also shown in Reference 15 that if $\bar{c}/L \to 0$ (a rudder of small chord in large-scale turbulence), then

$$\overline{C_L^2} \to 4\pi^2 \frac{\overline{V^2}}{U_0^2} = 4\pi^2\overline{\alpha^2}$$

that is, the rudder behaves as a quasi-stationary rudder. If, on the other hand, $\bar{c}/L$ becomes large (a rudder of long chord in small-scale turbulence), it follows from the previous equation that $\overline{C_L^2} \to 0$. Now $\bar{\alpha}$ is the mean angle of attack for turbulent flow, and, for turbulent flow, $v(t)$ will be negative as often as it is positive so that $\bar{v}(t) = 0$ and $\bar{\alpha} = 0$. Hence, in steady turning when the angle of attack of the rudder is α_R, the largest mean coefficient of rudder side force is expected to be $\overline{C_L} = 2\pi\alpha_0$, since $\alpha_R = \alpha_0 + \alpha$, and $\bar{\alpha}_R = \alpha_0 + \bar{\alpha} = \alpha_0$, where α_0 is the mean angle of attack of the rudder and α is the fluctuating angle of attack. Thus the lift is proportional to α_R but the mean lift is the same whether or not there is turbulence.

If we are interested in the *maximum* lift, then, strictly speaking, the mathematics furnishes no such maximum. We might, however, use $2\pi\sqrt{\overline{\alpha^2}}$ as a rough measure of the maximum increase due to turbulence. Then $(C_L)_{max} = 2\pi(\alpha_0 + \sqrt{\overline{\alpha^2}})$.

In another interesting phenomenon of possible major importance, let us suppose that all points on the rudder surface are sometimes within the wake of a turbulent region and sometimes outside the wake. If the probability of switching from one regime to the other is governed by a Poisson distribution, then it can be shown that (see Reference 15)

$$\overline{C_L^2} = 4\pi^2 \frac{\overline{V^2}}{U_0^2}\,\frac{2}{\pi}\,\frac{\eta \log \eta + \frac{\pi}{2}}{1 + \eta^2}$$

*This concept is discussed in the Appendix of Reference 15.

which becomes very large for small-scale turbulence ($\bar{c}/L$ large). This result is of possible interest if in steady turning, flow around a skeg and past a rudder is *intermittent* in time.

An experimental procedure that can throw light on the nature of the fluctuating loads on rudders is discussed briefly.

Several different types of experiments can be performed but only a particular series or class is discussed here. In this series, the rudder is deflected to a predetermined value for different approach speeds. The vessel then goes into a steady turn during which the speed is supposed to remain constant. The experiment is then repeated for a different approach speed and hence a new speed in steady turning is obtained. The experiment is repeated again and again. Thus the *speeds in turning are different*, but the *rudder deflection remains the same* for each experiment. In each of these experiments, the flow past the rudder may be given in terms of a Reynolds number $R_{\bar{c}}$ based on chord $\bar{c}$.

Suppose that a dynamometer is mounted at the root of the rudder stock and suppose further that it is capable of measuring the shear load $y(t)$ at the root as a function of time. Let the record of shear loads measured by the dynamometer be of the form indicated in Figure 6.

The value of the mean lateral force (in nondimensional form C_y) can be computed for each of the trials. The fluctuations about this mean are expressed in terms of $\sigma(t)$, a statistical measure of the buffeting intensity of shear loads

$$\sigma(t) = \left[\frac{1}{2T} \int_{t-T}^{t+T} y^2(t)dt \right]^{1/2}$$

where T is an arbitrary time, say of the order of 10 to 20 sec long.

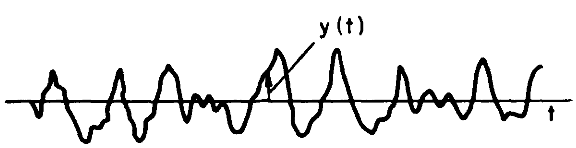

Figure 6 – Time History of Shear Loads Measured at Root of Rudder Stock

Next, suppose we plot C_y versus the Reynolds number $R_{\bar{c}}$. Suppose then that this plot is given by the solid line on which there

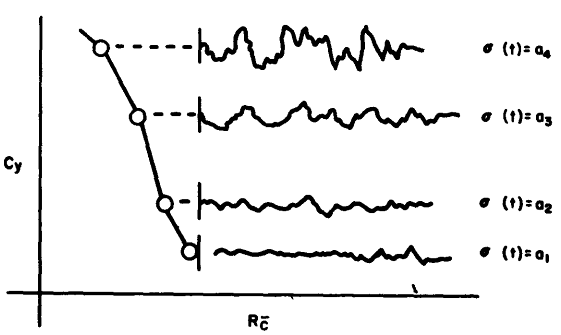

Figure 7 – Mean Lateral Force versus Reynolds Number for Different Intensities of Shear Load

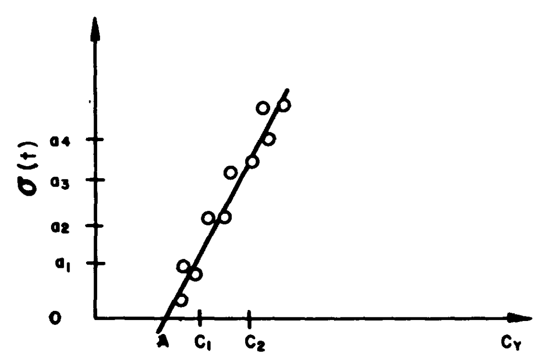

Figure 8 – Intensity of Shear Load versus Mean Lateral Force

are test points denoted by small circles, as in Figure 7. In this figure we also show samples of records of the fluctuating loads. Next, we plot C_y versus $\sigma(t)$, and obtain the graph, Figure 8.

This graph (Figure 8) indicates a strong correlation between C_y and σ. Furthermore, it shows that there exists a particular value of C_y, say A, up to which there is no buffeting, since σ is zero. Hence the value A is a buffet boundary for this value of $R_{\bar{c}}$. If C_y is greater than A, there is buffeting. Studies of this kind suggest that σ, the intensity of shear load, is primarily a function of C_y and $R_{\bar{c}}$. Thus, if C_y is below A, there is no buffeting, and hence the absence of dynamic loads transmitted to hull. For a discussion of other interesting avenues for research, References 16 and 17 are suggested.

SUMMARY

A procedure has been developed for predicting the static and dynamic loads on a rudder of a surface ship or submarine with sail in a steady horizontal turn as a function of the angle of attack.

The *static* force and moment exerted by the fluid are determined from equations derived in this report using an iterative technique presented in detail on page 10. The *dynamic* or *unsteady* loads associated with a rudder in both a turbulent and intermittently turbulent wake can be calculated from equations given on page 11.

Finally, an experimental method is given for determining the conditions under which rudder buffeting and, hence, the corresponding dynamic loads transmitted to the hull, are absent.

RECOMMENDATIONS

It is recommended that experimental tests be devised to provide certain data required by this theory and to compare theory and experiment.

ACKNOWLEDGMENTS

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