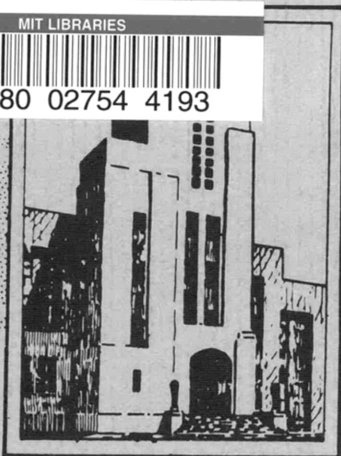

DEPARTMENT OF THE NAVY

DAVID TAYLOR MODEL BASIN

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WAVE RESISTANCE OF A MOVING PRESSURE DISTRIBUTION IN A CANAL

by

J.N. Newman and F.A.P. Poole

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The Wave Resistance of a Moving Pressure Distribution in a Canal

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Expressions are derived for the wave resistance of a pressure distribution which is moving with constant forward speed along the free surface of a canal of constant width and depth. Calculations are made for the case of rectangular and elliptic plan forms with constant pressure, as functions of speed, beam-length ratio, and the width and depth of the tank. Except in the vicinity of the critical Froude number $gh/c^2 = 1$, the tank walls are not generally important for widths greater than one or two lengths of the pressure distribution. The other parameters are more important and their effects on the wave resistance are shown in Figures 1—6.

Introduction

If a pressure distribution is moving along the free surface of a fluid, waves will be generated of nature similar to those generated by a ship. Since work must be done to generate these waves, it follows that the pressure distribution will experience a drag force, or wave resistance. In fact the earliest study of ship waves was made by Kelvin, who considered the waves generated by a moving pressure point. Subsequently Havelock [1] studied the wave resistance of a one-dimensional pressure distribution and [2], [3] of certain two-dimensional distributions with circular symmetry. Recently Wehausen [7] presented expressions for the wave resistance of an arbitrary pressure distribution in water of finite depth. Interest in this problem has been renewed by recent investigations of ground effect machines over water.

We consider here the problem of a pressure distribution which is moving with constant speed down the center of a canal of constant finite width and depth. The later stages of the analysis are restricted to a pressure distribution which is uniform and bounded by either a rectangle or an ellipse. For these particular distributions, calculations of the wave resistance are presented as functions of the Froude number, beam-length ratio, depth, and width of the canal. The analysis is based upon Wehausen's expression for an unbounded free surface, and the method of images is used to represent the canal walls, in the same manner as in similar studies of surface ships [4], [5]. The effect of the walls is introduced not only to determine the experimental error involved in towing tank experiments, but also to overcome the need for numerical methods of integration, for the resulting infinite series may be summed directly, and the case of an infinite free surface may be approached by increasing the canal width. The calculations have been made on an IBM digital computer and are presented here in graphical form. The effect of tank walls is found to be negligable in most cases and for this reason most of the computations involve a very wide, or essentially infinite, canal. We begin the analysis with the necessary modification of Wehausen's results to include the effects of walls, for an arbitrary distribution of pressure. This is followed by separate analyses for the rectangular and elliptic plan forms with constant pressure distribution. The last section presents the graphical results of calculations for these shapes for various beam-length ratios, depths, and widths of the canal.

Wave Resistance of a General Pressure Distribution

Let (x, y, z) be a Cartesian coordinate system with the plane $y = 0$ in the undisturbed free surface and y positive upwards.

If a pressure distribution $p(x,z)$ moves with constant velocity c in the $+x$ direction on the surface of a fluid of constant depth, the work W, done in overcoming the wave resistance is given by Wehausen [7] as

$$W = \frac{c}{\pi \varrho g} \int_{\Theta_0}^{\pi/2} \frac{k_0^3 \cos \Theta}{1 - \nu h \sec^2 \Theta \operatorname{sech}^2 k_0 h} \{[P(\Theta)]^2 + [Q(\Theta)]^2\} \, d\Theta \tag{1}$$

where

$$P(\Theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x,z) \cos [k_0 (x \cos \Theta + z \sin \Theta)] \, dx \, dz$$

$$Q(\Theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x,z) \sin [k_0 (x \cos \Theta + z \sin \Theta)] \, dx \, dz$$

$$\Theta_0 = \begin{cases} \cos^{-1} \sqrt{\nu h} & \text{if } \nu h < 1 \\ 0 & \text{if } \nu h > 1 \end{cases}$$

ϱ = fluid density
g = gravitational acceleration
h = depth of fluid
$\nu = g/c^2$

and $k_0 = k_0(\Theta)$ is the positive real root of
$k_0 - \nu \sec^2 \Theta \tanh k_0 h = 0$, $\Theta_0 < \Theta < \pi/2$.

First we shall change the variable of integration to $k = k_0 h$. It follows that

$$\cos \Theta = \sqrt{\nu h \frac{\tanh k}{k}}$$

$$\sin \Theta = \sqrt{1 - \nu h \frac{\tanh k}{k}}$$

$$\frac{d\Theta}{dk} = \frac{\sqrt{\nu h}}{2k \cosh^2 k} \, \frac{\cosh k \sinh k - k}{\sqrt{\tanh k} \, (k - \nu h \tanh k)}$$

and thus that

$$W = \frac{\nu c}{2 \pi \varrho g h^2} \int_{K_0}^{\infty} \frac{k^2 \tanh k}{\sqrt{k^2 - \nu h k \tanh k}} \{[P(k)]^2 + [Q(k)]^2\} \, dk \tag{2}$$

where

$$P(k) + i\,Q(k) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,z)\exp\left(\frac{i\,x}{h}\sqrt{vh\,k\tanh k} + \right.$$

$$\left. + \frac{i\,z}{h}\sqrt{k^2 - vh\,k\tanh k}\right) dx\,dz,$$

and K_0 is the real positive root of the equation $K_0 - vh\tanh K_0 = 0$ if $vh > 1$ while $K_0 = 0$ if $vh \leq 1$.

From energy considerations the wave resistance R is equal to W/c.

The above equations hold for an unbounded free surface, but they are readily adapted to the case of a canal of constant width, by using the "method of images". To simplify the algebra we shall assume that the original pressure distribution is symmetric with respect to the center of the canal, which we take to coincide with the x-axis. If the walls are a distance w apart, then the boundary condition on these walls may be satisfied by locating image pressure distributions along the z-axis, centered at the points $z = \pm\,nw$ $(n = 1, 2, 3, \ldots)$. Thus the total pressure distribution will be periodic, $p(x, z + wn) = p_0(x,z)$, say, where $|z| < w/2$ and $p_0(x,z)$ is the original pressure distribution located within the canal. It is necessary to start with a finite number of images, say 2N, so that $(n = 1, 2, \ldots N)$, to ensure convergence of the integrals. The functions P + iQ may then be written

$$P + iQ = \sum_{n=-N}^{N} (P_0 + iQ_0)\, e^{in\,w/h\,\sqrt{k^2 - vh\,k\tanh k}}$$

where

$$P_0 + iQ_0 = \iint_{S_0} p_0(x,z)\exp\left\{i\frac{x}{h}\sqrt{vh\,h\,k\tanh k} + \right.$$

$$\left. + i\frac{z}{h}\sqrt{k^2 - vh\,k\tanh k}\right\} dx\,dz \qquad (4)$$

and S_0 is the surface over which the pressure p_0 acts.

Since $P_0 + iQ_0$ does not depend on n, the sum in (3) may be evaluated from the formula

$$\sum_{n=-N}^{N} e^{inz} = \frac{\sin\left[\frac{1}{2}z(2N+1)\right]}{\sin\frac{1}{2}z}$$

Thus

$$P + iQ = \frac{\sin\left[(2N+1)\dfrac{w}{2h}\sqrt{k^2 - vh\,k\tanh k}\right]}{\sin\left[\dfrac{w}{2h}\sqrt{k^2 - vh\,k\tanh k}\right]}(P_0 + iQ_0)$$

and the total work done to overcome the wave resistance of all 2N + 1 pressure distribution is then

$$W = \frac{vc}{2\pi\varrho g h^2}\int_{K_0}^{\infty}\frac{k^2\tanh k}{\sqrt{k^2 - vh\,k\tanh k}}\;\frac{\sin^2\left[\dfrac{2N+1}{2h}\sqrt{k^2 - vh\,k\tanh k}\right]}{\sin^2\left[\dfrac{w}{2h}\sqrt{k^2 - vh\,k\tanh k}\right]}\cdot$$

$$\cdot\left\{[P_0(k)]^2 + [Q_0(k)]^2\right\} dk\,. \qquad (5)$$

The wave resistance of a single pressure distribution (say the central pressure, n = 0) in a canal is given by the average of the resistance of all the images in the limit $N \to \infty$, or

$$R = \lim_{N\to\infty}\frac{W}{(2N+1)c}$$

assuming that this limit exists.

Thus

$$R = \frac{v}{2\pi\varrho g h^2}\lim_{N\to\infty}\int_{K_0}^{\infty}\frac{k^2\tanh k}{\sqrt{k^2 - vh\,k\tanh k}}\;\frac{\sin^2\left[\dfrac{2N+1}{2h}w\sqrt{k^2 - vh\,k\tanh k}\right]}{(2N+1)\sin^2\left[\dfrac{w}{2h}\sqrt{k^2 - vh\,k\tanh k}\right]}\left\{[P_0(k)]^2 + [Q_0(k)]^2\right\} dk\,. \qquad (6)$$

The function

$$\Theta = \frac{w}{2h}\sqrt{k^2 - vh\,k\tanh k}$$

is a monotonic increasing function of k, which varies from 0 to ∞ as k goes from K_0 to ∞. Thus we may express (6) in the form

$$R = \lim_{N\to\infty}\frac{1}{2N+1}\int_{K_0}^{\infty} F(k)\frac{\sin^2[(2N+1)\Theta]}{\sin^2\Theta}\,dk$$

$$= \lim_{N\to\infty}\frac{1}{2N+1}\int_{0}^{\infty} f(\Theta)\frac{\sin^2[(2N+1)\Theta]}{\sin^2\Theta}\,d\Theta$$

where

$$f(\Theta) = \frac{F(k)}{d\Theta/dk} = \frac{vw}{4\pi\varrho g h^3}\;\frac{k^2\tanh k}{\Theta\,(d\Theta/dk)}\,(P_0^2 + Q_0^2)\,.$$

Subdividing the interval of integration, it follows that

$$R = \lim_{N\to\infty}\frac{1}{2N+1}\left\{\int_0^{\pi/2} f(\Theta)\frac{\sin^2(2N+1)\Theta}{\sin^2\Theta}\,d\Theta + \sum_{n=1}^{\infty}\int_{(2n-1)\pi/2}^{(2n+1)\pi/2} f(\Theta)\frac{\sin^2(2N+1)\Theta}{\sin^2\Theta}\,d\Theta\right\}$$

$$= \lim_{N\to\infty}\frac{1}{2N+1}\left\{\int_0^{\pi/2} f(\Theta)\frac{\sin^2(2N+1)\Theta}{\sin^2\Theta}\,d\Theta + \sum_{n=1}^{\infty}\int_{-\pi/2}^{\pi/2} f(\Theta + n\pi)\frac{\sin^2(2N+1)\Theta}{\sin^2\Theta}\,d\Theta\right\}$$

In this form the limit of the integrals as $N \to \infty$ may be obtained directly from a known result in the theory of Fourier series [8]. Thus

$$\lim_{N\to\infty}\frac{1}{2N+1}\int_0^{\pi/2} f(\Theta)\frac{\sin^2(2N+1)\Theta}{\sin^2\Theta}\,d\Theta = \frac{\pi}{2}f(0)$$

and

$$\lim_{N\to\infty}\frac{1}{2N+1}\int_{-\pi/2}^{\pi/2} f(\Theta + n\pi)\frac{\sin^2(2N+1)\Theta}{\sin^2\Theta}\,d\Theta = \pi f(n\pi)$$

and therefore

$$R = \frac{\pi}{2}\sum_{m=0}^{\infty}\varepsilon_m f(m\pi) \qquad (7)$$

where $\varepsilon_0 = 1$ and $\varepsilon_m = 2$ if $m \geq 1$, or

$$R = \frac{v}{\varrho g w h}\sum_{m=0}^{\infty}\varepsilon_m\frac{k_m^2\tanh k_m\left\{[P_0(k_m)]^2 + [Q_0(k_m)]^2\right\}}{|2k_m - vh\tanh k_m - vh\,k_m\,\mathrm{sech}^2 k_m|} \qquad (8)$$

where k_m is the positive real root of the equation

$$k_m^2 - vh\,k_m\tanh k_m = \frac{4\pi^2 m^2 h^2}{w^2}\,. \qquad (9)$$

For $vh < 1$, $k_0 = 0$ and the term with m = 0 in equation (8) vanishes. However as vh tends to one from above, that is

$$vh \to 1 + 0\,,$$

the term with m = 0 tends to the finite limit

$$\frac{k_0^2\tanh k_0\,(P_0^2 + Q_0^2)}{|2k_0 - vh\tanh k_0 - vh\,k_0\,\mathrm{sech}^2 k_0|} \to \frac{3}{2}\left[\iint_{S_0} p_0(x,z)\,dx\,dz\right]^2$$

Thus as $v h$ passes through one, there is a jump, ΔR, in the resistance, of magnitude

$$\Delta R = \frac{\frac{3}{2}\left[\iint_{S_0} p_0(x,z)\,dx\,dz\right]^2}{\varrho g w h^2} \tag{10}$$

It follows that this discontinuity will give rise to a sudden decrease in resistance for increasing speed, when passing through the critical Froude number $gh/c^2 = 1$. However this discontinuity vanishes as $w \to \infty$, and is thus a consequence of the canal walls. Except for this singular point, the denominator of each term in (8) is greater than zero. Therefore the terms in (8) are all bounded, and furthermore we may remove the absolute value sign from the denominator.

Thus we have obtained the wave resistance of a pressure distribution in a canal as an infinite series. For infinite depth, $h \to \infty$, the limit of (8) is readily found to be

R

$$R = \frac{v^2}{4\varrho g w}\sum_{m=0}^{\infty} \varepsilon_m \frac{\left[1+\sqrt{1+\left(\frac{4\pi m}{vw}\right)^2}\right]}{\sqrt{1+\left(\frac{4\pi m}{vw}\right)^2}}\left(P_{0\infty}^2 + Q_{0\infty}^2\right) \tag{11}$$

where

$$P_{0\infty} + iQ_{0\infty} =$$

$$\iint_{S_0} p_0(x,z)\exp\left(\tfrac{1}{2} i v x\sqrt{1+\sqrt{1+\left(\frac{4\pi m}{vw}\right)^2}} + 2\pi i y\, m/w\right)dx\,dz$$

The Rectangular Distribution

As a special case we now consider the pressure distribution

$$p_0(x,z) = p_0 \text{ for } |x| < L/2 \text{ and } |z| < B/2$$
$$p_0(x,z) = 0 \text{ for } |x| > L/2 \text{ or } |z| > B/2$$

where L and B are the length and beam respectively, and p_0 is a constant. Substituting in (4) and integrating with respect to x and z, it follows that

$$P_0(k) = \frac{4 p_0 h^2 \sin\left(\frac{L}{2h}\sqrt{vh\,k\tanh k}\right)\sin\left(\frac{B}{2h}\sqrt{k^2 - vh\,k\tanh k}\right)}{\sqrt{vh\,k\tanh k}\,\sqrt{k^2 - vh\,k\tanh k}} \tag{12}$$

and

$$Q_0 = 0.$$

Substituting (12) in (8) and using (9), we find that the wave resistance of a rectangular uniform pressure distribution is

$$R = \frac{4 p_0^2 w}{\pi^2 \varrho g}\sum_{m=0}^{\infty}\frac{\varepsilon_m}{m^2}\,\frac{k_m \sin^2\left(\frac{\pi B_m}{w}\right)\sin^2\frac{L}{2h}\sqrt{vh\,k_m\tanh k_m}}{2k_m - vh\tanh k_m - vh\,k_m\,\mathrm{sech}^2 k_m} \tag{13}$$

An interesting special case is $w = B$, where the pressure distribution extends completely across the canal and the problem is two-dimensional. The only non-zero term in (13) is the term with $m = 0$ and it follows that

$$R = \frac{4 p_0^2 w}{\varrho g}\,\frac{k_0 \sin^2\left(\frac{L}{2h}\sqrt{vh\,k_0\tanh k_0}\right)}{2k_0 - vh\tanh k_0 - vh\,k_0\,\mathrm{sech}^2 k_0}$$

or, since

$$k_0 - vh\tanh k_0 = 0,$$

$$R = \frac{4 p_0^2 w}{\varrho g}\,\frac{\sin^2\left(\frac{k_0 L}{2h}\right)}{1 - \frac{2k_0}{\sinh 2k_0}} \tag{14}$$

This expression is zero for $vh < 1$ and for the infinite set of velocities such that

$$c^2 = \frac{gL\tanh(2\pi n h/L)}{2\pi n} \qquad (n = 1, 2, 3, \ldots)$$

The Elliptic Distribution

As another special case we shall consider a pressure distribution which is constant over the interior of an ellipse and zero elsewhere, or

$$p_0(x,z) = p_0 \text{ for } \frac{4x^2}{L^2} + \frac{4z^2}{B^2} \leqslant 1$$
$$p_0(x,z) = 0 \text{ for } \frac{4x^2}{L^2} + \frac{4z^2}{B^2} > 1.$$

Before substituting in the integral for P_0 und Q_0 we make the following substitutions:

$$\xi = 2x/L$$
$$\eta = 2y/B$$
$$\alpha = \frac{L}{2h}\sqrt{vh\,k\tanh k}$$
$$\beta = \frac{B}{2h}\sqrt{k^2 - vh\,k\tanh k}$$

Then

$$Q_0 = 0$$

and

$$P_0 = \frac{1}{4} p_0 BL \int_{-1}^{1} d\xi \int_{-\sqrt{1-\xi^2}}^{\sqrt{1-\xi^2}} d\eta\, e^{i(\alpha\xi + \beta\eta)}$$
$$= \frac{1}{2}\frac{p_0 BL}{\beta}\int_{-1}^{1}\cos\alpha\xi\,\sin\beta\sqrt{1-\xi^2}\,d\xi$$
$$= \frac{p_0 BL}{\beta}\int_0^{\pi/2}\cos(\alpha\cos\varphi)\sin(\beta\sin\varphi)\sin\varphi\,d\varphi$$

where we have changed the variable of integration to $\xi = \cos\varphi$. Using the relations[1])

$$\cos(\alpha\cos\varphi) = \left(\frac{\pi\alpha\cos\varphi}{2}\right)^{1/2} J_{-1/2}(\alpha\cos\varphi)$$
$$\sin(\beta\sin\varphi) = \left(\frac{\pi\beta\sin\varphi}{2}\right)^{1/2} J_{1/2}(\beta\sin\varphi)$$

and Sonine's second finite integral[2]), it follows that

$$P_0 =$$
$$\frac{\pi}{2\beta} p_0 BL\sqrt{\alpha\beta}\int_0^{\pi/2} J_{-1/2}(\alpha\cos\varphi)\,J_{1/2}(\beta\sin\varphi)\cos^{1/2}\varphi\,\sin^{3/2}\varphi\,d\varphi$$
$$= \frac{\pi}{2} p_0 BL\,\frac{J_1(\sqrt{\alpha^2+\beta^2})}{\sqrt{\alpha^2+\beta^2}}$$

where J_1 is the Bessel function of the first kind or, substituting for α and β,

$$P_0(k) = \frac{\pi}{2} p_0 BL\,\frac{J_1\left(\sqrt{\left(\frac{L^2-B^2}{4h^2}\right) vh\,k\tanh k + \frac{B^2k^2}{4h^2}}\right)}{\sqrt{\left(\frac{L^2-B^2}{4h^2}\right) vh\,k\tanh k + \frac{B^2k^2}{4h^2}}}$$

Substituting in equation (8) and using (9), we find that the wave resistance of an elliptic uniform pressure distribution is

1) *cf. Watson [6] paragraph 3.4.*
2) *ibid, paragraph 12.13.*

$$R = \frac{\pi^2}{4}\,\frac{\nu p_o^2 B^2 L^2}{\varrho g w h} \sum_{m=o}^{\infty} \varepsilon_m \frac{k_m^2 \tanh k_m}{2 k_m - \nu h \tanh k_m - \nu h\, k_m \operatorname{sech}^2 k_m} \cdot \left\{ \frac{J_1\left(\sqrt{\dfrac{L^2 k_m^2}{4h^2} - \left(\dfrac{L^2 - B^2}{w^2}\right)\pi^2 m^2}\right)}{\sqrt{\dfrac{L^2 k_m^2}{4h^2} - \left(\dfrac{L^2 - B^2}{w^2}\right)\pi^2 m^2}} \right\}^2 \tag{15}$$

The Numerical Results

The infinite series given by equations (13) and (15) and their limiting forms for infinite depth have been evaluated using IBM 704 and 7090 type digital computers. The results of these computations are shown in Figures 1—8. Figures 1 and 2 show the normalized wave resistance of a rectangular pressure distribution in a very wide canal ($w/L = 10$) for infinite depth and for a finite depth $h = L/4$. Curves are shown for various beam-length ratios as functions of the reciprocal of the Froude number. Figure 3 shows the results for a single beam-length ratio ($B/L = 0.5$) and various values of the depth of the canal. Figures 4—6 show the corresponding results for the elliptic plan form. Figures 7—8 show the influence of canal width for an elliptic distribution of beam-length ratio $B/L = 0.5$, for infinite depth and for a depth $h = L/4$. One striking feature of these results is the severe "humps and hollows" at low Froude numbers, due to interference effects. This feature is more apparent than in Havelock's [3] computations since his assumed pressure distribution extends to infinity and therefore does not possess the same interference characteristics as a bounded pressure distribution. The "humps and hollows" of the elliptic plan form are less pronounced than those of the rectangle. This is presumably due to the very strong interference caused by the predominantly two-dimensional nature of the rectangle. In fact except for high Froude numbers or small beam-length ratios, the wave resistance of the rectangular distribution differs very little from the two-dimensional (infinite beam-

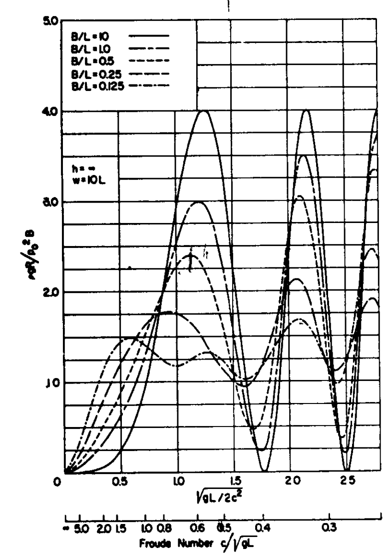

Figure 1 Wave Resistance of the Rectangular Pressure Distribution in a Canal of Infinite Depth and Various Beam-Length Ratios

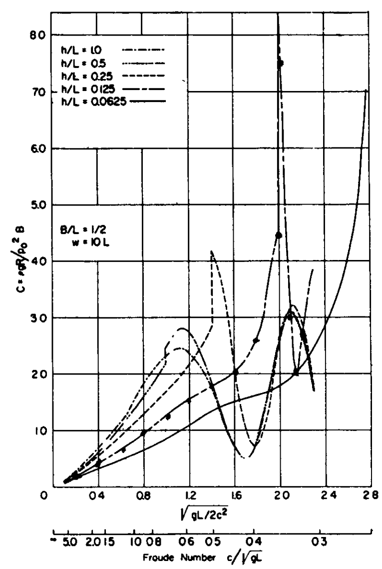

Figure 3 Wave Resistance of the Rectangular Pressure Distribution with Beam-Length Ratio 1/2 for Various Depths

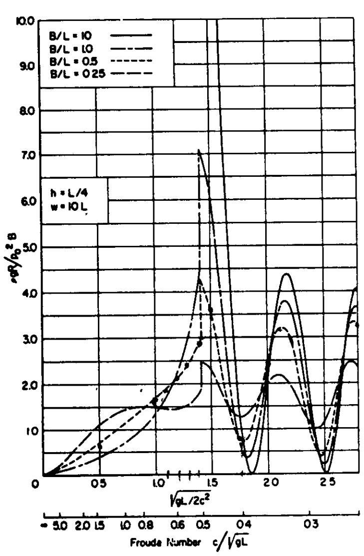

Figure 2 Wave Resistance of the Rectangular Pressure Distribution in a Canal of Depth h = L/4 and Various Beam-Length Ratios. The Curve for B/L = 10 is a Maximum of 24 at the Critical Froude Number 1/2, and is Zero for Higher Froude Numbers

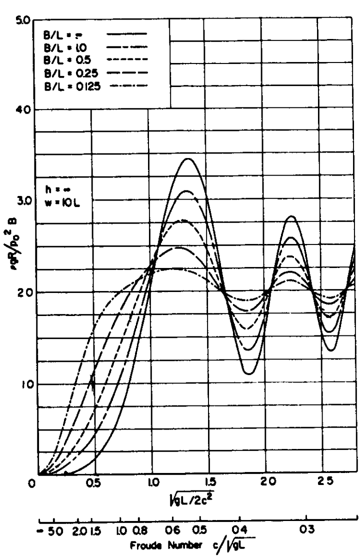

Figure 4 Wave Resistance of the Elliptic Pressure Distribution in a Canal of Infinite Depth and Various Beam-Length Ratios. The Curve for B/L = ∞ is Calculated for an Infinitely Wide Canal

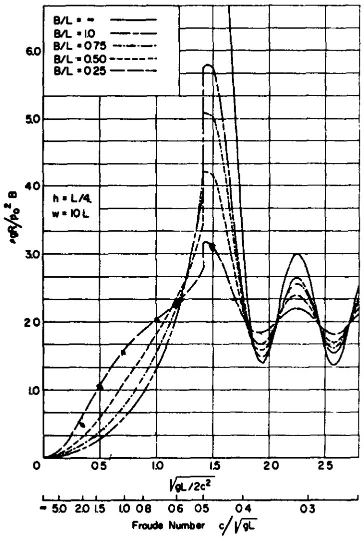

Figure 5 Wave Resistance of the Elliptic Pressure Distribution in a Canal of Depth h = L/4 and Various Beam-Length Ratios. The Curve for B/L = ∞ is Calculated for an Infinitely Wide Canal. This Curve is a Maximum of 16 at the Critical Froude Number ½, and is Zero for Higher Froude Numbers

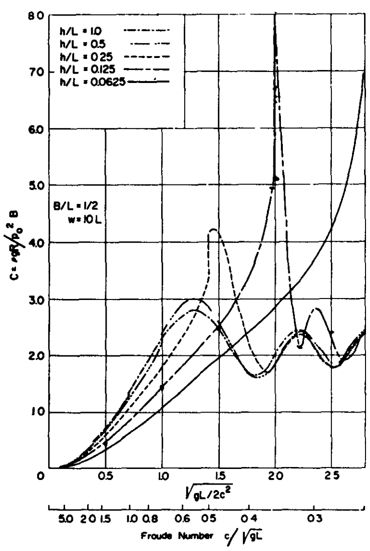

Figure 6 Wave Resistance of the Elliptic Pressure Distribution with Beam-Length Ratio ½ for Various Depths

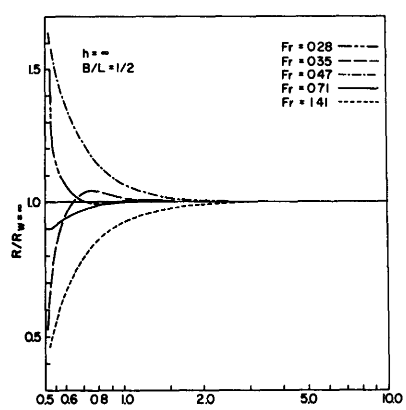

Figure 7 Wave Resistance of the Elliptic Pressure Distribution with Beam-Length Ratio ½ in a Canal of Infinite Depths, as a Function of Canal Width

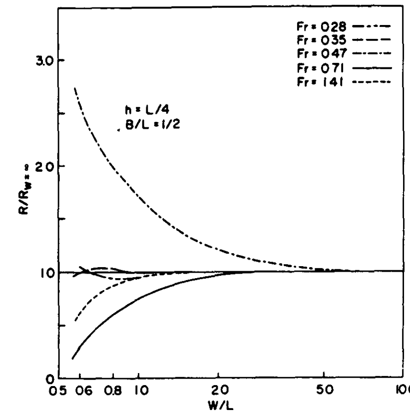

Figure 8 Wave Resistance of the Elliptic Pressure Distribution with Beam-Length Ratio ½ in a Canal of Depth h = L/4, as a Function of Canal Width

length ratio) case. This is also true, to a lesser extent, for the elliptic distribution. At the high Froude numbers, however, the departure from the two-dimensional results is greater, as a consequence of the fact that the waves are longer and thus the beam must become larger to maintain a two-dimensional flow.

These characteristics are also illustrated by the asymptotic approximations which may be obtained from equation (2) in the case of infinite width and depth. These are shown in Table 1.

Table 1

The asymptotic approximations for the wave resistance coefficient $\varrho gR/p_0^2B$ for infinite width and depth, in the limits of high and low Froude number and high and low beam-length ratio. Here γ is Euler's constant, $\gamma = 577\ldots$ and H_1 is the Struve function [6].

	Rectangle	Ellipse
$gL/c^2 \to 0$	$\frac{L}{\pi B}\left(\frac{gL}{c^2}\right)\left[3-\gamma+\log_e\left(\frac{B}{L}\frac{c^2}{gL}\right)\right]$	$\frac{\pi L}{4B}\left(\frac{gL}{c^2}\right)$
$gL/c^2 \to \infty$	2	2
$B/L \to 0$	1	2
$B/L \to \infty$	$4\sin^2\left(\frac{gL}{2c^2}\right)$	$\pi H_1\left(\frac{gL}{c^2}\right)$

The jump in resistance at the point $gh/c^2 = 1$ is apparent in Figures 2, 3, 5, and 6. As shown in equation (10) this jump vanishes if the tank width tends to infinity. In this case equation (2) shows that the resistance is finite and continuous at the point $gh/c^2 = 1$, but there is a discontinuity in the slope.

Figures 7 and 8 show that except for the neighborhood of the point $vh = 1$, the effects of the walls on the resistance of the elliptic distribution are negligible for tank widths greater than two model lengths, and not serious for widths greater than one model length. The corresponding results for the rectangular distribution are not included since these are not significantly different. It would appear therefore that the results shown in figures 1—6 may be used for any reasonable tank width up to infinity, or the case of an unbounded free surface, except for the vicinity of the critical Froude number $gh/c^2 = 1$, where the width of the tank is important.

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Nomenclature

B	beam of pressure distribution
c	forward elocity
g	gravitational acceleration
h	depth of the canal
J_n	Bessel function of the first kind
K_0	$= 0$ if $\nu h \leq 1$; the real positive root of $K_0 - \nu h \tanh K_0 = 0$ if $\nu h > 1$
k_m	root of equation (9)
L	length of the pressure distribution
m, n	indexes of summation
P, Q	integrals defined following equation (1)
P_0, Q_0	integrals defined following equation (3)
p	pressure
p_0	pressure of the central pressure distribution in the sequence of images
R	wave resistance
W	$= Rc$, work per unit time to overcome wave resistance
w	width of canal
ϵ_m	$= 1$ when $m = 0$, $= 2$ when $m \geq 1$
ν	$= g/c^2$
ρ	fluid density

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