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ACOUSTICS AND VIBRATION

COMPUTATION OF OSCILLATORY LOADS ON A SUPERCAVITATING HYDROFOIL

by

Jon Patton and Avis Borden, Ph.D.

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TABLE OF CONTENTS

	Page
ABSTRACT	1
ADMINISTRATIVE INFORMATION	1
INTRODUCTION	1
ANALYSIS	2
RESULTS	5
PERSONNEL	5
APPENDIX - EVALUATION OF INTEGRALS	30
REFERENCES	33

LIST OF FIGURES

Figure 1 - Hydrofoil with a Cavity Springing from an Arbitrary Point on the Upper Surface . . . 2

LIST OF TABLES

Table 1 - Frequency Functions for E = 0	6
Table 2 - Frequency Functions for E = 0.2	8
Table 3 - Frequency Functions for E = 0.4	10
Table 4 - Frequency Functions for E = 0.6	12
Table 5 - Frequency Functions for E = 0.8	14
Table 6 - Frequency Functions for E = 1.0	16
Table 7 - Unsteady Loads and Phase Angles for E = 0	18
Table 8 - Unsteady Loads and Phase Angles for E = 0.2	20

Page

Table 9 - Unsteady Loads and Phase Angles for E = 0.4 22

Table 10 - Unsteady Loads and Phase Angles for E = 0.6 24

Table 11 - Unsteady Loads and Phase Angles for E = 0.8 26

Table 12 - Unsteady Loads and Phase Angles for E = 1.0 28

NOTATION

a	Location of axis (positive aft of midchord)
b	Semichord length
C_L, C_M	Unsteady lift and moment coefficients
C'_L, C'_M	Complex unsteady lift and moment coefficients
c	Chord length, c = 2b
E	Location of separation on hydrofoil
F_1, F_2	Complex frequency functions
FR1, FR2	Real parts of the F_1 and F_2 frequency functions
FI1, FI2	Imaginary parts of the F_1 and F_2 frequency functions
h	Heave displacement
h_o	Maximum heave displacement
k	Reduced frequency, $k = \omega b/U$
L	Time-varying lift
L_o	Mean lift
M	Time-varying moment
M_o	Mean moment
P	Subscript for pitching motion
T	Subscript for heaving motion
t	Time
U	Flow velocity
α	Angle of attack
α_o	Maximum angular displacement
ρ	Fluid density
φ	Phase angle
ω	Circular frequency

ABSTRACT

Frequency functions and unsteady lift and moment coefficients for a hydrofoil oscillating sinusoidally in pitch and heave have been computed using a theory developed by L.C. Woods for an airfoil fitted with a spoiler. A cavity of infinite length is assumed to spring from an arbitrary point on the upper surface of the hydrofoil and extend aft to the trailing edge. Computed frequency coefficients and loading coefficients are tabulated for six locations of the separation point over a range of reduced frequencies from 0.01 to 20.0.

ADMINISTRATIVE INFORMATION

This work was authorized by Bureau of Ships letter S-F013 02 01 Serial 420-255 of 3 October 1960 and Serial 341-B-125 of 13 August 1963. The work was supported under SS600-000 Tasks 1702 and 1703.

INTRODUCTION

The theory of L. C. Woods[1] for the unsteady lift and moment on an airfoil, fitted with a spoiler, appeared to be suitable for predicting the loads on an oscillating hydrofoil operating at low cavitation numbers in an infinite fluid. The spoiler is located on the upper surface of the foil at an arbitrary point between the leading and trailing edges. With the spoiler at the two limiting positions, the flow over the hydrofoil may be considered fully wetted or supercavitating. The cavity is bounded by the free streamlines between the spoiler and trailing edge and extends infinitely far downstream. This corresponds to a zero cavitation number condition.

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[1] References are listed on page 34.

In the present analysis the lift and moment coefficients have been compared for two modes of pure sinusoidal motion. Inasmuch as the analysis is linear in amplitude, more complicated foil motions can be computed by the method of superposition which is described by Parkin.[2] With the use of the frequency functions $F_1(k,E)$ and $F_2(k,E)$, the response of a system to transients or other nonperiodic motions can be obtained.

A numerical analysis required for obtaining the frequency functions and the load coefficients was programmed in FORTRAN for the IBM 7090 computer. The results are presented in tabular form for the six positions of the spoiler and over a range of reduced frequencies. Future experimental studies will determine the usefulness of these data.

ANALYSIS

Woods' analysis gives expressions for the unsteady lift and moment on an oscillating hydrofoil on which the flow is forced to separate at some point E on its upper surface. Positive directions of lift, moment and displacement are shown in Figure 1. The pitch and moment axis is located at a distance ab aft of midchord, where b is the semichord. Positive

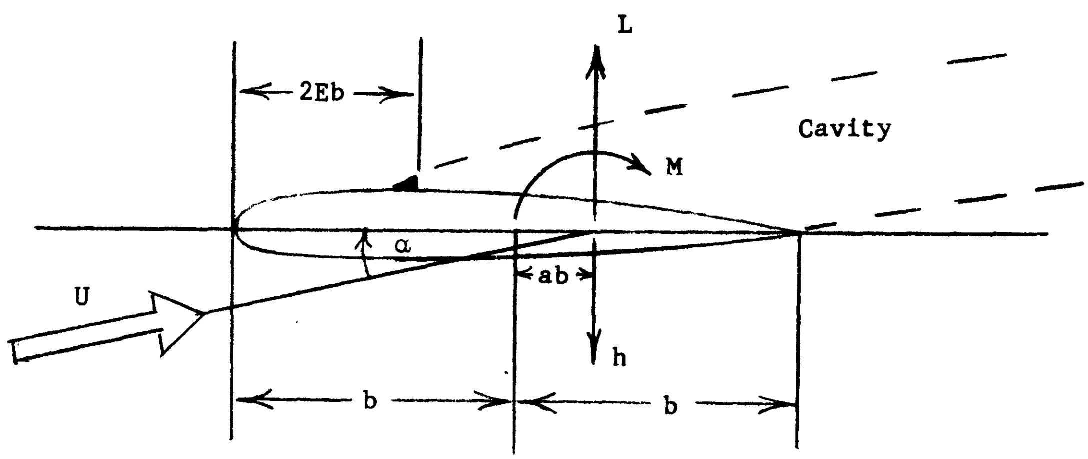

Figure 1 - Hydrofoil with a Cavity Springing from an Arbitrary Point on the Upper Surface

lift L is up, positive displacement h is down, and the pitching moment M and angular displacement α are positive when the leading edge is moving upward. The cavity springs at a distance 2bE from the leading edge.

In the present analysis, the axis of rotation is taken at midchord $(a = 0)$ rather than at the leading edge as in the original report. The unsteady lift and moment coefficients for unit displacements in heave T and pitch P are:

Lift coefficients

$$C_{LT} = \frac{L - L_o}{\pi \rho \omega^2 b^2 h_o e^{i\omega t}} \qquad [1]$$

$$C_{LP} = \frac{L - L_o}{\pi \rho \omega^2 b^3 \alpha_o e^{i\omega t}} \qquad [2]$$

Moment coefficients

$$C_{MT} = \frac{M - M_o}{\pi \rho \omega^2 b^3 h_o e^{i\omega t}} \qquad [3]$$

$$C_{MP} = \frac{M - M_o}{\pi \rho \omega^2 b^4 \alpha_o e^{i\omega t}} \qquad [4]$$

where $h_o e^{i\omega t}$ and $\alpha_o e^{i\omega t}$ are the amplitudes of oscillation of frequency ω in heave and pitch, respectively. L_o and M_o are the mean lift and moment. Expressions for the lift and moment for an arbitrary axis located at a distance a in semichords aft of midchord may be obtained from the following relations:

$$\frac{L - L_o}{\pi \rho \omega^2 b^3 e^{i\omega t}} = C'_{LT} \frac{h_o}{b} + \left[C'_{LP} + a\, C'_{LT} \right] \alpha_o \qquad [5]$$

$$\frac{M - M_o}{\pi \rho \omega^2 b^4 e^{i\omega t}} = \left[C'_{MT} + a\, C'_{LT} \right] \frac{h_o}{b} + \left[C'_{MP} + a \left(C'_{LP} + C'_{MT} \right) + a^2\, C'_{LT} \right] \alpha_o \qquad [6]$$

where the primed coefficients are complex functions of the phase angles φ by which the loads lead the displacements. Thus

$$C'_{LT} = C_{LT}\, e^{i\varphi_{LT}} \qquad C'_{MT} = C_{MT}\, e^{i\varphi_{MT}}$$
$$C'_{LP} = C_{LP}\, e^{i\varphi_{LP}} \qquad C'_{MP} = C_{MP}\, e^{i\varphi_{MP}} \qquad [7]$$

The definitions used here are somewhat different from those used by Woods.[1] Not only is the axis at midchord rather than at the leading edge but the basic length is the semichord b rather than the chord c, and the load coefficients differ by a factor πk^2, where k is the reduced frequency

$$k = \omega b/U \qquad [8]$$

For example, Woods' complex lift coefficient ℓ_{12} is

$$\ell_{12} = \pi k^2\, C'_{LT} \qquad [9]$$

The work of Kaplan and Henry was useful in formulating these equations.[3]

RESULTS

Woods' expressions for the unsteady load coefficients are algebraic combinations of two complex frequency functions $F_1(k, E)$ and $F_2(k, E)$. These parameters are complicated functions of three integrals I_0 , I_1 , and I_2 . The evaluation of these integrals by numerical methods is discussed in the Appendix. Computed values of the complex frequency functions $F_1(k, E)$ and $F_2(k, E)$ are presented in Tables 1 through 6 for six values of the point of flow separation E from 0 to 1.0 and for reduced frequencies ranging from 0.01 to 20. These functions are tabulated in terms of their real and imaginary parts

$$F_1(k, E) = FR1 + i\ FI1$$
$$F_2(k, E) = FR2 + i\ FI2 \quad [10]$$

When $E = 1.0$, the point of separation moves to the trailing edge, $F_2(k, 1) = 0$, and $F_1(k, 1)$ reduces to $2(1 - C(k))$, where $C(k)$ is the Theodorsen constant of unsteady airfoil theory.

Once the frequency functions were obtained, it was an easy matter to compute the unsteady load coefficients for an oscillating foil. The unsteady lift and moment coefficients, and phase angles for pitch and heave are presented as functions of k in Tables 7 through 12 for six values of E. For these calculations, the axis of rotation and loading are at midchord. By the use of Equations [5] and [6], the unsteady loads may be computed for any arbitrary axis of rotation.

PERSONNEL

The analysis and computations were made by Mr. Patton under the direction of Dr. Borden. The report was written by Dr. Borden.

TABLE 1

Frequency Functions for E = 0

K	FR1	FI1	FR2	FI2
0.01000	0.00686	0.03039	-0.00345	0.01370
0.02500	0.02019	0.06410	0.00109	0.02462
0.05000	0.04033	0.11002	0.00380	0.04816
0.07500	0.06016	0.14888	0.00757	0.07071
0.10000	0.07938	0.18325	0.01193	0.09261
0.15000	0.11631	0.24278	0.02231	0.13412
0.20000	0.15125	0.29379	0.03418	0.17301
0.25000	0.18437	0.33883	0.04703	0.20967
0.30000	0.21584	0.37937	0.06055	0.24433
0.35000	0.24583	0.41640	0.07451	0.27723
0.40000	0.27447	0.45058	0.08876	0.30859
0.45000	0.30191	0.48242	0.10318	0.33855
0.50000	0.32826	0.51229	0.11771	0.36725
0.55000	0.35361	0.54047	0.13229	0.39482
0.60000	0.37806	0.56720	0.14686	0.42135
0.65000	0.40167	0.59266	0.16140	0.44694
0.70000	0.42452	0.61699	0.17587	0.47166
0.75000	0.44667	0.64033	0.19026	0.49558
0.80000	0.46816	0.66278	0.20456	0.51877
0.85000	0.48905	0.68443	0.21875	0.54127
0.90000	0.50938	0.70536	0.23282	0.56314
0.95000	0.52919	0.72562	0.24677	0.58442
1.00000	0.54850	0.74528	0.26060	0.60515
1.05000	0.56736	0.76439	0.27430	0.62536
1.10000	0.58579	0.78297	0.28788	0.64509
1.15000	0.60380	0.80109	0.30132	0.66436
1.20000	0.62145	0.81877	0.31464	0.68321
1.25000	0.63873	0.83604	0.32783	0.70165
1.30000	0.65566	0.85292	0.34089	0.71970
1.35000	0.67228	0.86945	0.35382	0.73739
1.40000	0.68858	0.88564	0.36663	0.75475
1.45000	0.70460	0.90151	0.37932	0.77177
1.50000	0.72034	0.91708	0.39188	0.78848
1.60000	0.75103	0.94739	0.41666	0.82103
1.70000	0.78076	0.97668	0.44098	0.85250
1.80000	0.80961	1.00505	0.46485	0.88299
1.90000	0.83764	1.03259	0.48831	0.91257
2.00000	0.86492	1.05936	0.51135	0.94131
2.10000	0.89150	1.08543	0.53400	0.96928
2.20000	0.91743	1.11084	0.55628	0.99653

TABLE 1 (cont.)

Frequency Functions for E = 0

K	FR1	FI1	FR2	FI2
2.30000	0.94277	1.13565	0.57820	1.02312
2.40000	0.96753	1.15990	0.59978	1.04907
2.50000	0.99177	1.18363	0.62102	1.07444
2.60000	1.01551	1.20687	0.64194	1.09926
2.70000	1.03878	1.22965	0.66256	1.12357
2.80000	1.06160	1.25199	0.68288	1.14738
2.90000	1.08401	1.27393	0.70292	1.17074
3.00000	1.10602	1.29548	0.72269	1.19366
3.10000	1.12765	1.31666	0.74220	1.21616
3.20000	1.14892	1.33749	0.76145	1.23828
3.30000	1.16986	1.35799	0.78046	1.26002
3.40000	1.19046	1.37818	0.79923	1.28140
3.50000	1.21076	1.39807	0.81777	1.30245
3.60000	1.23075	1.41767	0.83610	1.32316
3.70000	1.25046	1.43699	0.85421	1.34357
3.80000	1.26990	1.45605	0.87211	1.36368
3.90000	1.28908	1.47486	0.88982	1.38351
4.00000	1.30800	1.49342	0.90733	1.40306
4.50000	1.39915	1.58290	0.99220	1.49708
5.00000	1.48521	1.66750	1.07300	1.58562
5.50000	1.56696	1.74793	1.15027	1.66953
6.00000	1.64499	1.82477	1.22443	1.74945
6.50000	1.71975	1.89847	1.29581	1.82591
7.00000	1.79163	1.96939	1.36469	1.89931
7.50000	1.86093	2.03781	1.43133	1.96999
8.00000	1.92792	2.10400	1.49592	2.03823
8.50000	1.99281	2.16815	1.55863	2.10426
9.00000	2.05579	2.23044	1.61963	2.16829
9.50000	2.11702	2.29103	1.67905	2.23049
10.00000	2.17662	2.35006	1.73698	2.29099
11.00000	2.29148	2.46384	1.84888	2.40746
12.00000	2.40117	2.57259	1.95599	2.51856
13.00000	2.50632	2.67690	2.05888	2.62496
14.00000	2.60747	2.77730	2.15803	2.72722
15.00000	2.70501	2.87415	2.25379	2.82575
16.00000	2.79933	2.96786	2.34650	2.92098
17.00000	2.89072	3.05870	2.43642	3.01321
18.00000	2.97943	3.14690	2.52380	3.10268
19.00000	3.06574	3.23273	2.60888	3.18969
20.00000	3.14976	3.31631	2.69179	3.27435

TABLE 2

Frequency Functions for E = 0.2

K	FR1	FI1	FR2	FI2
0.01000	0.01809	0.05527	0.00079	0.00745
0.02500	0.04599	0.11187	0.00213	0.01910
0.05000	0.09396	0.18123	0.00686	0.03630
0.07500	0.14094	0.23285	0.01313	0.05171
0.10000	0.18599	0.27283	0.02039	0.06546
0.15000	0.26908	0.32953	0.03652	0.08866
0.20000	0.34268	0.36582	0.05352	0.10709
0.25000	0.40754	0.38906	0.07050	0.12176
0.30000	0.46474	0.40356	0.08700	0.13343
0.35000	0.51533	0.41205	0.10280	0.14273
0.40000	0.56024	0.41632	0.11780	0.15011
0.45000	0.60028	0.41759	0.13197	0.15596
0.50000	0.63615	0.41672	0.14531	0.16056
0.55000	0.66840	0.41429	0.15785	0.16414
0.60000	0.69753	0.41076	0.16964	0.16689
0.65000	0.72394	0.40643	0.18072	0.16894
0.70000	0.74797	0.40153	0.19114	0.17043
0.75000	0.76992	0.39625	0.20094	0.17143
0.80000	0.79002	0.39071	0.21017	0.17204
0.85000	0.80850	0.38500	0.21887	0.17232
0.90000	0.82554	0.37922	0.22708	0.17232
0.95000	0.84128	0.37339	0.23483	0.17209
1.00000	0.85586	0.36758	0.24217	0.17166
1.05000	0.86941	0.36182	0.24911	0.17107
1.10000	0.88202	0.35611	0.25569	0.17033
1.15000	0.89378	0.35050	0.26192	0.16948
1.20000	0.90477	0.34497	0.26785	0.16853
1.25000	0.91506	0.33956	0.27348	0.16750
1.30000	0.92472	0.33425	0.27884	0.16640
1.35000	0.93379	0.32906	0.28394	0.16525
1.40000	0.94234	0.32399	0.28880	0.16405
1.45000	0.95039	0.31904	0.29344	0.16281
1.50000	0.95798	0.31420	0.29787	0.16154
1.60000	0.97197	0.30488	0.30615	0.15894
1.70000	0.98452	0.29602	0.31374	0.15628
1.80000	0.99585	0.28760	0.32070	0.15360
1.90000	1.00611	0.27960	0.32712	0.15092
2.00000	1.01545	0.27199	0.33304	0.14826
2.10000	1.02397	0.26476	0.33853	0.14563
2.20000	1.03177	0.25787	0.34362	0.14304

TABLE 2 (cont.)

Frequency Functions for E = 0.2

K	FR1	FI1	FR2	FI2
2.30000	1.03894	0.25131	0.34835	0.14049
2.40000	1.04554	0.24506	0.35275	0.13799
2.50000	1.05163	0.23909	0.35686	0.13555
2.60000	1.05728	0.23340	0.36071	0.13316
2.70000	1.06251	0.22795	0.36431	0.13083
2.80000	1.06739	0.22275	0.36768	0.12856
2.90000	1.07192	0.21777	0.37085	0.12635
3.00000	1.07616	0.21299	0.37384	0.12420
3.10000	1.08012	0.20842	0.37665	0.12210
3.20000	1.08384	0.20403	0.37930	0.12006
3.30000	1.08732	0.19981	0.38180	0.11807
3.40000	1.09059	0.19576	0.38416	0.11614
3.50000	1.09367	0.19187	0.38640	0.11426
3.60000	1.09657	0.18812	0.38852	0.11242
3.70000	1.09931	0.18451	0.39053	0.11064
3.80000	1.10190	0.18103	0.39244	0.10891
3.90000	1.10435	0.17768	0.39426	0.10722
4.00000	1.10666	0.17445	0.39598	0.10558
4.50000	1.11660	0.15984	0.40347	0.09800
5.00000	1.12438	0.14744	0.40943	0.09133
5.50000	1.13059	0.13677	0.41426	0.08545
6.00000	1.13564	0.12751	0.41823	0.08023
6.50000	1.13980	0.11939	0.42153	0.07558
7.00000	1.14326	0.11221	0.42431	0.07139
7.50000	1.14618	0.10583	0.42666	0.06763
8.00000	1.14867	0.10012	0.42867	0.06423
8.50000	1.15080	0.09497	0.43041	0.06113
9.00000	1.15263	0.09032	0.43192	0.05830
9.50000	1.15424	0.08609	0.43323	0.05571
10.00000	1.15564	0.08223	0.43439	0.05334
11.00000	1.15795	0.07544	0.43631	0.04912
12.00000	1.15979	0.06967	0.43784	0.04550
13.00000	1.16126	0.06470	0.43908	0.04237
14.00000	1.16246	0.06038	0.44008	0.03962
15.00000	1.16345	0.05659	0.44092	0.03720
16.00000	1.16428	0.05324	0.44161	0.03505
17.00000	1.16497	0.05027	0.44220	0.03313
18.00000	1.16556	0.04760	0.44271	0.03141
19.00000	1.16607	0.04519	0.44314	0.02985
20.00000	1.16651	0.04302	0.44351	0.02844

TABLE 3

Frequency Functions for E = 0.4

K	FR1	FI1	FR2	FI2
0.01000	0.02291	0.06686	0.00040	0.00585
0.02500	0.05964	0.13273	0.00195	0.01412
0.05000	0.12137	0.20952	0.00616	0.02638
0.07500	0.18051	0.26295	0.01156	0.03694
0.10000	0.23582	0.30147	0.01763	0.04598
0.15000	0.33397	0.35002	0.03057	0.06031
0.20000	0.41663	0.37522	0.04351	0.07072
0.25000	0.48615	0.38683	0.05586	0.07827
0.30000	0.54488	0.39021	0.06738	0.08370
0.35000	0.59482	0.38852	0.07802	0.08756
0.40000	0.63759	0.38368	0.08778	0.09022
0.45000	0.67449	0.37692	0.09672	0.09199
0.50000	0.70653	0.36900	0.10491	0.09308
0.55000	0.73455	0.36044	0.11242	0.09363
0.60000	0.75920	0.35157	0.11931	0.09378
0.65000	0.78100	0.34261	0.12564	0.09362
0.70000	0.80039	0.33372	0.13148	0.09321
0.75000	0.81771	0.32498	0.13687	0.09261
0.80000	0.83325	0.31646	0.14185	0.09186
0.85000	0.84726	0.30819	0.14647	0.09100
0.90000	0.85994	0.30020	0.15075	0.09005
0.95000	0.87145	0.29250	0.15474	0.08904
1.00000	0.88193	0.28509	0.15845	0.08798
1.05000	0.89151	0.27797	0.16192	0.08689
1.10000	0.90029	0.27112	0.16516	0.08577
1.15000	0.90835	0.26455	0.16820	0.08464
1.20000	0.91578	0.25825	0.17104	0.08350
1.25000	0.92264	0.25220	0.17371	0.08236
1.30000	0.92900	0.24639	0.17623	0.08122
1.35000	0.93489	0.24081	0.17859	0.08009
1.40000	0.94036	0.23545	0.18082	0.07897
1.45000	0.94546	0.23030	0.18293	0.07786
1.50000	0.95021	0.22535	0.18492	0.07677
1.60000	0.95882	0.21602	0.18858	0.07463
1.70000	0.96637	0.20737	0.19187	0.07256
1.80000	0.97305	0.19934	0.19485	0.07056
1.90000	0.97898	0.19188	0.19754	0.06864
2.00000	0.98428	0.18492	0.19998	0.06680
2.10000	0.98902	0.17842	0.20221	0.06503
2.20000	0.99330	0.17234	0.20424	0.06333

TABLE 3 (cont.)

Frequency Functions for E = 0.4

K	FR1	FI1	FR2	FI2
2.30000	0.99716	0.16664	0.20611	0.06170
2.40000	1.00065	0.16128	0.20782	0.06014
2.50000	1.00384	0.15625	0.20940	0.05865
2.60000	1.00674	0.15151	0.21085	0.05722
2.70000	1.00940	0.14703	0.21219	0.05584
2.80000	1.01183	0.14281	0.21344	0.05453
2.90000	1.01407	0.13880	0.21460	0.05326
3.00000	1.01614	0.13501	0.21567	0.05205
3.10000	1.01804	0.13142	0.21668	0.05088
3.20000	1.01981	0.12800	0.21761	0.04976
3.30000	1.02145	0.12475	0.21848	0.04869
3.40000	1.02297	0.12166	0.21930	0.04765
3.50000	1.02438	0.11870	0.22007	0.04666
3.60000	1.02570	0.11589	0.22079	0.04570
3.70000	1.02694	0.11320	0.22146	0.04477
3.80000	1.02809	0.11063	0.22210	0.04388
3.90000	1.02917	0.10817	0.22269	0.04303
4.00000	1.03018	0.10581	0.22326	0.04220
4.50000	1.03441	0.09538	0.22565	0.03847
5.00000	1.03760	0.08678	0.22748	0.03531
5.50000	1.04005	0.07957	0.22891	0.03261
6.00000	1.04197	0.07344	0.23006	0.03027
6.50000	1.04352	0.06817	0.23098	0.02823
7.00000	1.04477	0.06360	0.23174	0.02644
7.50000	1.04580	0.05959	0.23237	0.02485
8.00000	1.04666	0.05605	0.23290	0.02344
8.50000	1.04738	0.05290	0.23335	0.02218
9.00000	1.04800	0.05008	0.23373	0.02104
9.50000	1.04852	0.04754	0.23406	0.02001
10.00000	1.04897	0.04524	0.23434	0.01907
11.00000	1.04971	0.04125	0.23480	0.01743
12.00000	1.05028	0.03790	0.23516	0.01605
13.00000	1.05072	0.03505	0.23545	0.01487
14.00000	1.05108	0.03260	0.23568	0.01385
15.00000	1.05137	0.03046	0.23586	0.01295
16.00000	1.05161	0.02858	0.23601	0.01216
17.00000	1.05181	0.02693	0.23614	0.01147
18.00000	1.05198	0.02545	0.23625	0.01084
19.00000	1.05213	0.02412	0.23634	0.01029
20.00000	1.05225	0.02293	0.23642	0.00978

TABLE 4

Frequency Functions for E = 0.6

K	FR1	FI1	FR2	FI2
0.01000	0.02742	0.07608	0.00033	0.00386
0.02500	0.07122	0.14888	0.00148	0.00929
0.05000	0.14397	0.23029	0.00458	0.01713
0.07500	0.21225	0.28386	0.00845	0.02365
0.10000	0.27471	0.32015	0.01270	0.02905
0.15000	0.38212	0.36116	0.02145	0.03717
0.20000	0.46913	0.37782	0.02987	0.04266
0.25000	0.53985	0.38143	0.03765	0.04633
0.30000	0.59783	0.37783	0.04469	0.04872
0.35000	0.64583	0.37024	0.05104	0.05021
0.40000	0.68599	0.36052	0.05673	0.05106
0.45000	0.71990	0.34973	0.06185	0.05144
0.50000	0.74880	0.33853	0.06645	0.05148
0.55000	0.77363	0.32730	0.07061	0.05127
0.60000	0.79513	0.31628	0.07436	0.05089
0.65000	0.81386	0.30560	0.07777	0.05037
0.70000	0.83029	0.29533	0.08087	0.04976
0.75000	0.84478	0.28551	0.08370	0.04908
0.80000	0.85764	0.27615	0.08629	0.04835
0.85000	0.86909	0.26725	0.08866	0.04759
0.90000	0.87934	0.25879	0.09085	0.04681
0.95000	0.88855	0.25077	0.09286	0.04602
1.00000	0.89686	0.24315	0.09471	0.04523
1.05000	0.90438	0.23592	0.09643	0.04444
1.10000	0.91122	0.22906	0.09803	0.04366
1.15000	0.91744	0.22253	0.09951	0.04288
1.20000	0.92314	0.21634	0.10089	0.04212
1.25000	0.92835	0.21044	0.10217	0.04137
1.30000	0.93314	0.20483	0.10338	0.04063
1.35000	0.93755	0.19949	0.10450	0.03991
1.40000	0.94162	0.19439	0.10555	0.03921
1.45000	0.94538	0.18953	0.10654	0.03852
1.50000	0.94887	0.18489	0.10747	0.03785
1.60000	0.95513	0.17621	0.10916	0.03656
1.70000	0.96055	0.16826	0.11067	0.03534
1.80000	0.96530	0.16096	0.11201	0.03418
1.90000	0.96946	0.15424	0.11322	0.03307
2.00000	0.97315	0.14802	0.11430	0.03203
2.10000	0.97642	0.14227	0.11528	0.03104
2.20000	0.97933	0.13692	0.11616	0.03010

TABLE 4 (cont.)

Frequency Functions for E = 0.6

K	FR1	FI1	FR2	FI2
2.30000	0.98194	0.13195	0.11697	0.02921
2.40000	0.98429	0.12731	0.11771	0.02837
2.50000	0.98641	0.12297	0.11838	0.02756
2.60000	0.98833	0.11891	0.11899	0.02680
2.70000	0.99007	0.11509	0.11956	0.02607
2.80000	0.99166	0.11151	0.12008	0.02538
2.90000	0.99311	0.10814	0.12056	0.02472
3.00000	0.99443	0.10495	0.12100	0.02409
3.10000	0.99565	0.10194	0.12141	0.02349
3.20000	0.99677	0.09910	0.12179	0.02292
3.30000	0.99781	0.09640	0.12215	0.02237
3.40000	0.99876	0.09385	0.12248	0.02185
3.50000	0.99965	0.09142	0.12278	0.02135
3.60000	1.00047	0.08911	0.12307	0.02086
3.70000	1.00123	0.08691	0.12334	0.02040
3.80000	1.00194	0.08481	0.12359	0.01996
3.90000	1.00260	0.08281	0.12382	0.01954
4.00000	1.00322	0.08090	0.12404	0.01913
4.50000	1.00577	0.07252	0.12497	0.01731
5.00000	1.00765	0.06567	0.12567	0.01579
5.50000	1.00908	0.06000	0.12620	0.01450
6.00000	1.01018	0.05520	0.12662	0.01341
6.50000	1.01106	0.05111	0.12696	0.01246
7.00000	1.01177	0.04758	0.12723	0.01164
7.50000	1.01234	0.04450	0.12746	0.01091
8.00000	1.01282	0.04179	0.12765	0.01027
8.50000	1.01321	0.03939	0.12780	0.00969
9.00000	1.01355	0.03724	0.12794	0.00918
9.50000	1.01383	0.03532	0.12805	0.00872
10.00000	1.01407	0.03359	0.12815	0.00830
11.00000	1.01447	0.03058	0.12831	0.00757
12.00000	1.01477	0.02806	0.12843	0.00696
13.00000	1.01501	0.02592	0.12853	0.00644
14.00000	1.01520	0.02409	0.12861	0.00599
15.00000	1.01535	0.02250	0.12867	0.00559
16.00000	1.01548	0.02110	0.12872	0.00525
17.00000	1.01558	0.01987	0.12876	0.00495
18.00000	1.01567	0.01877	0.12880	0.00468
19.00000	1.01574	0.01779	0.12883	0.00443
20.00000	1.01581	0.01690	0.12886	0.00421

TABLE 5

Frequency Functions for E = 0.8

K	FR1	FI1	FR2	FI2
0.01000	0.03139	0.08410	0.00017	0.00193
0.02500	0.08165	0.16251	0.00081	0.00459
0.05000	0.16389	0.24708	0.00247	0.00836
0.07500	0.23959	0.30002	0.00449	0.01141
0.10000	0.30749	0.33388	0.00667	0.01385
0.15000	0.42121	0.36816	0.01102	0.01737
0.20000	0.51049	0.37802	0.01508	0.01960
0.25000	0.58114	0.37576	0.01874	0.02098
0.30000	0.63777	0.36737	0.02198	0.02178
0.35000	0.68375	0.35597	0.02484	0.02221
0.40000	0.72158	0.34326	0.02737	0.02236
0.45000	0.75305	0.33016	0.02961	0.02233
0.50000	0.77952	0.31719	0.03160	0.02218
0.55000	0.80199	0.30461	0.03337	0.02194
0.60000	0.82123	0.29257	0.03496	0.02163
0.65000	0.83784	0.28115	0.03639	0.02129
0.70000	0.85227	0.27035	0.03768	0.02092
0.75000	0.86490	0.26017	0.03884	0.02053
0.80000	0.87601	0.25059	0.03990	0.02013
0.85000	0.88583	0.24158	0.04086	0.01973
0.90000	0.89457	0.23310	0.04174	0.01933
0.95000	0.90237	0.22513	0.04255	0.01893
1.00000	0.90936	0.21762	0.04329	0.01853
1.05000	0.91566	0.21054	0.04397	0.01815
1.10000	0.92134	0.20386	0.04460	0.01777
1.15000	0.92650	0.19756	0.04518	0.01740
1.20000	0.93118	0.19160	0.04571	0.01705
1.25000	0.93546	0.18596	0.04621	0.01670
1.30000	0.93936	0.18062	0.04668	0.01636
1.35000	0.94295	0.17556	0.04711	0.01603
1.40000	0.94624	0.17075	0.04751	0.01571
1.45000	0.94927	0.16618	0.04789	0.01540
1.50000	0.95207	0.16184	0.04824	0.01510
1.60000	0.95706	0.15376	0.04888	0.01453
1.70000	0.96136	0.14641	0.04944	0.01399
1.80000	0.96509	0.13969	0.04994	0.01348
1.90000	0.96835	0.13354	0.05039	0.01300
2.00000	0.97122	0.12788	0.05079	0.01256
2.10000	0.97375	0.12266	0.05115	0.01214
2.20000	0.97599	0.11784	0.05147	0.01174

TABLE 5 (cont.)

Frequency Functions for $E = 0.8$

K	FR1	FI1	FR2	FI2
2.30000	0.97799	0.11336	0.05176	0.01137
2.40000	0.97978	0.10920	0.05203	0.01101
2.50000	0.98139	0.10533	0.05227	0.01068
2.60000	0.98283	0.10171	0.05248	0.01036
2.70000	0.98415	0.09833	0.05268	0.01006
2.80000	0.98534	0.09515	0.05287	0.00978
2.90000	0.98642	0.09217	0.05304	0.00951
3.00000	0.98741	0.08937	0.05319	0.00925
3.10000	0.98831	0.08672	0.05334	0.00901
3.20000	0.98914	0.08422	0.05347	0.00878
3.30000	0.98990	0.08186	0.05359	0.00856
3.40000	0.99061	0.07963	0.05371	0.00834
3.50000	0.99125	0.07751	0.05381	0.00814
3.60000	0.99186	0.07550	0.05391	0.00795
3.70000	0.99241	0.07359	0.05400	0.00777
3.80000	0.99293	0.07177	0.05409	0.00759
3.90000	0.99341	0.07003	0.05417	0.00742
4.00000	0.99386	0.06836	0.05425	0.00726
4.50000	0.99569	0.06114	0.05456	0.00655
5.00000	0.99704	0.05527	0.05479	0.00595
5.50000	0.99805	0.05041	0.05497	0.00546
6.00000	0.99883	0.04633	0.05511	0.00504
6.50000	0.99945	0.04285	0.05522	0.00467
7.00000	0.99994	0.03986	0.05531	0.00435
7.50000	1.00034	0.03725	0.05538	0.00408
8.00000	1.00067	0.03497	0.05544	0.00383
8.50000	1.00094	0.03294	0.05549	0.00362
9.00000	1.00117	0.03113	0.05553	0.00342
9.50000	1.00137	0.02952	0.05557	0.00325
10.00000	1.00153	0.02806	0.05560	0.00309
11.00000	1.00181	0.02553	0.05565	0.00282
12.00000	1.00201	0.02342	0.05569	0.00259
13.00000	1.00217	0.02163	0.05572	0.00239
14.00000	1.00230	0.02010	0.05575	0.00222
15.00000	1.00241	0.01876	0.05577	0.00208
16.00000	1.00249	0.01759	0.05578	0.00195
17.00000	1.00256	0.01656	0.05579	0.00183
18.00000	1.00262	0.01565	0.05581	0.00173
19.00000	1.00267	0.01483	0.05582	0.00164
20.00000	1.00272	0.01409	0.05582	0.00156

TABLE 6

Frequency Functions for E = 1.0

K	FR1	FI1	FR2	FI2
0.01000	0.03515	0.09131	-0.00000	-0.00000
0.02500	0.09132	0.17448	-0.00000	-0.00000
0.05000	0.18197	0.26129	-0.00000	-0.00000
0.07500	0.26392	0.31319	-0.00000	-0.00000
0.10000	0.33615	0.34461	-0.00000	-0.00000
0.15000	0.45441	0.37291	-0.00000	-0.00000
0.20000	0.54484	0.37725	-0.00000	-0.00000
0.25000	0.61489	0.37049	-0.00000	-0.00000
0.30000	0.67006	0.35864	0.	0.
0.35000	0.71420	0.34463	0.	0.
0.40000	0.75005	0.32997	-0.00000	-0.00000
0.45000	0.77955	0.31544	-0.00000	-0.00000
0.50000	0.80413	0.30142	-0.00000	-0.00000
0.55000	0.82482	0.28811	-0.00000	-0.00000
0.60000	0.84240	0.27557	-0.00000	-0.00000
0.65000	0.85747	0.26382	-0.00000	-0.00000
0.70000	0.87048	0.25284	-0.00000	-0.00000
0.75000	0.88180	0.24258	0.	0.
0.80000	0.89171	0.23300	0.	0.
0.85000	0.90043	0.22406	-0.00000	0.00000
0.90000	0.90814	0.21570	-0.00000	0.00000
0.95000	0.91500	0.20788	0.	-0.
1.00000	0.92113	0.20055	-0.00000	0.00000
1.05000	0.92662	0.19367	-0.00000	0.00000
1.10000	0.93157	0.18721	-0.00000	0.00000
1.15000	0.93604	0.18114	0.	-0.
1.20000	0.94009	0.17542	-0.00000	0.00000
1.25000	0.94377	0.17002	-0.00000	0.00000
1.30000	0.94713	0.16493	-0.00000	0.00000
1.35000	0.95020	0.16011	-0.00000	0.00000
1.40000	0.95301	0.15555	-0.00000	0.00000
1.45000	0.95559	0.15123	-0.00000	0.00000
1.50000	0.95797	0.14713	-0.00000	0.00000
1.60000	0.96220	0.13953	-0.	0.
1.70000	0.96583	0.13263	-0.00000	-0.00000
1.80000	0.96897	0.12636	-0.00000	-0.00000
1.90000	0.97170	0.12063	-0.	0.
2.00000	0.97409	0.11538	-0.00000	-0.00000
2.10000	0.97619	0.11055	-0.	0.
2.20000	0.97806	0.10610	-0.	0.

TABLE 6 (cont.)

Frequency Functions for E = 1.0

K	FR1	FI1	FR2	FI2
2.30000	0.97971	0.10197	-0.00000	-0.00000
2.40000	0.98119	0.09815	-0.00000	0.00000
2.50000	0.98251	0.09460	-0.	-0.
2.60000	0.98370	0.09128	-0.00000	0.00000
2.70000	0.98478	0.08818	-0.	-0.
2.80000	0.98575	0.08528	-0.	-0.
2.90000	0.98663	0.08256	-0.	-0.
3.00000	0.98744	0.08001	-0.00000	0.00000
3.10000	0.98818	0.07760	-0.00000	0.00000
3.20000	0.98885	0.07533	0.	0.
3.30000	0.98947	0.07319	-0.00000	-0.00000
3.40000	0.99004	0.07116	-0.00000	-0.00000
3.50000	0.99056	0.06924	-0.00000	-0.00000
3.60000	0.99105	0.06742	-0.00000	-0.00000
3.70000	0.99149	0.06569	0.	0.
3.80000	0.99191	0.06405	-0.00000	-0.00000
3.90000	0.99230	0.06248	0.	0.
4.00000	0.99266	0.06099	-0.00000	0.00000
4.50000	0.99413	0.05447	-0.00000	0.00000
5.00000	0.99520	0.04920	-0.00000	-0.00000
5.50000	0.99601	0.04484	0.	-0.
6.00000	0.99663	0.04119	-0.00000	0.00000
6.50000	0.99712	0.03808	0.	0.
7.00000	0.99751	0.03541	-0.00000	-0.00000
7.50000	0.99782	0.03308	0.	-0.
8.00000	0.99808	0.03104	-0.	0.
8.50000	0.99829	0.02924	-0.	0.
9.00000	0.99848	0.02763	-0.00000	0.00000
9.50000	0.99863	0.02619	-0.00000	-0.00000
10.00000	0.99876	0.02489	-0.00000	-0.00000
11.00000	0.99898	0.02265	-0.00000	-0.00000
12.00000	0.99914	0.02077	-0.00000	0.00000
13.00000	0.99927	0.01918	0.	0.
14.00000	0.99937	0.01782	-0.00000	0.00000
15.00000	0.99945	0.01664	-0.00000	0.00000
16.00000	0.99951	0.01560	-0.00000	-0.00000
17.00000	0.99957	0.01469	0.	-0.
18.00000	0.99962	0.01387	-0.00000	-0.00000
19.00000	0.99965	0.01314	-0.00000	-0.00000
20.00000	0.99969	0.01249	-0.00000	0.00000

TABLE 7

Unsteady Loads and Phase Angles for $E = 0$

k	C_{LT}	φ_{LT}	C_{LP}	φ_{LP}	C_{MT}	φ_{MT}	C_{MP}	φ_{MP}
0.010	49.65828	4.72221	4966.95551	0.01859	18.62096	4.71172	1862.35262	6.28080
0.025	19.82426	4.73327	793.97435	0.04269	7.43259	4.70699	297.54446	6.27338
0.050	9.84873	4.76257	197.96599	0.09340	3.68864	4.70966	73.98005	6.27117
0.075	6.53872	4.79470	88.15965	0.14615	2.44373	4.71496	32.77083	6.27117
0.100	4.89422	4.82858	49.90285	0.19951	1.82304	4.72197	18.40445	6.27246
0.150	3.27030	4.89844	22.72450	0.30383	1.20561	4.73855	8.19260	6.27651
0.200	2.47726	4.96861	13.26502	0.40137	0.89954	4.75699	4.64009	6.28109
0.250	2.01504	5.03717	8.89201	0.48992	0.71760	4.77628	3.00330	0.00219
0.300	1.71697	5.10289	6.50727	0.56876	0.59747	4.79580	2.11713	0.00576
0.350	1.51166	5.16504	5.05594	0.63815	0.51249	4.81522	1.58394	0.00845
0.400	1.36347	5.22323	4.10032	0.69886	0.44936	4.83435	1.23826	0.01023
0.450	1.25268	5.27734	3.43262	0.75185	0.40072	4.85305	1.00132	0.01110
0.500	1.16751	5.32739	2.94402	0.79811	0.36215	4.87125	0.83175	0.01114
0.550	1.10052	5.37355	2.57305	0.83856	0.33087	4.88889	0.70614	0.01040
0.600	1.04683	5.41603	2.28282	0.87404	0.30502	4.90595	0.61042	0.00899
0.650	1.00306	5.45509	2.05008	0.90526	0.28331	4.92243	0.53576	0.00699
0.700	0.96687	5.49102	1.85956	0.93283	0.26484	4.93832	0.47635	0.00449
0.750	0.93658	5.52408	1.70089	0.95728	0.24894	4.95365	0.42827	0.00158
0.800	0.91092	5.55453	1.56675	0.97903	0.23512	4.96841	0.38878	6.28150
0.850	0.88896	5.58262	1.45192	0.99845	0.22300	4.98264	0.35593	6.27796
0.900	0.87000	5.60856	1.35253	1.01586	0.21228	4.99635	0.32829	6.27421
0.950	0.85349	5.63258	1.26566	1.03150	0.20274	5.00956	0.30479	6.27031
1.000	0.83901	5.65484	1.18910	1.04561	0.19419	5.02228	0.28465	6.26629
1.050	0.82620	5.67551	1.12112	1.05836	0.18649	5.03456	0.26723	6.26221
1.100	0.81482	5.69476	1.06036	1.06992	0.17952	5.04640	0.25206	6.25810
1.150	0.80462	5.71270	1.00572	1.08043	0.17318	5.05782	0.23875	6.25398
1.200	0.79546	5.72946	0.95633	1.08999	0.16738	5.06885	0.22702	6.24989
1.250	0.78717	5.74515	0.91146	1.09872	0.16207	5.07950	0.21661	6.24584
1.300	0.77963	5.75985	0.87052	1.10669	0.15717	5.08979	0.20734	6.24186
1.350	0.77276	5.77366	0.83302	1.11399	0.15265	5.09974	0.19902	6.23795
1.400	0.76646	5.78666	0.79854	1.12068	0.14847	5.10936	0.19155	6.23413
1.450	0.76066	5.79890	0.76673	1.12682	0.14457	5.11868	0.18479	6.23041
1.500	0.75531	5.81046	0.73730	1.13247	0.14095	5.12770	0.17867	6.22680
1.600	0.74576	5.83173	0.68456	1.14243	0.13438	5.14491	0.16800	6.21991
1.700	0.73746	5.85084	0.63868	1.15088	0.12861	5.16110	0.15906	6.21347
1.800	0.73019	5.86811	0.59841	1.15806	0.12348	5.17637	0.15148	6.20750
1.900	0.72375	5.88378	0.56278	1.16415	0.11889	5.19079	0.14499	6.20199
2.000	0.71800	5.89808	0.53104	1.16931	0.11476	5.20443	0.13937	6.19692
2.100	0.71282	5.91117	0.50259	1.17368	0.11102	5.21737	0.13448	6.19228
2.200	0.70814	5.92321	0.47694	1.17736	0.10762	5.22966	0.13019	6.18803

TABLE 7 (cont.)

Unsteady Loads and Phase Angles for E = 0

k	C_{LT}	φ_{LT}	C_{LP}	φ_{LP}	C_{MT}	φ_{MT}	C_{MP}	φ_{MP}
2.300	0.70388	5.93431	0.45371	1.18043	0.10451	5.24135	0.12640	6.18416
2.400	0.69998	5.94460	0.43256	1.18298	0.10166	5.25249	0.12302	6.18063
2.500	0.69638	5.95414	0.41325	1.18506	0.09903	5.26312	0.12001	6.17742
2.600	0.69307	5.96304	0.39553	1.18672	0.09659	5.27328	0.11730	6.17451
2.700	0.68999	5.97134	0.37922	1.18802	0.09433	5.28301	0.11486	6.17187
2.800	0.68713	5.97912	0.36417	1.18898	0.09223	5.29234	0.11265	6.16948
2.900	0.68445	5.98641	0.35022	1.18966	0.09026	5.30128	0.11063	6.16733
3.000	0.68194	5.99327	0.33728	1.19006	0.08842	5.30988	0.10879	6.16539
3.100	0.67959	5.99974	0.32523	1.19023	0.08670	5.31815	0.10710	6.16364
3.200	0.67737	6.00584	0.31398	1.19018	0.08508	5.32611	0.10555	6.16207
3.300	0.67528	6.01161	0.30347	1.18993	0.08355	5.33378	0.10412	6.16066
3.400	0.67330	6.01709	0.29361	1.18950	0.08210	5.34119	0.10279	6.15940
3.500	0.67142	6.02228	0.28436	1.18891	0.08073	5.34834	0.10157	6.15828
3.600	0.66964	6.02721	0.27566	1.18817	0.07944	5.35526	0.10043	6.15728
3.700	0.66795	6.03191	0.26746	1.18728	0.07821	5.36194	0.09936	6.15640
3.800	0.66633	6.03639	0.25972	1.18628	0.07704	5.36842	0.09837	6.15563
3.900	0.66479	6.04066	0.25240	1.18515	0.07592	5.37470	0.09744	6.15495
4.000	0.66331	6.04474	0.24548	1.18392	0.07486	5.38078	0.09657	6.15436
4.500	0.65680	6.06273	0.21576	1.17641	0.07020	5.40870	0.09292	6.15250
5.000	0.65142	6.07751	0.19234	1.16716	0.06640	5.43311	0.09014	6.15197
5.500	0.64687	6.08990	0.17345	1.15668	0.06323	5.45474	0.08796	6.15233
6.000	0.64296	6.10047	0.15791	1.14531	0.06054	5.47411	0.08620	6.15326
6.500	0.63955	6.10960	0.14491	1.13330	0.05822	5.49162	0.08476	6.15456
7.000	0.63654	6.11760	0.13390	1.12081	0.05620	5.50759	0.08355	6.15610
7.500	0.63385	6.12467	0.12446	1.10797	0.05441	5.52223	0.08252	6.15778
8.000	0.63144	6.13096	0.11628	1.09487	0.05282	5.53574	0.08164	6.15955
8.500	0.62926	6.13662	0.10914	1.08160	0.05139	5.54828	0.08087	6.16136
9.000	0.62727	6.14174	0.10286	1.06820	0.05011	5.55996	0.08019	6.16318
9.500	0.62545	6.14639	0.09729	1.05473	0.04894	5.57089	0.07959	6.16499
10.000	0.62377	6.15064	0.09232	1.04123	0.04787	5.58115	0.07906	6.16677
11.000	0.62078	6.15815	0.08386	1.01425	0.04598	5.59994	0.07815	6.17023
12.000	0.61819	6.16459	0.07694	0.98743	0.04437	5.61681	0.07739	6.17351
13.000	0.61591	6.17018	0.07119	0.96094	0.04297	5.63207	0.07676	6.17660
14.000	0.61389	6.17510	0.06634	0.93485	0.04174	5.64601	0.07622	6.17951
15.000	0.61208	6.17946	0.06221	0.90926	0.04065	5.65880	0.07575	6.18223
16.000	0.61045	6.18336	0.05866	0.88421	0.03968	5.67063	0.07534	6.18479
17.000	0.60897	6.18687	0.05558	0.85973	0.03880	5.68161	0.07498	6.18719
18.000	0.60761	6.19006	0.05288	0.83586	0.03801	5.69185	0.07466	6.18944
19.000	0.60637	6.19297	0.05052	0.81261	0.03728	5.70144	0.07437	6.19156
20.000	0.60522	6.19563	0.04842	0.79000	0.03662	5.71045	0.07411	6.19356

TABLE 8

Unsteady Loads and Phase Angles for E = 0.2

k	C_{LT}	φ_{LT}	C_{LP}	φ_{LP}	C_{MT}	φ_{MT}	C_{MP}	φ_{MP}
0.010	103.82518	4.69311	10382.57471	6.26634	60.78741	4.68779	6079.55902	6.25571
0.025	40.98480	4.67717	1639.44492	6.25407	24.00647	4.66372	960.90970	6.22718
0.050	20.04081	4.66181	400.87517	6.24486	11.75286	4.63435	235.58775	6.19002
0.075	13.07078	4.65454	174.34205	6.24382	7.67723	4.61251	102.82356	6.16001
0.100	9.59693	4.65269	96.04033	6.24828	5.64664	4.59554	56.87807	6.13453
0.150	6.14784	4.66036	41.06842	6.26887	3.63018	4.57136	24.54637	6.09250
0.200	4.44847	4.67880	22.33640	0.01738	2.63490	4.55594	13.47508	6.05838
0.250	3.44818	4.70488	13.89727	0.05696	2.04669	4.54636	8.45499	6.02953
0.300	2.79639	4.73661	9.43552	0.10222	1.66093	4.54088	5.77996	6.00443
0.350	2.34283	4.77254	6.81673	0.15151	1.39002	4.53838	4.19558	5.98215
0.400	2.01237	4.81159	5.16159	0.20346	1.19031	4.53808	3.18412	5.96207
0.450	1.76334	4.85287	4.05590	0.25691	1.03759	4.53942	2.50117	5.94377
0.500	1.57079	4.89563	3.28458	0.31087	0.91744	4.54202	2.01947	5.92696
0.550	1.41886	4.93925	2.72734	0.36453	0.82073	4.54556	1.66763	5.91146
0.600	1.29700	4.98320	2.31287	0.41720	0.74138	4.54982	1.40316	5.89709
0.650	1.19795	5.02703	1.99686	0.46835	0.67525	4.55464	1.19955	5.88377
0.700	1.11653	5.07036	1.75070	0.51757	0.61938	4.55988	1.03958	5.87140
0.750	1.04895	5.11289	1.55527	0.56458	0.57163	4.56544	0.91167	5.85991
0.800	0.99238	5.15436	1.39750	0.60921	0.53040	4.57123	0.80783	5.84927
0.850	0.94469	5.19460	1.26817	0.65136	0.49448	4.57720	0.72241	5.83942
0.900	0.90421	5.23347	1.16069	0.69102	0.46294	4.58329	0.65129	5.83032
0.950	0.86965	5.27086	1.07026	0.72822	0.43504	4.58947	0.59147	5.82195
1.000	0.83999	5.30673	0.99330	0.76304	0.41020	4.59570	0.54066	5.81426
1.050	0.81439	5.34104	0.92712	0.79556	0.38797	4.60197	0.49715	5.80724
1.100	0.79220	5.37380	0.86967	0.82592	0.36796	4.60824	0.45959	5.80084
1.150	0.77287	5.40502	0.81937	0.85423	0.34986	4.61451	0.42694	5.79506
1.200	0.75598	5.43474	0.77498	0.88062	0.33343	4.62075	0.39839	5.78985
1.250	0.74115	5.46301	0.73552	0.90522	0.31844	4.62697	0.37326	5.78519
1.300	0.72808	5.48987	0.70021	0.92817	0.30472	4.63316	0.35103	5.78105
1.350	0.71653	5.51539	0.66842	0.94957	0.29212	4.63930	0.33126	5.77742
1.400	0.70627	5.53962	0.63964	0.96954	0.28051	4.64540	0.31360	5.77426
1.450	0.69715	5.56264	0.61346	0.98819	0.26977	4.65145	0.29776	5.77155
1.500	0.68900	5.58451	0.58953	1.00562	0.25983	4.65745	0.28349	5.76926
1.600	0.67515	5.62502	0.54732	1.03717	0.24198	4.66929	0.25888	5.76588
1.700	0.66392	5.66165	0.51121	1.06486	0.22643	4.68091	0.23850	5.76394
1.800	0.65472	5.69483	0.47993	1.08923	0.21276	4.69230	0.22142	5.76326
1.900	0.64712	5.72496	0.45253	1.11076	0.20067	4.70349	0.20695	5.76371
2.000	0.64077	5.75240	0.42831	1.12983	0.18989	4.71446	0.19459	5.76512
2.100	0.63544	5.77744	0.40671	1.14677	0.18023	4.72523	0.18392	5.76739
2.200	0.63092	5.80037	0.38732	1.16187	0.17152	4.73581	0.17466	5.77038

TABLE 8 (cont.)

Unsteady Loads and Phase Angles for E = 0.2

k	C_{LT}	φ_{LT}	C_{LP}	φ_{LP}	C_{MT}	φ_{MT}	C_{MP}	φ_{MP}
2.300	0.62706	5.82142	0.36979	1.17534	0.16364	4.74620	0.16656	5.77400
2.400	0.62375	5.84079	0.35387	1.18740	0.15647	4.75642	0.15943	5.77815
2.500	0.62089	5.85868	0.33933	1.19821	0.14992	4.76646	0.15312	5.78275
2.600	0.61840	5.87522	0.32599	1.20791	0.14392	4.77635	0.14752	5.78773
2.700	0.61623	5.89056	0.31371	1.21663	0.13840	4.78608	0.14251	5.79301
2.800	0.61432	5.90483	0.30236	1.22448	0.13331	4.79566	0.13801	5.79854
2.900	0.61265	5.91812	0.29183	1.23154	0.12859	4.80511	0.13396	5.80427
3.000	0.61116	5.93053	0.28204	1.23791	0.12422	4.81443	0.13030	5.81016
3.100	0.60984	5.94214	0.27291	1.24364	0.12015	4.82361	0.12697	5.81616
3.200	0.60866	5.95303	0.26437	1.24880	0.11635	4.83268	0.12395	5.82224
3.300	0.60760	5.96325	0.25637	1.25344	0.11281	4.84164	0.12119	5.82838
3.400	0.60665	5.97287	0.24885	1.25762	0.10949	4.85048	0.11866	5.83454
3.500	0.60580	5.98194	0.24178	1.26136	0.10637	4.85922	0.11634	5.84071
3.600	0.60503	5.99049	0.23511	1.26472	0.10344	4.86785	0.11421	5.84686
3.700	0.60432	5.99858	0.22881	1.26772	0.10068	4.87639	0.11224	5.85298
3.800	0.60369	6.00624	0.22285	1.27039	0.09808	4.88483	0.11042	5.85906
3.900	0.60311	6.01350	0.21720	1.27276	0.09563	4.89318	0.10874	5.86509
4.000	0.60257	6.02039	0.21184	1.27485	0.09330	4.90144	0.10718	5.87105
4.500	0.60050	6.05019	0.18867	1.28183	0.08335	4.94154	0.10083	5.89964
5.000	0.59910	6.07393	0.17020	1.28435	0.07554	4.97980	0.09627	5.92588
5.500	0.59811	6.09329	0.15512	1.28370	0.06927	5.01643	0.09287	5.94966
6.000	0.59739	6.10937	0.14258	1.28072	0.06413	5.05157	0.09027	5.97107
6.500	0.59684	6.12294	0.13200	1.27601	0.05987	5.08535	0.08824	5.99032
7.000	0.59643	6.13454	0.12295	1.26995	0.05627	5.11785	0.08663	6.00762
7.500	0.59610	6.14457	0.11513	1.26287	0.05321	5.14913	0.08533	6.02321
8.000	0.59584	6.15333	0.10830	1.25497	0.05058	5.17925	0.08426	6.03729
8.500	0.59563	6.16105	0.10229	1.24642	0.04830	5.20827	0.08338	6.05003
9.000	0.59545	6.16790	0.09697	1.23736	0.04631	5.23621	0.08263	6.06161
9.500	0.59531	6.17402	0.09222	1.22789	0.04456	5.26313	0.08201	6.07215
10.000	0.59519	6.17952	0.08796	1.21810	0.04301	5.28905	0.08147	6.08180
11.000	0.59500	6.18902	0.08065	1.19778	0.04041	5.33808	0.08061	6.09877
12.000	0.59485	6.19691	0.07460	1.17682	0.03832	5.38356	0.07995	6.11319
13.000	0.59474	6.20358	0.06953	1.15550	0.03661	5.42576	0.07944	6.12559
14.000	0.59466	6.20930	0.06522	1.13402	0.03520	5.46491	0.07904	6.13634
15.000	0.59459	6.21424	0.06152	1.11253	0.03403	5.50127	0.07871	6.14576
16.000	0.59454	6.21857	0.05832	1.09115	0.03304	5.53505	0.07844	6.15405
17.000	0.59449	6.22238	0.05552	1.06997	0.03219	5.56646	0.07822	6.16142
18.000	0.59446	6.22577	0.05307	1.04904	0.03147	5.59570	0.07803	6.16801
19.000	0.59443	6.22880	0.05090	1.02843	0.03084	5.62295	0.07788	6.17392
20.000	0.59440	6.23153	0.04897	1.00817	0.03030	5.64838	0.07774	6.17927

TABLE 9

Unsteady Loads and Phase Angles for E = 0.4

k	C_{LT}	φ_{LT}	C_{LP}	φ_{LP}	C_{MT}	φ_{MT}	C_{MP}	φ_{MP}
0.010	131.78234	4.68465	13178.28052	6.25747	78.99242	4.68050	7900.31500	6.24917
0.025	51.80511	4.65940	2072.25540	6.23525	31.06804	4.64886	1243.58614	6.21420
0.050	25.14927	4.63240	503.04406	6.21336	15.10273	4.61077	302.75740	6.17017
0.075	16.28413	4.61618	217.18768	6.20233	9.79667	4.58288	131.22973	6.13594
0.100	11.87199	4.60718	118.79303	6.19860	7.15722	4.56163	72.11238	6.10798
0.150	7.50469	4.60423	50.11788	6.20647	4.54495	4.53248	30.74733	6.06445
0.200	5.36495	4.61532	26.92384	6.22875	3.26354	4.51520	16.70313	6.03177
0.250	4.11324	4.63620	16.56349	6.26109	2.51153	4.50572	10.38626	6.00609
0.300	3.30268	4.66423	11.12980	0.01748	2.02189	4.50161	7.04529	5.98522
0.350	2.74194	4.69757	7.96420	0.06225	1.68048	4.50130	5.07995	5.96777
0.400	2.33561	4.73486	5.97715	0.11060	1.43049	4.50376	3.83303	5.95285
0.450	2.03090	4.77504	4.65813	0.16121	1.24058	4.50823	2.99578	5.93984
0.500	1.79630	4.81726	3.74338	0.21299	1.09206	4.51418	2.40821	5.92835
0.550	1.61190	4.86079	3.08614	0.26500	0.97319	4.52124	1.98098	5.91807
0.600	1.46450	4.90503	2.59984	0.31650	0.87618	4.52910	1.66117	5.90880
0.650	1.34505	4.94947	2.23089	0.36686	0.79572	4.53756	1.41588	5.90040
0.700	1.24710	4.99368	1.94485	0.41562	0.72806	4.54646	1.22381	5.89275
0.750	1.16599	5.03731	1.71880	0.46242	0.67048	4.55567	1.07072	5.88578
0.800	1.09824	5.08007	1.53712	0.50705	0.62095	4.56509	0.94680	5.87942
0.850	1.04123	5.12173	1.38883	0.54935	0.57797	4.57465	0.84512	5.87363
0.900	0.99292	5.16213	1.26612	0.58927	0.54035	4.58429	0.76069	5.86836
0.950	0.95173	5.20113	1.16328	0.62679	0.50719	4.59397	0.68983	5.86359
1.000	0.91642	5.23865	1.07611	0.66197	0.47776	4.60365	0.62978	5.85929
1.050	0.88600	5.27464	1.00143	0.69488	0.45148	4.61331	0.57846	5.85544
1.100	0.85965	5.30908	0.93684	0.72561	0.42790	4.62293	0.53426	5.85201
1.150	0.83673	5.34198	0.88049	0.75428	0.40663	4.63249	0.49591	5.84900
1.200	0.81670	5.37335	0.83092	0.78101	0.38736	4.64198	0.46242	5.84636
1.250	0.79914	5.40324	0.78701	0.80592	0.36982	4.65139	0.43301	5.84410
1.300	0.78368	5.43169	0.74783	0.82912	0.35381	4.66071	0.40703	5.84220
1.350	0.77002	5.45875	0.71266	0.85073	0.33913	4.66994	0.38397	5.84062
1.400	0.75790	5.48447	0.68092	0.87088	0.32563	4.67908	0.36340	5.83937
1.450	0.74712	5.50893	0.65211	0.88966	0.31317	4.68813	0.34498	5.83842
1.500	0.73750	5.53219	0.62585	0.90717	0.30165	4.69708	0.32841	5.83776
1.600	0.72116	5.57533	0.57969	0.93875	0.28103	4.71468	0.29991	5.83723
1.700	0.70791	5.61438	0.54038	0.96630	0.26312	4.73190	0.27637	5.83767
1.800	0.69705	5.64979	0.50647	0.99037	0.24744	4.74875	0.25669	5.83896
1.900	0.68807	5.68197	0.47688	1.01146	0.23359	4.76524	0.24007	5.84100
2.000	0.68058	5.71129	0.45080	1.02995	0.22129	4.78138	0.22589	5.84368
2.100	0.67427	5.73807	0.42763	1.04621	0.21030	4.79718	0.21370	5.84693
2.200	0.66892	5.76260	0.40688	1.06051	0.20042	4.81267	0.20313	5.85064

TABLE 9 (cont.)

Unsteady Loads and Phase Angles for E = 0.4

k	C_{LT}	φ_{LT}	C_{LP}	φ_{LP}	C_{MT}	φ_{MT}	C_{MP}	φ_{MP}
2.300	0.66435	5.78513	0.38819	1.07309	0.19149	4.82786	0.19391	5.85476
2.400	0.66042	5.80587	0.37124	1.08418	0.18339	4.84275	0.18582	5.85922
2.500	0.65702	5.82503	0.35581	1.09394	0.17601	4.85737	0.17867	5.86394
2.600	0.65406	5.84275	0.34168	1.10252	0.16927	4.87173	0.17232	5.86889
2.700	0.65147	5.85920	0.32870	1.11006	0.16308	4.88583	0.16666	5.87402
2.800	0.64919	5.87450	0.31673	1.11668	0.15738	4.89968	0.16159	5.87928
2.900	0.64718	5.88875	0.30565	1.12245	0.15212	4.91331	0.15703	5.88464
3.000	0.64539	5.90206	0.29537	1.12749	0.14725	4.92671	0.15291	5.89007
3.100	0.64380	5.91452	0.28580	1.13184	0.14273	4.93989	0.14918	5.89554
3.200	0.64237	5.92621	0.27686	1.13559	0.13852	4.95287	0.14579	5.90103
3.300	0.64109	5.93718	0.26850	1.13879	0.13460	4.96565	0.14270	5.90652
3.400	0.63994	5.94751	0.26067	1.14149	0.13094	4.97823	0.13987	5.91199
3.500	0.63890	5.95725	0.25330	1.14374	0.12751	4.99062	0.13728	5.91743
3.600	0.63796	5.96644	0.24637	1.14557	0.12430	5.00283	0.13489	5.92282
3.700	0.63710	5.97514	0.23983	1.14702	0.12128	5.01486	0.13270	5.92815
3.800	0.63631	5.98337	0.23366	1.14812	0.11844	5.02672	0.13067	5.93342
3.900	0.63560	5.99117	0.22781	1.14890	0.11576	5.03841	0.12880	5.93862
4.000	0.63494	5.99859	0.22227	1.14938	0.11324	5.04994	0.12706	5.94375
4.500	0.63236	6.03065	0.19844	1.14815	0.10250	5.10524	0.12001	5.96808
5.000	0.63058	6.05624	0.17957	1.14225	0.09419	5.15693	0.11494	5.99015
5.500	0.62931	6.07711	0.16428	1.13312	0.08761	5.20533	0.11117	6.00998
6.000	0.62837	6.09447	0.15166	1.12167	0.08229	5.25069	0.10830	6.02774
6.500	0.62766	6.10913	0.14108	1.10856	0.07793	5.29323	0.10605	6.04363
7.000	0.62710	6.12167	0.13209	1.09426	0.07430	5.33315	0.10426	6.05788
7.500	0.62666	6.13252	0.12437	1.07911	0.07126	5.37062	0.10282	6.07068
8.000	0.62630	6.14201	0.11769	1.06335	0.06868	5.40581	0.10164	6.08222
8.500	0.62601	6.15037	0.11185	1.04719	0.06647	5.43888	0.10065	6.09266
9.000	0.62577	6.15779	0.10671	1.03078	0.06456	5.46996	0.09983	6.10213
9.500	0.62556	6.16442	0.10216	1.01423	0.06291	5.49921	0.09913	6.11075
10.000	0.62539	6.17039	0.09811	0.99764	0.06147	5.52674	0.09853	6.11863
11.000	0.62512	6.18069	0.09124	0.96464	0.05908	5.57713	0.09758	6.13249
12.000	0.62491	6.18926	0.08564	0.93220	0.05720	5.62199	0.09684	6.14427
13.000	0.62475	6.19651	0.08102	0.90061	0.05570	5.66208	0.09627	6.15439
14.000	0.62463	6.20272	0.07716	0.87005	0.05448	5.69802	0.09582	6.16317
15.000	0.62453	6.20810	0.07390	0.84063	0.05348	5.73038	0.09546	6.17086
16.000	0.62444	6.21280	0.07111	0.81240	0.05264	5.75961	0.09516	6.17763
17.000	0.62438	6.21695	0.06873	0.78540	0.05194	5.78612	0.09491	6.18365
18.000	0.62432	6.22063	0.06666	0.75960	0.05135	5.81023	0.09470	6.18902
19.000	0.62427	6.22393	0.06485	0.73500	0.05084	5.83224	0.09453	6.19386
20.000	0.62423	6.22690	0.06328	0.71156	0.05040	5.85240	0.09438	6.19822

TABLE 10

Unsteady Loads and Phase Angles for E = 0.6

k	C_{LT}	φ_{LT}	C_{LP}	φ_{LP}	C_{MT}	φ_{MT}	C_{MP}	φ_{MP}
0.010	155.39515	4.67884	15539.72668	6.25220	90.28841	4.67509	9030.17651	6.24471
0.025	60.87890	4.64805	2435.36938	6.22528	35.39082	4.63850	1416.70723	6.20620
0.050	29.38256	4.61497	587.86809	6.19869	17.10596	4.59528	342.98876	6.15936
0.075	18.91941	4.59494	252.47918	6.18519	11.03597	4.56448	147.89321	6.12446
0.100	13.72226	4.58358	137.44647	6.18043	8.02242	4.54173	80.88409	6.09716
0.150	8.59704	4.57867	57.54435	6.18889	5.05071	4.51220	34.21105	6.05735
0.200	6.10175	4.59002	30.74605	6.21376	3.60181	4.49662	18.46762	6.03012
0.250	4.65164	4.61241	18.84983	6.24960	2.75679	4.48994	11.42685	6.01068
0.300	3.71861	4.64262	12.64383	0.00977	2.20986	4.48923	7.72120	5.99631
0.350	3.07703	4.67848	9.04423	0.05817	1.83060	4.49262	5.55029	5.98536
0.400	2.61472	4.71843	6.79268	0.10973	1.55427	4.49890	4.17767	5.97676
0.450	2.26980	4.76124	5.30197	0.16299	1.34528	4.50721	3.25864	5.96982
0.500	2.00549	4.80596	4.26995	0.21675	1.18249	4.51697	2.61520	5.96408
0.550	1.79862	4.85182	3.52911	0.27007	1.05265	4.52775	2.14826	5.95923
0.600	1.63389	4.89816	2.98098	0.32220	0.94703	4.53925	1.79929	5.95505
0.650	1.50085	4.94446	2.56487	0.37260	0.85968	4.55124	1.53198	5.95139
0.700	1.39210	4.99028	2.24185	0.42088	0.78640	4.56355	1.32290	5.94816
0.750	1.30228	5.03527	1.98613	0.46677	0.72418	4.57607	1.15641	5.94528
0.800	1.22744	5.07916	1.78012	0.51014	0.67077	4.58869	1.02175	5.94270
0.850	1.16456	5.12173	1.61154	0.55093	0.62450	4.60135	0.91133	5.94039
0.900	1.11137	5.16284	1.47166	0.58914	0.58407	4.61399	0.81970	5.93832
0.950	1.06608	5.20237	1.35410	0.62484	0.54848	4.62658	0.74283	5.93649
1.000	1.02728	5.24028	1.25415	0.65812	0.51695	4.63908	0.67773	5.93487
1.050	0.99387	5.27653	1.16829	0.68909	0.48883	4.65148	0.62212	5.93346
1.100	0.96494	5.31112	1.09383	0.71789	0.46362	4.66376	0.57423	5.93225
1.150	0.93978	5.34408	1.02870	0.74464	0.44090	4.67591	0.53271	5.93124
1.200	0.91779	5.37544	0.97127	0.76949	0.42034	4.68792	0.49647	5.93041
1.250	0.89849	5.40526	0.92027	0.79257	0.40165	4.69979	0.46465	5.92977
1.300	0.88149	5.43359	0.87469	0.81400	0.38459	4.71151	0.43655	5.92931
1.350	0.86645	5.46051	0.83371	0.83391	0.36897	4.72308	0.41163	5.92902
1.400	0.85310	5.48607	0.79666	0.85241	0.35462	4.73450	0.38941	5.92889
1.450	0.84121	5.51034	0.76299	0.86960	0.34139	4.74578	0.36951	5.92893
1.500	0.83058	5.53339	0.73226	0.88559	0.32916	4.75691	0.35163	5.92913
1.600	0.81247	5.57612	0.67817	0.91430	0.30729	4.77875	0.32089	5.92996
1.700	0.79775	5.61476	0.63206	0.93920	0.28833	4.80004	0.29552	5.93133
1.800	0.78563	5.64978	0.59225	0.96081	0.27174	4.82079	0.27434	5.93320
1.900	0.77557	5.68159	0.55750	0.97961	0.25712	4.84104	0.25647	5.93551
2.000	0.76713	5.71058	0.52687	0.99595	0.24415	4.86081	0.24124	5.93820
2.100	0.76000	5.73707	0.49966	1.01018	0.23258	4.88011	0.22816	5.94122
2.200	0.75392	5.76134	0.47532	1.02256	0.22219	4.89898	0.21684	5.94454

TABLE 10 (cont.)

Unsteady Loads and Phase Angles for E = 0.6

k	C_{LT}	φ_{LT}	C_{LP}	φ_{LP}	C_{MT}	φ_{MT}	C_{MP}	φ_{MP}
2.300	0.74871	5.78363	0.45340	1.03333	0.21282	4.91743	0.20697	5.94809
2.400	0.74420	5.80417	0.43355	1.04266	0.20433	4.93548	0.19832	5.95186
2.500	0.74028	5.82315	0.41548	1.05074	0.19661	4.95314	0.19068	5.95579
2.600	0.73685	5.84072	0.39896	1.05771	0.18956	4.97044	0.18391	5.95985
2.700	0.73384	5.85704	0.38380	1.06368	0.18311	4.98738	0.17788	5.96402
2.800	0.73118	5.87222	0.36983	1.06877	0.17717	5.00398	0.17248	5.96827
2.900	0.72882	5.88638	0.35692	1.07307	0.17171	5.02025	0.16763	5.97257
3.000	0.72671	5.89961	0.34495	1.07665	0.16666	5.03620	0.16325	5.97690
3.100	0.72482	5.91201	0.33382	1.07960	0.16198	5.05185	0.15929	5.98125
3.200	0.72313	5.92363	0.32344	1.08196	0.15763	5.06719	0.15569	5.98560
3.300	0.72160	5.93456	0.31375	1.08380	0.15359	5.08225	0.15241	5.98994
3.400	0.72022	5.94486	0.30466	1.08516	0.14982	5.09703	0.14942	5.99425
3.500	0.71897	5.95456	0.29614	1.08608	0.14630	5.11153	0.14668	5.99853
3.600	0.71783	5.96374	0.28812	1.08661	0.14301	5.12577	0.14416	6.00277
3.700	0.71678	5.97241	0.28057	1.08678	0.13992	5.13974	0.14184	6.00695
3.800	0.71583	5.98063	0.27345	1.08662	0.13702	5.15346	0.13969	6.01108
3.900	0.71496	5.98843	0.26672	1.08616	0.13430	5.16694	0.13772	6.01516
4.000	0.71415	5.99584	0.26034	1.08542	0.13173	5.18017	0.13588	6.01916
4.500	0.71095	6.02794	0.23300	1.07826	0.12090	5.24290	0.12845	6.03817
5.000	0.70872	6.05361	0.21148	1.06677	0.11260	5.30032	0.12313	6.05536
5.500	0.70711	6.07459	0.19413	1.05236	0.10610	5.35294	0.11917	6.07079
6.000	0.70590	6.09205	0.17988	1.03595	0.10092	5.40123	0.11616	6.08460
6.500	0.70497	6.10682	0.16800	1.01820	0.09671	5.44558	0.11381	6.09695
7.000	0.70424	6.11947	0.15797	0.99956	0.09325	5.48638	0.11194	6.10802
7.500	0.70365	6.13042	0.14940	0.98038	0.09037	5.52396	0.11043	6.11797
8.000	0.70318	6.14000	0.14201	0.96092	0.08795	5.55862	0.10919	6.12694
8.500	0.70279	6.14845	0.13559	0.94135	0.08590	5.59063	0.10817	6.13505
9.000	0.70246	6.15595	0.12998	0.92184	0.08414	5.62026	0.10731	6.14241
9.500	0.70219	6.16267	0.12503	0.90248	0.08263	5.64772	0.10658	6.14911
10.000	0.70196	6.16870	0.12066	0.88337	0.08132	5.67322	0.10595	6.15524
11.000	0.70158	6.17913	0.11328	0.84611	0.07917	5.71903	0.10495	6.16601
12.000	0.70130	6.18782	0.10733	0.81041	0.07750	5.75892	0.10419	6.17516
13.000	0.70108	6.19517	0.10247	0.77644	0.07618	5.79390	0.10360	6.18303
14.000	0.70091	6.20146	0.09844	0.74428	0.07511	5.82475	0.10313	6.18985
15.000	0.70077	6.20691	0.09507	0.71392	0.07424	5.85214	0.10275	6.19582
16.000	0.70066	6.21169	0.09222	0.68532	0.07352	5.87658	0.10243	6.20109
17.000	0.70056	6.21590	0.08979	0.65843	0.07292	5.89850	0.10218	6.20577
18.000	0.70048	6.21964	0.08771	0.63315	0.07242	5.91827	0.10196	6.20995
19.000	0.70042	6.22298	0.08590	0.60939	0.07198	5.93617	0.10178	6.21371
20.000	0.70036	6.22600	0.08433	0.58707	0.07161	5.95245	0.10162	6.21710

TABLE 11

Unsteady Loads and Phase Angles for $E = 0.8$

k	C_{LT}	φ_{LT}	C_{LP}	φ_{LP}	C_{MT}	φ_{MT}	C_{MP}	φ_{MP}
0.010	176.75511	4.67447	17675.96240	6.24889	96.49204	4.67045	9650.94092	6.24085
0.025	69.03058	4.64005	2761.67255	6.21992	37.70833	4.62979	1509.72499	6.19940
0.050	33.14782	4.60381	663.40230	6.19279	18.13891	4.58255	363.89515	6.15031
0.075	21.24415	4.58263	283.69844	6.18076	11.65206	4.54960	156.31243	6.11490
0.100	15.34446	4.57136	153.88546	6.17869	8.43861	4.52582	85.22010	6.08808
0.150	9.54888	4.56889	64.09693	6.19465	5.28145	4.49611	35.88132	6.05069
0.200	6.74479	4.58438	34.15966	6.22842	3.75078	4.48159	19.31671	6.02666
0.250	5.12553	4.61171	20.93620	6.27354	2.86278	4.47644	11.93542	6.01049
0.300	4.09006	4.64720	14.06570	0.04276	2.29068	4.47735	8.06079	5.99917
0.350	3.38223	4.68838	10.09303	0.09957	1.89552	4.48227	5.79499	5.99091
0.400	2.87503	4.73348	7.61349	0.15865	1.60855	4.48989	4.36401	5.98461
0.450	2.49866	4.78115	5.97378	0.21834	1.39207	4.49934	3.40650	5.97960
0.500	2.21175	4.83033	4.83885	0.27742	1.22381	4.51001	2.73624	5.97544
0.550	1.98831	4.88019	4.02355	0.33494	1.08981	4.52149	2.24978	5.97185
0.600	1.81123	4.93003	3.41933	0.39024	0.98094	4.53347	1.88608	5.96866
0.650	1.66886	4.97932	2.95948	0.44289	0.89098	4.54576	1.60733	5.96574
0.700	1.55298	5.02761	2.60135	0.49262	0.81556	4.55821	1.38916	5.96303
0.750	1.45765	5.07459	2.31672	0.53931	0.75153	4.57070	1.21529	5.96049
0.800	1.37850	5.12000	2.08642	0.58295	0.69658	4.58316	1.07454	5.95810
0.850	1.31224	5.16369	1.89708	0.62361	0.64896	4.59554	0.95902	5.95583
0.900	1.25635	5.20554	1.73919	0.66139	0.60735	4.60779	0.86307	5.95370
0.950	1.20888	5.24550	1.60582	0.69644	0.57070	4.61990	0.78251	5.95170
1.000	1.16831	5.28355	1.49187	0.72894	0.53821	4.63184	0.71421	5.94983
1.050	1.13343	5.31971	1.39349	0.75904	0.50923	4.64361	0.65581	5.94810
1.100	1.10329	5.35403	1.30776	0.78693	0.48322	4.65520	0.60548	5.94651
1.150	1.07710	5.38655	1.23242	0.81277	0.45977	4.66660	0.56180	5.94506
1.200	1.05424	5.41736	1.16570	0.83673	0.43852	4.67781	0.52363	5.94376
1.250	1.03419	5.44652	1.10621	0.85894	0.41919	4.68884	0.49010	5.94262
1.300	1.01653	5.47413	1.05283	0.87956	0.40154	4.69969	0.46046	5.94162
1.350	1.00092	5.50025	1.00465	0.89872	0.38536	4.71036	0.43415	5.94078
1.400	0.98706	5.52499	0.96095	0.91652	0.37047	4.72086	0.41067	5.94008
1.450	0.97471	5.54842	0.92112	0.93309	0.35673	4.73120	0.38963	5.93954
1.500	0.96367	5.57062	0.88466	0.94851	0.34403	4.74137	0.37071	5.93915
1.600	0.94485	5.61163	0.82024	0.97630	0.32127	4.76125	0.33814	5.93880
1.700	0.92952	5.64858	0.76506	1.00051	0.30149	4.78054	0.31122	5.93900
1.800	0.91688	5.68198	0.71723	1.02167	0.28415	4.79928	0.28872	5.93971
1.900	0.90637	5.71226	0.67535	1.04021	0.26883	4.81752	0.26971	5.94089
2.000	0.89753	5.73980	0.63833	1.05649	0.25522	4.83528	0.25349	5.94251
2.100	0.89004	5.76493	0.60537	1.07080	0.24304	4.85261	0.23955	5.94451
2.200	0.88364	5.78793	0.57581	1.08341	0.23208	4.86952	0.22746	5.94685

TABLE 11 (cont.)

Unsteady Loads and Phase Angles for E = 0.8

k	C_{LT}	φ_{LT}	C_{LP}	φ_{LP}	C_{MT}	φ_{MT}	C_{MP}	φ_{MP}
2.300	0.87813	5.80905	0.54915	1.09453	0.22218	4.88604	0.21691	5.94949
2.400	0.87336	5.82850	0.52497	1.10432	0.21319	4.90220	0.20765	5.95239
2.500	0.86920	5.84645	0.50292	1.11296	0.20499	4.91802	0.19948	5.95552
2.600	0.86556	5.86308	0.48275	1.12055	0.19750	4.93351	0.19222	5.95883
2.700	0.86234	5.87851	0.46420	1.12723	0.19061	4.94869	0.18575	5.96230
2.800	0.85949	5.89287	0.44710	1.13309	0.18428	4.96358	0.17995	5.96591
2.900	0.85696	5.90626	0.43128	1.13821	0.17843	4.97820	0.17473	5.96961
3.000	0.85470	5.91878	0.41659	1.14266	0.17301	4.99254	0.17003	5.97340
3.100	0.85267	5.93050	0.40292	1.14651	0.16798	5.00663	0.16576	5.97725
3.200	0.85084	5.94150	0.39017	1.14982	0.16330	5.02047	0.16188	5.98114
3.300	0.84918	5.95184	0.37824	1.15263	0.15894	5.03407	0.15835	5.98505
3.400	0.84769	5.96158	0.36706	1.15499	0.15486	5.04744	0.15512	5.98898
3.500	0.84632	5.97077	0.35656	1.15695	0.15105	5.06059	0.15216	5.99291
3.600	0.84508	5.97945	0.34668	1.15852	0.14747	5.07352	0.14943	5.99683
3.700	0.84394	5.98766	0.33736	1.15975	0.14411	5.08625	0.14693	6.00073
3.800	0.84290	5.99545	0.32856	1.16067	0.14095	5.09877	0.14461	6.00460
3.900	0.84194	6.00283	0.32024	1.16129	0.13798	5.11110	0.14247	6.00843
4.000	0.84106	6.00985	0.31236	1.16164	0.13517	5.12323	0.14048	6.01223
4.500	0.83754	6.04027	0.27847	1.16003	0.12325	5.18114	0.13243	6.03047
5.000	0.83506	6.06461	0.25168	1.15411	0.11404	5.23483	0.12664	6.04728
5.500	0.83326	6.08453	0.23000	1.14514	0.10677	5.28467	0.12233	6.06256
6.000	0.83190	6.10111	0.21211	1.13400	0.10092	5.33098	0.11904	6.07637
6.500	0.83085	6.11515	0.19712	1.12126	0.09613	5.37405	0.11647	6.08883
7.000	0.83003	6.12717	0.18440	1.10737	0.09217	5.41412	0.11443	6.10007
7.500	0.82937	6.13759	0.17348	1.09265	0.08886	5.45143	0.11278	6.11022
8.000	0.82883	6.14670	0.16402	1.07732	0.08606	5.48620	0.11143	6.11941
8.500	0.82838	6.15474	0.15575	1.06159	0.08367	5.51863	0.11030	6.12775
9.000	0.82801	6.16189	0.14848	1.04559	0.08161	5.54890	0.10936	6.13534
9.500	0.82770	6.16828	0.14205	1.02943	0.07984	5.57719	0.10856	6.14227
10.000	0.82743	6.17403	0.13632	1.01322	0.07829	5.60365	0.10787	6.14862
11.000	0.82700	6.18396	0.12658	0.98089	0.07574	5.65167	0.10677	6.15982
12.000	0.82668	6.19224	0.11865	0.94902	0.07375	5.69399	0.10594	6.16936
13.000	0.82643	6.19924	0.11210	0.91791	0.07216	5.73145	0.10528	6.17758
14.000	0.82623	6.20524	0.10661	0.88773	0.07087	5.76479	0.10477	6.18472
15.000	0.82607	6.21044	0.10198	0.85860	0.06982	5.79460	0.10435	6.19098
16.000	0.82594	6.21499	0.09802	0.83058	0.06895	5.82136	0.10400	6.19651
17.000	0.82583	6.21900	0.09462	0.80370	0.06822	5.84551	0.10372	6.20142
18.000	0.82574	6.22257	0.09167	0.77798	0.06760	5.86737	0.10348	6.20582
19.000	0.82566	6.22576	0.08910	0.75339	0.06707	5.88725	0.10328	6.20977
20.000	0.82560	6.22863	0.08685	0.72991	0.06661	5.90539	0.10311	6.21335

TABLE 12

Unsteady Loads and Phase Angles for E = 1.0

k	C_{LT}	φ_{LT}	C_{LP}	φ_{LP}	C_{MT}	φ_{MT}	C_{MP}	φ_{MP}
0.010	196.65359	4.67103	19666.10400	6.24683	98.34874	4.66595	9837.31934	6.23667
0.025	76.58089	4.63423	3063.97375	6.21754	38.33272	4.62123	1535.25427	6.19154
0.050	36.60524	4.59669	732.83153	6.19254	18.36706	4.56964	368.88816	6.13855
0.075	23.36459	4.57604	312.24437	6.18448	11.76069	4.53391	158.11719	6.10054
0.100	16.81733	4.56642	168.88106	6.18746	8.49580	4.50816	86.09673	6.07172
0.150	10.41053	4.56915	70.09733	6.21529	5.29980	4.47564	36.23966	6.03095
0.200	7.32957	4.59118	37.33051	6.26193	3.75816	4.45872	19.54582	6.00345
0.250	5.56135	4.62561	22.91816	0.03676	2.86760	4.45102	12.11692	5.98335
0.300	4.43745	4.66833	15.45432	0.10144	2.29575	4.44899	8.21832	5.96759
0.350	3.67375	4.71657	11.14844	0.16951	1.90169	4.45052	5.93725	5.95446
0.400	3.12978	4.76836	8.46438	0.23856	1.61597	4.45429	4.49497	5.94297
0.450	2.72854	4.82220	6.68983	0.30683	1.40061	4.45947	3.52836	5.93256
0.500	2.42450	4.87694	5.46051	0.37306	1.23327	4.46548	2.85046	5.92290
0.550	2.18911	4.93166	4.57569	0.43641	1.09999	4.47197	2.35742	5.91380
0.600	2.00368	4.98564	3.91802	0.49636	0.99163	4.47869	1.98797	5.90516
0.650	1.85544	5.03833	3.41557	0.55262	0.90200	4.48546	1.70414	5.89692
0.700	1.73544	5.08934	3.02244	0.60513	0.82677	4.49217	1.48145	5.88906
0.750	1.63726	5.13838	2.70838	0.65393	0.76281	4.49876	1.30354	5.88157
0.800	1.55614	5.18527	2.45284	0.69918	0.70783	4.50517	1.15915	5.87446
0.850	1.48853	5.22990	2.24151	0.74105	0.66010	4.51137	1.04035	5.86773
0.900	1.43175	5.27223	2.06422	0.77978	0.61831	4.51735	0.94141	5.86139
0.950	1.38371	5.31229	1.91358	0.81560	0.58144	4.52309	0.85812	5.85545
1.000	1.34279	5.35011	1.78410	0.84875	0.54868	4.52860	0.78732	5.84991
1.050	1.30773	5.38578	1.67167	0.87946	0.51938	4.53388	0.72662	5.84477
1.100	1.27750	5.41939	1.57317	0.90793	0.49305	4.53893	0.67417	5.84003
1.150	1.25130	5.45104	1.48615	0.93436	0.46925	4.54376	0.62852	5.83568
1.200	1.22848	5.48084	1.40871	0.95894	0.44764	4.54837	0.58853	5.83173
1.250	1.20850	5.50890	1.33933	0.98183	0.42793	4.55279	0.55329	5.82817
1.300	1.19092	5.53534	1.27681	1.00319	0.40989	4.55701	0.52207	5.82497
1.350	1.17540	5.56025	1.22016	1.02314	0.39331	4.56104	0.49427	5.82215
1.400	1.16164	5.58375	1.16857	1.04181	0.37803	4.56490	0.46939	5.81967
1.450	1.14939	5.60592	1.12139	1.05931	0.36390	4.56859	0.44704	5.81754
1.500	1.13844	5.62686	1.07805	1.07574	0.35079	4.57212	0.42688	5.81573
1.600	1.11978	5.66539	1.00114	1.10573	0.32723	4.57875	0.39203	5.81304
1.700	1.10459	5.69994	0.93492	1.13241	0.30666	4.58483	0.36308	5.81149
1.800	1.09207	5.73105	0.87725	1.15627	0.28854	4.59044	0.33873	5.81096
1.900	1.08165	5.75917	0.82654	1.17772	0.27246	4.59561	0.31805	5.81134
2.000	1.07288	5.78468	0.78157	1.19710	0.25809	4.60039	0.30031	5.81253
2.100	1.06544	5.80791	0.74140	1.21468	0.24518	4.60482	0.28498	5.81443
2.200	1.05908	5.82914	0.70528	1.23071	0.23351	4.60894	0.27162	5.81694

TABLE 12 (cont.)

Unsteady Loads and Phase Angles for E = 1.0

k	C_{LT}	φ_{LT}	C_{LP}	φ_{LP}	C_{MT}	φ_{MT}	C_{MP}	φ_{MP}
2.300	1.05360	5.84860	0.67261	1.24537	0.22291	4.61277	0.25991	5.81999
2.400	1.04885	5.86650	0.64291	1.25884	0.21324	4.61635	0.24957	5.82349
2.500	1.04470	5.88301	0.61579	1.27124	0.20438	4.61969	0.24040	5.82739
2.600	1.04106	5.89829	0.59091	1.28270	0.19623	4.62281	0.23222	5.83161
2.700	1.03785	5.91246	0.56800	1.29332	0.18871	4.62575	0.22490	5.83611
2.800	1.03500	5.92563	0.54684	1.30319	0.18176	4.62850	0.21831	5.84083
2.900	1.03246	5.93792	0.52723	1.31239	0.17530	4.63109	0.21236	5.84573
3.000	1.03019	5.94939	0.50900	1.32098	0.16929	4.63354	0.20697	5.85078
3.100	1.02815	5.96014	0.49201	1.32901	0.16368	4.63584	0.20207	5.85594
3.200	1.02631	5.97022	0.47614	1.33655	0.15843	4.63802	0.19759	5.86118
3.300	1.02465	5.97969	0.46127	1.34363	0.15351	4.64009	0.19350	5.86648
3.400	1.02314	5.98861	0.44731	1.35030	0.14889	4.64204	0.18975	5.87180
3.500	1.02177	5.99703	0.43419	1.35659	0.14454	4.64390	0.18629	5.87715
3.600	1.02051	6.00498	0.42182	1.36254	0.14044	4.64566	0.18311	5.88249
3.700	1.01936	6.01250	0.41015	1.36816	0.13657	4.64734	0.18017	5.88782
3.800	1.01831	6.01963	0.39911	1.37348	0.13291	4.64894	0.17745	5.89312
3.900	1.01734	6.02639	0.38866	1.37854	0.12944	4.65046	0.17492	5.89838
4.000	1.01644	6.03282	0.37874	1.38334	0.12615	4.65192	0.17258	5.90359
4.500	1.01287	6.06067	0.33595	1.40415	0.11193	4.65829	0.16300	5.92872
5.000	1.01035	6.08295	0.30190	1.42081	0.10060	4.66347	0.15604	5.95196
5.500	1.00850	6.10118	0.27415	1.43444	0.09136	4.66775	0.15082	5.97316
6.000	1.00712	6.11637	0.25110	1.44580	0.08368	4.67136	0.14682	5.99236
6.500	1.00604	6.12921	0.23163	1.45542	0.07720	4.67443	0.14367	6.00972
7.000	1.00520	6.14023	0.21497	1.46366	0.07165	4.67708	0.14116	6.02540
7.500	1.00452	6.14977	0.20056	1.47080	0.06685	4.67939	0.13912	6.03959
8.000	1.00396	6.15811	0.18796	1.47705	0.06265	4.68141	0.13744	6.05244
8.500	1.00350	6.16548	0.17685	1.48256	0.05895	4.68321	0.13604	6.06412
9.000	1.00312	6.17202	0.16699	1.48746	0.05566	4.68481	0.13487	6.07477
9.500	1.00280	6.17788	0.15817	1.49185	0.05272	4.68624	0.13387	6.08449
10.000	1.00252	6.18315	0.15024	1.49580	0.05008	4.68753	0.13302	6.09340
11.000	1.00208	6.19225	0.13654	1.50261	0.04551	4.68977	0.13164	6.10913
12.000	1.00175	6.19983	0.12514	1.50830	0.04171	4.69164	0.13059	6.12255
13.000	1.00149	6.20624	0.11549	1.51310	0.03850	4.69322	0.12977	6.13411
14.000	1.00128	6.21174	0.10723	1.51723	0.03574	4.69459	0.12912	6.14417
15.000	1.00112	6.21651	0.10007	1.52080	0.03336	4.69576	0.12859	6.15299
16.000	1.00098	6.22068	0.09381	1.52392	0.03127	4.69680	0.12816	6.16078
17.000	1.00087	6.22435	0.08828	1.52668	0.02943	4.69771	0.12780	6.16771
18.000	1.00077	6.22762	0.08337	1.52913	0.02779	4.69853	0.12750	6.17391
19.000	1.00069	6.23055	0.07898	1.53132	0.02633	4.69925	0.12724	6.17948
20.000	1.00063	6.23318	0.07503	1.53330	0.02501	4.69991	0.12702	6.18453

APPENDIX

EVALUATION OF INTEGRALS

Woods' analysis contains the following integrals which must be evaluated numerically

$$I_n = \frac{1}{2\pi}\int_0^\infty e^{-i\omega\left(\cosh x - \alpha \cosh \frac{x}{2}\right)} \cosh \frac{nx}{2}\, dx \qquad [11]$$

where $\quad n = 0, 1, 2$

$$\alpha = -4\left[\frac{1 - \sqrt{E}}{1 + \sqrt{E}}\right] \qquad [12]$$

and

$$\omega = \frac{k}{4}(1 + \sqrt{E}\,)^2$$

The integrand of Equation [11] is finite and well behaved in the vicinity of $x = 0$. At large values of x and for n greater than zero, the integrand becomes very large and oscillates wildly. Therefore, the integral is divided into two parts

$$I_n = I_n' + I_n'' \qquad [13]$$

The function I_n' includes an integration out to $x = x_o$.

$$I_n' = \frac{1}{2\pi}\int_0^{x_o} e^{-i\omega\left(\cosh x - \alpha \cosh \frac{x}{2}\right)} \cosh \frac{nx}{2}\, dx \qquad [14]$$

Over this range I_n' is evaluated by Simpson's rule.

In order to evaluate the integral from x_o to infinity, the following change of variable is made:

$$p = 2\left(\cosh \frac{x}{2} - \frac{\alpha}{4}\right)^2$$

Then

$$I_n'' = \frac{e^{i\omega_o\left(\frac{\alpha^2}{8}+1\right)}}{2\pi} \int_{p_o}^{\infty} \frac{G_n\, e^{-i\omega p}}{\sqrt{p\left(p+\alpha\sqrt{\tfrac{1}{2}p}+2\left(\frac{\alpha^2}{16}-1\right)\right)}}\, dp \qquad [15]$$

where $G_o = 1$

$$G_1 = \frac{\alpha}{4} + \sqrt{\tfrac{1}{2}p} \qquad [16]$$

$$G_2 = p + \alpha\sqrt{\tfrac{1}{2}p} + 2\left(\frac{\alpha^2}{16} - 1\right) + 1$$

and

$$p_o = 2\left(\cosh\frac{x_o}{2} - \frac{\alpha}{4}\right)^2 \qquad [17]$$

In order to improve the convergence of the integrals in Equation [15], a partial integration is performed. The resulting three integrals become

$$I_o = I_o' + \frac{i\, e^{i\omega\left(\frac{\alpha^2}{8}+1\right)}}{4\pi\omega}\left[-\frac{2\, e^{-i\omega p_o}}{(p_o\, Q)^{\frac{1}{2}}} + \int_{p_o}^{\infty} \frac{\left[p + \frac{3}{4}\alpha\sqrt{\tfrac{1}{2}p} + \left(\frac{\alpha^2}{16} - 1\right)\right] e^{-i\omega p}}{(p\, Q)^{3/2}}\, dp\right] \qquad [18]$$

$$I_1 = I_1' + \frac{\alpha}{4} I_o'' + \frac{i\sqrt{2}\, e^{i\omega\left(\frac{\alpha^2}{8}+1\right)}}{8\,\pi\,\omega}\left[-\frac{2\, e^{i\omega p_o}}{Q_o^{\frac{1}{2}}} + \int_{p_o}^{\infty} \frac{\left(1 + \frac{\alpha}{2\sqrt{2p}}\right) e^{-i\omega p}}{Q^{3/2}}\, dp\right] \qquad [19]$$

$$I_2 = I_2' + I_o'' + \frac{i\, e^{i\omega\left(\frac{\alpha^2}{8} + 1\right)}}{2\pi\omega} \left[- \left(\frac{Q_o}{p_o}\right)^{\frac{1}{2}} e^{-i\omega p_o} \right.$$

$$\left. + \lim_{p \to \infty} \left(\frac{Q}{p}\right)^{\frac{1}{2}} e^{-i\omega p} + \int_{p_o}^{\infty} \frac{\left[\frac{\alpha}{4}\sqrt{\tfrac{1}{2}p} + \left(\frac{\alpha^2}{16} - 1\right)\right] e^{-i\omega p}}{(p^3\ Q)^{\frac{1}{2}}}\, dp \right] \qquad [20]$$

where

$$Q = p + \alpha\sqrt{\tfrac{1}{2}p} + 2\left(\frac{\alpha^2}{16} - 1\right)$$

$$Q_o = p_o + \alpha\sqrt{\tfrac{1}{2}p_o} + 2\left(\frac{\alpha^2}{16} - 1\right) \qquad [21]$$

Although the second term in the bracket of Equation [20] has an oscillating finite value as p tends to infinity, it is customary in airfoil theory to assume that the oscillation frequency has a small negative imaginary part. That is,

$$\omega = \omega' - i\epsilon$$

On this basis the second term becomes zero in the limit.

The integrands of Equations [18], [19], and [20] are now defined over the complete range of p, and a numerical integration is possible. Due to the oscillatory nature of the integrands, the integrals were evaluated by Filon's method of quadratures.[4] Increment sizes and terminal points in the numerical integrations were chosen in such a manner that the results had an accuracy of five significant figures.

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5 AUTHOR(S) *(Last name, first name, initial)*

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2. Spoilers--Location--Effectiveness
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