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LONGITUDINAL STRENGTH AND MINIMUM WEIGHT

by

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[redacted]

STRUCTURAL MECHANICS LABORATORY

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NOTATION

a	Vertical acceleration at the c. g.
B	Ship beam
C_B	Block coefficient
d	Draft (design)
D_{HD}	Ship Depth
f	Risk factor
g	Acceleration due to gravity
H_{mdl}	Average height of two consecutive waves
H_{char}	Characteristic wave height
H_{max}	Maximum wave height
H_o	Wave amplitude (regular)
H	Wave height
h	Heave amplitude
L	Ship length
l_f	Distance between c. g. of whole ship and centroid of δ_f
l_a	Distance between c. g. of whole ship and centroid of δ_a
M	Bending moment
M_{max}	Max bending moment
M_{sag}	Bending moment in sag with Smith correction
M_{hog}	Bending moment in hog with Smith correction
M_{var}	(Total) Bending moment variation with Smith correction
M_a	Pitching moment about c. g. by inertia forces
m	Bending moment coefficient

N	Number of variations
n	Exponential of risk factor
P_c	Critical buckling load
R	Repair cost in percent of hull cost
r_f	Radius of gyration of forebody about c. g. of ship
r_a	Radius of gyration of afterbody about c. g. of ship
t	Plate thickness
V	Weight of entrained water
w	Plate weight
W	Ship's weight
$\dot{\omega}$	Angular acceleration
x_a	Moment arm of afterbody virtual mass
x_f	Moment arm of forebody virtual mass
Z	Section modulus
Z_o	Section modulus without unfairness
$Z_{meas.\ pt.}$	Section modulus (refer to location of measured point)
λ	Wave lengths
Δ	Displacement
δ	Plate deflection
δ_o	Initial plate deflection
δ_f	Displacement force of forebody
δ_a	Displacement force of afterbody
σ	Applied or working stress
σ_c	Critical buckling stress

σ_y	Critical yield stress
σ_{logx}	Standard deviation (log.normal)
σ_χ	Standard deviation (normal)
α	Phase angle
$\sqrt{E}$	Root-mean-square of the variate
γ	Density of salt water

FOREWORD

A research program in surface ship structures sponsored by the Bureau of Ships has been underway at the David Taylor Model Basin for many years. The general objective of the entire program is to provide realistic design targets for the naval architect and rational design procedures by which these targets may be attained. A prime target for optimum design is that of minimum weight. This is so partially because of the association of weight reduction with economy and more recently because of weight-critical ships. Consequently, the Model Basin welcomed the research efforts in this area made by Dr. Jan Getz, presently Director of Research, The Ship Research Institute of Norway, Trondheim, while he was at the Model Basin and at the University of California under the auspices of the National Research Council, U.S.A.

Dr. Getz presented some of his findings in a paper given before the 1960 Scandinavian Ship Technical Conference in Oslo, Norway. This paper was later published in the 1960 issue of European Shipbuilding, Vol. IX, No. 5. However, the continuing interest in minimum weight design among naval ship designers, warrants a further distribution of his findings within the Navy. In addition, the comments by the discussers of the paper, both at the conference and by correspondence, are considered by the author to add to the value of the paper. With the author's permission, therefore, the material presented in his paper, together with the resultant discussions and the author's replies, is republished as a Model Basin report. The assistance of Mr. S. E. Lee and Mrs M. K. Cook in preparation of the original material for publication in standard Model Basin format is gratefully acknowledged. Dr. Getz has recognized that the material which might be published under the heading of longitudinal strength and minimum weight is more comprehensive than could be presented in a single paper. The material presented herein was limited therefore to the philosophy of design and a discussion of longitudinal stresses. Later Dr. Getz plans to publish a critical examination of the problems of buckling, the optimization of stiffeners and girders as standard supporting elements, and typical minimum weight calculations for assembled panels in deck and bottom.

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ABSTRACT

The rational calculation of ship strength is now in sight, and the question of systematic optimisation is one of immediate interest. The variation of both the free and the leading dimensions must be studied on an economic basis, but first the design criteria and size and nature of the loading must be clarified. The danger of brittle fracture and fatigue and the importance of built-in or thermal stresses are discussed, while the calculations are based on a tough material whose yield point is the maximum effective strength under tension and is determined by the plastic buckling strength under compression.

The advantages of working beyond the buckling strength are discounted on the basis of the actual plate thickness and the shape of the initial distortions. The corrosion allowance is kept separate from the strength norm and a length-dependent working stress is not used.

The longitudinal stresses are based on statistic measurements and calculations with an economy-based risk of damage or need for change of speed and course. The danger of complete structural failure is discussed. As a basis for these observations, measurements from ships are compared with oceanographical data and statistical calculations based on them. The influence on the bending moment of ship motions at sea is discussed.

INTRODUCTION

Obviously, it is not a new aim for a designer of transport equipment to attempt to attain minimum weight and material consumption, and maximum pay load. However, it is only in recent years that an optimisation of designs has been subjected to systematic analysis. Aircraft designers, for obvious reasons, took the lead in this development. Later, the builders of vehicles and rolling stock on roads and railways followed to a considerable extent, while naval architects have on the whole neglected the systematic study of the problems of weight. The reason for this is partly that the economic importance of the hull steel weight has not been so obvious, but more important, perhaps, is the fact that both the load and the stress distribution in the structure present such complex problems that dimensioning is based to a considerable extent on empirical rules.

The results of research in many countries have, however, now brought us to the threshold of an epoch in which the strength of a hull may be calculated on a rational basis. This opens the way for the construction of better and cheaper ships. It is worth emphasizing that accumulated experience will continue to play a large role. However, an improved analysis and interpretation of this material will be possible on the basis of rational calculation.

The object of this work is to outline some of the knowledge which is now at our disposal in this field and to suggest how this material may be used in a purely rational longitudinal strength calculation where the necessary strength is obtained with a minimum consumption of material and building costs and an optimum load-carrying capacity.

PART 1 PHILOSOPHY OF DESIGN

THE MINIMUM WEIGHT PRINCIPLE

When the aim is to undertake a weight strength optimisation of a ship hull, it is necessary to remember that the structural arrangement is a compromise between the functional and the strength considerations, and that the former are steadily becoming more important in the total economy. The size, proportions, shape, and arrangement of a ship are mainly determined by the transport requirements--the propulsion, stability, and seagoing qualities-and the requirements for cargo handling. Minimum weight calculations may then be made for the various panels with the ship's main dimensions, bulkhead spacing, deck height, and deck openings as "leading" dimensions.

The remaining dimensions, such as stiffener spacing, choice of profiles, plate thickness and shape, and spacing of the girders, are then regarded as "free" dimensions which can be determined from a minimum weight analysis. Even these dimensions, however, are far from independent of practical restrictions. Cubic capacity, cargo stowing, production, cleaning, and maintenance must also be taken into account.

Minimum weight is used as the prime criterion in an optimum design. This is based on the assumption that both material price and the cost of production are approximately proportional to the weight. Further--and what is most important for the design of transport equipment--there is a maximum pay load.

The assumption that the building costs vary with the weight does not entirely hold true. When the design is given a more refined form or workmanship to save weight, it generally means an increase in production costs. Reduction in weight achieved by means of lightening holes does not effect any particular saving in costs. Finally, maintenance costs must also be taken into account.

In the second place, it is also worth analysing the influence of the leading dimensions on the weight and cost of the structure and balancing this against the other economic factors. This applies both to length, which is a very expensive dimension, and to the depth of the hull girder. An increase in the latter, without an increase in the draught, may increase the cubic capacity without much increase in the weight as the larger girder depth makes possible a smaller midship sectional area. Before beginning a study of the total economy and the economic main dimensions, it is appropriate, however, to have made analyses for the individual component panels.

Weight reductions in a structure may be potentially obtained by three essentially different methods:

a) By systematic variation of the "free" dimensions so that a given utilization of material (a determined nominal stress level) produces a prescribed carrying capacity (loading) with minimum weight.
b) By a raising of the nominal stress level justified by more certain determination of the existing loads and of the strength properties of the material, and further by a lowering of the stress concentrations.
c) By use of other materials with higher strength/weight ratios.

This work is mainly concerned with parts a) and b), and alternative materials will be limited to different grades of constructional steel.

The study of longitudinal strength does not enter into what is generally understood by minimum weight calculations. Nevertheless, the greatest latent possibilities for weight saving lie in precisely this field. Moreover, in order to achieve results of lasting values in the study of the panels which contribute to the longitudinal strength, it is not sufficient to operate with an arbitrarily assumed allowable stress.

It is not possible to avoid the problem of plastic dimensioning and plastic buckling, and it is therefore necessary to have a complete design philosophy for longitudinal strength, and to establish clear calculation criteria.

This work is therefore concerned with a discussion of the actual basis of calculation and of the bending loads which actually arise.

DESIGN PHILOSOPHY AND CALCULATION CRITERIA

There are three main problems in all strength calculations:

1. Determination of the load.
2. Calculation of the corresponding stresses.
3. Establishment of the strength properties of the material and of the buckling strength of the structure.

The calculation of the stresses arising from known longitudinal moments no longer presents any serious problem as far as the mean stresses are concerned. The determination of the stress concentrations at openings and changes of cross section, however, require further study in connection with the possibility of fatigue failure and brittle fracture.

The dominant load on a large vessel is the bending moment on the hull girder produced by the varying longitudinal distribution of weight and buoyancy and of the dynamic forces. The conventional method of placing the ship on a standard wave can be considered only as a comparative calculation, even though measurements at sea have shown that the process -- more or less accidentally -- produces stresses of about the right size. This by no means signifies that experience has led us to a final strength norm precluding further improvements. To progress further today, however, it is necessary to tackle the problems in a radical fashion and to analyse the structural problem on a purely rational basis.

The legacy from former practice which must first be discarded is the fictitious allowable nominal stress -- and its variation with ship length. We must have faith that it will soon be possible to determine the actual loading with reasonable accuracy and probability. The corresponding stresses which can be allowed should depend exclusively on the material, the method of joining and the buckling strength. The decisive material properties for carrying out strength calculations will be the yield point in tension and the plastic, eccentric buckling strength in compression, which, again, is dominated by the yield point. Of at least equal importance, however, is the toughness of the material--which is a decisive factor for the actual basis of calculation.

A very difficult problem in connection with the determination of ship static collapse strength and fatigue strength is the influence of the stresses not due to external load. This concerns built-in stresses during the manufacture of the material or caused by welding, straightening and mounting, and furthermore thermal stresses arising from temperature gradients over the structure.

As far as the static strength is concerned, these stresses have no influence on the maximum plastic carrying capacity in tension, provided that the material is sufficiently tough. Nor is it probable, as far as compressive stresses are concerned, that built-in or thermal stresses have a dominant influence on the load at point of collapse, but it is reasonable to assume that they have an influence on the load when the damage first becomes visible. With reference to the danger of fatigue it is clear that these stresses have an influence on the average level about which the stresses vary at a certain point of the structure, and this will to some extent affect the fatigue strength. These questions have not yet been fully elucidated, but their solution is essential for the full application of a rational strength calculation.

The question of corrosion allowance should be kept completely separate from the actual strength calculation and should in the future be left in principle to the owners. The classification societies should basically prescribe the scrapping thicknesses which would thus be identical with the design thicknesses. Such a practice would provide full stimulus to the employment of the corrosion protection methods which are available today and under rapid development.

When we have thus reached a rational basis for calculation, there is the danger that the well-known "safety factor" will raise its head. Let it be said at once that in principle there would not be room for any such factor in an advanced strength calculation since it is assumed that the determination of the greatest strain for which we design represents the absolute maximum combined load which will occur at a chosen low probability. It is also assumed that the yield point and the buckling strength of the welded material are determined as probable minimum values. The safety will then be in the choice of low probability values or, if preferred, in low risk figures. We should distinguish here between the risk of damage to the structure, which can be chosen on a purely economic basis, and the risk of total failure and loss of life, which must be evaluated from combined humane and economic viewpoints.

It is worth noting carefully that we introduced above, the plastic buckling strength. Conventional practice based on elastic buckling calculations affords no opportunity for the optimum utilization of the material in panels subjected to high loads. Ships which sail today would undoubtedly break after buckling of the deck or bottom panels at a considerably lower bending moment than that which the built-in material in itself makes possible. In other words, if the safety of these ships against structural failure of this type is considered satisfactory, then there is room for big savings.

The ultimate plastic strength in compression is, in addition to the yield point, dominated by the eccentricity and initial deflection of the structure. The determination of a minimum collapse strength is therefore indissolubly connected with the determination and limitation of these quantities in practice. This will perhaps encounter considerable indolence, but it is necessary to face the fact that inaccuracies in construction can be equally significant for the total strength of the ship as material properties.

As far as local deflection of the plates between longitudinal stiffeners is concerned, the question can justifiably be raised whether the minimum weight criterion can be associated with the theoretical buckling limit. It is well known that aircraft designers do not follow this principle. They find it economical to exceed the buckling limit, let the plate buckle between the stiffeners, and merely reckon with a reduced effective plate width. And as a plate in practice is never ideally flat, a gradual buckling takes place as the axial load increases, and the theoretical buckling limit displays no characteristic point at all on the stress-deflection curve (Figure 1). The effect of such a practice on the weight of an initially plane plate of given breadth under axial stress is shown in Figure 2. It will be seen that the weight decreases, although slowly, with the working stress even after the buckling limit has been passed and correction is made for the reduced effective width.

This observation applies, however, only to a plate of a given unstiffened width. It is easy to show that it would be worthwhile preventing buckling of a stiffened panel by means of more closely spaced stiffeners. Only when this cannot be done for practical reasons or due to production cost does the constant width case become applicable. In the appraisement of a ship design, however, there are also several other factors which make the principle of supercritical stresses inapplicable.

a) For larger ships, the axial load to be transferred per unit width of a panel is so great that it would involve excessive practical difficulties to apply the necessary section area mainly to the stiffeners.

b) The lateral pressure on the plates necessitates a considerable plate thickness if the distance between the stiffeners is to be kept at a practical level at the same time that the local bending stresses are kept moderate so that the material can contribute fully to the longitudinal strength.

c) As a consequence of a) and b), the plate thickness becomes so great in relation to the distance between stiffeners that the deflection must be kept small in relation to the plate thickness so that the stresses shall not result in local yielding arising from combined bending and axial stresses.

d) As a result of a), b), and c), the margin in excess of the buckling limit which might be permitted is very small and the corresponding weight reduction inconsiderable.

e) With regard to the effect of the initial deflection, it may even be a question of keeping well below the theoretical buckling limit. However, the form of the initial buckles in welded ships might be of help. These distortions will mainly be caused by welding shrinkage from the attachment of the sections, and are expected to pull the plates inward between the profiles (Figure 3). (Observations recently made on two ships, one with and one without plates straightened by heating, show a deflection pattern of a more confused character). The further deflection caused by axial force will hypothetically occur in the same pattern so long as this is a stable distortion form. However, as this form corresponds to clamped plate edges with a theoretical buckling stress of 7/4 of the minimum buckling stress for freely supported edges (alternate buckling), the deflection will grow up to the minimum buckling stress. This means that the effective width will not be substantially reduced up to this point. As soon as the minimum buckling stress is passed, however, this form of distortion becomes unstable* and one must allow for the possibility that the pattern will suddenly switch over to alternate buckling with considerable deflections and substantially reduced effective plate width. Then the section modulus is also reduced as is indicated, with some exaggeration, in Figure 4.

The moment carrying capacity of the hull girder will vary with the working stress, as indicated in Figure 5. If the minimum buckling stress is much below the yield point, the static carrying capacity will theoretically achieve its maximum after alternate deflection has taken place (Figure 5 a), but the local yielding and incipient distortions could not be accepted for a ship with the repeated and reversed stresses to which it is subjected. When the buckling limit lies near to the yield point, as it should (Figure 5 b), the hull girder has reached its maximum bending strength when the buckling stress is reached, and this moment is the useful design strength of the ship. Certainly, in this case the ship will also have some energy absorption capacity after the buckling limit is reached. Even if the corresponding distortions involve severe damage, this reserve is of substantial importance for the safety of the ship against complete structural failure.

It may altogether be concluded that the plastic buckling stress

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*In a recent test designed to study this possibility, alternate deflections developed gradually long before the critical load was reached.

for the plates between the longitudinal stiffeners must be reckoned to determine the theoretical upper limit for the longitudinal strength of the hull girder. We then assume that stiffeners and girders have at least corresponding strength. In practice, the sudden drop in strength at the buckling stress will seldom occur if the curves are rounded off.

We have so far discussed the material from the point of view of its tough, static qualities. In one way, this is highly unrealistic. In recent years the majority of total losses arising from insufficient strength have been due to brittle fracture, and the greater number of failures causing economic concern are of a fatigue nature.

All the same, the maximum static strength is the natural design basis. The epidemic of brittle fracture which was experienced with vessels built during World War II has now been overcome. Brittle fracture is still a latent problem, but the probability of disastrous fracture has already been decisively reduced by the material and design requirements introduced by the classification societies. There is every reason to believe that future developments will further reduce the danger of fractures of this nature. On the other hand, we do not know of a very dominating connection between the nominal stresses and the risk of brittle fracture. It will most probably be uneconomic in the future to tackle the brittle fracture problem by limiting the nominal stresses. For a material with sufficiently low transition temperature and a design with sufficiently smooth transitions and careful workmanship, we may therefore base the quasi-static tensile strength on the yield point.

The importance of fatigue for longitudinal strength has been discussed in ship technical circles for many years, and the most divergent opinions have been expressed. There is still too little factual material data available to settle the problem numerically, but in principle, there should no longer be any need for disagreement. Were it possible to design a hull as an ideal box girder, our present knowledge of the load spectrum and ship steel fatigue strength -- including few but high stresses -- would seem to indicate that fatigue fracture should not occur. But with the stress concentrations which are unavoidable in practice, the full utilization of the static strength of the hull could very easily involve fatigue fracture. It thus becomes the task of future research to determine how great stress concentrations can be tolerated without fatigue fracture occurring too often. Today it seems clear that the notch effects are so great that fatigue is decisive for the longitudinal strength. This is reflected among other things in the modest demands for buckling strength of the deck and bottom panels. This is undoubtedly an uneconomic practice, and it would pay to raise the buckling stress and reduce the stress peaks by means of more subtle design if this is compensated by higher allowable nominal stresses. The final aim is to bring the fatigue strength up to a level where dimensioning from static and dynamic material properties would result in equal scantlings.

In the foregoing, the longitudinal strength has been discussed independently of the local stresses. This can be justified only on the assumption that the design is carried out so that the longitudinal stresses arising from the local loads are kept very moderate. We shall here assume that such a principle leads to an economic result, but it is clear that the choice here also is a compromise which could be optimised.

PART II LONGITUDINAL BENDING MOMENT

FORCES ACTING ON A SHIP

The forces acting on a ship are partly weights and inertia forces arising from ship acceleration in a seaway and any vibrations which may be present, and partly static and dynamic fluid pressure, including wave impact and slamming, depending on the motions of the sea and the vessel. The resultant loading along the hull girder gives the shearing force and the bending moment both in the vertical and horizontal planes of the ship as well as a torsion couple.

The horizontal moment has not so far been especially considered in the determination of the longitudinal strength but has been taken care of through the empirically determined working stresses. When we come to a rational strength calculation, the horizontal moment must be taken into consideration in the determination of the midship section modulus. The few measurements so far available indicate that simultaneously occurring stresses from the horizontal moment can lie between 10 and 50 percent of the stresses from the greatest vertical moment (Reference [1], pp 14, 29), (Reference [2], pp 57--59). Even though these stresses add themselves fully only at the corners and thus do not have a full effect on the "collapsing moment," the combined stresses have a fairly direct influence on the "damage moment."

The shearing forces in the two planes have no substantial influence on the stress amidships. When optimising the longitudinal distribution of the materials, it may, on the other hand, be necessary to study the influence of the shearing stress more closely.

The nominal torsion stresses also have only a small influence with the deck openings which are normal today. If substantially bigger hatch openings come into use to facilitate cargo handling, however, torsion may become a dominant problem.[3, 4]

Hull vibrations initiated by machinery and propellers can be limited today so that they have no appreciable influence on the longitudinal strength. The impulses must be kept small so that the forced vibrations do not become noticeable, and resonance must be avoided by advance calculation of the various natural frequencies of the hull and suitable choice of engine and rpm.

On the other hand, it may be impossible to avoid the effects of slamming, wave impact, and rapid immersion of the bow flare. Such impulses can result in big momentary stresses and subsequent powerful vibrations which must definitely be included in a rational strength calculation. In extreme cases, the additional stresses may emerge to over 100 percent;[5] values of 20 to 50 percent of the wave stresses are, however, more normal.

It is convenient to divide the vertical bending moment into a still-water bending moment and a wave bending moment. The calculation

[1]References are listed on page 70.

of the still-water bending moment presents no problem, and several investigations have clarified the influence of different ship arrangements on the bending moment in loaded and ballasted condition.[6, 7] On the whole, it is a question of keeping the still-water bending moment as small as possible, but from the point of view of optimisation, it may eventually prove to be an advantage to have a certain hogging moment in still water. This should not only compensate for a sagging moment from the waves which is greater than the hogging moment, but also for the possible position of the neutral axis under half the moulded depth. In sagging we have the compressive stresses in the deck, and as the destructive compressive stresses are necessarily lower than the dangerous tensile stresses in a tough material, it is worth while keeping the total sagging moment somewhat lower than the total hogging moment. Here may also be included the sagging moment arising from the changed pressure distribution round a ship moving ahead.

Another small correction which has been indicated is the bending moment arising from an eccentric attack of the axial water pressure. There is no point in including this in the still-water calculation, however, as this moment changes in a seaway. And as the axial pressure is greatly reduced during extreme hogging and the eccentricity small during extreme sagging, the effect can safely be ignored compared with other uncertain factors.

In the following, we shall concentrate on the wave bending moments. We have three ways of determining these:

a) Measurements aboard ships.
b) Model tests.
c) Calculations.

Each of these methods has its advantages and drawbacks. Statistical measurements aboard ships over long periods of time provide directly the information required, but only for a certain type of ship, and perhaps only for a certain route. Model tests open up a simpler way of studying many ship types, and particularly for the study of the effect of systematic variations. Calculations are able potentially to provide fuller information in a shorter time. Both models tests and calculations must, however, build on an assumed wave condition, and this condition must be selected from oceanographic wave spectra.

The result of these studies of wave bending moments is now beginning to be a source of information which can be used for practical calculation, at least as far as the vertical moment is concerned. The results of measurements made at sea must therefore constitute the solid basis, with calculations and model tests used as tools for interpolation between the measured ship types and sizes.

STATISTICAL MEASUREMENTS ABOARD SHIPS

During the last few years, a number of statistically planned and analyzed measurements have been carried out aboard ships in service [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. The results of these measurements are collected in Figures 6 and 7, and data of the vessels are given in Table 1. Figure 6 shows the probability of exceeding a given vertical wave bending stress variation. In Figure 7, the bending stress variation is replaced by the effective value (root-mean-square $= \sqrt{E}$) of this quantity over a large number of variations at the same weather, course, and speed condition.

The most important problem is now the determination of the probability to be applied when reading off such load spectra for design purposes. In using the $\sqrt{E}$ diagram, we must also choose a risk factor in the transition from the effective value to a probable maximum value. These questions will be discussed more closely in later sections.

Figures 6 and 7 are drawn on logarithmic probability paper on the assumption that the distribution is linear (logarithmic normal distribution). It may be mentioned in this connection that data from some ships can be reproduced with equally good or better approximation on arithmetical probability paper (simple normal distribution). This is explained by Figure 8 which shows that for a standard variation $\sigma_{\log x}$ up to 0.15, the normal and the log-normal distribution may plot well within the confidence limits applicable. For most ships, the stress distribution function is narrow or steep enough to give a standard variation of less than 0.15, and the amount of measured data will not generally be sufficient to give information to distinguish between the two types of

TABLE 1

Principal Characteristics of Ships Discussed in Text

No.	Ship	Type	Δ tons	L ft	B ft	d ft	D_{HD} ft	C_B	$Z_{meas.pt.}$ ft in^2
1.	ESSEX	Aircraft carrier	41,500	820	92	29	60.8	0.66	167,000
2.	SPERRY	Destroyer	3,350	369	40.6	14.5	24.2	0.54	7,750
3.	FESSENDEN	Destroyer escort	1,630	306	36	--	20.7	--	--
4.	UNIMAC	Coast Guard cutter	2,500	300	41.7	11.6	24.6	0.56	10,700[1]
5.	GOPHER MARINER	C-4 cargo ship	18,700	528	76	27	44.5	0.61	--
6.	MOREMAC	C-3 cargo ship	17,600	465	69.5	28.5	42.5	0.67	47,000
7.	OCEAN VULCAN	Cargo ship	13,750	416	56.9	37.3	37.3	0.75	--
8.	CANADA	Cargo ship	14,300	465	64	26.7	39.8	0.65	29,600
9.	MINNESOTA	Cargo ship	11,500	440	59	25.4	37.5	0.62	22,100
10.	ESSO ASHVILLE	T-2 tank ship	21,900	503	68	30	39.3	0.75	47,700

[1]Including effective deck house.

distribution.

This leaves us in an awkward position as far as the long-term distribution of the stress itself is concerned. If such a diagram is to be used for design purposes, there is a need for extrapolation outside the measured range, and here the two types of distribution will give completely different results. Such an extrapolation therefore cannot be considered permissible. Using instead the distribution of the root-mean-square value of the stress, the need for extrapolation vanishes, and the type of distribution function is of no importance. Admittedly, we have now delayed the solution of the problem confronting us, and in determining the maximum value corresponding to the $\sqrt{E}$ value and a chosen risk factor, we must assume some short-term distribution function. Jasper[10] has shown, however, that the Rayleigh distribution is well suited to determine the probable maximum values, and the author has therefore chosen to work with a log-normal $\sqrt{E}$ distribution and a Rayleigh short-term distribution.

MODEL TESTS AND CALCULATIONS

The results of a number of model tests have been published in recent years. The majority were carried out in regular waves of moderate height. The results so far have a limited design value, to a large extent because the results have been somewhat confusing as far as the effect of ship speed is concerned.

Model data have a particular interest for the control of theoretical calculations. To carry out a relatively complete theoretical calculation of the bending moment, it is necessary to know the ship motions in the sea. It has been possible to calculate with good approximation both the pitching and heaving amplitudes and phases of regular waves for various speeds at sea. It has also been possible to calculate by means of a strip method the bending moments in hogging and sagging with a reasonable degree of agreement with the model test data.[13, 14]

The author has not investigated whether this method of calculation has inherent possibilities of simplification to a practical design formula. The first difficulty that will be encountered is, of course, the transition to an irregular seaway where both the amplitude and the phase of the motions will vary continually. The only method which presents itself is linear superposition of the harmonic components of which the irregular sea is made up. This is a big task in itself, but it is also doubtful whether the linear superposition will be sufficiently accurate when the bow and aft flares are deeply immersed, not to mention the effect of shipped water.

At the present stage of development, we must satisfy ourselves with a far simpler method of calculation -- of the kind already in use by Norske Veritas. This method does not take into account the vertical accelerations of the ship but particular attention is later paid to their effects.

STATIC CALCULATION

We will here rely on the Norske Veritas' investigations for the design of tankers and cargo ships.[6, 7] Trochoidal-shaped waves were used here and Smith's correction was included. The bending moment was found to vary nearly in proportion to the wave height and linearly with the block coefficient within the range investigated ($H < L/15$, $0.6 < C_B < 0.8$). Normal variations in draught were seen to have little influence on either the hogging or the sagging moment, particularly for the fuller shapes, when Smith's correction was included.

The longitudinal bending moment can thus be written in the following simple form:

$$M = m \cdot \gamma \cdot f(C_B)HBL^2$$

where H is the wave height.

In the above-mentioned publications, a conservative influence of C_B is finally used. If the values which can be read from the diagrams are used instead, we obtain the values for the bending moment coefficient m which are given in Table 2. As the table shows, the coefficients are almost identical for the two ship types, and the following general formulas can be used:

TABLE 2

Wave Bending Moment Coefficients

	Tankers			Dry Cargo Ships		
	Hog	Sag	Var	Hog	Sag	Var
m	0.0125	0.015	0.0275	0.0125	0.015	0.275
$f(C_B)$	$\frac{C_B - 0.1}{0.7}$	$\frac{C_B + 0.3}{1.1}$	$\frac{C_B + 0.1}{0.9}$*	$\frac{C_B - 0.1}{0.7}$	$\frac{C_B + 0.4}{1.2}$	$\frac{C_B + 0.15}{0.95}$*
*Somewhat conservative for smaller C_D values.						

$$M_{sag} = 0.015 \frac{C_B + 0.35}{1.15} \quad HBL^2$$

$$M_{hog} = 0.0125 \cdot \frac{C_B - 0.1}{0.7} \quad HBL^2$$

$$M_{var} = 0.0275 \frac{C_B + 0.13}{0.93}$$

$$\approx 0.0325\, C_B \; HBL^2$$

The relationship between the sagging moment and the total moment variation varies with the block coefficient.

$$\frac{M_{sag}}{M_{var}} = 0.44 \frac{C_B + 0.35}{C_B + 0.15} \approx 0.44 \left(1 + \frac{0.185}{C_B}\right)$$

This expression gives the figures shown in Table 3. The tendency shown appears reasonable when compared with observations at sea and model test results.

TABLE 3

Relation Between Block Coefficient (C_B) and Hog to Sag Variation

C_B	0.6	0.7	0.8
$\frac{M_{sag}}{M_{var}}$	0.578	0.557	0.542

Most measurements made at sea are worked out statistically from, moment variations from, for example, a hogging value to the following sagging value. Extreme values of hogging and sagging moments do not necessarily follow immediately after one another, but the difference between $(M_{sag.\ max} + M_{hog.\ max})$ and $(M_{sag} + M_{hog})_{max}$ is not particularly great.[1]

In a more exact calculation of the bending moment, a number of effects are involved in addition to Smith's correction. Most of these are connected with the ship motions and cannot be introduced directly into a simplified calculation. The biggest correction, which arises from the disturbance of pressure caused by the presence of the hull in the wave, may however, be roughly taken into account through a general reduction of the moment of at least 20 percent. If we include this correction and further assume a proportional variation with the block coefficient (which gives only ± 2 percent maximum error for $0.6 < C_B < 0.8$), we obtain the following simple expression for the total moment variation -- excluding slamming etc:

$$M_{var} = 0.026\ C_B\ HBL^2$$

This expression takes no account of the inertia forces arising from the ship vertical accelerations, and this question will be discussed below. There also remains the choice of wave height as a decisive factor in the calculation.

PITCHING AND HEAVING MOTIONS OF A SHIP

The motions of a ship in regular waves have been studied fairly thoroughly through model tests, and it has also been possible to achieve good agreement with theoretical calculations. For wave lengths which do not differ too much from the ship length, and thus cause the greatest bending moments, the ship will heave and pitch in the period of encounter, and the amplitudes will be approximately proportional to the wave height, depending very much, however, on the tuning factor. Near resonances between the period of encounter and the natural periods, the amplitudes become great, and slamming may frequently occur with large waves and a small draught.

The phase angle between the motion and regular waves also depends on the period of encounter. With low frequencies of encounter, the ship will behave in about the same way as a plank, following the level and slope of the surface without any great "physical" phase lag. (In practice, however, it is desirable to measure the phase angle for both motions from one and the same point on the wave profile, and it is unusual to use the center of the wave trough as the point of reference (Figure 9). In the above-mentioned case, we thus obtain 0 degree phase lag for pitching and 90 degree phase lag for heaving -- measured relative to the ship center of gravity).

If the ship speed is increased so that the frequency of encounter becomes higher, the inertia forces will delay the motions in relation to the impulses. When the period of encounter coincides with the pitching or heaving period, the phase lag should, according to simple theory, be 90 degrees (or 180 degrees for heaving with the above definition of the zero point), but this does not agree so very well with the model tests. The phase lag tends to be considerably less for heaving.

For very high speeds in head seas, the phase lag increases theoretically to 180 degrees (270 degrees for heaving) and the position of the ship is in direct antiphase to the impulses. The motion corresponds to that of a telegraph pole floating vertically in short waves. The center of gravity is in the lowest position as the wave top passes. Further, a ship lowers its bow as it goes into a wave front. The model tests show a certain amount of dispersion in the phase lag for different models at high speeds.

These observations are not directly applicable to a ship in an irregular seaway. But as we are only concerned here with the greatest effect of the motions on the longitudinal stresses, we may note that the maximum motion amplitudes occur in regular waves in the vicinity of the resonance ranges. In an irregular seaway, short wave trains with approximately constant period may occur, and the motions may then assume amplitudes approaching those in regular waves, while at the same time the phase angle adjusts itself in the corresponding direction. The phase angle will, however, depend on the state of motion at the beginning of the wave train, and it will presumably be necessary to take into account that the phase lag may be the least favourable from the point of view of strength. We are not concerned here with the supercritical speeds, but assume that cargo ships are propelled at or near the synchronous periods of encounter. The biggest additional stresses in sagging ($\alpha = 0$ degree, see Figure 9) will generally be obtained when the ship is lying in its lowest position (90 degree phase lag) with maximum upward acceleration and with the bow down (90 degree phase lag). In hogging also ($\alpha = 180$ degrees), the accelerations from heaving in this phase will increase the bending moment.

Both model tests in irregular waves and observations at sea show that maximum accelerations often occur at about the same time as the largest bending moments. Heaving and pitching accelerations are strongest when the ship is sailing in head or nearly head seas and is proceeding at the maximum service speed which conditions permit. It is also at these comparative headings that the largest wave bending moments occur. The amplitudes of the motions have also been studied, and normal maximum amplitudes for larger ships appear to be between $\pm$ 4 to 5 degrees in pitching and a heaving acceleration of about $\pm$ 0.2 g. Extreme values may be 50 to 100 percent greater, but as the violence of the motions is to a certain extent under the control of the ship's master, it should not be necessary to base the strength on the most extreme values which can be obtained. It must, however, be assumed to be more economical to design for the more usual maximum values, so that the ship makes its journeys without frequent or long delays.

Accordingly, it seems possible to draw certain general conclusions concerning the amplitude of the motions as well as the phase angle. The numerical choice may be made the subject of closer studies, but the order of magnitude is already known, as indicated above. The question then arises whether it is possible to reach a generalization of what effects these motions have on the longitudinal stresses.

THE EFFECT OF HEAVING AND PITCHING ON THE MIDSHIP BENDING MOMENT

When a vessel is subjected to vertical oscillations, this will generally result in a bending moment along the hull girder. For the sake of simplicity, we will satisfy ourselves with considering the bending moment at the ship center of gravity at the moment when the vessel is in an extreme position where the accelerations and inertia forces are maximum, but where the oscillation speeds and resistance (damping forces) are nil.

As heaving and pitching are strongly coupled at larger motion amplitudes, it is simplest to study the combined effect for a specific case where the waterline profile is given (Figure 10). VL 1 is the wave profile for the ship in static equilibrium and VL 2 is the wave profile at given heaving and pitching amplitudes. In accordance with the above discussion, we choose to study the case with the ship in the lowest position and with the bow down in sagging, together with the ship in the highest position with the bow up in hogging.

It is now a purely geometrical task to determine the additional displacements (possibly with correction for Smith's effect) and their centers of attack on the forebody and afterbody.[1] The weight distribution of the ship and the weight of entrained water are also determined, and the resultant center of gravity (x) and radius of inertia (r) are established for each half (Figure 11).

The displacement force which gives the ship center of gravity a vertical acceleration is $(\delta_f + \delta_a)$ and with the ship weight W and weight of entrained water V, the following acceleration is obtained:

$$\frac{a}{g} = \frac{\delta_f + \delta_a}{W + V}$$

At the same time, an external moment with respect to the center of gravity arises of $M = \delta_f l_f + \delta_a l_a$, which gives the ship an angular acceleration of

$$\dot{\omega} = \frac{\delta_f l_f + \delta_a l_a}{\frac{(W+V)_f}{g} r_f^2 + \frac{(W+V)_a}{g} r_a^2}$$

When the accelerations are thus determined for the ship as a rigid body, the inertia forces exerted on the two halves of the ship can be calculated and the moment at the point of gravity determined

$$\begin{aligned} M_a &= \delta_f l_f - \frac{(W+V)_f}{g} (ax_f + \dot{\omega} r_f^2) \\ &= \delta_a l_a - \frac{(W+V)_a}{g} (ax_a - \dot{\omega} r_a^2) \end{aligned}$$

if the forces are taken as positive in the direction in which they are drawn, a positive moment will imply a sagging moment.

To be able to draw general conclusions as to the size of the correction of the static wave bending moment arising from an empiric size of the vertical motions, it is necessary to calculate some typical cases. Such examples are given in Figures 12 and 13.

In order that the wave profile should not exceed the ship profile, the pitch angle is taken as $\pm$ 3.5 degrees, the heave acceleration as $\frac{a}{g} = \pm 0.15$ for the cargo ship and ± 0.076 for the tanker. In addition, the tanker is studied at reduced draught. It should be mentioned, also, that the accelerating forces and moments are not large enough to correspond to harmonic oscillations at the period of encounter. Even these reduced amplitudes seem, therefore, to involve very severe if not exaggerated conditions.

These dynamic additions to the wave bending moment seem to fit in well with the trend of model tests results. The great influence of the still-water bending moment is clearly brought out, and the overall addition goes from almost nothing to 20--30 percent of the wave moment. The additions can be assumed to grow with motion amplitudes up to resonance speed.

WAVE HEIGHTS

If the bending moment is to be calculated on the basis of static and dynamic formulas, a physically probable or possible wave height and wave profile must be introduced. The latter is perhaps the most vexing question. A number of actual wave profiles have been established on the basis of stereoscopic photography, but very few include extreme wave heights. The most we can conclude from these profiles is that they are extremely irregular and that a particularly deep wave trough seldom has extreme wave crests on both sides -- which favours the ship in the sagging condition somewhat, as an average height may be allowed for discussion (see Figure 14).

As the wave profile has a great influence on the bending moment, and as it is so difficult to determine the unfavourable profile which should be combined with the extreme wave heights, the bending moment calculation is already doomed to be approximate.

We can undertake a final adjustment of the wave height to be used in the calculation through comparison with measured stresses on ships in service. A condition for a sound comparison, however, is that it is based on a corresponding probability.

We have no regular wave measurements of heights and lengths which are comprehensive enough for this purpose, but must take as a basis the visual observations made from the weather ships in the North Atlantic.[15] As long as the route or operational area of the ship is not determined, the choice of the observation material to be used is necessarily somewhat arbitrary. Here it has been decided to use the average for all the stations in the North Atlantic, and this should be quite representative for ships sailing in this sea area.

The observed height is the characteristic height which, according to the instruction, is the average height of the larger well-formed waves. The characteristic* height provides a good measure of the roughness of the sea. Comparisons between these approximate, visual observations and more stringent measurements show, as we shall see, that the characteristic height is fairly proportionate to the root-mean-square $\left(\sqrt{E}\right)$ height. A wave observation of several minutes is made every fourth hour. At the same time, the dominant period of these well-formed waves is observed so that it is possible to tabulate a number of wave observations both for height and for length. The following analysis is based on some 10 years of observations at 10 positions or about 20,000 characteristic heights.

In studying the probability of the occurrence of a seaway which may cause large bending moments, we are interested only in those waves whose length does not differ too much from the ship length. Until a more exact method is developed, we will content ourselves with assuming that waves within a certain length interval have the same maximum effect on the bending moment, and that other waves have no effect.

Figure 15 shows the dependence of the static wave bending moment on the wave length for regular waves and constant wave height for a block coefficient of 0.80. The choice of the wave length interval is very arbitrary. For a heading directly into the waves, the interval $0.7L < \lambda < 1.4L$ would appear to be reasonable. If we next include the effect of up to 60 degree oblique heading, wave lengths down to $\lambda = 0.4L$ will come into consideration. For irregular seas, the top of the curve will be flatter and better suit the stepped function to be

*According to Jasper's terminology, the expression "characteristic" is used for the visually determined heights, but the word "significant" is reserved for the statistically exact concept H 1/3 which is the average height of the "largest third" of the waves.

employed. If we include the dynamic effects, we may obtain maximum bending moments for wave lengths which are quite different from the ship length. The dynamic additions cannot, however, be generalized and should be dealt with separately.

In assuming that the ship acts as a fairly broad filter, we have assumed that the ship is always sailing at an unfavourable angle in relation to the waves. We shall return later to the effect of an arbitrary course in relation to the dominant wave direction.

Figure 16 is compiled on the basis of these wave observations and shows the probability of the characteristic wave height exceeding given values for various length intervals, which correspond to the observed wave periods. Figure 17 shows corresponding curves where the length interval is adjusted to the response interval $0.4\ L < \lambda < 1.4\ L$ for certain values of the ship length L.

The characteristic wave height which is exceeded a given percentage of the time in the North Atlantic is taken out of this figure and reproduced in Figure 18 as a function of the ship length. The probability of exceeding a large characteristic wave height reaches a maximum, and there is no reason to operate with a wave height which increases with the ship length beyond L = 600 feet. On the contrary, as long as the economic considerations are valid, we shall see that a probability level between 1 and 5 percent is applicable, and the actual wave heights thus decrease with length after 600 to 800 feet are passed.

When the roughness of the sea is given -- for example, by the characteristic wave height -- we are faced with the determination of the largest individual wave height which can be expected to occur. The observations made on the weather ships are not concerned with the individual maximum heights, and as the observations are based on human judgment and the maximum values few in number, any attempt in this direction would produce unreliable values. However, aided by the experience that $H_{char} \sim \sqrt{E}$, we may calculate statistically the probability of the occurrence of a certain wave height in a given seaway. There is today a sound basis for assuming that the Rayleigh distribution can be used for statistic treatment of the frequency of wave heights up to quite large waves. The choice of the "largest probable wave height" depends on the approval of the chance of a still larger wave height occurring. We shall return later to the numerical choice of the risk factor (f), and will here merely examine the effect of a given risk. When $f < 0.1$, we can use the following expression

$$H_{max} = \sqrt{E(y + \ln N)}$$

Where E is the mean square of the variable (here the wave height) and y is a function of the risk. With $f = 10^{-n}$, y may be written $y = n \ln 10 = 2.3\, n$. N is the number of variables (waves).

For the average of a large number of observations we may, as mentioned, determine a comparatively constant relation between the characteristic wave height H_{char} and $\sqrt{E}$. Jasper (Reference 16 p. 46) indicates the ratio 1.88 when E is determined from wave records, but the scatter is considerable and the relation is based only on moderate wave heights. Comparison of the probability distribution for characteristic and maximum wave heights at the same positions indicates a somewhat lower proportion (Figure 19). The measurements[17] were carried out for 10 to 15 minutes every third hour for 3 years. Each measurement thus covers about 100 waves (somewhat fewer for the long waves), and the most probable recorded maximum value in these samples will be

$$H_{max} = 2.5 \sqrt{E}$$

In Figure 19 the ratio H_{max}/H_{char} decreases with increasing heights, which indicates that the visual observations underestimate the small waves or, more probably overestimate the large. For the area with a characteristic wave height of 20 to 30 feet, Figure 19 shows maximum values of 135 to 140 percent of the characteristic heights, which at the most, gives a ratio of,

$$\frac{H_{char}}{\sqrt{E}} = \frac{2.15}{1.35} = 1.6$$

This more conservative figure is used in Table 4 which gives $H_{max} / \sqrt{E}$ and H_{max} / H_{char} as a function of the risk factor (f) and number of waves (N) for which the maximum value is to be estimated. Figure 18 includes a scale for H_{max} based on

$$\frac{H_{max}}{H_{char}} = 2.5$$

which we shall later see is a reasonable practical choice as long as the waves do not break. It should be stressed that these maximum heights are fairly extreme for the corresponding wave intensity.

Turning now from economical considerations to the evaluation of the risk of total loss and safety of life, we are obliged, it may seem, to come to a decision about the maximum wave heights in the most violent sea condition to which we may reasonably expect the ship to be exposed. For extremely large waves, the relationship between the visual observations and the measured values seem to fail (Figure 19) presumably because a large proportion of the shorter waves reaches the breaking point. The observations from the weather ships indicate that characteristic wave heights of 45 to 50 feet do occur up to 0.1 percent of the time at the most exposed stations west of the British Isles and south of Iceland (Positions I and J). The roughest sea so far (May 1960) reported by the weather ships was observed in December 1959 southwest of Iceland (Position K). The characteristic wave height was here estimated at 59 feet. Using a factor of 2.5 to calculate the height of extreme waves, we would have to design ships of 1,000-feet length for almost 150-foot waves, which is obviously unrealistic. Figure 19 shows maximum measured wave heights of 51 feet and the largest actually registered instrumentally are about 60 feet. As regards the influence of length on the wave height, Figure 18

TABLE 4

Estimated Maximum Values of Wave Heights

N	10^3		10^4		10^5		10^6	
f	$\frac{H_{max}}{\sqrt{E}}$	$\frac{H_{max}}{H_{char}}$	$\frac{H_{max}}{\sqrt{E}}$	$\frac{H_{max}}{H_{char}}$	$\frac{H_{max}}{\sqrt{E}}$	$\frac{H_{max}}{H_{char}}$	$\frac{H_{max}}{\sqrt{E}}$	$\frac{H_{max}}{H_{char}}$
0.63*	2.63	1.64	3.02	1.89	3.39	2.12	3.71	2.32
0.1	3.02	1.83	3.39	2.12	3.71	2.32	4.01	2.51
0.01	3.39	2.12	3.71	2.32	4.01	2.51	4.29	2.68
0.001	3.71	2.32	4.01	2.51	4.29	2.68	4.56	2.85
0.0001	4.01	2.51	4.29	2.68	4.56	2.85	4.80	3.00

*Most probable.

indicates that a maximum height exists for each probability level, the height then dropping off for increasing lengths. Only for rare sea conditions, occurring less than 0.5 percent of the time in the North Atlantic, does there seem to be a very slight increase within the length range of interest.

These are the scanty factual data on extreme wave heights, and the author can see no foundation for establishing any particular figure or relationship beyond the known limitation $H_{max} < L/7$. A formula such as $H = 0.45L^{0.6}$ (m) or even $H = L^{0.5}$ (m) cannot be regarded as giving really extreme heights. We shall, however, return later to the question of whether it is strictly necessary for a rational design to fix an absolute maximum wave height to be associated with complete failure.

EXTENT OF DAMAGE FROM CASUAL OVERLOADING

To evaluate the risk arising from overloading, it is necessary to know the extent of the damage as a function of the stress. We are far from being in a position to claim that our knowledge in this field is complete, but some experimental data and experience are available.

The damage will as a rule -- in a tough material -- start on the compression side with permanent deflection of the plates and possibly of the stiffeners arising from high combined stresses from the longitudinal load, and from local bending. The latter is caused partly by initial deflection and eccentricities and partly by lateral loading. Such damage has quite often occurred as a result of too low buckling strength, and in some cases with transverse beams, total failure has apparently started in this way. In the great majority of cases, however, the damage has not resulted in total loss. This also applies to cases where the whole deck area near the midship has buckled extensively.[5]

Large-scale tests with stiffened panels and columns of plate/stiffener combinations[18] also show that the buckling strength is not entirely exhausted when the first permanent distortions occur, and, in particular, the structure still has a big energy absorption capacity before the compression strength decreases seriously as indicated in Figure 20. The reserve between the "damage load" and the ultimate load must be assumed to decrease with an increasing ratio between the buckling stress and the yield point. This problem will be discussed in greater detail in a subsequent paper on buckling problems.

In the tests until destruction which have been carried out with destroyers in docks, the failure was initiated by local buckling. The collapse occurred fairly suddenly, according to the reports, so the difference between the collapse load and the obviously harmful load have not been so great in spite of the fact that the buckling stress was comparatively low. This shows partly longitudinally stiffened panels do not have any particularly big strength reserve due to membrane effect after buckling of the stiffeners (as columns) has started. The probable reason is that the membrane stresses become so large locally (at the "anchorage points") that the material yields or is torn apart.

For a girder built up of a rolled section and plating, the plastic reserve strength in bending is about 17 percent reckoned from the time the yielding first occurs at the outermost fibers. If the girder is loaded as a column, however, the plastic reserve in the axial strength is small. If the buckling stress lies near the yield point, the reserve strength may decrease to a few percent.

We may sum up the above by saying that the hull girder can hardly carry a bending moment which is substantially greater than the moment which causes discernible damage, but the structure can sustain considerably larger deformations without reducing materially the static strength. This conclusion is of decisive value for the validity of the considerations underlying the following calculations.

ECONOMIC STRENGTH NORM

In the estimation of the wave height to be used as a basis for the determination of the ship scantlings, we are faced with two essentially different types of risk (excluding brittle types of fracture):

1. Risk of damage: The risk of exceeding the damage load beyond which the permanent distortions become comprehensive and unacceptable, so that expensive and time-consuming repairs result, but without lives, cargo, or ship being exposed to immediate danger.
2. Risk of total failure: The risk of exceeding the ultimate load of the

ship with occurrence of extensive buckling and/or ductile fractures leading to probable loss of ship and cargo and highly endangering human lives.

We will first consider the risk of damage and analyse it on an economic basis. We will than discuss the risk of total loss which a given risk of damage may entail.

We shall prove below that it is economical to build a ship strong enough to maintain its speed and course in most weather conditions. Certainly it will be beneficial -- with regard to the unavoidable reduction of speed in a seaway -- to take advantage of the meteorological services and set the course of the voyage according to a continually adjusted "weather routing." It would not, however, be advisable to build a ship so weak that it would frequently have to reduce engine power or heave to. Our first task is therefore to determine the degree of bad weather which the ship should be able to sustain under normal running without undertaking voluntary reduction of speed or radical change of course.

CHOICE OF WAVE INTENSITY FOR FULL PROPULSION.

The roughness of the sea is characterized here by the root-mean-square ($\sqrt{E}$) of the wave height or of the bending moment on the hull girder. When the weather is so stormy that the chosen design value of $\sqrt{E}$ is exceeded, the captain must take steps to lessen the loads, that is, change course, reduce speed, heave to, or run before the storm.

We arbitrarily assume that half the time such measures must be taken is time lost. If we build the ship stronger -- but at greater cost and with reduced carrying capacity -- the loss of time will be reduced, and the problem is to find the economic optimum. The midship section modulus is proportional to the total bending moment and approximately proportional for small still water moments also to the $\sqrt{E}$ value of the wave height if we compare the probability distribution for the $\sqrt{E}$ values of the bending moment ($\sqrt{E}$ value of stress in Figure 7) with the mean probability distribution of the characteristic wave height in the North Atlantic for the ship response interval (Figure 17) (or the proportionate

$\sqrt{E}$ value), we find no direct agreement (Figure 21). The relative decrease in the bending stress is not so great as the decrease in the wave height when we pass from one weather condition to one exceeded more often. There may be several reasons for this.

One such effect might be thought to be connected with the fact that the bending moment in a given seaway is a function of the relative heading of the ship. An example of this dependence is shown in Figure 22.[2] Other measurements in irregular storm seas also tend to show that the reduction in the bending moment variation through the adjustment of the course may be fairly limited, but there is as yet no generally accepted relationship. If all relative courses are assumed to be equally probable, it may be estimated, purely arbitrarily, that about one-fifth of the time that a given seaway occurs, the ship will be subjected to stresses of approximately maximum $\sqrt{E}$ values for that wave condition. In service there will be a tendency not to take very rough seas on the beam, and the fraction of time that the maximum $\sqrt{E}$ value occurs is therefore probably higher in the measurements made.

Taking this heading effect into consideration will not, however, improve the agreement. On the contrary, the lesser decrease in bending stress than in wave height will be more pronounced when we shift the readings to lower levels of probability (Figure 21), but maintain the ratio between the two probability figures.

A more useful explanation is the fact that the probability distribution for the wave heights in the ship's area of operation is not the same as for the mean of the 10 weatherships. This explains a considerable dispersion between ships of the same length, and a probable mean tendency in the direction observed as the most exposed sea areas are avoided when possible, partly from experience and partly on the basis of gale warnings.

Finally, there is also the factor that conventional speed reduction in severe weather reduces the stress dependence on the wave height, and this will be reflected in the long-term recordings of the bending moment.

In the economic analysis to be made here, it is reasonable to take into account the stress reduction arising from navigation outside storm centers, as this is a timesaving maneuver. The further stress reduction caused by voluntary reduction in speed is timewasting, and it would be contrary to the whole idea of the analysis to take this reduction into account in the calculation of the necessary bending moment. We therefore choose a mean curve in Figure 21 for use in this analysis. Assuming logarithmic normal distribution of $\sqrt{E}$, we obtain the steepness of the distribution curve and thus the relative bending moments to be used in the calculation.

To proceed further, it is necessary to make a number of rather arbitrary assumptions, so the procedure must be regarded to some extent as an example, but the result is not too sensitive to changes in the assumptions.

To find the effect of changes in the section modulus on ship economy, we assume that interest and amortization of the capital amounts to 10 percent per annum. We assume the cost of the steel hull in "bare" condition to be proportional to the weight. The author lacks data on the dependence of the weight on the midship section modulus, and it is assumed here that the weight varies by one third of the variation in the modulus. This preassumes that the position of the neutral axis and the still-water bending moment are adjusted so that both the deck and bottom flanges are fully utilized. (Otherwise, the ratio should be reduced to about (1:10).[12] The limited influence of the midship scantlings on the thickness towards the ship ends and on the thickness of the ship sides, bulkheads, and secondary decks is then taken into account.

An increase in strength also means a loss in deadweight carrying capacity.* We assume that 1-percent increase in the steel weight means 1/4 percent loss in the carrying capacity. With an annual freight income

*This may be important in the future for "cubic ships" also, as a deeper hull girder may increase the cubic capacity at small cost because of the favourable effect on the section modulus.

of two thirds of the cost of the steel hull, this means that every percent increase in the steel weight involves an annual freight loss of 1/6 percent of the cost of the steel hull.

On the basis of these assumptions, the difference in the annual economic result is calculated for various ship lengths and for deadweight cargo and cubic cargo. The results, given in Figure 23, show marked minima in the calculated "loss" when deviating from the optimum strength norm. For the sake of simplicity, an arbitrary point of reference was chosen as the basis for comparison, this being the seaway intensity only exceeded by (1-P) = 2 percent of the time. From the point of view of safety, it would be correct to choose a strength norm somewhat on the upper side of the economic optimum. For ships with cubic cargoes, values for (1-P) of 1 to 2 percent would thus be a natural choice, depending on the ship length. For deadweight ships, the loss in cargo-carrying capacity with heavier hulls would force the economic optimum down to a strength norm corresponding to (1-P) equal to 3 to 5 percent of the time. How far this is consistent with safety will be discussed later, but it should be remembered that the wave intensity in question here is that through which the ship may proceed at full engine power with some given risk of damage.

CHOICE OF RISK OF DAMAGE

We will now assume that the ship is in the most intense seaway which it is built to endure, without taking safety measures. The ship must be provided with an instrument so that the $\sqrt{E}$ value of the bending stresses can be determined with certainty. From our knowledge of the statistical nature of the waves, we know that we cannot indicate an absolute maximum wave height, and we must accept a certain risk of the damage load being exceeded, as already discussed in the section on wave heights.

The problem now is to find the risk factor which is economically acceptable. We must then estimate the economic consequences of loads which exceed the damage level. Some guidance is provided by the statistical observation that the increase in the probable maximum load due to changing from a risk factor of 0.01 to 0.001 is only 7 percent. Damage to the actual hull girder will therefore as a rule be due to a bending moment which is only a few percent above that which the design can bear, and the average damage will therefore be moderate. When the peak load is caused by slamming or wave impact with short impulse duration, this part of the load will have limited working capacity and the energy can probably be absorbed without the damage becoming too extensive.

All the same, it is difficult to evaluate the extent of damage in monetary value, and the analysis is based on a damage (R) equivalent to 5, 10, 20, or 100 percent of the cost of the steel hull. These figures are assumed to cover both repair costs and the operational loss incurred during repair time. The remaining assumptions are retained unchanged from the preceding section.

In the calculation of extreme values on a statistical basis, it is necessary to know the number of variations (N). We found above that a ship should be able to proceed 95 to 99 percent of the time in the open sea without taking any particular precautions. For the remaining 5 to 1 percent of the time, we assume that the $\sqrt{E}$ value of the bending moment is kept constant at the design level. If we reckon about 5×10^7 stress variations in the course of the lifetime of a ship, we get an N value of between 5×10^5 and 2.5×10^6. We can then read the relative wave height or section modulus from Table 4.

Figure 24 shows the result of the economic variation for $N = 10^5$ and 10^6, and it will be noted that a multiplication of N by 10 has no marked influence on the optimum value of f. Neither do quite large variations in the assumed repair costs change the order of magnitude of f. The probable variation range of N and R gives optimum risk values of between 1 and 5 percent, which corresponds to values of $M_{max}/\sqrt{E}$ or $H_{max}/\sqrt{E}$ of 4.0 to 4.3, and H_{max}/H_{char} of 2.5 to 2.7.

EVALUATION OF RISK OF TOTAL STRUCTURAL FAILURE

People show in many ways that they are willing to take a risk on life and health if only it is small enough. It is obvious that such a risk is present in a number of technical constructions and devices, and that it is economically dictated. It is generally difficult, however, to express the risk numerically, and we have no generally approved standard. In practice, a greater risk will certainly be accepted if its reduction is particularly expensive.

As far as the strength of a ship is concerned, a numerical evaluation of the risk is particularly difficult. The total structural risk is composed of several danger factors, of which the most important groups are material defects, faulty workmanship, and exceeding the calculated load. We shall deal here only with the last problem.

As far as the total strength of the hull girder against collapse is concerned, we are not yet in a position to calculate this with any great accuracy, but we can probably estimate the strength within a reasonable interval.

A less known item is the effect of an extremely large wave on the bending moment. A linear variation with the wave height naturally does not hold good when the wave breaks over the deck, nor can an extrapolation of the statistically measured curve be assumed to be justified. For waves which rise higher than the hull profile, the influence of the wave height is flattened out. A certain guidance may be expected from the calculation of the limiting case with vertical wave fronts[19] which, in an example for a MARINER ship, gives extreme values of about 160 percent of the standard moment for H = L/20. However, even this is such an extreme load that it corresponds approximately to a linear extrapolation of the actual moment (Smith's effect etc. included) to a wave height of L/7. For large ships, this information is of no real help. Model tests are, however, suited to clarify the effect of waves which break over the ship and will probably soon settle the matter.

The last factor of uncertainty to be dealt with is the actual wave height. It is doubtful if or when we shall obtain any real statistical data for the occurrence of the most extreme waves in the most exceptional gales. Even if we could calculate this effect, the evaluation of their frequency and of the probability that a ship will meet them, taking gale warnings and weather routing into account, will be very approximate only. We are therefore in a very weak position with regard to the risk of complete structural failure, but certain evaluations can be made nevertheless.

As long as the wave intensity (or the effective value $\sqrt{E}$ of the moment variations) remains below the design value, the risk of total loss is practically nonexistent. When the design limit is based on a risk factor of 1 percent, only one ship in a thousand will, in the whole of its lifetime, exceed the design value by as much as 7 percent. It must be assumed that such a load would lead not to total failure but to serious damage. This is based not only on the static moment reserve but also on the energy absorption capacity of the structure.

Such a low risk figure as 0.1 percent is already beyond the limit of what is permissible in many cases in the way of extrapolation. We cannot, therefore, commit ourselves to any general numerical estimate of the risk of total destruction, but it already seems clear that this risk is small compared with the danger of other disasters, especially fire.

Finally, we must face the fact that the wave intensity will exceed the design value 1 or more percent of the time in the open sea. We assume, as mentioned, that the ship is equipped with a special instrument for the determination of the effective value of the stress variations and the question then is whether the measures the captain can take are sufficiently effective to prevent structural damage from occurring. The answer, in the author's opinion, is a fairly unconditional "Yes."

There are three separate though connected effects which the captain instinctively employes in practice when changing course and speed. In the first place, he reduces the frequency of slamming and wave impact very considerably, which immediately reduces the risk of the coincidence of maximum wave bending moment and high impact effect with powerful subsequent hull vibrations. It is not possible, however, to count on the complete elimination of slamming with speed reduction nor with any decisive decrease in intensity on the rare occasions when it occurs.

The other two effects are associated with the quasi-static bending moment and the dynamic bending moment due to vertical accelerations. Due to changes in both these quantities, we may count on the $\sqrt{E}$ value of the moment variations being controlled effectively through changes in course and speed (Figure 22). At a rough estimate, it should be possible by these means to lower the $\sqrt{E}$ value to about one half of the value corresponding to full engine power and an unfavourable heading. This means that we can handle a wave condition twice as severe as the design value, and this certainty corresponds to a seaway which only very few ships indeed would encounter in their lifetimes, even if they sailed without the least regard to gale warnings.

COMPARISON OF THE STATISTICAL MEASUREMENTS, STATIC CALCULATIONS, AND MODEL TESTS

In the foregoing, a basic principle has been established for the comparison of measurements at sea with simple static and dynamic calculations. The effect of different hull forms and weight distributions on vertical accelerations should be studied more closely. However, there is no information available on weight distribution in ships where statistical measurements have been taken, and it is obvious that this distribution is not constant, particularly not for dry cargo ships. We may, however, assume that the centers of gravity for the two ship halves in loaded condition vary according to a normal distribution about the centers of gravity for homogeneous cargo. The probability distribution of the bending moment will then be the same for the varying actual cargo distribution as for a homogeneous cargo distribution. There is no basis for any similar conclusions for partly loaded or ballasted ships, but the most reasonable procedure will be to use the centers of gravity for homogeneously loaded ships in all cases.

Until the effect of heaving and pitching is studied more systematically and numerically, we cannot establish any definite addition to the static bending moment, but we may obtain some general information from the model tests which have been made. The variation with vertical motions and hence with the period of encounter is very much dependent on the ship block or waterline coefficient and of the wave length ratio λ/L. A good example of this is given, for regular waves, in Figure 12 of Reference 20. The variation of the total wave moment from the "static" value for $v = 0$ and $\lambda/L = 1$ seldom exceeds $\pm$ 15 percent for the actual range of speed of cargo ships. The distribution on hogging and sagging moments varies, however, and the separate variations of these can be

TABLE 5

Comparison of Statically Calculated and Statistically Determined Bending Moments

Calculated							Measured and Calculated				
Ship	L ft	B ft	C_B	$(1\text{-}P)$* Percent	H_{max}** ft	Calculated Variable M_{max}*** tons-ft	$(1\text{-}P)$† Percent	$\sqrt{E}$ kips/in.^{2}	Variable $\sigma_{max} = 4\sqrt{E}$ tons/in.^{2}	Z ft-in.^{2}	Measured Variable $M_{max} = 4\sqrt{E}\,Z$ tons-ft
UNIMAC	300	41.6	0.563	5	41	64,000	1	1.8	3.22	10,700	34,000
							3	1.35	2.42		26,000
SPERRY	369	40.6	0.538	5	44	97,000	1	6.3	11.3	7,300	83,000
							3	5.3	9.5		71,000
MINNESOTA	440	59	0.62	5	46	242,000	1	6.5	11.6	22,100	256,000
							3	5.7	10.2		225,000
CANADA	465	64	0.647	5	46.5	310,000	1	5.8	10.4	29,600	308,000
							3	4.7	8.4		248,000
MOREMAC	465	69.5	0.67	5	46.5	365,000	1	6.1	10.9	47,000	513,000
							3	4.9	8.8		413,000
ESSEX	820	92	0.662	5	43	1,310,000	1	5.3	9.5	167,000	1,580,000
							3	3.7	6.6		1,100,000

* Probability of exceeding given wave height, for $f = 0.01$.

** From Figure 18.

*** $M_{max} = \dfrac{0.026}{35} C_B HBL^2$ for measurement in feet no heaving and pitching effect included.

† Probability of exceeding given response.

somewhat greater (Figure 12c Reference 20), and this is, of course, decisive for the static dimensioning.

The few model tests carried out in irregular waves show that the maximum stresses increase under these conditions, presumably due to an occasionally more unfavourable phase angle between wave and ship motion.

The decisive test for the usefulness of the calculation is, of course, comparison with measurements made aboard ships in service. For those ships were the long-term distribution only exists as direct stress variations, the extrapolation basis is too weak for a direct comparison. We must go to probabilities in the order of 10^{-7} to 10^{-9} to achieve the calculated bending moments. This corresponds to a maximum load which occurs on an average between 5 and 0.05 times in the service life of a ship. This does not sound unreasonable, taking into consideration ship navigation and handling, but such an extrapolation is too great to plead any reliability. For the few ships where the long-term distribution is available for the $\sqrt{E}$ value of the bending moment, the basis for comparison is much better, and Figure 25 in Table 5 and statistically measured and purely statically calculated values are compared.

The area of operation of the ships varies, and the basis of comparison (average wave conditions in the North Atlantic) is therefore more or less good. As discussed above, a ship will not be subjected to the largest possible stress the whole time the typical waves are within the response interval, and in the table the probability of maximum response is stipulated between 1/5 and 3/5. With the exception of the 300-ft. ship, the agreement is completely within what may be expected without the inclusion of dynamic correction for heaving and pitching. (For the M/S MINNESOTA, the results contain a somewhat unknown effect of horizontal bending and slamming).

The $\sqrt{E}$ value found for the 300-foot ship gives a maximum bending moment of only one half the calculated value. By using the long-term distribution of the actual stresses, we find agreement at the comparatively high probability of 5×10^{-7}. The ship served as a weathership at an exposed station (B) and was thus unable to avoid rough seas, and the probability of maximum response should be rather higher than usual. There can therefore be reason to doubt whether the $\sqrt{E}$ distribution is quite correct in this case.

The author would like to conclude by saying that the largest vertical bending moments can be determined with reasonable accuracy on the basis of a static calculation plus a correction for heaving and pitching, provided the maximum wave height is chosen on a statistical basis. In the determination of the total loading, due regard must be paid to the horizontal moments and to slamming as well as to the still-water bending moment.

The practical result of this conclusion will be that we reckon with other wave heights than is now usual, but combine the moments with the actual strength of the material against yielding and buckling. When allowance is made for both Smith's effect and interaction effect, the static vertical bending can be based on a wave height of $H = L/7$ up to 15 meters and which is thereafter kept constant. For ships of over 700 to 800 feet, a reduced wave height may even be discussed, as this would be economical, and a small reduction would appear to be justifiable without any danger of complete structural failure if the ship is correctly handled. When both wave height and the corresponding design stress are taken into account, this will lead to smaller section moduli and reduced steel weights for the very large ships. Before any use can be made of this conclusion, however, the conditions for ductile failure must be met. In other words, the buckling strength should lie near the yield point and material, design details and workmanship should be of such good quality that brittle fracture and fatigue failures are avoided.

ACKNOWLEDGMENTS

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The author would like to express his gratitude for the grant and for the hospitality which these institutions showed him, and, not least, for the inspiring discussions with staff members, especially with Dr. R. W. Clough, Dr. N. H. Jasper, and Dr. H. Schade.

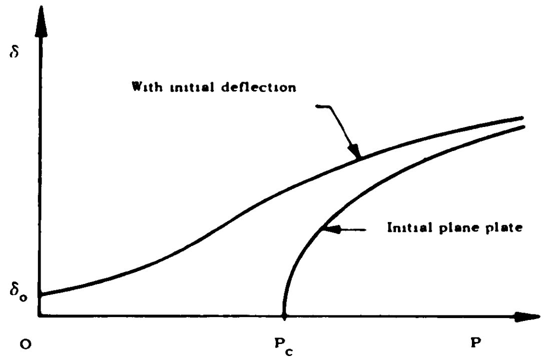

Figure 1 – Fixed Plate Deflection under Axial Load

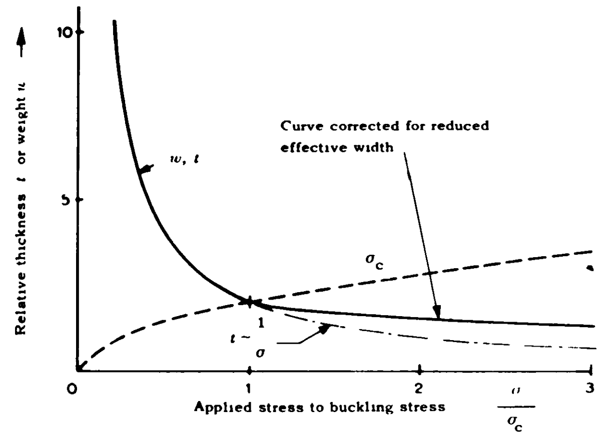

Figure 2 – Relative Thickness or Weight of Plate for Working Stress beyond Buckling

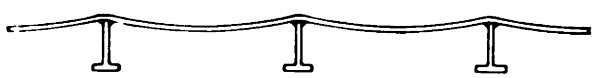

Figure 3 – Sample of Plate Unfairness Caused by Welding

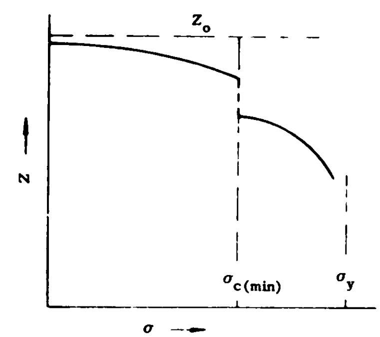

Figure 4 – Change of Section Modulus with Working Stress

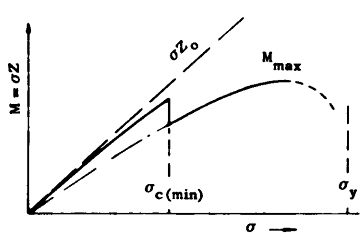

Figure 5a – Low Buckling Stress Limit

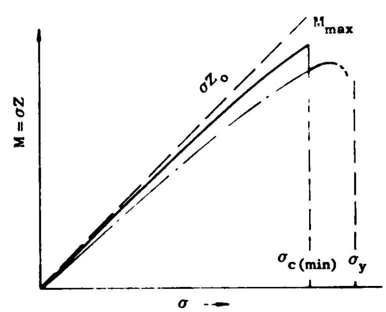

Figure 5b – High Buckling Stress Limit

Figure 5 – Effects of Minimum Buckling Stress Limits on Bending Moment

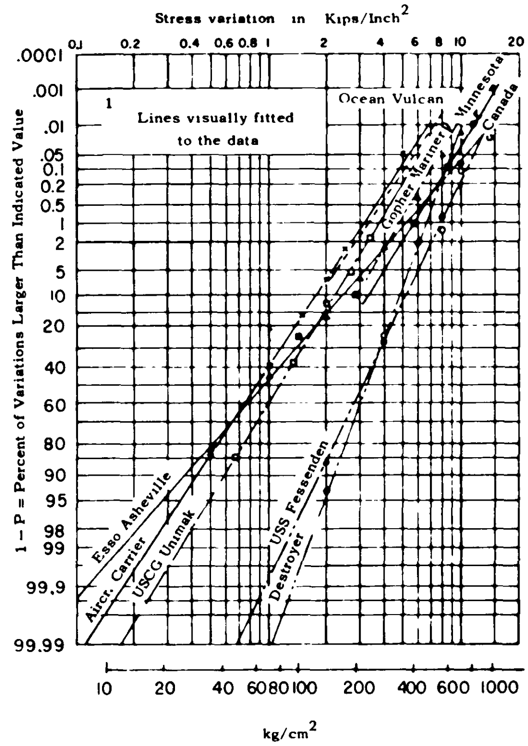

Figure 6 – Log-Normal Distribution of Wave Bending Stress Variations

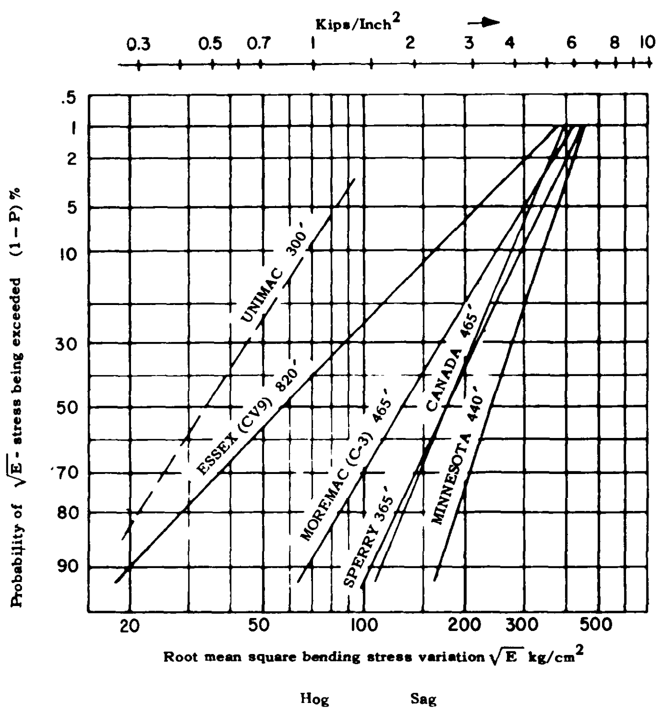

Figure 7 – Log-Normal Distribution of RMS Values for Wave Bending Stress Variation

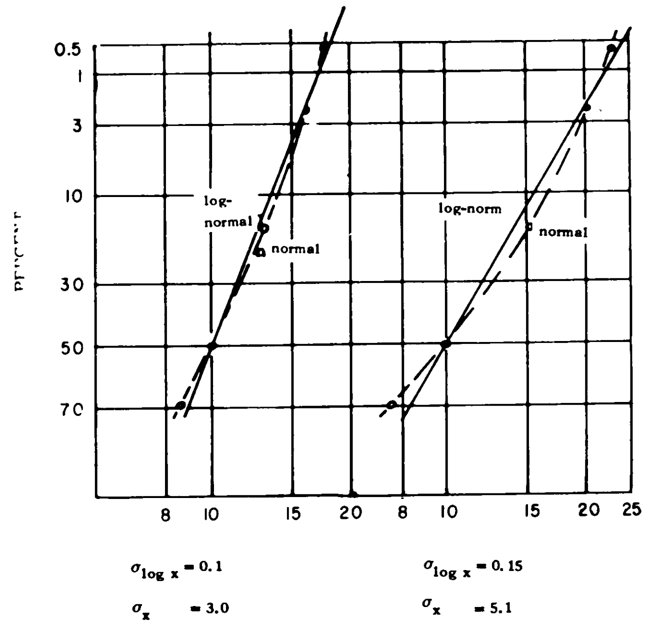

Figure 8 – Normal and Log-Normal Distribution Functions on Logarithmic Probability Paper

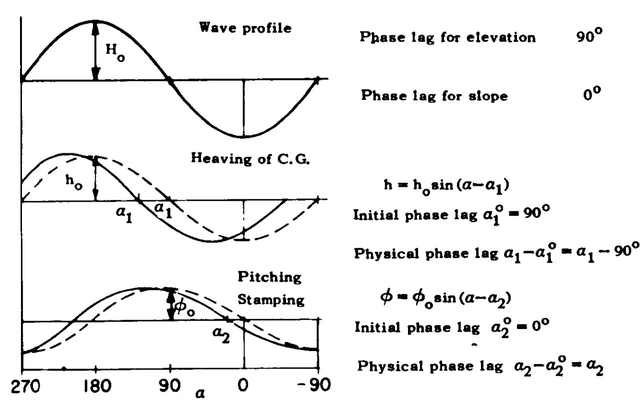

Figure 9 – The Position of the Center of Gravity of the Ship Relative to the Wave Trough

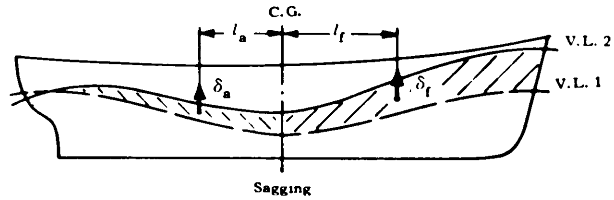

Figure 10a – Ship in Sagging Condition (Center of Gravity and Bow Down)

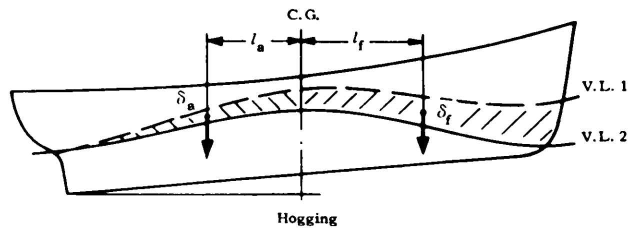

Figure 10b – Ship in Hogging Condition (Center of Gravity and Bow Up)

Figure 10 – Relative Positions of Ship and Regular Wave for Maximum Bending Moments

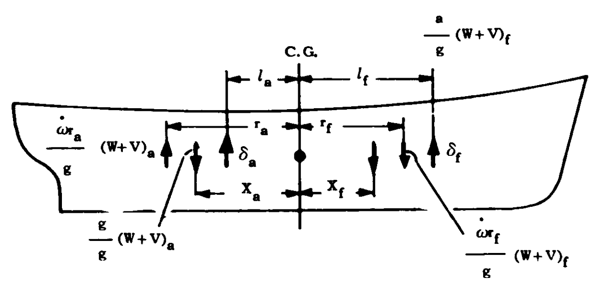

Figure 11 – Forces Due to Heaving and Pitching of the Ship

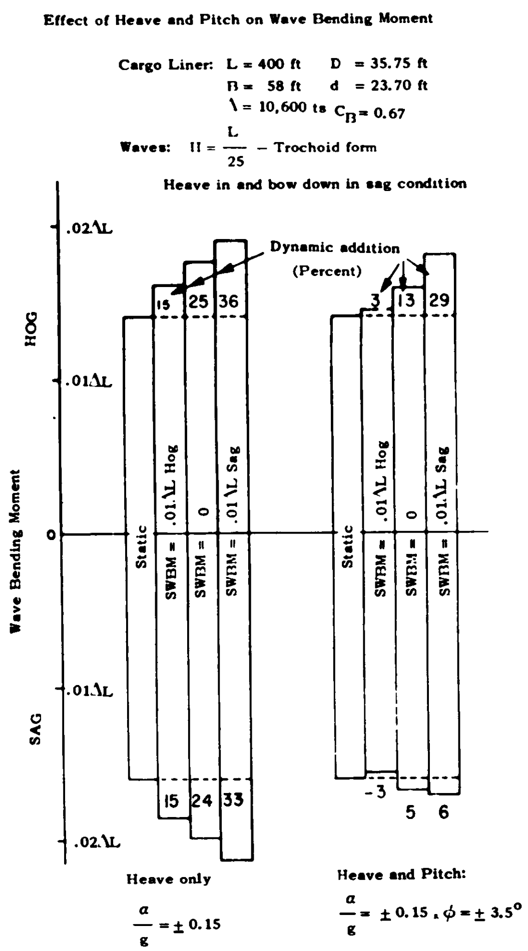

Figure 12 – Effect of Pitching and Heaving on Wave Bending Moment for Cargo Ship

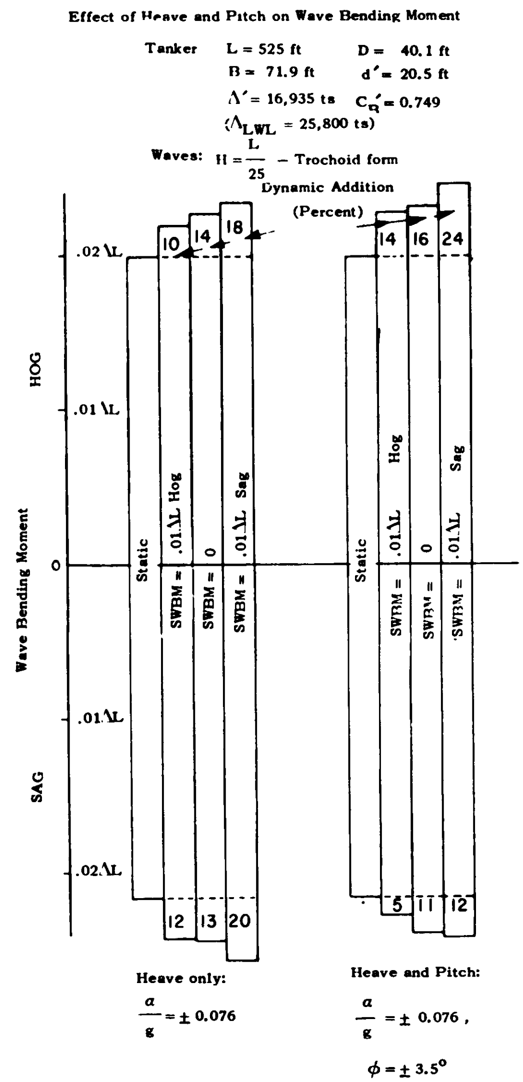

Figure 13 – Effect of Pitching and Heaving on Wave Bending Moment for Tanker

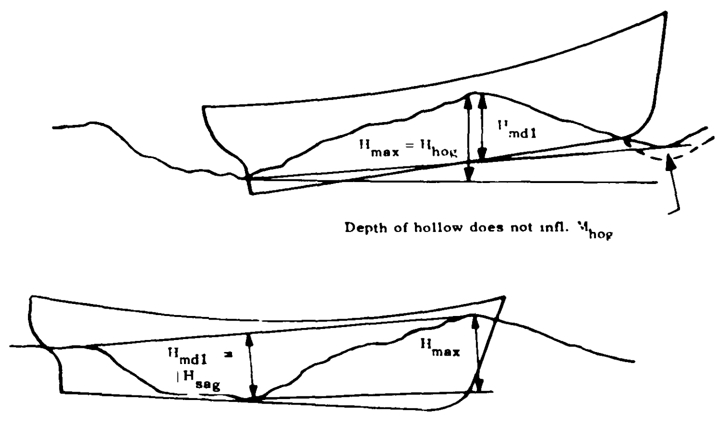

Figure 14 – Sagging and Hogging Conditions of Ship in Realistic Wave Profile

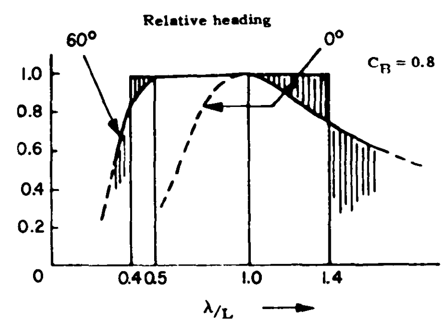

Figure 15 – Effect of Wave Length on Wave Bending Moment

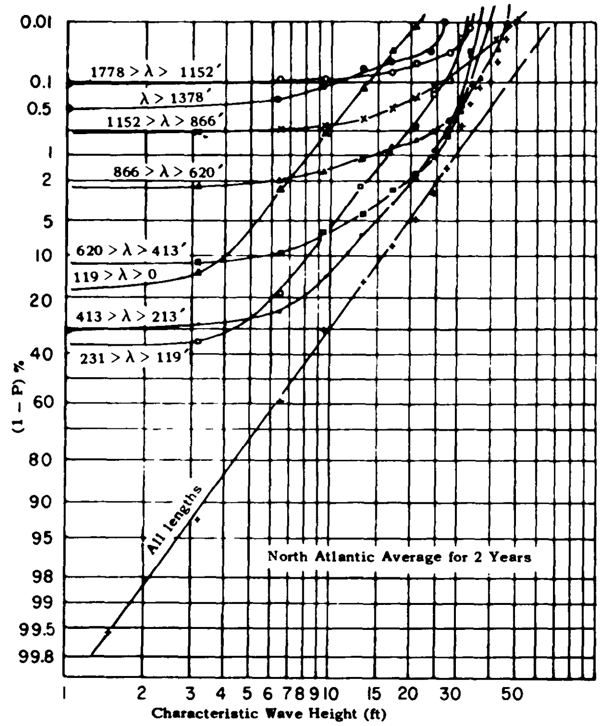

Figure 16 – Distribution of Characteristic Wave Height for Various Wave Lengths

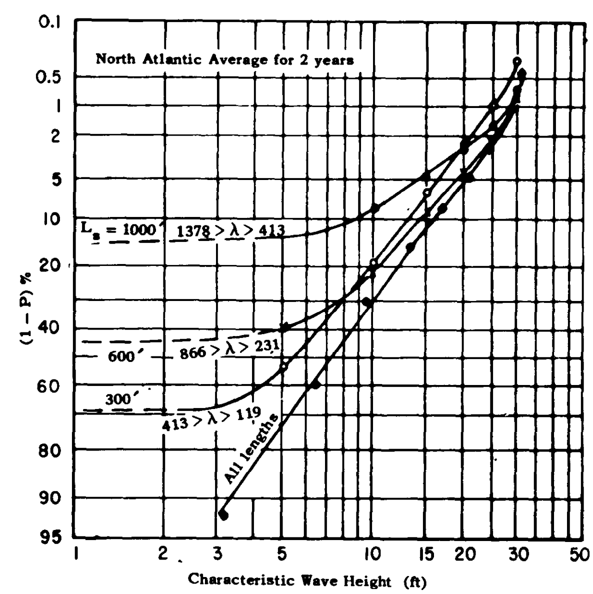

Figure 17 – Distribution of Characteristic Wave Height for Wave Lengths Proportional to Ship Lengths

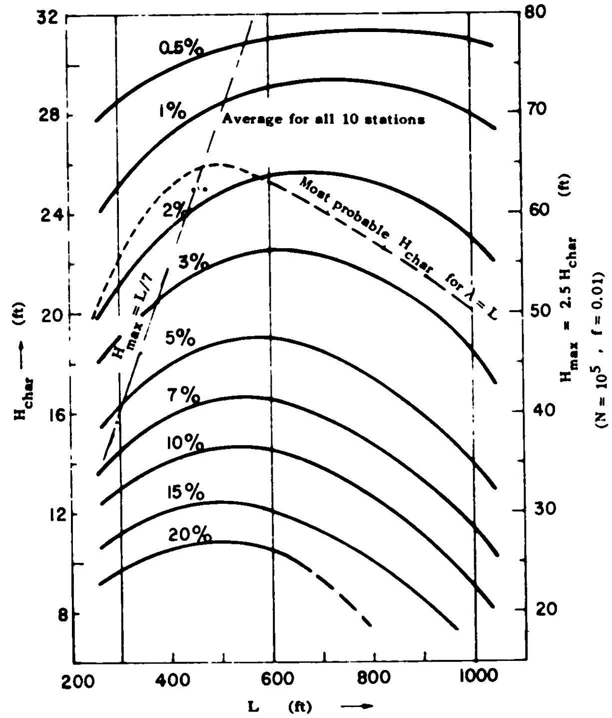

Figure 18 – Probable Characteristic Wave Heights as Function of Ship Length

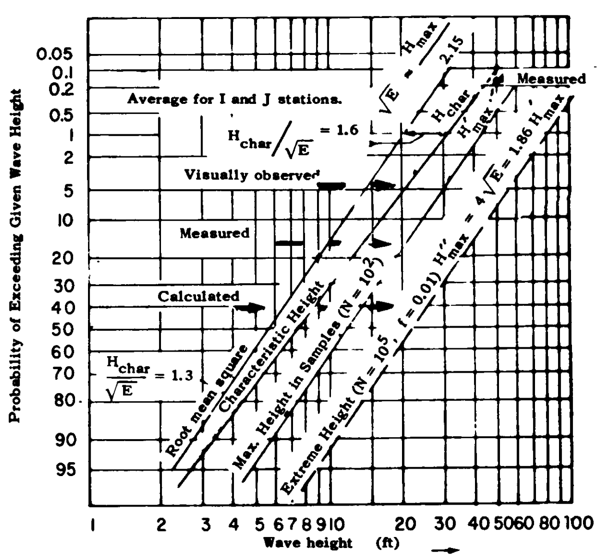

Figure 19 – Distribution of Maximum Wave Heights

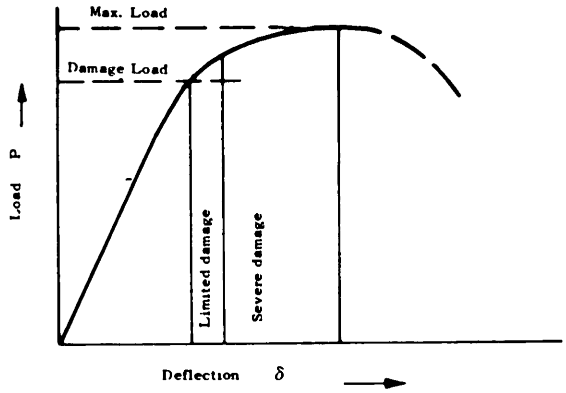

Figure 20 – Load Carrying Capacity beyond Critical Buckling Point

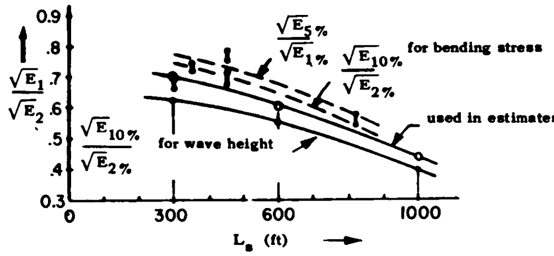

Figure 21 – Slopes of Probability Distribution Curves of Bending Stress and Wave Heights (From Figures 7 and 17)

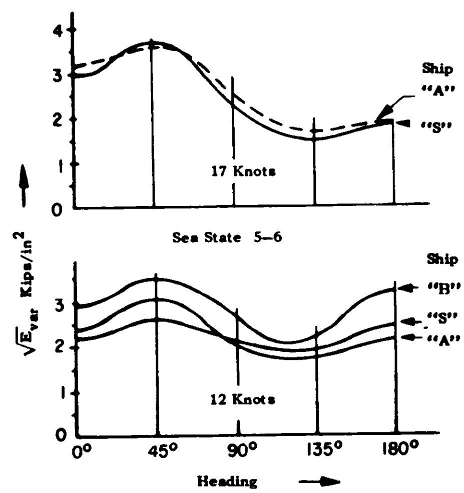

Figure 22 – Effect of Ship Relative Headings on Bending Stress

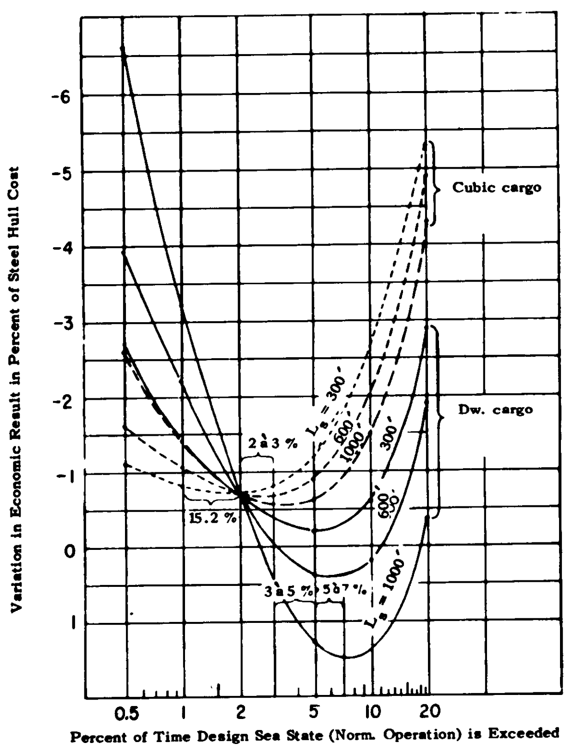

Figure 23 – Annual Financial Realization for Ships Operating in Sea Greater Than Design Sea State

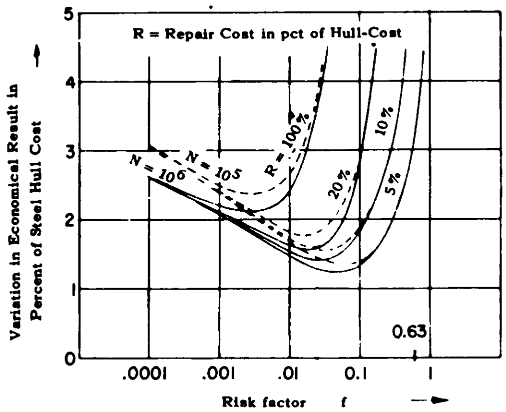

Figure 24 – Annual Damage Cost as Function of Design Risk Factor

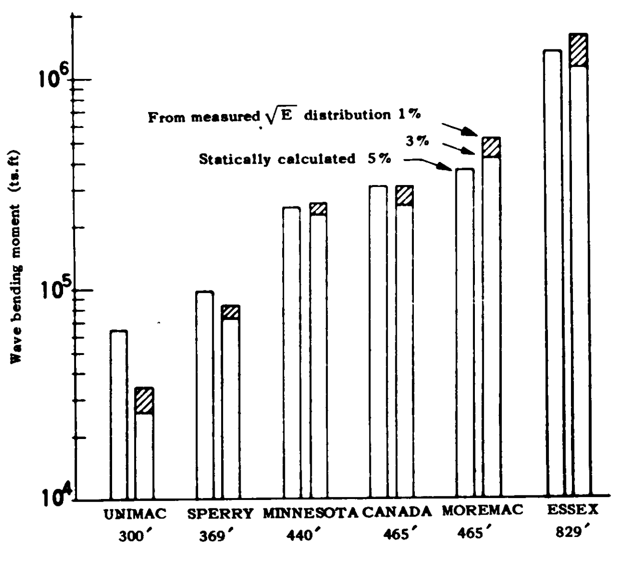

Figure 25 – Comparison Between Trial Determined and Calculated Wave Bending Moments

APPENDIX A

SUMMARY OF DISCUSSION GIVEN AT CONFERENCE

Professor E. Steneroth, Technical University in Stockholm, stressed the many detailed problems that will have to be clarified before the proposed design criteria can be accepted. He especially amplified the brittle fracture problem and the stress concentration problem for repeated tension and for repeated compression buckling. He pointed out the economical advantages of better detail design and of the application of special materials at points of stress raisers if this application is followed by an increase in the nominal stresses. But he asked how this could be incorporated in normal construction work. Are the general requirements to be raised, or are we to have different classes of design and workmanship with corresponding permissible nominal stress levels?

Mr. L. Swenson, Director of The Shipbuilding Research Foundation, Goteborg, indicated his appreciation of the value of discussion on the future application of the increasing knowledge of stresses in ships at sea. He emphasized the need for study of the hogging and sagging part of the bending moment variation in cargo ships. Swedish measurements at sea have indicated a somewhat steeper variation of M_{sag}/M_{hog} with C_B than the theoretical value given in Table 3 in the paper. He also pointed out that today the most dangerous stresses are the tensile stresses at points of stress concentration and therefore the total hogging moments should not exceed the total sagging moment.

Mr. N. Flensburg, Lloyds Register, Goteborg, discussed the rust allowance of the plate thicknesses and concluded that for practical reasons, a certain minimum addition should be kept as a classification requirement. He further drew attention to the buckling problem and stated that there is no serious problem today for longitudinally stiffened plating of relative normal design.

Mr. E. Abrahamsen, Head of Research Department, Norske Veritas, summarized the main conclusion of the paper and compared it with present Norske Veritas practice. He emphasized that with the present limited knowledge, it seems unsafe to work with declining height of the longest waves. He also stated that the formula $H = 0.45\ L^{0.6}$ used by Norske Veritas is not meant to be a maximum height but a "damage mean" for fatigue considerations. He suggested a formula $M_B = C_1C_2C_3C_4L^2B$ where C_1 depends on ship length, C_2 on the ship geometrical form, C_3 on the ship mass distribution (still-water bending moment and mass inertia) and C_4 on ship speed. If, now, C_1 is chosen so the calculated wave bending moment occurs with a certain probability once in the lifetime of the ship, then it will be possible by use of suitable factors to find the values to be used in the estimation of cumulative damage, buckling strength, etc.

He further pointed out that Figures 25 and 26 in the paper are based on a zero still-water bending moment, and he gave a diagram which included a still-water bending moment equal to half the wave bending moment for a given risk. When he also assumed that the weight of the ship structure varies with one fifth of the section modulus variation (not one third as the author has done), he found that the optimum economy corresponded to a risk factor which was almost one tenth of the author's value, but the curves are very flat at the bottom.

When ships are "hove to," this is most often to avoid local damages - and an economical analysis of this question might be of greater importance. Figure 25 indicates that the large ships should be brought to delaying maneuvers more frequently than the shorter ships - if strength is determined on an economical - statistical basis. This seems doubtful, and if the effect on the steel weight of an increased section modulus is modified as indicated above and the effect of local damages are included, this result may be altered.

Mr. Abrahamsen stressed that buckling and tensile strength of tough materials are not dominating problems today. The greater proportion of structural damage to the main hull is due to fatigue fractures, and the problem is to improve design and workmanship to exclude such damage. Once we have been successful in this respect, the question of higher nominal stresses will occur and give rise to buckling considerations. The practical hindrance to a development in this direction lies in the fact that present classification practice does not give any reward for sound details. The possibility of using a better quality of steel should also be examined with special emphasis on the low cycle fatigue properties. And whereas a ten factor variation in N has but slight influence on the risk factor f for the maximum static wave bending moment, it has an important effect on the fatigue strength of a locally high stressed detail.

The use of a $\sqrt{E}$-meter may be of help for the handling of a ship in rough weather, but the load history of the ship may be equally important.

Mr. Chr. Mürer, Norske Veritas, stated that tensile stresses are the dominating problem today, and that buckling damage rarely occurs. According to Norske Veritas' rules, the critical stress will be about 2000 kg/cm^2 (12.8 t/sq.in.) for a yield point of 2600 kg/cm^2 (16.6t/sq.in.) for longitudinal stiffening and somewhat less for transverse stiffening. He pointed out that the test to destruction of three destroyers proved that basically transversly stiffened ships may have a large reserve strength above the theoretical critical load.

Mr. Mürer further stressed that the simple static calculation can never give a reliable estimate of the actual bending moment, and actual calculations based on the yield point and ultimate buckling load would have to start out with a more advanced calculation of the bending moment. Such calculations showed that the bending moment was the result of a small difference between a large hydrodynamic moment and a large moment from the inertia forces. Thus the bending moment will be strongly influenced by changes in the distribution of masses and damping forces.

Mr. M. Loetveit, Norske Veritas, drew attention to the importance of the longitudinal distribution of the bending moment on the weight saving problem. Tests in Delft and Trondheim showed that the maximum wave bending moment often occurred at some distance away from midships, and the same applied to the still-water bending moment.

When studying the effect of ship motions on the bending moment, it has been shown by Szebehely[21] that it is advantageous to refer the motions to the axis of least vertical movement in each particular case, (the apparent pitching axis). If this is done, there will always be 90 degree phase difference between pitching and heaving, and the bending moment amidships can be written.

$$M_{\otimes} = \ddot{\psi} \cos \omega t (I_f + \frac{m}{g} b) \quad \ddot{z} \sin \omega t \frac{m_f}{g} + \int_0^{L/2} p(x)dx$$

Here I_f is the mass moment of inertia of the foreship about and m_f is the weight moment of the forebody about $\otimes$. The apparent pitching axis lies a distance b aft of $\otimes$ and the heaving amplitude at this point is z, also in the equation ψ is the pitching angle, ω is the frequency of wave encounter and p is the resulting vertical component of the hydrodynamic pressure per unit length at a given time t.
(t = 0 corresponds to "bow up").

When the bending moment is dominated by the hydrodynamic moment, as normally is the case, then the author's conclusion that an increase in hogging still-water moment reduces the bending moment amidship is valid. This, however, is certain only as long as the motions are kept constant by a constant moment of inertia of the masses.

Dr. techn. G. Vedeler, Managing Director of Norske Veritas, showed two slides illustrating the type of damage occuring in shell and deck plating. Two hundred and ten tankers more than 500 feet long were examined over a 4-year period. Among these, 66 ships had damages in these regions; and 49 of the damaged ships were built before the new rules of 1954 were put into force. Only in one case (a ship built in 1945), was there indication of brittle fracture; all the other cracks were of a fatigue type and at points of intersecting stiffeners and at discontinuities and openings.

Fifty-five of 122 dry cargo ships of over 400 feet in length built after the last war, had suffered such damage. Two of these cases were classed as brittle fracture (built in 1949), all the others as fatigue cracks with hatch corners frequently the crack starters.

The classification socieies have not wanted to lower the nominal stresses to cure brittle fractures, but they have worked for better materials, better design details, and better workmanship. There is still much to be done to overcome the fatigue problem, but the brittle fractures seems to be generally well controlled at normal temperatures.

Dr. Vedeler gave a word of warning regarding the basing of strength calculations on the yield point for tensile strength. If this leads to the application of high strength steel, then there is a danger of brittle fracture. The built-in stresses, which are up to the magnitude of the yield point, will then be higher and come closer to the ultimate strength of the material. It is not obvious that built-in stresses will have no influence on the buckling strength. During the 1960 meeting of I. I. W., there were no less than six papers dealing with open profiles. For these structures, the ultimate load was only 60 to 65 percent of the theoretical column load, this result is easily explainable. For a closed boxformed section like a ship, the influence is not known and deserves more investigation.

Mr. J. Oervig, Technical University, Trondheim, was in agreement that the design details are the bottleneck for further improvement. He raised the question of whether inspection can deal with this problem successfully, if a higher nominal stress is dependent on approval of the details. He thought it could be done if both design and workmanship are prescribed in detail.

M. R. Bennett, The Shipbuilding Research Foundation, Goteborg, expressed his appreciation of the need for a well-established design philosophy in order to advance rational hull design. When searching for the maximum probable bending moment, there are two problems to be solved; what is the response of the vessel to a given sea condition and how frequently will the vessel meet this sea condition? How can we now define the sea condition? Probably the energy spectrum will be the dominating property, but so far, the oceanographic science has not been able to cope with the determination of the probability of encountering a given energy spectrum. In the paper, the sea condition is characterized by a single extreme wave, the height of which is not exceeded with a certain probability within a given length interval. This Mr. Bennett thought was a sound procedure but the problem was how to apply this wave height.

He pointed out that the analysis of the effect of the motions on the bending moment is very valuable, and it is certainly correct as stated, that the heaving amplitude and phase angle are of major importance; then the phase lag between heaving and pitching, and then determining whether the pitching motion will increase or decrease the bending moment. The form of the frame sections will determine the numerical value. In short regular waves, the maximum sagging moment occurs after the bow reaches the wave crest; in long waves, the maximum occurs before this. The ratio between hog and sag reaches a marked minimum near the resonance wave-length where the phase angle between wave and maximum sag is almost zero. Here the sagging moment is almost 70 percent of the total variation. It is probable, therefore, that it will prove necessary to study two different conditions, one to obtain maximum sagging and one to arrive at maximum hogging. In order to carry out such calculations for different ship forms and weight distribution, we need numerical values for the motion amplitudes and phase angles. The only way to obtain such values so far is by direct measurements at sea; and then the measurement of the resulting bending moment seems simpler and more directly to the point.

Mr. Bennett indicated that the agreement in Table 5 between static calculation and statistically determined bending moments, based on corresponding probabilities, is good, but the number of ships is too small to permit definite conclusions. The principle of assuming the probability that the response will exceed a certain value is less than the probability that the wave height will exceed the corresponding height is new and correct. The figures given in the last column of Table 5, for (1 - P) = 1 percent agree surprisingly well with the figures derived for MINNESOTA and CANADA in SSF Report No. 15 by a completely different method of extrapolation.

Mr. B. Larson, Kockums Mekanuska Verkstads, Malmö, Sweden, drew attention to the need for some compensation for better details in order to stimulate this development. If the nominal stresses could be increased by 10 percent, this would mean a weight saving of 250 tons for a 40,000-ton deadweight tanker and 600 tons for 90,000-tons deadweight tanker. Cost savings all included should amount to $50,000 and $120,000

(European prices) respectively. This money would then be available for improvements of details. Alternatively, the material in deck and bottom plating could be changed to a better quality (from C to C-normalized according to The American Bureau of Shipping which would cost about $25,000 and $60,000 respectively. A similar amount could also be used for better design, workmanship, and inspection.

Mr. Larson thought that the classification societies should consider the quality of the details and lay down the permissible nominal stress correspondingly. Norske Veritas has, in fact, already started this in principle by not requiring compensation for elliptical hatch openings with a length/breadth ratio of 2.0. This may increase the nominal stresses by as much as 4 percent.

Mr. Larson also recommended a more scientific distribution of the material in the longitudinal direction. He thought that a constant section modulus over half the ship length was conservative and that a considerable weight saving might be obtained when the actually needed strength distribution had been determined.

The Author replied that in general, it is worth remembering that the main idea of this paper has been to discuss a design philosophy on which minimum weight analysis may soundly be based. The basic question was: Is a static strength calculation based on plastic strength in tension and compression at all meaningful? If problems of fatigue and brittle fracture and complications of built-in stresses, thermal stresses, etc., do upset a static consideration, then optimisation will be very complicated. The conclusion of the author was that a static calculation will become valid in the future, provided certain conditions are fullfilled to keep the risk of tough static failure, brittle fracture, and fatigue cracks to a reasonably balanced level, and an optimum design will require just such a balance. Most of the remarks or criticisms given by the discussers dealt with these conditions, and the author agreed that there are many difficult problems to be solved before we can reach the aim. But there was a misunderstanding on the part of those who thought the author was recommending the static calculation described as an independently applicable method of determining ship scantlings.

Some of the discussers seemed to think that the static damage calculation is of little interest because the present problem lies in the fatigue cracks at stress concentrations. The author will support those discussers who want to encourage the improvement of such details. It remains to be proved, but the author has good faith in the hope that the fatigue and brittle fracture problems can both be suppressed within economical limits to bring them on the same level as the problem of semistatic failure. But even with the present section modulus and design details, it would add to the safety of the ship to build in the maximum buckling strength that can reasonably be obtained without adding material.

Professor Steneroth questioned many of the assumptions made in the paper and thus also, the statistical distribution functions. Of course these have not been definitely proven valid, but, their application is not unreasonable. The Rayleigh distribution has been used to predict extreme values with an astonishing accuracy in the measurements made so far. And any error in the $\sqrt{E}$-distribution-log-normal or whatever it may be - is of no importance when large extrapolations are avoided.

Regarding Mr. Larsen's remarks, the author agreed that an increase of the stress level will raise the risk of brittle fracture although we are not able to calculate the relationship. However, it will pay to fight brittle fracture by improving details and materials rather than by lowering the stress level.

The kind remarks from the author's colleagues at the Swedish Shipbuilding Research Foundation were highly appreciated, and the importance of separate study of hogging and sagging moments was acknowledged. It should be sufficient, however, that this question was studied in a limited number of cases, and the value of more simple measurements giving only the total variation will not be reduced. The ratio between hog and sag in Table 3 is included to demonstrate that even a simple static calculation gives the right tendence, although it is obvious that the real ratio will be influenced by the dynamic corrections. When the author stated that the total sagging moment in an optimum design should be somewhat larger than the hogging moment, this again was referring to a future situation where fatigue and brittle fracture had been overcome.

It was most interesting to note that the Swedish Foundation will widen their activity on stress measurements at sea. We in Norway are now also taking measurements on one ship, will fit out two more, and plan to concentrate on large tankers. This action was taken with the view that for the time being, only direct observation at sea can yield reliable design information.

Mr. Bennett's discussion of the effect of ship motion was most welcome. Unfortunately, there was not sufficient information available on the weight distribution to include a dynamic correction in Table 5. However, the examples given show that the order of magnitude is such that the inclusion might improve the agreement although a much better agreement can hardly be expected from the crude static calculations.

Although direct measurements at sea yield the bending moment for the actual loading conditions, measurements of motions and information about weight distribution would be useful to form an opinion of the effect of another loading condition.

Mr. Larson's comment regarding the weight and cost savings to be obtained from a 10-percent increase in the stress level is very helpful and strongly supports the author's views. In addition to the direct savings, there is the increased earning capacity which calculated for a 20-year service life may easily amount to four times the direct savings.

The history of cracks in ships in recent years presented by Dr. Vedeler includes cracks started at points where the geometrical stress raises are not obvious. Such cracks may be difficult to fight as they must be caused by poor workmanship, residual stresses, and poor qualities of material and welding.

Mr. Abrahamsen pointed out correctly that Figures 25 and 26 are valid only for zero still-water bending moment. When a still-water bending moment of about half the wave bending moment is included, the economic value of the latter will be increased by only 6 to 7 percent. In this case a reasonable risk figure for large ships will be about 1 percent. The risk figure may be decreased to 0.3 percent for small ships. The effect on the optimum points of Figure 25 will be to reduce the (1-P) values to about two thirds for a 1000-foot ship and to about one third for a 500-foot ship.

There is hardly any information available today on which to base an economical analysis for local rough water damage, but the author is of the opinion that it can hardly pay to build ships in such a way that the probability of local damage will govern the ship speed.

Mr. Abrahamsen was puzzled by the finding in Figure 25 that large ships should economically be built to a more frequent slowdown and change of course. This is caused by the oceanographic fact brought out in Figure 19 that the long waves have a much flatter statistical distribution curve. This means that the relative increase in strength requirement is larger for long ships when the sea state is increased to levels which occur less frequently.

As to the longitudinal distribution of the material, mentioned by Mr. Larson and Mr. Loetveit, there is still too little information available on this, but both model tests and calculations have shown that with the dynamic effects included, the maximum bending moment may occur outside the midship of the vessel. Any saving of material will therefore depend on restrictions on weight distribution.

Based on Figure 6 the author reached several conclusions. Most of the curves converge towards a stress variation of about 600 kg/cm^2 (8.6 kips/in^2) at (1-P) = 0.01 percent. With $N = 5 \times 10^7$ variations in a service life, this means 5000 variations above this level. Assuming a still-water bending moment at the maximum equal to the wave bending moment, the stress variations will be from somewhere near zero to plus or minus something around 600 kg/cm^2. This means that there must be a stress concentration factor of about 4 to cause the stress to come up to the yield point 5000 times in the service life. In addition, there exists, of course, horizontal bending, slamming, and vibrations, and the many variations at a lower level. Nevertheless, it seemed reasonable that a stress concentration factor much smaller than 4 can be obtained by careful design and production.

APPENDIX B

DISCUSSION BY CORRESPONDENCE

Dr. N. H. Jasper, Superintendent of Scientists of Mine Defense Laboratory, Panama City, Florida, formerly head of Ship Dynamics Division of the David Taylor Model Basin: Following are comments on specific items in your paper.

Part 1 of the paper is very well written, and I am, as you know in complete agreement with its philosophy and could not improve thereon if I so wished. I have attempted on various occasions to convince others of many of the criteria given but have not been too successful in influencing ship designers, I fear. For example, I have been told repeatedly that our ships have no structural weaknesses and that it would be inadmissible to put any gages on the ship with the view of guiding the captain in changing course of speed under abnormally severe conditions because our ships are built strong enough to make this unnecessary. The present paper should go far toward convincing the designers that the extreme condition for which a ship is to be built can be reasonably estimated with a good degree of confidence.

I consider Part 11 of your paper a very great step toward an actual design procedure but feel that some of the assumptions and developments used in arriving at estimates of the probability levels for entering the statistical distribution curves of bending moments are open to considerable question. Nevertheless, due to the insensitivity of the estimate of extreme value to the risk level, I do not think that the "license" taken will appreciably affect the conclusions with which I am in general agreement.

The relationship $H_{char} = 1.88 \sqrt{E}$ derived by me in Reference 16 is based on a consistent set of values. That is, both the visual estimate of the characteristic wave height and the E value were obtained from the same sea condition. Furthermore, the visual observations were made in the way that weatherships have generally made them. The time for the visual observation is generally believed to be considerably less than

10 to 15 minutes and thus one might expect the reported characteristic height to be less than if it were based on longer observations.

The formula $H_{max} = 1.6 \sqrt{E}$ shown in Figure 19 is based (1) on values of H_{max} measured by an instrument from a sample of 15-minute duration and related to E by a theoretical statistical law and (2) on characteristic wave height from weathership observations, and therefore does not seem a consistent development.

It is possible that for severe sea states, the coefficient should be lowered, perhaps to 1.6, to allow for visual overemphasis of the larger waves. I would not think that this is a significant factor because mathematical fitting of the long term distribution, such as Figure 19, will not be responsive to the large waves.

Regarding the slope of distribution curves (Figure 21), I have been unable to accept reasoning derived therefrom and therefore cannot evaluate the method of estimating the design bending moments. If this is valid the figure would permit one to estimate the slope of the log-normal distributions in Figure 7 for any length of ship.

I disagree with the inference of the statement that slamming loads will have limited damage capacity. I see no significant difference between ordinary and slamming-induced stress as far as failure is concerned. The maximum stress is likely to occur during the first cycle, and even with partial buckling, the natural period is not appreciably affected. The buckling failure of the main deck of the TICONDEROGA was undoubtedly associated with severe whipping, such as illustrated in Figure 5a of David Taylor Model Basin Report 1216.[22]

Referring to Table 5 of the paper, presumably the static moment M_{max} was calculated without allowance for heaving and pitching accelerations by the formula $M = 0.026\ C_B\ HBL^2$. If so, then it is difficult to see why the measured moment variations should check against this value. I should think that allowances for ship motions, as described on page 13, could and should have been included in the "quasi-static" calculation.

The distribution, Figure 7, of the 300-ft ship was derived for the operational condition of a weathership and therefore could not be expected to fit into the "normal" pattern. Its predicted bending moment would be expected to be relatively low.

In conclusion, it seems to me that the most important problems here are (1) definition of the long-term distribution of $\sqrt{E}$ for bending moment for each ship as in Figure 7 and (2) determination of probability (1-P) required for estimating the expected extreme bending moment from Figure 7. The estimate of design value could then be made by applying the risk factor (1-P) as read from the abscissa of Figure 23 and to Figure 7 to get $\sqrt{E}$ max. Next the risk factor f from Figure 24 is applied to obtain the expected maximum (design) bending moment corresponding to $\sqrt{E}$ max. Perhaps Figure 21 does the same thing in another way, but I do not follow the reasoning.

Professor E. V. Lewis, Webb Institute of Naval Architecture, formerly of Davidson Laboratory, Stevens Institute of Technology: Information on horizontal (or lateral) bending moments is also obtainable from model tests in oblique waves, although the interpretation in terms of full-scale ships remains in doubt.[23]

The application of linear superposition to predict bending moments in irregular seas is not really such a "big task" as suggested on page 9. I am convinced that recourse to simpler approaches must be considered as interim only.

Simple calculations are presented for determining the dynamic additions to the wave bending moment due to ship motions. Yet model tests and detailed calculations show a reduction. Results of calculations compared with experiments are given by Jacobs and Dalzell[24] and the numerical procedure for the detailed calculation is given by Jacobs, Dalzell, and Lalangas.[25]

I am doubtful about expecting the captain to take steps to reduce the loads to prevent structural damage, especially since present knowledge does not permit us to advise him how to accomplish this. We should work toward this objective, however.

You indicate that extreme stresses occur in irregular waves because of unfavorable phase relationships. Actually, it appears to result more from unusually high waves in the irregular pattern (see Reference 13).

Finally, a reduced design wave height is recommended for very large ships. I am convinced that the amount of such reduction can be determined satisfactorily only by considering realistic irregular wave patterns. First we need more data on actual ocean wave spectra. Then we must apply superposition theory to predict bending moment spectra for ships of different size. We are hoping to do some work along this line at the Davidson Laboratory in the near future.

Regarding the Author's reply to Dr. Jasper, derivation of $H_{char} = 1.6\ E$, (Figure 19), the author would think that it is permissible to derive a relationship between two quantities belonging to different samples of the same family, providing each sample is large enough. In this case, the visual observations covered 10 years and the measurements 3 years. The author therefore feels that this relation can also be said to be based on the "same sea condition."

It must be admitted that the ratio $H'_{max}/\sqrt{E}$ from Derbyshire's measurements is based on a theoretical statistical law, but one which is reasonably well established, and in any case the same law on which the extreme wave height prediction $H''_{max}/\sqrt{E}$ is made. No new error is introduced, therefore, by plotting also this $\sqrt{E}$ curve in Figure 19 and comparing it with the H_{char} curve.

It would be advisable, of course, not to mix the visually observed wave heights into the calculations and to proceed directly from measured maximum wave height to predicted extreme heights. The reason for dealing with the visually observed heights is the fact that information on wave height distribution for different wave lengths (Figure 16) is only available in sufficient quantity from weathership observations of H_{char}. The influence of the ratio $H_{char}/\sqrt{E}$ is therefore limited to the splitting of the frequency of occurrence of the different wave heights into certain length intervals (Figure 17). As soon as this splitting has been done, we may consider that we revert to the basis of measurements and theory, and thus basically remain on a consistant basis.

The curve given in Figure 21 for the slope of the wave height distribution is taken directly from Figure 17, using the straight part of the curves. The curves for the slope of the bending stress distribution (or rather the black points in the figure), are derived directly from Figure 7. The curves are visually fitted to the points. The scatter between the points is natural, but the main observation is that they all fall above the curve for the waves and that some average slope value seems reasonable for the economic calculation, as explained in the paper. Figure 21 serves only to help select a mean and unbiased slope of the distribution curve to be used in the examples of choice of "design sea state" in Figure 23.

I did assume in general in my paper that allowance was given for slamming loads. But as the numerical value of the allowance was difficult to assess with any certainty values in excess of a reasonable design allowance were realized to be possible. Contrary to your opinion, however, I think this was less serious than if the same amount of excess stress had been due to quasi-static wave bending moment. In my opinion, the hull girder may absorb the part of the slamming shock exceeding the hull strength and cause only more or less extensive plastic deformation, whereas a quasi-static bending moment of the same magnitude might cause complete failure. (Brittle fracture is assumed to be excluded). The risk of such damage should, eventually, be brought into the economic criteria, but first we need much more information on slamming intensity and frequency for a given ship and size.

In the preparation of Table 5 and Figure 25, allowance for heaving and pitching was not included because the necessary information on weight distribution was not available. Referring to Figures 12 and 13, however, the probable correction is of the order of 10 to 20 percent and I think a comparison is interesting even without this refinement included.

As to the distribution curve of the USCGC UNIMAC, I find that the total measuring period was only about 2 months. This I had forgotten, otherwise I would not have included this vessel in may paper - it is no true long-term distribution. I cannot find any comment in the report as to the average severity of the weather during the test, but moderate sea conditions would of course explain the result.

Your conclusion as to the application of Figures 23 and 24 in combination with Figure 7 is exactly the way I have done it. Figure 21 was only indirectly involved as the steepness of the $\sqrt{E}$-distribution will affect the position of minimum points of Figure 23. With the risk factor between 0.01 and 0.001 and the (1 - P) value about 2 percent, we get N about 10^6 in the most severe sea condition and from Table 4, we have

$$\frac{M\ max}{\sqrt{E}} = 4.3 \text{ to } 4.5$$

In Table 5, I have used the ratio 4.0 which is consistant with the right-hand scale of Figure 18. I feel that the $\frac{M\ max}{\sqrt{E}}$ - ratio as determined by the Rayleigh distribution may be reduced by physical factors for severe sea conditions, but it remains to be shown how much. I understand that the Davidson Laboratory is presently working on this problem.

Author's reply to Professor Lewis: I am glad to have your opinion that the linear superposition method will not cause great difficulties. In my remark that this was a "big task," I meant to include the problem of defining a truly representative sea spectrum. There remain also limitations of the linearity of the physical phenomena. But I certainly agree that simple calculations are interim only.

As to your comments on possible increase or reduction of the wave bending moment due to the ship motions, it may be that you are speaking of the total effect of the ship motion, including the hydrodynamic effects of "interaction" etc., while I am talking about the separate effect of the inertia forces on ship and entrained water due to the ship vertical accelerations.

You are doubtful as to the possibility of advising the captain how to reduce the structural load on his vessel. I can see that in certain very irregular sea conditions, available methods will be less efficient, but not quite useless. Further study of the problem is naturally necessary.

I meant to indicate that the phase relationship is an additional cause for extra large bending moments in an irregular sea.

I fully agree that more data on actual ocean wave spectra are needed, not only for determining a possible reduction in design wave height for long ships but also for all model and theoretical treatment of the problem. A reduced probable wave height can, of course, also be studied by direct long time statistical measurements of ship response.

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