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GRAPHICAL ANALYSIS FOR MAXIMUM STRESSES IN SANDWICH CYLINDERS UNDER EXTERNAL UNIFORM PRESSURE

by

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STRUCTURAL MECHANICS LABORATORY

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ABSTRACT

A simplified solution is presented for computing the axial and transverse forces in sandwich cylinders with annular webs loaded under external uniform pressure. A series of graphs is shown for the determination of Lamé deflection coefficients and transcendental stress functions. With these forces and stress functions, a procedure is developed for computing maximum stresses in both outside and inside shells of the sandwich cylinders. Stresses can be computed more rapdily by this procedure than by a more exact solution.

INTRODUCTION

In recent years, the Navy has shown great interest in the exploration of the depths of the ocean by means of deep-diving manned submarines. Since the pressure hulls of these submarines are weight critical, new types of structures are being investigated to reduce fabrication difficulties and to produce hulls with high strength-to-weight ratio characteristics. One of these structures is a cylindrical sandwich shell consisting of two concentric cylinders connected by annular webs. Figure 1 illustrates a deep-diving submarine with a sandwich shell as the hull structure.

To evaluate the structural strength of a cylindrical sandwich shell, the locations and magnitudes of maximum stresses must be determined. Rigorous analyses for stress distributions in sandwich shells loaded under external hydrostatic pressure were carried out by Pulos[1] and Raetz.[2] These analyses illustrate that the maximum stresses occur in the inner and outer shells at locations next to the annular webs and midway between the two webs that separate these shells. Therefore, the stresses in the inside and outside shells at these two locations are of interest for the purposes of structural design and evaluation.

This report presents a procedure for determining stresses in the sandwich cylinder. The basic equations of equilibrium and compatibility are presented in a simplified form to enable rapid determination of forces within the structure. A series of graphs are presented to determine values for deflection and stress functions. The difference between maximum stresses computed by these simplified equilibrium equations and graphs and by more exact solutions is negligible. The method of analysis is the same as discussed in References 1 and 2.

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[1]References are listed on page 19.

STRESS EQUATIONS

To compute the stresses in a sandwich cylinder, it is necessary to determine the axial and transverse forces (V_o, V_i, H_o, and H_i), which act at the intersections of the annular webs and cylinders; see Figure 2. The method for determining these loads, as shown in the Appendix, involves the solution of three simultaneous equations. This method may be simplified by assuming that the axial membrane strain in the outer shell is equal to the axial membrane strain in the inner shell. This assumption may be approximated by $V_o/Eh_o = V_i/Eh_i$. With this relationship and Equation [11] in the Appendix, the axial forces V_o and V_i are:

$$V_o = \frac{\frac{R_o}{2}}{1 + \frac{h_i}{h_o}\,\frac{R_i}{R_o}} \quad \text{and} \quad V_i = \frac{h_i}{h_o} V_o \qquad [1]$$

By substituting Equations [1] into Equations [12] of the Appendix, the transverse forces H_o and H_i can be expressed in terms of two simultaneous equations:

$$(g_o - M)\,H_o - N\,H_i = -\frac{R_o^2}{h_o}\left(1 - \frac{\frac{\nu}{2}}{1 + \frac{h_i}{h_o}\,\frac{R_i}{R_o}}\right) + \frac{b}{2} M$$

$$-P\,H_o + (g_i - Q)\,H_i = \frac{\nu}{2}\,\frac{R_o^2}{h_i}\left(1 - \frac{1}{1 + \frac{h_i}{h_o}\,\frac{R_i}{R_o}}\right) + \frac{b}{2} P \qquad [2]$$

Equations [1] and [2] are a system of equations for the forces when the sandwich structure is under an externally applied uniform load of 1.0 psi The functions M, N, P, and Q in Equations [2] are the Lamé deflection coefficients for the annular webs as determined from Equations [10] in Reference 1, and can be expressed in the following form:

$$M = \frac{R_w}{b}\left(1 + \frac{d}{2R_w}\right)\left(\frac{2R_w}{d} + \frac{d}{2R_w} - 2\nu\right)$$

$$N = \frac{R_w}{b}\left(1 + \frac{d}{2R_w}\right)\left(\frac{2R_w}{d} + \frac{d}{2R_w} - 2\right) \qquad [3]$$

$$P = \frac{R_w}{b}\left(1 - \frac{d}{2R_w}\right)\left(\frac{2\,R\,w}{d} + \frac{d}{2R_w} + 2\right)$$

$$Q = \frac{R_w}{b}\left(1 - \frac{d}{2R_w}\right)\left(\frac{2R_w}{d} + \frac{d}{2R_w} + 2\nu\right) \quad [3]$$

The functions g_o and g_i are edge coefficients for the outside and inside shells, respectively, and are:

$$g_o = -\frac{2\,R_o^2}{h_o l}\,\bar{F}_{10} \quad \text{and} \quad g_i = -\frac{2R_i^2}{h_i l}\,\bar{F}_{1i} \quad [4]$$

where F_{10} and F_{1i} are transcendental functions shown in Equation [8].

The notation used in Equations [1] through [4] is illustrated in Figure 2 and is as follows:

h_o is the outside shell thickness in inches,
h_i is the inside shell thickness in inches,
b is the annular web thickness in inches,
d is the total depth of the annular web and shells in inches,
R_o is the mean radius of the outside shell in inches,
R_i is the mean radius of the inside shell in inches,
R_w is the average radius of the inside and outside shells in inches,
l is the unsupported shell length in inches,
ν is Poisson's ratio (dimensionless), and
E is Young's modulus in psi.

Maximum stresses in cylindrical sandwich shells loaded under external uniform pressure occur at locations adjacent to the annular webs and midway between the webs. To determine these stresses, the limits $x = 0$ and $x = l/2$ (see Figure 2) are substituted into Equations [14] of the Appendix. Equations [14] and [15] can then be written in the following form for the stresses adjacent to the annular webs and midway between the webs for both inside and outside shells:

$$\frac{\sigma_{\text{OMOO}}}{\sigma_{uo}} = 1 - \frac{H_o}{l}\left(B_{10}\right) \qquad \frac{\sigma_{\text{XMOO}}}{\sigma_{uo}} = \frac{V_o}{R_o} + \frac{H_o}{l}\left(B_{30}\right)$$

$$\frac{\sigma_{\text{OMOI}}}{\sigma_{uo}} = 1 - \frac{H_o}{l}\left(B_{20}\right) \qquad \frac{\sigma_{\text{XMOI}}}{\sigma_{uo}} = \frac{V_o}{R_o} - \frac{H_o}{l}\left(B_{30}\right) \quad [5]$$

$$\frac{\sigma_{OMIO}}{\sigma_{ui}} = -\frac{H_i}{l}\left(B_{1i}\right) \qquad \frac{\sigma_{XMIO}}{\sigma_{ui}} = \frac{V_i}{R_i} + \frac{H_i}{l}\left(B_{3i}\right)$$

$$\frac{\sigma_{OMII}}{\sigma_{ui}} = -\frac{H_i}{l}\left(B_{2i}\right) \qquad \frac{\sigma_{XMII}}{\sigma_{ui}} = \frac{V_i}{R_i} - \frac{H_i}{l}\left(B_{3i}\right)$$

$$\frac{\sigma_{OFOO}}{\sigma_{uo}} = 1 - \frac{H_o}{l}\left(B_{50}\right) \qquad \frac{\sigma_{XFOO}}{\sigma_{uo}} = \frac{V_o}{R_o} - \frac{H_o}{l}\left(B_{40}\right)$$

$$\frac{\sigma_{OFOI}}{\sigma_{uo}} = 1 - \frac{H_o}{l}\left(B_{60}\right) \qquad \frac{\sigma_{XFOI}}{\sigma_{uo}} = \frac{V_o}{R_o} + \frac{H_o}{l}\left(B_{40}\right) \qquad [5]$$

$$\frac{\sigma_{OFIO}}{\sigma_{ui}} = -\frac{H_i}{l}\left(B_{5i}\right) \qquad \frac{\sigma_{XFIO}}{\sigma_{ui}} = \frac{V_i}{R_i} - \frac{H_i}{l}\left(B_{4i}\right)$$

$$\frac{\sigma_{OFII}}{\sigma_{ui}} = -\frac{H_i}{l}\left(B_{6i}\right) \qquad \frac{\sigma_{XFII}}{\sigma_{ui}} = \frac{V_i}{R_i} + \frac{H_i}{l}\left(B_{4i}\right)$$

where

$$\sigma_{uo} = -p\,\frac{R_o}{h_o} \quad \text{and} \quad \sigma_{ui} = -p\,\frac{R_i}{h_i} \qquad [6]$$

in which p is the externally applied pressure on the outside shell in psi. The

The B functions are determined by the following:

$$B_{10} = 2\,\overline{F}_{20} - \nu\overline{F}_{30}$$
$$B_{20} = 2\,\overline{F}_{20} + \nu\overline{F}_{30}$$
$$B_{30} = \overline{F}_{30}$$
$$B_{40} = \overline{F}_{40} \qquad [7]$$
$$B_{50} = 2\,\overline{F}_{10} + \nu\overline{F}_{40}$$
$$B_{60} = 2\,\overline{F}_{10} - \nu\overline{F}_{40}$$

where

$$\bar{F}_{10} = \frac{\theta_o}{2}\left[\frac{\sinh\theta_o + \sin\theta_o}{\cosh\theta_o - \cos\theta_o}\right]$$

$$\bar{F}_{20} = \theta_o\left[\frac{\cosh\frac{\theta_o}{2}\sin\frac{\theta_o}{2} + \sinh\frac{\theta_o}{2}\cos\frac{\theta_o}{2}}{\cosh\theta_o - \cos\theta_o}\right]$$

$$\bar{F}_{30} = \frac{6\,\theta_o}{\sqrt{3(1-\nu^2)}}\left[\frac{\cosh\frac{\theta_o}{2}\sin\frac{\theta_o}{2} - \sinh\frac{\theta_o}{2}\cos\frac{\theta_o}{2}}{\cosh\theta_o - \cos\theta_o}\right] \quad [8]$$

$$\bar{F}_{40} = \frac{3\,\theta_o}{\sqrt{3(1-\nu^2)}}\left[\frac{\sinh\theta_o - \sin\theta_o}{\cosh\theta_o - \cos\theta_o}\right]$$

The function θ_o is the shell flexibility parameter for the outside shell and is:

$$\theta_o = \sqrt[4]{3(1-\nu^2)}\;\frac{l}{\sqrt{R_o h_o}} \quad [9]$$

The functions F_{1i}, F_{2i}, F_{3i}, F_{4i}, B_{1i}, B_{2i}, B_{3i}, B_{4i}, B_{5i}, and B_{6i} are similarly determined by substituting the shell flexibility parameter for the inside shell (θ_i) into Equations [7] and [8]. This shell flexibility parameter for the inside shell is:

$$\theta_i = \sqrt[4]{3(1-\nu^2)}\;\frac{l}{\sqrt{R_i h_i}} \quad [10]$$

The locations of the stresses in Equations [5] are as follows:

σ_{OMOO} is the circumferential stress at midbay on the outer fiber of the outside shell,

σ_{OMOI} is the circumferential stress at midbay on the inner fiber of the outside shell,

σ_{OMIO} is the circumferential stress at midbay on the outer fiber of the inside shell,

σ_{OMII} is the circumferential stress at midbay on the inner fiber of the inside shell,

σ_{XMOO} is the longitudinal stress at midbay on the outer fiber of the outside shell,

σ_{XMOI} is the longitudinal stress at midbay on the inner fiber of the outside shell,

σ_{XMIO} is the longitudinal stress at midbay on the outer fiber of the inside shell,

σ_{XMII} is the longitudinal stress at midbay on the inner fiber of the inside shell,

σ_{OFOO} is the circumferential stress next to the web on the outer fiber of the outside shell,

σ_{OFOI} is the circumferential stress next to the web on the inner fiber of the outside shell,

σ_{OFIO} is the circumferential stress next to the web on the outer fiber of the inside shell,

σ_{OFII} is the circumferential stress next to the web on the inner fiber of the inside shell,

σ_{XFOO} is the longitudinal stress next to the web on the outer fiber of the outside shell,

σ_{XFOI} is the longitudinal stress next to the web on the inner fiber of the outside shell,

σ_{XFIO} is the longitudinal stress next to the web on the outer fiber of the inside shell, and

σ_{XFII} is the longitudinal stress next to the web on the inner fiber of the inside shell.

The positions of these stresses are illustrated in Figure 2.

GRAPHICAL ANALYSES

Graphical representations of the Lamé deflection coefficients and transcendental functions F and B are shown in Figures 3 through 9. When a value of 0.3 is used for Poisson's ratio, the stresses are computed by the following procedure:

1. Compute θ_o and θ_i from Equations [9] and [10].
2. Determine $\overline{F}_{1o}$ and $\overline{F}_{1i}$ from Figure 4.
3. Compute g_o and g_i from Equations [4].
4. Determine M, N, P, and Q from Figure 3.
5. Compute V_o and V_i from Equations [1].
6. Compute H_o and H_i from Equations [2].
7. Determine B_{1o}, B_{2o}, B_{3o}, B_{4o}, B_{5o}, B_{6o}, B_{1i}, B_{2i}, B_{3i}, B_{4i}, B_{5i}, and B_{6i} from Figures 7 through 9.
8. Compute stresses from Equations [5].

This procedure is illustrated by a numerical example in Table 1.

When any selected value is used for Poisson's ratio, the stresses are computed by the following procedure:

1. Compute θ_o and θ_i from Equations [9] and [10].
2. Determine $\overline{F}_{1o}$, $\overline{F}_{2o}$, $\overline{F}_{3o}$, $\overline{F}_{4o}$, $\overline{F}_{1i}$, $\overline{F}_{2i}$, $\overline{F}_{3i}$, and $\overline{F}_{4i}$ from Figures 4 through 6.
3. Compute g_o and g_i from Equations [4].
4. Compute M, N, P, and Q from Equations [3].
5. Compute V_o and V_i from Equations [1].
6. Compute H_o and H_i from Equations [2].
7. Compute B_{1o}, B_{2o}, B_{3o}, B_{4o}, B_{5o}, B_{6o}, B_{1i}, B_{2i}, B_{3i}, B_{4i}, B_{5i}, and B_{6i} from Equations [7].
8. Compute stresses from Equations [5].

Calculations were made on a FORTRAN 7090 computer to determine the differences between stresses computed by these procedures and stresses computed by using the more exact equations for the axial and transverse forces (see Equations [11] and [12] in the Appendix). These calculations were made on a series of sandwich geometries in a practical range in which the inside flexibility parameter (θ_i) varied from 1.0 to 3.0, and the ratios of outside and inside shell thicknesses (h_o/h_i) varied from 1.0 to 3.0. The maximum differences between the stresses computed from the simplified graphical solution and the same stresses as determined from the FORTRAN 7090 computer occurred at locations next to the annular webs. For the largest stresses at the webs, the greatest difference between the two solutions occurred for $\theta_i = 3.0$ and $h_o/h_i = 3.0$. This difference was approximately 2 percent. Many geometries used in current structures have values ranging from 1.0 to 2.0 for θ and values of 1.0 for h_o/h_i. For these geometries, an expected error of approximately 1 percent can be expected.

ACKNOWLEDGMENTS

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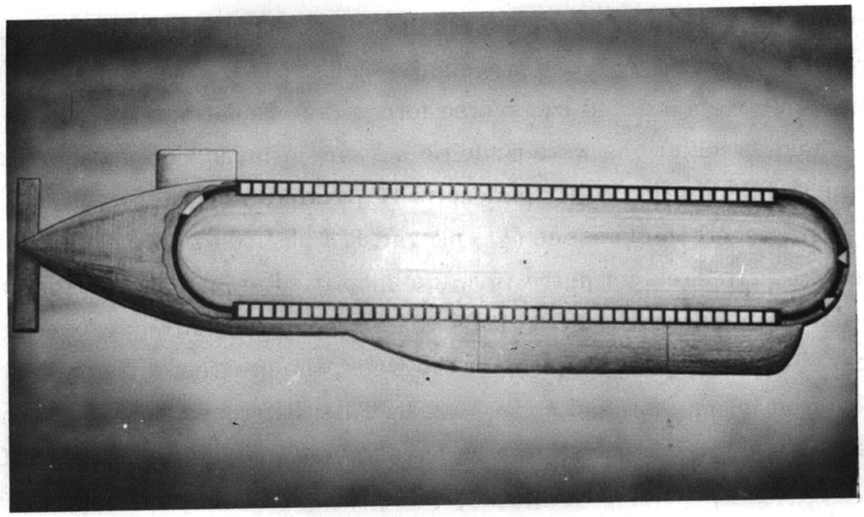

Figure 1 – Deep-Diving Submarine with Sandwich Hull

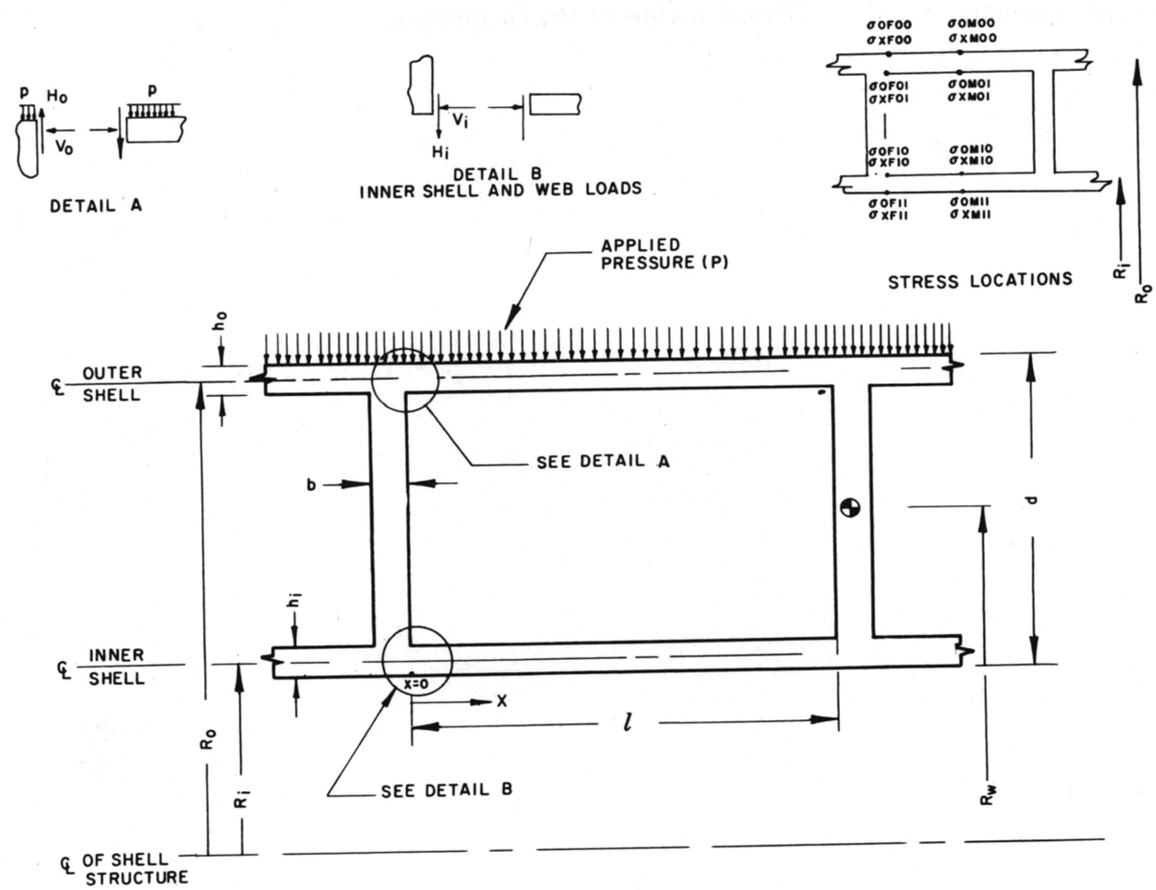

Figure 2 – Cross Section of Sandwich Shell

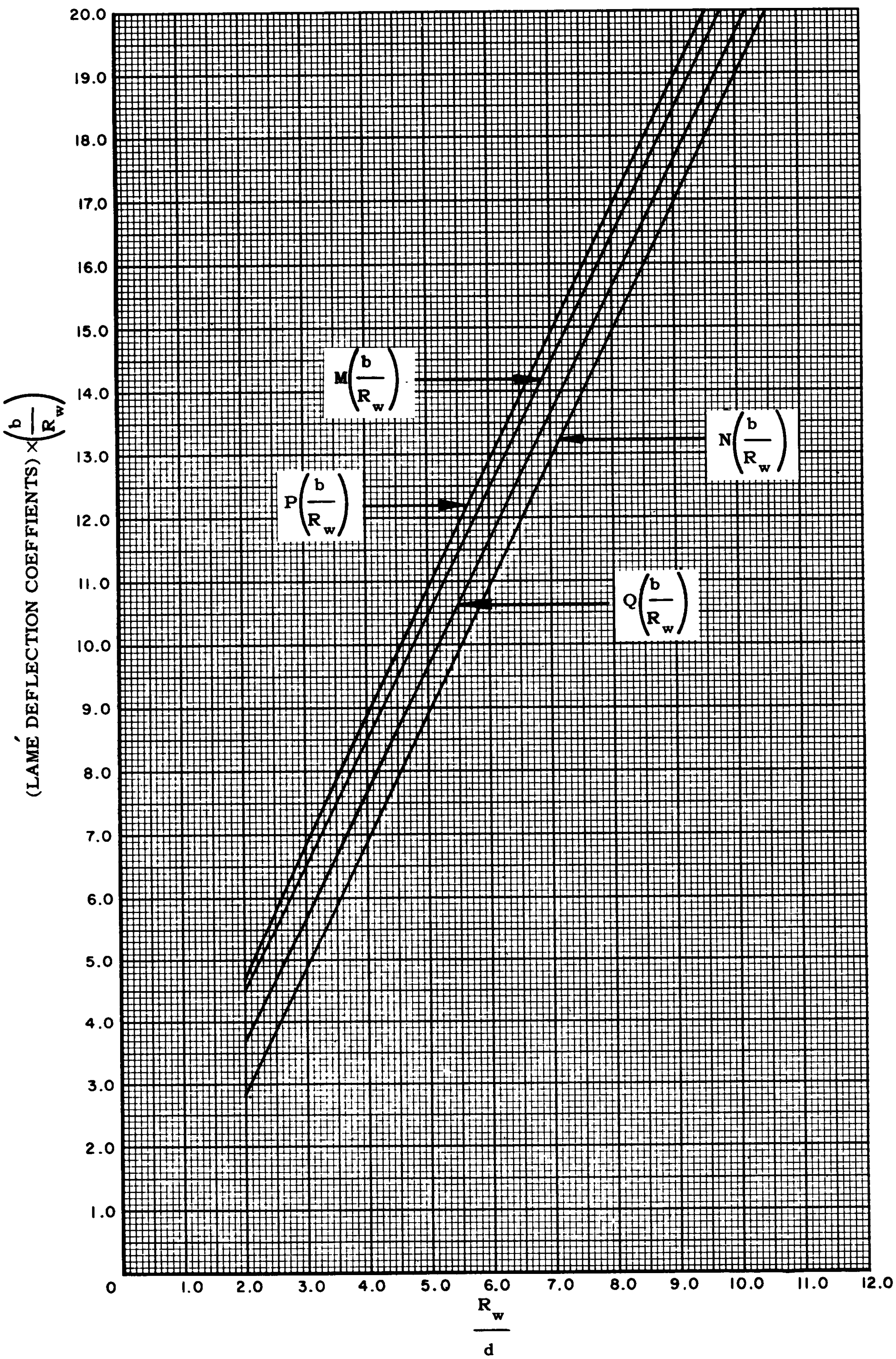

Figure 3 – Lamé Deflection Coefficients

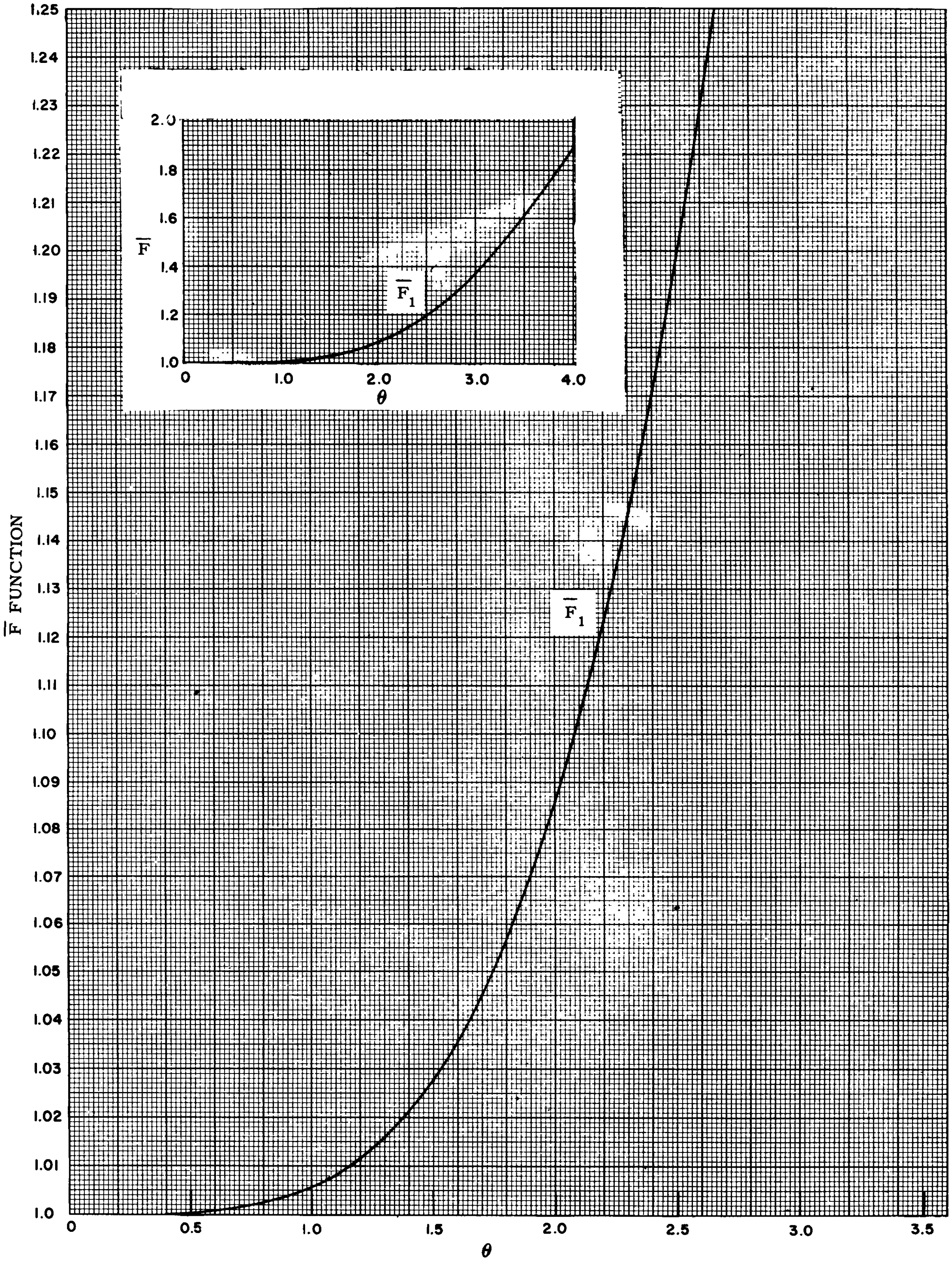

Figure 4 – Function $\overline{F}_1$

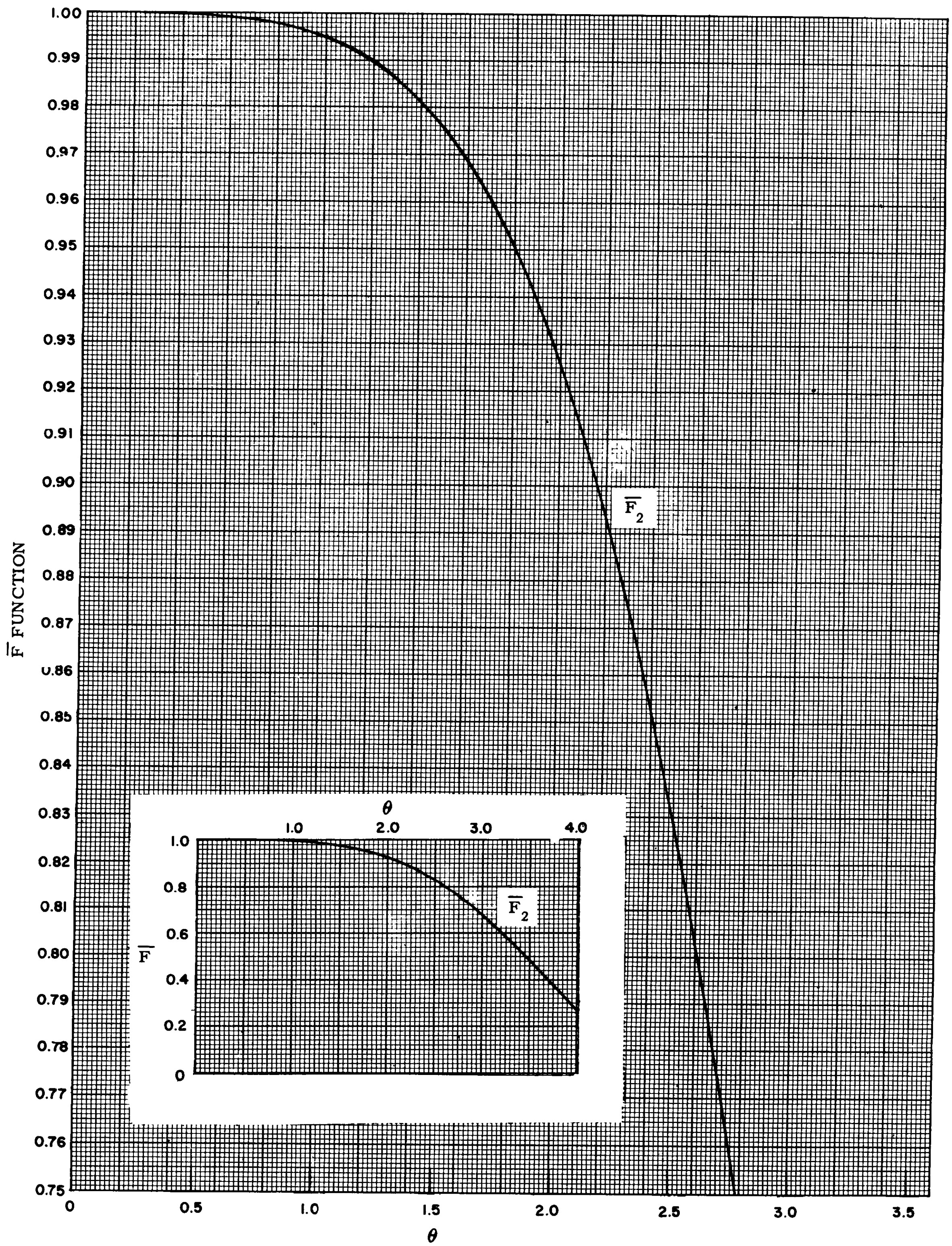

Figure 5 – Function $\overline{F_2}$

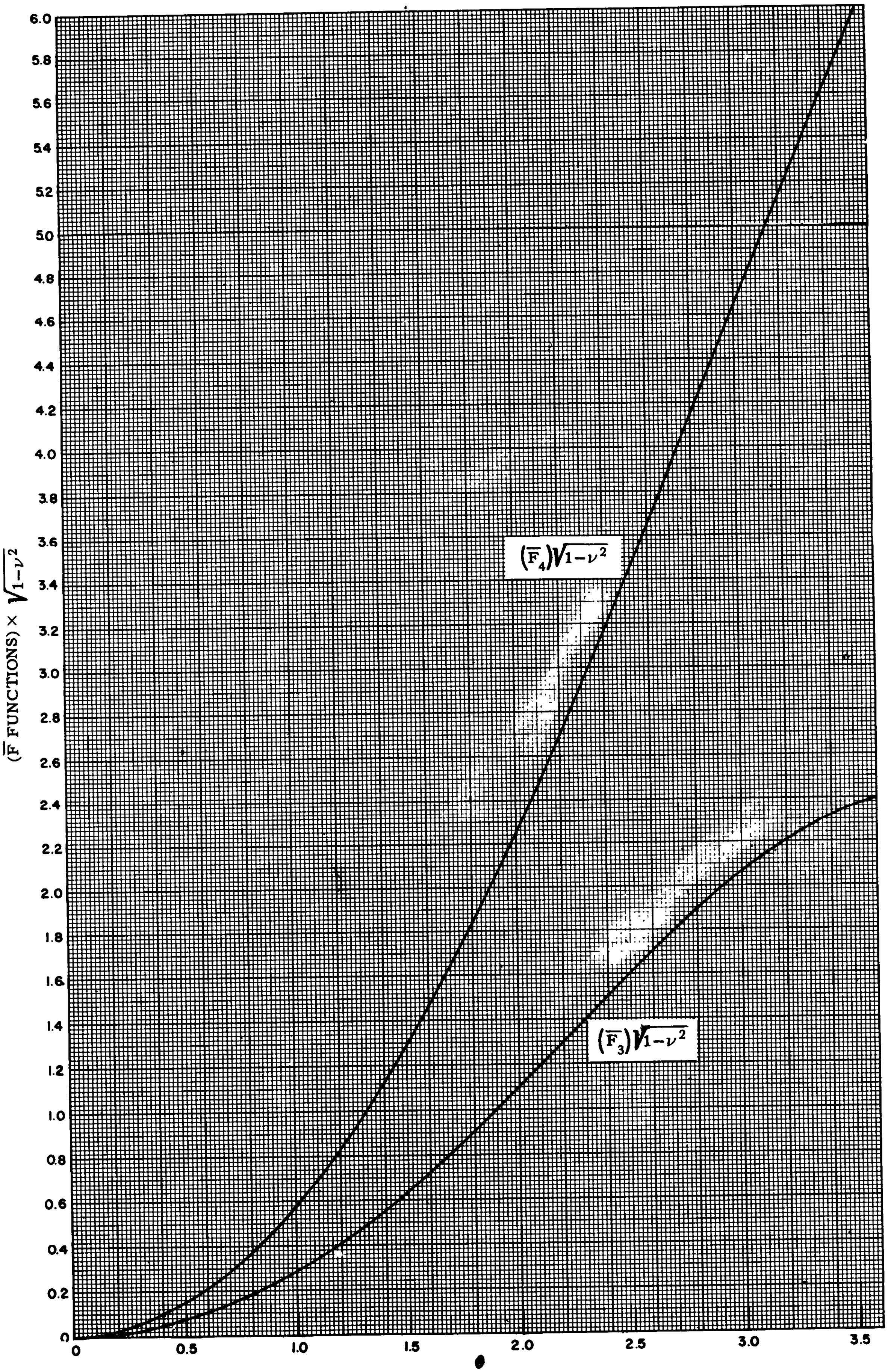

Figure 6 – Functions $\overline{F}_3$ and $\overline{F}_4$

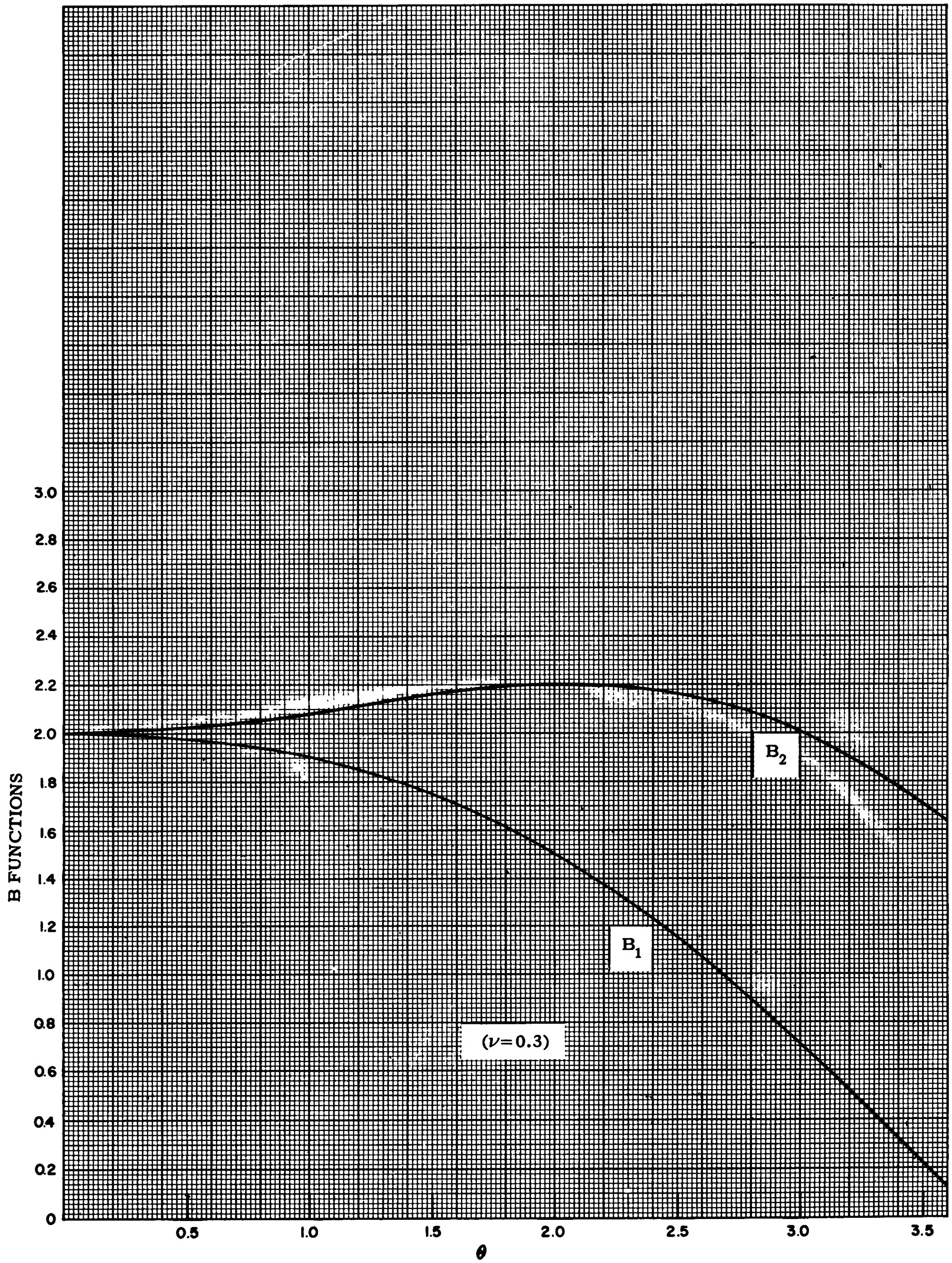

Figure 7 – Functions B_1 and B_2

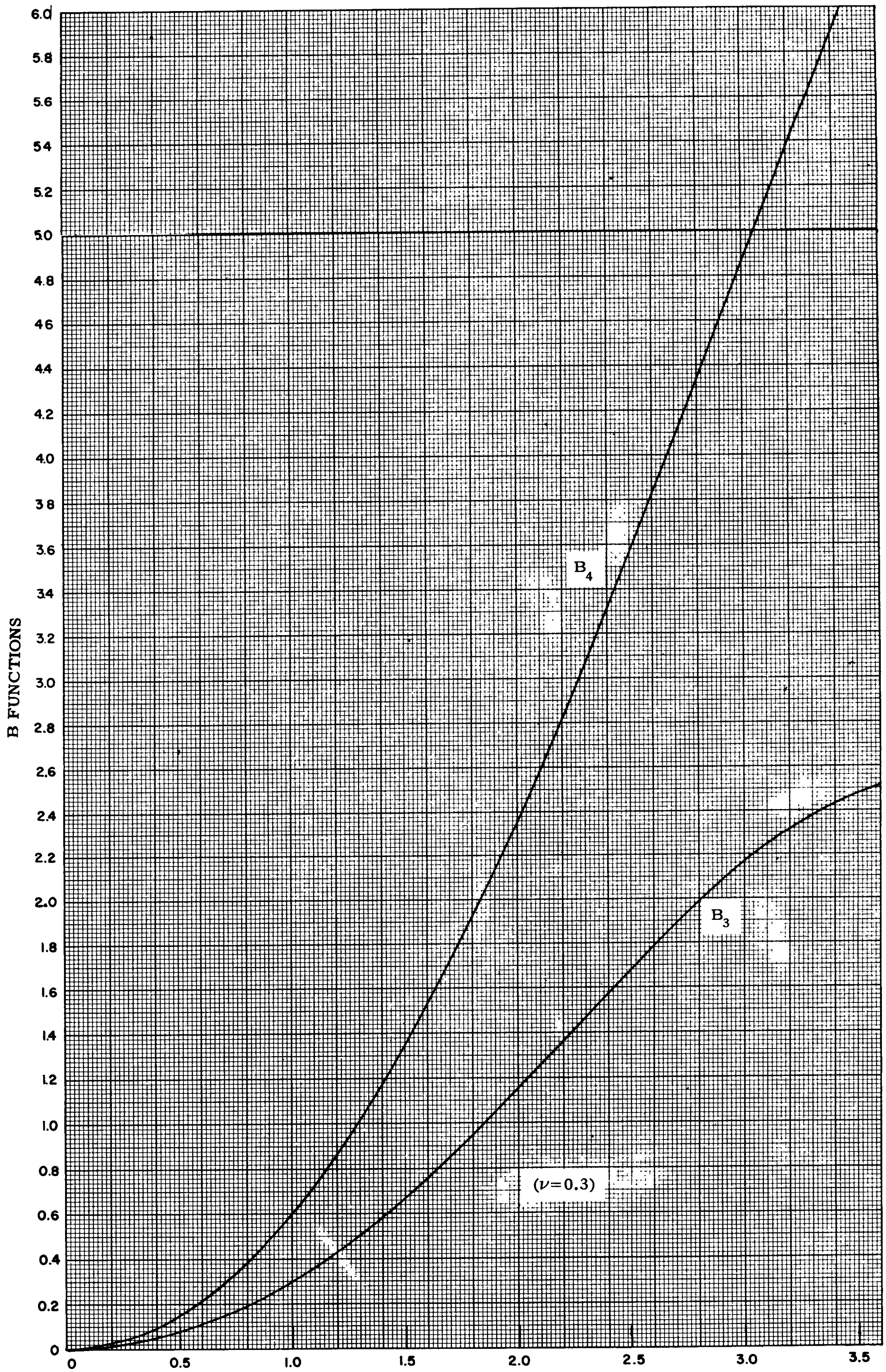

Figure 8 – Functions B_3 and B_4

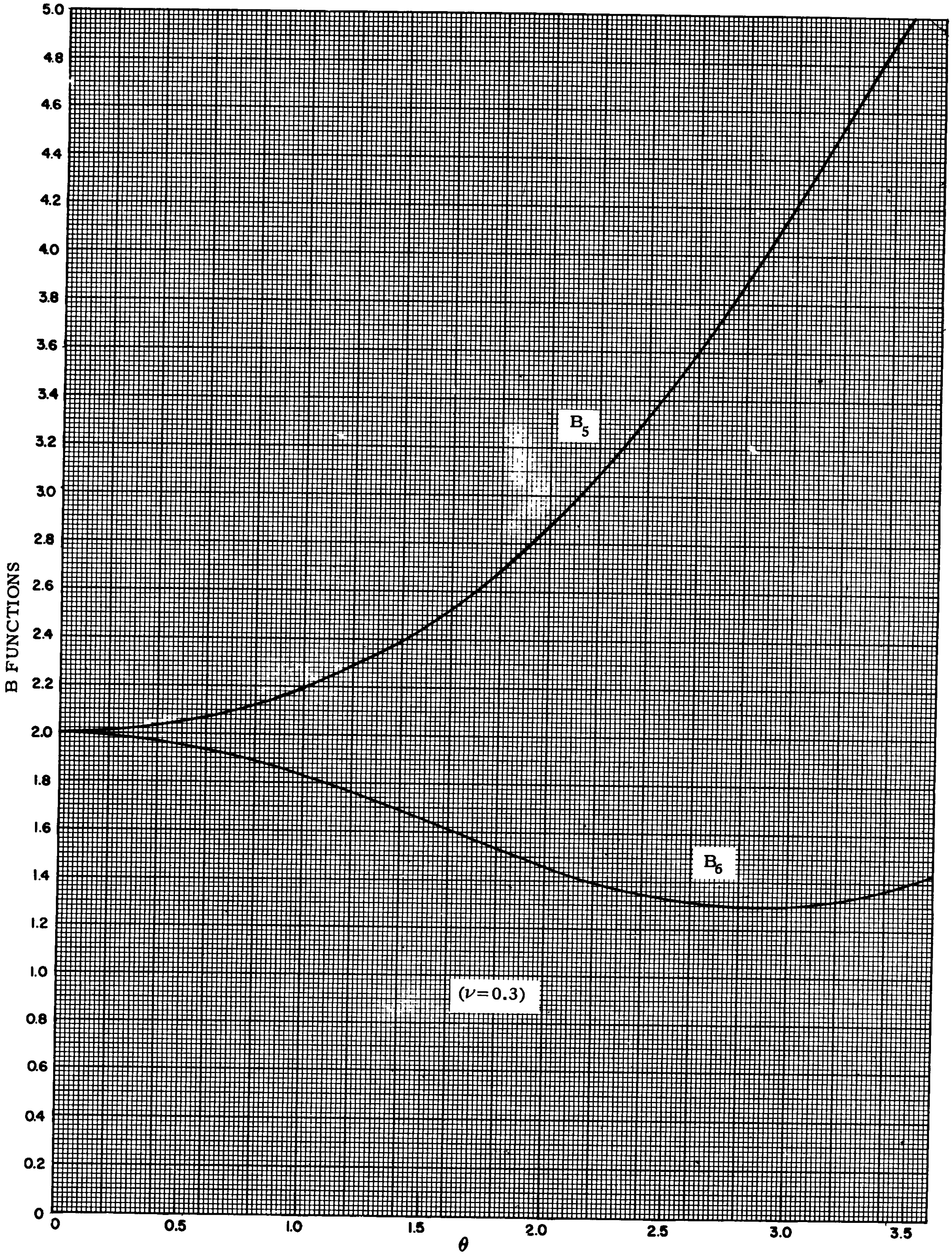

Figure 9 – Functions B_5 and B_6

TABLE 1

Calculations for Stresses in Sandwich Cylinders

GEOMETRY DESIGNATION			MODEL OV-5	
Item	Operation		Notation	Result
(1)	Input Data		R_o	53 19 in
(2)			R_i	41.91 in.
(3)			h_o	1.625 in
(4)			h_i	1.625 in.
(5)			b	1.375 in.
(6)			l	8.625 in.
(7)			p	1.0 psi
(8)	Equations [9] & [10]	(1)(3)		86.434
(9)		(2)(4)		68.104
(10)		$\sqrt{(8)}$		9.2969
(11)		$\sqrt{(9)}$		8.2525
(12)		1.2854(6)/(10)	θ_o	1.193
(13)		1.2854(6)/(11)	θ_i	1.343
(14)	Equations [8]	Figure 4	$\bar{F}_{10}$	1.011
(15)		Figure 4	$\bar{F}_{1i}$	1.018
(16)	Equations [4]	(3)(6)		14.016
(17)		(4)(6)		14.016
(18)		$(1)^2$		2829.2
(19)		$(2)^2$		1756.4
(20)		2.0(18)/(16)		403.71
(21)		2.0(19)/(17)		250.63
(22)		−(14)(20)	g_o	−408.15
(23)		−(15)(21)	g_i	−255.15
(24)	Equations [3]	[(1)+(2)]/2.0	R_w	47.55
(25)		(1)−(2)+[(3)+(4)]/2.0	d	12.90
(26)		(24)/(25)	R_w/d	3.686
(27)		(24)/(5)	R_w/b	34.58
(28)		Figure 3	$M(b/R_w)$	7.85
(29)			$N(b/R_w)$	6.25
(30)			$P(b/R_w)$	8.22
(31)			$Q(b/R_w)$	7.00

Item	Operation		Notation	Result
(32)	Equation [3]	(27)(28)	M	271.4
(33)		(27)(29)	N	216.1
(34)		(27)(30)	P	284.2
(35)		(27)(31)	Q	242.1
(36)	Equations [7]	(4)/(3)		1.0
(37)		(2)(36)/(1)		0.7879
(38)		1.0+(37)		1.7879
(39)		(1)/[2.0(38)]	V_o	14.87
(40)		(36)(39)	V_i	14.87
(41)		(18)/(3)		1741.0
(42)		0.15/(38)		0.0839
(43)		1.0−(42)		0.9161
(44)		(41)(43)		1595.0
(45)		1.0/(38)		0.5593
(46)		1.0−(45)		0.4407
(47)		0.15(18)(46)		187.0
(48)		(47)/(4)		115.1
(49)		(5)(32)/2.0		186.6
(50)		(5)(34)/2.0		195.4
(51)		(49)−(44)		− 1,408.4
(52)		(48)+(50)		310.5
(53)		(22)−(32)		− 679.6
(54)		(23)−(35)		− 497.2
(55)		(53)(54)−(34)(33)		276,500.
(56)		(51)(54)+(52)(33)		767,400.
(57)		(53)(52)+(34)(51)		−611,300.
(58)		(56)/(55)	H_o	2.775
(59)		(57)/(55)	H_i	− 2.211
(60)		(39)/(1)	V_o/R_o	0.2796
(61)		(40)/(2)	V_i/R_i	0.3548
(62)		(58)/(6)	H_o/l	0.3217

Item	Operation		Notation	Result
(63)		(59)/(6)	H_i/l	− 0.2563
(64)		−(7)(1)/(3)	σ_{uo}	−32.73
(65)		−(7)(2)/(4)	σ_{ui}	−25.79
(66)	Equations [7]	Figure 7	B_{10}	1.85
(67)			B_{1i}	1.81
(68)			B_{20}	2.11
(69)			B_{2i}	2.13
(70)		Figure 8	B_{30}	0.42
(71)			B_{3i}	0.54
(72)			B_{40}	0.85
(73)			B_{4i}	1.08
(74)		Figure 9	B_{50}	2.29
(75)			B_{5i}	2.36
(76)			B_{60}	1.76
(77)			B_{6i}	1.71
(78)	Equations [6]	(64)[1.0−(62)(66)]	σ_{OMOO}	−13.2*
(79)		(64)[1.0−(62)(68)]	σ_{OMOI}	−10.5
(80)		−(65)(63)(67)	σ_{OMIO}	−11.9
(81)		−(65)(63)(69)	σ_{OMII}	−14.1
(82)		(64)[1.0−(62)(74)]	σ_{OFOO}	− 8.6
(83)		(64)[1.0−(62)(76)]	σ_{OFOI}	−14.2
(84)		−(65)(63)(75)	σ_{OFIO}	−15.6
(85)		−(65)(63)(77)	σ_{OFII}	−11.3
(86)		(64)[(60)+(62)(70)]	σ_{XMOO}	−13.6
(87)		(64)[(60)−(62)(70)]	σ_{XMOI}	− 4.7
(88)		(65)[(61)+(63)(71)]	σ_{XMIO}	− 5.6
(89)		(65)[(61)−(63)(71)]	σ_{XMII}	−12.7
(90)		(64)[(60)−(62)(72)]	σ_{XFOO}	− 0.2
(91)		(64)[(60)+(62)(72)]	σ_{XFOI}	−18.1
(92)		(65)[(61)−(63)(73)]	σ_{XFOI}	− 16.3
(93)		(65)[(61)+(63)(73)]	σ_{XFII}	− 2.0

*Units of stress results in Items (78) through (93) are in psi/psi. Locations of stress results on cylinders are shown in Figure 2. Minus signs denote compression.

APPENDIX

EQUILIBRIUM AND STRESS CONDITIONS IN SANDWICH SHELLS

The stresses in sandwich shells are determined, in part, from the internal forces which act in the structure. This appendix shows the method of analysis for these forces and stresses.

EQUILIBRIUM IN SANDWICH SHELLS

To determine the axial and transverse forces (V_o, V_i, H_o, H_i) which act on the outside and inside shells of a cylindrical sandwich structure, it is necessary to consider their conditions of equilibrium and compatability. Equations [57] of Reference 2 show that the equations for equilibrium of forces and compatibility of deformations for a tube-core sandwich cylinder can be expressed by a series of four equations. These equations are in terms of the four unknown axial and transverse forces V_o, V_i, H_o, and H_i.

From the equilibrium conditions of the end forces on a sandwich cylinder with annular webs loaded under an external uniform pressure, the relationship between the outside and inside axial forces V_o and V_i is

$$V_o R_o + V_i R_i = \frac{R_o^2}{2} p \qquad [11]$$

By applying Equations [57] of Reference 2 to the case of a sandwich cylinder with annular webs and utilizing Equation [11], the forces H_o, H_i, and V_o can be expressed in terms of three simultaneous equations, namely:

$$\begin{aligned}
(g_o - M) H_o - N H_i - \nu \frac{R_o}{h_o} V_o &= -\frac{R_o^2}{h_o} + \frac{b}{2} M \\
-P H_o + (g_i - Q) H_i + \nu \frac{R_o}{h_i} V_o &= \frac{\nu}{2} \frac{R_o^2}{h_i} + \frac{b}{2} P \qquad [12] \\
\nu \frac{R_o}{h_o} H_o - \nu \frac{R_i}{h_i} H_i + \frac{l}{2h_o}\left(1 + \frac{h_o}{h_i}\frac{R_o}{R_i}\right) V_o &= \frac{R_o l}{2h_o}\left(\nu + \frac{1}{2}\frac{h_o}{h_i}\frac{R_o}{R_i}\right)
\end{aligned}$$

and from Equation [11], the force V_i is:

$$V_i = \frac{R_o}{R_i}\left(\frac{R_o}{2} - V_o\right) \qquad [13]$$

Equations [12] and [13] represent a series of equations for determining H_o, H_i, V_o, and V_i in a cylindrical sandwich shell loaded under a uniform external pressure of 1.0 psi. The equations assume that the sandwich cylinder is constructed from an isotropic and homogeneous material. Therefore, the resultant forces are independent of the mechanical properties of the material. The coefficients of the forces in Equations [12] are defined in Equations [3] and [4].

STRESSES AND STRAINS IN SANDWICH SHELLS

From the differential equations of equilibrium for cylindrical shells loaded under external uniform pressure, Equations [66] through [82] of Reference 2 show that the circumferential strain ϵ_θ and the axial stress σ_x in cylindrical shells are:

$$\begin{aligned}\epsilon_\theta = \frac{H}{4RD\alpha^3}\Bigg[& \frac{\sinh(\alpha l) + \sin(\alpha l)}{\cosh(\alpha l) - \cos(\alpha l)} \cos(\alpha x)\cosh(\alpha x) \\ & + \sin(\alpha x)\cosh(\alpha x) - \cos(\alpha x)\sinh(\alpha x) \\ & - \frac{\sinh(\alpha l) - \sin(\alpha l)}{\cosh(\alpha l) - \cos(\alpha l)} \sin(\alpha x)\sinh(\alpha x)\Bigg] \\ & + \frac{\nu V}{Eh} - p\frac{R}{Eh}\end{aligned} \qquad [14]$$

$$\begin{aligned}\sigma_x \begin{cases}\text{Inner Fiber} \\ \text{Outer Fiber}\end{cases}, = -\frac{V}{h} \pm \frac{3H}{\alpha h^2}\Bigg[& \frac{\sinh(\alpha l) + \sin(\alpha l)}{\cosh(\alpha l) - \cos(\alpha l)} \cos(\alpha x)\cosh(\alpha x) \\ & - \sin(\alpha x)\cosh(\alpha x) - \cos(\alpha x)\sinh(\alpha x) \\ & + \frac{\sinh(\alpha l) + \sin(\alpha l)}{\cosh(\alpha l) - \cos(\alpha l)} \sin(\alpha x)\sinh(\alpha x)\Bigg]\end{aligned}$$

By Hooke's law, the circumferential stress σ_θ and axial strain ϵ_x are:

$$\begin{aligned}\sigma_\theta &= E\,\epsilon_\theta + \nu\sigma_x \\ \epsilon_x &= \frac{1-\nu^2}{E}\,\sigma_x - \nu\epsilon_\theta\end{aligned} \qquad [15]$$

The parameters in Equations [14] and [15] are:

$$\alpha = \sqrt[4]{\frac{3(1-\nu^2)}{R^2h^2}} \qquad \text{and} \qquad D = \frac{Eh^3}{12(1-\nu^2)}$$

and

E is Young's modulus in psi,

ν is Poisson's ratio (dimensionless),

x is the axial coordinate (dimensionless); see Figure 2,

R is shell radius in inches,

h is shell thickness in inches,

l is unsupported shell length in inches, and

H and V are transverse and axial forces, respectively, in pounds/inches.

The stresses and strains on the outside shell of sandwich structures are determined by substituting H_o, V_o, and the geometric properties of the outside shell into Equations [14] and [15]. Similarly, stresses and strains on the inside shell are determined by substituting H_i, V_i, and the geometric properties of the inside shell into the same equations.

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GRAPHICAL ANALYSIS FOR MAXIMUM STRESSES IN SANDWICH CYLINDERS UNDER EXTERNAL UNIFORM PRESSURE, by James A. Nott. May 1964. ii, 21p. illus., diagrs., graphs, tables, refs. UNCLASSIFIED

A simplified solution is presented for computing the axial and transverse forces in sandwich cylinders with annular webs loaded under external uniform pressure. A series of graphs is shown for the determination of Lamé deflection coefficients and transcendental stress functions. With these forces and stress functions, a procedure is developed for computing maximum stresses in both outside and inside shells of the sandwich cylinders. Stresses can be computed more rapdily by this procedure than by a more exact solution.

1. Cylindrical shells (Stiffened)--Stresses--Graphical analysis
2. Cylindrical shells (Stiffened)--Sandwich construction--Stresses

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