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THE EFFECT OF FLUID INJECTION ON THE DRAG OF FLAT PLATES AT HIGH REYNOLDS NUMBERS

by

Paul S. Granville

HYDROMECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

September 1961

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NOTATION

a	Constant in Equation [5]
a_1	Constant in Equation [7]
C_f	Coefficient of frictional resistance
D	Frictional drag or resistance
h	Height of momentum-control surface
L	Length of flat plate
m	Exponent in Equation [9]
n	Exponent in Equation [5]
R_L	Reynolds number, $R_L \equiv UL/\nu$
u	Velocity in x-direction
u_τ	Friction velocity
U	Free-stream velocity
v_w	Injected velocity normal to wall
x	Distance along plate
y	Distance from plate
δ	Boundary-layer thickness
ζ	Constant in Equation [9]
θ	Momentum thickness
ν	Kinematic viscosity of fluid
ρ	Density of fluid
τ_w	Shearing stress on wall
o	Subscript indicating impervious wall

ABSTRACT

An analysis of the effects of fluid injection on the drag of flat plates with turbulent boundary layers indicates no reduction in overall drag even though the local skin friction is less with fluid injection than without. This is accounted for by the drag due to the momentum changes of the injected fluid when taken from the ambient flow.

INTRODUCTION

The recent aerodynamic investigations[1] of fluid injection or blowing into turbulent boundary layers mainly for cooling purposes have also indicated reductions in local skin friction. This naturally suggests the prospect of using fluid injection for the reduction of frictional resistance, especially at high Reynolds numbers for which the boundary-layer flow is turbulent.

A convenient approach to the analysis of the frictional drag of bodies with fluid injection is provided by the work of Turcotte,[2] wherein the local skin friction of a surface with fluid injection is related to that of an impervious surface. Since Turcotte's relation is unwieldy for drag analysis, a simpler power-law relation is developed which satisfactorily correlates existing experimental data. This power-law relation is used in integrating the change in momentum thickness over a flat plate with constant fluid injection. The drag of the flat plate is related to the momentum thickness of the wake by also considering the momentum changes of the injected fluid obtained from the ambient flow.

A comparison of the drag of a flat plate with fluid injection to one without shows no reduction in drag; in fact, it shows a large increase in drag. Furthermore, this drag of a flat plate with fluid injection does not

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[1] References are listed on page 7

include the added internal losses of the injection flow through the ducting and the porous shell.

LOCAL SKIN FRICTION OF TURBULENT BOUNDARY LAYER WITH FLUID INJECTION

If fluid of the same density ρ as the outer flow is injected at velocity v_w normal to the wall into a turbulent boundary layer previously existing on an impervious wall with shearing stress τ_{w_o}, the new wall shearing stress or local skin friction τ_w is then stated as

$$\tau_w = f\ (\tau_{w_o},\ v_w,\ \rho) \qquad [1]$$

A convenient dimensionless grouping as given by Turcotte[2] is

$$\tau_w/\tau_{w_o} = f\ (v_w/u_{\tau_o}) \qquad [2]$$

where $u_{\tau_o} = \sqrt{\tau_{w_o}/\rho}$ is the shear velocity on the impervious surface.

A specific relation for Equation [2] was obtained by Turcotte[2] by an analysis of the laminar and transitional sublayers which incorporated an hypothesis of Rott that the shearing stress at the outer edge of the transitional sublayer is unaffected by fluid injection, or

$$\tau_w/\tau_{w_o} = \exp\left[\ -\ 6.94\ (v_w/u_{\tau_o})(1 + \sqrt{\tau_{w_o}/\tau_w})\right] \qquad [3]$$

At small values of v_w/u_{τ_o}, Equation [3] reduces to

$$\tau_w/\tau_{w_o} = 1\ -\ 13.89\ (v_w/u_{\tau_o}) \qquad [4]$$

Turcotte's relation, Equation [3], agrees very well with the experimental results of Mickley and Davis,[3] as seen in Figure 1.

The marked decrease in local skin friction with fluid injection, shown in Figure 1, consequently prompted this investigation into the possibilities of fluid injection in providing low drags for streamlined bodies at high Reynolds numbers.

For convenience of analysis, a power law of the form

$$v_w/u_{\tau_o} = a\,(1-\tau_w/\tau_{w_o})^n \qquad [5]$$

is now proposed. This power law provides a good fit to the experimental data of Mickley and Davis, as seen in Figure 1, for $a = 0.0916$ and $n = 1.315$.

MOMENTUM THICKNESS OF A FLAT PLATE WITH FLUID INJECTION

The momentum equation for a flat plate with fluid injection in a zero pressure gradient is obtained by integrating the equation of motion across the boundary layer,[3] or

$$d\theta/dx = \tau_w/\rho U^2 + v_w/U \qquad [6]$$

where $\theta \equiv \int_0^\delta (u/U)\,[1-(u/U)]\,dy$ is the momentum thickness,

U is the velocity outside the boundary layer,

u is the velocity in the x-direction,

y is the normal distance from the plate,

x is the distance along the plate from the leading edge, and

δ is the boundary-layer thickness.

A positive v_w/U-term or fluid injection acts in Equation [6] to give a reduced local coefficient of skin friction $\tau_w/\rho U^2$. On the other hand, suction will have the opposite effect.

Equation [5] may be rewritten to give the local coefficient of skin friction for fluid injection as

$$\tau_w/\rho U^2 = (\tau_w/\rho U^2)_o - a_1 (vw/U)^{1/n} (\tau_w/\rho U^2)^{1-(1/2n)} \qquad [7]$$

where $a_1 = (1/a)^{1/n}$ and $u_{\tau_o} = (\tau_{w_o}/\rho)^{\frac{1}{2}}$. Then Momentum Equation [6] incorporating Equation [7] becomes

$$d\theta/dx = d\theta_o/dx - a_1 (v_w/U)^{1/n} (d\theta_o/dx)^{1-(1/2n)} \qquad [8]$$

since

$$d\theta_o/dx = (\tau_w/\rho U^2)_o \text{ for } v_w/U = 0$$

The power law for a flat plate without fluid injection[4] is

$$d\theta_o/dx = \zeta/(U\theta_o/\nu)^m \qquad [9]$$

For a turbulent boundary layer starting at the leading edge, $\theta_o = 0$ at $x = 0$, there results

$$\frac{\theta_o}{x} = \frac{[(1+m)\zeta]^{1/1+m}}{(Ux/\nu)^{m/1+m}} \qquad [10]$$

or

$$\frac{d\theta_o}{dx} = \frac{\zeta^{1/1+m}}{[(1+m)(Ux/\nu)]^{m/1+m}} \qquad [11]$$

Substituting Equation [11] into Equation [8] and integrating from $x = 0$ yields for uniform injection or constant v_w/U.

$$\frac{\theta}{x} - \frac{\theta_o}{x} - a_1 \left[\frac{2n\ (1+m)^{1/2n}}{2n+m}\right] \left(\frac{vw}{U}\right)^{1/n} \left(\frac{\theta_o}{x}\right)^{1-(1/2n)} + \frac{v_w}{U} \qquad [12]$$

DRAG OF A FLAT PLATE WITH FLUID INJECTION

The drag of a flat plate can be evaluated from the momentum changes encountered by the incoming stream flowing past the flat plate. In the case of the injection of fluid which originates in the free stream, the drag of the flat plate has to include the momentum change of the injected fluid. The additional losses suffered by the injected fluid in the ducting and through the porous surface will not be considered in this analysis, since they depend on specific designs and materials.

The sketch in Figure 2 shows the momentum control surface about the flow past a idealized flat plate with fluid injection, which is similar to that presented by Schlichting[5] and for an impervious plate. The overall change in the rate of momentum from the free stream to the wave is equal to the drag of both sides of the flat plate, or

$$D = \rho \int_{\infty}^{\infty} u\ (U-u)\ dy \qquad [13]$$

for a unit width of plate and h → ∞ . Since the momentum thickness of the wake $\theta \equiv \int_0^\delta (u/U)[1-(u/U)]\ dy$

$$D = 2\rho U^2 \theta \qquad [14]$$

Then for drag coefficient $C_f \equiv D/\tfrac{1}{2}\rho U^2 2L$

$$C_f = 2\theta/L \qquad [15]$$

the same as that for an impervious plate of length L.

From Equation [12], the drag coefficient of a flat plate with fluid injection becomes

$$C_f/2 = C_{f_o}/2 - a_1 \, [2n(1+m)^{1/2n}/2n+m](v_w/U)^{1/n} \, (C_{f_o}/2)^{1-(1/2n)} + v_w/U \qquad [16]$$

With a = 0.0916, a_1 = 6.15, n = 1.315, and m = 0.1686 from Reference 4, Equation [16] becomes

$$C_f = C_{f_o} - 8 \, (v_w/U)^{0.76} C_{f_o}^{\;0.62} + 2 \, v_w/U \qquad [17]$$

The values of the drag coefficient C_f as a function of Reynolds number $R_L \equiv UL/\nu$ are plotted in Figure 3 for constant rates of fluid injection v_w/U. The higher values with fluid injection make it evident that fluid injection does not lead to any overall drag reduction. This is due to the added drag arising from the momentum changes of the injected fluid being obtained from the ambient flow. The injection fluid can also be taken from other regions of the ambient flow, such as those by the trailing edge in recirculation schemes, without, however, affecting the overall drag.

ACKNOWLEDGMENT

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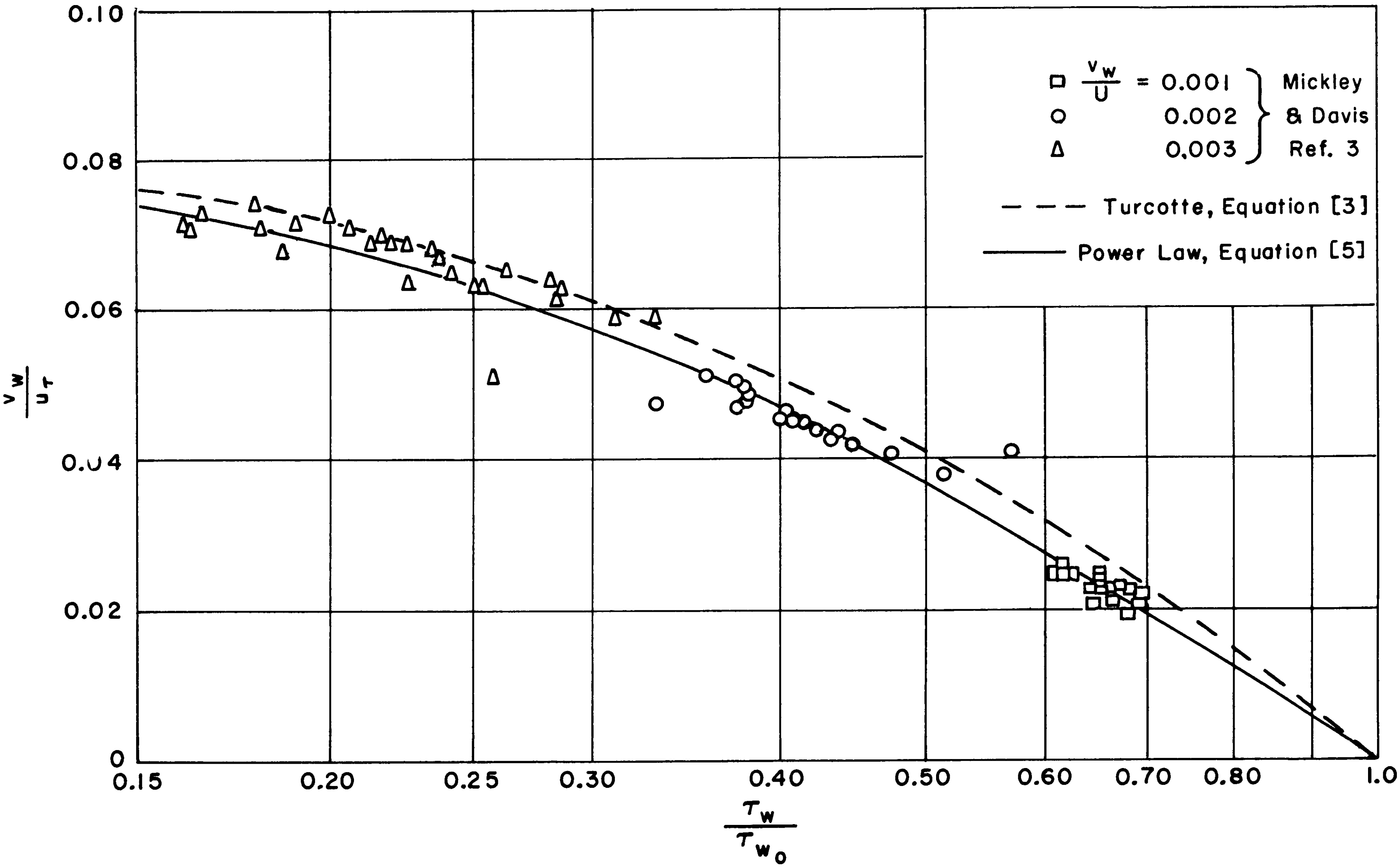
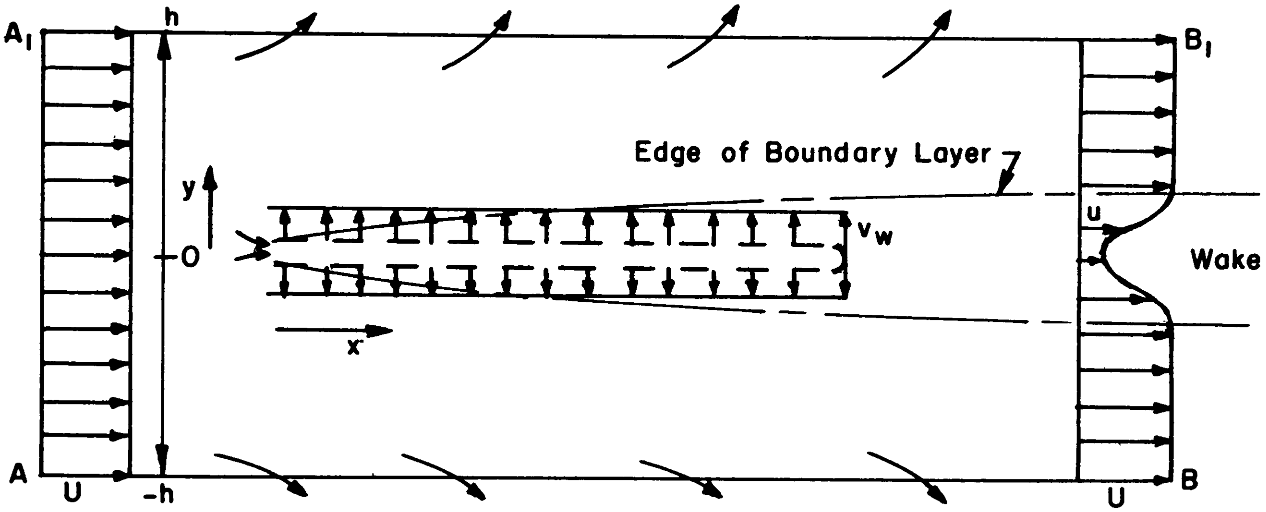


Figure 1 - Effect of Fluid Injection Velocity v_w on Wall Shearing Stress τ_w



Cross Section	Rate of Flow	Rate of x-Momentum
AA_1	$\int_{-h}^{h} U dy$	$\rho \int_{-h}^{h} U^2 dy$
BB_1	$-\int_{-h}^{h} u dy$	$-\rho \int_{-h}^{h} u^2 dy$
A_1B_1 and AB	$-\int_{-h}^{h} (U-u)\, dy$	$-\rho \int_{-h}^{h} U(U-u)\, dy$
Control Surface	0	$\rho \int_{-h}^{h} u(U-u)\, dy$

(Unit width of plate)

Figure 2 - Momentum Control Surface about Flat Plate with Fluid Injection through Porous Surface

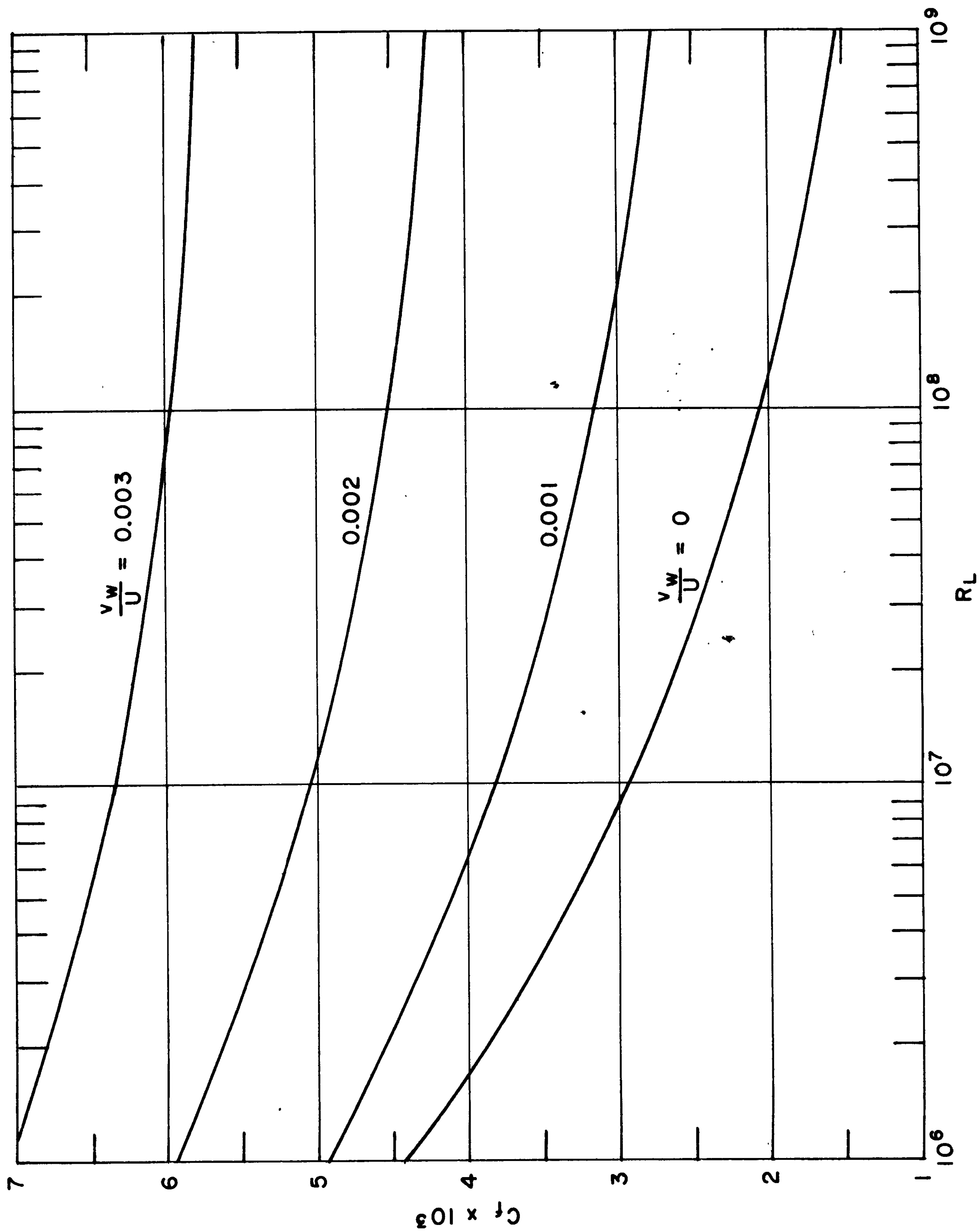


Figure 3 - Variation of Drag Coefficient C_f with Reynolds Number R_L and Injection Velocity Ratio v_w/U

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