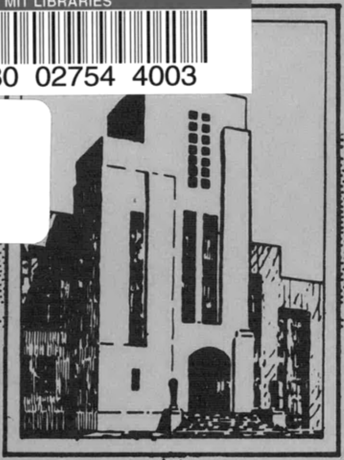
[redacted]

jak Prof Abkowitz #1



DEPARTMENT OF THE NAVY

DAVID TAYLOR MODEL BASIN

HYDROMECHANICS

○

AERODYNAMICS

○

STRUCTURAL
MECHANICS

○

APPLIED
MATHEMATICS

SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS

[redacted]

by

Margaret D. Bledsoe, Otto Bussemaker
and
William E. Cummins

SNAME Spring Meeting, 1960

RESEARCH AND DEVELOPMENT REPORT

November 1961

Report 1559

DTMB-R-1559

SEAKEEPING TRIALS ON THREE
DUTCH DESTROYERS

by

Margaret D. Bledsoe, Otto Bussemaker
and
William E. Cummins

Reprint of Paper Presented at
Spring Meeting
The Society of Naval Architects and Marine Engineers
Washington, D.C.
26–28 May 1960

November 1961

Report 1559

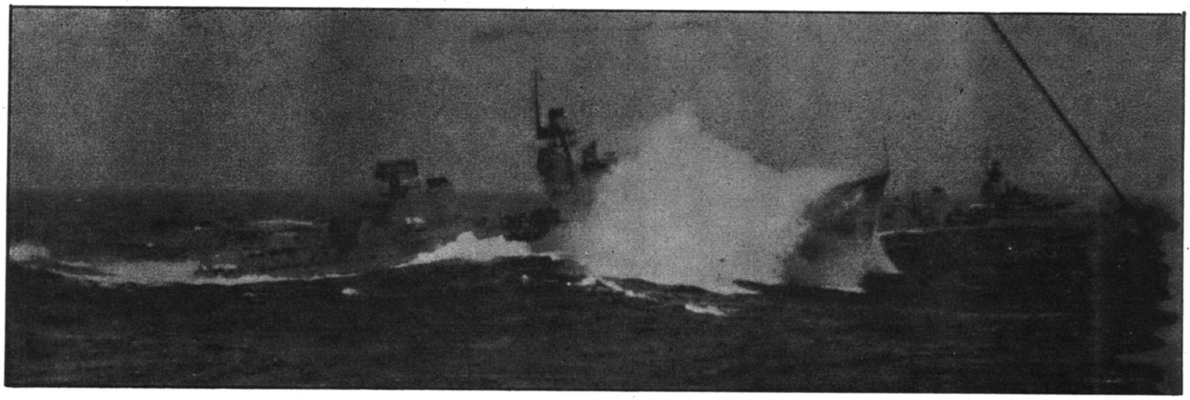


Fig. 1 Ship B slamming in rough seas—speed 17 knots

Seakeeping Trials on Three Dutch Destroyers

By Margaret D. Bledsoe,[1] *Visitor,* Otto Bussemaker,[2] *Visitor,*
and W. E. Cummins,[3] *Member*

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.

---

[1] Specialist, Seaworthiness and Fluid Dynamics Division, David Taylor Model Basin, Washington, D. C.

[2] Naval Architect, Bureau of Ships, Royal Netherlands Navy, The Hague, Holland.

[3] Head, Seaworthiness and Fluid Dynamics Division, David Taylor Model Basin, Washington, D. C.

Presented at the Spring Meeting, Washington, D. C., May 26–28, 1960, of THE SOCIETY OF NAVAL ARCHITECTS AND MARINE ENGINEERS

1-Trials

Introduction

History

THE Netherlands Naval Authorities have been interested in research on motions of ships in waves, dating from the time of the development of plans for the reconstruction and rebuilding of the Royal Netherlands Navy. In the Dutch Bureau of Ships it was realized that the general behavior of ships in waves was a design criterion of prime importance, and that the design of new weapons and instruments and the development of new functions made it necessary to obtain accurate estimates of the maximum motions and accelerations. The theoretical efforts of Weinblum, Kent, St. Denis, Pierson, and others looked promising, and adequate facilities for model tests would be available in a short time. Nevertheless, it was decided that the Royal Netherlands Navy could make a contribution to the entire field of research in seakeeping by conducting full scale trials in an attempt to get the desired data.

These early shipboard measurements were all made under the direct supervision of Mr. W. H. Warnsinck, in most cases with the aid of the crew of the ship. He used rather simple instruments of his own design. The trials gave some very useful results, such as stresses due to slamming, [1],[4] but it soon became clear that it would be difficult to derive general information from the trials because of the lack of precise knowledge of the "input" causing the motions. All knowledge of the wave portion of the wave-ship system was based on visual estimates, which were too inaccurate to provide a proper base for comparison. The possibilities of obtaining a precise measure of the waves during very extreme weather conditions are quite limited, and it is impossible to avoid the introduction of inaccuracies which tend to nullify the value of the data obtained on motions and stresses. Therefore, these trials did little to alter the fact that the comparison of the behavior at sea of different types of ships is a question of personal feeling rather than objective judgment. However, to avoid the difficulties arising from the imprecise knowledge of the sea state, at least as far as comparing different ships is concerned, an alternative procedure is available; namely, to conduct synchronous trials of different ships in the same sea. Thus, the idea of comparative seakeeping trials was born.

The successful execution of such trials with more than one ship is a major undertaking. An extensive instrumentation system is essential, and a large staff of skilled people is required to operate the equipment. Finally, the analysis of the large amount of trial data is a job of no small magnitude. Both instrumentation and staff were lacking at the Dutch Bureau of Ships. Thus, when the Chief of Naval Operations gave permission to carry out comparative seakeeping tests on three ships, it was necessary to obtain assistance in order to exploit fully the opportunity.

It proved to be a fortunate coincidence that, in the course of discussions between representatives in the RNN and the United States Navy on various aspects of seaworthiness, it was found that a collaboration in this field of research was feasible and would be of interest to both navies, and should supply useful general information. There was agreement on the importance of more reliable data on ship motions, slamming, speed loss, and stresses, of ships of the destroyer type. The RNN suggested therefore, that the David Taylor Model Basin of the USN co-operate in instrumenting and manning these trials and in the analyis of the data. An agreement was reached that the trials should be carried out under Dutch supervision with Dutch ships, instrumented with American and Dutch equipment, and that TMB would carry out the bulk of the analysis of the trial results.

Purpose

The purpose of the trials included the following aspects:

1 Comparison of the general behavior of three ships in order to obtain design information for immediate use on ships of the same type. Special emphasis was placed on:

(*a*) Rolling, pitching, and heaving.

(*b*) Slamming.

(*c*) Spray, green water on deck, and forefoot emergence.

2 To obtain information about ships' hull stresses in waves and additional stresses due to slamming.

3 To obtain information about the influence of speed, course and sea condition on the differences of the behavior of the three ships.

4 To collect data for a correlation study of the behavior of the corresponding models in a wave-making tank.

Results of the Trials

In earlier reports, [2] and [3], Warnsinck and Szebehely, and St. Denis have described the special instruments and the methods used to collect the data. Also preliminary results

---

[4] Numbers in brackets designate References at the end of the paper.

Table 1 Ships' Main Characteristics

	Ship A	Ship B	Ship S
Length, overall, m	111	116	110
Length, BP, m	108	112	103
Breadth, m	11.3	11.8	10.9
Depth, m	6.35	6.60	6.24
Draft, loaded, m	3.85	4.01	3.89
Displacement trial cond, tons	2,663	2,988	2,570
SHP	45,000	60,000	40,000
Block coefficient	0.562	0.563	0.569
Midship coefficient	0.815	0.827	0.798
Waterplane coefficient	0.795	0.801	0.803
GM transverse, m	0.72	0.72	0.88
GM longitudinal, m	271.6	282.8	249.6
Longitudinal radius of gyration, m	25.7	26.2	24.4
Natural period of roll, sec	9.11	11.96	9.54
Natural period of pitch, sec	4.3	4.6	4.0
Natural period of heave, sec	4.8	4.7	4.8
Section modulus topside for vertical bending, cu cm	1004 × 10^3	836 × 10^3	1410 × 10^3
Section modulus topside for transverse bending, cu cm	1212 × 10^3	1333 × 10^3	1196 × 10^3
Calculated frequency of 2-node vertical vibrations, cpm	75	72.3	

were disclosed together with some comments on seakeeping trials in general. Since the present paper is mainly a data report, some of the facts and data that were given before are repeated here in a more detailed form for completeness.

None of the present authors took an active part in the preparation and effectuation of the trials, and an important part of the analysis of the measured data was carried out under supervision of people not fully aware of all the motives for the choices of instrumentation and decisions about the execution of the trials. For this reason:

1 The analysis of the data took much more time than estimated beforehand, and the analysis could not be completed according to the original plan.

2 Not all available data were completely analyzed because of questions as to their validity and usefulness.

3 Some of the statements in earlier reports [3], which were based on preliminary results, are not supported by the presently available facts and figures.

The paper deals with the analysis of the trial results, and presents a compilation of the data collected. The prime purpose is (*a*) to make the data accessible in a form which makes it possible quickly to compare the motions, accelerations, and stresses of ships of the same type under comparable weather conditions, and (*b*) to provide data for comparison with other ships and models. It is felt that this objective has been achieved, with the reservation that a quantitative comparison of the stresses is not considered completely valid. However, even here a qualitative comparison is believed justifiable, and in particular it is possible to compare stresses due to slamming with wave-induced stresses.

Data on Ships

The ships available for the trials were three different types of destroyers of approximately the same dimensions, identified here as ships A, B, and S. It may be pertinent to state the advantages underlying the choice of this type of ship for trials on sea behavior. Of all vessels capable of sustained operation in high seas, the destroyer, because of her small size and great speed potential, experiences a larger range of operating conditions than any other vessel. Moreover, in modern warfare one of the destroyer's main opponents, the submarine, is not affected by the sea conditions which tend to limit the fighting efficiency of the hunter. Thus, information on the sea behavior of this type of ship is of prime importance.

Ships A and B, type AS destroyers, were commissioned shortly before the trials and ship S was a wartime built utility-type destroyer. A and B are practically all-welded ships of modern construction with extensive use of high-tensile steel. Ship S is a riveted ship of prewar construction and of British Naval Standard D Quality Steel.

Table 1 lists the ships' main characteristics. The body plans and curves of transverse and longitudinal static stability (righting arms for changes in heel and trim) are given in Figs. 2 and 3.

The longitudinal radii of gyration of ships A and B were calculated from the available detailed weight distribution. Since such a detailed weight distribution for ship S was not available, the radius of gyration for this ship was estimated from

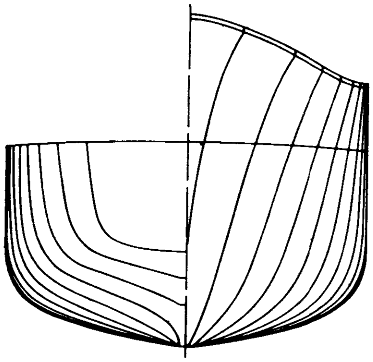


Figure (2-a) Body Plan, Ship A

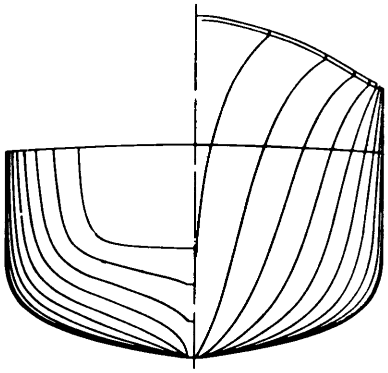


Figure (2-b) Body Plan, Ship B

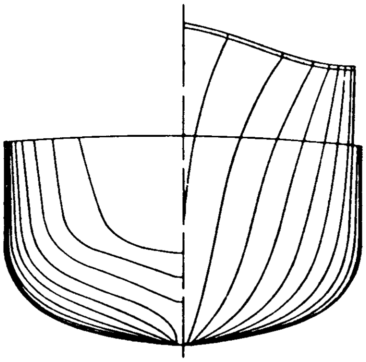
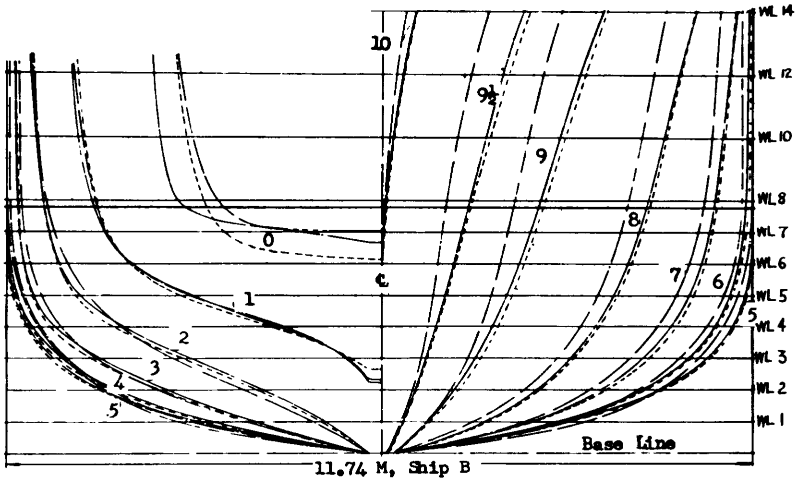


Figure (2-c) Body Plan, Ship S

Figure (2-d) Comparison of Ship Forms



	Ship A	Ship B	Ship S
Breadth	11.300 M	11.740 M	10.893 M
Draft	3.825 M	3.900 M	3.505 M

Fig. 2 Body plans and comparison of ship forms

the available schematic weight-distribution curve. The natural periods of pitch and heave are taken from the model trials at zero speed. Natural periods for roll are taken from heeling experiments in still water at zero speed. Frequency of the two-node vertical vibrations of ships A and B was computed according to the methods of Lockwood Taylor with added mass from Prohaska [4]. Measurements on a sister ship of ship A have shown a frequency of about 78 cpm for the two-node vertical bending vibration.

Description of Trials

Instrumentation

To obtain the desired information about the behavior of the ships in sea waves, it was decided to use recording instruments which were available at the time of planning the trials (early 1955) and also had proved their reliability before. The majority of the instruments were furnished by TMB. RNN supplied instruments for recording the motions, the cameras, and the spare accelerom-

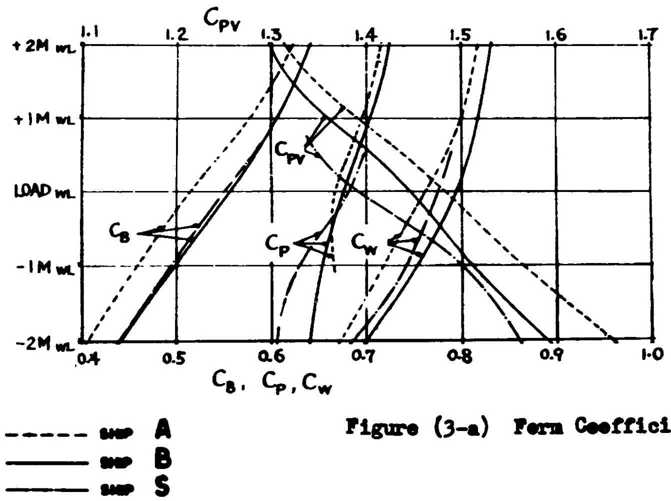


Figure (3-a) Form Coefficients

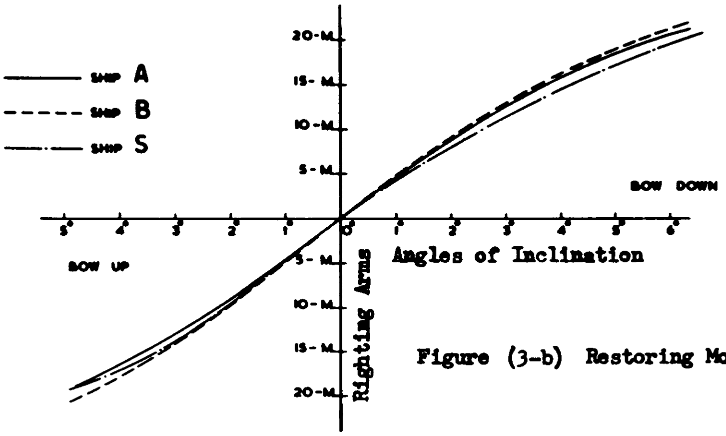


Figure (3-b) Restoring Moment for Pitch

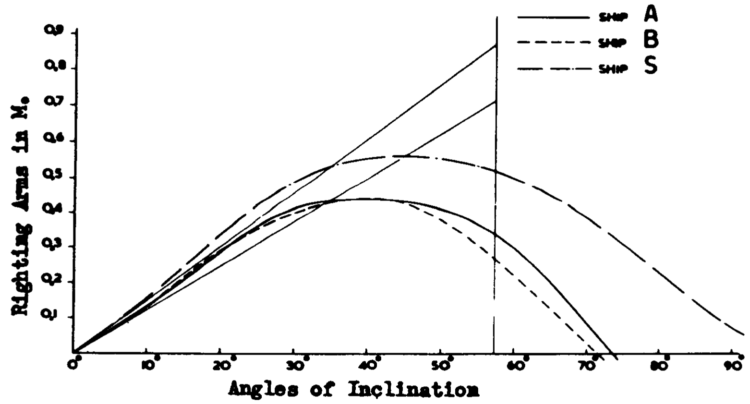


Figure (3-c) Curves of Stability

Fig. 3 Form coefficients, restoring moments for pitch and curves of stability for the three ships

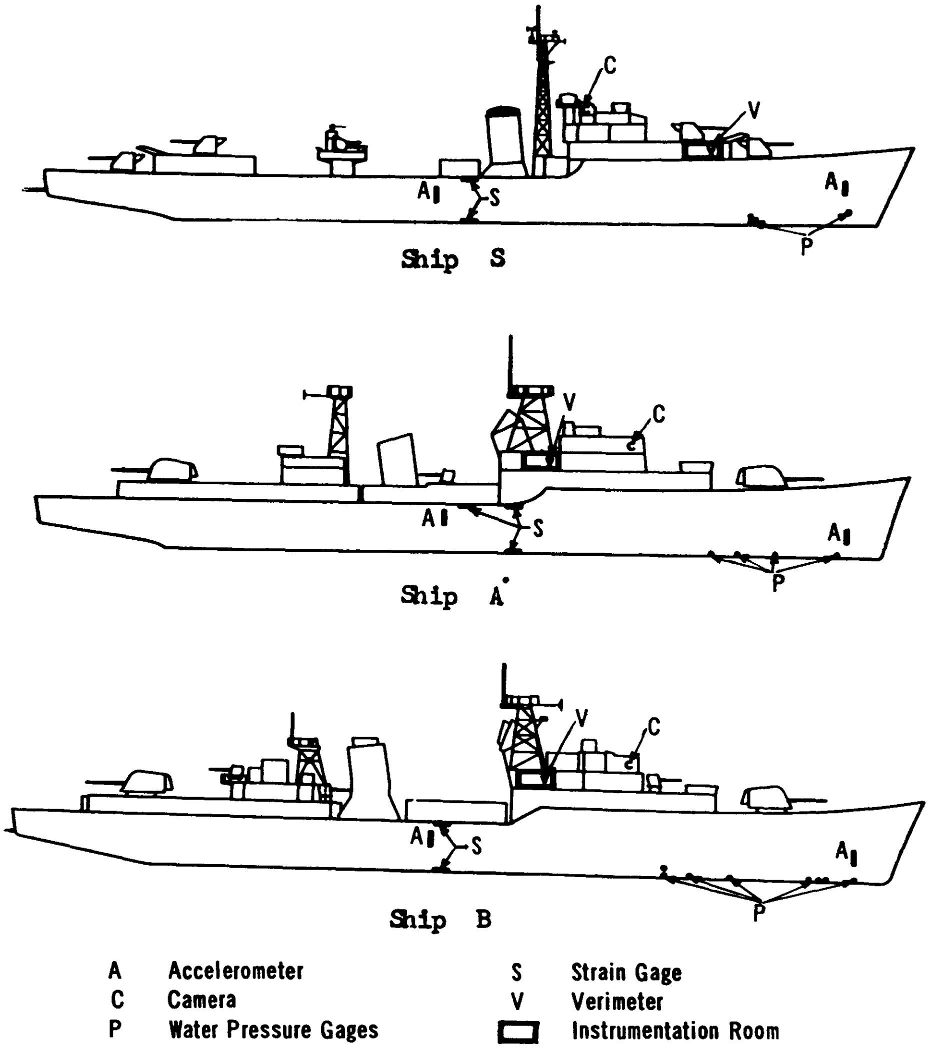


Fig. 4 Location of instruments

eters. In later stages of the planning it was decided to pay more attention to slamming, and the necessity for recording slams arose. It was expected that this could be done by recording the stresses and vibration due to slamming in the main structural members, and so at the same time providing means for judging the effect of slamming on the hull. Thus, strains were recorded in co-operation with the Institute for Applied Physical Research (TNO) Delft and the University of Gent, Belgium, who furnished additional instrumentation and personnel for the strain measurements. For the distribution of the instruments over the ships and their location, see Tables 2, 3, and 4 and Fig. 4.

Ship motions were recorded on "Verimeters" built by the RNN for ship-motion measurements. They record simultaneously roll and pitch angles and linear accelerations in three directions. The roll and pitch angles are taken from a gyroscope of approximately 18,000 rpm. Linear accelerations are taken from mass-spring systems, which unfortunately have the disadvantage that only low-frequency accelerations can be picked up. Recording was done on a single roll of paper by mechanically operated pens. Because of the limited paper speed of about 1 mm per sec it is not possible to record high-frequency components.

A disadvantage of the Verimeter for this purpose was only recognized in the later stage of analysis. It was found that the pitch and roll records suffer from the precession of the gyro units which was set off by the hard turns during the maneuvers. The degree of damping of the motion of precession varies with each run of the trials and it is quite small, with the result that a large spurious com-

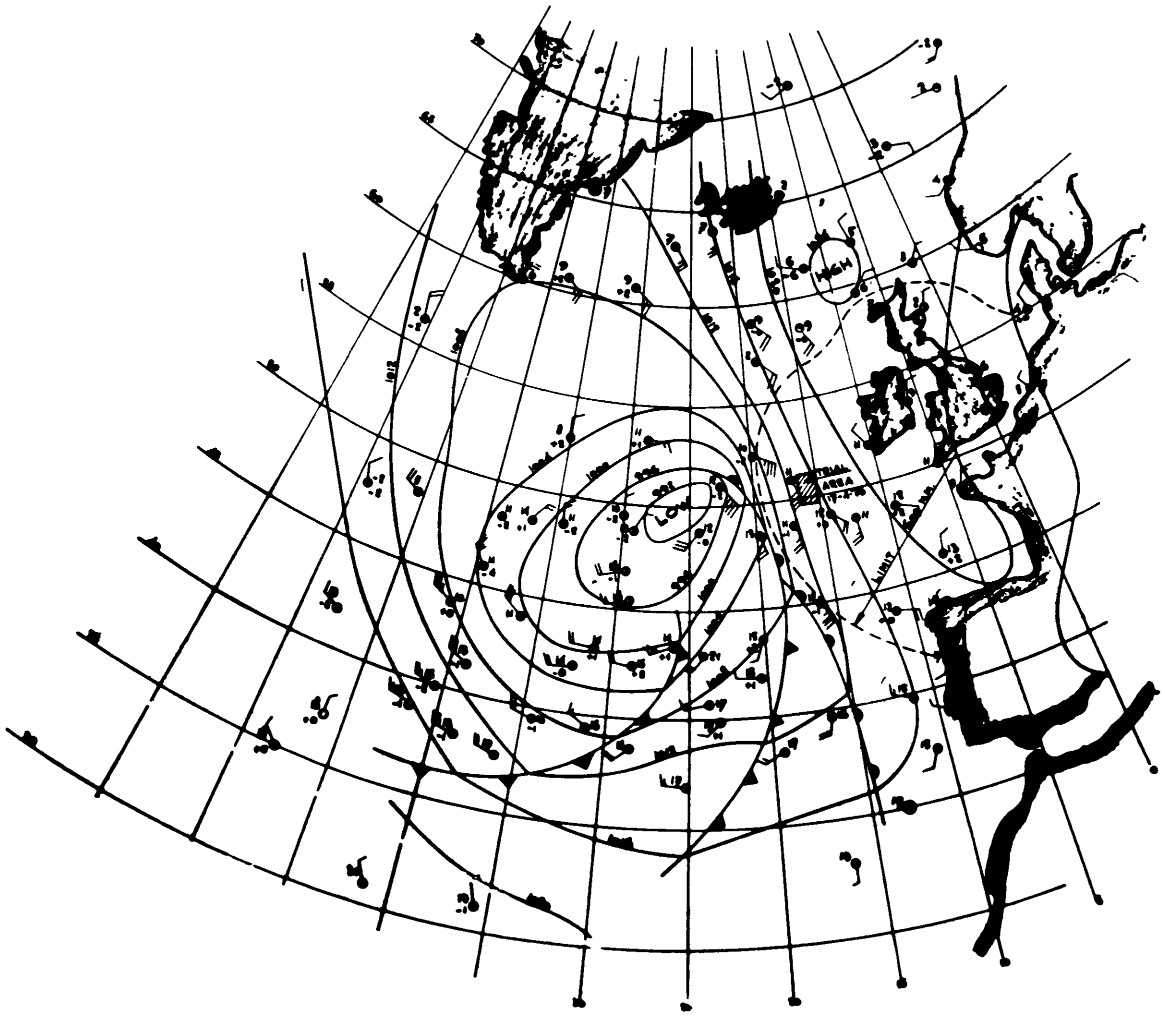


Fig. 5 Weather chart in operating area on April 17, 1956

ponent is introduced. The Verimeters were calibrated in the workshop on turning tables.

Vertical accelerations at the bow and near the center of gravity were measured with 10-g Statham accelerometers. The natural frequency of the accelerometers was 250 cps. The accelerometers were calibrated aboard, statically.

Water pressures in the ship's bottom were measured with two types of pressure gages:

1 *Dynisco gages:* These membrane-type instruments operate with an unbonded strain gage. The capacity of the gage is 500 psi; the natural frequency is 14 kc. The gages were calibrated on board in drydock.

2 *TNO pressure gages:* These instruments were manufactured by TNO, especially for RNN purposes. The gages operate with a four arm bridge of bonded strain gages on a membrane and are designed for 150 psi maximum load. The gages were calibrated in the laboratory and installed in the ship's bottom in the docking-plugs.

Longitudinal stresses in the ships' hulls were measured with strain gages of the Baldwin A1 type, prefabricated with rubber cover and cable by TNO method. Stresses were measured in the midship section at the rider plates of the center keel and at port and starboard sheerstrakes.

Recording of the data on accelerations, at the bow and CG, pressures, and stresses amidships was done on string oscillographs of three types, Tables 2, 3 and 4. The principle of the oscillographs is the same. Each channel consists of a thermostatically stabilized sensitive galvanometer. Light rays reflecting from mirrors on the galvanometers produce the amplified signals on photographic paper. During most runs a paper speed of 0.45 ips was used. During the special slamming runs, the paper speed was 1 fps. This high speed was chosen in order to obtain records of the high-frequency phenomena during impact between the hull and water surface. The *amplifiers* forming the link between the gages and the oscillographs were of four different types, Tables 2, 3 and 4. Since, during the initial stage of the planning of the trials a reliable wave-height meter was not available, it was not possible to collect exact data on the waves. However, to obtain at least some information on the sea, photographs were taken of the ships during the runs. The photographs indeed give some information about relative ship and wave position and bow emergence and submergence. The pictures were taken with special cameras with automatic film transport and electrically controlled shutters. Radio transmitters and receivers located in the three ships operated the electromagnetic shutters, thus permitting the center ship to be photographed simultaneously from both sides. The photographs were synchronized with the data records.

Table 2 Instrumentation List, Ship A

Quantity	Measuring Instrument		Location	Recording Instrument	Amplifiers
Roll	Verimeter No 1		Instrumentation Room	Verimeter No 1	
Pitch					
Longitudinal Acceleration					
Transverse Acceleration					
Vertical Acceleration					
Vertical Bow Accel	Statham		Fr 187, C L 2 7 m above keel	Consolidated String Oscillograph (TMB) Type 5-114-P4, No 355383	Gent Carrier Amplifier Type 1 113B Carr Freq 3 K.C.
Vertical C G. Accel	Statham		Fr 93, C L 4 7 m above keel		
Pressure	Dynesco pressure gage		Fr 184-185 P side of keel		Carrier Amplifier Type 1-127 No 157969 Carr Freq 20 K C.
Pressure			Fr 169-170 P side of keel		
Pressure			Fr 160-161 P side of keel		
Pressure		157	Fr 153-154 P side of keel		
Strain	Baldwin A-1 TNO-prefabricated	22	Fr 111-112, P sheerstrake	Consolidated String Oscillograph (Gent) Type 5 114P3 No	Consolidated Carrier Amplifier Type 1 113B Carrier Freq 3 K.C.
Strain		25	Fr 99-100, P sheerstrake		
Strain		51	Fr 111-112, SB sheerstrake		
Strain		52	Fr 99-100, SB sheerstrake		
Strain		50	Fr 111-112 Keel		
Strain		50	Fr 111-112 Keel		

Sea State

The sea condition was not recorded during these trials. Therefore, an attempt was made to perform a hindcast of the sea based on weather charts obtained in the area during the trial period. An example of the weather charts is shown in Fig. 5. Two different sea conditions were met during the trials. The first was encountered on April 11, 1956 when a moderate sea generated by a wind of force 4–5 was superimposed on a medium large swell. It was encountered at 250 miles WSW from Torbay. This is referred to in the present report as the Y-sea condition. The next significantly different sea (designated as X) was encountered some 400 miles west of Lands End on April 17, 1956. This was a state 5 to 6 sea again superimposed over a moderate swell. On both dates the local winds were from the southeast while the swell reached the operating area from a southwesterly direction. The results of the hindcasts for these two areas on the appropriate dates are listed in Table 5.

From the data of Table 5, spectra of the seaway were computed for the appropriate wind conditions following the procedure of Neumann. For the Y-sea condition, wave spectra were computed only for the swell, since a comparison of the *E*-values for sea and swell shows that the effect of the sea was comparatively small. Motions measured in the Y-sea state are then assumed due to the presence of swell only, and heading angle is referred to the direction of swell. The spectra describing the Y-sea state are shown in Figs. 6(*a*) and (*b*). Shown also are the spectra modified for the case when the ships were travelling into the waves at speeds of 16, 22, and 26 knots. These were obtained from the original spectra by multiplying the ordinate by the Jacobian and modifying the frequency to the appropriate frequency of encounter [5]. Fig. 6(*a*) describes the sea at the beginning of the tests; Fig. 6(*b*) at the end.

For the X-sea state, spectra were computed for both the swell and sea separately and the combination of the two was obtained by adding the components at corresponding frequencies. The sea spectrum for a 24-knot wind and modifications for appropriate ship speeds are shown in Fig. 7(*a*) by the solid lines, for the fully developed sea. The dashed line corresponds to the partially developed sea which existed at the time of the first hindcast. However, since the sea was within ½ hr of being fully developed at the initial phase of the tests, the spectra modified for ship speeds were computed for both the partially and fully developed sea for the 12-knot speed. In the 17-knot speed the modified spectrum is based on the fully developed sea only. Since the swell characteristics did not change appreciably between the

Table 3 Instrumentation List, Ship B

Quantity	Measuring Instrument		Location	Recording Instrument	Amplifiers
Roll	Verimeter, No 3		Instrumentation Room	Verimeter, No 3	–
Pitch					–
Longitudinal Acceleration					–
Transverse Acceleration					–
Vertical Acceleration					–
Vertical Bow Accel	Statham 158049		Frame 195, C L 2 9 m above keel	Consolidated String Oscillograph Type 5-114-Pr-18	Carrier Amplifier Type 1-118 Carr Freq 3 K.C.
Vertical C G Accel	Statham 158065		Frame 96, C L 5 5 m above keel		
Pressure	Dynesco pressure gage	727	0 8 m aft Fr 195 SB 7 m above keel		Consolidated Carrier Amplifier Type 1-127 Carr Freq 20 K.C.
Pressure		729	27 m aft Fr 157 75 m outbd, SB		
Pressure		597	1 13 m fwd 150 72 m outbd, SB		
Pressure		756	39 m fwd Fr 150 2 84 m outbd, SB		
Pressure	TNO pressure gage	177	25 m aft Fr 189 45 m outbd, P	Hathaway String Oscillograph R 12 - B	Hathaway Stain Gage Control Unit R 12 - A
Pressure		175	10 m aft Fr 188 45 m outbd, SB		
Pressure		174	45 m aft Fr 183 45 m outbd, SB		
Pressure		176	45 m aft Fr 166 65 m outbd, P		
Strain	Baldwin A-1 TNO-prefabricated	24	Fr 98-99 P sheerstrake		
Strain		84	Fr 98-99 SB sheerstrake		
Strain		46	Keel, Fr 98-99		

Table 4 Instrumentation List, Ship S

Quantity	Measuring Instrument		Location	Recording Instrument	Amplifiers
Roll	Verimeter No 2		Instrumentation Room	Verimeter No 2	
Pitch					
Longitudinal Acceleration					
Transverse Acceleration					
Vertical Acceleration					
Vertical Bow Accel	Statham		Frame 9 C L	Consolidated String Oscillograph Type 5-114 P 2 18 Channel	Consolidated Carrier Amplifier Type 1-118 4 channel carr freq 3 K.C.
Vertical C.G. Accel	Statham		Frame 5 C.L		
Strain	Baldwin A-1 TNO-prefabricated		Fr 52-53 P side sheer-strake		
Strain			Fr 52-53, SB side Sheerstrake		
Strain			Fr 52-53 keel		
Pressure	Dynesco Pressure gage		Fr 8-9 1 m outbd		Consolidated Carrier Amplifier Type 1-127 4 channel Carr freq 20 K.C.
Pressure			Fr. 30-31 1 m outbd		
Pressure			Fr 32-33 1 m outbd		
Pressure			Fr 32-33 21 m outbd		

Table 5 Sea and Swell Hindcasts in Operating Area of Destroyer Trials

	Y		X	
Sea	1230Z	1830Z	1230Z	1830Z
Location of fetch	47°N-8°W	48°N-7°W	47.5°N-15°W	47°N-14°W
Fetch width, nm	360	220	210	270
Fetch length, nm	240	270	300	300
Wind speed, knots	12*	10*	24	24*
Effective duration, hrs	9	15	12	18
Wave direction	SE	SSE	SE	SE
Total energy, *E*, sq ft	0 6	0.25	14.0	19 2
Period band, sec	1–7	1–6	3–10	3.7–13.5
Significant height, ft	2.2	1.4	10 6	12.4
Swell				
Location of generating area	42.5°N-21°W		45°N-41°W	
Date swell was generated	10 April, 1230Z		15 April, 1230Z	
Fetch width, nm	200		250	
Fetch length, nm	200		300	
Decay distance, nm	400		700	
Total energy, *E*, sq ft	4.8	4.0	5.6	6.4
Period band, sec	8 5–13.5	7.3–10.0	9.7–13 8	8.8–12.2
Significant height, ft	6.2	5.7	6.7	7.2
Direction of swell	SW	SW	SSW	SSW

---

* Fully developed.

time of the first and second hindcast, swell spectra were computed from the data of the second hindcast only, that corresponding to the fully developed sea.

In arriving at a swell spectrum the assumption is made that the distribution of swell energy throughout its bandwidth is similar to the distribution of energy in the sea over its period band.

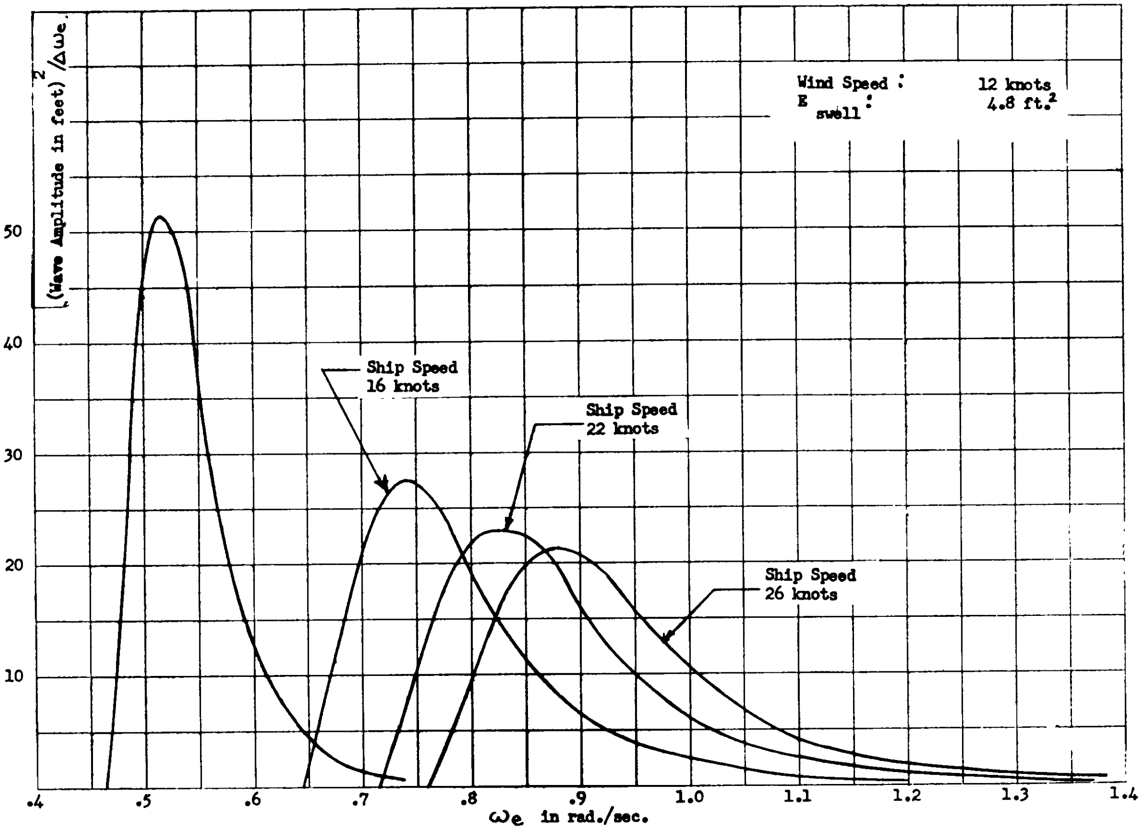


Fig. 6(a) Neumann spectra for Y sea state in operating area at beginning of test period, swell only. Modified spectra derived for ships heading into swell

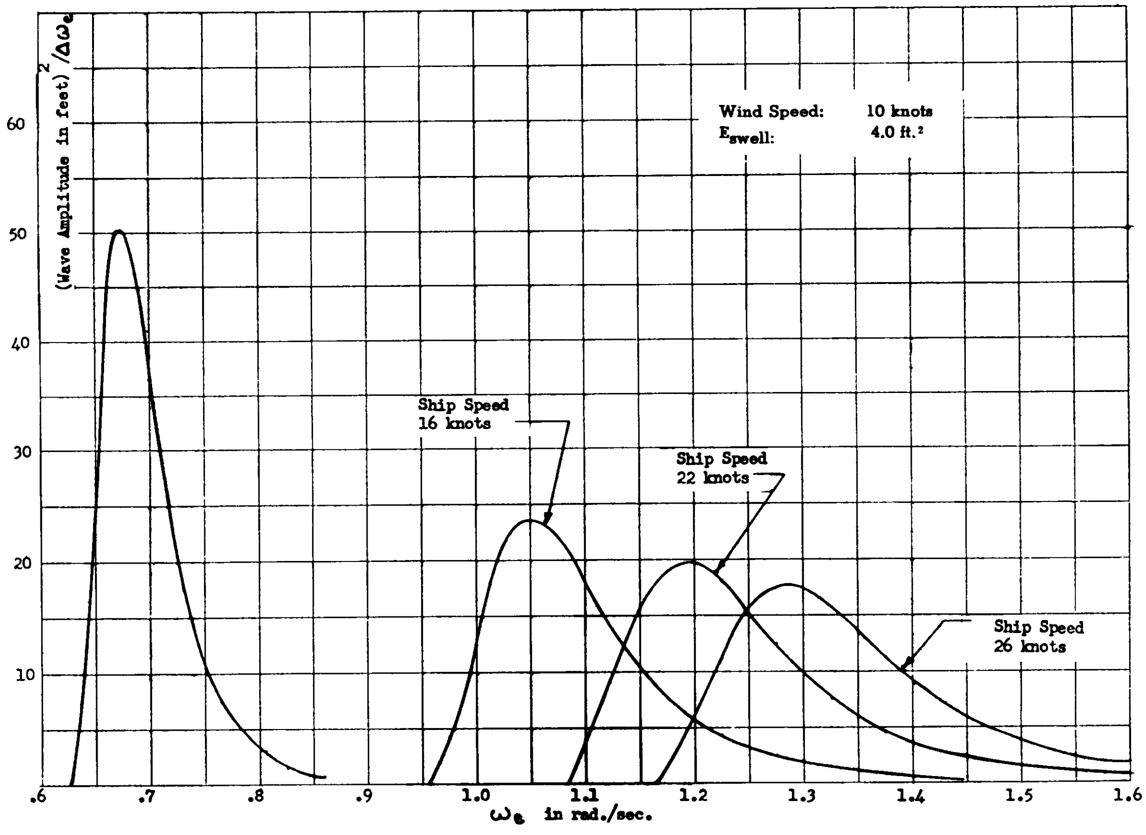


Fig. 6(b) Neumann spectra for Y sea state in operating area at end of test period, swell only. Modified spectra derived for ships heading into swell

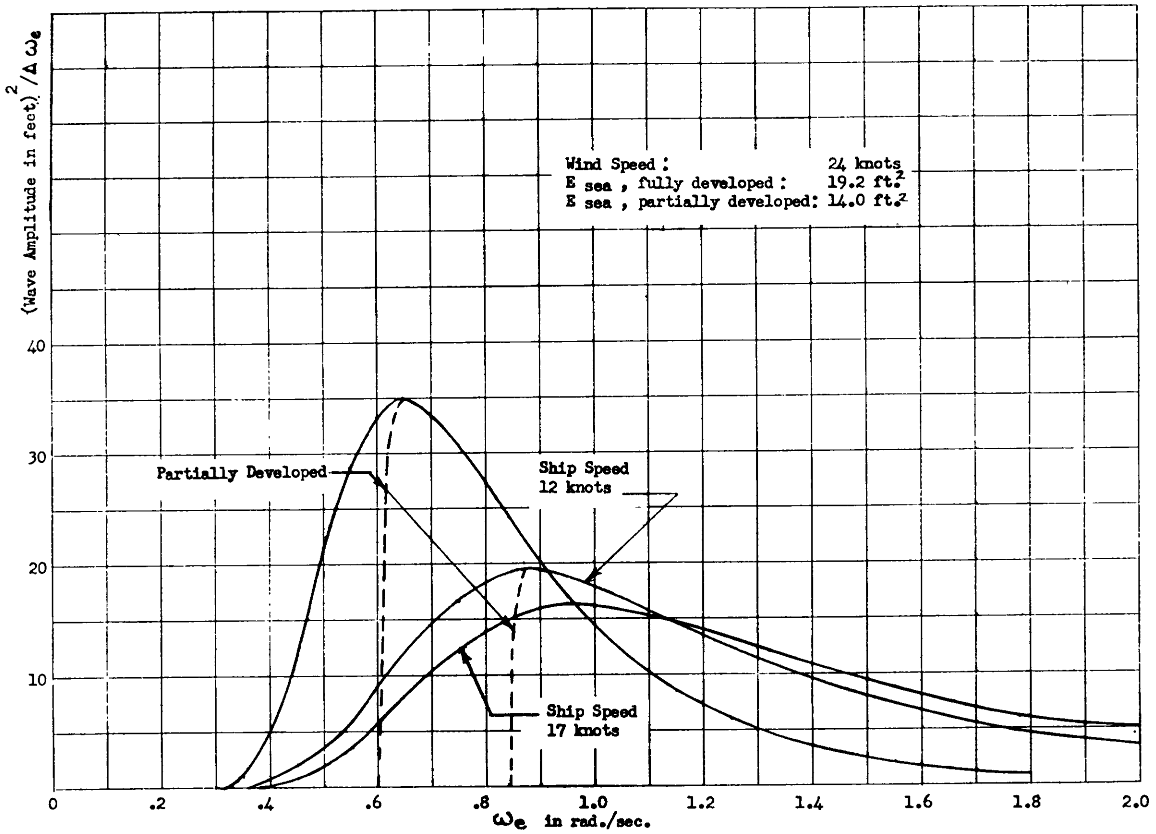


Fig. 7 (a) Neumann spectra for fully developed X sea state in operating area, sea only. Modified spectra for ships heading into the sea

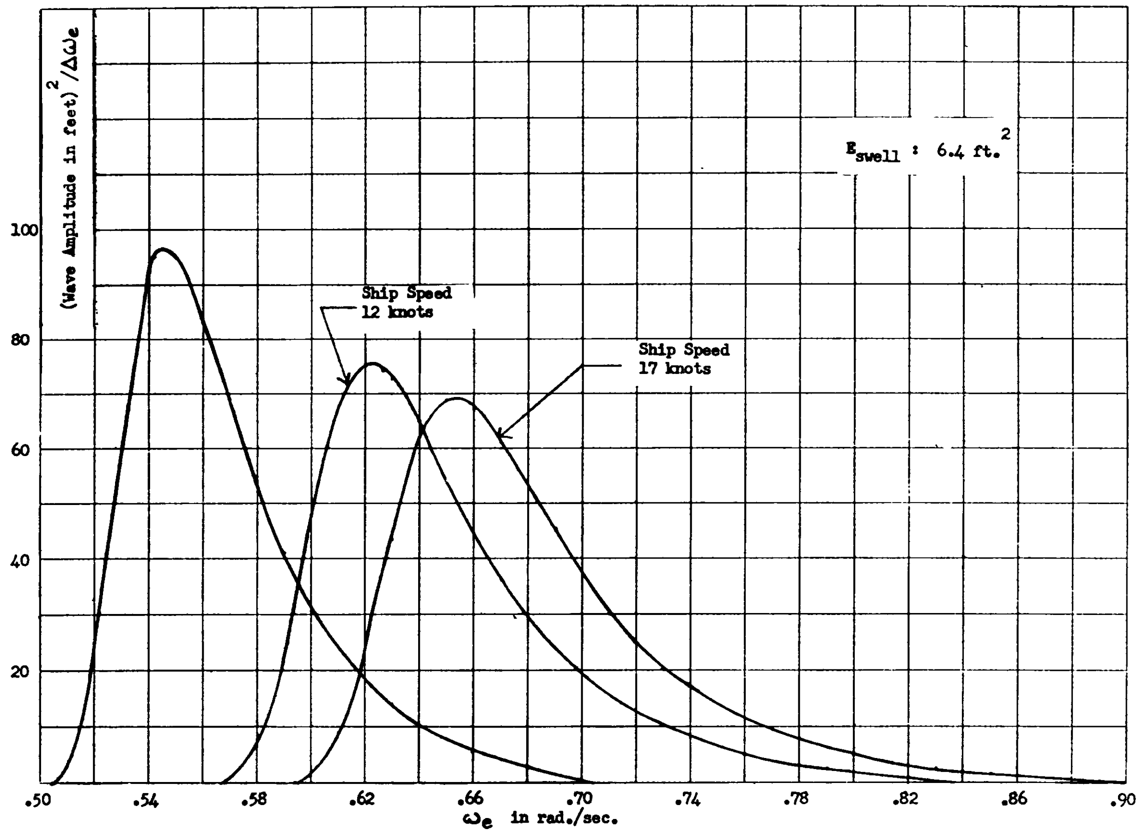


Fig. 7 (b) Neumann spectra for swell in fully developed X sea state in operating area. Modified spectra for ships heading into the sea (67.5 deg to swell)

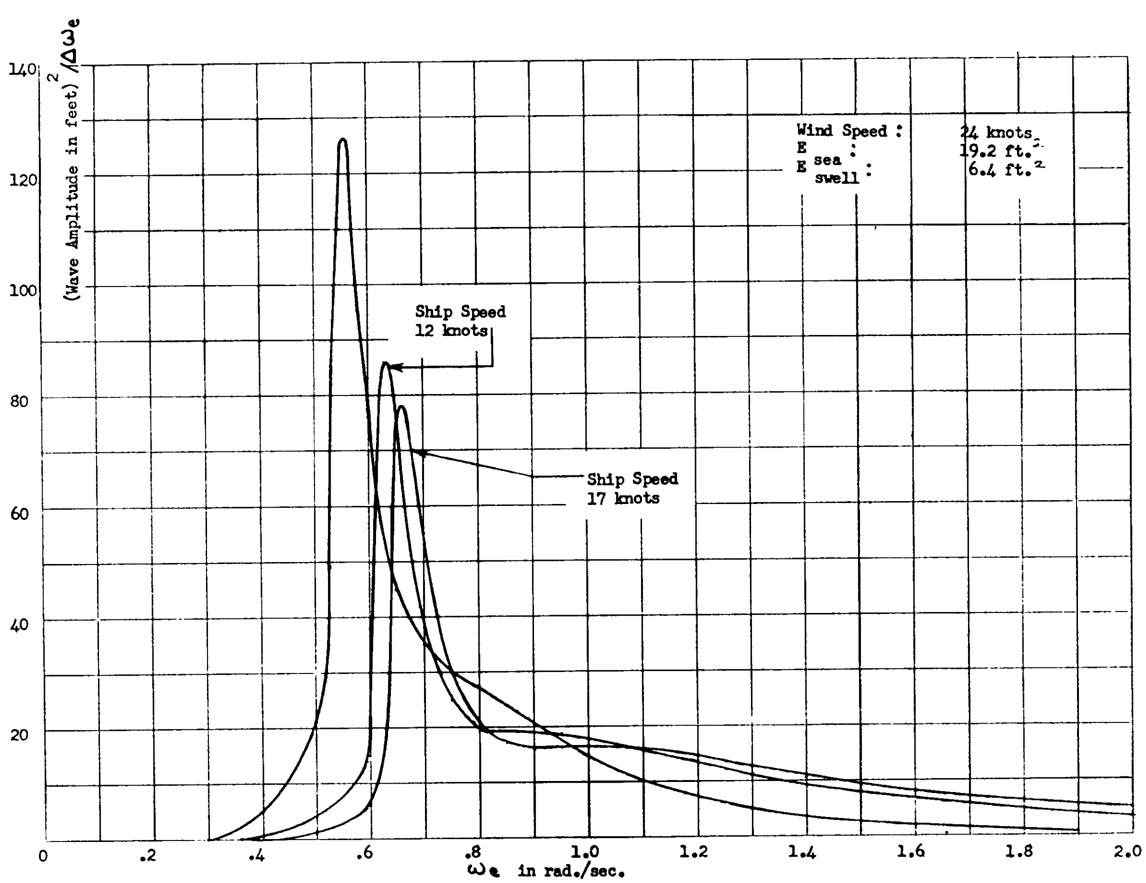


Fig. 7 (c) Neumann spectra for fully developed sea and swell in X sea state. Modified spectra for ships heading into sea.

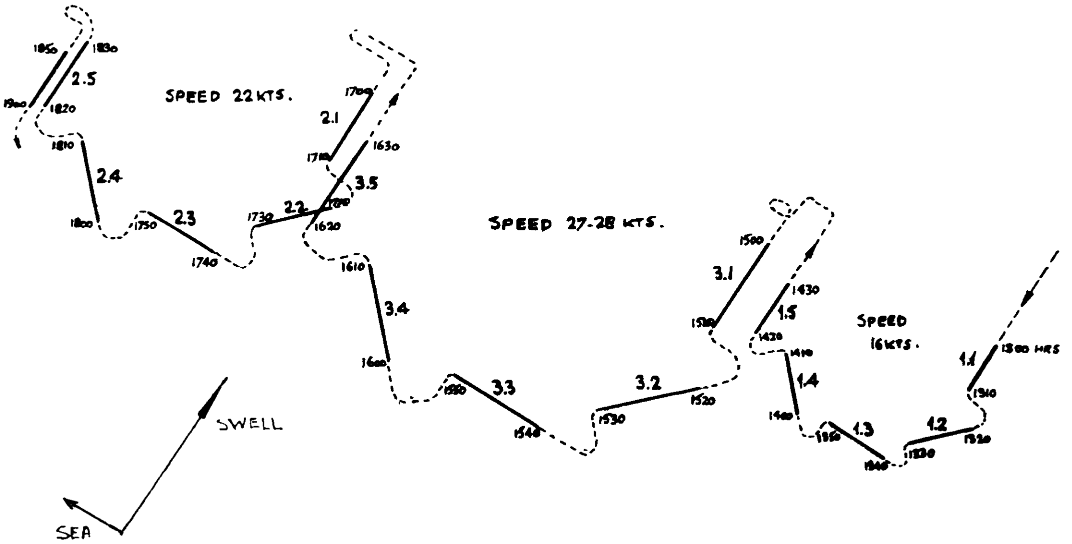


Fig. 8 (a) Trial pattern Y, moderate sea condition, April 11, 1956

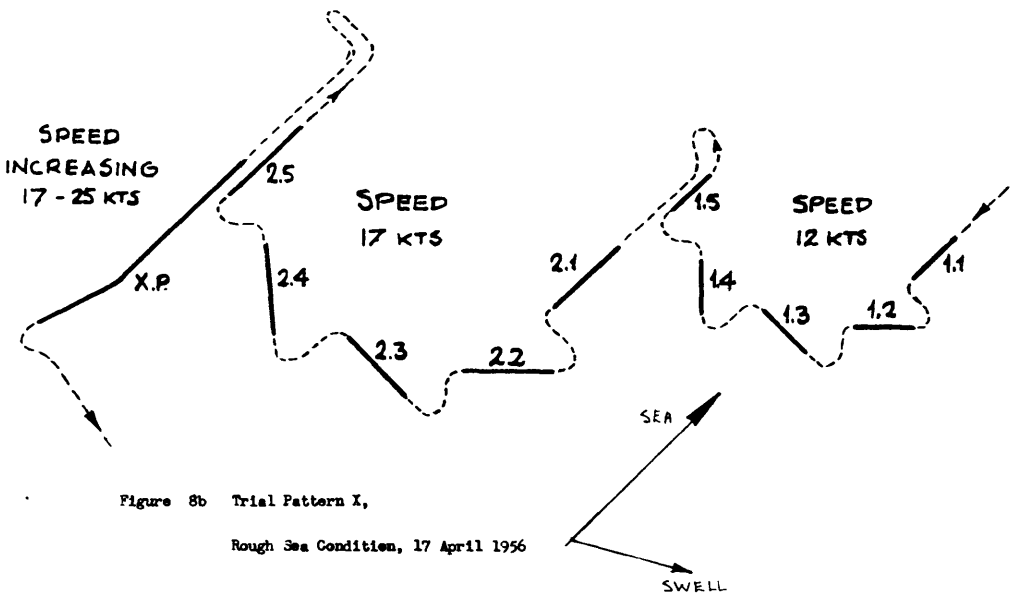


Fig. 8 (b) Trial pattern X, rough sea conditions, April 17, 1956

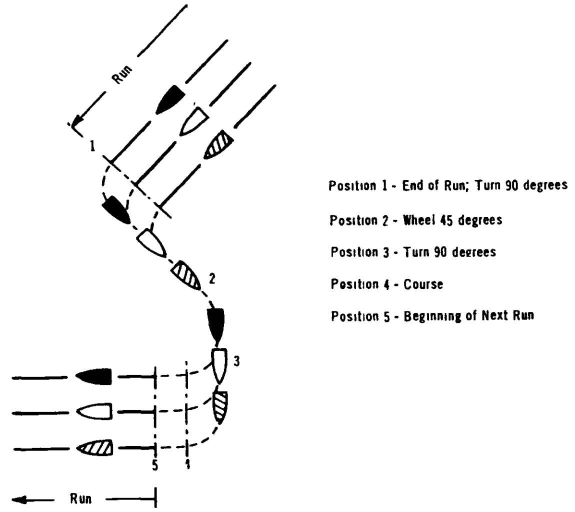


Fig. 8 (c) Maneuvering scheme

In other words, the E_i in the swell spectrum have the same ratio to each other as those in the sea spectrum, where E_i represents the energy density associated with $\Delta\omega_i$ or the ith frequency band. Then if E_k is the maximum of the E_i, E_i/E_k for the swell spectrum equals E_i/E_k for the sea spectrum where the $\Delta\omega_i$ in the swell spectrum is determined by dividing the swell-frequency range into the same number of equal parts as that used in establishing the E_i/E_k ratio from the sea spectrum. To establish the swell spectrum we know that

$$E_T = \sum_{i=1}^{N} C_i E_k \tag{1}$$

where

E_T = total energy density

E_k = maximum energy density per $\Delta\omega$

C_i = E_i/E_k, this ratio being obtained from the sea spectrum, or

$$\frac{E_T}{\sum_{i=1}^{N} C_i} = E_k = \frac{[r(\omega)]_2}{\Delta\omega}\Delta\omega \tag{2}$$

where $r(\omega)$ measures the average squared value of the wave amplitude associated with the frequency, $\Delta\omega$. Therefore

$$\frac{[r(\omega)]_k^2}{\Delta\omega} = \frac{E_T}{\Delta\omega \sum_{i=1}^{N} C_i} \tag{3}$$

or

$$\frac{[r(\omega)]_i^2}{\Delta\omega} = \frac{E_T C_i}{\Delta\omega \sum_{1}^{N} C_i} \tag{4}$$

A plot of equation (4) for various frequencies then yields the swell spectrum, its area being the E-value obtained from the hindcasts. The spectrum due to the swell for the fully developed

X-sea condition is plotted in Fig. 7(*b*) along with modifications for ship speeds of 12 and 17 knots. Fig. 7(*c*) was obtained by adding ordinates of Figs. 7(*a*) and 7(*b*) at corresponding frequencies and represents the cumulative effect of sea and swell present for the fully developed X-sea condition.

Location of the Trials

The three ships were available for the trials from April 7–21, 1956; that is, during this period the trials had priority over all other exercises, if favorable sea conditions were encountered within the assigned operating area of the sector described by a 600-mile radius from Torbay, England.

During the planning of the trials it was decided to make three series of tests in different sea conditions. During the allowed time, however, only two sufficiently different sea conditions were actually met. The first condition was met on April 11, 1956, at 250 miles WSW from Torbay (49°N-9°30′W). Trials started at 1200 hours and lasted till 1900 hours. A second condition did not occur till April 17, 1956. Here the trials area was some 400 miles west of Lands End (50°N-16°W). Trials started at 1200 hours and the last run (a special slamming run) started at 1715 hours.

Plan of Tests and Speeds

In the first sea condition the trial pattern consisted of three series of runs, each at approximately constant propeller RPM with five different headings. In the second sea condition there were only two series of five headings each. Figs. 8(*a*) and (*b*) show the trial patterns. For each test condition data were recorded for approximately 10 min. To obtain the new course after each run in the shortest possible time and with the least speed loss, a rather complicated maneuvering scheme, Fig. 8(*c*), consisting of a sequence of standard turns and wheels, was used. The analysis of the records showed (unfortunately too late) that the gyros used for recording roll and pitch angles were not free from the precession induced by these 90-deg turns.

To obtain good photographic pictures, a line abreast formation of the ships stationed as closely to each other as possible was requested. This resulted in a ship-to-ship distance of 400 yd; this distance being short enough so that the ships were in the same sea but large enough that the interference of the waves propagated by the ships was avoided.

As others involved in full-scale trials have stated before [6, 7] it is difficult to keep speed constant during a series of runs. This is mainly due to the difference in heading, but the methods for recording the true speed and speed changes during a run are also far from reliable and exact. This was found again during the trials discussed here. Especially in the heavy sea state, readings of the ship's log show a rather wide scatter during each series of runs. Before the start of the first run of a series, the number of revolutions of ship B was adapted to the desired speed. In the moderate sea-state tests, this RPM remained constant during each series. Ships A and S kept station to ship B. In the low and medium-speed runs, the log readings on ship B did not show any significant difference in speed due to change of heading. The small changes in speed of ships A and S can probably be attributed to the need for recovering the speed losses resulting from the maneuvers.

For the low-speed runs in the heavy sea state, the RPM was not the same for all runs since, after a 45-deg change in heading from head to bow seas, an increase in speed of ½ knot with the same RPM was observed. It was therefore decided to reduce RPM in an attempt to keep constant speed as much as possible. In head seas log readings show a small variation in speed with a mean of 11 knots. For beam and following sea headings RPM was reduced about 10 per cent. Log readings show a speed variation from 11.5 to 13 knots for beam seas and from 10 to 14 knots for quartering and following seas. To keep station to ship B, ship S had to change RPM. For the high-speed runs it was possible to keep RPM constant for all runs without great speed variations. The mean of the log readings changed from 17½ knots for head seas to 18½ knots for following seas.

Unfortunately data concerning correction factors for the ships' logs during the trials are not available, thus all speeds connected to a series of runs are only approximate

In sea state X, instead of a 22-knot series, a special "slamming run" was performed, when the three ships were running line abreast in head seas. Speed was gradually increased from 17 knots to 20 and 22 knots, until some serious slams occurred. Ship A could not increase speed because of slight engine trouble and left formation. The other two ships continued increasing their speed to about 25 knots and then changed heading angle so that the waves were met at approximately 10 deg from the port bow. The run was completed when one ship ran into a succession of three large waves, resulting in a combination of heavy slams and green water shipped over the forecastle deck.

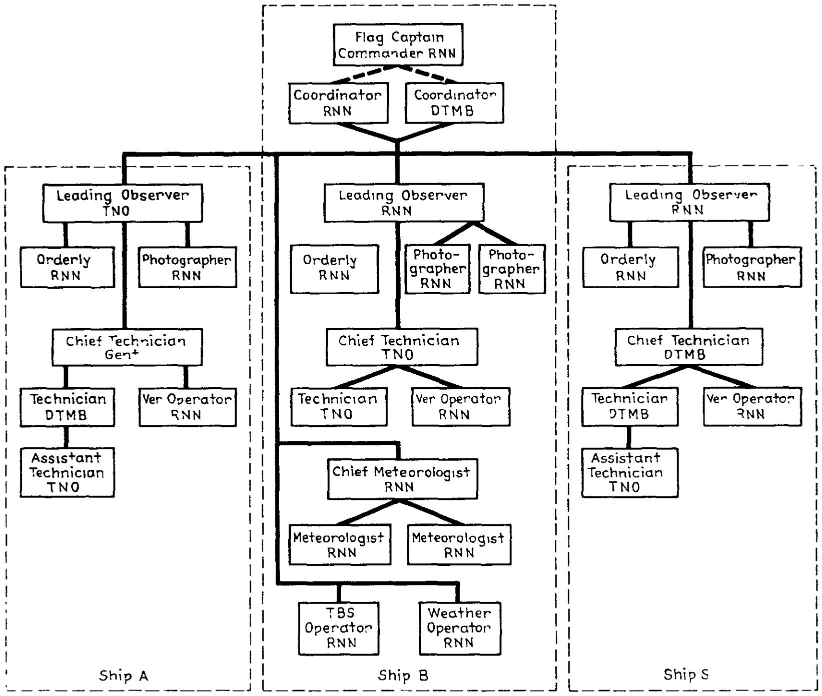


Fig. 9 Personnel organization

Personnel

The personnel participating in the trials were of varied background regarding nationality, status (civilian or military), and field of specialization. Their organization is shown in Fig. 9. The total number of persons involved directly in the trials, under supervision of the trial co-ordinators, was approximately 30. When needed, additional assistance was received from the ships' officers and crews. The commanding officers, under the captain of the flagship, were in charge of the nautical undertakings. The recording of data was supervised by the leading observers on each ship. Nautical orders were given by flags or signal lamps, trial orders by radio and telephone, and internal ship communication by telephone and the public address system. The international co-operation was on a high level, no language difficulties were encountered, and a perfect understanding was established. Owing to the high professional pride of all trial personnel, no records were lost because of seasickness.

It is not possible to mention all who were directly or indirectly involved with the preparation and the execution of the trials but it is felt that, in addition to the previously mentioned co-ordinators and originators of the trials, Messrs. Warnsinck and Szebehely, much of the success of the trials can be attributed to:

Mr. C. Drayer: Institute of Applied Physical Research, Delft, Leading Observer, ship A.
Mr. C. D. Donath: Royal Netherlands Navy, Leading Observer, ship S.
Mr. E. Vlierboom: Royal Netherlands Navy, Leading Observer, ship B.
Mr. R. J. Singleton: David Taylor Model Basin, U. S. Navy, Chief Technician, ship S.
Mr. de Vries: Institute of Applied Physical Research, Delft, Chief Technician, ship B.
Mr. S. v. d. Velde: University of Gent, Belgium, Chief Technician, ship A.
Mr. Paardekooper: Royal Netherlands Navy, Chief Meteorologist.

Collected Data

The following data were obtained:

1 Roll angle.
2 Pitch angle.
3 Surge acceleration.
4 Transverse acceleration.
5 Vertical acceleration:
  (*a*) At the bow.
  (*b*) Near the center of gravity.
  (*c*) Near the bridge.
6 Longitudinal stresses from strain gages located:
  (*a*) Amidships on the sheerstrake, port and starboard.

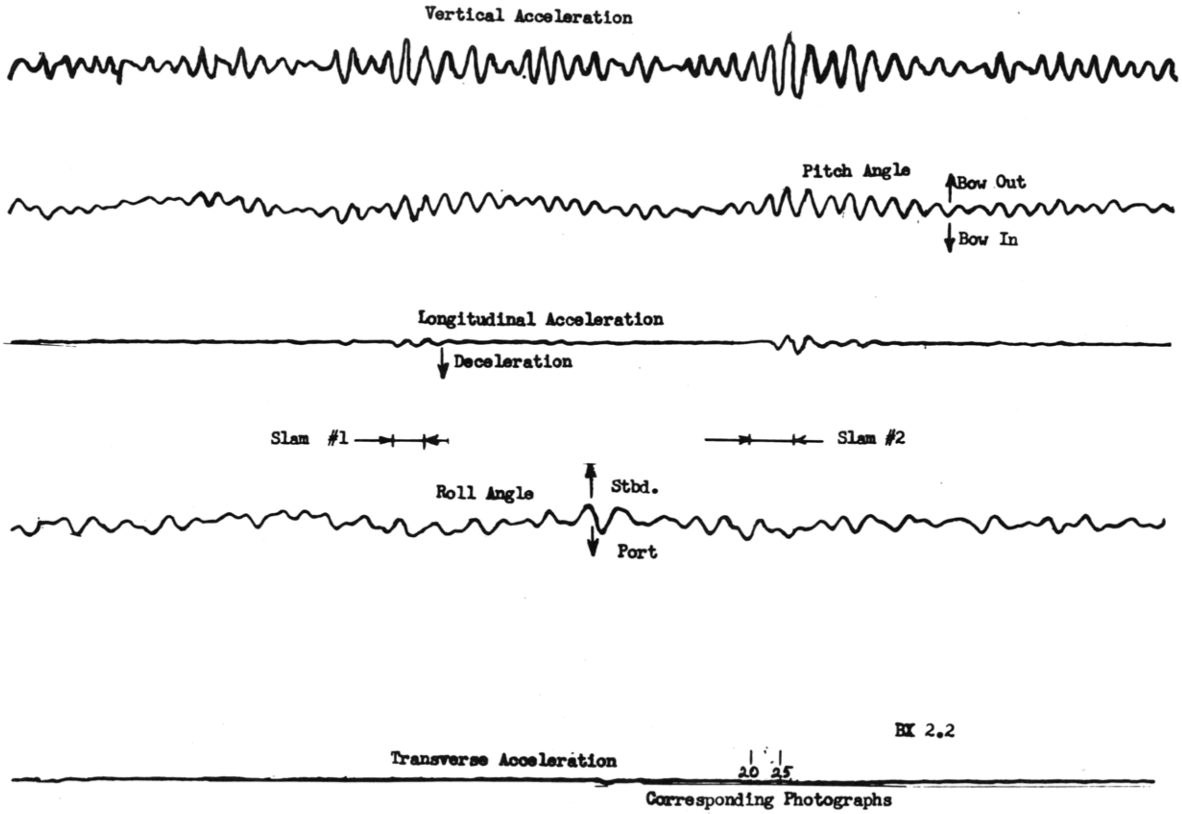


Fig. 10(a) Sample of Verimeter record

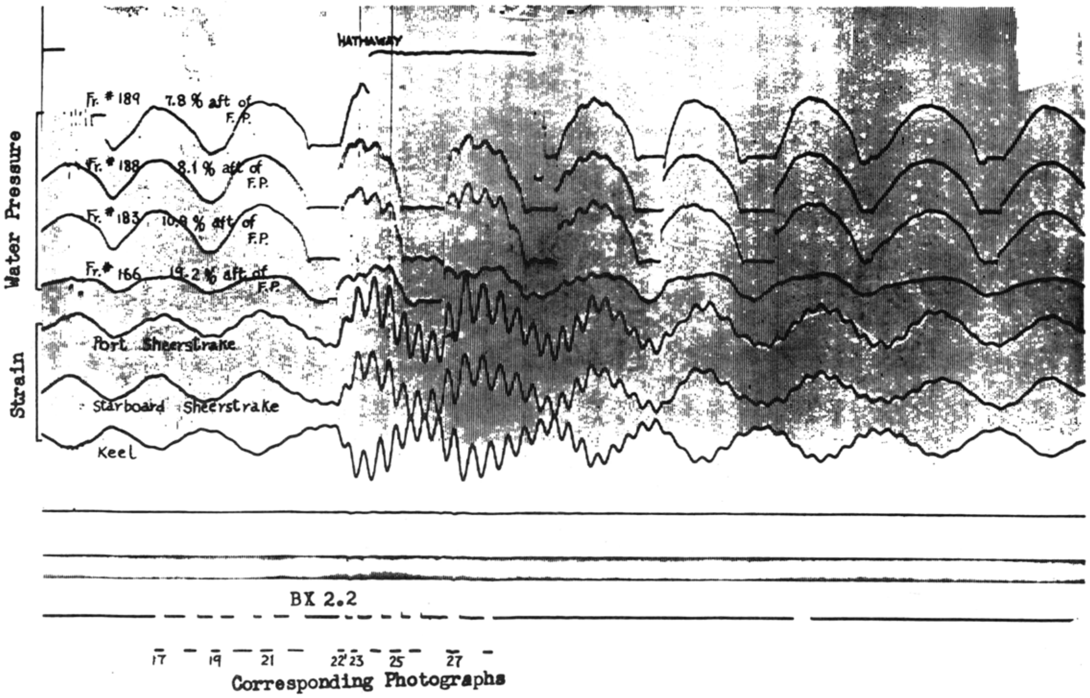


Fig. 10(b) Sample of oscillograph record

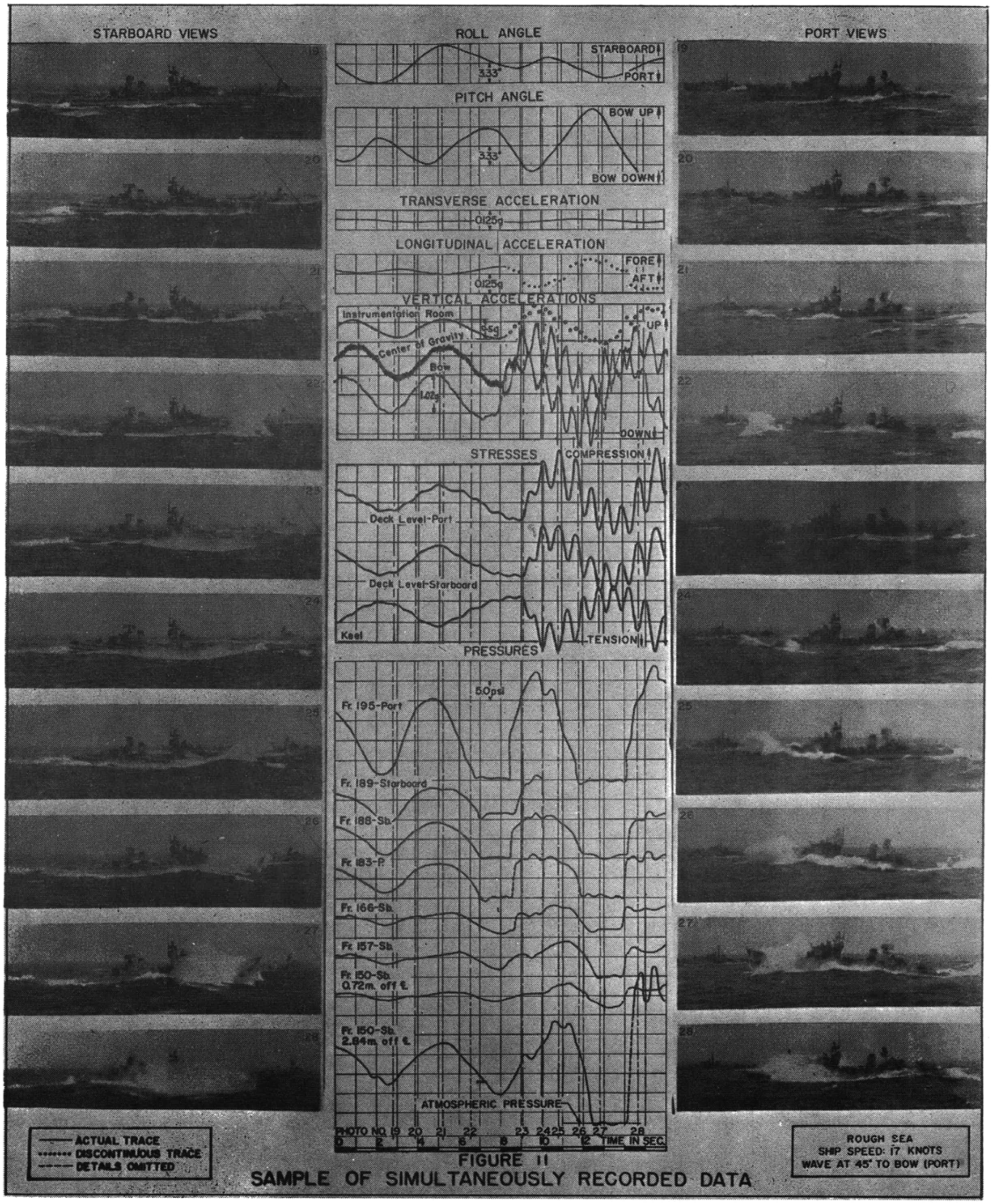


Fig. 11. **Sample of Simultaneously Recorded Data**

(*b*) Amidships on the centerline keel rider plate.

7 Water pressures on the ship bottom in the forebody on the centerline.

8 Synchronized photographs of the three ships.

Sample records of the recorded data are shown in Figs. 10(*a*) and (*b*). Fig. 11 is an illustration of the synchronization of the photographs and the recorded data.

All roll and pitch-angle records for the two sea states have been read and analyzed. Histograms for double amplitudes and periods are available. For a selected number of cases strip reading for the computation of the power spectra was performed, Part 2.

The records for surge, transverse, and vertical accelerations near the bridge were not analyzed. The accelerometers for measuring the longitudinal and transverse accelerations possessed rather high internal friction and most of the records show a significant deviation from the mean only during slamming or lurching. Since the recording instrument did not permit recording of the high-frequency phenomena which occur during slamming, the authors did not feel the time required to read these records was justified by the results to be extracted from them. For the transverse acceleration there was also the complication that it was not possible to determine which part of the acceleration was due to sway and which was due to yaw, since there were no records of the steering and rudder angle.

The accelerometer for the vertical acceleration at 24 per cent of the length aft of the forward perpendicular had less internal friction, but here difficulty arose from the inadequate synchronization with the records for the vertical accelerations at the bow and near the center of gravity.

During the "slamming run," the oscillographs were run at a high paper speed (1 fps) for short intermittent periods, so there are no continuous records of accelerations, stresses, and pressures for this run. These records are only suitable for detailed study of slamming and only a limited statistical analysis was performed on these data.

2—Analysis of the Data

General

The determination of a program for the analysis of the very large amount of data collected during these trials was a matter of some difficulty There are many quantities which might be of interest both to the design office and to the research laboratory, and it would be very fine if we could present them all. However, for reasons of economy in both time and money this was not possible, and it was necessary to select those parameters which reveal a maximum of useful information, and which could be determined with a reasonable amount of work.

In order to describe fluctuating quantities of the sort measured, there are two aspects which should be characterized. There are (*a*) the amplitude of the oscillations, and (*b*) the frequencies which they contain. For statistically stationary, Gaussian phenomena (the rigid body motions and nonvibratory stresses are assumed to be of this type), the energy-spectrum technique has been found well suited to this purpose [5]. In principle, the spectrum can be used to compute any statistical quantity derivable from the original record. However, the computation of the spectrum from a graphical record is quite laborious, as it requires a very large number of readings at small spacing. In addition, there was an additional complication during these trials arising from the precessions of the gyros in the Verimeters after the vessels changed course. This resulted in a large amplitude variation of the mean line for all angular records. The period of this oscillation was several times greater than any believed to exist in the spectrum of the variable. The net effect of the precession was a spurious peak in the computed spectrum at very low frequencies. This peak has arbitrarily been faired out in all data presented here. Spectra were computed for 46 cases of pitch and roll, and for two cases of heave acceleration.

For all of its merits, the spectrum has one serious disadvantage. It is presented in the form of a curve for each quantity for each run, and while these curves are adapted to a detailed analysis of the phenomena, they are not convenient for exhibiting gross variations between runs, such as changes due to speed or heading. If attention is restricted to amplitudes only, there is a single parameter derivable from the spectrum which fulfills this need very well. This is the area under the spectrum, which we designate by E. Using the Pierson-St. Denis definition of the spectrum, E is identically equal to twice the variance of the variable; that is, if $r(t)$ is the measured quantity,

$$E = 2\overline{[r(t) - \bar{r}]^2} \tag{5}$$

This parameter has some important properties which greatly enhance its value for our purpose. As $r(t)$ is normally distributed, $E(t)$ completely determines its distribution. Specifically, for $a > 0$

$$P[|r(t)| > a] = \frac{2}{(\pi E)^{1/2}} \int_a^\infty e^{-x^2/E}\, dx \tag{6}$$

Further, evidence has been presented that the double-amplitude oscillations (as measured from peak to trough) seem to obey a Rayleigh distribution. For the asymptotic case of a very narrow spectrum, this is an immediate consequence of a result due to Rice [8], and Jasper [9] has presented empirical evidence that it is

Table 6 *E* for Pitch and Roll Angle and Heave Acceleration

Run No.	Pitch, deg^2	Roll, deg^2	Heave acceleration, g^2
Ship A			
AY 1.1	1.03	3.46	
AY 1.2	1.28	2.69	0.0027
AY 1.3	0.88	12.89	0.0042
AY 1.4	0.57	15.71	0.0012
AY 1.5	0.31	3.06	0.0007
AY 2.1	0.68	2.94	0.0092
AY 2.2	1.40	4.54	0.0104
AY 2.3	0.63	18.62	0.0080
AY 2.4	0.57	11.43	0.0026
AY 2.5	0.55	9.17	0.0020
AY 3.1	1.30	3.37	0.0038
AY 3.2	1.39	4.40	0.0075
AY 3.3	0.85	12.49	0.0029
AY 3.4	0.39	14.16	
AY 3.5	0.37	8.51	0.0015
AX 1.1	1.60	5.07	0.0040
AX 1.2	3.05	4.40	0.0057
AX 1.3	2.28	17.17	0.0092
AX 1.4	1.48	46.43	0.0045
AX 1.5	1.17	22.52	0.0012
AX 2.1	4.05	9.86	0.0118
AX 2.2	4.94	5.18	0.0117
AX 2.3	3.90	25.45	0.0105
AX 2.4	1.80	40.97	0.0043
AX 2.5	1.59	21.70	0.0017
Ship B			
BY 1.1	0.85	1.57	0.0016
BY 1.2	1.30	1.31	0.0027
BY 1.3	0.77	6.29	0.0026
BY 1.4	0.47	12.77	0.0006
BY 1.5	0.30	2.31	0.0004
BY 2.1	0.96	1.25	0.0030
BY 2.2	1.09	2.27	0.0055
BY 2.3	0.68	10.57	0.0028
BY 2.4	0.56	10.88	0.0010
BY 2.5	0.41	4.81	0.0010
BY 3.1	1.03	2.87	0.0059
BY 3.2	1.38	3.45	0.0092
BY 3.3	0.91	7.45	0.0055
BY 3.4	0.73	11.68	0.0016
BY 3.5	0.28	5.22	0.0012
BX 1.1	1.77	5.99	0.0028
BX 1.2	2.69	3.39	0.0037
BX 1.3	1.59	14.08	0.0063
BX 1.4	0.81	31.66	0.0055
BX 1.5	0.95	35.72	0.0016
BX 2.1	3.98	8.42	0.0114
BX 2.2	3.87	3.69	0.0090
BX 2.3	3.88	14.60	0.0092
BX 2.4	1.17	48.51	0.0038
BX 2.5	0.99	15.93	0.0010
Ship S			
SY 1.1	1.15	1.15	0.0017
SY 1.2	0.90	1.16	0.0033
SY 1.3	0.63	3.02	0.0022
SY 1.4	0.53	7.17	0.0008
SY 1.5	0.58	1.75	0.0004
SY 2.1	1.45	1.43	0.0039
SY 2.2	1.40	2.52	0.0058
SY 2.3	0.84	9.64	0.0031
SY 2.4	0.58	7.53	0.0010
SY 2.5	0.42	3.91	0.0009
SY 3.1	1.17	1.18	0.0070
SY 3.2	1.68	1.94	0.0067
SY 3.3	1.21	5.78	0.0071
SY 3.4	0.65	9.99	0.0019
SY 3.5	0.37	7.33	0.0011
SX 1.1	2.49	4.76	0.0041
SX 1.2	2.55	1.58	0.0046
SX 1.3	2.53	9.59	0.0054
SX 1.4	1.92	29.28	0.0052
SX 1.5	1.47	14.25	0.0011
SX 2.1	5.52	5.92	0.0146
SX 2.2	5.99	2.70	0.0134
SX 2.3	4.19	12.86	0.0122
SX 2.4	1.54	46.77	0.0063
SX 2.5	1.69	16.21	0.0010

more generally true. Unfortunately, many of the records obtained during these trials deviate significantly from the Rayleigh law. Still, it will be shown that in most cases the Rayleigh law can be used to estimate a number of parameters which are of importance, even when the entire double-amplitude distribution cannot be adequately approximated by such a function.

The Rayleigh distribution is of the form

$$P(r, E_J) = P[(r_{\max} - r_{\min}) < R] = [1 - \exp(-R^2/E_J)] \quad (7)$$

where $r_{\min}$ is the minimum value immediately succeeding $r_{\max}$. Evidence is presented in Appendix 1 that

$$E_J = 4E \quad (8)$$

Thus, the parameter E can be used to estimate certain important statistics associated with the double amplitudes. For instance:

Standard deviation of $r(t) = (E/2)^{1/2}$

Most frequent double amplitude of oscillation $= 1.414\sqrt{E}$

Average double-amplitude oscillation $= 1.77\sqrt{E}$

Characteristic double oscillation (average of $\frac{1}{3}$ highest) $= 2.83\sqrt{E}$

Average of $\frac{1}{10}$ highest double oscillation $= 3.60\sqrt{E}$

Most probable maximum double oscillation for N oscillations $= 2(E \ln N)^{1/2}$

There was some difficulty in determining E, because the spectrum was not computed in all cases, and some other technique was required which would not be affected by the precession. The method chosen is virtually unaffected by the oscillation of the mean line. It is described and justified in Appendix 1.

The situation with respect to the frequencies and periods is not nearly as satisfactory. There is no single parameter which is as meaningful as

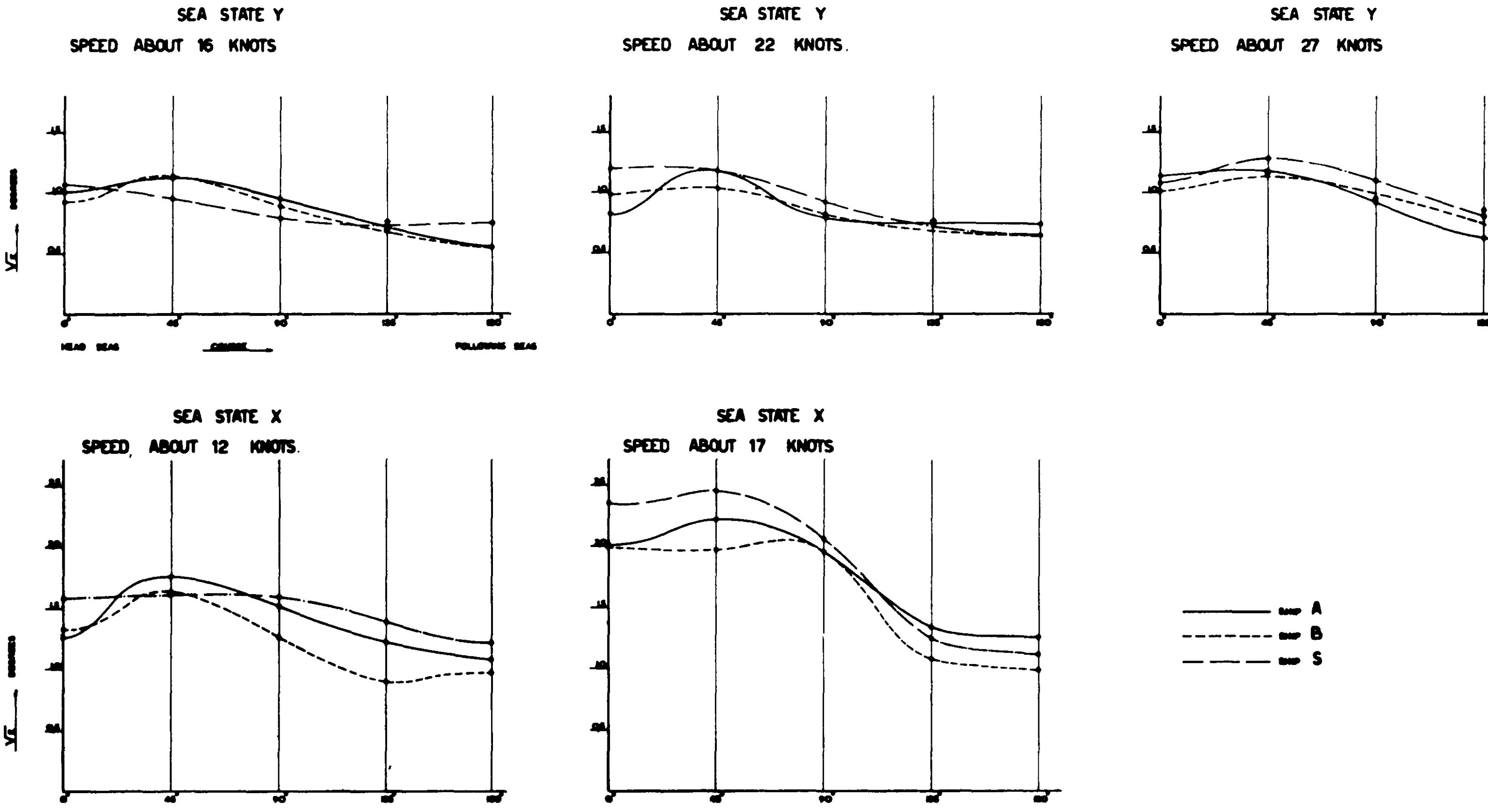


Fig. 12 (*a*) Variation in $\sqrt{E}$ for pitch with respect to heading

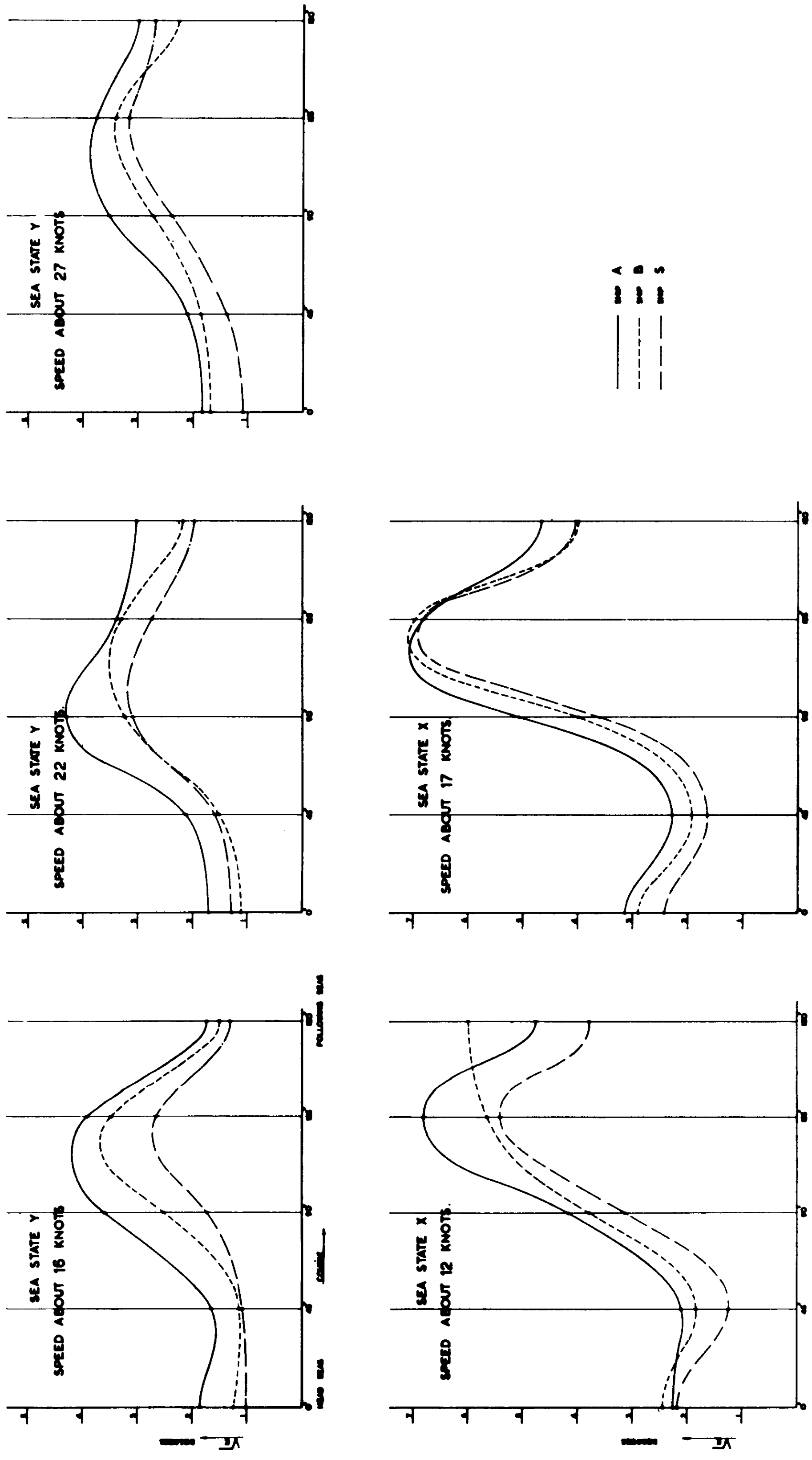


Fig. 12(*b*) Variation in $\sqrt{E}$ for roll with respect to heading

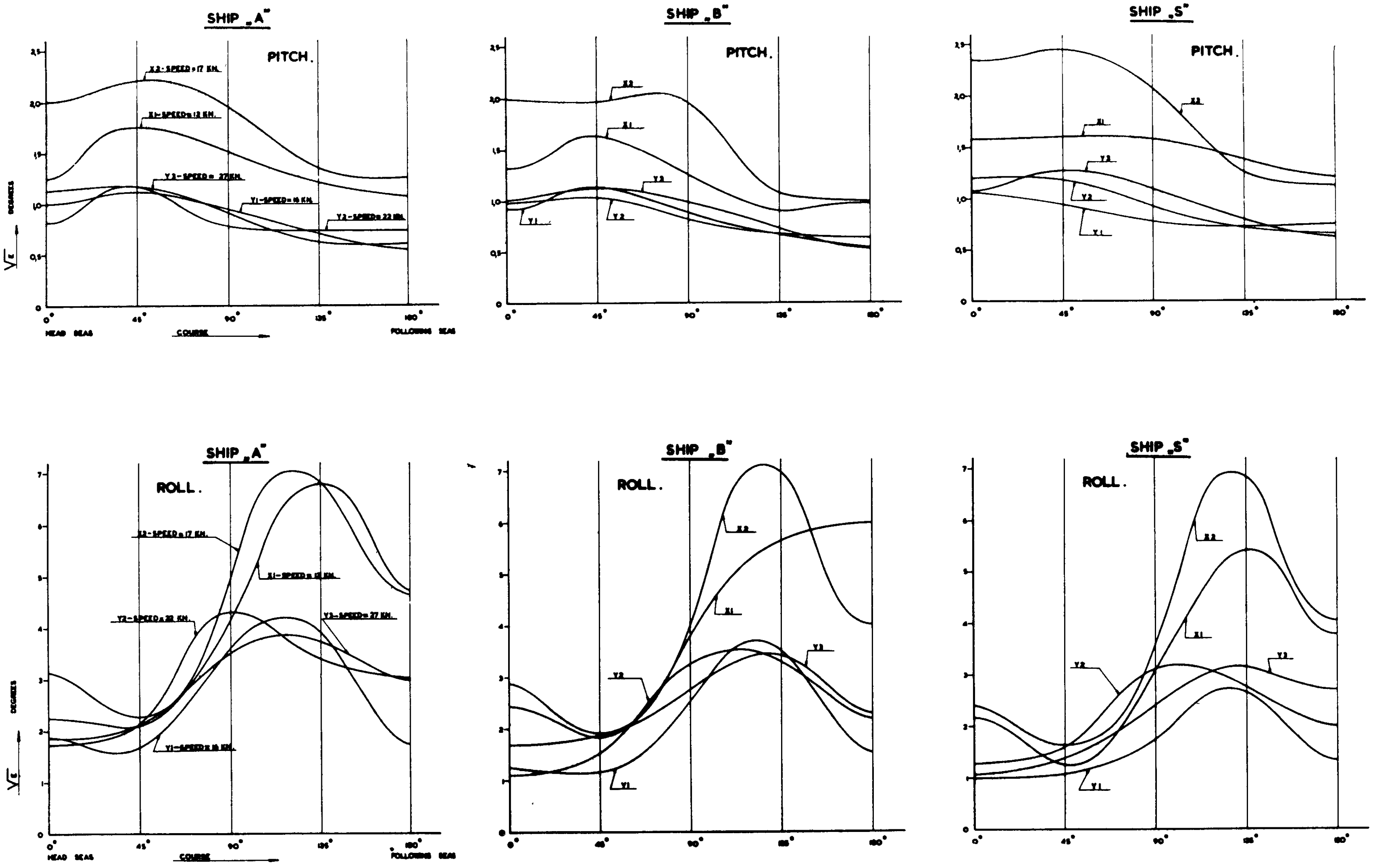


Fig. 12(*c*) Variation in $\sqrt{E}$ for pitch and roll with respect to heading

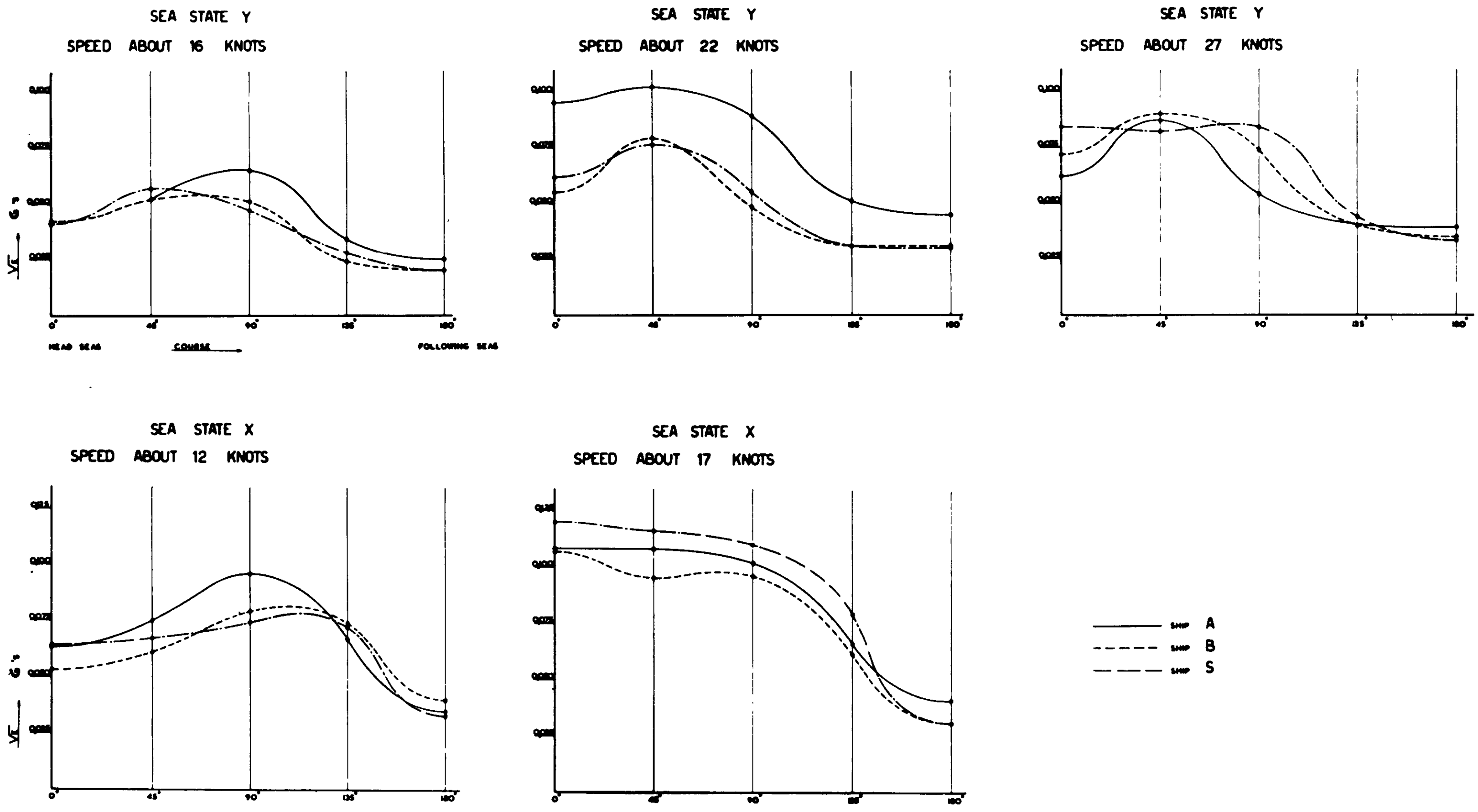


Fig. 12(*d*) Variation in $\sqrt{E}$ for heave acceleration with respect to heading

E. Certainly, the frequency corresponding to every well-defined peak of the spectrum (and there are frequently more than one) is of particular interest. Also, the breadth of spectrum, and the frequencies at which the response becomes negligible are important. However, short of computing the spectrum in every case, there is no way this information can be obtained.

The cumulative frequency-distribution diagrams and histograms were prepared for periods of pitch, roll, and heave accelerations for all cases. These periods were obtained by taking the time between successive peaks. From these data it has seemed most reasonable to present only the average period and the periods defining the upper limit of the lowest sextile and the lower limit of the greatest sextile.

The mean period is precisely the quantity which would be obtained by an observer with a stop watch, counting cycles as the variable reaches successive peaks. The relation between this mean and any period evident in the spectrum is not very close. Even for a sharply tuned oscillation such as roll, high frequencies in the spectrum tend to shift the mean period downward from the period of maximum energy. This will be shown in the discussion of the spectra.

The limits of the sextiles give an idea of the spread of the periods. A wide spread is some indication of a broad spectrum, but a small spread may mean only a high-frequency peak which obscures all low frequencies.

It is fully recognized that these quantities have serious deficiencies. It might be better to exhibit all histograms for the periods. However, these histograms are in themselves misleading, and it is not felt that they are of sufficient value to warrant the space they would require, since there are 225 of them.

The phenomena related to slamming must be presented in an entirely different way, since they are nonstationary. The statistics of occurrence of slamming are reported elsewhere [3], and will be repeated here for completeness. Some data relating to particular slams are also given.

Rigid Body Motions

Pitch, Roll, and Heave Acceleration

Heaving accelerations, and pitching and rolling motions of the three ships were evaluated by reading the peak-to-peak oscillations and computing the E-value as described in Appendix 1. These are tabulated in Table 6 and $\sqrt{E}$ is plotted with respect to heading in Figs. 12(*a*), 12(*b*), 12(*c*) and 12(*d*). The cumulative frequency distributions of the double amplitudes were also determined and these are shown in Figs. 13(*a*), 13(*b*) and 13(*c*). The highest 10 per cent of the recorded motions are plotted in the polar diagram in Figs. 14(*a*), 14(*b*) and 14(*c*). Spectra indicating the band of frequencies present were determined for selected cases; these results are discussed in the next section in relation to the period distribution.

Figs. 12(*a*), 12(*b*), 12(*c*) and 12(*d*) show the variation in $\sqrt{E}$ for heave acceleration, pitch, and roll with the significant variables of sea condition, ship, speed, and course. Since the parameter $\sqrt{E}$ is statistically related to the various characteristic properties of the motions, the latter are immediately derivable, once $\sqrt{E}$ is known (see preceding section). Some interesting observations can be made concerning the effect of the four variables on the resulting motions:

1 *Effect of Course.* Maximum pitch was generally obtained in bow seas, the pitching in head and beam seas being of the same order of magnitude and slightly less than that in bow seas. Minimum pitching was experienced in following seas.

Heaving accelerations generally followed the same trend, with maxima occurring in head, bow or beam seas and minima in following seas.

Maximum roll occurred in beam and quartering seas with minimum in head seas.

2 *Effect of Speed.* The effect of speed on roll was for the most part negligible in both sea states, as was its effect on pitch in the mild sea state. However, some increase in pitching was observed in the rough sea condition with increase in speed. Heaving was increased somewhat with increase in speed in both sea states.

3 *Effect of Sea.* In going from moderate to rough seas, rolling was little affected in head to beam seas while in quartering seas the motion was increased by as much as a factor of 2. The opposite trend was observed for pitch, it being little affected in following seas but increased by a factor of 2 in head to beam seas. For comparable speeds heaving was more pronounced in the rough sea, it too being roughly twice that in the mild sea.

4 *Effect of Ship.* No marked differences existed between the three ships in heave. (The accelerations in ship A in the mild sea at a speed of 22 knots appear to be in error, probably due to the calibration factor. As a consequence, these data are undergoing further study.) However, ship S appears to show a slight tendency to pitch somewhat more than ships A and B. In roll, ship A was the worst, ship S the best with ship B falling between the two. While ship A has the smallest roll period, it is not significantly different from that of ship S which proved to be the best in

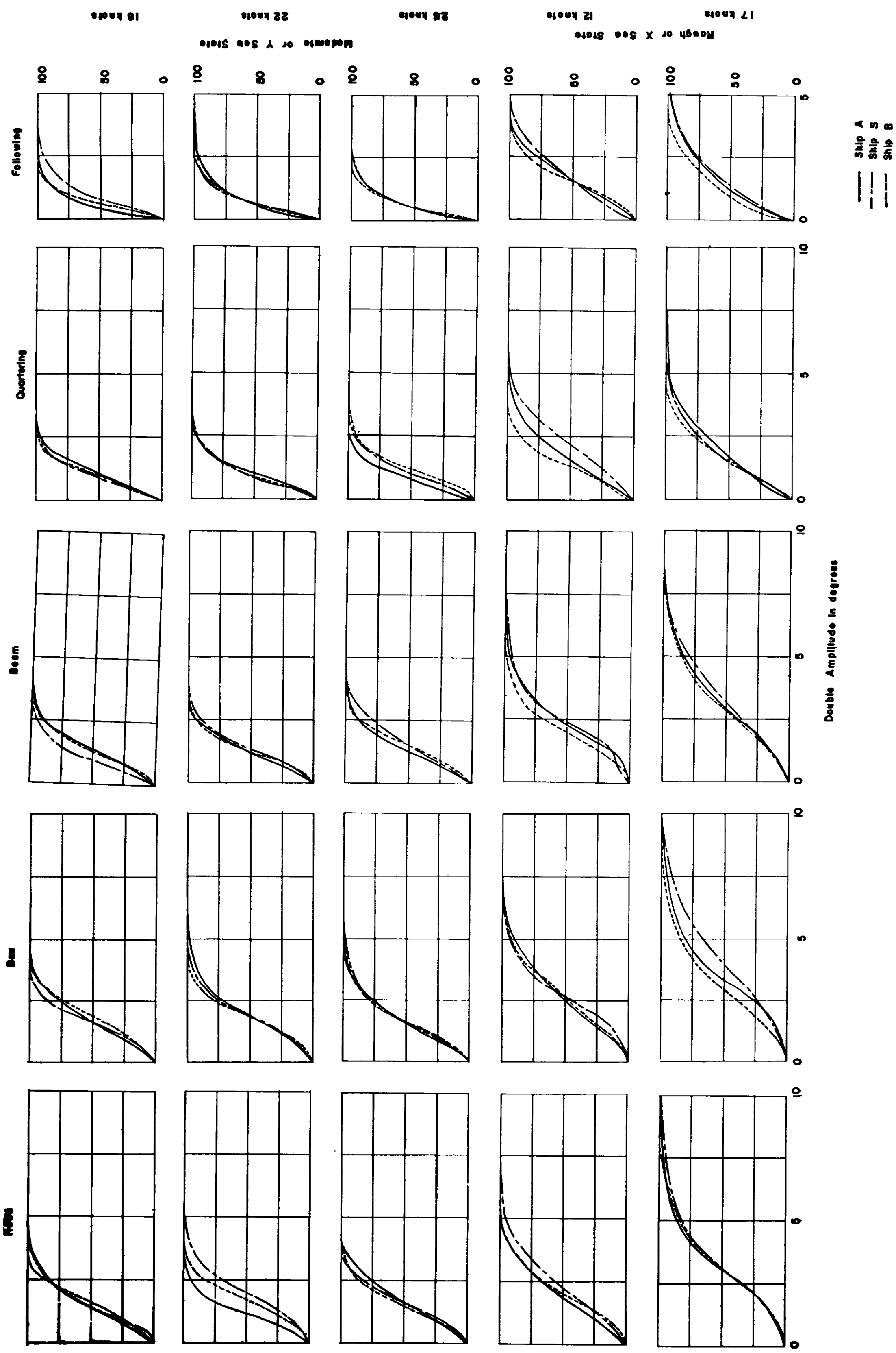


Fig. 13(a) Cumulative distribution function for pitch amplitude

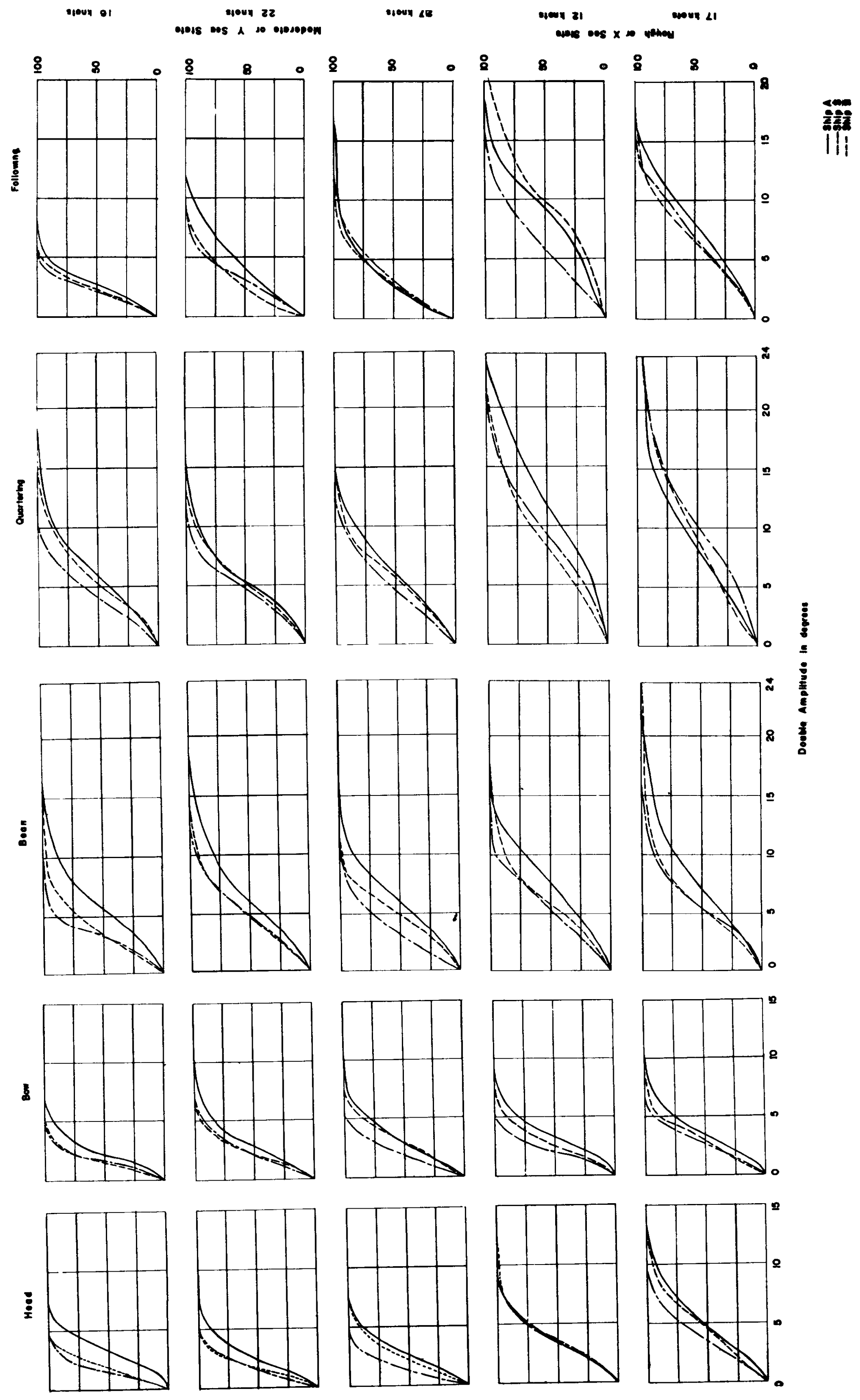


Fig. 13 (*b*) Cumulative distribution function for roll amplitude

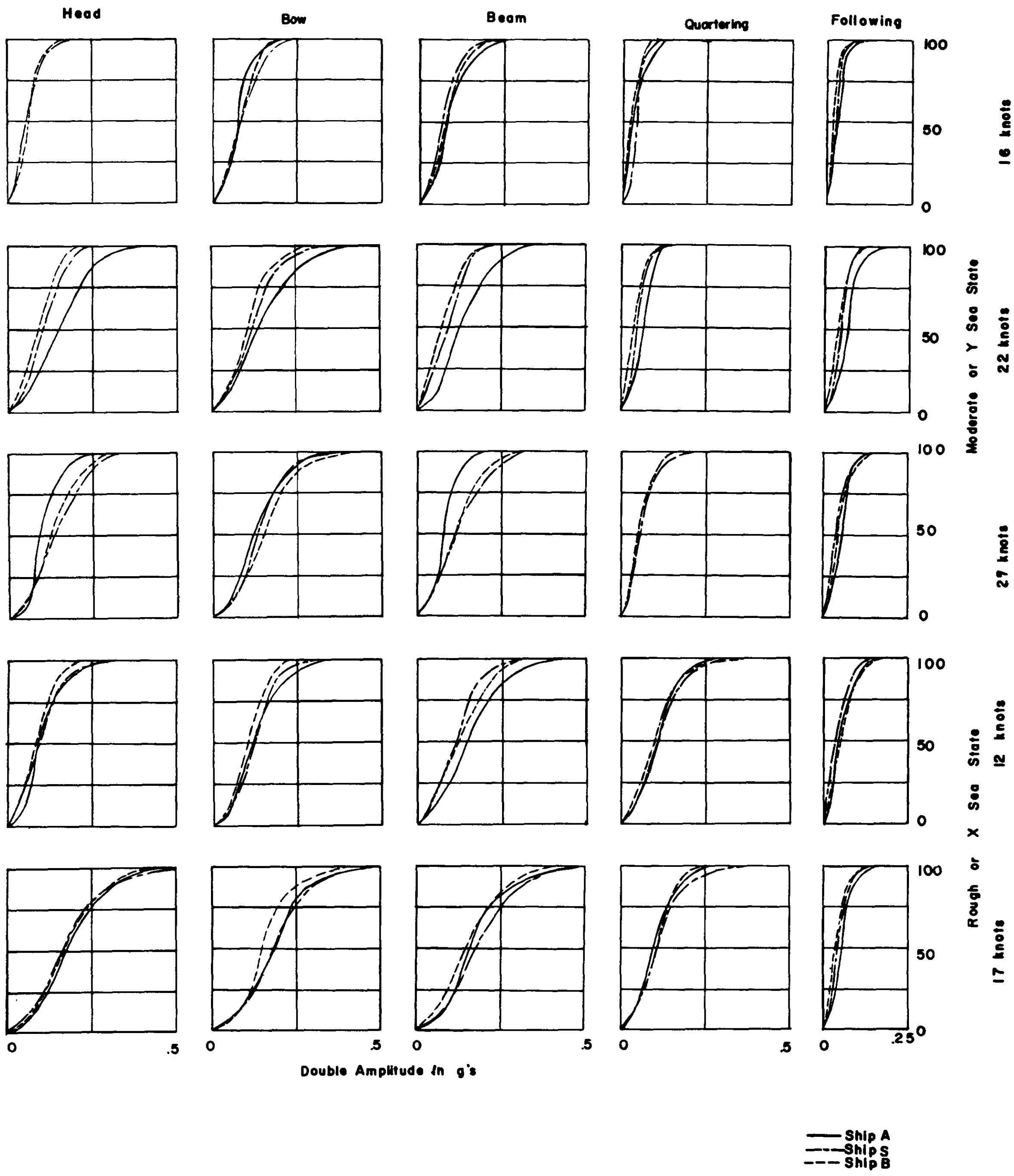


Fig. 13(*c*) Cumulative distribution function for heave acceleration amplitude

rolling. This improvement in rolling characteristics of ship S over ship A can probably be attributed to ship S having 60 per cent greater bilge-keel area than ship A.

Another method of analyzing the data is shown in the cumulative distribution functions plotted in Figs. 13(*a*), 13(*b*) and 13(*c*). These diagrams apply to peak-to-peak variations of the motions and show the cumulative probability of occurrence associated with a particular event. Immediately derivable from the plots is an estimate of the probability of the response exceeding a given

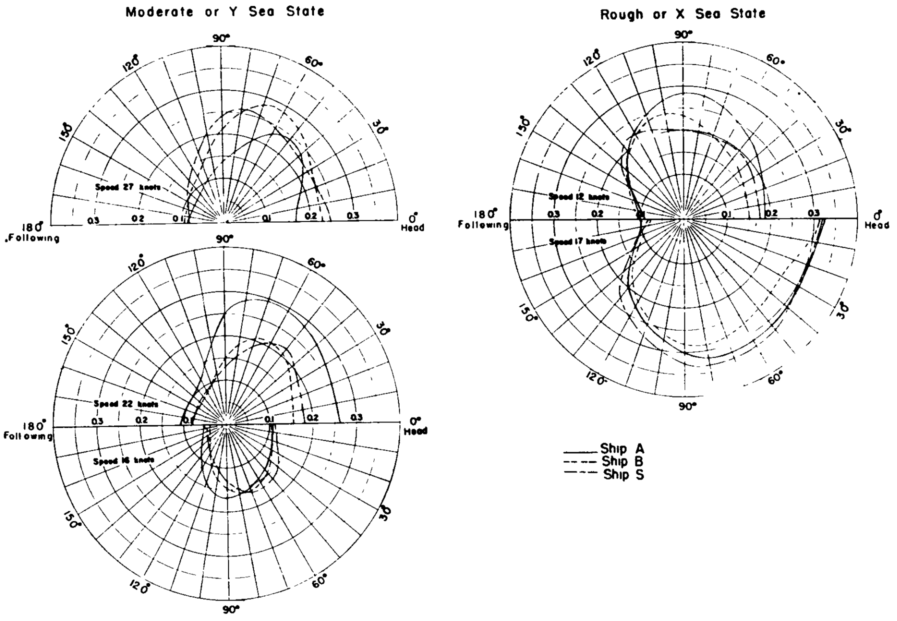


Fig. 14(*a*) Highest 10 per cent of heave acceleration double amplitudes in g's

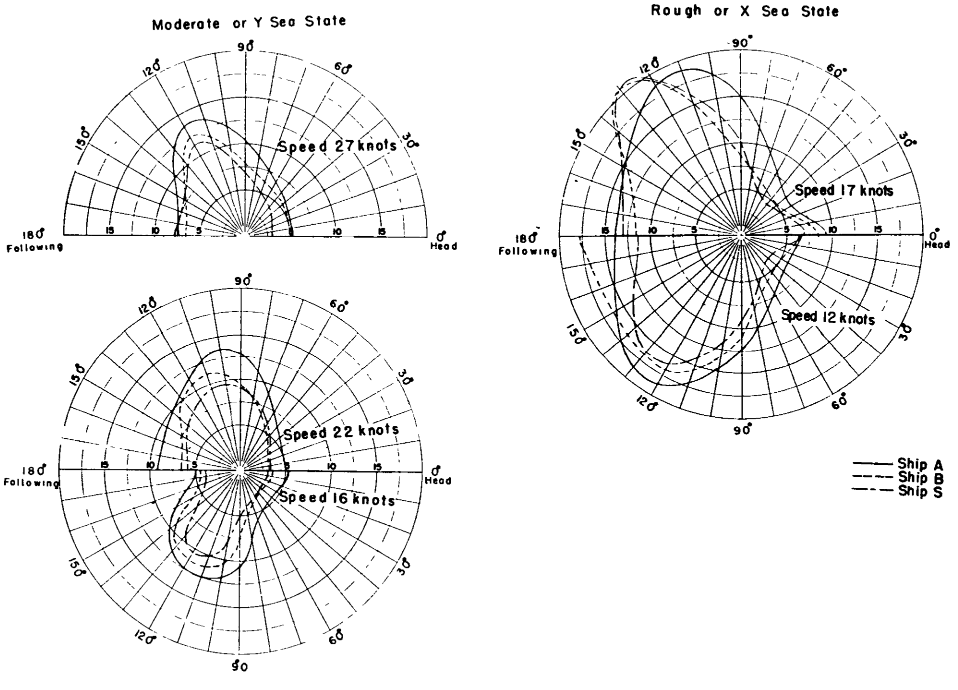


Fig. 14(*b*) Highest 10 per cent of roll double amplitudes in degrees

value. For example, the cumulative distribution-function diagram for ship B in X (rough) sea state at speed of 17 knots and 45 deg heading to the waves shows that the probability of the pitch double amplitude exceeding 5 deg is 16 per cent Figs. 14(*a*), 14(*b*) and 14(*c*) were prepared from the cumulative distribution-function diagrams and show the lower limit of the highest 10 per cent

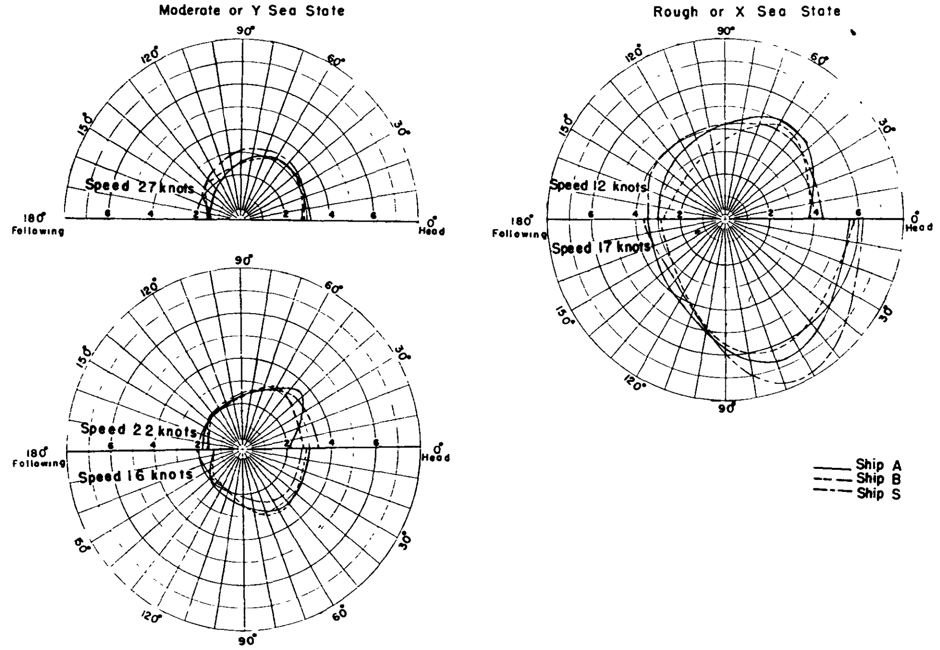


Fig. 14(*c*) **Highest 10 per cent of pitch double amplitudes in degrees**

of the motion double amplitudes. This should not be confused with the average of the highest 10 per cent obtained from the relation $h_{1/10} = 3.60\sqrt{E}$ and shown in Table 10.

The cross correlation between pitch and roll for ship S in sea state X, beam seas to the port, at a speed of 17 knots was computed. The results were expressed by the correlation coefficient

$$\rho = \frac{\frac{1}{N}\Sigma x_i y_i}{(\sigma_x^2 \sigma_y^2)^{1/2}}$$

where

$$\frac{1}{N}\Sigma x_i y_i$$

is the covariance of the two variables, and σ_x^2 and σ_y^2 are the variances or mean square values of the respective variables. The correlation coefficient is a measure of the functional dependence between the two variables.

Values associated with the correlation coefficient are $-1 \leq \rho \leq +1$.

The degree of correlation reaches its maximum then for $\rho = \pm 1$; large values mean the linear dependence is high. A positive correlation signifies that positive (negative) values of one variable usually occur with positive (negative) values of the other. Conversely, a negative correlation indicates that positive values of one are associated with negative values of the other. If $\rho = 0$ then no correlation exists between the two variables. If their joint distribution is normal, the probability density function can be expressed as the product of two one-dimensional normal density functions. The variables are then said to be independent.

In order to obtain the cross correlation between pitch and roll, the deviations from the means of each of the variables were taken simultaneously at 150 points, equally spaced over the entire sample. As a check on the accuracy of the variances obtained from this limited number of readings, a comparison was made with the variances obtained by spectral analysis. The difference in variance between the two methods was 3 per cent for roll and 2 per cent for pitch. The correlation coefficient or normalized covariance was found to be −0.314. In order to use this properly as a measure of the degree of dependence between the two variables in question, the histograms of the observed values of the two variables were compared with the normal density function, Figs. 15(*a*) and 15(*b*). Shown also in the figures are the experimental and analytical cumulative distribution functions. These figures indicate that the observations (variations about the mean) are random and that the associated probability density functions are normally distributed for both pitch and roll.

The indication of a degree of interdependence

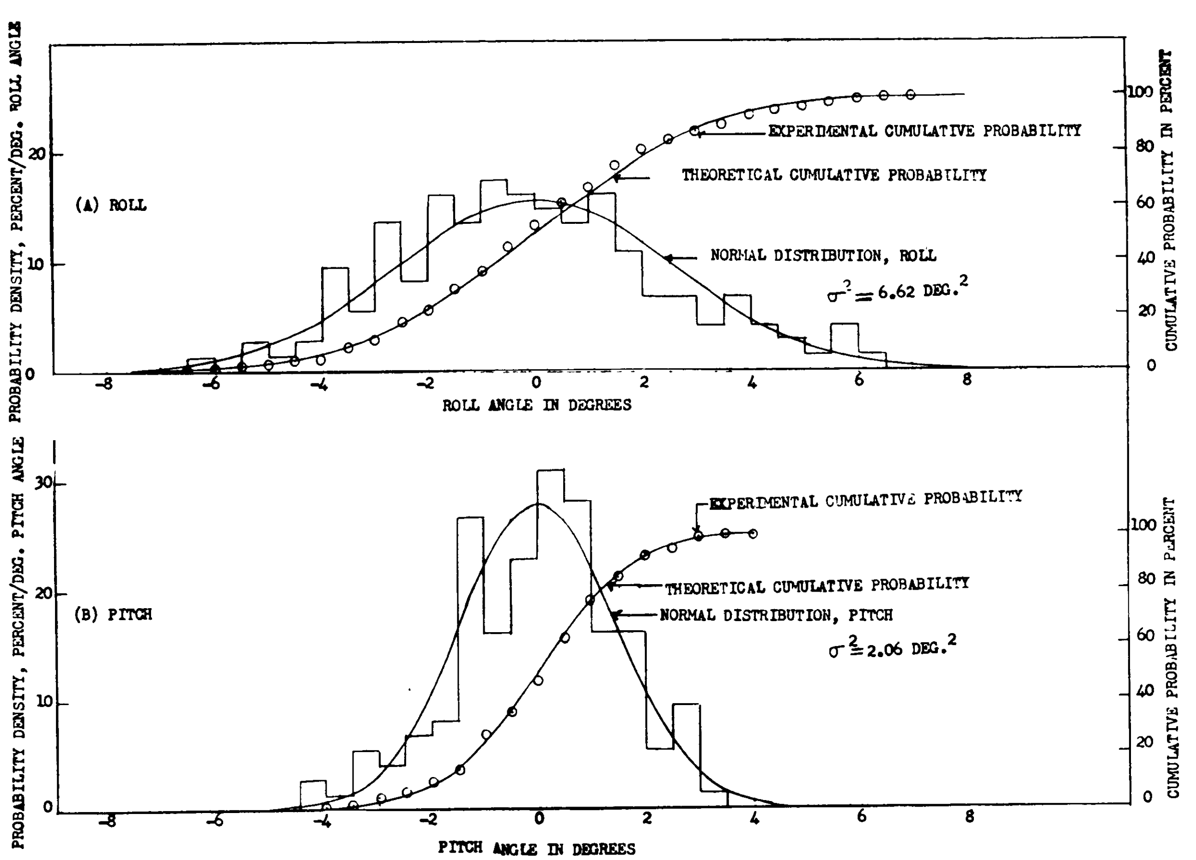


Fig. 15 Normal distribution for pitch and roll angle, ship S at a 17-knot speed, sea state X (rough), beam seas

between the pitching and rolling motion of the ship in this particular case then is apparent. However, the correlation coefficient should not be considered as an exact indication of the interdependence, since the effect of errors in measurement, as well as the effect of other factors which may have introduced a spurious correlation, were not evaluated.

Using the correlation coefficient and the respective variances, the joint distribution of pitch and roll was determined. These results are shown in Fig. 16. The curves formed by the intersection of vertical planes parallel to the pitch axis with the distribution surface are normal distribution curves representing the probability density distribution of pitch for constant angles of roll. Those formed by the intersection of vertical planes parallel to the roll axes with the distribution surface represent the probability density distribution of roll for constant angles of pitch. These curves are not, in general, symmetric about zero [as is the case for the associated one-dimensional distributions in Figs. 15(*a*) and 15(*b*)]. The axis of symmetry for pitch for a given roll angle, for example, depends upon the amount of roll, and *vice versa*. In general, for increasing roll to port, the axes of symmetry of the pitch probability density curves shift toward increasing positive values of pitch (bow up), while for increasing starboard roll the axes of symmetry shift toward increasing bow down pitch. The same trend is found for the roll probability density curves as the pitch angle is allowed to vary over its range. Information which may be obtained from the plot in Fig. 16 is the joint probability of any combination of pitch and roll angles being exceeded. For example, it may be desired to determine the probability that roll will exceed 3 deg and pitch exceed 2 deg simultaneously. Evaluation of the area under the distribution surface determined by these limits gives 6 per cent as the probability of a pitch angle of 2 deg or greater occurring with a 3-deg or greater roll angle.

Figs. 15(*a*) and 15(*b*), on the other hand, show that 24 per cent of the roll angles are greater than 3 deg while 16 per cent of the pitch angles are

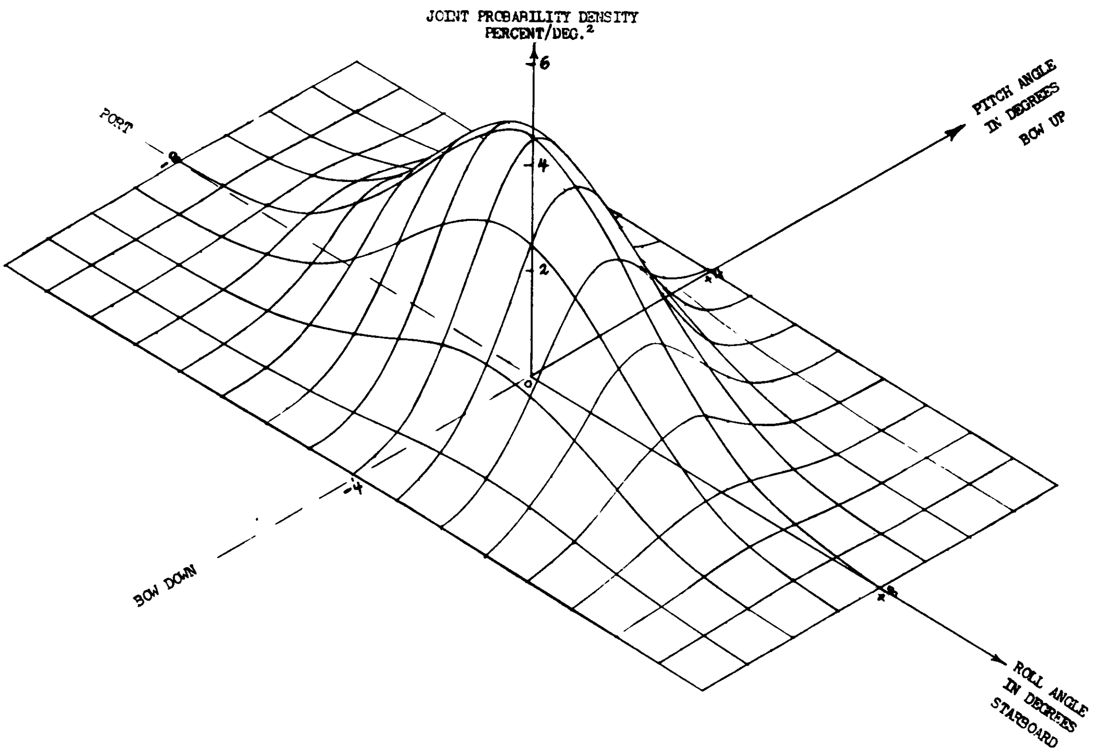


Fig. 16 Joint distribution of pitch and roll, ship S at a speed of 17 knots, sea state X (rough), beam seas

Table 7 Natural Periods and Frequencies

	Roll		Pitch		Heave	
Ship	T, sec	ω, rps	T, sec	ω, rps	T, sec	ω, rps
A...	9.11	0.69	4.3	1.46	4.8	1.31
B...	11.96	0.52	4.6	1.37	4.7	1.34
S...	9.54	0.66	4.0	1.57	4.8	1.31

rps = radians per second.

greater than 2 deg if only the one-dimensional distributions are considered.

If roll and pitch were completely uncorrelated it would be expected that the joint probability of roll exceeding 3 deg and pitch exceeding 2 deg would be only 4 per cent. Therefore, we can conclude that even low orders of correlation can have a significant effect upon the joint occurrence of large magnitudes (statistically speaking) of the two modes of motion.

Discussion of Spectra and Average Periods

A complete set of roll and pitch spectra was obtained for all three ships, for all headings, at the 22-knot speed in the Y (moderate) sea state. These are shown in Figs. 17 and 18. Additional computations were made for ship B in certain other head and beam-sea conditions, and these spectra are presented in Figs. 19 and 20. In judging these spectra, it must be remembered that the runs lasted only 10 min. The total number of cycles was not sufficient to keep the statistical sampling error small, particularly for following seas.

The values of ω corresponding to the natural periods of the three vessels in roll, pitch, and heave are given in Table 7.

The roll spectra are quite uniform in their general character, except for following seas. They are dominated by one large peak at or very near the natural frequency, frequently to the exclusion of all secondary peaks. Rolling in head and bow seas is moderate, as one should expect from the lack of energy at these frequencies in the spectrum of encounter for the waves, Figs. 6(*a*) and 6(*b*). However, for beam seas the spectrum of encounter will be nearly identical with the spectrum at a fixed point, and at the time of the beam sea, 22-knot run, there should have been a peak near $\omega = 0.67$. Thus, there was excitation near the natural frequency of all three vessels, and the roll was virtually synchronous. The spectra show this very clearly, with the curve having the form of a single sharp spike. In quartering seas, all

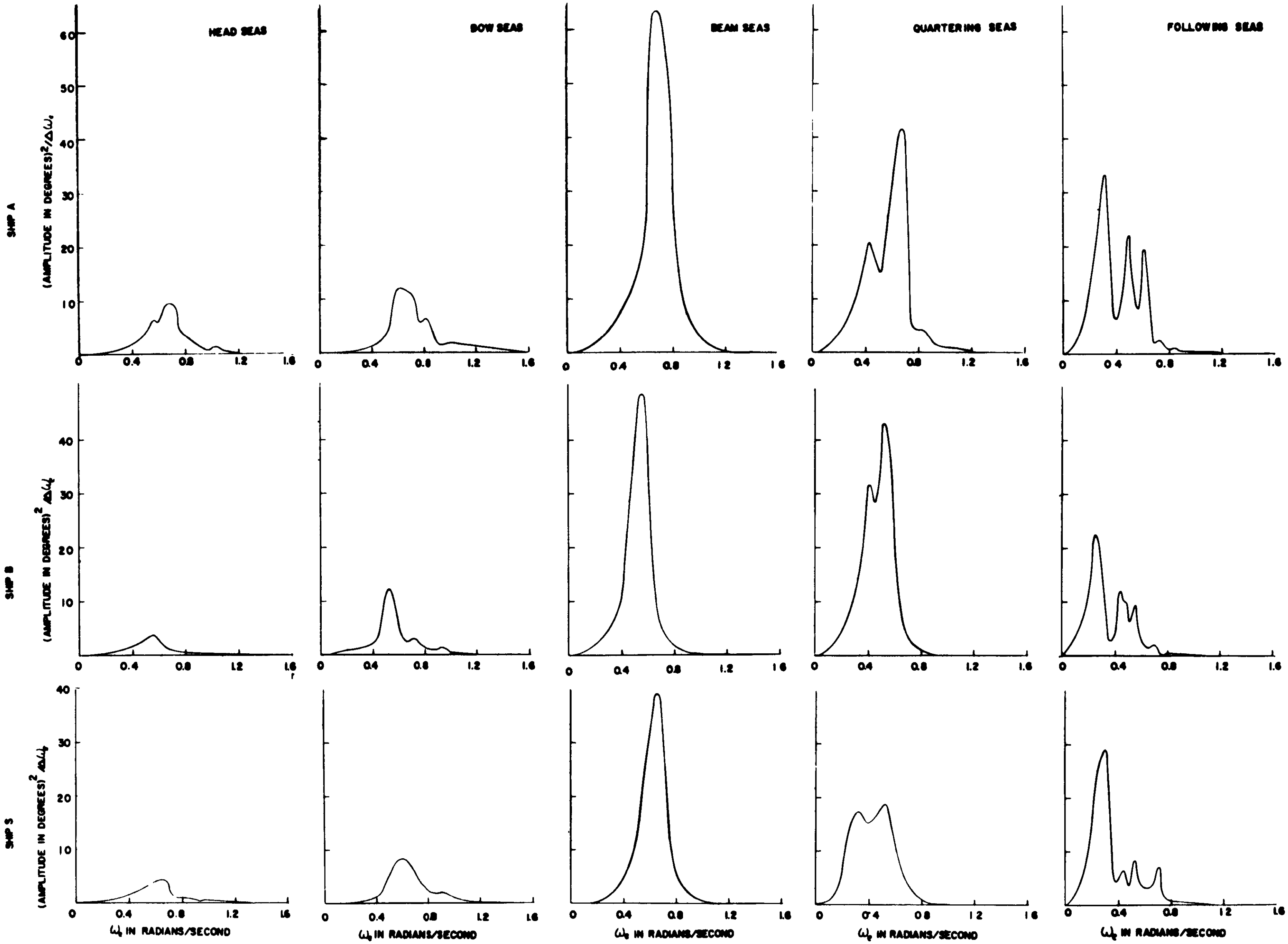


Fig. 17 Roll energy spectra for the three ships in moderate or Y sea state at a speed of 22 knots

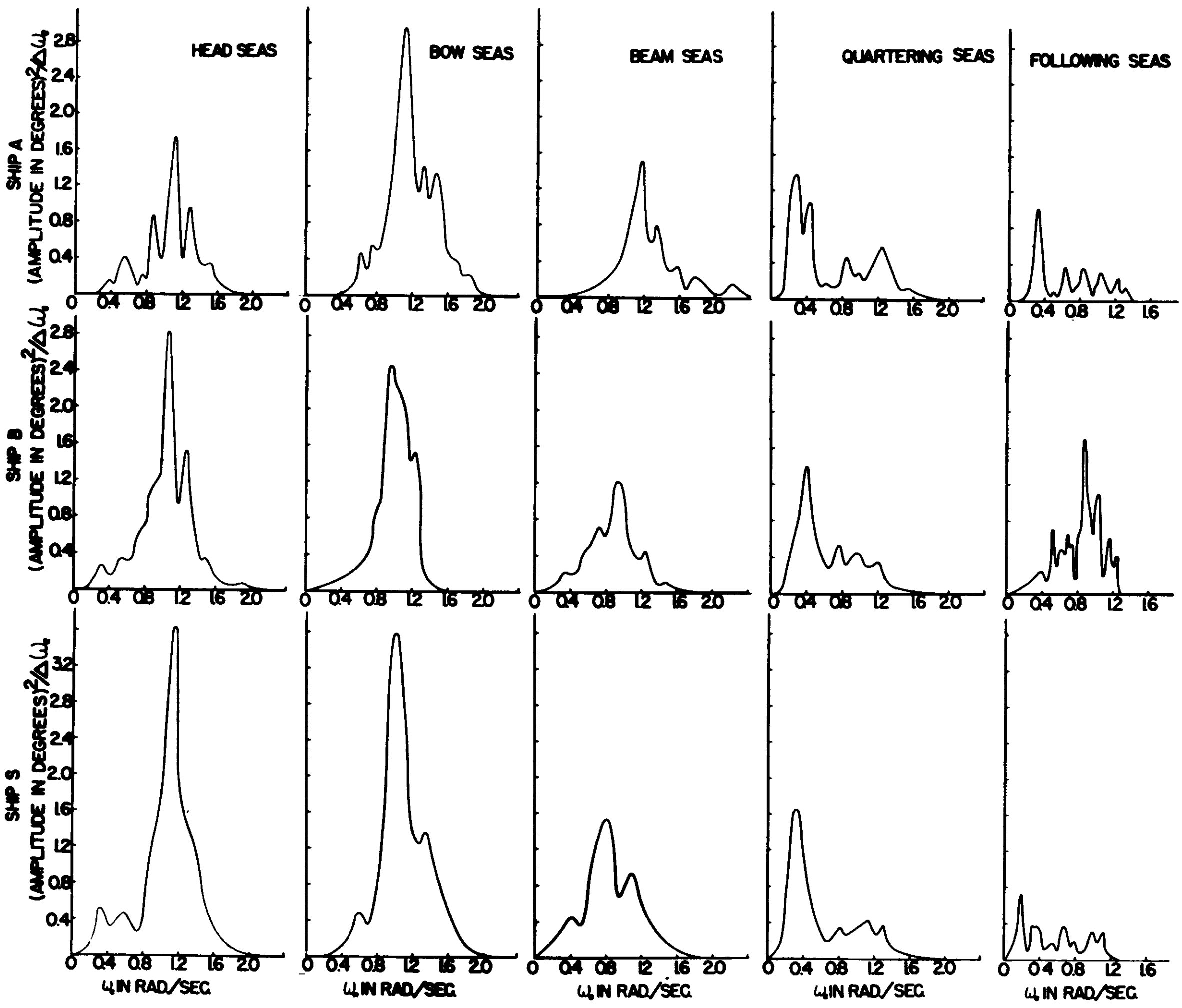


Fig. 18 Pitch energy spectra for the three ships in moderate or Y sea state at a speed of 22 knots

spectra exhibit a double peak. For ships A and B, one of these is near the natural frequency, and the second is a lower peak near $\omega = 0.4$. Ship S shows a peak somewhat below the natural frequency, and a second, slightly lower peak near 0.33, which is about where the peak of the spectrum of encounter would occur at this heading. In following seas, the rolling remains substantial, and the spectra have a ragged appearance, probably attributable to the inadequate length of the sample. There is the possibility that some of the very low-frequency response has been lost in the removal of the precession. All three cases show a strong, dominant peak at low frequency which is not clearly attributable to any well-defined component in the wave spectrum. It corresponds roughly to the frequency of waves of half the ship length, which the ships would be overtaking at this speed, and the assumed swell spectrum is deficient in energy for such waves. Under the circumstances, further analysis is not justifiable.

Fig. 19 shows a similar pattern for other speeds for the Y-sea state. At 16 knots, peaks are again at the natural frequency, and the rolling is slightly heavier for head seas than is the case for 22 knots. This is attributable to the fact that the spectrum of encounter has not shifted quite so far from the natural frequency. At 27 knots there is evidence that the character of roll is changing somewhat. In head seas, a low-frequency roll appears which is much greater in magnitude than that due to synchronous rolling, and in beam seas, secondary peaks are evident.

Rolling in sea state X follows the same general pattern. Rolling in head seas is much greater, because the spectrum of encounter has a peak near the natural frequency, Fig. 7(*c*). The higher peaks for 17 knots are not believed significant, and

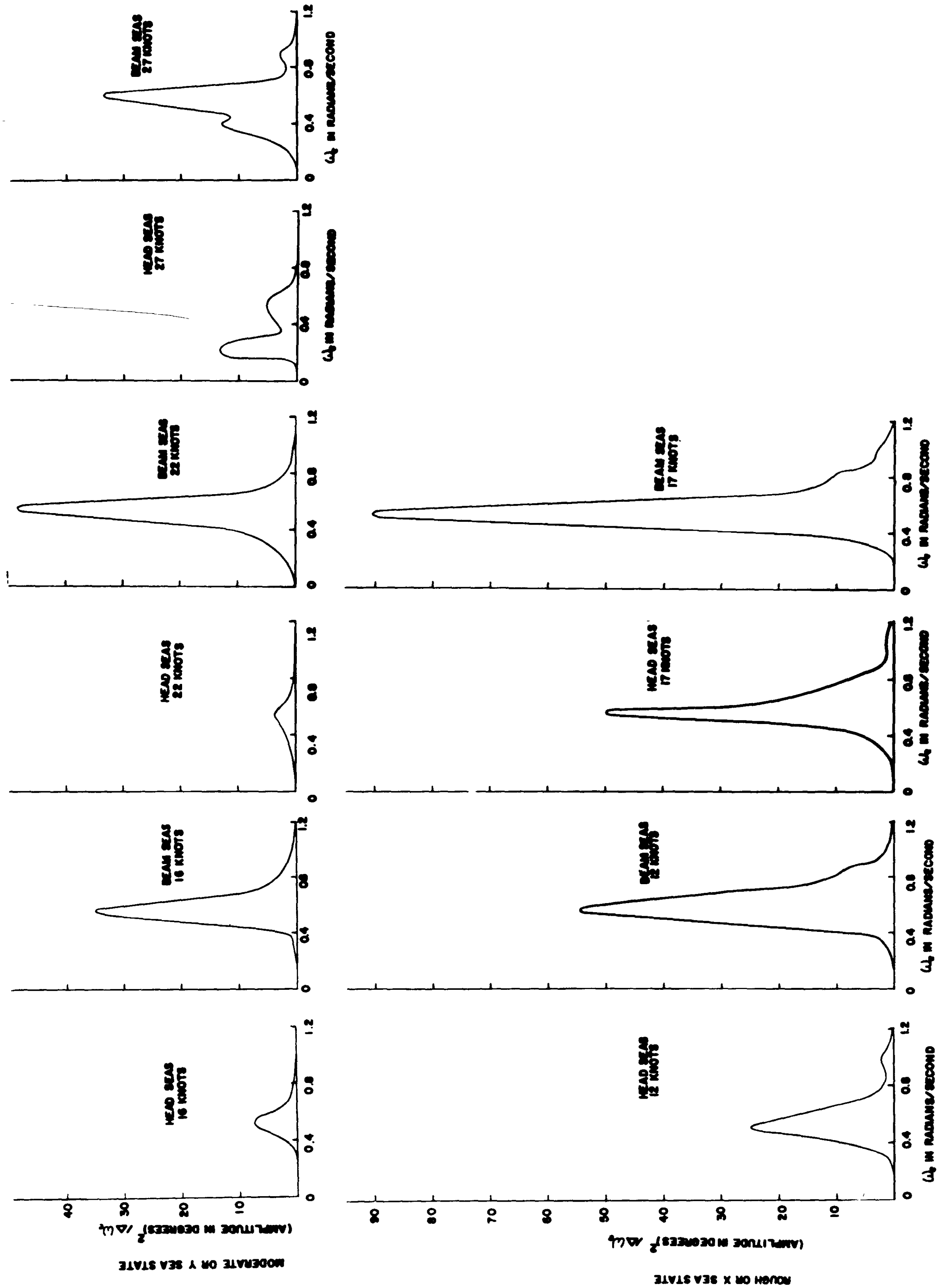


Fig. 19 Roll energy spectra for ship B obtained in head and beam seas

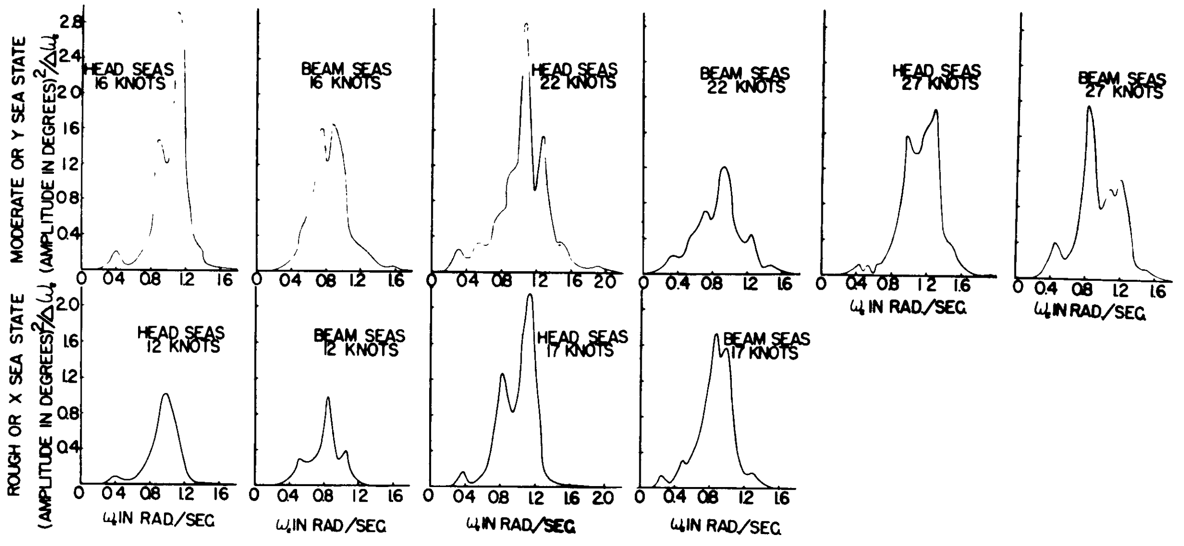


Fig. 20 Pitch energy spectra for ship B obtained in head and beam seas

indicate only slightly sharper tuning. The areas under the curves are roughly the same for both 12 and 17 knots.

The pitch spectra are much broader than roll spectra, as would be expected, Figs. 18 and 20. The greater damping in pitch prevents synchronous pitching from dominating the motion, and in fact none of the peaks is clearly associated with the natural frequency. Thus, the oscillation is almost completely forced, and a detailed analysis of the variation with speed and heading requires a more detailed knowledge of the wave spectrum than is available. On the other hand, the observed pitch spectra can be used to check the hindcast, and this has been attempted in a later section.

Spectra for heave acceleration and vertical acceleration at the bow were computed for two runs. These are shown in Figs. 21(*a*) and 21(*b*). They were used in the analysis of the variation of vertical acceleration along the length.

The mean periods of roll, pitch, and heave acceleration are given in Figs. 22, 23 and 24. These data were obtained from histograms of the time intervals between successive peaks. The range of periods appearing in the histograms is indicated by curves showing the limits of the first and last sextiles.

It is not easy to correlate these results with the spectra. Even in the case of roll, which is the purest motion, there is a uniform shift of the mean to a higher frequency than that of the well-defined peak of the spectrum. Thus, for roll of ship B in beam seas at 22 knots, the spectral peak occurs near $\omega = 0.55$, corresponding to a period of 11.5 sec. The mean is given in Fig. 23(*b*) as 9.8 sec. And this response was as sharply tuned as any which were recorded!

While the mean periods for pitch are not too inconsistent with dominant peaks in the spectra, the spread between the sextiles is not a good measure of the breadth of the spectrum. In fact, the data in these figures indicate that pitch might be more sharply tuned than roll, which is certainly not the case.

It can be concluded that stop-watch observations of motions are quite limited in their value, particularly if the distribution of energy with frequency is desired.

Distribution of Vertical Accelerations

At any point along the length of the ship, the vertical acceleration can be written as

$$a_t = \ddot{z}_t + X\ddot{\psi}_t \tag{9}$$

where

$\ddot{z}_t$ = heave acceleration at CG
$\ddot{\psi}_t$ = angular pitch acceleration
X = distance from observed point to the CG

For fixed conditions, that is, for a certain sea state, speed, and course, the heave and pitch deviations have zero means; thus a_t has a zero mean also.

The variance of a_t, or the mean square deviation from zero, has a nonzero value and is

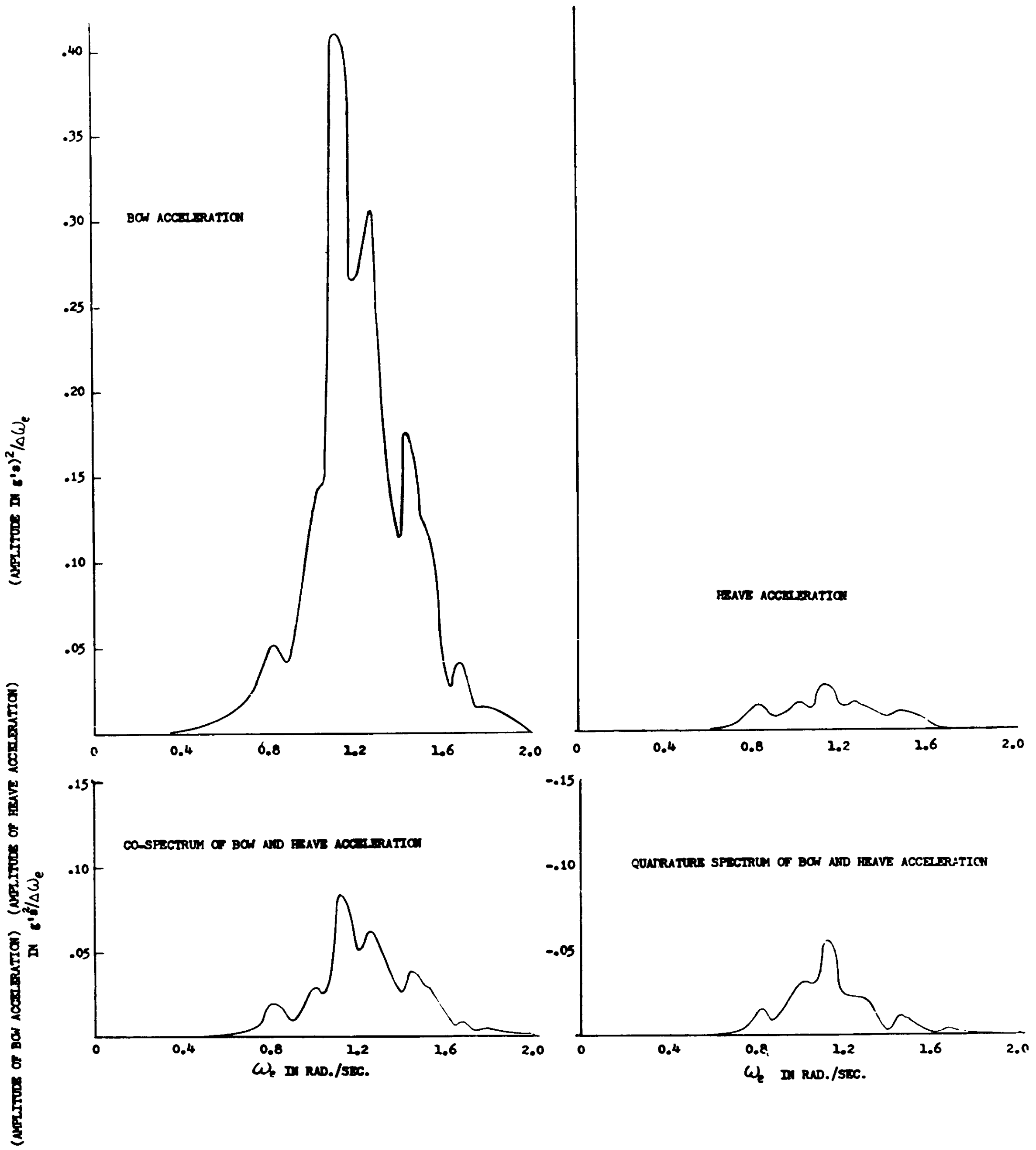


Fig. 21 (*a*) Cross spectra for bow and heave acceleration for ship A at a speed of 17 knots, sea state X (rough), head seas

$$\sigma_a^2 = \overline{(\ddot{z}_i + X\ddot{\psi}_i)^2} = \frac{\sum\limits_1^n \ddot{z}_i^2}{n} + 2X\frac{\sum\limits_1^n \ddot{z}_i\ddot{\psi}_i}{n} + X^2\frac{\sum\limits_1^n \ddot{\psi}_i^2}{n} \qquad (10)$$

Thus the time variance of a_i is a quadratic function of X and has a single minimum value. This minimum occurs at

$$X = -\frac{\sum\limits_1^n \ddot{z}_i\ddot{\psi}_i}{\sum\limits_1^n \ddot{\psi}_i^2} \qquad (11)$$

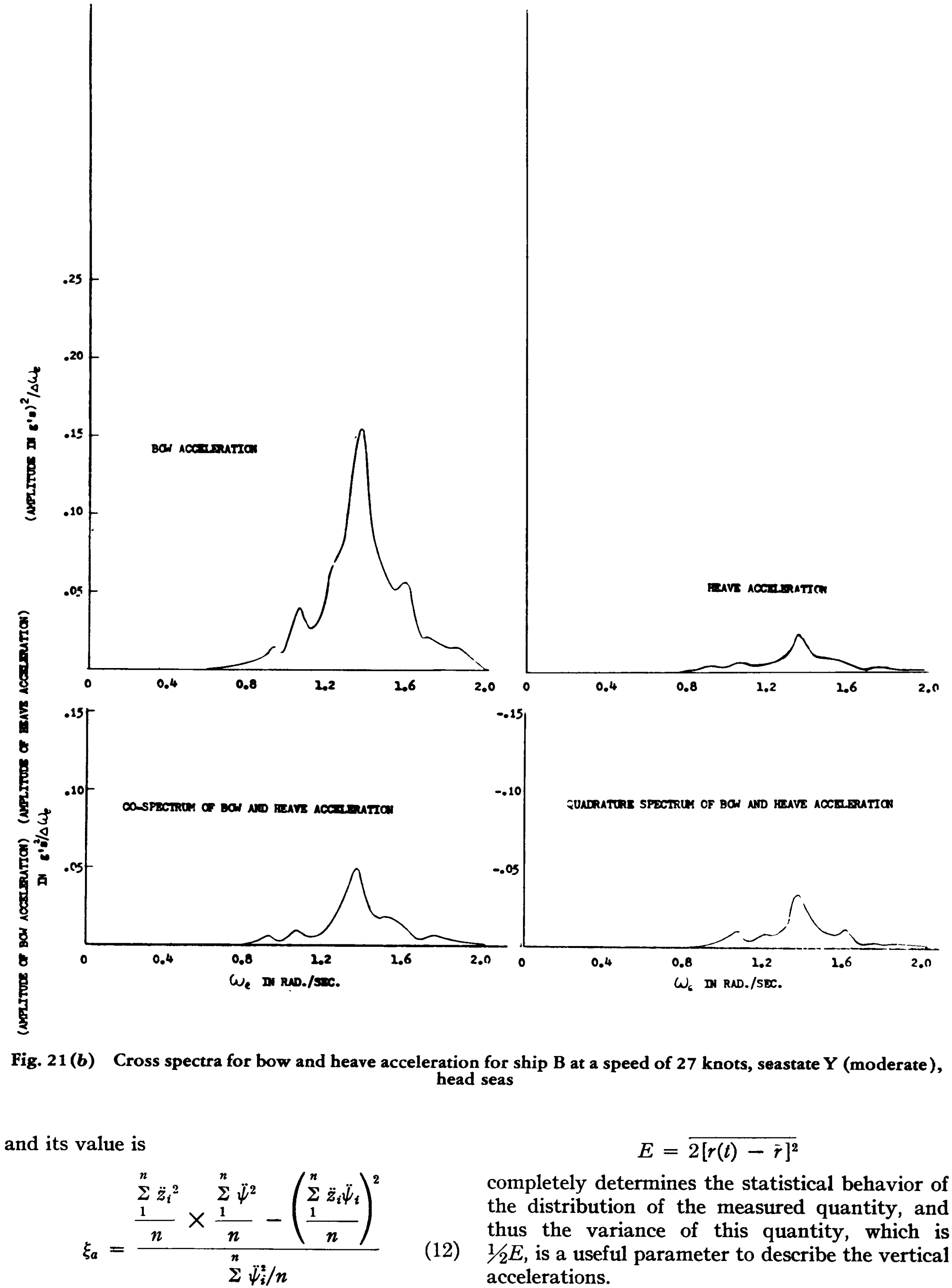


Fig. 21 (*b*) Cross spectra for bow and heave acceleration for ship B at a speed of 27 knots, seastate Y (moderate), head seas

and its value is

$$\xi_a = \frac{\dfrac{\sum_1^n \ddot{z}_i^2}{n} \times \dfrac{\sum_1^n \ddot{\psi}^2}{n} - \left(\dfrac{\sum_1^n \ddot{z}_i\ddot{\psi}_i}{n}\right)^2}{\sum_1^n \ddot{\psi}_i^2/n} \tag{12}$$

As stated before, the parameter

$$E = \overline{2[r(t) - \bar{r}]^2}$$

completely determines the statistical behavior of the distribution of the measured quantity, and thus the variance of this quantity, which is $\frac{1}{2}E$, is a useful parameter to describe the vertical accelerations.

Since the vertical accelerations have been measured at two locations on the ships, the vari-

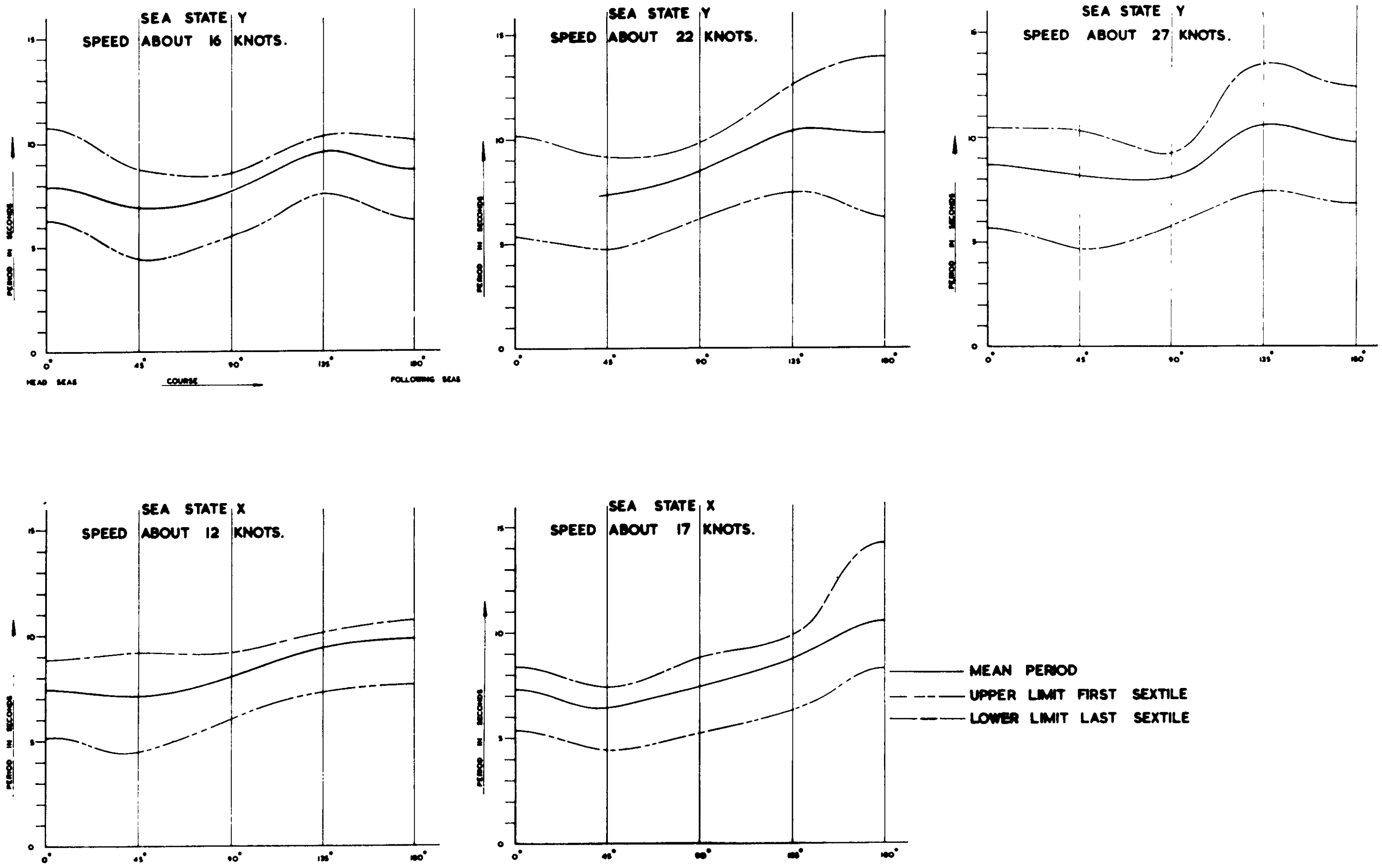


Fig. 22 (*a*) Distribution of periods of roll with respect to heading, ship A

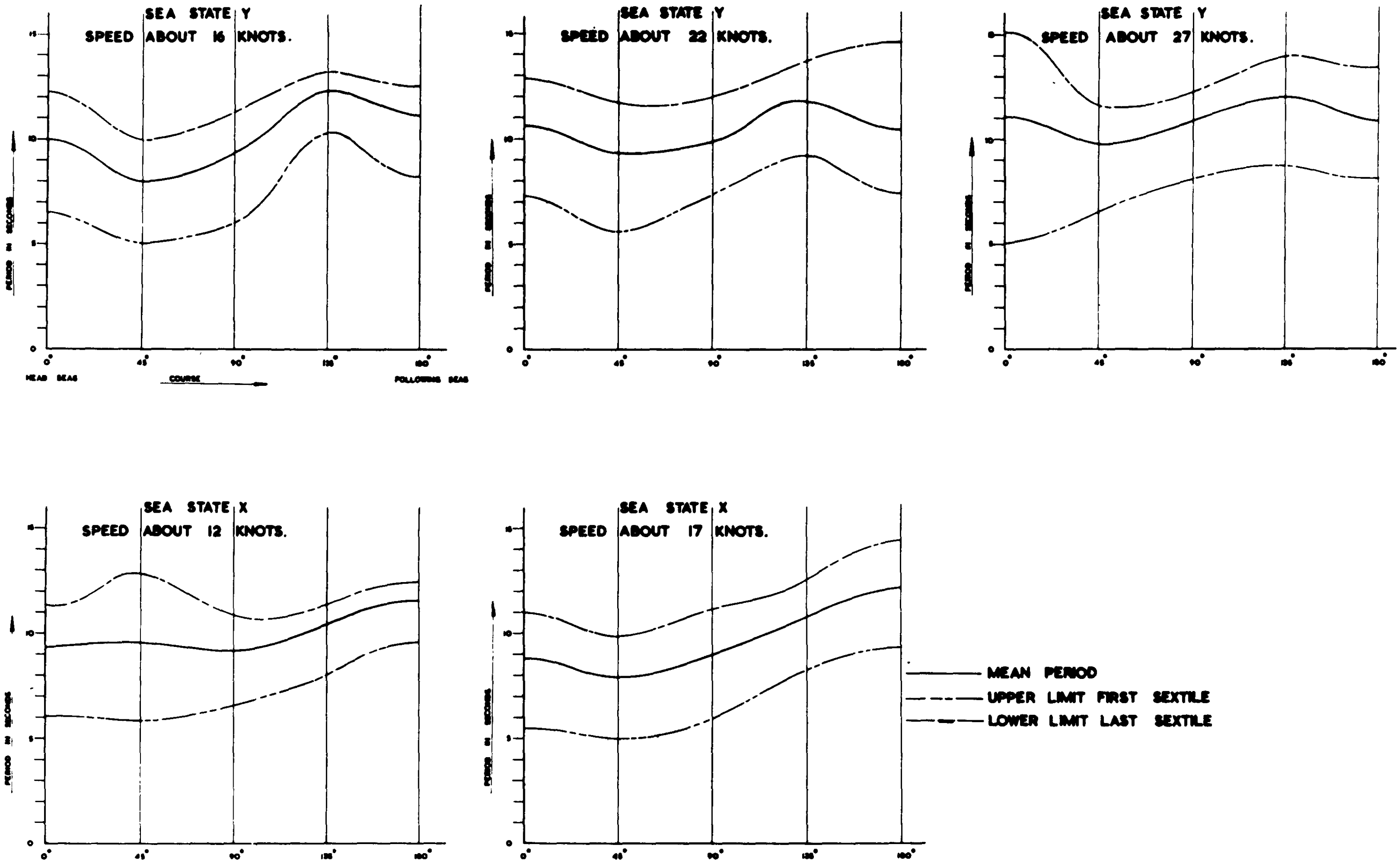


Fig. 22 (*b*) Distribution of periods of roll with respect to heading, ship B

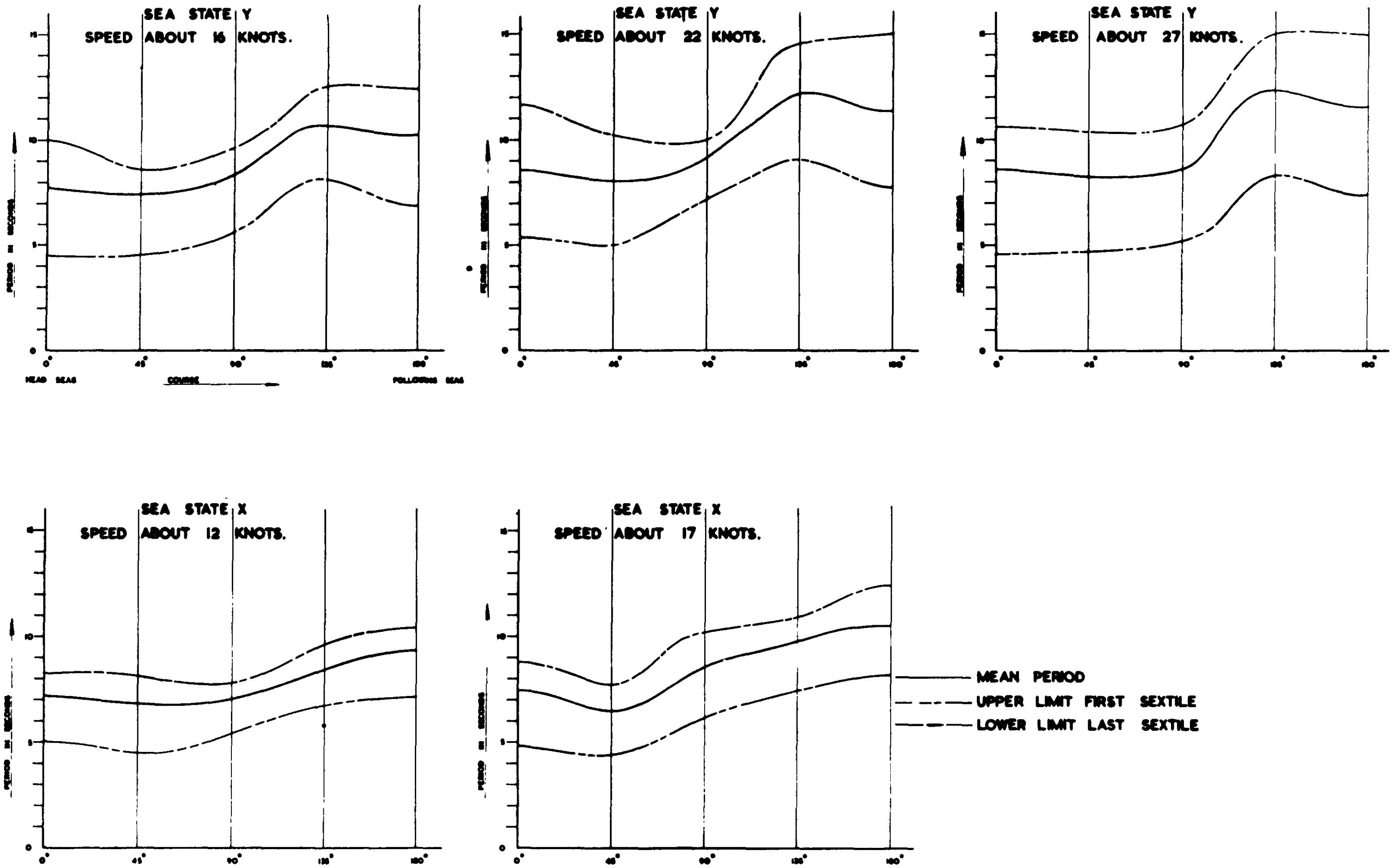


Fig. 22(*c*) Distribution of periods of roll with respect to heading, ship S

Table 8 Variances and Covariances of Vertical Accelerations

Run No.	$10^4 \times$ variance of vertical acceleration at bow in g^2	$10^4 \times$ covariance of vertical acceleration at bow and CG in g^2	$10^4 \times$ variance of vertical acceleration near CG in g^2
AY 2.1..	234	87.3	42.8
AY 3.1..	112	34.9	20.3
AX 1.1..	212	38.0	17.7
AX 2.1..	686 (838)[a]	104.0 (184)	46.0 (59.0)
BY 1.1..	117	25.3	9.1
BY 2.1..	183	41.7	16.5
BY 3.1..	251 (265)	66.9 (88.9)	35.9 (34.1)
BX 1.1..	201	42.5	18.6
SY 1.1..	125	28.8	15.4
SY 2.1..	223	56.1	24.9
SY 3.1..	732	138.0	48.3
SX 1.1..	237	43.6	20.3
SX 2.1..	556	130.1	50.1
AY 1.2..	79.9	20.7	23.4
AY 2.2..	208	77.8	46.4
AY 3.2..	139	47.6	29.7
AX 1.2..	483	80.1	26.9
AX 2.2..	854	138.0	57.5
BY 1.2..	154	33.4	15.7
BY 2.2..	203	58.8	30.0
BY 3.2..	306	89.7	43.8
BX 1.2..	289	45.6	18.3
SY 1.2..	151	33.0	17.5
SY 2.2..	240	59.1	28.7
SY 3.2..	335	88.6	46.5
SX 1.2..	365	54.6	20.6
SX 2.2..	785	183.0	78.5

---

[a] Numbers in parentheses are variance and covariance as determined from cross-spectral analysis.

ance of the vertical acceleration at any point can be computed.

From the variance and covariance of the measured vertical accelerations it is also possible to calculate the variance of the angular pitch acceleration. This calculation is derived in Appendix 2.

According to Szebehely [10] the vertical motion of ships in head seas in regular waves is composed of a vertical translation and a rotation about a fixed axis, "the apparent pitching axis." This axis is defined as the position of least vertical motion. For simple harmonic motion the position of the apparent pitching axis would be identical with the position of minimum mean square acceleration. At sea, these two positions are in general not coincident, but Cartwright [6] found that in head seas the two positions are close together. Moreover, both the instantaneous axis of rotation and the instantaneous position with minimum mean square acceleration have a concentrated distribution about their mean.

From previous calculations of Donath and Bussemaker [11] it is known that in head seas, the position of minimum mean square vertical acceleration moves generally in a narrow region aft of the center of gravity. This region has a tendency to become larger with change of the course from head to beam seas.

Thus only in head seas is it possible to define the position of minimum mean square vertical acceleration, as computed from the time variance of the measured vertical accelerations over the entire run.

Therefore, only for head and bow seas have the time variances of the vertical accelerations been computed and the location of minimum mean square vertical acceleration derived. The calculation is described and discussed in Appendix 2. The variances and covariances were computed from 200 simultaneous strip readings, equally spaced over the records.

Table 8 gives the computed variances and covariances of the measured accelerations, and Table 9 compares these data with the variances of heave and pitch accelerations as derived from:

1 The assumed Rayleigh distribution for heave acceleration.

2 The mean value of double amplitudes of heave acceleration.

3 Pitch acceleration energy spectra.

The position of the axis of minimum vertical acceleration for the various speeds and sea conditions is situated in a region aft of the CG for head and bow seas. For ships B and S this region is rather small. For ship B the mean distance from the CG amounts to 13.10 ± 2.50 m. For ship S the mean distance is 10.58 m and the standard deviation for all cases amounts to 3.65 m. The mean value of this distance for ship A is 10.99 m but here the standard deviation is 7.1 m. In general, however, it can be said that over the ship's length, the region with the least vertical motion is situated between 0.04 and 0.16 of the length aft of the CG.

A comparison of the calculated variance of pitch acceleration with that derived from the available pitch-angle spectra shows that in most cases there is a significant difference. However, if the accelerations are compared, the differences are not too disturbing. For the observed cases the maximum is 37.5 per cent and the mean is 18 per cent. A possible explanation for this difference (the accelerations from the spectra are usually too small) may be the manner in which the pitch acceleration spectra were computed from the pitch spectra. In the conversion, the high-frequency components are of more importance than the low-frequency components and as stated before, the instruments used for recording the angles of motion were not suitable for obtaining the high-frequency components. The method used for transforming the spectra is accurate only

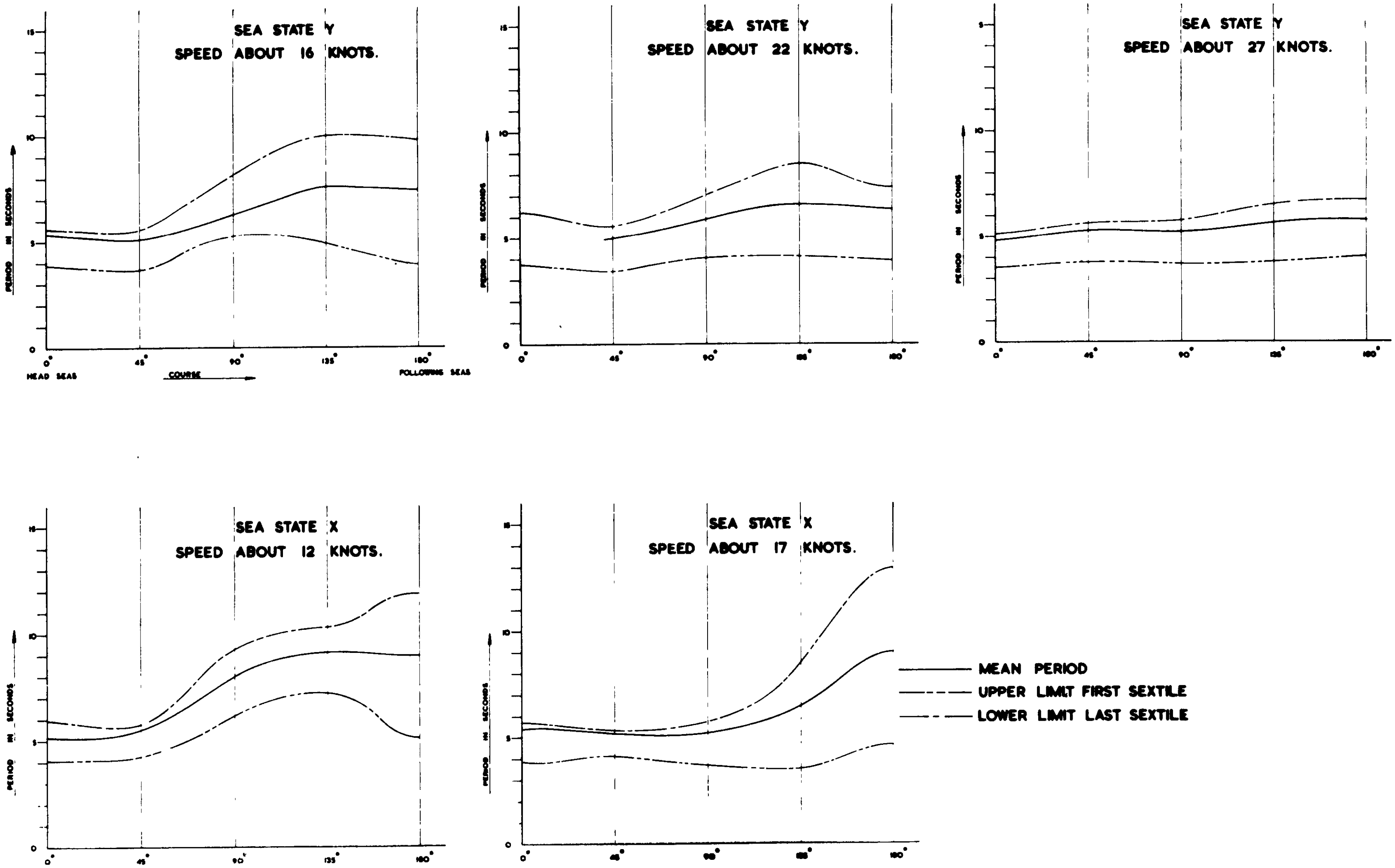


Fig. 23(*a*) Distribution of periods of pitch with respect to heading, ship A

Table 9 Distribution of Heave and Pitch Accelerations

	10^5 × variance of heave acceleration from strip, readings in g^2	10^5 × variance of heave acceleration from E_p in g^2	10^5 × variance of heave acceleration from $E = (h/1.77)^2$ in g^2	10^5 × variance of minimum vertical acceleration in g^2	Distance of axis of minimum vertical acceleration aft CG in meters	10^5 × variance of pitch acceleration from accelerometers in rad^2/sec^4	10^5 × variance of pitch acceleration from pitch spectra in rad^2/sec^4
AY 2.1	428	460	435	234	23.70	36.6	19.3
AY 3.1	203	190	199	168	13.26	22.5	
AX 1.1	177	210	213	150	8.01	55.3	
AX 2.1	460 (590)[a]	593	614	395 (294)	6.90 (13.45)	188.7 (190)	
BY 1.1	90.7	78.8	90.5	56.2	13.69	23.8	18.5
BY 2.1	165	148	161	110	13.82	37.6	31.0
BY 3.1	359 (341)	296	316	297 (93.4)	13.03 (26.52)	49.6 (39.7)	28.0
BX 1.1	186	141	144	144	11.67	43.6	28.0
SY 1.1	154	86	79	132	10.97	29.0	
SY 2.1	249	192	217	177	14.54	47.6	44.5
SY 3.1	483	351	371	400	11.81	142.9	
SX 1.1	203	202	185	171	4.67	59.8	
SX 2.1	501	730	558	368	9.59	121.9	
AY 1.2	234	135	120	184	−1.08	22.3	
AY 2.2	464	502	414	364	17.61	35.6	59.6
AY 3.2	297	375	326	253	13.77	26.5	
AX 1.2	269	285	215	188	9.05	125.9	
AX 2.2	575	585	573	474	7.73	228.8	
BY 1.2	157	126	135	127	11.37	33.3	
BY 2.2	300	275	242	228	15.65	37.7	
BY 3.2	438	461	455	315	16.76	55.9	
BX 1.2	183	183	204	148	8.88	70.0	
SY 1.2	175	155	165	152	10.41	35.9	
SY 2.2	287	291	299	256	13.07	52.7	51.6
SY 3.2	465	336	355	378	13.29	71.6	
SX 1.2	206	231	290	188	3.91	106.0	
SX 2.2	785	671	645	565	13.55	174.3	

---

[a] Numbers in parentheses were obtained from cross spectral analysis.

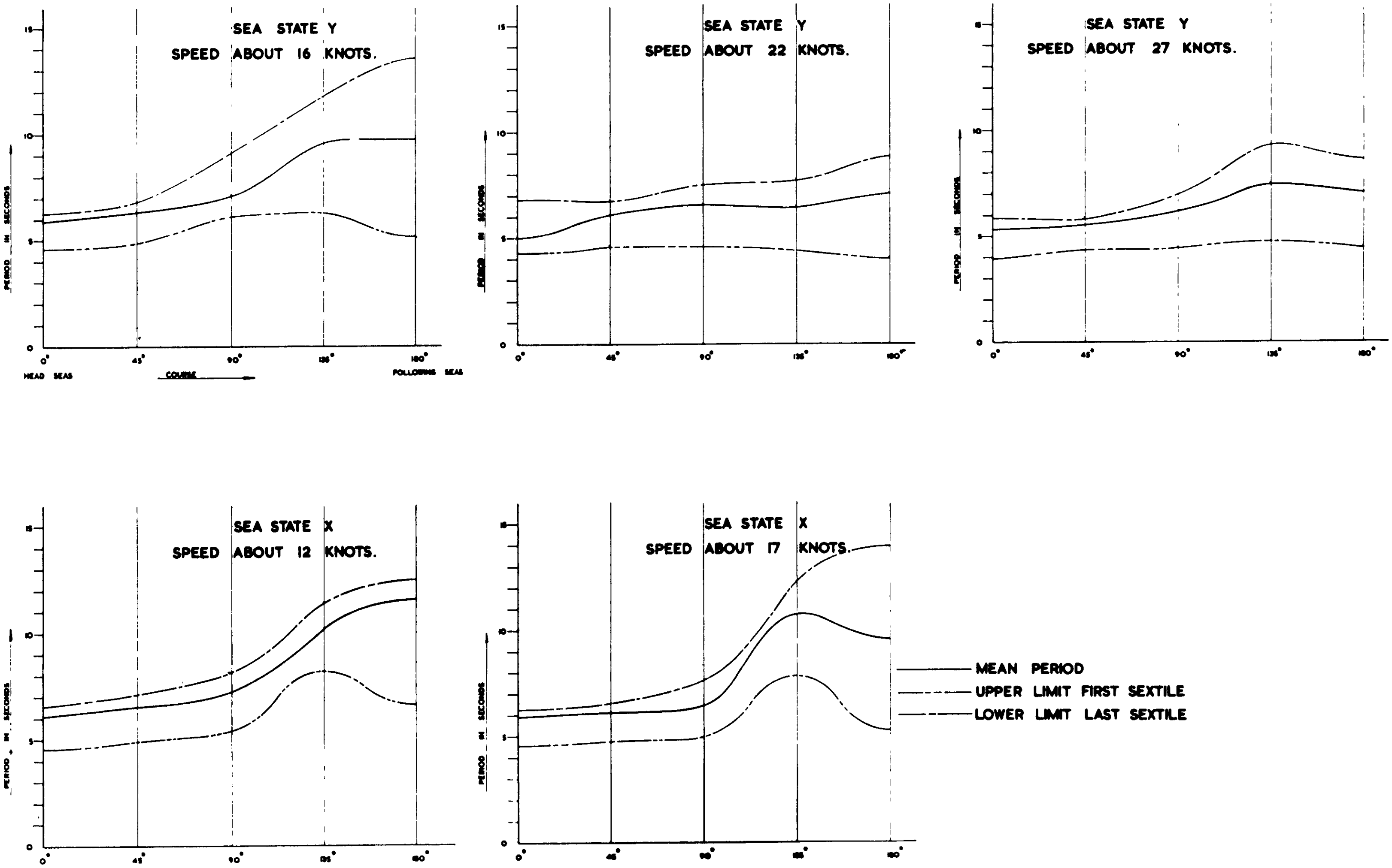


Fig. 23(*b*) Distribution of periods of pitch with respect to heading, ship B

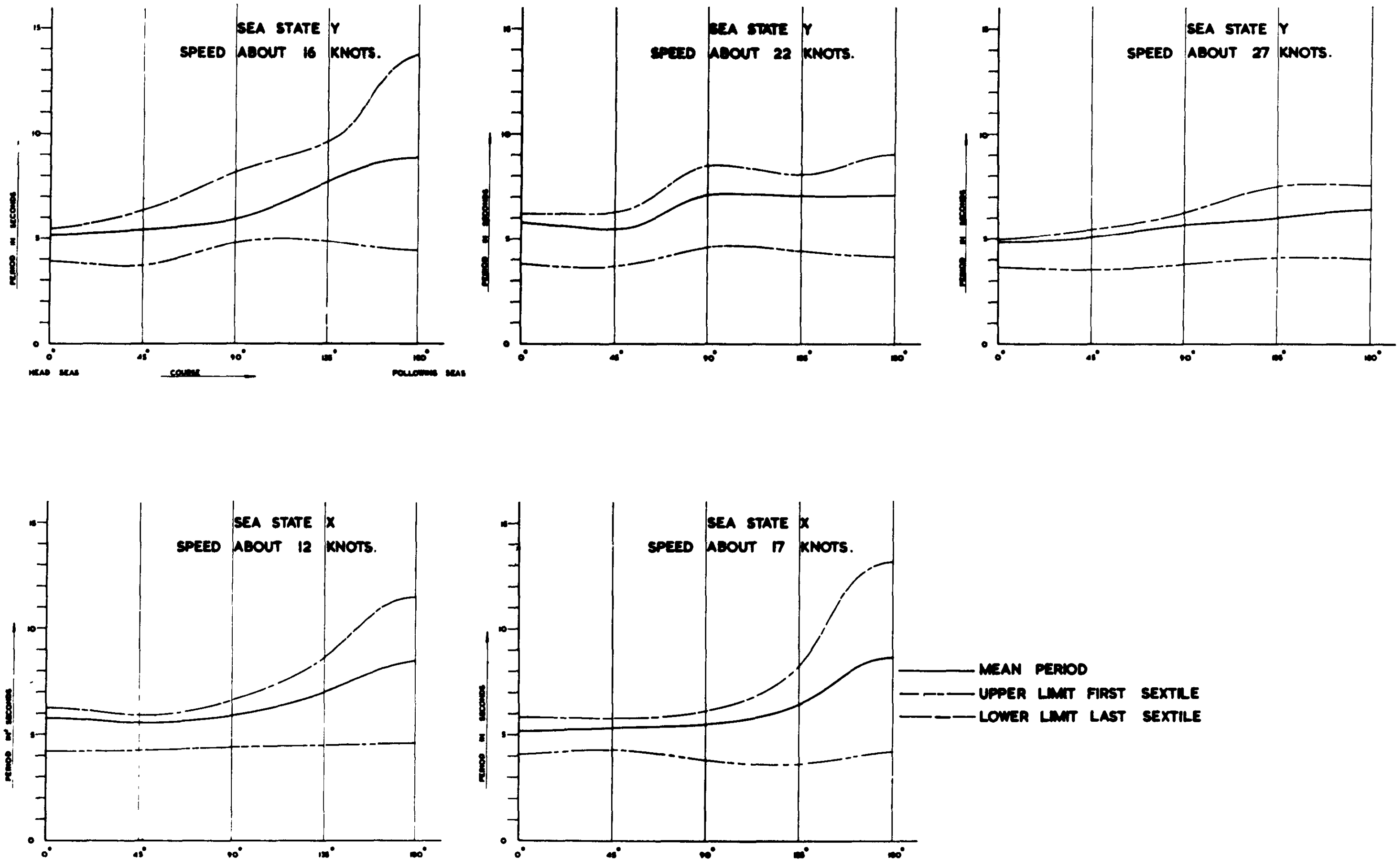


Fig. 23(*c*) Distribution of periods of pitch with respect to heading, ship S

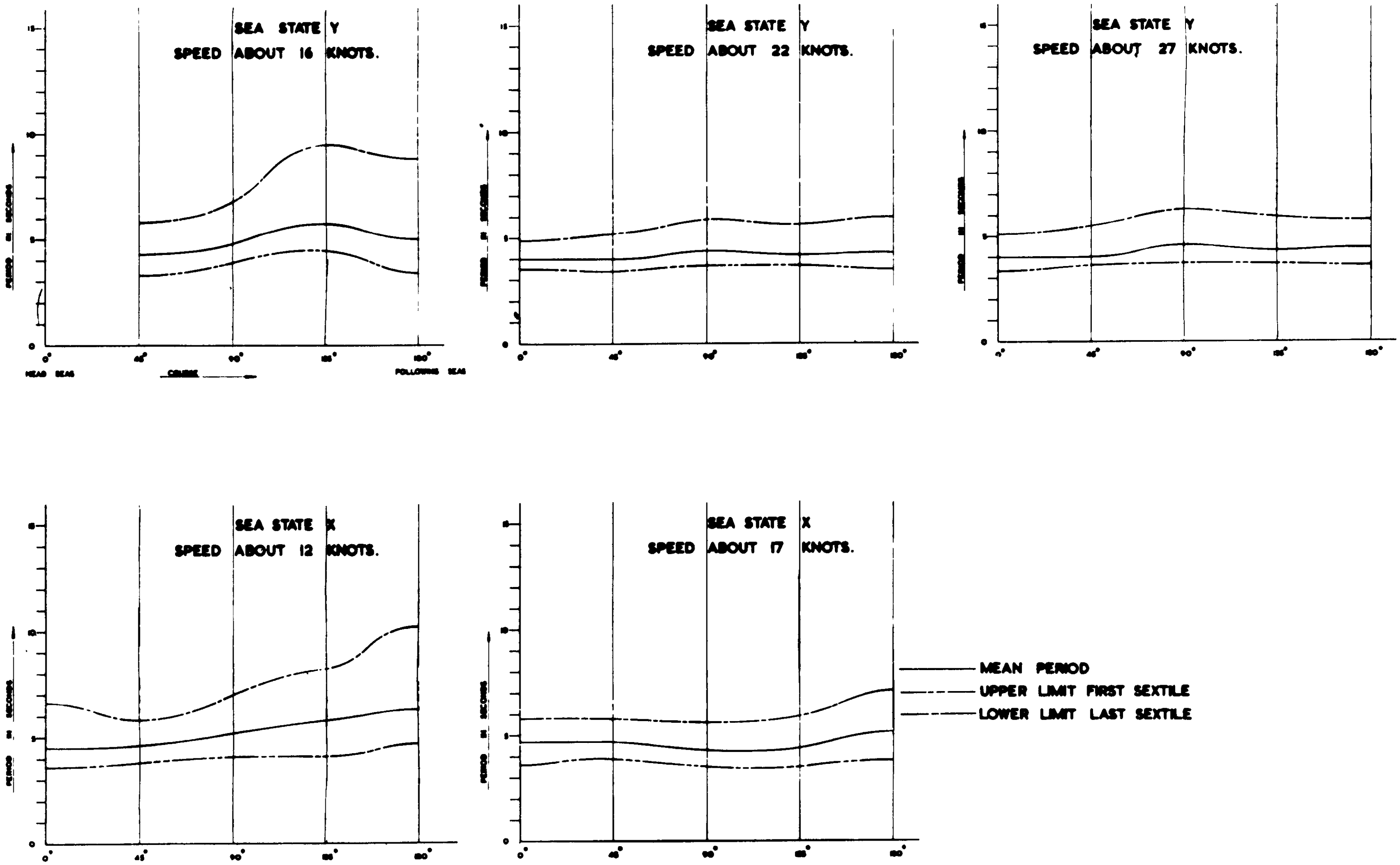


Fig. 24(*a*) Distribution of periods of heave acceleration with respect to heading, ship A

Table 10 Comparison of Computed and Observed Double Amplitudes for Roll in Degrees

Run No.	Average of highest 10 per cent		Average of highest one third		Average		Maximum	
	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed
AY 2.1....	6.15	6.2	4.84	5.0	3.02	2.8	7.84	10.7
AY 2.2....	7.78	7.5	6.12	5.7	3.82	3.3	9.76	11.2
AY 2.3....	14.34	14.4	12.26	11.7	7.65	7.0	19.28	18.7
AY 2.4....	12.15	12.4	9.49	9.4	5.94	6.0	14.70	15.7
AY 2.5....	10.81	9.5	8.56	8.3	5.36	4.6	13.10	13.7
BY 1.1....	4.50	4.2	3.53	3.4	2.21	2.1	5.52	4.7
BY 1.3....	9.00	10.2	7.05	7.4	4.43	4.1	11.16	15.7*
BY 2.1....	4.04	3.8	3.16	3.0	1.98	1.8	4.88	5.2
BY 2.2....	5.43	5.4	4.31	4.2	2.67	2.6	6.70	6.7
BY 2.3....	11.70	11.8	9.18	9.0	5.74	5.3	14.16	15.7
BY 2.4....	11.90	11.6	9.34	9.2	5.84	5.6	14.16	13.2
BY 2.5....	7.87	8.0	6.20	6.1	3.87	3.3	9.46	9.2
BY 3.1....	6.08	6.2	4.79	4.7	2.98	2.7	7.34	7.7
BY 3.3....	9.81	9.6	7.71	8.0	4.84	4.1	11.70	11.7
BX 1.1....	8.81	9.3	6.90	6.8	4.32	4.0	10.74	13.2
BX 1.3....	13.87	13.5	10.89	10.0	6.80	6.1	16.74	17.7
BX 2.1....	10.49	10.5	8.23	8.1	5.15	4.8	12.98	13.2
BX 2.3....	13.75	16.1	10.83	11.4	6.76	6.4	17.03	25.7*
SY 2.1....	4.28	4.1	3.37	3.4	2.10	2.0	5.26	5.2
SY 2.2....	5.62	5.7	4.44	4.7	2.78	2.5	6.98	7.2
SY 2.3....	11.13	10.7	8.74	8.5	5.46	5.3	13.56	12.7
SY 2.4....	9.87	9.4	7.75	7.4	4.85	4.9	11.58	12.2
SY 2.5....	7.10	7.6	5.57	6.0	3.49	3.6	8.42	11.2*
Comparison of Computed and Observed Double Amplitudes for Pitch in Degrees								
AY 2.1....	2.99	2.9	2.22	2.2	1.47	1.4	3.57	3.7
AY 2.2....	4.25	4.4	3.34	3.2	2.09	2.0	5.60	6.2
AY 2.3....	2.85	2.8	2.24	2.2	1.39	1.4	3.66	3.7
AY 2.4....	2.74	2.6	2.15	2.1	1.35	1.5	3.42	3.2
AY 2.5....	2.67	2.7	2.09	1.8	1.31	1.0	3.38	3.7
BY 1.1....	3.28	3.6	2.61	2.8	1.63	1.8	4.26	3.7
BY 1.3....	3.20	3.2	2.49	2.4	1.56	1.6	4.02	3.7
BY 2.1....	3.56	3.4	2.78	2.9	1.75	1.9	4.54	4.2
BY 2.2....	3.75	3.6	2.95	3.1	1.84	2.1	4.86	4.7
BY 2.3....	2.96	2.8	2.33	2.2	1.47	1.5	3.76	3.7
BY 2.4....	2.69	2.6	2.12	2.0	1.34	1.2	3.44	3.2
BY 2.5....	2.29	2.3	1.79	1.6	1.14	1.0	2.90	2.7
BY 3.1....	3.71	3.6	2.92	2.8	1.82	1.8	4.66	4.2
BY 3.3....	3.44	3.2	2.70	2.6	1.71	1.8	4.36	4.7
BX 1.1....	4.79	4.6	3.77	3.8	2.36	2.4	6.10	5.2
BX 1.3....	4.54	4.4	3.57	3.4	2.22	2.2	5.76	5.7
BX 2.1....	7.19	7.0	5.65	5.6	3.52	3.6	9.28	8.2
BX 2.3....	7.14	7.2	5.64	5.4	3.50	3.4	9.04	9.2
SY 2.1....	4.33	4.2	3.40	3.6	2.12	2.2	5.51	5.2
SY 2.2....	4.29	4.2	3.34	3.2	2.09	2.1	3.05	4.7*
SY 2.3....	3.29	3.2	2.68	2.4	1.64	1.6	4.13	4.2
SY 2.4....	2.64	2.6	2.07	2.0	1.29	1.2	3.43	3.2
SY 2.5....	2.33	2.5	1.81	1.3*	1.15	1.0	2.90	4.7*
Comparison of Computed and Observed Double Amplitudes for Heave Acceleration in g's								
AX 2.1....	0.39	0.42	0.31	0.33	0.19	0.21	0.50	0.57
BY 3.1....	0.28	0.27	0.22	0.22	0.14	0.14	0.35	0.36

if in the recording and the reading of the records, the high-frequency components were taken into account. A sample of a transformed spectrum is given in Fig. 25.

When the variance and covariance used in the foregoing calculation, Table 8, are compared with the more reliable values determined from cross-spectral analysis for the two cases in which this was performed, the percentage differences in the covariances are at least twice the differences in the variances. This suggests that the estimated covariance from the 200 pairs of points taken from the records may be subject to a more serious sampling error than the estimated variance. Since a small change in the covariance has a large influence on the minimum vertical acceleration, its position, and other derived data, this could be another reason for the unsatisfactory results. These aspects require further analysis before positive conclusions can be drawn.

Comparison of Estimated and Observed Double Amplitudes of Motion

Using E and the statistical relations given previously, the average, characteristic, average of highest $1/10$ and most probable maximum

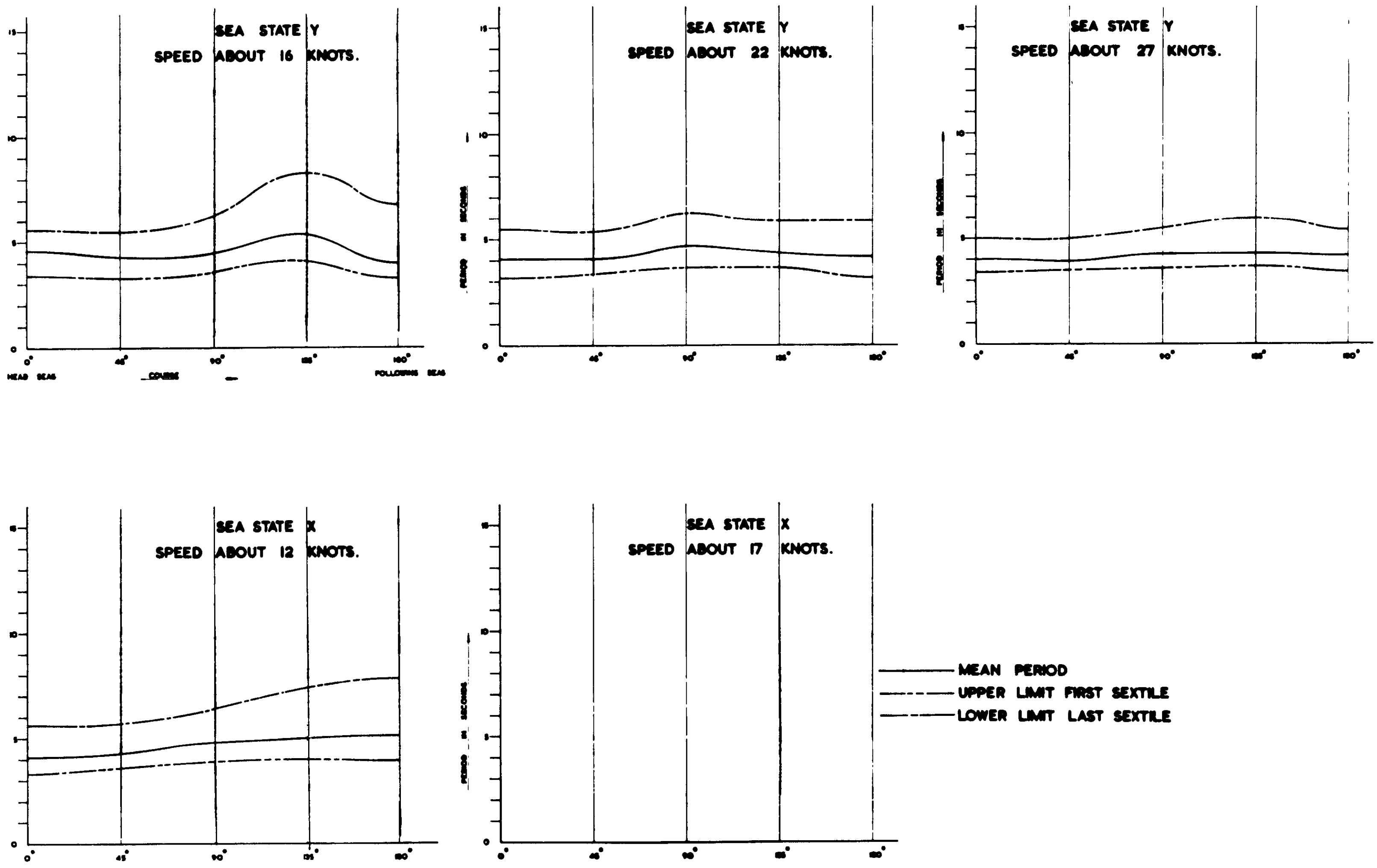


Fig. 24(*b*) Distribution of periods of heave acceleration with respect to heading, ship B

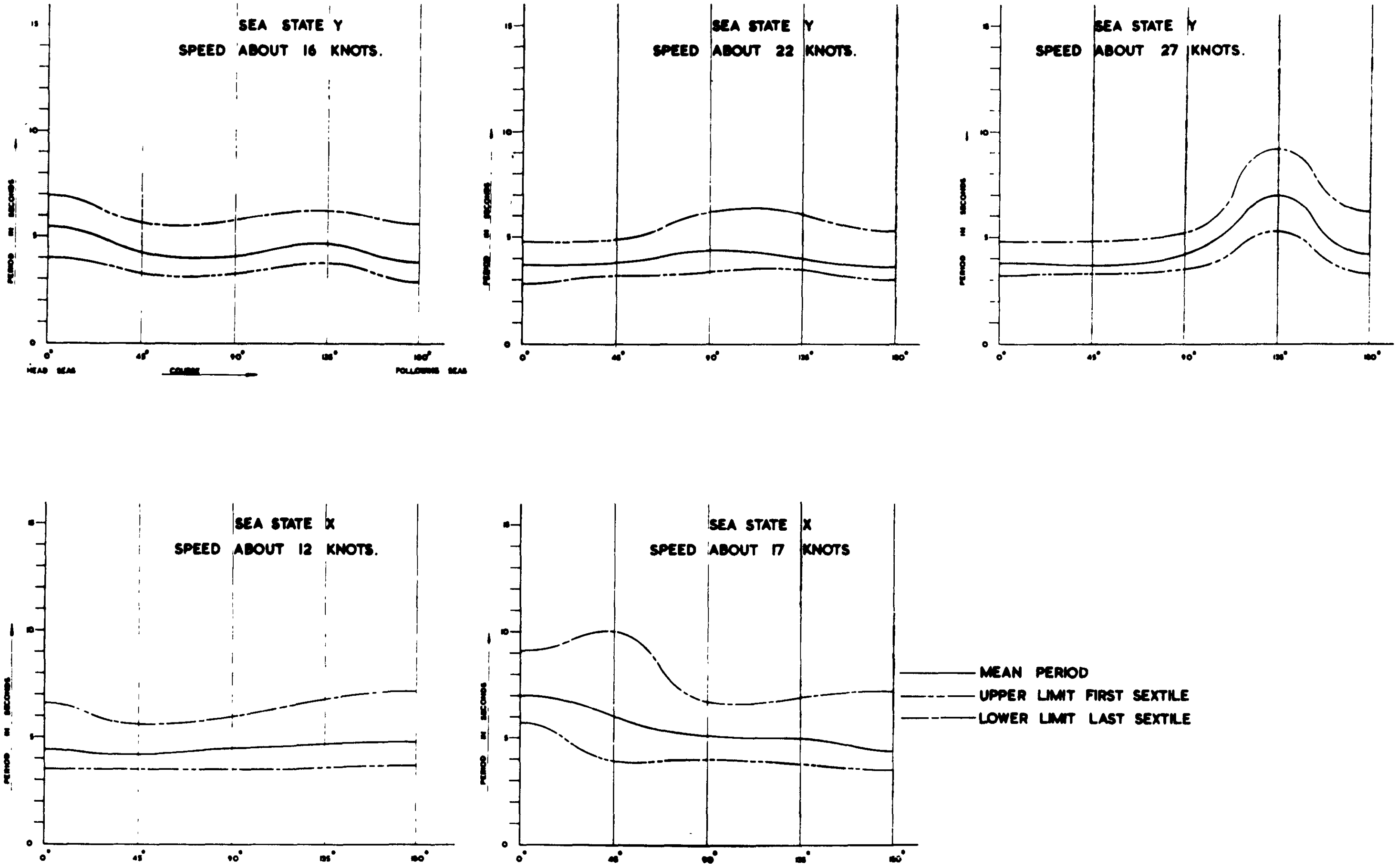


Fig. 24(*c*) Distribution of periods of heave acceleration with respect to heading, ship S

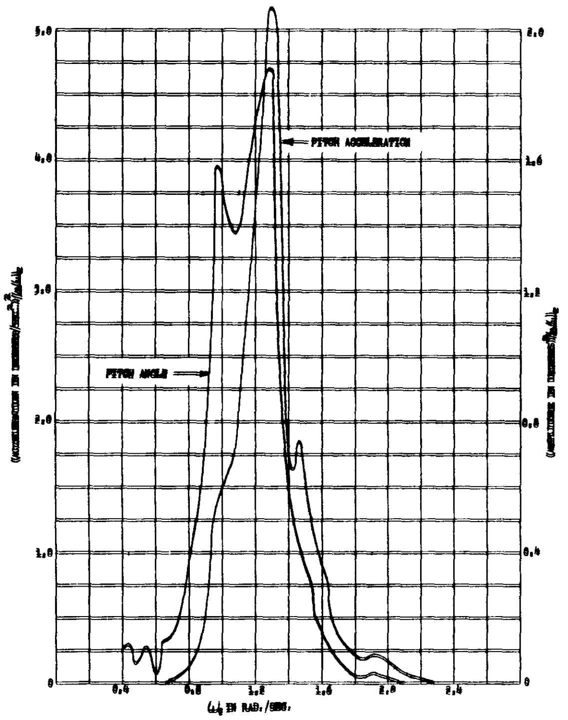


Fig. 25 Pitch angle and converted acceleration spectra for ship B in moderate (Y) head seas at a speed of 27 knots

double amplitudes were computed for representative cases of pitch, roll, and heave acceleration. The predicted values are shown in Table 10 along with the corresponding experimentally obtained data. A comparison shows that in general the agreement between the two is good indeed. In the majority of the cases the predicted and observed values are within 10 per cent, there being few cases where the discrepancy was between 10 and 20 per cent and only 6 cases where the error was 20 to 40 per cent. These are indicated in the table by the asterisks. In these six cases where the error was greater than 20 per cent, five were in the prediction of the maximum motions.

It is interesting to note that the theoretically derived relations for estimating the motions in general gave better agreement for pitch than for roll. However, for both roll and pitch the largest discrepancies appeared in the prediction of the maxima. The validity of the assumption of the Rayleigh distribution to describe the pitch, roll and heave acceleration amplitudes was investigated through application of the Chi-square test. This analysis is discussed in detail in Appendix 1. However, it may be interesting to review Table 10 in light of the results found in Appendix 1.

Of the 23 roll runs considered in Table 10, 14 were found to fail the Chi-square test, only two of which (BY 1.1 and BX 1.1) failed in a random fashion. In the case of maximum roll for BX 1.1 an error greater than 10 per cent arose in predicting the motion. Of the other 12 runs which failed the Chi-square test, only two, BY 1.3 and BX 2.3 resulted in significant differences between the predicted and observed maxima. The other run in which the discrepancy between observed and predicted maxima is large is SY 2.5. This particular case passed the Chi-square test but it too showed the same tendency which resulted in the other 12 failures; namely, a high concentration of observations in the lower class intervals. While E_p agreed well with E_h for this case, there was a significant difference between these values of E and that obtained from the power spectrum (Table 21). However, using the E from the power spectrum instead of E_p would result in errors greater than 20 per cent in the other predicted motions.

While nine of the pitch cases in Table 10 failed the Chi-square test, only two (SY 2.2 and SY 2.5) showed serious errors in predicting the maximum motions. (SY 2.5 also showed poor prediction of the significant amplitude.) A significant deviation between E_p and E from the power spectrum was observed. Both E_p and E for this case are probably in error; E_p due to the wide breadth of the spectrum and E due to the difficulty of eliminating the precession in the following sea tests.

In summary, it might be said that comparison of the observed and predicted maxima in Table 10 appears to justify the statistical relations for estimating the motions even in cases where the Rayleigh distribution does not hold. However, in estimating the maxima, caution should be exercised, especially in the case of roll when the motion is large. In these cases the extreme values can be underestimated by a significant amount.

The predicted motions in pitch can be expected to be reasonably accurate even when the Rayleigh distribution does not hold.

Comparison of Measured and Estimated Pitch Spectra

Using the derived Neumann sea spectra an attempt was made to predict the pitch-response spectra for ship B in head seas at speeds of 12 and 17 knots using the method of reference [5]. This approach required the assumption that the response of a ship to a seaway is the sum of the responses to the individual components of which it is comprised. It also assumes linearity of response to wave height, which has been found to be approximately correct if the motions are not so severe that bow emergence and shipping of green water occur. It required a knowledge of the

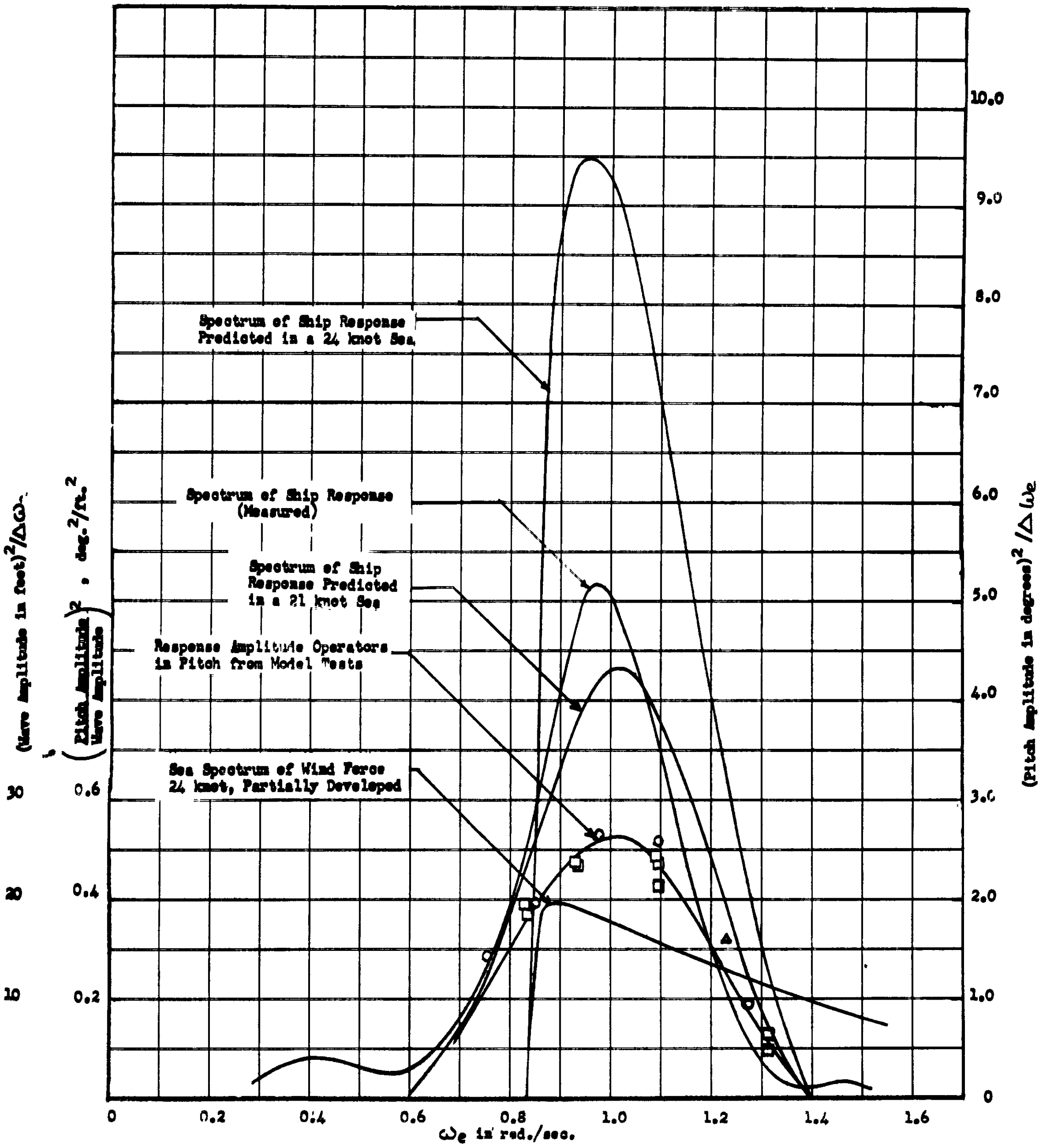


Fig. 26(a) Comparison of predicted and measured pitch response spectra for ship B in sea state X, head seas, at a ship speed of 12 knots

frequency-response functions or so-called "response amplitude operators." These are dependent upon the geometric and dynamic properties of the ship and may be obtained from model tests in regular or irregular waves. When the operators are obtained from model tests, the amplitudes of motion at constant speed in various wave lengths are determined, expanded to ship scale, divided by the wave amplitude, squared and plotted against frequency of encounter. The response spectra for the ship are then obtained by multiplying the ordinates of the sea spectrum and those of the response amplitude operator at corresponding frequencies.

Complete model data for ship B were not available for the 12-knot speed; therefore, the response amplitude operators were obtained from model tests on the DLG-9, reference [12], and the

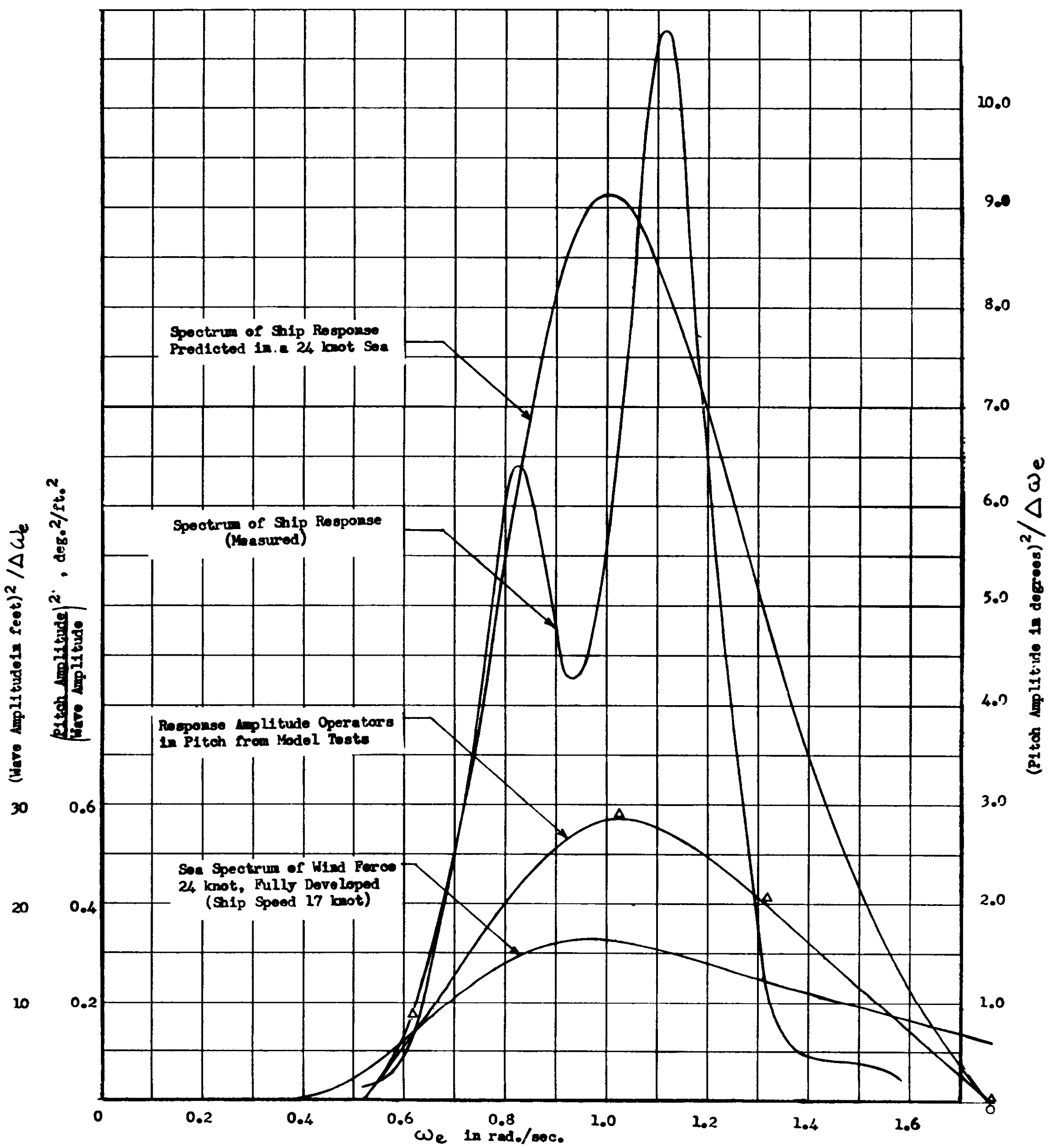


Fig. 26(*b*) Comparison of predicted and measured pitch response spectra for ship B in sea state X, head seas, at a speed of 17 knots

destroyer tests reported in reference [13], both of which have characteristics similar to those of ship B. The response amplitude operators so obtained are plotted in Fig. 26(*a*). The circles refer to data from reference [13] and the squares to data from reference [12]. The response amplitude operator from model tests on ship B was available for one frequency. This is shown in the figure by the triangle. Shown also in the figure is the Neumann sea spectrum (partially developed) corrected to the 12-knot speed and the derived and measured pitch response spectra for ship B. Since the response amplitude operators apply in head seas only, the swell components (coming from almost abeam) were omitted in the derived sea spectra in Fig. 26(*a*). Furthermore, it can be assumed that these components coming from abeam are of less significance in the pitching motion than are the head-sea components. Indeed, the measured spectrum shows no effect of the swell whatsoever. In general, the agreement between the derived and measured pitch response spectra is poor as far as magnitude is concerned but excellent agreement is found concerning the frequency of maximum energy. This frequency corresponds to the period of encounter determined by taking the average time interval between successive peaks.

Since the derived spectrum depends heavily on the sea spectrum which, in this case, represents an idealized sea for a 24-knot wind, an attempt was made to determine the effect of slight inaccuracies in estimating the wind velocity from a hindcast of weather charts. The Neumann sea was therefore derived for a 21-knot wind and the resulting motion spectrum obtained on this basis. The predicted pitch response curve for the 21-knot sea is also shown in Fig. 26(*a*). The agreement between these results and the measured ones is good both as to magnitudes and distribution. It might therefore appear that the wind force was not accurately estimated from the hindcast and that the 21-knot speed is more realistic in this case.

Fig. 26(*b*) shows the theoretical and measured results for pitch for ship B in head seas at a ship speed of 17 knots. The response amplitude operators were available from model tests on ship B for this speed. They are shown in the figure by the triangles. Using these and the Neumann spectrum for a 24-knot fully developed sea (the hindcast showed the sea to be fully developed at the time of this test) the predicted response was obtained. The comparison between the measured and derived results in this case is acceptable. The computed curve does not show a double peak and while its peak is slightly displaced, it is in closer agreement with the average period of encounter than the experimentally obtained peak. The total energy density is 28 per cent greater than that indicated by the experimental results, which would effect a 14 per cent difference in prediction of the associated statistical properties. However, one would be inclined to accept the theoretical curve as an indication of what was experimentally obtained.

The question arises as to why the 24-knot sea apparently produces good agreement with the experimental results at the 17-knot speed while considerable disagreement exists at the 12-knot speed. The actual time lapse between the two runs was approximately 1 hr so it is possible that a change in sea state could be responsible; however, it is not to be expected that a significant change occurred during this period. It is virtually impossible to pinpoint the exact cause of the discrepancy. Errors are obviously present in the determination of the sea, the response amplitude operators, as well as the experimentally measured values. However, the greatest source of error can be expected to exist in the determination of the sea: (*a*) The Neumann representation is a highly idealized sea and is characterized by the wind velocity. For the present analysis this was obtained by estimations from a hindcast of weather data. (*b*) The assumption was made in the analysis that the sea was unidirectional; therefore, those wave components in the system which were not coming from the dominant direction were incorrectly evaluated when the spectrum was expressed in a co-ordinate system moving with the ship traveling into the waves. However, if this were the source of error then it should have appeared in both cases, since both runs were made at the same relative headings, speed being the only difference.

The most likely major source of error would appear to lie in the estimation of the wind velocity. This can be seen from the great difference in the predicted results for the two wind forces ($V = 21$ and 24 knots) in Fig. 26(*a*). It is possible that a more accurate description of the sea would have been obtained if a 22-knot wind were assumed. This would result in an overprediction of the response shown in Fig. 26(*a*) and an underprediction of the results in Fig. 26(*b*).

At any rate, the present analysis indicates that meaningful results, by application of the principle of linear superposition of responses to the component waves in an irregular sea, can be obtained only if a more accurate description of the sea is obtained, both as to amplitudes and frequency distribution. Of possible importance also is the directionality of those wave components whose directions differ from the dominant direction of the system.

Stresses and Bending Moments Due to Waves

Discussion of E Values for Stresses

From the measured strains, the stresses in the different hull-structure members can be calculated. The strains and stresses are the result of both local loads and wave-induced bending moments. In order to measure the wave-induced bending moments and stresses, the gages were placed on the sheerstakes, which should be relatively free from local bending. However, the location of the gages was not the same for the three ships and the construction of the ships at the gage locations varied.

Another difficulty is the distinction between longitudinal stresses due to hogging and longitudinal stresses due to sagging. From the records

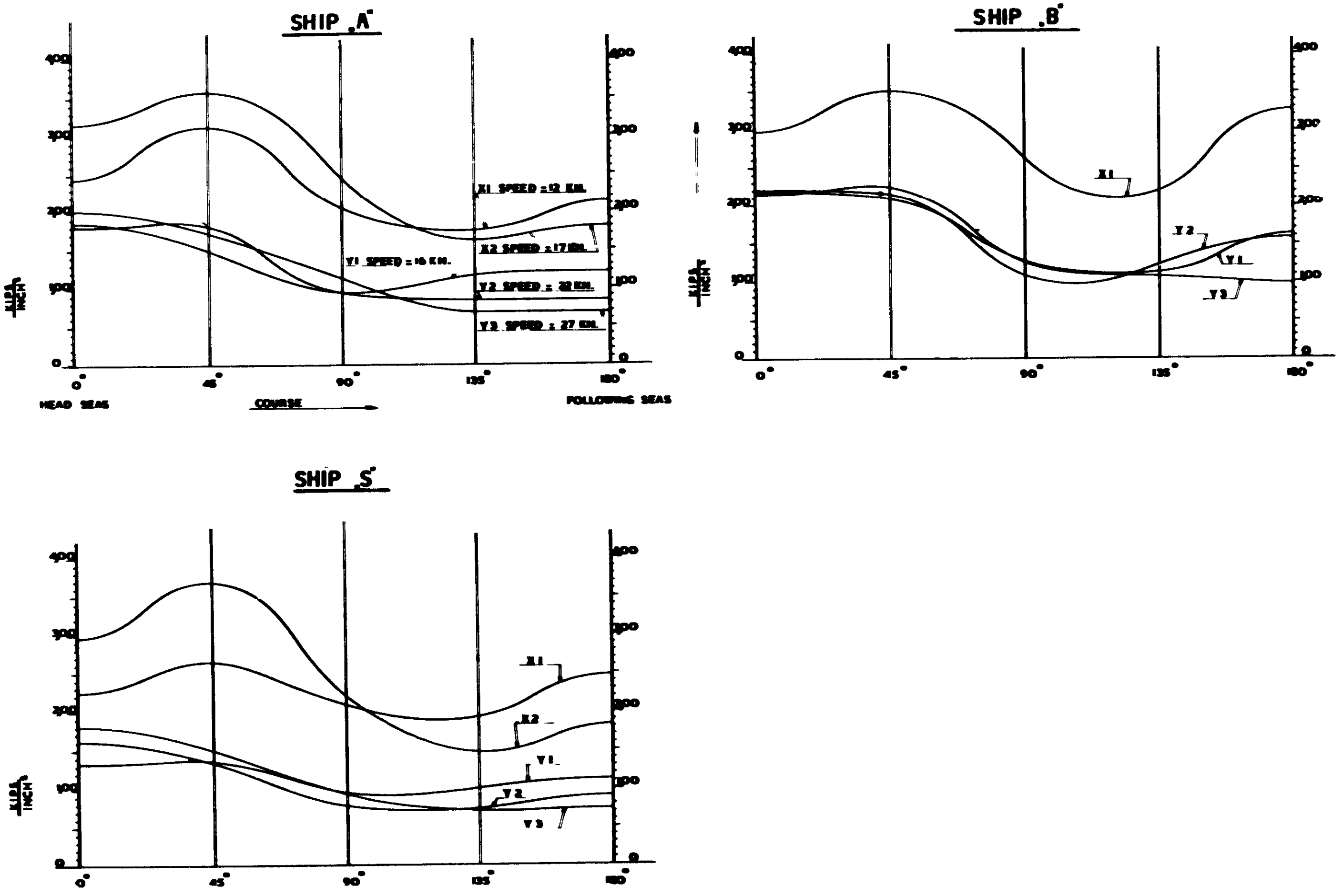


Fig. 27 Variation in $\sqrt{E}$ for longitudinal stress with respect to heading

Table 11 Tabulation of *E* Values for Longitudinal and Transverse Stresses

Run No.	Ship A		Ship B		Ship S	
	Long. stress, $\left(\frac{\text{kips}}{\text{in}^2}\right)^2$	Trans. stress, $\left(\frac{\text{kips}}{\text{in}^2}\right)^2$	Long. stress, $\left(\frac{\text{kips}}{\text{in}^2}\right)^2$	Trans. stress, $\left(\frac{\text{kips}}{\text{in}^2}\right)^2$	Long. stress, $\left(\frac{\text{kips}}{\text{in}^2}\right)^2$	Trans. stress, $\left(\frac{\text{kips}}{\text{in}^2}\right)^2$
Y1.1	3.45	0.096	4.81	0.015	1.77	0.053
Y1.2	2 22	0.039	4.40	0.092	1.84	0.047
Y1.3	0.88	0.027	1.60	0.051	0.88	0.046
Y1.4	1.33	0.049	1.25	0.048	1.00	0.056
Y1.5	1.45	0.045	2.65	0.039	1.26	0.033
Y2.1	3.26	0.050	4.94	0.057	2.68	0.065
Y2.2	3.28	0.062	4.68	0.070	1.79	0.047
Y2.3	0.86	0.027	1.16	0.052	0.61	0.035
Y2.4			1.50	0.048	0.53	0.028
Y2.5	0.70	0.023	2.46	0.118	0.82	0.051
Y3.1	4.02	0.065	4.71	0.078	3.35	0.085
Y3.2	2.96	0.042	5.05	0.071	2.52	0.059
Y3.3	1.26	0.036	1.54	0.064	0.84	0.028
Y3.4	0.46	0.026	1.12	0.037	0.50	0.039
Y3.5	0.46	0.030	0.94	0.053	0.53	0.076
X1.1	5.92	0.148	8.89	0.138	5.10	0.133
X1.2	9.73	0.124	12.39	0.327	7.04	0.237
X1.3	4.19	0.105	6.84	0.159	4.42	0.162
X1.4	3.04	0.068	4.79	0.126	3.70	0.161
X1.5	4.60	0.067	10.64	0.121	6.13	0.137
X2.1	9.87	0.121	—	—	8.77	0.204
X2.2	12.77	0.164	—	—	13.66	—
X2.3	5.99	0.085	—	—	4.76	0.116
X2.4	2.54	0.055	—	—	2.18	0.145
X2.5	3.25	0.084	—	—	3.34	0.187

it was not possible to trace the zero stress line. Therefore only stress variations were considered during the analysis; that is, the magnitudes of the variations of the stresses and bending moments. In order to obtain the total stress in the ships' girders for design purposes the maximum amplitudes of the wave-induced stresses should be added to the maximum stresses induced by the loading condition of the ship.

During several runs in the rough sea the records of the stresses showed an important additional vibratory stress caused by the slamming of the ship. Since slamming phenomena cannot be analyzed in a manner similar to that for the rigid body motions and stresses, the superimposed high-frequency components of the stresses were not taken into account.

From the records it soon became clear that the stresses due to the transverse bending moments could not be neglected. The transverse bending was in most cases the reason for the differences which existed between the readings of the starboard and port deck strain gages. Therefore, the mean value of the starboard and port strain-gage readings was taken as the characteristic value for the stress due to longitudinal bending of the hull. Half of the difference between the two readings was taken as a measure of the stress due to horizontal bending.

It was assumed that, since for both waves and motions the Rayleigh distribution provides a workable tool for approximating the distribution of the observed variables, this distribution would also be useful for defining the distribution of the stress double amplitudes. Therefore the calculation of E for stresses due to longitudinal bending was also done by the method described in Appendix 1 and $\sqrt{E}$ is plotted for each ship separately in Fig. 27 (Table 11 also).

The E for the transverse stresses were computed from the relation

$$E = \frac{1}{4N} \sum_{1}^{N} \left[\frac{X_{ip} - X_{is}}{2} \right]^2$$

where X_{ip} is the peak-to-peak value of the port strain-gage reading and X_{is} is the peak-to-peak value of the starboard strain-gage reading.

In this computation it was assumed that the stress variations in the port and starboard sheerstrakes are in phase, so that the differences of the peak-to-peak readings of the two topside gages give the peak values for stresses due to horizontal bending. This is not believed a too bold assumption since the records actually showed that the port and starboard variations were almost always in phase, Fig. 10(*b*).

The effect of torsional bending is also neglected since generally the torsional bending moment in waves is small [14]. It is recognized that even though the torsional moment is small, the induced stress may be large at points of discontinuities; however, in these trials the gages were located on continuous members so that these effects should be at a minimum.

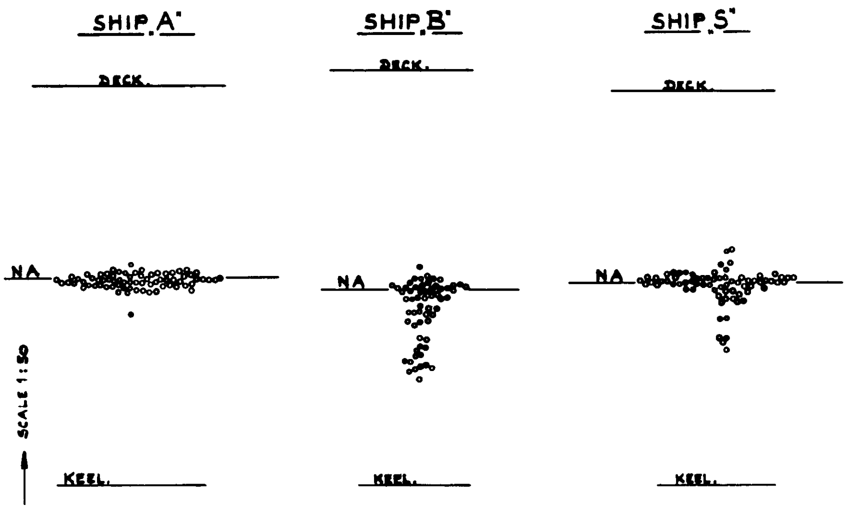


Fig. 28 'Instantaneous position of neutral axis

Table 12 Comparison of Stress Magnitudes and Section Moduli Ratios for the Three Ships

	X1.1	X1.2	X1.3	X1.4	X1.5	X2.1	X2.2	X2.3	X2.4	X2.5	Section moduli ratio
$\sqrt{E_A}/\sqrt{E_S}$.	1.08	1.18	0.98	0.91	0.87	1 06	0.97	1.13	1.07	0.98	1.41(S/A)
$\sqrt{E_B}/\sqrt{E_A}$.	1.23	1.13	1.27	1.26	1.52						1.20(A/B)
$\sqrt{E_B}/\sqrt{E_s}$.	1.32	1.33	1.24	1.14	1.32						1.69(S/B)

Calculation of Bending Moments

In order to make a nondimensional comparison, bending moments were computed for the three ships. The bending moments can be derived from the stresses by multiplying by the section modulus at the location of the strain gages. In the conversion of stresses to bending moments, the assumption is made that the ship's hull behaves like a beam with a linear stress distribution. This assumes that the position of the neutral axis remains constant during positive and negative bending. From simultaneous stress measurements at the deck and keel of the ship it is possible to check this assumption. From simultaneous readings taken at random from all available records, the instantaneous positions of the neutral axes were calculated and are shown in Fig. 28. This figure shows that for ship A the position of the neutral axis remains about the same for hogging and sagging conditions. Ships B and S have a wider scatter. Since ship S has a riveted hull it is possible that the position of the neutral axis is different in hogging and sagging conditions. In the calculation of the section modulus and vertical moments the mean value of the computed position of the neutral axis was used. As a result of this, the true hogging moment will be slightly greater than that computed from the stresses at the deck and the sagging moment will be somewhat smaller than the computed moment. It was thought that the scatter in ship B may have been due to the interference of local stresses and bending which disturbed the linear stress distribution. This of course would result in errors in converting the stress into bending moments. In order to determine the possible connection between the scattering and the measured stresses, the following analysis was performed: In the same seas, ships of the same geometric dimensions can be expected to have equal bending moments. Therefore, the ratio of the stresses measured on the ships may be assumed to be inversely proportional to the ratio of their respective section moduli. While in these trials the geometric construction of the ships was not identical, it was sufficiently similar to permit an approximate comparison in this respect. Since $\sqrt{E}$ is directly proportional to the stress magnitudes, the ratios of these values obtained in the rough sea were computed and compared with the respective section moduli ratios. The results are given in Table 12.

Part of the difference between the ratios which

Table 13

	Stress	Bending moment
Ship A	1356 kg/cm^2 (19.4 kpsi)	13570 ton-m (43800 ft-tons)
Ship B	1530 kg/cm^2 (21.9 kpsi)	12800 ton-m (41400 ft-tons)
Ship S	1134 kg/cm^2 (16.2 kpsi)	16200 ton-m (52400 ft-tons)

is apparent in Table 12 is probably attributable to differences in the ships' dimensions and possible differences in wave input.

With the exception of $\sqrt{E_A}/\sqrt{E_S}$ for runs X1.1, X2.1 and X2.2, those cases where discrepancies occurred of greater than about 20 per cent were associated with relatively small stress. In general, where stress variations were large the discrepancy was not greater than 20 per cent. This suggests that the stress pattern may be distorted either by local effects or measuring errors when the stresses are small but that when the stresses are large the effect is in general not significant.

The comparison in the table shows that ship B is no worse in this respect than the other two ships, even though she experienced a wider scatter in the position of the neutral axis. In summary, one might conclude that, for all three ships, when the stresses and moments due to vertical bending are relatively small, they may be somewhat influenced by local disturbances or measuring errors. However, when they are large, these effects are less significant and the measured stresses may be considered as a measure of the true bending stresses.

To compare the bending moments in dimensionless form the derived moments were divided by

$$\rho g h L^2 B$$

where

ρg = specific gravity of water
h = wave height
L = ship's length
B = ship's breadth

The wave heights are based on the hindcast of the sea condition. Significant wave heights of 6 ft for sea state Y and 12 ft for sea state X were arbitrarily taken as the wave heights which induced the significant bending moments, computed from

$$M_Y = 2.83\sqrt{E} \cdot \text{(section modulus)}$$

The variations of the bending-moment coefficients for the three ships for the different speeds and headings are plotted in Figs. 29 and 30. These figures can be directly compared to the results of the model tests [15] and the results of the theoretical calculation due to Swaan [16]. In evaluating the vertical bending moments of the three ships, the limitation mentioned earlier concerning local effects should be considered. From Fig. 27 it is clear that the stresses caused by longitudinal bending moments have their maximum values in bow and head seas. The magnitude of the wave-induced bending stresses is relatively high and the expected maximum double amplitude of stresses and corresponding vertical moments for the ships running during a 12-hr period at a speed of 12 knots in a state 5 to 6 sea would be as given in Table 13.

However, they do not exceed the calculated $L/20$ static value of the wave bending moments, as might be expected from the model work of Lewis [17]. Fig. 27 shows that in moderate sea there is no marked increase in stresses with change in speed, while in the rough sea the stresses apparently increased about 15 to 38 per cent when the speed was increased from 12 to 17 knots. In both sea conditions the stresses in head seas are the smallest for low speeds while in following seas the stresses are smallest for the high speeds. This trend suggests that the stresses were more influenced by the period of encounter than by speed. The nondimensionalized plots in Figs. 29 and 30 show that the magnitude of the bending moment in both rough and moderate seas is almost entirely independent of speed. This indicates that the increase in stress with speed in the rough sea, Fig. 27, may be as much due to some change in wave height as to change in speed.

The horizontal bending moments are approximately the same for all headings. For head and bow seas the bending-moment coefficients for horizontal bending are about 16 per cent of those for vertical bending. In other headings this percentage increases to about 20 per cent.

Slamming

Discussion of Pressures and Stresses Due to Slamming

The three ships were instrumented with pressure gages at various points in the forebody to record the slamming pressures experienced during the trials. The types and characteristics of these gages are given in the section "Description of Trials." Their locations are given in Tables 14, 15 and 16 for ships A, B, and S, respectively.

No slamming was observed in the moderate or Y-sea condition (state 3–4) for any heading or

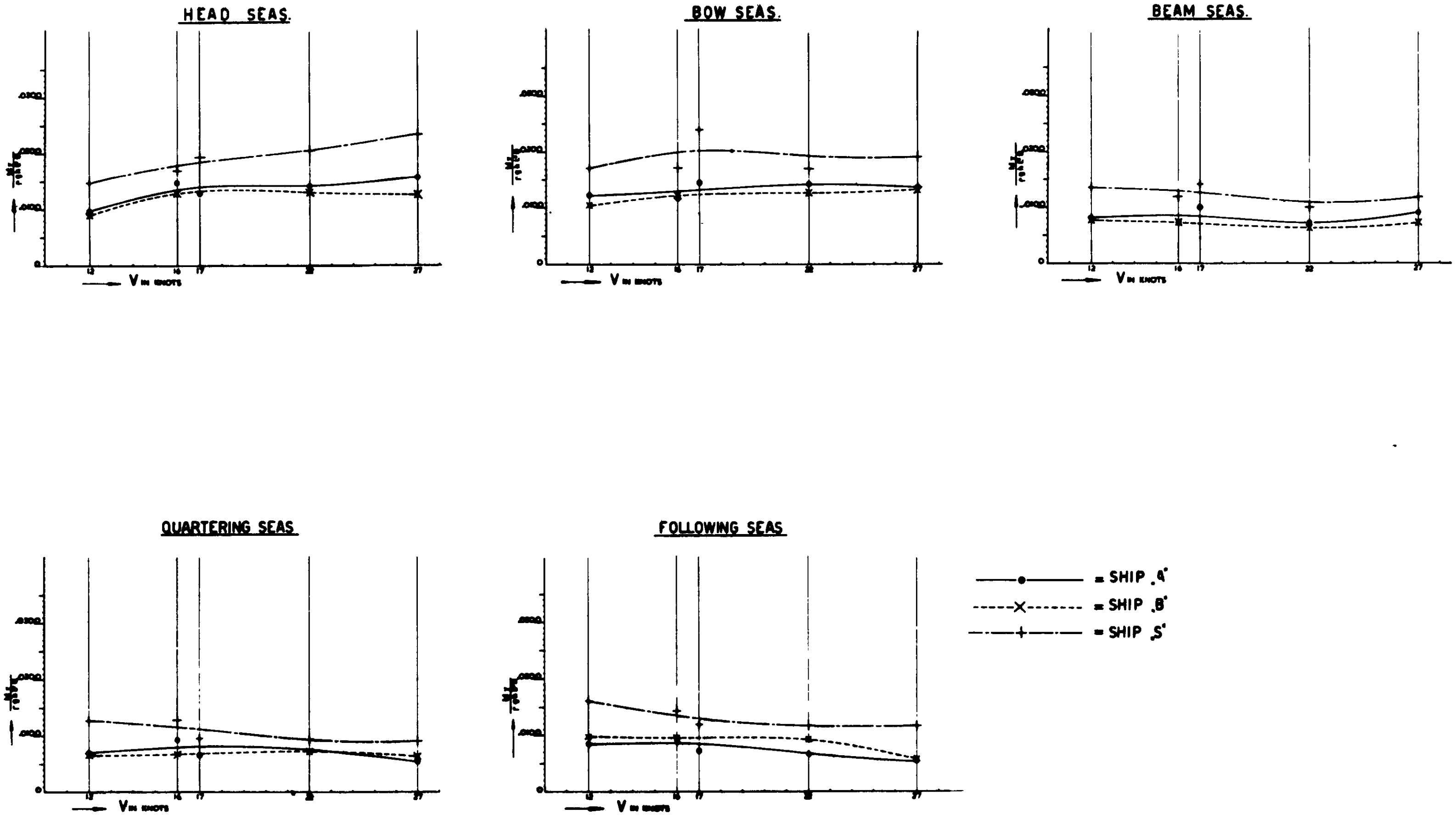


Fig. 29 Variation in vertical midships bending moments

Table 14 Pressure-Gage Locations in Ship A

Pressure gage no. 1....	6.02 per cent aft FP, port side of keel
Pressure gage no. 2...	13.63 per cent aft FP, port side of keel
Pressure gage no. 3..	18 23 per cent aft of FP, port side of keel
Pressure gage no. 4..	21.85 per cent aft FP, port side of keel

Table 15 Pressure-Gage Locations in Ship B

Pressure gage no. 5..	5.35 per cent aft FP, 0.7 m above keel, starboard
Pressure gage no. 1..	7.80 per cent aft FP, 0.45 m outboard, port
Pressure gage no. 2..	8.10 per cent aft FP, 0.45 m outboard, starboard
Pressure gage no. 3..	10.9 per cent aft FP, 0.45 m outboard, starboard
Pressure gage no. 4.	19.2 per cent aft FP, 0.65 m outboard, port
Pressure gage no. 6	23.4 per cent aft FP, 0.75 m outboard, starboard
Pressure gage no. 7..	25.6 per cent aft FP, 0.72 m outboard, starboard
Pressure gage no. 8..	.26.2 per cent aft FP, 2.84 m outboard, starboard

Table 16 Pressure-Gage Locations in Ship S

Pressure gage no. 1...	9.56 per cent aft FP, 1.0 m outboard
Pressure gage no. 2...	34.3 per cent aft FP, 1.0 m outboard
Pressure gage no. 3...	36.5 per cent aft FP, 1.0 m outboard
Pressure gage no. 4...	36.5 per cent aft FP, 2.1 m outboard

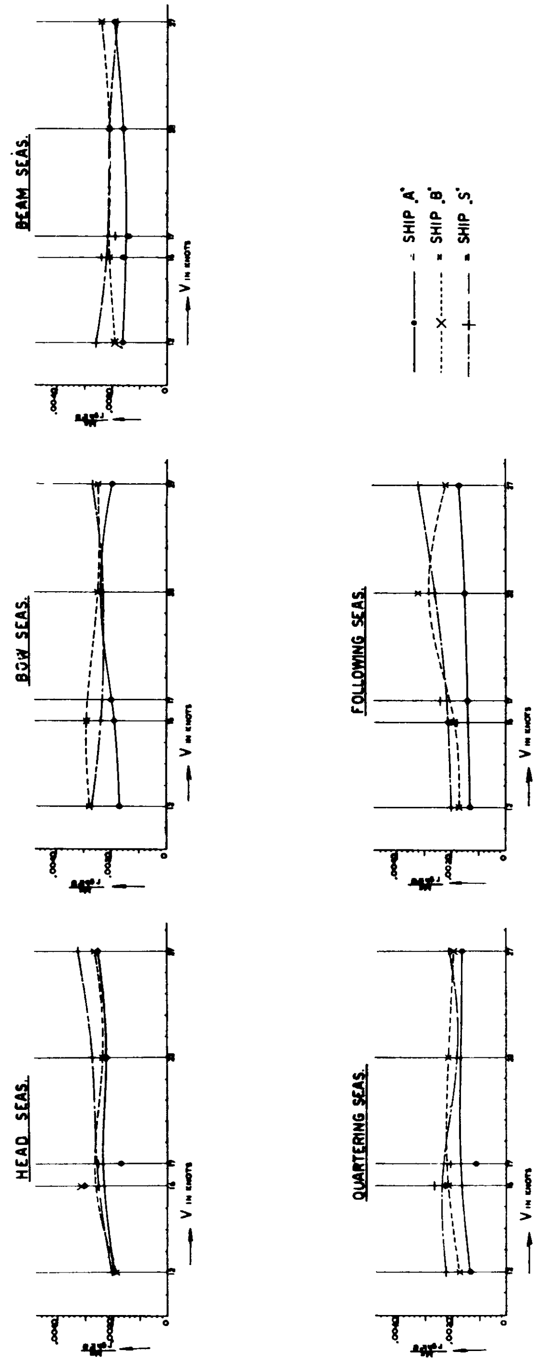


Fig. 30 Variation in horizontal midships bending moments

speed. In the rough or X-sea condition (state 5–6) slamming was observed at both speeds for which the tests were made (V = 12 and 17 knots).

It might be well to mention here the basis on which the presence of a slam was determined in the data analysis. When a ship bow suffers a heavy blow there is usually associated with it large impact pressures, sudden vertical deceleration, and superimposed vibratory stresses in the hull. Any one of these may be used as a measure of the severity of a slam; however, the impulsive stress produced by the impact appears to provide the most reliable criterion for determining the occurrence of a slam. These impulsive stresses take the form of added high-frequency stresses superimposed on the basic stresses imposed by the wave action and ship motions. For purposes of the analysis, the presence of these superimposed stresses in the data recorded at the sheerstrake was considered indicative of the occurrence of a slam. The total number of slams evaluated on the foregoing basis was counted for each ship and these are recorded in Table 17 for the various test conditions. Since the duration of each test run was approximately 10 min the slamming rate can be easily determined from the data of this table.

The extent and frequency of bow emergence associated with the slams may be determined by noting which gages came out of the water and the frequency with which they emerged. These statistics are tabulated in Tables 18 and 19 for ship speeds of 12 and 17 knots, respectively. It is

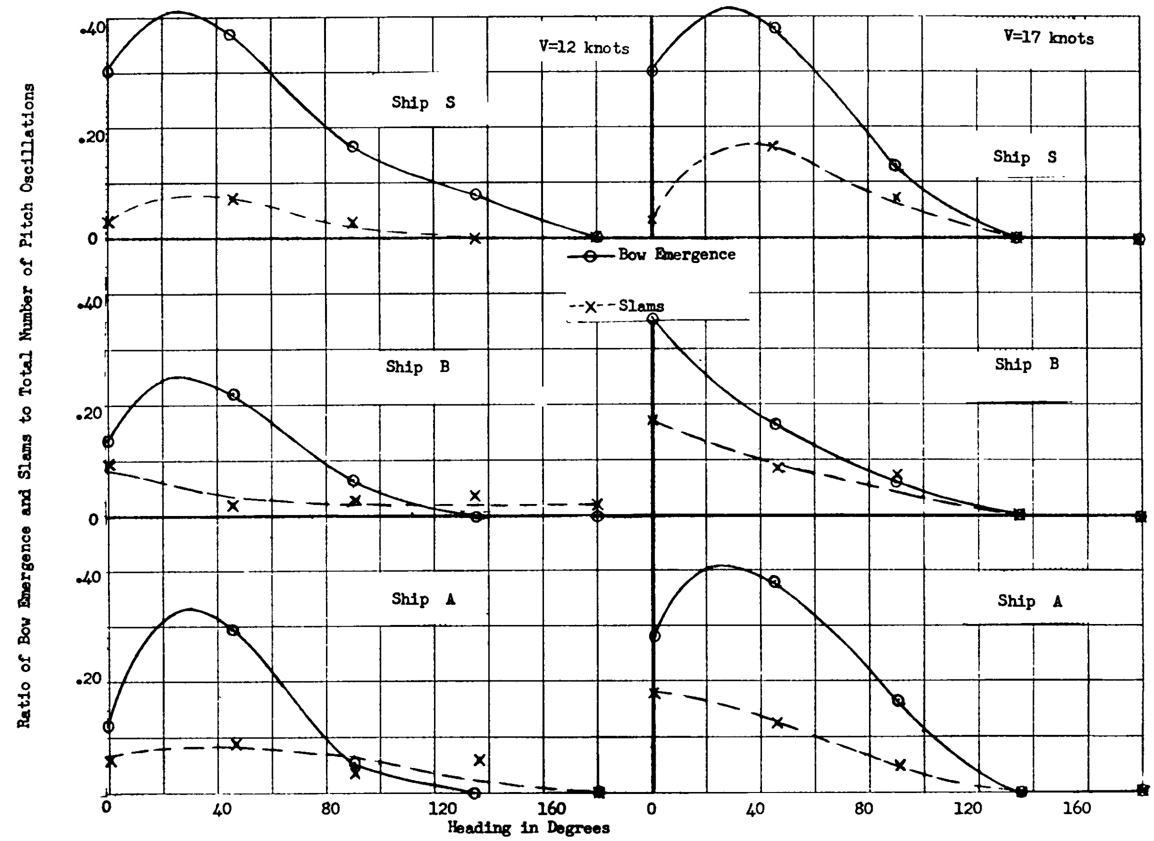


Fig. 31 Ratio of bow emergence and slams to total number of pitch oscillations, as a function of heading

Table 17 Number of Slams per Run per Ship in Rough or X-Sea State

		Ship A		Ship B		Ship S	
Speed	Heading	Slams	Cycles of encounter	Slams	Cycles of encounter	Slams	Cycles of encounter
12 knots....	Head	7	120	9	95	3	97
12 knots....	Bow	10	110	2	105	7	97
12 knots....	Beam	4	105	2	92	2	89
12 knots....	Quartering	6	93	3	87	0	75
12 knots....	Following	0	60	1	47	0	62
17 knots....	Head	19	102	19	112	3	100
17 knots....	Bow	14	112	9	112	16	100
17 knots....	Beam	5	103	7	97	7	100
17 knots....	Quartering	0	94	0	96	0	87
17 knots....	Following	0	71	0	70	0	62
Total.......		65	970	52	913	38	869

more meaningful, however, to relate these data to the total number of pitch oscillations which the ship experienced during the various test conditions. Using the data of Tables 18 and 19 for the foremost gages on each of the ships as indicative of bow emergence, the percentage of the total number of pitch oscillations for which the bow emerged was determined at each heading. These results are shown in Fig. 31. Shown also in the figure is the ratio of the number of slams in each test condition to the total number of pitch oscillations observed for that same condition. This figure shows the interesting result that usually bow emergence was more frequent in bow than head seas. Ship S slammed most frequently in bow seas also. However, for ships A and B, the high-

Table 18 Number of Times Each Gage Came out of Water in Rough Seas at Ship Speed of 12 Knots

Ship	Heading	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8
A....	Head	15	0	0	Inoperative				
B....	Head	3	0	0	0	13	0	0	0
S....	Head	30	2	0	0				
A....	Bow	33	7	2	Inoperative				
B....	Bow	7	—	4	0	23	0	0	0
S....	Bow	36	1	0	0				
A....	Beam	6	0	0	Inoperative				
B....	Beam	4	3	1	0	6	0	0	0
S....	Beam	13	1	0	0				
A....	Quartering	0	0	0	Inoperative				
B....	Quartering	0	0	0	0	0	0	0	0
S....	Quartering	6	0	0	0				
A...	Following	0	0	0	Inoperative				
B....	Following	0	0	0	0	0	0	0	0
S....	Following	1	0	0	0				
A....	TOTAL	54	7	2	Inoperative				
B. ..	TOTAL	14	8	5	0	42	0	0	0
S....	TOTAL	86	4	0	0				

Table 19 Number of Times Each Gage Came out of Water in Rough Seas at Ship Speed of 17 Knots

Ship	Heading	Gage 1	Gage 2	Gage 3	Gage 4	Gage 5	Gage 6	Gage 7	Gage 8
A.....	Head	29	11	4	Inoperative				
B.....	Head	23	19	14	2	39	0	0	0
S.....	Head	30	5	2	5				
A.....	Bow	43	16	8	Inoperative				
B.....	Bow	14	14	8	2	18	0	0	0
S.....	Bow	38	21	9	17				
A.....	Beam	17	3	2	Inoperative				
B.....	Beam	6	5	4	0	6	0	0	0
S.....	Beam	13	4	0	4				
A.....	Quartering	0	0	0	Inoperative				
B.....	Quartering	0	0	0	0	0	0	0	0
S.....	Quartering	0	0	0	0				
A.....	Following	0	0	0	Inoperative				
B.....	Following	0	0	0	0	0	0	0	0
S.....	Following	0	0	0	0				
A.....	TOTAL	89	30	14	Inoperative				
B.....	TOTAL	43	38	26	4	63	0	0	0
S.....	TOTAL	82	30	11	26				

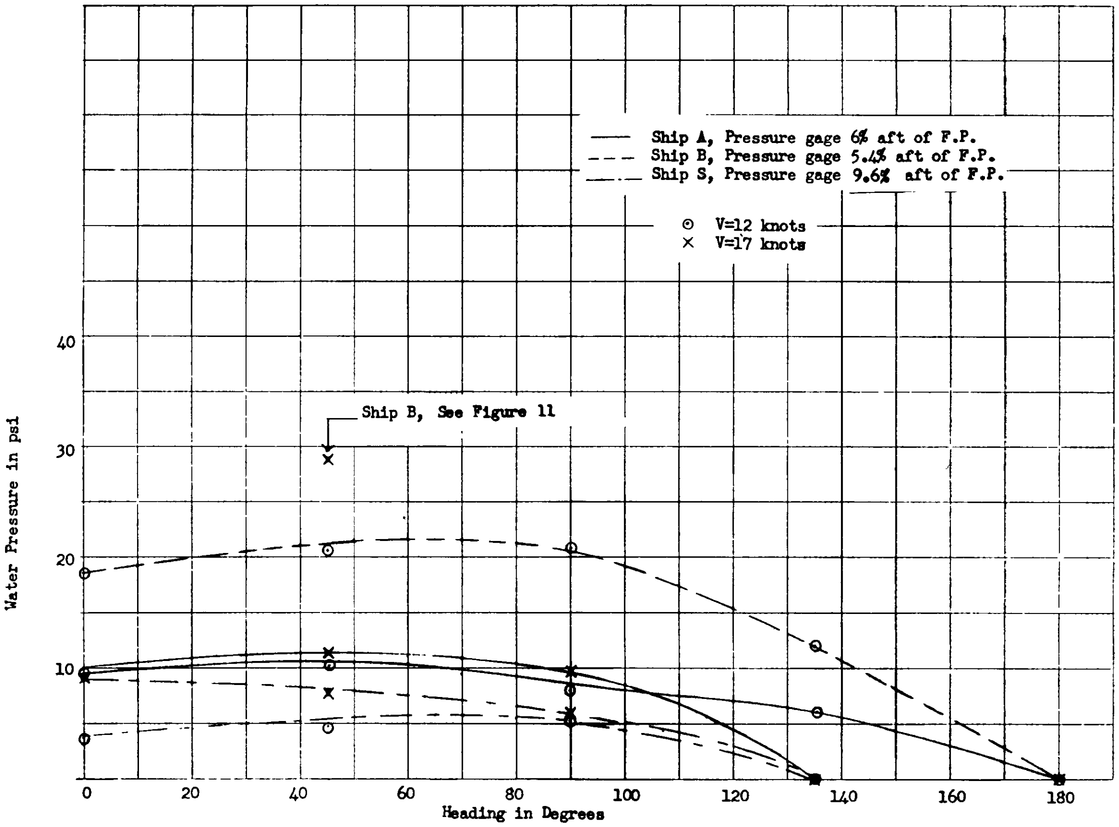


Fig. 32 Maximum water pressures occurring in X sea state at speeds of 12 and 17 knots

est rate of slamming generally occurred in head seas though bow emergence was more frequent in bow seas.

Pressures associated with these slams were recorded on a string oscillograph; however, the paper speed was not high enough to provide sufficient resolution for detecting the high-frequency impact pressures. Only the low-frequency water pressures produced by the waves and ship motions were obtained during this series of tests. The maximum water pressures measured on each ship during each slam were tabulated and their average is plotted in Fig. 32 for constant speed as a function of heading angle. These maximum pressures occurred at 6, 5.4 and 9.6 per cent of the length aft of the forward perpendicular for ships A, B, and S, respectively, this being the foremost location of the gages on each of the ships.

The original records containing water-pressure data for ship B at a speed of 17 knots were unfortunately not available for analysis, so to this extent the data of Fig. 32 are incomplete. However, Fig. 11 shows data for two slams which occurred at this speed in bow seas. The singular point in Fig. 32 for ship B at the 17-knot speed is the average of the maximum pressures recorded at 5.4 per cent of the ship length for the two slams shown in Fig. 11. The graph in Fig. 11 shows that this was the location of highest pressure with one exception. This exception occurred at 26 per cent of the ship length where a pressure of some 40 psi was observed during one slam.

Although Fig. 31 shows that the occurrence of slamming in beam seas is relatively small compared to that in head and bow seas, Fig. 32 shows that when slamming did occur, the magnitude of the associated water pressures is of the same order as that in head and bow seas.

Low pressures were observed in quartering seas and in following seas. Ship B apparently suffered the highest water pressures. All three ships showed an increase in pressure magnitude with an increase in speed at all headings where slamming occurred.

The maximum stress accompanying each slam (i.e., that stress superimposed on the primary stress due to wave action) was also determined

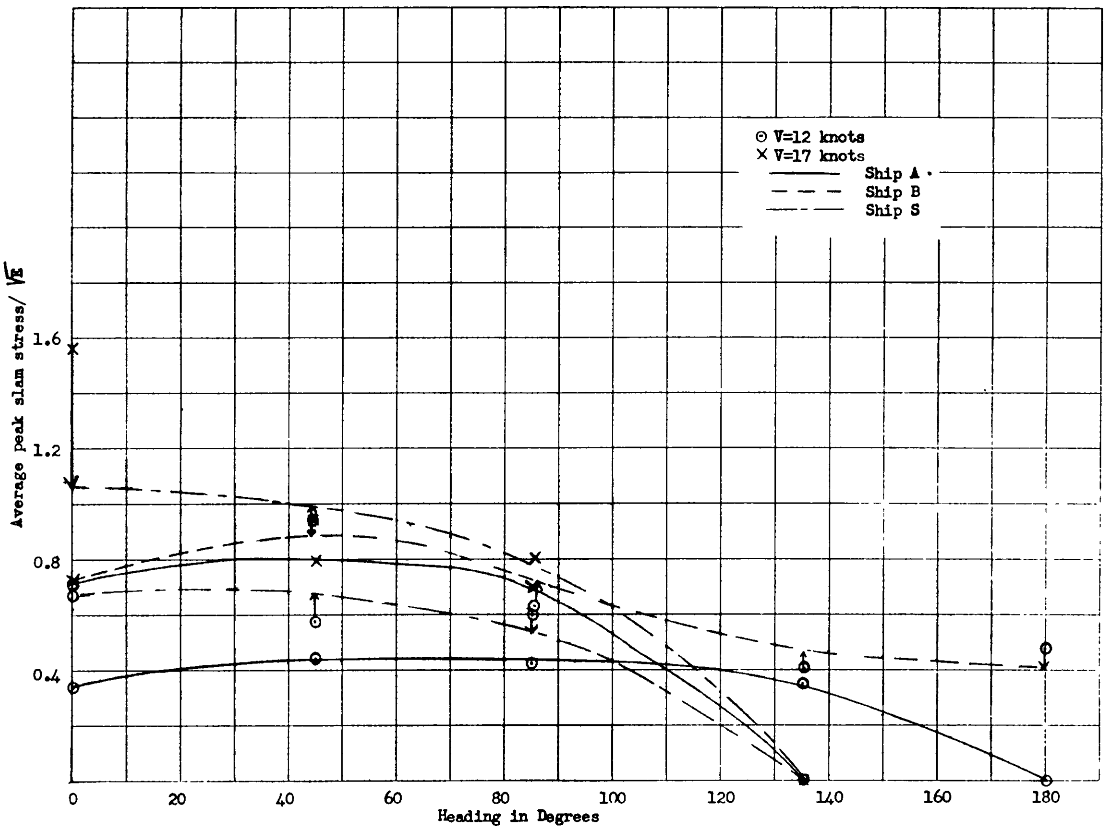


Fig. 33 Ratio of average peak slam stress to $\sqrt{E}$ in X sea state at speeds of 12 and 17 knots

from the records. These maximum stresses were averaged for each heading and the ratio of this average to $\sqrt{E}$ for the motion-induced stresses at that heading is plotted in Fig. 33. In this figure, as in Fig. 32, data for ship B at the 17-knot speed are not included. Since from a knowledge of the E-value such statistical properties as the average wave stress can be determined, this presentation permits an evaluation of the increase in the basic stress due to slamming.

Fig. 33 shows that for a particular ship at constant speed this ratio in head, bow and beam seas is of the same order of magnitude but decreases rapidly in quartering and following seas. For a particular ship this ratio increases with speed. This statement applies to ships A and S; sufficient data were not available for ship B to permit this comparison.

Discussion of "Special Slam" Tests

In addition to the slamming pressure and stress measurements made during the regular maneuvers in the heavy sea condition, special tests designed to study slamming were conducted. Motions, accelerations, strains and pressures were recorded during this special slamming run. The pressures were recorded at a high paper speed in order to provide better time resolution. The paper speed used was 1 fps, except for ship B where a paper speed of 5 ips was used to record the output of gages nos. 1, 2, 3 and 4. A sample of such a high-speed slam record is shown in Fig. 34. Because of the high paper speed, however, the oscillographs were run only intermittently. Therefore, in the analysis of the data it was not possible in general to relate a particular slam with the ship speed.

The pressures (both water and impact) experienced by each ship for all slams during this special test are presented statistically in Fig. 35. This figure shows the cumulative distribution of the magnitudes irrespective of ship speed and gage location. It can be seen that 50 per cent of the pressures were less than 10 psi for all three ships and 80 per cent were less than 25 psi. Only 5 per cent were greater than 50 psi. The maximum pressure for ship S was about 30 psi, for ship A 50 psi, and for ship B approximately 100 psi.

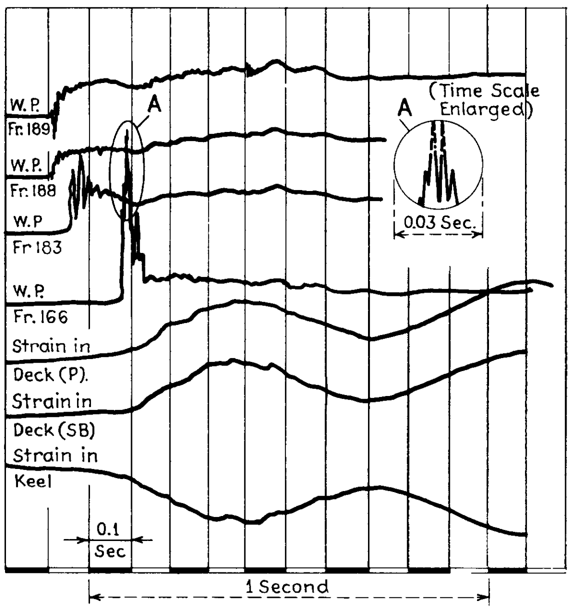


Fig. 34 Sample high-speed slam record

The heaviest slamming encountered during this specially conducted test was experienced by ship B at a speed of 25 knots when she encountered a succession of large waves coming from 10 deg to the bow (port). The impact pressures resulting from these slams have been plotted in Fig. 36 as a function of the distance from the forward perpendicular. The average duration of these slams (time for pressure rise to peak and return to zero) was 0.02 sec. The figure shows that the pressure magnitudes increase with this distance and reach a maximum somewhere aft of the quarter length. Model tests on destroyer-type vessels in regular waves indicate the region of maximum pressure to be around 20 per cent of the length aft of the forward perpendicular [13]. In irregular seas, however, the phase relations vary randomly and this in turn greatly affects the impact pressures. For instance, regular wave tests [13] would not have indicated a pressure as high as the 100 psi which was recorded during one slam at 0.26 *L* aft of the FP. In irregular seas, however, random-phase combinations can occur which produce very serious slams and high pressures.

Comparison of Estimated and Observed Pressures

An attempt was made to derive theoretically the pressures which ship B suffered during this last series of slams using the method of reference [18]. These slams which occurred at a ship speed of 25 knots and heading of 10 deg were the most severe of any encountered during the trials and were therefore chosen for the theoretical analysis. In such an analysis, a knowledge of the wave dimensions and phase relations between bow and wave in addition to the bow motion is necessary. Since no wave measurements were recorded during the trials and since no photographs of ship B were taken during these particular slams, some assumptions and theoretical estimates have been necessarily introduced in the analysis. These involve wave length, wave height, and phase relation between bow motion and wave. The wave length was estimated to be 590 ft. This was obtained from a knowledge of the encounter period (6.09 sec, taken from the pitching record), the ship speed (25 knots) and heading angle (10 deg). Wave height was estimated to be 22.1 ft. This was obtained from a knowledge of the total energy density derived from the sea spectrum discussed in the section, "Description of Trials," and application of the Longuet-Higgens' distribution function for the amplitude of the highest wave in *N*-oscillations shown in Fig. 37, which was derived from data in reference [19]. In this case the severe slamming occurred after 267 oscillations. No data were available which permitted estimation of the phase lag between bow motion and wave. Therefore, the value most critical for slamming was assumed, namely 180 deg. The amplitude of pitch and heave and phase lag between pitch and heave were taken from the records.

In addition, the analysis was also made for other wave heights $(h/\lambda = 1/30, 1/20, 1/17.5$ and $1/15)$ for this same wave length (590 ft). This first required estimation of the pitch and heave motions for these wave heights. For this purpose, the study made of the effect of wave height on motions of the *Mariner* [20] was used as a basis. These results are shown in nondimensional form in Fig. 38 by the solid line. They were obtained at design draft at the speed most critical for slamming $(F = 0.18)$ for the *Mariner* vessel. While the curves of Fig. 38 were obtained in waves of length equal to ship length, they are assumed applicable for any other wave length. Shown also in the figure are the nondimensional amplitudes of pitch and heave measured during the tests for ship B plotted for the wave 590 ft in length by 22.1 ft in height. Then the nondimensional motion values for other wave heights are obtained by drawing curves through these points so that a constant ratio between the destroyer curves and *Mariner* curves is maintained throughout. These are shown by the dashed line in the figure.

With these variables known or assumed the relative velocity between ship bow and waves was computed following the method in reference

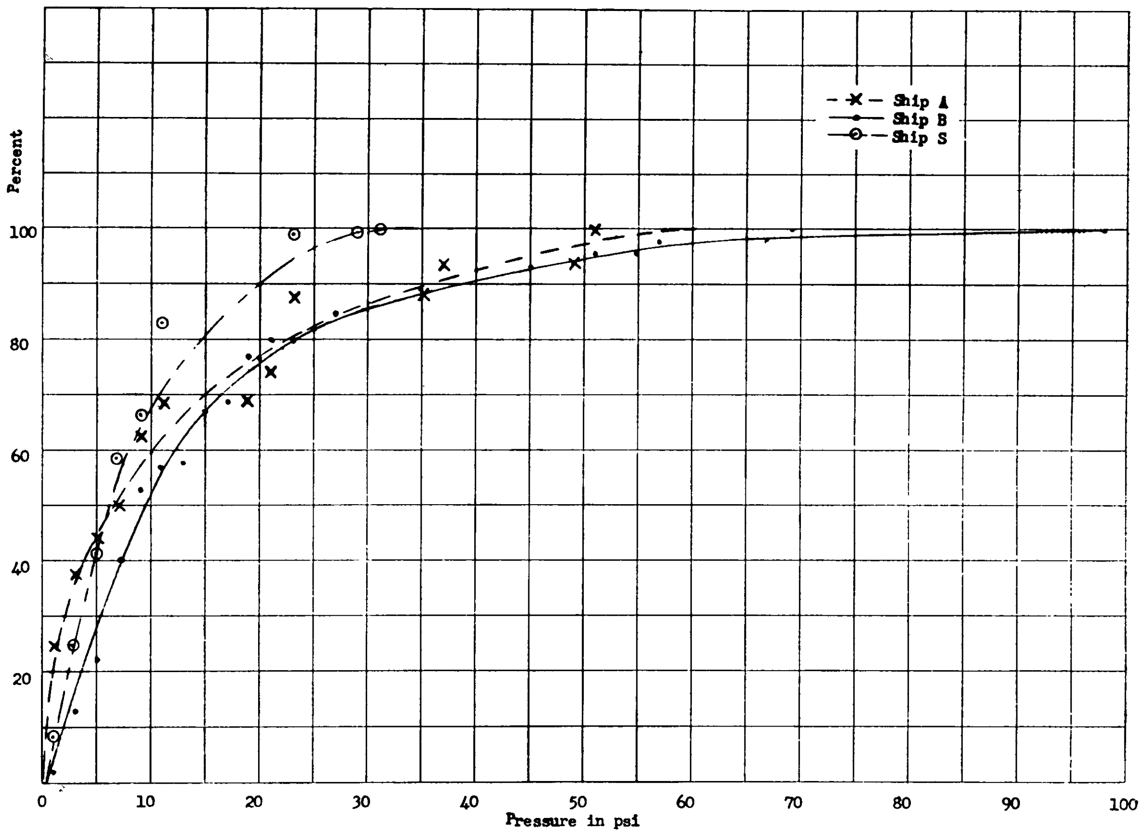


Fig. 35 Cumulative distribution function of water and slam pressures during special slam runs in X sea state

Table 20 Ship B: Theoretical Pressure at Pressure Gage No. 4 Location in Waves of λ/L = 1.6

Wave height, ft	19.7	22.1	29.5	33.7	39.4
Wave length λ, ft	590	590	590	590	590
λ/L	1.61	1.61	1.61	1.61	1.61
h/λ	1/30	1/26.7	1/20	1/17.5	1/15
Ship speed V, knots	25	25	25	25	25
Encounter frequency, ω_e	1.03	1.03	1.03	1.03	1.03
Wave frequency, ω_0	0 584	0.584	0.584	0.584	0.584
Pitch coefficient, ϕ_m/ϕ_ω	0.705	0.705	0.668	0.630	0.580
Wave slope, ϕ_ω, deg	6 00	6.70	8.95	10.23	11.97
Pitch amplitude, ϕ_m, deg	4.22	4.74	5.98	6.44	6.93
Heave coefficient $2z_m/h$	0.660	0.660	0.655	0.642	0.620
Wave amplitude, $h/2$, ft	9.85	11 05	14.75	16.85	19.70
Heave amplitude z_m ft	6.50	7.30	9.67	10.80	12.20
Phase lag between pitch and heave, deg	53	53	53	53	53
Amplitude of bow motion, ft	13.65	15.25	19.70	22.00	23.75
Phase lag between bow and wave, deg	180	180	180	180	180
Amplitude of relative velocity, fps	12.8	14.4	18.15	19.4	20.5
Pressure magnitude at no. 4 pressure gage, psi	11.6	14.7	23.2	26.5	29.4

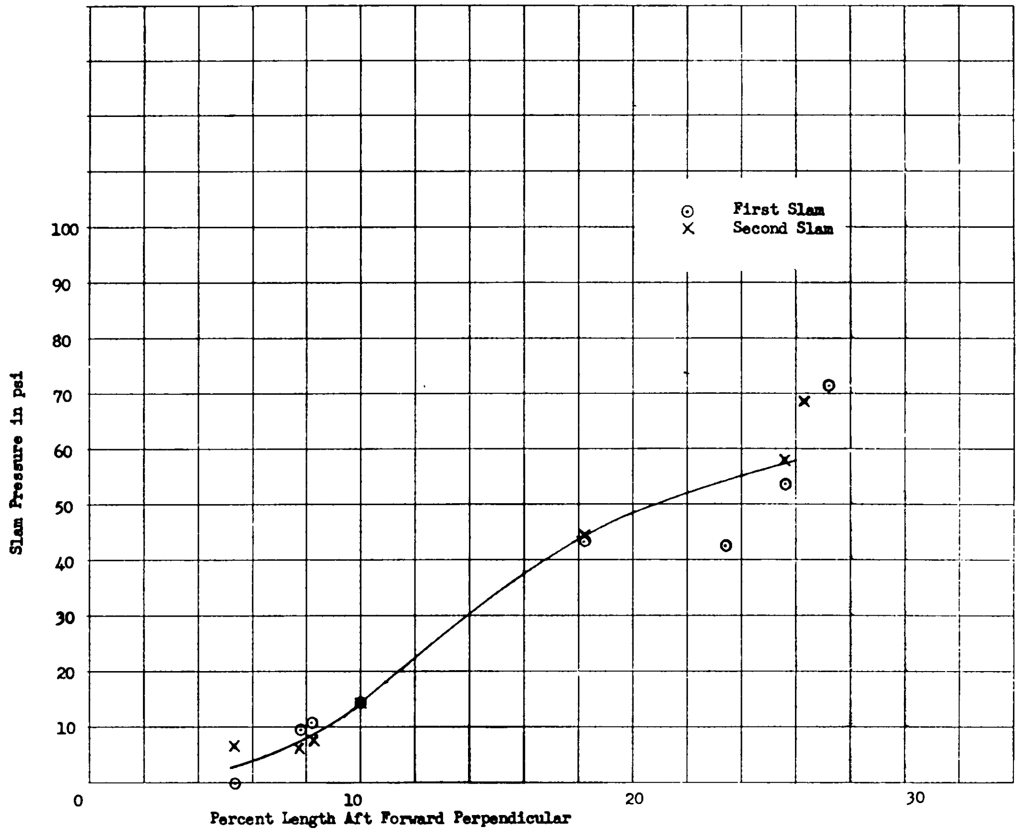


Fig. 36 Peak slam pressures on ship B recorded during two severe slams at a speed of 25 knots with waves at 10 deg to the bow (port)

Table 21 Ship B: Theoretical Pressure at Pressure Gage No. 4 Location in Waves of $\lambda/L = 1.0$

Wave height, ft	12.3	18.4	24.5
Wave length λ, ft	368	368	368
λ/L	1.00	1.00	1.00
h/λ	1/30	1/20	1/15
Ship speed, V, knots	25	25	25
Encounter frequency, ω_e	1.47	1.47	1.47
Wave frequency, ω_o	0.74	0.74	0.74
Pitch coefficient, ϕ_m/ϕ_ω	0.705	0.668	0.580
Wave slope, ϕ_ω, deg	6.00	8.95	11.97
Pitch amplitude, ϕ_m, deg	4.22	5.98	6.95
Heave coefficient, $2z_m/h$	0.660	0.655	0.620
Wave amplitude, $h/2$, ft	6.15	9.20	12.25
Heave amplitude, z_m, ft	4.06	6.02	7.59
Phase lag between pitch and heave, deg	53		
Amplitude of bow motion, ft	11.65	16.6	19.9
Phase lag between bow and wave, deg	180		
Amplitude of relative velocity, fps	14.8	21.4	26.2
Pressure magnitude at no. 4 pressure gage, psi	15.4	32.0	48.0

[21]. Since the instant of impact could not be determined due to shortage of data, the amplitude of relative velocity was used as the estimated magnitude of relative velocity in the calculations of the pressure magnitudes. Calculations of pressures were made for ship B at the location of gage no. 4 (0.45 m aft of frame 166 and 0.65 m outboard, starboard). The significant parameters used in this calculation and the computed pressures are listed in Table 20.

Calculations were also made for a wave of length equal to ship length (368 ft or $\lambda/L = 1$) for wave

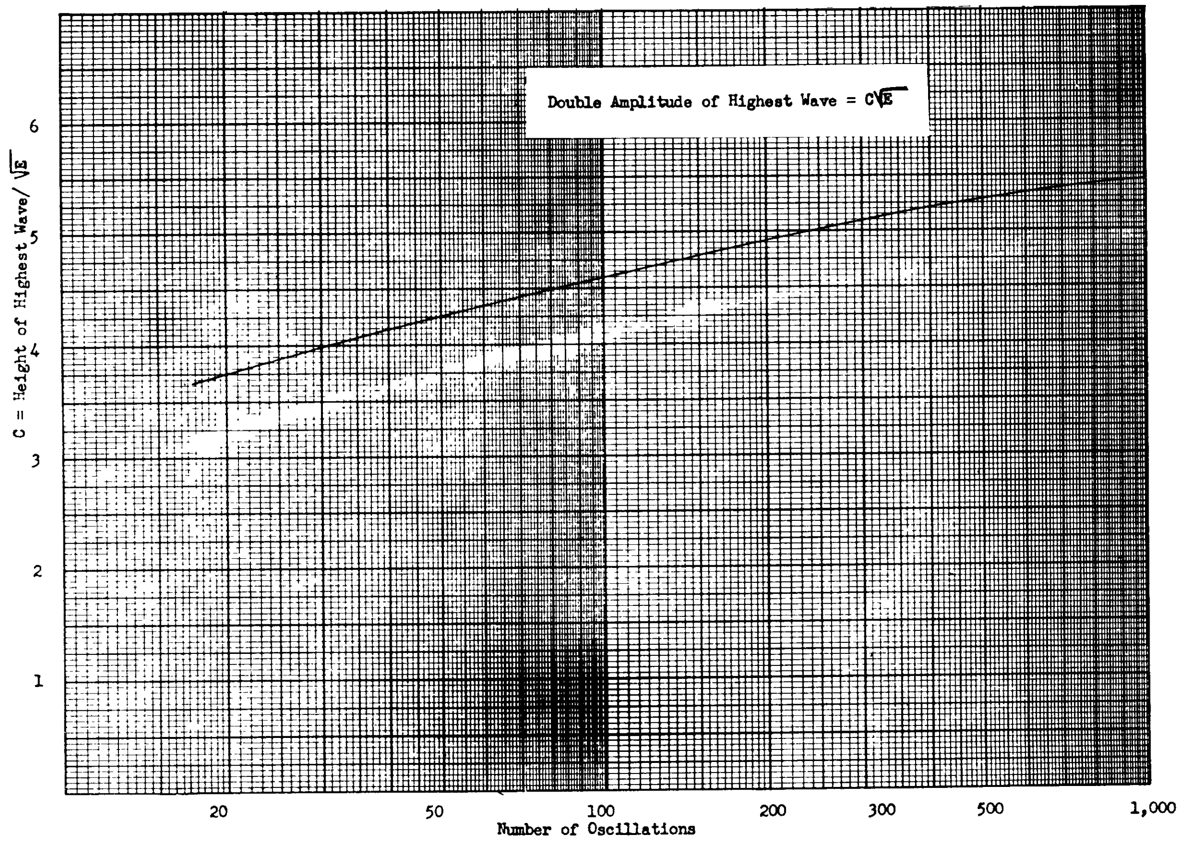


Fig. 37 Longuet-Higgins distribution for height of highest wave in N-oscillations

length-to-height ratios of 1/30, 1/20 and 1/15. The appropriate parameters and evaluated pressures for this condition are tabulated in Table 21.

The pressures are plotted in Fig. 39 as a function of wave height for constant wave length. The figure shows that the theoretical pressure magnitude in the 560-ft wave is less than the measured value, even for the very steep waves. It may therefore be concluded that the wave length based on the period of encounter was overestimated. On the other hand, for a $\lambda/L = 1$ wave, the measured pressure corresponds to a theoretical pressure computed for a wave whose height is $\lambda/16$.

Experimental studies in regular waves generally indicate that slamming is severe in waves of $\lambda/L = 1.0$ and h/λ of approximately 1/20. The value of 1/16 indicated by the present analysis is in close agreement with these findings.

The curve in Fig. 39 for $\lambda/L = 1$ also indicates that the minimum wave height for which slamming would occur is about 7.5 ft or $(h/\lambda)_{\min} = 1/50$. This result also agrees well with experimental results obtained in regular seas of wave length equal to ship length at the critical speed for slamming [22].

In addition to computing the pressure at the location of gage no. 4, the pressure distribution over the entire station was computed for the regular wave of $\lambda/L = 1.0$ and $h/\lambda = 1/20, 1/17$ and $1/15$. These results are shown in Fig. 40. Shown also is the pressure measured by gage No. 4. The order of magnitude of the theoretically computed pressure for this wave condition agrees well with the measured pressure. This would indicate that the theoretical procedure for estimating the maximum slamming pressure a vessel may encounter does give the proper order of magnitude.

3—Summary

The results of the destroyer seaworthiness trials may be summarized as follows:

1 Little difference exists between the three ships in pitching and heaving characteristics although ship S showed a slight tendency to pitch

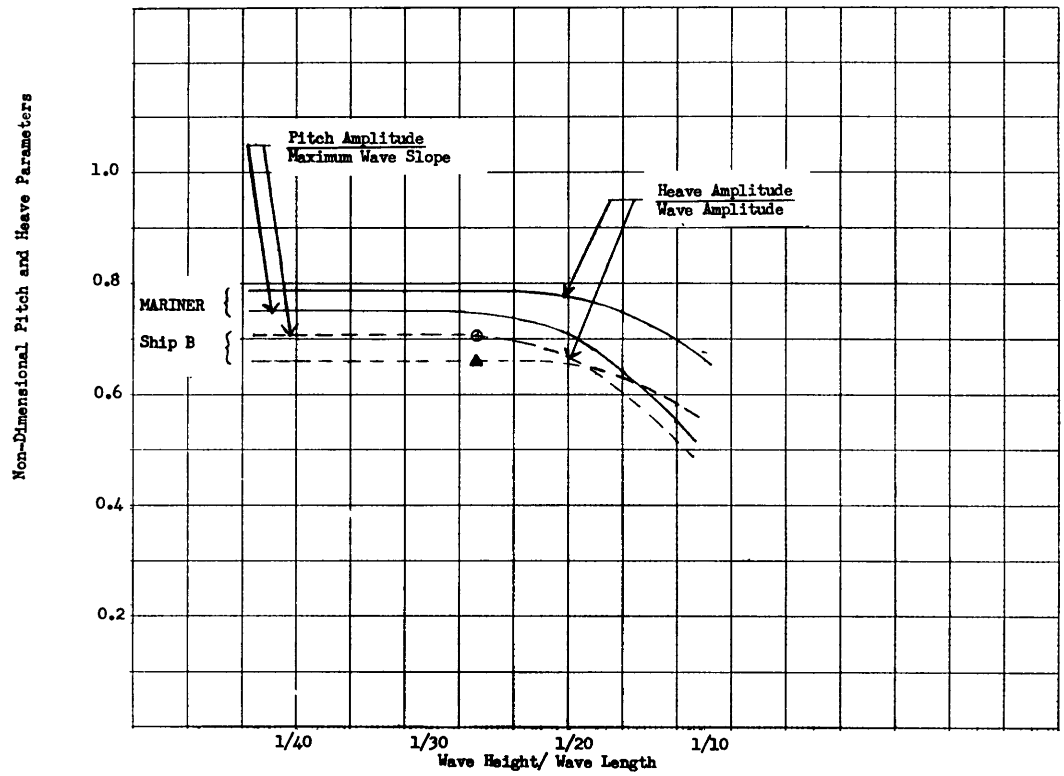


Fig. 38 Nondimensional pitch and heave parameters for ship B as a function of wave height; estimated from mariner model tests

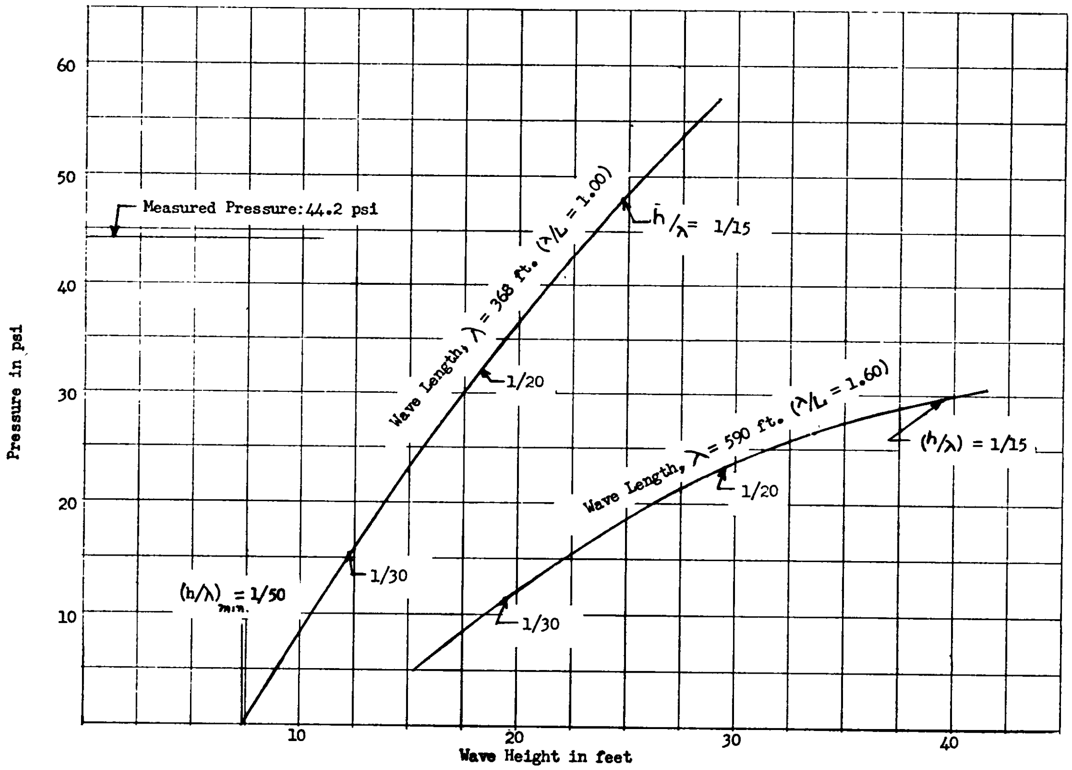


Fig. 39 Computed pressure at 19.2 per cent of the length aft of forward perpendicular and 0.65 meter outboard, ship B, speed 25 knots

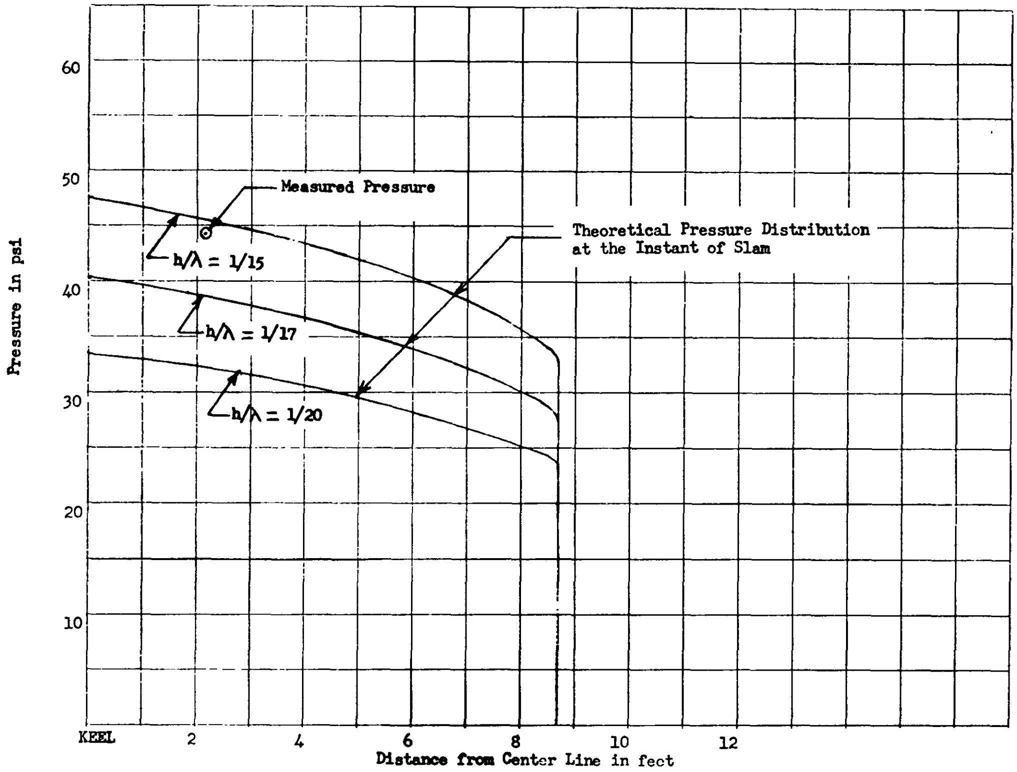


Fig. 40 Theoretical pressure distribution over station at 19.2 per cent aft of forward perpendicular at instant of slam, ship B. Wave length = ship length and wave height to length ratio = 1:20 1:17 and 1:15

more than ships A and B. The largest pitching and heaving occurred in head, bow and beam seas with minima in following seas.

2 Pitching was not influenced and heaving only slightly influenced by speed in the moderate sea. However, in the rough sea these motions increased with speed.

3 Maximum roll occurred in beam and quartering seas and in general could not be reduced by changing speed. The effect of going from moderate to rough seas was to increase rolling in some cases by as much as 100 per cent.

4 Ship A, which possessed the shortest natural rolling period, proved to be the heaviest roller while ship S was the best in this respect. Though rolling could be decreased by change in course, those headings which were favorable for roll, proved unfavorable for pitching.

5 Longitudinal stresses due to vertical bending were at a maximum in head to bow seas and generally minimum in beam to quartering seas. Though it appears that the effect of speed was more pronounced in the rough sea than in the mild sea, the larger stresses are believed due to the higher waves encountered during the high-speed runs. The highest stresses were observed at 17 knots in rough seas at a 45-deg heading. The significant double amplitudes of stresses for this condition were 10.1, 12.0 and 10.5 kpsi for ships A, B, and S, respectively. For comparison, the standard statical calculation for topside stress for the average warship is given by Biles [23] as 23.4 to 33.4 kpsi.

6 In both sea conditions the stresses in head seas are the smallest for low speeds, while in following seas they are smallest for high speeds, indicating a greater influence by period of encounter than by speed.

7 The effect of transverse bending on the longitudinal stresses cannot be neglected, especially in following and quartering seas. At these headings the horizontal bending moments increase the stresses due to vertical bending by about 20 per cent.

8 For ship A the average of the peak slam stresses obtained in the rough sea was 20 to 25 per cent of the wave-induced stresses due to vertical bending at 12 knots, and 40 to 45 per cent

at 17 knots. For ship S this ratio was 30 to 40 per cent at 12 knots and 50 to 90 per cent at 17 knots. For ship B the percentage was 25 to 50 per cent at 12 knots.

9 During the rough sea maneuvers ship A slammed most frequently and ship S the least. Bow emergence was most frequent in bow seas for all three ships. Though ship S slammed the least, her highest rate of slamming occurred in bow seas. Ship A and B, on the other hand, slammed most frequently in head seas. Though bow emergence occurred for 30 to 40 per cent of the pitch oscillations in head and bow seas, slamming occurred in less than 20 per cent of these oscillations.

10 Maximum water pressures associated with slamming (not high-frequency impact pressures) occurred on ship B. These were of the order of 20 to 30 psi at five percent of the length aft of the forward perpendicular. A pressure of 40 psi was observed during one slam, occurring at 26 per cent of the length aft of the perpendicular.

11 Though slamming occurred less frequently in beam seas than in head and bow, when it did occur the water pressures were of the same magnitude.

12 The maximum impact pressure measured during the specially conducted slam tests was 100 psi. This occurred on ship B at 0.26 of the length aft of the forward perpendicular. However, only 10 per cent of the slam pressures exceeded 40 psi for both ships A and B. The maximum impact pressure measured in ship S was 30 psi. The average duration of the slam pressures was 0.02 sec.

13 An analysis of the slamming pressure for ship B as a function of distance from the forward perpendicular shows that the pressure magnitudes increase with this distance and reach a maximum somewhere aft of the quarter length.

14 More generally, there is strong evidence that the double-amplitude oscillations do not always follow the Rayleigh distribution. In roll, the failure is attributable to nonlinear damping for small oscillation and nonlinear restoring force for large oscillation. In pitch and heave, the reason probably lies in the great breadth of the spectrum. The results indicate that the single-parameter Rayleigh distribution is just not adequate for all ships under all conditions.

15 In spite of the failure of the Rayleigh law for describing the entire distribution of double amplitudes, estimates of averages, extreme values, and other statistics are surprisingly good in most cases.

Acknowledgments

The authors wish to express their appreciation to the Flag Officer Material RNN, Rear Admiral W. J. Kruys, for permission to publish the data in this paper although responsibility for the statements contained herein rests with the authors.

In addition to the previously mentioned personnel who made fruition of this work possible, the authors would also like to give thanks to Dr. K. Ochi and Mr. V. Monacella of the David Taylor Model Basin; to Dr. Ochi for consultation in regard to the slamming analysis and to Mr. Monacella for the Chi-square analysis performed in connection with the statistical presentation of the motion data.

Since there were some six miles of records obtained during these trials, the task of data reduction was a formidable one. Without the assistance of the firm of M. Rosenblatt and Son of New York City in data reading and reduction, the work presented herein would have been even more laborious. The authors therefore wish to express their appreciation to the Rosenblatt firm for their effort in this respect.

References

1 W. H. Warnsinck, Discussion of "On Slamming" by V. G. Szebehely, Proceedings of the Seventh International Conference on Ship Hydrodynamics, 1954.

2 W. H. Warnsinck and V. G. Szebehely, "Joint Royal Netherlands Navy and United States Navy Seakeeping Trials," Mededelingen Van De Marine Staf, GS 110 (9/56).

3 W. H. Warnsinck and M. St. Denis, "Dutch Destroyer Trials," Proceedings of the Symposium on the Behavior of Ships in a Seaway, vol. 1 Wageningen, The Netherlands, 1957, pp. 439–467.

4 C. W. Prohaska, "Vibrations Verticales du Navire," *Bulletin de l'Association Technique Maritime et Aeronautique*, vol. 46, 1947, pp. 171–219.

5 M. St. Denis and W. J. Pierson, "On the Motions of Ships in Confused Seas," TRANS. SNAME, vol. 61, 1953, p. 280.

6 D. E. Cartwright, "On the Vertical Motions of a Ship in Sea Waves," Proceedings of the Symposium on the Behavior of Ships in a Seaway, vol. 1, Wageningen, The Netherlands, 1957, pp. 1–27.

7 D. E. Cartwright and L. J. Rydill, "The Rolling and Pitching of a Ship at Sea," *Transactions of the Institution of Naval Architects*, London, England, vol. 99, 1957, p. 100.

8 S. O. Rice, "Mathematical Analysis of Random Noise," *Bell System Technical Journal*,

vol. 23, July 1944, pp. 282–332; vol. 24, January, 1945, pp. 46–156.

9 N. H. Jasper, "Statistical Distribution Patterns of Ocean Waves and of Wave-Induced Ship Stresses and Motions, with Engineering Applications," TRANS. SNAME, vol. 64, 1956.

10 V. G. Szebehely, "Apparent Pitching Axis," *Forschungsheft für Schifftechnik*, February, 1956.

11 O. Bussemaker, Discussion of "On the Vertical Motions of a Ship in Sea Waves," Proceedings of the Symposium on the Behavior of Ships in a Seaway, vol. 2, Wageningen, The Netherlands, 1957, p. 791.

12 M. D. Bledsoe, "A Study of the Seaworthiness of the CA-68, CB-1 and DLG-9 Classes of Ships," DTMB Report 1163, in review.

13 Committee No. 34 of the Shipbuilding Research Association of Japan, "Experiments on a Destroyer Model Moving in Regular Waves," Translation by Experimental Towing Tank, Stevens Institute of Technology Note 496, September, 1958.

14 Admiralty Ship Welding Committee, "SS Ocean Vulcan Sea Trials," Reports No. R 9 and R 10, 1953.

15 G. Vossers, W. A. Swaan and H. Rijken, "Seakeeping Tests with Models of Three Destroyers," Publication No. 173 of the Netherlands Ship Model Basin, July, 1959.

16 W. A. Swaan, "Amidship Bending Moments for Ships in Waves," *International Shipbuilding Progress*, vol. 6, September, 1959.

17 E. V. Lewis and J. F. Dalzell, "Motion, Bending Moment and Shear Measurements on a Destroyer Model in Waves," Experimental Towing Tank, Stevens Institute of Technology, Report No. 656, April, 1958.

18 K. M. Ochi and M. D. Bledsoe, "A Theoretical Consideration of Impact Pressure at Ship Slamming," David Taylor Model Basin Report 1321, in review.

19 M. S. Longuet-Higgins, "On the Statistical Distribution of the Height of Sea Waves," *Journal of Marine Research*, vol. 11, no. 3, 1952, pp. 245–266.

20 K. M. Ochi, "Model Experiments on the Effect of a Bulbous Bow on Ship Slamming," David Taylor Model Basin Report 1360, in review.

21 V. G. Szebehely, "Ship Slamming in Head Seas," David Taylor Model Basin Report 913, February, 1955.

22 K. M. Ochi, "Model Experiments on Ship Strength and Slamming in Regular Waves," TRANS. SNAME, vol. 66, 1958.

23 J. H. Biles, "Design and Construction of Ships," vol. 1, Charles Griffin and Company, Ltd., London, England, 1908.

24 D. E. Cartwright and M. S. Longuet-Higgins, "The Statistical Distribution of the Maxima of a Random Function," *Proceedings of the Royal Society*, series A, vol. 237, 1956, pp. 212–232.

25 N. H. Jasper, "Service Stresses and Motions of the Esso Asheville, a T-2 Tanker, Including a Statistical Analysis of Experimental Data," David Taylor Model Basin Report 960, September 1955.

26. N. H. Jasper and R. L. Brooks, "Sea Tests of the USCGC Unimak, part 2—Statistical Presentation of the Motions, Hull Bending Moments and Slamming Pressures for Ships of the AVP Type," David Taylor Model Basin Report 977, April 1957.

27. N. H. Jasper, R. L. Brooks, and J. T. Birmingham, "Statistical Presentation of Motions and Hull Bending Moments of Essex Class Aircraft Carriers," David Taylor Model Basin Report 1251, February 1959.

28 Milton Martin, "Roll Damping Due to Bilge Keels," Iowa Institute of Hydraulic Research, University of Iowa, Contract Nonr 1611-(01), November, 1958.

APPENDIX 1

Distribution of Double Amplitudes

Theory

The theory for the distribution of the double-amplitude oscillation is quite deficient. Rice has discussed the case of a variable with a very narrow spectrum [8], where the signal has the form of a sinusoidal oscillation with slowly changing amplitude and frequency, the apparent frequency remaining near the center of the spectrum. There are well-defined upper and lower envelopes, which are symmetrical about the mean line. He found that r_{env} is distributed according to the Rayleigh law:

$$P(r_{\text{env}} < R) = P(R,E) = 1 - \exp(-R^2/E) \quad (13)$$

Pierson and St. Denis concluded that $r_{\max}$ should have the same distribution and surmised that estimates based on this distribution should still be reasonable when the spectrum is not narrow.

Cartwright and Longuet-Higgins examined the distribution of $r_{\max}$ for broad spectra, and found that it deviates from the Rayleigh distribution [24]. For very broad spectra it tends to the Gaussian distribution. While they did not develop the theory for the distribution of double amplitudes, they did compare observed distributions with the Rayleigh distribution for five records. The χ^2-test was applied, and in two of the five cases, corresponding to the broadest spectra, it was found that the probability of the sample distribution arising from a Rayleigh population was extremely low. In both cases, the histograms suggest that the number of observations falling in the lowest class interval is excessive, a point of possible significance in the later discussion.

Jasper has examined the distribution of double amplitudes from a large number of records taken aboard ships [9], [25], [26], [27]. He concluded, in all the cases examined, that the observed distributions could have come from a Rayleigh population:

$$P[(r_{\max} - r_{\min}) < R] = P(R,E_J) = 1 - \exp(-R^2/E_J) \quad (14)$$

He did not discuss the relation between E_J, which he estimated from the mean square of the double amplitude, and E.

For the narrow spectrum case, where $r_{\text{env}}(t)$ changes very slowly,

$$r_{\max} = -r_{\min}$$

very nearly. Therefore,

$$\begin{aligned} P[(r_{\max} - r_{\min}) < R] &= P(r_{\max} < R/2) \\ &= 1 - \exp(-R^2/4E) \\ &= P(R,4E) \quad (15) \end{aligned}$$

Thus, for the narrow spectrum case

$$E_J = 4E \quad (16)$$

By formal reasoning, this is as far as we can proceed. We can only hope, with St. Denis and Pierson, that for wide spectrum variables the distribution will not be significantly different. Noting the success that Jasper achieved, we conducted tests for a number of cases to see if the distributions were of the Rayleigh type. All those cases in which spectra were available were further investigated to see if the parameter was indeed $4E$. The results of these tests will be described.

Estimation of E

The selection of E as the basic parameter for presentation of all data concerning amplitudes introduced a serious difficulty. Since E is defined as the area under the spectrum, or equivalently, as twice the variance, it was clear that the effect of the precession of the gyros on the spectra must be removed, or else that an alternate method not affected by the presence of precession must be found. The foregoing hypothesis seemed to provide such an alternative; that is, if the double amplitudes do follow a Rayleigh distribution, and if the defining parameter is equal to $4E$, nearly, then E can be estimated from the experimental cumulative distribution functions. Further, a slow oscillation of the mean line should have little effect upon these functions, and hence upon E. A test was carried out for the 48 cases for which spectra were available, and the independent estimates of E compared.

The simplest parameter to estimate from a set of random observations is the mean, which we designate by h. Also, this quantity can ordinarily be estimated with a fairly high degree of statistical reliability. From the relations between the mean and the defining parameter for a Rayleigh distribution, we have the estimate:

$$E_h = \left(\frac{h}{1.77}\right)^2 \quad (17)$$

This procedure gives equal weight to all oscillations.

The region of greatest interest to the ship designer is that of the large oscillations. Therefore, an alternative estimate for E was developed which gave greater weight to this region. This estimate was developed from the following reasoning: From equation (15) we have

$$\frac{R^2}{4E} = -\ln[1 - P(R,4E)]$$

Therefore, if the cumulative distribution function for the Rayleigh distribution is plotted on inverted semi-log paper, with abcissa equal to R^2, it becomes a straight line passing through the origin, with slope proportional to $1/E$. The points for an empirical cdf, arising from observations of such a Rayleigh distribution, should fall very nearly along this straight line, and they can be used to estimate the slope.

This was done, taking the slope to be that of the line through the origin which resulted in the best least squares fit to the data. The resulting estimate for E is then

$$E_p = \frac{\Sigma t_i^2}{4\Sigma y_i t_i} \quad (18)$$

where

t_i = X_i^2
X_i = double amplitude corresponding to upper end of ith class interval
y_i = $-\ln[1 - F(X_i)]$

Table 22 Comparison of Estimates of E

Run	No. of observations	E spectrum	E_p	E_h	E_p/E spectrum
			Roll		
AY 2.1	186	3.22	2.94	2.68	0.91
AY 2.2	188	4.33	4.54	3.39	1.05
AY 2.3	147	17.60	18.62	15.63	1.06
AY 2.4	113	11.90	11.43	11.41	0.97
AY 2.5	108	8.78	9.17	7.02	1.04
BY 1.1	130	1.72	1.57	1.49	0.91
BY 1.3	140	7.17	6.29	5.35	0.88
BY 2.1	118	1.03	1.25	1.20*	1.21
BY 2.2	139	2.58	2.27	2.06	0.88
BY 2.3	115	10.36	10.57	8.81	1.02
BY 2.4	101	11.01	10.88	10.16	0.98
BY 2.5	106	5.04	4.81	3.12	0.96
BY 3.1	108	3.54	2.87	2.28	0.81
BY 3.3	99	8.00	7.45	8.37*	0.93
BX 1.1	124	6.43	5.99	5.07	0.93
BX 1.3	145	14.78	14.08	11.78	0.95
BX 2.1	149	9.78	8.42	7.39	0.87
BX 2.3	144	20.45	14.60	13.21	0.71
SY 2.1	126	1.51	1.43	1.17	0.95
SY 2.2	124	2.74	2.52	2.01	0.92
SY 2.3	118	9.55	9.64	8.85	1.01
SY 2.4	86	7.48	7.53	7.67	1.01
SY 2.5	93	6.65	3.91	3.73	0.59
			Pitch		
AY 2.1	209	0.66	0.68	0.65	1.03
AY 2.2	276	1.39	1.40	1.26	1.01
AY 2.3	199	0.70	0.63	0.67*	0.90
AY 2.4	174	0.66	0.57	0.56	0.86
AY 2.5	174	0.26	0.55	0.32	2.12
BY 1.1	216	0.92	0.85	1.02	0.92
BY 1.3	186	0.83	0.77	0.80*	0.93
BY 2.1	214	1.13	0.96	1.18*	0.85
BY 2.2	226	1.07	1.09	1.35	1.02
BY 2.3	181	0.60	0.68	0.74	1.13
BY 2.4	194	0.59	0.56	0.49	0.95
BY 2.5	170	0.52	0.41	0.28	0.79
BY 3.1	199	0.92	1.03	0.96*	1.12
BY 3.3	188	0.86	0.91	0.93	1.06
BX 1.1	190	1.86	1.77	1.79*	0.95
BX 1.3	184	1.51	1.59	1.62	1.05
BX 2.1	224	4.11	3.98	3.88	0.97
BX 2.3	193	3.64	3.88	3.43*	1.07
SY 2.1	188	1.54	1.45	1.61*	0.94
SY 2.2	189	1.66	1.40	1.43*	0.84
SY 2.3	160	0.94	0.84	0.83	0.89
SY 2.4	159	0.70	0.58	0.50	0.83
SY 2.5	149	0.32	0.42	0.29	1.31
			Heave Acceleration		
AX 2.1	199	0.0118	0.0118	0.0123	1.00
BY 3.1	199	0.0068	0.0059	0.0063*	0.88

$F(X_i)$ = value of empirical cdf corresponding to X_i

The basic idea behind this estimate was that, if there was any Rayleigh distribution which gave a reasonable estimate to the data, this method would produce it. Actually, as just mentioned, it gives much greater weight to the larger values of X_i, since these points have much greater leverage about the origin. This introduced a difficulty. The highest class intervals contain very few observations, so $1 - F(X_i)$ is subject to large absolute variation, and because of the great leverage of these points, this can mean a poorer estimate for E. In particular, the value of y_i at which $F(X_i)$ reaches 1 is infinite, and this must always occur for a finite number of observations. Therefore, the rule was arbitrarily adopted that if the total number of observations was not greater than 150, no value of X_i for which $F(X_i) > 0.98$ should be included in computing E_p. If the number of observations exceeded 150, values up to 0.985 were included.

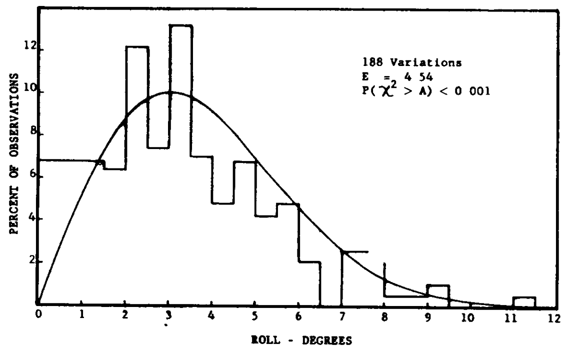
In Table 22 these two estimates for E are compared with the value obtained from the spectrum. If the E from the spectrum is assumed to be most reliable, it is clear that E_p is a much better estimate than E_h. In only those cases marked with an asterisk is E_h better, and in none of these is the difference significant.

The agreement in almost all cases is remarkably good. In only four cases is the agreement definitely bad (greater than 25 per cent). Three of these are for following seas, and the whole question of the following-sea records will be discussed later in detail. The fourth case is the heaviest rolling condition for which a spectrum was computed, and there is evidence that nonlinear effects are significant. For other runs, the difference is usually less than 10 per cent.

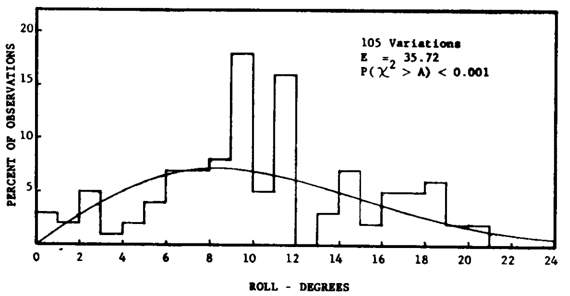
The last column suggests that there is a tendency for E_p to be less than the E obtained from the spectrum, since the average ratio is about 0.98. It could be expected that E_p would be off in this direction, because of the insensitivity of the distribution of the double amplitudes to the lower frequency components of the spectrum. In fact, it is this very effect which permits the removal of the influence of precession.

Most of the quantities in which we are interested are proportional to $\sqrt{E}$, and the error in this quantity will usually be less than 5 per cent. In only five cases will the error be greater than 10 per cent. Actually, it may be argued that E_p is better for estimating many of these quantities than E, since it was specifically selected to give a good fit to the data in the upper range. Thus, even when the empirical distribution deviates from the Rayleigh distribution, E_p still defines a distribution of this type which gives a reasonable fit in the region of greatest interest. In this report E_p is used throughout as the estimate for E, and for consistency, this includes even these 48 cases.

The selection of E_p to characterize the amplitude data was based on the assumption that the double-amplitude data would be approximately Rayleighian. This selection was necessarily made before the data were reduced to a form suitable for analysis. As will be seen in the next section, this



(a) Run AY 2 2



(b) Run BX 1.5

Fig. 41 Sample histograms and theoretical Rayleigh distributions for roll. (*a*) **Run AY 2.2.** (*b*) **Run BX 1:5**

basic assumption cannot be considered generally valid for the records obtained. Therefore, the justification for the use of E_p rests squarely upon the empirical evidence in Table 22.

The Chi-Square Test

As the reduction of the data proceeded and plots of the histograms and cumulative distribution functions became available, it became clear that all was not well with the Rayleigh assumption. Visual checks suggested that the data might be departing seriously from this working hypothesis. It was decided that tests should be performed to establish the validity (or lack of it) of the Rayleigh distribution.

A χ^2 test was carried out for all roll, pitch, and heave acceleration records which seemed to deviate from the Rayleigh distribution. This statistical test measures the probability that a certain parameter, χ^2, computed for a given record, will be exceeded by the value of χ^2 for a random sample chosen from a Rayleigh population having the estimated value of E_p. A low value of this probability is an indication that the Rayleigh hypothesis may be in error.

The sample was considered to have passed the test if $P(\chi^2 > A) > 0.05$ where A was the value of χ^2 from the sample. Of the 224 cases of roll, pitch, and heave acceleration, it would then be expected that about 11 would fail even if the samples were from a Rayleigh population. Actually 107 were found to fail. The chance that such a large number would fail is so small as to be completely negligible (probability less than 10^{-70}). Thus, it is a necessary conclusion that the samples of recorded data did not all come from populations which obey the Rayleigh distribution, or that the test is invalid.

There are a number of reasons why these samples might have failed the χ-square test. These include:

(*a*) A certain number of failures is to be expected for purely statistical reasons (about eleven in this collection).

(*b*) The record might be too short for the data to fit the proper theoretical distribution accurately, Rayleigh or otherwise. This could be expected for cases in which the number of oscillations is small, since there is a tendency for a sequence of large (or small) oscillations to occur in succession. There is a further effect upon the statistical test due to the correlation between successive oscillations.

(*c*) There might be a systematic error due to instrumentation (unfortunately, no checks could be made of this possibility).

(*d*) There might be a systematic error in reading.

(*e*) The samples might truly come from a non-Rayleigh population. This could happen because of the breadth of the spectrum or possibly because of some sort of nonlinearity in the phenomenon being recorded.

The histograms and other available data were examined to seek clues as to which of the foregoing possibilities might apply. It was realized, of course, that any particular failure might be due to more than one of these factors.

A number of cases showed no systematic deviation from the assumed distribution. The observations in various class intervals were large or small in an apparently random fashion. Such cases may be purely statistical failures, though an inadequate sample size could also be responsible. The records in Table 23 appeared to fail in this way.

Table 23

Roll	Pitch	Heave
AY 1.2	AY 2.4	SY 3.4
AY 1.3	SY 3.3	SX 1.3
BY 1.1	SX 2.1	SX 1.5
SY 1.2		SX 2.1
BX 1.1		
SY 1.3		

Roll. Roll is the motion with the narrowest spectrum, and hence would be expected to provide the best match with the Rayleigh distribution; 39 failures were found, a surprisingly large number, and of these only six seemed to be random. The remaining cases showed two sorts of systematic deviation.

The first and largest group had an excessive number of observations falling into the lower class intervals. In all of these cases, the major contributions to the χ^2-value for the sample came from these intervals. These records are given in Table 24. Of these 27 records, 19 failed at the 1 per cent level, and 11 failed at the 0.1 per cent level. A typical case is AY 2.2, in which 26 of the 188 observations were less than 1 deg, Fig. 41(a). If the Rayleigh distribution were to hold with the value of E_p estimated from the sample there would be only 11 observations expected in this interval.

Table 24

AY 2.1
AY 2.2
AY 2.5
AY 3.2
AY 3.5
AX 2.3
AX 2.4
BY 1.3
BY 2.2
BY 2.5
BY 3.1
BY 3.2
BY 3.5
BX 1.2
BX 1.3
BX 1.4
BX 2.1
BX 2 2
BX 2.3
SY 1.1
SY 2.1
SY 2.2
SY 3.1
SY 3.2
SY 3.3
SY 3.4
SY 3.5

A spot check of the records indicated that a systematic reading error was not the explanation, as the records were free from visible noise, and these small-amplitude oscillations were quite well defined.

The shortness of sample is not likely to be the answer, either, for the pattern is much too consistent. In fact, many of the runs which passed the test showed the same tendency.

The method of estimating E from E_p is admittedly one which tends to give an inaccurate fit in this region of small values since it gives greatest weight to the larger angles. However, a significant improvement would require a reduction of E_p of at least 50 per cent, and this is in direct conflict with the evidence of Table 22. Twelve of the foregoing runs are included there, and in none of them is E_p significantly greater than E from the spectrum.

The evidence is very strong then, that the Rayleigh distribution does not hold for these cases. The spectra for 3 runs, AY 2.5, BY 2.5, and BY 3.1, are not typical of roll spectra in that they have multiple peaks and are somewhat broader than usual, but the remaining 9 cases for which spectra are available do not appear at all unusual. Therefore, the explanation is not believed to lie in the shape of the spectrum.

It is suggested that the explanation might be a nonlinear damping condition at small angles of roll. Martin has carried out experiments which show that bilge keels are several times more effective in damping very small angular motions than large [28]. Thus, during periods of slight excitation, the oscillation would be much smaller, proportionately, than during periods of heavy excitation. This would cause just such a shift of the distribution.

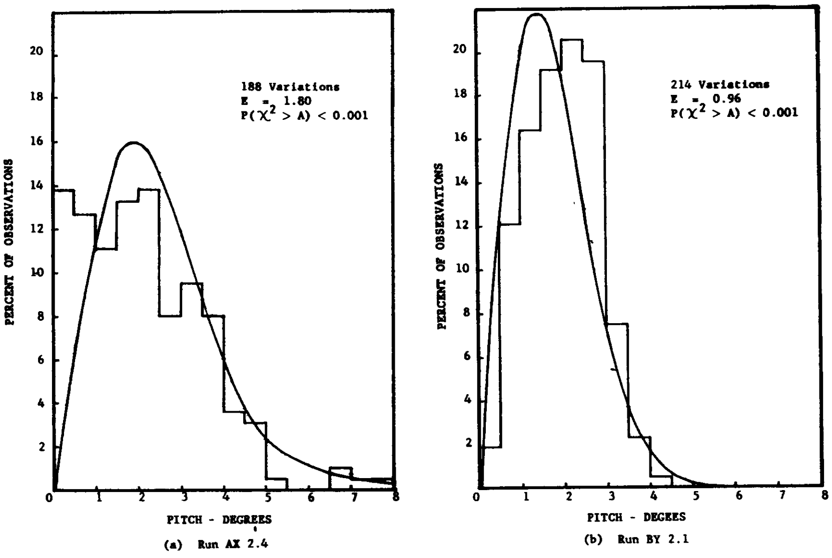
There remain 6 cases of roll in which the histograms show quite a different pattern. These are AX 1.4, AX 1.5, BX 1.5, BX 2.4, SY 1.3 and SX 2.5. In all these cases the number of observations falling in the middle and upper-class intervals are much greater than would be expected, Fig. 41(b). Even though the value of E_p is chosen to give a good fit in this range, no Rayleigh distribution provides an adequate fit. All but one of these cases occurred in the rough sea state, at headings in which the mean roll angle was large: beam, quartering, and following seas. While the other runs at these headings in the rough sea did not fail the test in this fashion, the histograms exhibited strong tendencies toward the same sort of behavior. Here again, the pattern is so consistent that shortness of record is not believed to be the answer. The explanation must lie in the nonlinear nature of roll to large angles. If the evidence collected during these trials is valid, roll to large angles deviates seriously from the Rayleigh distribution, in that large magnitudes are far more probable than would be predicted. Therefore, the use of the Rayleigh distribution is nonconservative, and possibly dangerous.

Pitch. Pitch is, in general, a broad spectrum response, and this might be expected to exhibit strong non-Rayleigh tendencies. This was found to be true though, surprisingly, no more so than roll. There were 30 failures out of 75 runs, three of which seemed to be for statistical reasons. Again, there were two well-defined classes of systematic deviation, corresponding roughly to the two classes of failure in roll, though there are important differencies in detail.

The first group included a preponderance of observations in the lower class intervals, with the number in each interval tending to decrease as the angle increases. Thus, at first glance, these failures seem to be similar to those in the first group in roll. However, the trend is even more strongly developed, Fig. 42(a). Table 25 gives the runs which were of this type. All of these records gave probabilities less than 1 per cent, and 7 gave probabilities less than 0.1 per cent.

Table 25

AY 1.5
AY 2.5
AY 3.1
AY 3.2
AY 3.5
AX 1.4
AX 2.4
AX 2.5
BY 2.5
BX 2.5
SY 1.3
SY 1.5
SY 2.5
SY 3.5
SX 1.5
SX 2.4
SX 2.5



(a) Run AX 2.4

(b) Run BY 2.1

Fig. 42 Sample histograms and theoretical Rayleigh distributions for pitch. (a) Run AX 2.4. (*b*) Run BY 2.1

Note that 14 of the 17 runs were in quartering or following seas, a condition which broadens the spectrum and amplifies the low-frequency components. Therefore, it is believed that these failures are primarily due to the breadth of the spectrum, and are similar to the failures observed by Cartwright and Longuet-Higgins.

It might be argued that the small number of oscillations which would be expected in a short following sea run would contribute to the statistical variability of the sample, and hence to large values of χ^2. This argument would have some weight, if the pattern of failure were not so consistent. Fifteen of the thirty runs in quartering and following seas failed the test, 14 falling into the group under discussion, and one showing no systematic deviation. Therefore, the short length of the sample cannot be a significant factor in this result.

Spectra are available for three of these records: AY 2.5, BY 2.5, and SY 2.5. It may be noted that these are the only cases for pitch in which E_p deviates by more than 20 per cent from the E obtained from the spectrum. It may be surmised, then, that E_p may be a fairly poor estimate of E for all the pitch records which fail in this fashion. It is also possible that E is inaccurate, because of the difficulty of removing the effects of precession from the spectrum when there is sea excitation in this frequency range. This is the case in following seas when the longer waves are overtaking the ship.

The second group of failures had a deficiency of observations in the lower class intervals and an excess in the class intervals just above the middle. The histograms frequently showed a fairly sharp cut-off at the upper end, rather than the long tail which is characteristic of the Rayleigh distribution. This latter tendency was accentuated by the excess in the upper middle range, Fig. 42(*b*). Table 26 gives the pitch records for this pattern of failure. Seven of these runs gave probabilities less than 1 per cent, and four gave probabilities less than 0.1 per cent.

Table 26

BY 2.1	SY 1.2
BY 2.1	SY 2.1
BY 2.2	SY 2.2
BY 2.3	SY 3.1
	SX 1.2
	SX 1.3

There is no evident reason for these failures. The pitching in some cases was heavy, but no more so than during many other runs which passed the test. The most evident characteristics are that most of the runs are in head and bow seas, and that none of the runs for ship A are included.

It may be noted that spectra were computed for 6 of the above runs, and the agreement between E_p and E is quite good, Table 22. The agreement between predicted and observed properties is usually good, Table 10. Thus, even though the Rayleigh distribution is not correct, the parameter E_p still provides an entirely suitable measure of the pitching properties for this class of failures.

Heave Acceleration. The heave acceleration records faired no better than roll or pitch. There were 37 failures, of which 28 failed at the 1 per

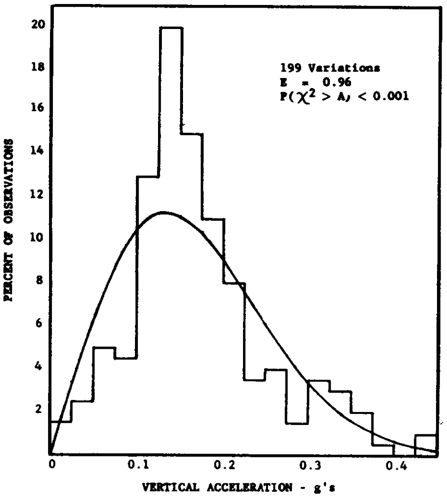


Fig. 43 Sample histogram and theoretical Rayleigh distribution for vertical acceleration—run BX 2.2

cent level, and 11 at the 0.1 per cent level. Four records had no clear pattern of failure, and for want of sufficient evidence are considered to have failed for statistical reasons. The remaining 33 records were minor variations from a very well-defined pattern. This was typically a deficiency of observations in the lower or upper class intervals, (usually both) and an excess of observations in the middle range. The histograms frequently resembled the symmetrical bell shape of the normal distribution more closely than the skew shape of the Rayleigh distribution. Fig. 43 shows an example. The failures are listed in Table 27.

Table 27

AY 1.2	AX 1.1	BY 1.1	BX 1.2	SY 1.4	SX 1.2
AY 2.4	AX 1.2	BY 1.5	BX 1.4	SY 1.5	SX 2.5
AY 2.5	AX 1.4	BY 2.1	BX 1.5	SY 2.1	
AY 3.1	AX 1.5	BY 2.2	BX 2.2	SY 2.4	
AY 3.2	AX 2.1	BY 2.4		SY 2.5	
AY 3.3	AX 2.2	BY 3.5			
AY 3.5	AX 2.3				
	AX 2.4				
	AX 2.5				

The reason for this rather remarkable behavior is not known. The deficiency in the lower class intervals is quite strange. The higher frequencies are emphasized in an acceleration spectrum much more than in a motion spectrum, and this may in some way be responsible. The whole question is open, and perhaps deserves further study.

Summary. The foregoing discussion is hardly very strong support for the theory that double amplitudes follow the Rayleigh distribution. In fact, it is clear that this single-parameter distribution is just not adequate for all variables for all conditions.

This conclusion is based not so much on the large number of χ^2 failures as upon the consistent patterns which examination of the histograms reveals. The χ^2 test has provided a tool for identifying questionable runs. The argument then, is to some extent subjective. However, this does not weaken its force to any significant degree.

This partial repudiation of the Rayleigh distribution might be assumed to invalidate the use in this paper of E_p for characterizing amplitudes of response. This, however, is not the case. Table 22 includes many runs which failed the χ^2 test, yet in all but the pitch records in following seas and the one case of very heavy rolling, E_p is a good estimate of E. E_p was chosen to provide a good fit to the frequency functions in the range of greatest interest, and the fit here is generally good, even when discrepancies are greater at small values of the variable. Therefore, it is considered that E_p provides a suitable measure of the amplitudes, including the cases in which the data is definitely non-Rayleigh.

This conclusion is further supported by the successful use of E_p for estimating extreme values of double amplitude oscillations to be expected during a run, Table 10. The general agreement between estimated and observed values is one of the strongest arguments for the validity of equation (18) for estimating E. However, caution is advisable in using E_p to estimate probable extreme values of roll under heavy rolling conditions, as the nonlinearities of the motion apparently become serious.

Appendix 2

The location of the accelerometers in the ships is described in the first section of the paper. The exact positons are as follows:

Ship A...... Forward accelerometer: 50.52 m fwd of CG
Center accelerometer: 1.18 m fwd of CG
Ship B...... Forward accelerometer: 52.45 m fwd of CG
Center accelerometer: 2.00 m fwd of CG
Ship S....... Forward accelerometer: 49.91 m fwd of CG
Center accelerometer: 2.49 m fwd of CG

Since the accelerometers were fixed to the ships, the records also show components due to gravity acceleration. The linear displacements on the record, S_1 and S_2, are equal to

$$S_1 = \beta_1[a_{c_t} - (1 - \cos\varphi_t \cos\psi_t)g] \qquad (19)$$

$$S_2 = \beta_2[a_{b_t} - (1 - \cos\varphi_t \cos\psi_t)g] \qquad (20)$$

where

β_1 and β_2 are calibration factors
a_{c_t} = acceleration near CG perpendicular to ships' reference plane at time t

a_{b_t} = acceleration at bow perpendicular to ships' reference plane at time t

φ = roll angle

ψ = pitch angle

g = gravity acceleration

Now

$$a_{c_t} = \ddot{Z}_t \cos \varphi_t \cos \psi_t \tag{21}$$

and

$$a_{b_t} = (\ddot{Z}_t + l\ddot{\psi}_t \cos \psi_t)\cos \varphi_t \cos \psi_t \tag{22}$$

where l is the distance between accelerometers.

Since the maximum measured double amplitude of pitch was 8 degrees, the approximation $\cos \psi \approx 1$ may be used.

Thus

$$\ddot{Z}_t = \frac{1}{\cos \varphi}\left[\frac{S_1}{\beta_1} + (1 - \cos \varphi)g\right] \tag{23}$$

$$\ddot{\psi}_t = \frac{1}{l \cos \varphi}\left[\frac{S_2}{\beta_2} - \frac{S_1}{\beta_1}\right] \tag{24}$$

The variance and covariance of $\ddot{Z}_t$ and $\ddot{\psi}_t$ may be evaluated as follows:

$$\frac{\sum_1^n \ddot{Z}_i^2}{n} = \frac{1}{n}\sum_1^n \frac{1}{\cos^2 \varphi}\left[\left(\frac{S_1}{\beta_1}\right)^2 + (1 - \cos \varphi)^2 g\right.$$
$$\left. + 2\frac{S_1}{\beta_1}(1 - \cos \varphi)g\right] \tag{25}$$

In head seas, there is no coupling between roll and pitch, or between roll and heave and pitch acceleration.

Thus $\cos \varphi$ and S_1 are noncorrelated and equation (25) can be written

$$\frac{1}{n}\sum_1^n \ddot{Z}_i^2 = \left(\frac{1}{n}\sum_1^n \frac{1}{\cos^2 \varphi}\right)\frac{1}{n}\sum_1^n \left(\frac{S_1}{\beta_1}\right)^2$$
$$+ \frac{g^2}{n}\sum_1^n \frac{(1 - \cos \varphi)^2}{\cos^2 \varphi} \tag{26}$$

since

$$\frac{1}{n}\sum_1^n \frac{S_1}{\beta_1} = 0$$

Thus the influence of roll on the computation of the variance of the heave acceleration can be calculated as follows:

$$\frac{1}{n}\sum_1^n \frac{1}{\cos^2 \varphi_i} \approx \frac{1}{n}\sum_1^n (1 + \varphi_i^2) \tag{27}$$

because

$$\cos^2 \varphi = \left(1 - \frac{\varphi^2}{2} + \frac{\varphi^4}{4!}\right)^2 \qquad \text{since } \varphi \ll 1$$

$$\cos^2 \varphi \approx 1 - \varphi^2$$

$$\frac{1}{\cos^2 \varphi} \approx 1 + \varphi^2$$

Now since $E = 2\sigma_\varphi^2$, equation (26) may be evaluated as follows:

$$\frac{1}{n}\sum_1^n \frac{1}{\cos^2 \varphi_i} = 1 + \frac{1}{2}E\left(\frac{\pi}{180}\right)^2$$
$$= 1 + 1.52 \times 10^{-4}E \tag{28}$$

$$\frac{1}{n}\sum_1^n \frac{(1 - \cos \varphi)^2}{\cos^2 \varphi} = \frac{1}{n}\sum_1^n \frac{1}{\cos^2 \varphi}$$
$$- \frac{2}{n}\sum_1^n \frac{1}{\cos \varphi} + 1$$

$$\frac{1}{n}\sum_1^n \frac{1}{\cos \varphi} = 1 + \frac{1}{2n}\sum_1^n \varphi^2$$
$$= 1 + 0.76 \times 10^{-4}E \tag{29}$$

Using equations (28) and (29) equation (26) gives the variance of heave acceleration as

$$\frac{1}{n}\sum_1^n \ddot{Z}_i^2 = \frac{1 + 1.52 \times 10^{-4}E}{\beta_1^2}\,\frac{\sum_1^n S_1^2}{n} \tag{30}$$

From equation (24) the variance of pitch acceleration is found to be

$$\frac{\sum_1^n \ddot{\psi}_i^2}{n} = \frac{1 + 1.52 \times 10^{-4}E}{l^2}$$
$$\times\left[\frac{1}{\beta_1^2}\,\frac{\sum_1^n S_1^2}{n} + \frac{1}{\beta_2^2}\,\frac{\sum_1^n S_2^2}{n} - \frac{2}{\beta_1\beta_2}\,\frac{\sum_1^n S_1S_2}{n}\right] \tag{31}$$

From (23) and (24) the covariance of pitch and heave acceleration is

$$\frac{\sum \ddot{Z}_i\ddot{\psi}_i}{n} = \frac{1 + 1.52 \times 10^{-4}E}{l}$$
$$\times\left[\frac{1}{\beta_1\beta_2}\,\frac{\sum_1^n S_1S_2}{n} - \frac{1}{\beta_1^2}\,\frac{\sum_1^n S_1^2}{n}\right] \tag{32}$$

since $\frac{1}{n}\sum_1^n \frac{S_1}{\beta_1} = 0$ and $\frac{1}{n}\sum_1^n \frac{S_2}{\beta_2} = 0$

The result is that the variances and covariances of pitch and heave accelerations can be computed from the variances and covariances of the linear displacements S_1 and S_2 from the record.

The effect of roll can be calculated with the available E-values for roll angles.

Roll has no influence on the position of the axis of minimum vertical acceleration.

Neglecting the influence of roll on the acceleration, the maximum error, for all cases considered, amounts to 0.16 per cent, which is small compared to the errors in the recording and reading.

Thus equations (30), (31) and (32) become

$$\frac{1}{n}\sum_{1}^{n}\ddot{Z}_i{}^2 = \frac{1}{\beta_1{}^2}\frac{\sum_{1}^{n} S_1{}^2}{n} \tag{33}$$

$$\frac{1}{n}\sum_{1}^{n}\ddot{\psi}_i{}^2 = \frac{1}{l^2}\left[\frac{1}{\beta_1{}^2}\frac{\sum_{1}^{n} S_1{}^2}{n} + \frac{1}{\beta_2{}^2}\frac{\sum_{1}^{n} S_2{}^2}{n} - \frac{2}{\beta_1\beta_2}\frac{\sum_{1}^{n} S_1S_2}{n}\right] \tag{34}$$

$$\frac{1}{n}\sum_{1}^{n}\ddot{\psi}\ddot{Z} = \frac{1}{l}\left[\frac{1}{\beta_1\beta_2}\frac{\sum_{1}^{n} S_1S_2}{n} - \frac{1}{\beta_1{}^2}\frac{\sum_{1}^{n} S_1{}^2}{n}\right] \tag{35}$$

The position with the minimum mean square vertical acceleration, equation (11) is defined by

$$X = -l\,\frac{\dfrac{1}{\beta_1\beta_2}\dfrac{\sum_{1}^{n} S_1S_2}{n} - \dfrac{1}{\beta_1{}^2}\dfrac{\sum_{1}^{n} S_1{}^2}{n}}{\dfrac{1}{\beta_1{}^2}\dfrac{\sum_{1}^{n} S_1{}^2}{n} + \dfrac{1}{\beta_2{}^2}\dfrac{\sum_{1}^{n} S_2{}^2}{n} - \dfrac{2}{\beta_1\beta_2}\dfrac{\sum_{1}^{n} S_1S_2}{n}}$$

From equation (12) the minimum vertical acceleration is found to be

$$\xi_a = \frac{\left(\dfrac{1}{\beta_1{}^2}\dfrac{\sum_{1}^{n} S_1{}^2}{n}\right)\left(\dfrac{1}{\beta_2{}^2}\dfrac{\sum_{1}^{n} S_2{}^2}{n}\right) - \left(\dfrac{1}{\beta_1\beta_2}\dfrac{\sum_{1}^{n} S_1S_2}{n}\right)^2}{\left(\dfrac{1}{\beta_1{}^2}\dfrac{\sum_{1}^{n} S_1{}^2}{n}\right) + \left(\dfrac{1}{\beta_2{}^2}\dfrac{\sum_{1}^{n} S_2{}^2}{n}\right) - \left(\dfrac{2}{\beta_1\beta_2}\dfrac{\sum_{1}^{n} S_1S_2}{n}\right)}$$

Discussion

Kazuo M. Ochi,[5] ***Visitor:*** This paper on seakeeping trials carried out on three destroyers gives not only a great deal of valuable data on those particular types of ships but also gives undoubtedly a most comprehensive and noteworthy contribution to the subject of seaworthiness of ships in general.

It is particularly gratifying to find that the tests on slamming were conducted with painstaking care and that they succeeded in measuring the two kinds of pressures associated with slamming; one a high pressure of very short duration of time as seen in Fig. 34, the other the sinusoidal form pressure of relatively long duration as seen in Fig. 11. The latter is due to bow immersion and is particularly significant for ships with large bow flare. It is really important for a ship's designer to have knowledge of the magnitude and duration of time of these two kinds of pressures associated with slamming.

Some interesting and important results concerning the frequency of occurrence of slamming and bow emergence in rough seas are shown in Fig. 31. The difference in the frequency of occurrence of slamming and frequency of occurrence of bow emergence in the figure shows that bow emergence is not the only criterion which causes slamming.

In connection with Fig. 31, it may be mentioned that bow and beam seas often hit diagonally against the ship's forebody with sufficient impact to cause vibratory stresses in the hull. This wave impact is often mistaken for true slamming. It is thought, therefore, that some of the slams indicated in the figure in bow and beam seas may really be due to diagonal-wave impact on the ship's forebody and not to true slamming.

The statements concerning the scatter of the positions of the neutral axes are thought to be somewhat misleading. Inasmuch as the shear-lag phenomenon is usually associated with bending of the ship's outside plating, it would induce errors in estimating the position of the neutral axis by connecting only two readings measured on the sheer strake and on the keel. Therefore, the scatter as seen in Fig. 28 is not surprising, and it is believed that the scatter depends neither on interference of local stress nor on riveted hull construction.

One question arises concerning the comparison of the bending moment for the three ships. As mentioned in the paper, ships of the same geometric dimensions can be expected to have nearly equal bending moments in the same seas and at the same ship speeds. If the bending moments of the three ships are computed from Table 12 and compared, it is found that the ratio of the bending moment between Ships A and B gives the proper order of magnitude. However, the ratio between Ships A and S, and Ships B and S are less than 1, by approximately 20 per cent. This suggests

---

[5] Taylor Model Basin, Washington, D. C.

that the bending moment of Ship S may be in doubt. This reasoning is further verified in the comparison of the bending moments of the three ships given in Table 13.

The effect of ship speed on bending moment as shown in Fig. 29 is significant. However, a period of more than 2 or 3 hr was required for completion of the tests at different speeds, so it is expected that the sea states might have been changed. Therefore, the bending moment measured in the tests may include both the effects of ship speed and of sea state. The speed effect on bending moments should be treated separately for each sea state, since the bending moment may not be a function of the single parameter, wave height. The writer would like to know the physical meaning of the dimensionless expression of bending moment mentioned in the paper.

It is generally known from model tests in regular waves that the measured bending moment is always less than the bending moment calculated for a given wave height irrespective of ship speeds. This is due to the effect of body-wave interference on the dynamical strength of ships in waves, and this is an important problem for ship's strength among waves. This writer suggests that in general, it might be significant to compute the energy spectrum for bending moment for a given sea state and then obtain the ratio between measured and calculated bending moment. This ratio may be less than 1, and it will give the effect of body-wave interference on the dynamical strength of ships at seas.

John Vasta, *Member:* To organize an expedition that requires a flotilla of three destroyers to go to sea seeking rough weather is an experimental undertaking of no small proportions. Much credit is due those who initially conceived these trials, those who successfully carried them out, and the authors of the paper, who have analyzed the data.

With modern instrumentation, it is relatively easy to accumulate a large body of experimental data. The truth of this is attested by the 6 miles of records collected during the 2 weeks of testing. The difficulty lies in the amount of time it takes to analyze the data and report the results. This is not a condemnation of these particular trials. It is a complaint of a general nature and applies, more or less, to all full-scale tests. What is needed is the development of reliable methods of analyzing experimental data expeditiously.

The authors state in the introduction that "it would be difficult to derive general information from the trials because of lack of precise knowledge of the input causing the motion." This no doubt refers to the sea input. Since the properties of the sea encountered in the operating area have been determined from visual estimates and from "hind-casting," the authors express concern that such a procedure may be too inaccurate to provide a basis for comparison. Continuing along this pessimistic vein, they further state that in the analysis of the data it was impossible "to avoid the introduction of inaccuracies which tend to nullify the value of the data obtained on motion and stress." After such a sweeping condemnation, are we to conclude that the results are of questionable value?

Although no direct measurements of the sea were taken, I believe that the estimates based on weather charts for the trial period do provide a basis for much useful discussion. From Table 5 it is noted that for the most severe seas encountered, the significant wave height was 12.4 ft, and the height of the swell 7.2 ft. The authors state that the spectra computed separately for swell and sea were combined by adding the components at corresponding frequencies. This implies that the resultant significant wave height was greater than 12.4 ft. The text does not give a resultant wave height, nor does it give the associated wave length. Since, by tradition, structural strength is related to a wave height, usually $L/20$ or $1.1\sqrt{L}$, it would be interesting to know the properties of the sea associated with the highest stresses. More specifically, what were the wave length and wave height that induced maximum stress?

In item 5 of the summary, mention is made of the highest significant stress intensities measured. These, however, differ from the expected maximum stress values listed in Table 13. Is this difference due primarily to the fact that in one case we are dealing with a measured quantity and in the other with a statistically derived value? How do the estimated extreme stress values compare with actual measurements? Some clarification of this point would be appreciated.

For the benefit of ship designers who are less conversant with statistical techniques it would be helpful if the authors were to give several numerical examples which would show, for example, how one derives the pitch and roll amplitudes listed in Table 10 from the data of Table 6.

In the analysis of the experimental data, the authors have made use of statistical methods. This is a proper approach, especially when a broad generalization of the test results is sought, but some confusion may have been created by not emphasizing which are the measured quantities and which are the statistically derived values. The discussion does not have a common denominator. While it is entirely proper to speak in terms of swell and sea spectra, energy densities, cumulative frequency distributions, normal density functions, variants and covariants, one must remember that these terms are foreign and frightening to the ship designer. He has been steeped in the tradition of simple $L/20$ waves, and he needs help in understanding the statistical approach. This paper has introduced a brand new technical jargon which may confuse the uninitiated. Eventually, I am sure, it will provide a sound basis for a better understanding of ship design. I extend congratulations to the authors for the monumental task which they have performed.

Prof. Edward V. Lewis, *Council Member*: This paper provides a comprehensive report on an ambitious and interesting project. Clarification is obtained of a number of questions regarding the behavior of ships at sea, some of which confirm model test findings previously reported. At the same time the paper shows the handicaps under which researchers work when records of the sea surface are not available. The authors undoubtedly fervently wished for such data, but even without it they have made an excellent analysis and presentation.

One of the conclusions of the paper is that there is little difference in the pitching and heaving characteristics of the three ships. This is in agreement with the findings of model research and motion theory that relatively large hull changes are necessary to make significant effects on amplitudes of motion. Nevertheless, it is noted that "Ship S showed a slight tendency to pitch more than Ships A and B." The reason for this may be that it is the smallest of the three ships, both in length and displacement. Looking for other possible differences, Table 1 shows the shortest pitching period for Ship S. Taking length into account by comparing period-length ratios, $T/\sqrt{L}$, the period still appears to be somewhat shorter than the others. This is hard to understand, for the greater pitching of this ship, particularly in the rough X-sea, Figs. 12(a) and c, would suggest a relatively longer period. A question arises then as to how the natural pitching periods were determined.

A comparison of the body plans in Fig. 2 indicates a much finer load waterline for Ship S. And Table 2 shows that the $\Delta/(L/100)^3$ of Ship S is about 10 per cent greater than the other two. Both of these characteristics would indicate a longer pitching period for Ship S.

The second summary point notes that speed had much more effect on pitching and heaving in the rough X-sea than in the moderate Y-sea case. It is believed that the reason for this is that increasing speed in the Y-sea resulted in an approach to the "supercritical" condition; i.e., the frequency of encounter with the longest important wave in the sea was too high to cause serious motions. Referring to Fig. 26(a), an increase of speed to 27 knots would move the peak of the estimated sea spectrum over to the right of the response-amplitude-operator peak. This would mean a lower pitch spectrum, as shown in Fig. 25.

Fig. 18, showing pitch spectra for all three ships, is interesting from several points of view. First of all, it is stated, "the greater damping in pitch prevents synchronous pitching from dominating the motion, and in fact none of the peaks is clearly associated with the natural frequency." This statement may be only partly true, for it is important to note that the peaks of the response amplitude operators plotted in Fig. 26 likewise do not correspond to the stated natural pitching frequency. Perhaps it would be better to say that damping and other effects (such as coupling between pitch and heave, wave length in relation to ship length, and so on) cause the greatest pitching response to occur at a frequency considerably lower than the natural frequency—in both regular and irregular waves.

Fig. 18 shows then that the principal peaks of pitching response occur at $\omega_e = 1.0$, the frequency which corresponds to the peak of the response-amplitude curve. Only in quartering seas do significant forced oscillations appear, at a frequency of about 0.4. The highest peaks occur in head and bow seas where presumably wave-spectrum peaks also occur at a frequency of about 1.0.

At first glance, a comparison of the pitch spectra in Fig. 18 suggests quite different behavior for the three ships at any particular heading. Actually, however, it is much more reasonable to assume that the differences are mainly differences in sampling. In other words, if longer records than 10 min had been obtained, the spectra for the three ships might have shown better agreement. The spectral variations shown in head seas, for example, look very much like those obtained from separate model runs in different parts of an irregular-wave program. The wave spectra then show similar variations, but when pitch spectra are divided by the corresponding wave spectra, the resulting response operators usually agree very well. In short, without wave measurements to provide the wave spectra to go with the pitch spectra of Fig. 18, it is impossible to draw detailed conclusions.

The data on slamming in the paper are also of particular interest. Fig. 11 illustrates a phenomenon which is different from the ordinary concept of slamming. The local bottom pressures and the accompanying "whipping" stresses build up slowly as the flare enters the water. In contrast to this, Fig. 34 shows a case of true bottom slamming in which the local pressure builds up quickly to much higher values and then drops. The effect on hull stresses is hardly perceptible in this case.

Consideration of model and full-scale data suggests that large whipping stresses, such as are shown in Fig. 11, are characteristic of very flexible hulls. The less rigid the hull, the lower the natural vibration frequency, and apparently short wave components of the sea then provide some additional excitation. For, example, I estimate that for the destroyer at 17 knots a wave 26 ft in length approaching 45 deg from the bow would be in synchronism with the natural two-node vertical vibration frequency. It does not seem unreasonable that such waves would be present in the spectrum of the sea.

One or two minor points: Mention is made of "a state 5 to 6 sea." Since there are a number of sea scales available, it might be well to define the condition more explicitly. The discussion of pitching is not entirely clear. In describing ship motion in head seas the usual convention is to assume that it is composed of a vertical motion of the CG (heave) and an angular motion (pitch). No axis need be specified. It is my understanding that the concept of "apparent pitching axis" is only applicable to harmonic motion with pitch and heave exactly in phase (or 180 deg out of phase).

Samuel M. Y. Lum,[6] ***Visitor:*** There is one phase of the over-all problem about which I would like to arouse some author comment. This pertains to the question of the role of the transfer function or response-amplitude characteristics of a given ship in random seas as denoted by

$$\Phi_{00}(\omega) = |KG(\omega)|^2 \Phi_{ii}(\omega) \qquad (36)$$

where

$\Phi_{00}(\omega)$ refers to measured response spectrum

$\Phi_{ii}(\omega)$ refers to (supposedly) measured input spectrum

$KG(\omega)$ is the frequency-response function which characterizes the uniqueness of a given ship

The full-scale sea trials provide an opportunity for checking input-output relationships of a ship operating in its natural environment to clarify the uniqueness of a given vessel's "signature" through its transfer function.

This is the problem of synthesis:

$$|KG(\omega)|^2 = \frac{\Phi_{00}(\omega)}{\Phi_{ii}(\omega)} \qquad (37)$$

The transfer function or frequency-response characteristics obtained in this manner is often referred to as the inverse problem of dynamics, using in this case random sea measurements and the methods of spectral analysis. It also can be obtained by controlled model tests by frequency-response techniques using regular waves or by convolution methods involving an arbitrary transient input. This will provide another way of isolating the transfer function

$$KG(\omega) = \frac{q_0(j\omega)}{q_i(j\omega)} \qquad (38)$$

where

$q_0(j\omega)$ = response

$q_i(j\omega)$ = input

If the data, as obtained, sufficiently met all the requirements of (*a*) stationary process, (*b*) Gaussian distribution, (*c*) linear model and the principle of superposition, then either method of arriving at the transfer function, by the "probabalistic" approach, equation (37), or the "deterministic" approach, equation (38), should result in equivalent solutions. It is desirable that the study be extended to see whether this can be demonstrated by an additional small investment of model tests to bulwark the full-scale results at hand. This in essence, when demonstrated, will effect a convincing marriage of the probabilistic school and the deterministic school. The proposed mode of investigation is based on the following premise:

If the foregoing sea tests are compatible with the ideal assumptions involved in spectral analysis, then equation (36) herewith, applies so that the sea input, $\Phi_{ii}(\omega)$, although not measured quantitatively, can be considered mutual to all three vessels at a given test period. This mutual input is expressed by the following relationship and equalities:

$$\Phi_{ii}(\omega) = \text{const} = \left.\frac{\Phi_{00}(\omega)}{|KG(\omega)|^2}\right]_{\text{Ship A}}$$

$$= \left.\frac{\Phi_{00}(\omega)}{|KG(\omega)|^2}\right]_{\text{Ship B}} = \left.\frac{\Phi_{00}(\omega)}{|KG(\omega)|^2}\right]_{\text{Ship S}} \qquad (39)$$

---

[6] Code 421, Bureau of Ships, Navy Department, Washington, D. C.

It can be assumed that the transfer characteristics $|KG(\omega)|^2$ linked with each ship can be readily reconstructed by controlled model tests using regular sea inputs. To reduce the complexity of deducing the transfer function through model experiments, it is suggested to take on only the head-seas case since this minimizes the cross-coupling effects and the resulting higher order characteristic equation associated with inclusion of all the degrees of freedom. Then using the measured responses for each ship obtained at sea and the respective transfer functions from model tests, it is possible to verify the equivalencies in equation (39).

This should then show, if the basic hypothesis outlined in a spectral treatment are not violated, that

1 The input spectrum $\Phi_{i_1}(\omega)$ of the seaway for the given run under surveillance is unique and can be reconstructed,

2 The methods of arriving at the transfer function by either of the previously mentioned two methods, the "probabilistic" approach or the "deterministic" approach, complement each other and

3 The full-scale response data are unimpeachable and future seakeeping studies can effectively be done in the laboratories with ready interpretation for full-scale performance.

General. It is somewhat unfortunate that more quantitative data of the wave-height spectrum representing the input are not available with the measured outputs. Although hindcast weather information was used, and then taking a Neumann spectrum based on a measured wind velocity and the frequency-response characteristics of a "similar" ship model tested in regular seas, it was shown that theoretical approximations of the random-sea response compared favorably with that measured at the full-scale trials. This implies that the seaway encountered met the requirements of a time-stationary process and hence the ergodic hypothesis if the results as shown are sufficient proof of this supposition. If the seaway as the generating input proved to be nonstationary, as may well happen within a 7-hr test program associated with a given sea condition, how much of a discrepancy would occur in the statistical results if the stationary assumptions were still retained? Furthermore, if the assumption of a Gaussian input distribution did not hold, would the E-values of the responses denoting their variances be sufficient to qualify the statistical properties completely? Would a piece-wise treatment offer better results? Would the authors elaborate on the above questions?

The ship dynamics people and the world of science are indebted to the participants of the U.S.—Dutch team involved in such a momentous task as this. The authors and collaborators are to be congratulated for their efforts and noteworthy achievements in making this worthwhile contribution.

R. N. Newton,[7] ***Visitor:*** The authors, and indeed all those associated with the trials described in this paper, are to be congratulated on the outcome of their efforts. Within the admitted limitations of the trials, and bearing in mind that they were carried out early in 1956, the authors have presented a very interesting and complete report. In particular, the very frank way in which they have discussed the shortcomings of the trial procedures and instrumentation used will be of great benefit to others who are trying to plan such trials.

For instance, it is clear from the statement that one of the greatest problems faced was the lack of accurate data on the sea conditions. The statement reads:

"At any rate, the present analysis indicates that meaningful results, by application of the principle of linear superposition of responses to the component waves in an irregular sea, can be obtained only if a more accurate description of the sea is obtained, both as to amplitudes and frequency distribution. Of possible importance also is the directionality of those wave components whose directions differ from the dominant direction of the system."

This statement has been fully borne out by later experience. Reliable quantitative conclusions can only be drawn if the sea conditions are accurately known and if the seas are long-crested or very nearly so. As such conditions were not met and not possessing equipment for measuring the sea, the authors have attempted to obtain sea spectra from the general weather charts. As we now know this approach can give misleading results particularly when based upon conditions such as those indicated in Fig. 5, and Fig. 26(a) shows how important is the accurate estimation of wind speed. Not the least of the problems is the fact that there is no universal agreement among the experts on the wave spectra due to given wind conditions, especially when the sea is only partially developed. All of which is being wise after the event and the authors are to be commended for their diligence in pursuing more approximate methods.

In any ship motion trial one very important object is usually to obtain response amplitude

---

[7] Superintendent, Admiralty Experiment Works, Haslar, England.

operators for the ship for comparison with model data from regular and irregular wave tests. To make this comparison meaningful it is necessary to have an accurate measurement of ship's speed. This is very difficult under the conditions pertaining to this sort of trial and it would be of value if the authors could comment on the speed measurements made and any suggestions they may have for future trials.

Another difficulty can arise from the interdependence of motions, particularly between rolling and yawing in head and following seas. Because of this it is suggested that the rolling spectrum can vary with the response of the helmsman and that consistent results can probably only be obtained by using an automatic steering gear. This again would help in ship-model comparisons as the model could be fitted with an automatic gear with similar characteristics. Certainly, rolling is a difficult quantity to analyze and I would agree with the authors' opinion that this is largely due to the non-linear response of ships in rolling.

There are lessons of principle as well as detail to be learned from the paper. Bearing in mind that the original purpose of the trials was to compare the behavior of the three ships under the same operating conditions, it is a pity that their responses were so similar. This suggests that in any similar trials it should be considered whether more would not be learned by comparing the behavior of ships of more widely different form. There would, of course, remain the problem of deciding which design features were responsible for any differences in response.

A point of more immediate interest, however, is that consideration could well be given to following these trials with trials of one of the ships in which the sea surface is accurately measured. Such trials could, given suitable long crested seas, provide realistic ship response operators for comparison with model studies. They would also provide a reliable datum against which the behavior of the other two ships could be gaged using the results of this present trial. What is more they could provide a basis for prediction of the behavior of any ships of similar form in other sea spectra.

It may be interesting to record that A.E.W. are carrying out similar trials with a view to setting up the corresponding sea condition in the new maneuvering tank and so enable reliable ship-model correlation to be achieved. A number of other authorities are planning, or have carried out, such trials and this brings me to my last point.

The study of ship motion is both wide and complex. The problems posed can be more quickly and accurately solved by a complete and frank exchange of data obtained by all investigators in the field. By this I mean not just the exchange of final results because the subject is so complex that to be useful to others such results need to be supplemented with details for instance of the methods of recording and analyzing data, the method of selecting the data presented, and sufficient information to enable the reader to calculate confidence limits if spectral analysis is used.

The paper presented by the authors is, I submit, an excellent example in this respect and one which all who are interested in the subject would do well to follow. Finally, although the fact is obvious from the contents of the paper it is worth recording in the discussion that the investigations described are an outstanding example of the value to be derived from international cooperation.

Norman H. Jasper, *Member:* The analysis of ship trials presented in the paper is the result of a considerable effort. Much of the ship-response data in this paper are given in terms of the parameter E of the corresponding statistical distribution which was assumed to be of the Rayleigh type. The numerical value of E was obtained from the peak-to-peak variations of the measurements, and it was then used to estimate characteristic and extreme values of ship response for comparison with measured values.

The general agreement between statistical estimations and measurements is shown to be quite good even in the case of extreme values. In fact, the deviations often appear to lie within the accuracy of measurement and analysis. Nevertheless, the authors do appear to conclude, on the basis of chi-square tests, that the assumption of a Rayleigh distribution is probably statistically not valid. This discussion is primarily directed at examining the suitability as well as the validity of use of the chi-square test in this paper and intends to show that the tests as applied here cannot form the basis for a rejection of the Rayleigh hypothesis.

Ship-response data have been collected, analyzed and reported over the past several years in a manner analogous to that given here for a number of ship types, and a wide variety of operating conditions. The ships were aircraft carriers [27], escort vessels [26], destroyers (TMB Report 1198), and Swedish merchant ships.[8]

---

[8] R. Bennet, "Stress and Motion Measurements on Ships at Sea," Report 15, The Swedish Shipbuilding Research Foundation.

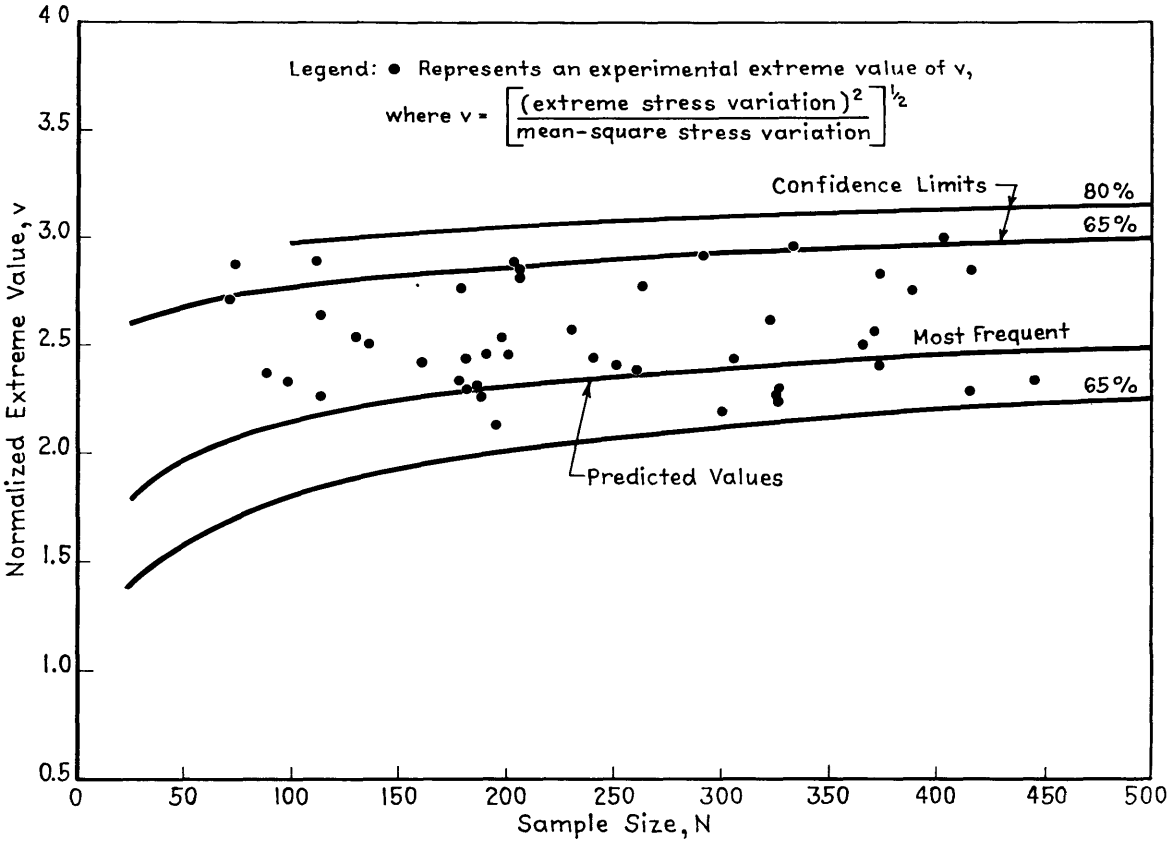


Fig. 44 Essex Class Carriers—scatter of measured extreme hull-girder stress values (main deck—amidship)

For each case the comparison of measured extreme values with statistical estimates, made on the assumption of the Rayleigh distribution, were usually good, Figs. 44 and 45 which plots the extreme values for carrier and escort vessels together with the expected variations. Fig. 46 is a similar plot for the Dutch destroyer. The presentation in the figures is in effect a "goodness-of-fit" test if one is interested in the prediction of extreme values. Apparently the Rayleigh assumption results in a moderate bias. If the parameter E had been derived from a truncated distribution, the bias would probably have been reduced appreciably.

The manner of estimation of the parameter E is somewhat different in the present paper than was the practice in the foregoing papers. Heretofore, H was estimated as the mean-square value of the peak-to-peak variations to which the symbol E_J was assigned in the present paper. E_J is the most efficient statistical estimate of E. The alternative estimate E_p used in this paper appears to be without statistical foundation. The authors in their procedure arbitrarily reject some of the largest measurements though they could appreciably affect E_p; other large observations are retained in order to weigh appreciably the estimate because "the region of greatest interest to the ship designer is that of the largest values." If low values are not of interest, then it would seem more appropriate to deal with a truncated distribution in which measurements below some value are discarded and treating the remainder of the data by valid statistical methods. The authors use their E_p-value, though derived on an arbitrary basis, to define the theoretical Rayleigh distribution against which the experimental distribution is compared by means of the chi-square test. The entire procedure seems questionable. If estimates of large values are of interest then a test of significance such as shown in Figs. 44 through 46 is more suitable. If the *entire* distribution is to be tested, then a chi-square is theoretically a suitable test. It would of course reject the Rayleigh assumption for a lack of fit anywhere within the distribution, even though there may be no practical interest in that questionable region.

Statistical conclusions drawn from the chi-

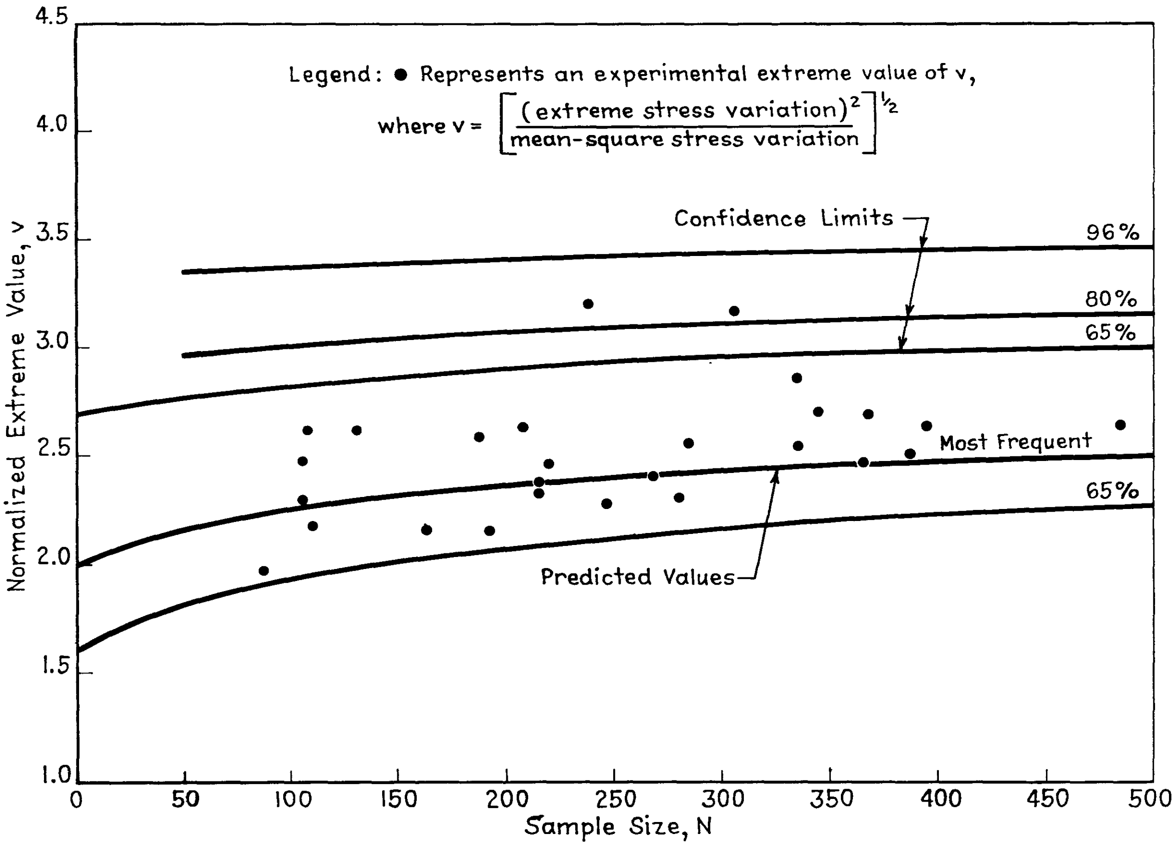


Fig. 45 AVP-type Escort Ships—scatter of measured extreme hull-girder stress values (main deck—amidship)

square test applied in this paper are believed to be invalid for the following reasons:

(*a*) The use of E_v as an estimate of E is arbitrary and thus without statistical foundation, as discussed before. The best estimate is believed to be E_J obtained, if desired, from a truncated distribution.

(*b*) The chi-square test is meaningful for a rejection of the hypothesis only if applied to discrete, exact, independent events. Unfortunately, the variables to which the test has been applied here are neither exact nor independent. The dependence of individual variations on preceding ones is very high for roll and less for other variables. We have computed the autocorrelation function for typical samples and find that variations as much as 6 cycles are "reasonably independent." This dependence was pointed out in [9]. The effect of dependence is striking. Assume that only $1/E$ of the sample observations are independent and that the remainder are repetitive values. Then the chi-square value would be $1/m$ times the value that was obtained on the assumption that all observations were independent values. The probability value for chi-square in the case of 20 degrees of freedom and $m = 5$ would have risen from 0.001 to 0.975 when this allowance for independence is made. For $m = 2$ the probability would increase from 0.001 to 0.70. Even slight correlation between successive cycles may have considerable effect on this goodness-of-fit test.

(*c*) Errors in the original measurement, as well as in the reading and classification of the peak-to-peak values may affect the chi-square value appreciably. These errors need not be systematic. May I suggest that the authors take at least one typical record and have it independently read and classified by say five different persons, allowing each person to choose his own classification intervals. The variation in the probabilities associated with the resulting chi-square values may be instructive. If there is much scatter in those probabilities, as I suspect, then not much faith can be placed in such tests which do not evaluate the contribution of errors.

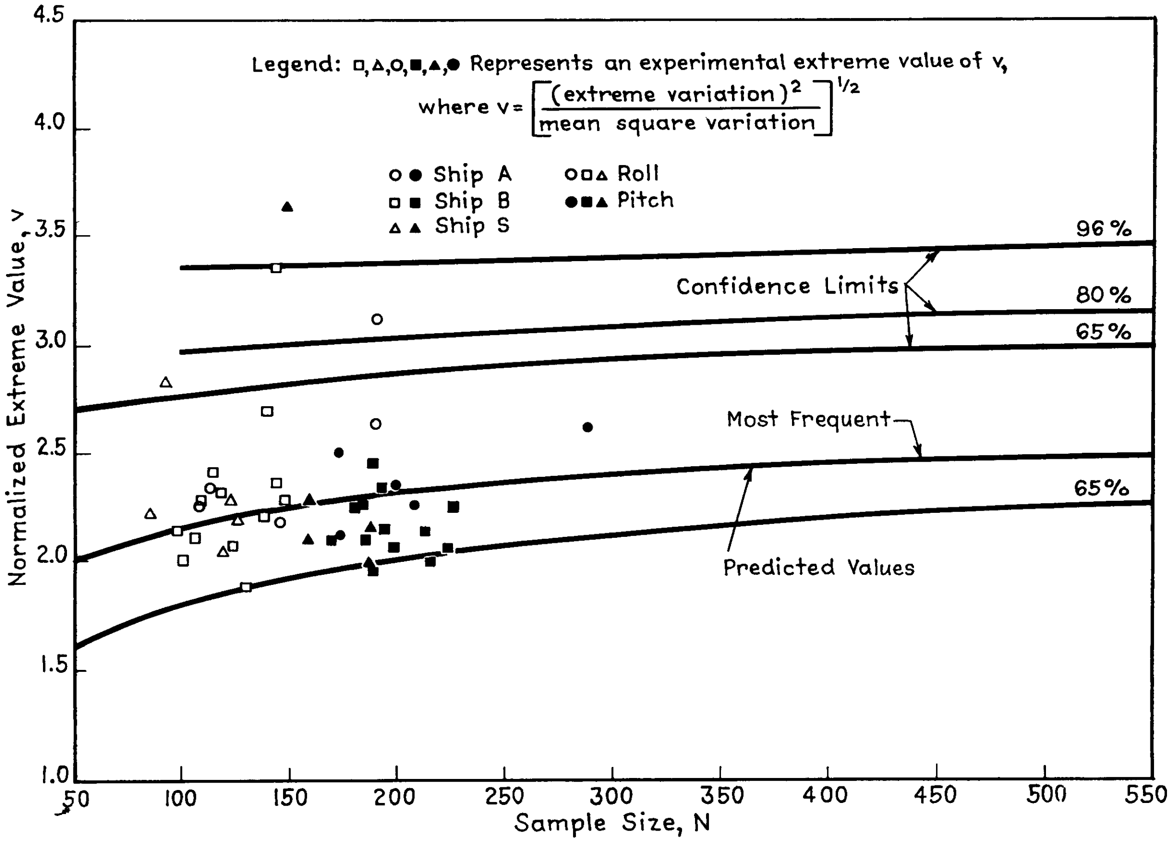


Fig. 46 Dutch Destroyer—scatter of measured extreme motion

From a practical standpoint, and I suppose we should be, there is little interest in small values. The small values are also subject to the greatest errors of measurement and analysis. Consequently, it would be best to deal with a distribution truncated at some low value and make our calculations on the basis of a truncated Rayleigh distribution. Whenever the data from automatic statistical counters are used such a truncation is automatically made. Mr. Bennet of the Swedish Shipbuilding Research Association has used such truncated data and obtained good agreement with power spectral analysis. The chi-square test would then omit the contributions of the very small values. A truncation should also be made at the upper end because of insufficient sample sizes. Incidentally, Mr. Bennet would accept tests for chi-square at the 0.1 per cent level.

In general I believe that the present paper gives added support to using the assumption of a Rayleigh distribution in predicting characteristic and extreme values of ship response.

Comments on other specific points follow:

1 The authors conclude that correlation between variables may have significant effect upon the joint occurrence of large magnitudes. How useful is this effect in the light of the large statistical variations in the extreme value?

2 The position of the neutral axis in bending was obtained from simultaneous strain measurement in the keel and the sheerstrakes. The gage location in the "keel" may be subject to appreciable local stresses due to variation in loads on the inner bottom unless the gage is located at the neutral axis of the bottom structure. Possibly some of the scatter of points in Fig. 28 for ships B and S may be caused by local strains.

3 Use of the " impulse" vibratory stress as a response criterion of slam seems more useful than the others noted. None of the "response" type criteria will identify the cause of the "slam" if such is to be the term. The response may be a result of bottom impact at the bow, bow-flare immersion or similar actions at the stern or other protuberances. The term "hull whipping" would

appear to be more descriptive of ship response whereas slamming suggests some form of bottom impact.

4 The magnitudes of the "low-frequency pressures" observed in these agree with those reported by Greenspon on the basis of extensive slamming tests on Unimak (TMB Report 978). Greenspon also reports occasional pressures as high as 300 psi of very short duration; he compares simultaneous stress measurements with stresses calculated on the basis of the measured high-intensity pressures. The agreement was fair and suggests that these high pressure loads can be considered as static loads for design of small local plate panels.

5 The comparison of estimated and observed pressures is made on rather broad assumptions. For example, it is apparently assumed that the most severe slam occurs in the highest wave. This is doubtful. On the extensively instrumented *Essex* trials (TMB Report 1216), the most severe slam (vibratory stress about 19,000 psi) occurred in a 26-ft-high wave although much lower whipping stresses were observed for waves of about the same height under the same operating conditions.

The authors have produced a fine analysis of the complex and voluminous test data which should receive careful consideration by those interested in the behavior of ships in a seaway.

Prof. B. V. Korvin-Kroukovsky, *Member:* The subject of this interesting paper can be considered as composed of two separate projects; the planning, organization and conduct of trials by one group of people, and the analysis of test data by another group, which did not participate in trials. This separation of activities appears to be a sad consequence of the increasing complexity of instrumentation and analysis. It tends to reduce drastically the potential value of the improved instrumentation. It is a strong opinion of the writer that the most useful results of full-scale sea trials could be obtained by furnishing on the spot the quantitative information to the test personnel aboard, who would appraise it in connection with their immediate qualitative experience. The instruments should be designed with this end in view. A lesser amount of information roughly analyzed with due regard to personally experienced physical conditions, and made immediately available, will be more useful to naval architects than a volume of impersonal statistical data several years later.

The first project, that of organizing and conducting trials, was excellently executed. However, a fatal error was made in not providing shipborne wave meters. Without measured wave data, a good deal of the information obtained lost much of its value. The authors state that ". . . during the initial stage of the planning of the trials a reliable wave-height meter was not available. . ." The writer finds it difficult to accept this statement since such a meter was described by Tucker in 1952[9] and was in use on the British ship *Weather Explorer* since 1953; that is, three years before the trials on the project which is under discussion.

In the second project, that of the analysis, the primary interest of the authors appears to be in statistics with only a limited attention given to the physical aspects of ship behavior, and their connection with ship properties. Of course, naval architects now have to accept mathematical statistics as a necessary part of their activity, since without it ship behavior in an irregular sea is incomprehensible. The spectral analysis is a most useful tool in evaluating ship motions and stresses in a given sea state. But what is the value of the spectral analysis applied to ship motions by themselves without regard to sea waves?

A review of many previously conducted ship trials at sea indicates that the data obtained are seldom sufficiently complete to permit a well-defined analysis. The problem is just too difficult and the scope of trials necessarily has to be limited. Therefore, it should be a rigid rule that model tests in towing tanks should also be conducted in the conditions matching as nearly as possible those at sea. Such tests can help in the interpretation of the trial data during the analysis, and also can fill in some of the data which were missed. In the present project such model tests would be particularly useful because of the lack of the sea-wave data. It is regretted that the model tests were not conducted as a part of the analysis here discussed.

The part of the paper dealing with slamming appears to the writer to be of greatest interest and usefulness. The publication of the preliminary data in reference [3] indicated for the first time that addition of the slam-excited vibratory stress may double the bending stress in ships of the destroyer type. Subsequently Jasper and Birmingham[10] found even higher vibratory stresses in aircraft carriers.

In discussing slamming, separate consideration must be given to:

---

[9] M. J. Tucker, "A Wave Recorder for Use in Ships," *Nature*, vol. 170, 1952, p. 657.

[10] N. H. Jasper and J. T. Birmingham, "Strains and Motions of USS Essex (CVA 9) During Storms Near Cape Horn," DTMB Report 1216, August 1958.

Conditions leading to a slam.

Impact pressure.

Vibratory bending stress.

These now will be discussed in turn. The comparison of three ships in identical wave conditions was one of the most important initial objectives of the project here described. At first sight the results appear almost as a failure, since no significant differences in motions of three ships were indicated. However, the amplitudes of motions by themselves do not represent the most important characteristics of a ship's behavior. Phase relationships of ship motions to the wave profile determine whether the motions will lead to such speed-limiting events as slamming and shipping of water. In the absence of either wave records or model tests these phase relationships cannot be evaluated explicitly. However, a plausible picture can be formed on basis of examination of Tables 1, 17 and 19.

Table 17 clearly indicates the superiority of Ship S, which slammed only 38 times in 869 wave-encounter cycles, as against 52 times out of 913 for the, apparently worst, Ship B. Table 19 shows that a small part of the bow, up to pressure gage 1, was exposed in Ship S just as often as in the others. The exposure of a sizable part of the bottom, as shown by the gage 3, was 11, 14 and 26 times. respectively, for Ships S, A and B. This is observed to be the order of the natural periods in pitching, which are shown in Table 1. E. V. Lewis[11] indicated that the synchronous speed for pitching increases as the ship's natural period decreases. Conversely, at any speed, identical for three ships, the ship with the shortest period will be farthest away from the synchronism and will have more favorable phase relationship to waves. The trial data presented in the foregoing tables appear to corroborate E. V. Lewis's conclusion that for the best seakeeping ability a ship must have a low displacement/length ratio leading to a short natural period of pitching. Ship S is shown to be better than others in this respect.

The detailed discussion of the impact pressure must wait publication of references [12] and [18] on which the authors' data are based. However, attention should be called to the fact that pressures at the instant of impact are shown in Fig. 11 by the heights of vertical lines on lower diagrams at 8.5 sec on the time scale. The pressures at this instant are low and, after initial and practically instantaneous jump, they continue to rise for about a second and a half. The maximum pressure at 10 sec corresponds to the full bow submersion and appears to be composed of roughly equal amounts of hydrodynamic and hydrostatic pressures.

The initial load, causing ship vibration and stress, results from simultaneously occurrent pressures over a large part of a ship's bottom, as indicated in Fig. 11 by the pressure jump at 8.5 sec at frames 166, 183, 189 and 195. The bending stress, resulting from such a sudden load application, cannot be determined by the rules of statics and it is necessary to consider elastic properties of the ship. A rough mental picture can be formed by neglecting higher modes of the slam-excited vibration and concentrating attention on the two-node one, the natural period of which is about 0.8 sec, as is shown in Table 1 of the paper. At the instant of impact vertical accelerations are imparted to the loaded ship sections. Quarter of a cycle later, i.e. in 0.2 sec, these sections develop maximum vertical velocity, and half a cycle later, i.e. in 0.4 sec, the maximun deflection and stress are reached. This first maximum is barely discernible since it represents a sagging stress imposed on the hull which at this instant experiences the hogging stress due to the normal wave encounter. The first significant maximum occurs at the next vibration cycle 0.8 sec later, or theoretically at 9.7 sec on the time scale. The diagram in Fig. 11 shows it at 10 sec. The 1.5 sec, which passed from the initial impact, represent about 1/4 of the ship pitching cycle and the bow is now in a fully submerged condition, as this is shown by the second-from-the-top diagram in Fig. 11 and in Fig. 24. The maximum of the vibratory stress is now superposed on the near maximum sagging stress due to wave encounter. The vibration started by the initial bottom impact is now aggravated by the further load application resulting from the water penetration by the flared bow. Once excited, vibrations last through several normal sagging-hogging cycles, as is shown by Fig. 10(*b*). thus producing a large number of strain applications and bringing about the danger of fatigue failure at the points of stress concentration.

Fig. 11 clearly demonstrates the different nature of destroyer slamming as compared to that of cargo ships. The large bottom deadrise in destroyers limits the initial impact pressure and force to small values, while the maximum force results from the immersion of the flared bow. This force predominates to such an extent that the emersion of the bow is not a necessary condition for a slam, and it was demonstrated in [3] that a significant proportion of slams occurred without bow emersion. The rate of the slamming-

---

[11] E. V. Lewis, "Ship Speeds in Irregular Seas," Trans. SNAME, vol. 63, 1955, pp. 134–202.

load applications is relatively slow; about ⅔ of the maximum appears to have been applied in 0.4 sec; i.e., in about half of the vibratory period. Frankland[12] shows that this is in the region of maximum susceptibility of the structure to absorb the vibratory energy.

In cargo ships, characterized by a small bottom deadrise and a large amount of flat area, the initial impact causes high pressure and loads. These drop abruptly in a fraction of a second when the edges of the wetted area reach the turn of the bilge. The bow emergence is a necessary prerequisite to a slam in this case. The shortness of the load-application time, in comparison to the natural period of two-noded vibration, is such that a relatively small vibratory energy can be absorbed. Despite the large impact force, the resulting vibration adds only some 30 per cent to the sagging bending moment as against 100 to 200 per cent in destroyers and aircraft carriers.

Commander Peter Du Cane, RN (Ret), *Member:* This paper now amplifies that presented at Wageningen in 1957 and poses some interesting questions which would seem difficult to answer.

One notices immediately that ship S, a rather old type, possesses some inherent characteristics which make her all around a more seakindly form. A search for the reasons is not altogether revealing. The beam is a fraction less and the C_{pv} is less than the others.

Despite the natural period of roll being the same as for ship A the rolling amplitudes recorded are substantially less.

The body plan does not reveal much difference but perhaps the buttock lines would be more informative. Otherwise we are forced to the somewhat unscientific conclusion that it is no more than a matter of nationality!

It was reported in the Wageningen paper that the incidence of slamming was greater with waves on the bow than from ahead. However, to some extent this is now modified excepting for the case of ship S where the waves are reported as being 10 deg on the bow, which is a rather small angle to measure with accuracy from observation rather than by recording methods.

What is perhaps worthy of note is that in the discussion on slamming in the paper the authors associate this phenomenon statistically with amplitude of pitch or at least with closing velocity between stem and wave.

Although not all of the greatest pitching amplitudes lead to slamming and indeed a probability percentage is put forward, yet it is significant that no slamming is reported, or apparently expected, associated with small relative velocities between ship and local wave. This is contrary to observed phenomena, as most of us will have experienced the sudden slam appearing unexpectedly in relatively quiescent conditions, certainly as far as ship motion is concerned. Kent in discussing this matter relative to an article appearing recently under the authorship of E. P. Thayer entitled "Can Design Eliminate Forebody Damage"[13] even goes so far as to confirm the possibility of slamming when the closing velocity between wave and ship is negative; i.e., when the ship is rising away from the wave.

It will be observed that for the case of ship which produces the greatest slamming pressure this occurs at 0.26 length. However great the pitching amplitude, it is doubtful if relative velocity would be great at this point.

It would be interesting to know whether the experiments carried out at Wageningen in the seakeeping basin confirm any or all of the conclusions here recorded, especially as regards the incidence of slamming and the relative merits of hull S. Presumably in theory the pressures which are mainly proportional to velocity and mass density (P) of water could also be reproduced to scale, though the instrumentation would be difficult as the time constant is speeded up between model and full scale in relation to the square root of their lengths.

Just to round off this contribution could the authors please state any conclusions they may have reached as to how they would alter to improve these hulls if they were asked to design them again.

Fendall Marbury, Jr., *Associate Member:* The seakeeping qualities of destroyers are important and have received considerable attention from scientists and engineers concerned with seakeeping, the trials reported here being a good example. It should be mentioned that an equally important and closely related quality of destroyers has been neglected, to the detriment, one supposes, of their effectiveness. This is the phenomenon known as sonar quenching, or rather to the resistance of destroyers thereto.

One of the principal duties of destroyers is to fight submerged submarines, and for this purpose they are equipped with echo-ranging gear which enables them to locate submarines by transmitting sound in the submarines' direction and timing the return of their echoes. This gear is usually

---

[12] J. M. Frankland, "Effect of Impact on Simple Elastic Structures," DTMB Report 481, April 1942.

[13] Marine Engineering Log, June 1960.

installed in the region of the ship where the authors of this paper put their water pressure gages; Fig. 4, that is, the part of the bottom where slamming occurs.

When the ship is slamming, the sonar is, of course, useless. The transducer is either out of water or in flow so turbulent that all that can be heard from the sonar is loud noise. It is when the ship's motions are less violent but still appreciable that sonar-quenching occurs, degrading but not precluding the operation of the sonar.

From what little is known of quenching, it is probable that more than one mechanism is involved, though all are results of bad weather and ship motions. Quenching manifests itself to the sonar operator as periodic blasts of noise loud enough to drown out the echoes for which he is listening. It is also known that under quenching conditions sound transmissions are sometimes attenuated, presumably by air bubbles near the sonar dome.

Whatever the mechanisms of quenching may be, the phenomenon is not correlated in an obvious manner with any particular ship motion. It has been the writters experience that it can occur during pitches, during rolls, and during periods of less-than-average motion. This suggests that, though related to ship motions, it has a random aspect, and that one approach to its study would be by statistical analysis similar to that used by the authors. The data for such an analysis could be collected with little instrumentation other than what is already installed on sonar-equipped ships. Equally important but probably more expensive would be study of the mechanisms of quenching.

It should be emphasized that sonar-quenching is fully as serious a limitation on the anti-submarine performance of destroyers in rough weather as slamming and shipping water. It can be considered more serious, because it occurs in weather that is less rough. It would be desirable to know with more exactness when, how and to what degree quenching occurs, because such knowledge might suggest modifications to the ship-sonar design which would reduce it.

It can be contended, because it has not been proved true or false, that any modification which improves the seaworthiness of a destroyer will also reduce its sonar-quenching. The writer believes, however, that design modifications will be found which will improve the one, as measured by any given criterion, more than the other. More frequent and quantitative observations of quenching should shed light on this question, also.

In so far as known by the writer, until now, opportunities to make such observations have been passed up. For example, once the considerable investment of money, effort, and ship time required for these tests had been decided upon, quantitative observations of sonar-quenching could have been made with little additional trouble. This was not done, however.

It is recommended that, for warships, study of the related qualities of seakeeping and resistance to sonar-quenching proceed hand in hand henceforth. Those at the Taylor Model Basin are in a uniquely favorable position to make important advances in this field. Though the results of such study might be classified, they could result in important improvements to our rough-weather anti-submarine operations.

LCDR W. H. Warnsinck, RNNR,[14] ***Visitor:*** Having had something to do with the trials, it was a great pleasure reading this paper throughout. The many familiar photographs and illustrations seemed to bring back the sting of the spray and the admiration for three slender destroyers, wheeling and turning in close formation, and I was thankful for the revival of memories of the happy co-operation between the rather heterogeneous naval and civilian crew. In bad Naval slang: "them were the days!"

I may be forgiven to add a few nontechnical remarks to the very factual data report presented by the authors, who deserve congratulations with their tedious and capable further data reduction and analysis. Thanks to them none of the "some six miles of records," gathered in a few hours by three good ships, have stayed on dusty shelves or in dirty drawers.

First I would like to pay tribute to two men, who, in my recollection, were mainly responsible for successfully starting and maintaining the original idea of co-operation between the two Navies; viz, Dr. Todd and Dr. de Rooij.

It was Dr. Todd who, discussing "various aspects of seaworthiness" in early 1955 in his office at the Taylor Model Basin, almost at once fell for the idea of co-operating in the intended sea trials with three of Her Netherlands Majesty's destroyers. And, once back in Holland, it was Dr. de Rooij, then Deputy Head of RNN Material Department, who warmly welcomed and confirmed the proposed scheme. Both of them backed the project during the preparatory stage, and they backed it with their full support.

I might also put some emphasis on present-day meterological knowledge, and point out to the

---

[14] Nederland Line Royal Dutch Mail, Amsterdam, Holland.

profession the vast possibilities this improving science renders sea-trial organizers. It suffices the authors to state that "two different sea conditions were met" during the trials. This was not, and never needs to be in future, a matter of "fortunately coming across."

It was solely due to excellent long-term wind and wave forecasting by a group of RNN meteorologists[15] using every bit of information they could squeeze out of their receiver sets and applying it to experience and to forecasting methods as given in H.O. publication no. 603 by Pierson, Neumann, and Jarres.

I would like strongly to advise everyone who is planning to run ship-performance tests to make use of the very best of maritime meteorologic advice they can acquire. In the United States this very best is near at hand.

In connection with the foregoing it might be of interest that since last Fall Nederland Line is involved in quite an extensive "optimum routing" test program for her transocean crossings, and the favorable experience with long-term forecasts during the 1956 destroyer-trials is largely confirmed. For masters of vessels and for those who conduct trials alike, the time of "passive luck" in meeting with certain favorable or unfavorable weather and sea conditions seems to have ended; possibilities for activity, action and decision have replaced the necessity of passive waiting for what is ahead.

Egil Abrahamsen, ***Member:*** The authors are to be congratulated on carrying through their tedious and extensive work in connection with the analyses of the records from the ambitious sea trials on the three Dutch destroyers. The conclusions drawn from the tests are quite realistic, and the authors have shown great ingenuity in deriving reasonable and consistent results from their vast amount of data. Even if the authors seem to find the discrepancies between predicted and observed motions to be large I feel that the statistical results given are encouraging, even if especially the extreme values leave something to be desired. I would like to confine my remarks to the question of longitudinal bending.

It is not stated in the paper if the strain gages were mounted in such a way that they showed true heart-of-plate stresses, and it is not excluded that they might have shown some stresses of a more local nature. Was any effort made to calibrate the strain gages statically prior to or after the tests by adding weights (filling up tanks) to the ship and by calculating the resulting change in still-water bending moment, which could be checked against the simultaneous change in the strain-gage readings? This might probably have been done with simple means and would have excluded a number of speculations regarding local effects, shear lag, corrections and structural efficiency of longitudinal material; also the influence of houses amidships could have been properly taken into account.

The authors state that the stresses apparently increased about 15 to 38 per cent when the speed was increased from about 12 to 17 knots. It would perhaps be more correct to say that the amplitude of stress variations increased as mentioned. It is possible that the sagging stresses increased more than the hogging stresses, for instance due to the sagging moment imposed on the ship as a result of the ship's own wave system. It is not likely, however, that this additional bending moment will be of any significance at speeds less than 15–17 knots in this case, and the effect should have been noticeable in moderate sea as well as in rough sea. Do the authors have any means of determining whether sagging stresses increased much more than hogging stresses (which might even decrease) at speeds in excess of say 20 knots?

In this connection it might be of interest to mention that the maximum wave bending moment in sagging which is usually found near amidships at moderate speeds will have its maximum value shifted forward as speed is increased. This is clearly borne out by Fig. 47, showing some preliminary results from tests carried out in the Trondheim Tank on a 10-ft model of a T-2-SE-A1 tanker. The model was split at amidships and at the quarter lengths where both vertical bending moments and shear forces were measured in regular waves. Thus we got five bending-moment values (including the zeroes at ends) and three shear-force values (i.e., slopes of the bending-moment curve at 3 points) to define the moment curve along the ship length.

From Fig. 4 in the paper it appears that the strain gages on Ship A are located somewhat farther forward than on Ships B and S. If the ships had been identical except for the position of strain gages, I would then have expected larger sagging-moment values on Ship A than on B and S at greater speeds. The opposite seems to be the case with the variation in vertical midship bending moments according to Fig. 30. Did the authors try to bring the bending-moment values for the different ships into line by correcting for differences in shape, mass moment of inertia (i.e., for differences in motions) and for mass

---

[15] In the flagship.

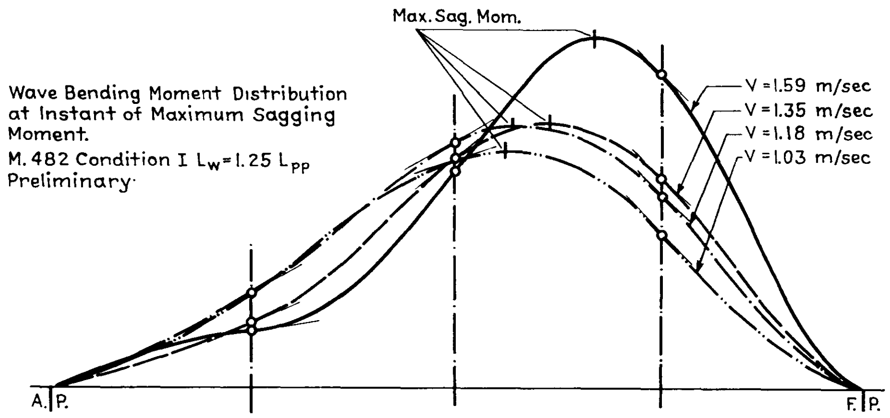


Fig. 47

distribution? This is a formidable task and perhaps not a rewarding one, but might help to clarify the problems further. It should be emphasized that the same mass moment of inertia may be obtained by different load distributions, and that load distribution also has an effect on the wave bending-moment distribution along the ship's hull.

Prof. Willard J. Pierson, Jr.,[16] *Visitor:* This paper is an excellent illustration of the wealth of data that can be efficiently extracted and analyzed when ship motions in waves are studied as a vector Gaussian process. The fact that the observations were planned in 1955 suggests that the subsequent series of observations for other vessels made at TMB will provide even more interesting information as these new observations will undoubtedly have profited from the experience gained in this exercise.

In the hope of contributing a little to the improvement of future analyses, a few comments will be made concerning the determination of the sea state, the effects of sampling variability, the use of additional more powerful statistical tools, and the applicability of the Rayleigh distribution to the study of the motions.

The sea state was not measured in this exercise. Certain analyses based on the Neumann spectrum have been given. These analyses were either inconclusive or depended on accurate wind-speed estimation within 3 knots. The Neumann spectrum appears to work fairly well in the range of wind speeds encountered, but it is extremely sensitive to wind speed in this same range. Moreover, recent new results lead me to believe that no currently available theoretical spectral form is correct. This all serves to point out the value of concurrent wave measurements in such trials. The Splashnick developed at TMB will undoubtedly prove to be most useful in future trials.

I agree with all of the conclusions of the paper except perhaps item 14. These conclusions summarize the bare facts obtained in the data reduction of the actual records. There can be but little doubt that the three vessels were quite similar in their characteristics and one would be hard pressed to show that any one of them was definitely superior.

However in those cases where comparison leads to statements to the effect that one vessel might prove to be definitely superior to another, it is felt that the use of somewhat more sophisticated statistical concepts might be useful. For example, point estimates of the various E-values are given. By means of the spectra, confidence intervals in these point estimates could have been found. They would probably prove to be quite broad. If two intervals for two different E-value estimates overlapped broadly, then the apparent superiority would be quite doubtful. The F-test would also be useful in some of these comparisons.

Some of the comments concerning the predicted and observed maxima in Table 10 are misleading. The most probable value is not necessarily the value that has to be observed, and in fact for the values of N that were used, between 100 and 200, there ought to be about 5 per cent of the observed maxima that exceed 25 per cent of the most probable value. Seven or eight percent might well exceed 20 per cent of the most probable value. The tabulation shows that about 10 per cent of the observations exceeded the most probable value by 20 per cent, and this is not

---

[16] Associate Professor of Meteorology, College of Engineering, New York University, New York, N. Y.

particularly unusual (see H. O. Pub. 603, page 12). After all the highest oscillation in 1000 has to occur some time and it could have happened in the 100 to 200 oscillation record actually analyzed.

Finally correspondence with Dr. Cummins has suggested that the double amplitudes were defined in a way not optimum for testing against the Rayleigh distribution. If superimposed oscillations that do not pass through the mean were included, this one fact alone would be enough to account for those cases in which the distributions would have an excess at low values.

Although reservations as to the correctness of statement 14 in the conclusions have been expressed, there must be many Gaussian processes for which the Rayleigh distribution will not hold for double amplitudes and even for half amplitudes defined in a still more favorable way with respect to the applicability of the theory. One should not despair if this proves to be the case. The distribution of maxima would still be theoretically exact if properly applied and certain truncated and folded distributions of this distribution of maxima would give useful statistics, only slightly modified from those obtainable from the Rayleigh distribution, concerning extreme values.

G. Vossers,[17] ***Visitor:*** The writer congratulates the authors on the accomplishment of this important program, it being one of the first extensive full-scale measurements on model mctions and slamming which really can be used for a correlation study with model tests. Of course, however comprehensive a paper may be, there will be always desiderata left, and in this case one of the main items is a measurement of the wave conditions. This is the more needed, since the original objective of the test—a comparison between three different ships in a given, although unknown, sea—appears to be rather difficult with ships so similar in main dimensions and section shape. The correlation will therefore have to be carried out mainly as a comparison between the behavior of a full-scale destroyer in general (the results of the sea tests for all three destroyers lumped together) with the behavior of a model of these destroyers; and for this type of correlation a measurement of the wave conditions would have been desirable.

It is true the hindcast of the wave conditions with the Neumann theory helps somewhat in this direction, but, a unidirectional spectrum, which, as the writer understands, was used for this hindcast, gives only a poor substitute for full-scale conditions. This is especially clear from Fig. 12, where a unidirectional spectrum would have predicted a clearly defined minimum pitch angle in beam seas with maxima in head and following seas; this does not follow at all from this figure and it can only be explained with a multidirectional spectrum. The writer, being in the happy circumstances of having both the full-scale as well as the model measurements in hand, is tempted to make a rough comparison between the two types of tests, but he should refrain from it, a preliminary result being liable to give other results than a detailed investigation, as has been demonstrated with the different accounts of the full-scale results themselves. A detailed comparison is being made at this time and the results will be published in due course. However, three points of agreement between the full-scale and the model tests are certain:

1 Slamming pressures in the model are in the same order of magnitude as the full-scale tests.

2 The vertical wave bending moments of prototype Ship A as well as of the model are smaller than those of the other ships.

3 Rolling amplitudes of the ship with the smallest natural period of roll are the largest for the prototype as well as for the model.

The writer agrees with the proposal of the authors to use the midship high-frequency stresses, induced by the impact pressure, associated with a slam, as an indication of the occurrence and a measure of the severity of the slam. At the NSMB this method is also used for model work and is to be preferred over the method which uses the sudden deceleration of the ship, provided the model is cut in halves and midship bending moments are recorded. In many cases the deceleration cannot be observed, while the high-frequency vibration is still present, indicating that somewhere an impact has taken place.

The writer disagrees with the meaning of the comparison of estimated and observed pressures during slamming, as presented in the last part of the paper. He is of the opinion that the input data for the calculation (phase lags, wave height, motion amplitudes) were derived from such a variety of sources unrelated to the ship in question, that the agreement, although being reasonable, is a mere matter of chance. A comparison for such a phenomenon, highly sensitive to small changes in the phase differences, has meaning only in relation to accurate input data.

Prof. G. P. Weinblum, ***Member;*** It has been realized from the beginning that the lack of seaway measurements would constitute a severe handicap. Therefore, it has been decided to compensate for

---

[17] Netherlands Ship Model Basin, Wageningen, Holland.

this shortcoming by using the splendid idea of comparative trials which as such can yield independently useful results.

Obviously, however, neither this sound approach nor the painstaking conjectures introduced by the authors could fill out the basic gap. Consequently, in the otherwise excellent summary, occasionally trivial statements occur.

However, the authors present results and methods which, in my opinion, completely justify the strenuous efforts invested. As such, I draw attention to:

1 The finding of the importance of "transverse bending" or, to put in a clearer form, the longitudinal bending in a horizontal plane.

2 Statistical data on slamming. In this field, only sporadic full-scale data were available. Again, of course, the lack of an adequate seaway description makes generalization of results somewhat more difficult, but here the lack is of minor importance. I think the profession is extremely gratified by the statements made under points 8–13 of the summary.

3 The general methods of evaluating statistical data. In this respect, we do not have much experience in our profession so that the present work may serve as a blueprint for future investigations. Further, the continuous efforts made by the authors to bolster up meager experimental data by results of appropriate theoretical considerations must be mentioned.

The authors themselves point out the great amount of work involved in the evaluation of the experiments. This again emphasizes how important a *thorough planning* of such projects is lest a discrepancy arises between possible results and the work invested.

Authors' Closure

The authors wish to express their sincere thanks to those who participated in the discussions through their written or oral contributions. It is realized that it was no simple task to discuss on short notice a paper which took some years to prepare. That so many had the courage to take the challenge is gratefully acknowledged. In the discussions, many subjects have been covered, but before going into details one important item should be mentioned. This paper is primarily a data report and describes only a part of a large research program on the seakeeping characteristics of destroyers. This program also includes extensive model tests which have been run in the Netherlands Ship Model Basin at Wageningen and probably will be extended as a result of this paper. The results of the model tests together with the results of a correlation study will be made available as soon as possible.

Lack of Wave Measurements And Its Effect on Data Analysis

Dr. Weinblum, Professors Korvin-Kroukovsky and Pierson and Messrs. Newton, Lum and Vossers, all discuss the problem connected with the lack of precise wave measurements. This deficiency in the present trials is acknowledged. However, the description of a wave-height meter by Tucker in 1952 does not alter the fact that during the trials a meter was not available. For this reason, additional trials on a sister ship of Ship B are now being planned in which wave-height data will be obtained. This will indeed augment the value of these present trials and make more meaningful the correlation studies between model and full-scale results. In regard to Prof. Korvin-Kroukovsky's remark on the usefulness of model tests to fill some of the lacking data of the full-scale trials, it should be pointed out that correlation between full-scale and model tests must first be established. With our present knowledge it seems hardly reasonable to use model test results in the interpretation of the trial data.

Mr. Lum has very rightly shown us the way to obtain the maximum information from these present trials, regardless of the lacking sea-state data, and this was exactly what the originators of the trials had in mind when they planned the trials with three ships. The only reason that an analysis of the data as suggested by Mr. Lum has not yet been fully worked out is the lack of sufficient model data, and the correlation study which must first be done. However, it is felt that it is now too early and optimistic to suggest that through a spectral treatment as described, full-scale trials will be unnecessary in future seakeeping studies.

Stresses and Bending Moments

Mr. Vasta would like to have more details and design data on the measured stresses. When talking about the particular stress measurements, we should not forget that the main reason for having strain gages in the ships was to have an indication on vertical vibrations due to slamming. However, the urgent wish for more useful and reliable design data to replace the presently used $L/20$ static-wave calculation for stresses is understood.

To use the data which have been collected so far, it is necessary to go into statistical details, however difficult and unreal it may appear to the naval architect in the design office. He must try to understand the statistical approach, which he may even find, as the authors did, so interesting that he will like it and use it.

For design data, it is necessary to consider the problem of extreme values. This is a field where even statisticians pause, and although the authors would like to give the design offices some more definite design figures, the answer cannot be given on the basis of the relatively small amount of trial results presented here.

In reply to Mr. Vasta's question concerning comparison of stresses in Table 13 and those listed in the summary, point 5; both values were obtained statistically; however, those in the summary are characteristic double amplitudes obtained at 17 knots in a condition which resulted in the largest stresses during the entire trials. Those in Table 13 are the most probable maximum stresses expected from the number of oscillations encountered in 12 hr operation (7000 oscillations) at 12 knots in head seas.

These data are compared to the expected and measured values during the 10-min runs which resulted in almost 200 oscillations as follows:

In Table 13, the estimated most probable maximum stresses for a 12-hr period are as follows:

BX1.2...21.9kpsi
SX1.2...16.2kpsi
AX1.2...19.4kpsi

The actual measured maxima are:

BX1.2...12.8kpsi
SX1.2....9.0kpsi
AX1.2...12.6kpsi

The estimated maxima for the number of oscillations which occurred during the test are:

BX1.2...15.3kpsi
SX1.2...13.0kpsi
AX1.2...17.2kpsi

Dr. Ochi also discusses stresses and the ratio between measured and calculated bending moments. From the results of these trials it appears that this ratio is considerably less than 1 due to the effect of body wave interference on the dynamical strength of ships in waves. However, it is not advisable to review the calculation on this basis because dynamical effects can include slamming which in some cases increases the measured stresses by as much as a 100 per cent so that the mentioned ratio has a tendency to be very nearly 1 or even higher.

Dr. Ochi and Dr. Jasper have both commented on the position of the neutral axis and their remarks are gratefully acknowledged. The calculation of the instantaneous position of the neutral axis when only two strain-gage readings are available is indeed questionable but the calculations were performed only as a check to investigate the validity of the conversion from measured stresses to bending moments; which conversion assumes a well-defined neutral axis and I/y. Since for Ship B the calculated instantaneous positions of the neutral axis are considerably below that position which follows from the computation of the section modulus, the assumption of the existence of local stresses does not seem to be unrealistic, and consequently the bending moments might be somewhat in error. The assumption however, that the bending moments in Ship S may be too small by approximately 20 per cent are not supported by the model test results. However, the discrepancies which still exist are not clarified and during further analyses the suggestion of Dr. Ochi certainly merits further attention.

The same applies for his remarks in regard to the effect of speed on bending moments. The computation of energy spectra and response-amplitude operators for stresses might possibly show that apart from the wave height, wave period is an important parameter for the description of bending moments. However, the bending moment appears to be primarily dependent on the wave height and independent of the ship's draft, this being the physical meaning of the used dimensionless expression.

In reply to Mr. Abrahamsen's remarks concerning the mounting of the strain gages so that true stresses could be obtained, the gages were mounted on continuous members so as to keep local effects at a minimum. However, the gages were not mounted in pairs, therefore some local effect may distort the stress pattern for relatively small variations. This is mentioned in the paper; however, these effects should not be significant in determining the higher stresses which are of prime concern to the ship designer.

The hogging and sagging stresses could not be separated in the analysis of the data; therefore their relative magnitudes cannot be evaluated. For this reason reference to stresses in the paper refer to magnitude of stress variations. This is discussed in the second paragraph of the section on "Stresses and Bending Moments Due to Waves."

Calibration of the strain gages of the type Mr. Abrahamsen suggests was, to the knowledge of the authors, not undertaken. This would indeed have simplified the data analysis assuming the method is practical for application to a destroyer whose tank capacities are somewhat limited. However the primary purpose of the stress study, as envisioned by the originators of the trials, was to determine the additional stresses due to slamming. For this purpose the ratio between the slam and wave-induced stresses is sufficient. It is therefore doubtful if such an extensive calibration procedure was deemed necessary.

Slamming

In connection with Commander DuCane's remarks on slamming, the occurrence of slamming was related to bow emergence, not pitch amplitude. Bow emergence is a necessary prerequisite for the occurrence of a slam. This statement makes a distinction between slamming and whipping or bow hitting.

Pitch amplitude is important in so far as it contributes to the relative velocity between ship and wave, but other factors must be considered too. In addition to relative velocity which also depends on phase relations between bow and wave, form effect must be considered. Commander DuCane doubts if relative velocity was high at 0.26 L where the maximum pressure was measured. This is probably true, but it should be pointed out that a relatively smaller velocity could produce a more serious slam at this location than that required at more forward locations where the sections are much finer.

In reply to Dr. Jasper's and Mr. Vosser's comment concerning the comparison of estimated and observed slamming pressures, some assumptions were necessarily introduced since the waves were not measured. These assumptions are all clearly stated in the paper. It should be pointed out that the comparison was made to illustrate the accuracy with which slamming pressure can be estimated if realistic parameters are known or assumed. Does the theory at least give a proper order of magnitude or is it far afield? For the authors at least it is encouraging that the theory does give reasonable results for realistic relative velocities. The problem of course is estimation of the relative velocity in a confused sea and this problem is currently under study at the Model Basin.

Dr. Ochi suggests that some of the slams in Fig. 31 in bow and beam seas may have resulted from side hitting which induced vibratory stresses in the hull rather than bottom impact. This may be. Had the oscillograph been operated at sufficiently high speed during these tests to detect the high-frequency impact pressures, this question could be resolved more easily. However, bow emergence was associated with these slams and another explanation can be made for the slamming at these relative headings. Heading in the figure refers to relative direction to the sea. However, significant swell was present also and this swell had a direction of about 65 deg to the sea. This means that where the sea was approaching the ship on the bow or beam, the swell components actually had an angle of only about 20 deg. Therefore some of the slams for these headings could have resulted from encounter with the swell rather than the sea.

Professor Korvin-Kroukovsky discusses Fig. 11 which he says clearly demonstrates the different nature of a destroyer slamming compared to that of cargo ships. It should be pointed out that slamming due to bottom impact is not shown in this figure due to insufficient resolution in the oscillograph record. Thus the curves in this region are dotted in Fig. 11. These pressures are the result of the combined wave and ship motion and are of relatively long duration–about 3.5 to 4 sec. For such pressures the emergence of the bow is not necessary since they can result from bow flare as Professor Korvin-Kroukovsky suggests. But as Dr. Ochi pointed out in his discussion there is another type of pressure which is also recognized. This is the pressure resulting from bottom impact when the bow emerges and then reenters. This takes the form of sharp peak pressures of very short duration. During the "special slam test," where the oscillograph was operated at sufficiently high speed, many such pressures were observed. A typical example is shown in Fig. 34 as Mr. Lewis points out. The average duration of pressures of this type obtained during the trials was 0.02 sec.

Therefore, the pressures associated with the immersion of the flared bow are not necessarily greater than those due to impact, for a destroyer. It is agreed, however, that the resultant stresses may be greater for the former type of load because it is of relatively longer duration and acts over a greater area.

Individual Problems

In reply to Mr. Lewis' questions, the natural pitching periods were determined from model scale tests. The longitudinal radius of gyration was calculated on the basis of the given weight distribution, and Ship S has a relatively smaller radius, the latter being caused by the great difference in weight of the guns on this ship as compared to those on Ships A and B.

The discussion of pitching axis is based on an operational need to know the place in the ship with the least vertical motion. It may be true that the concept of "apparent pitching axis" was introduced to define motions in regular seas and we agree that the expression was not used in the way it was originally intended. But it has been proved that in irregular seas there is a well-defined region of minimum vertical accelerations, and this was the reason for analyzing the vertical motion records in the chosen manner.

To Mr. Newton's comment concerning speed measurement, the difficulty of maintaining constant speed is admitted as well as the need for more accurate means of speed measurements. In the present trials an attempt was made to keep constant speed by varying RPM as heading dictated. This is believed preferable to maintaining RPM irrespective of heading. In future trials it is recommended that in addition to the visual observations of speed, a continuous recording be obtained. The Model Basin has already done this in connection with the Liberty ship trials and intends to use an electromagnetic log for speed measurements in the forthcoming trials of a sister ship of Ship B.

Mr. Marbury discusses the problem of sonar quenching and suggests effort should be made to study this problem when seaworthiness tests are conducted. He points out that quenching does not appear to be associated with any particular ship motion and can occur without slamming. Since the study of sonar quenching was not among the objectives of the originators of the trials the ships were not specifically instrumented for this measurement. As Mr. Marbury suggests slamming or bow emergence is not a necessary requirement. At any time when the sonar gear is sufficiently near the surface to be in turbulent waters, quenching will result. This depends on the relative position of location of the sonar gear to the wave. The motion of this point is a resultant of various modes of motion of the ship and their phase relationships. Combining this motion with that of the wave and the appropriate phase gives the relative position of this point with respect to the wave. Such an analysis cannot be made from the data of the present trials since the waves were not measured. However, in future trials where the waves are measured this type of analysis can be performed, though tedious, in order to determine the possibility of quenching even if quantitative measurements are not made.

In reply to Mr. Vasta's questions, the estimates of amplitudes of motions and stresses for the given E-values are based on the relations given on page 57. Incidentally, in Table 10, the measured quantities are those identified as "observed" and the statistically derived quantities as those identified as "predicted."

Commander DuCane suggests that Ship S is apparently a more sea-kindly form than the other two ships. Actually, the difference in the pitching and heaving amplitudes of the three ships was not marked, although it was stated in the paper that "Ship S showed a slight tendency to pitch more." Ship S was the best in roll though her natural period was close to that of Ship A which proved to be the worst in this respect. However Ship S was equipped with larger bilge keels than Ship A.

Professor Korvin-Kroukovsky points out that Ship S slammed less frequently than Ships A and B. Of course, in slamming, not only is the magnitude of vertical motion (or velocity) important but also the phase between wave and bow motion. As Professor Korvin-Kroukovsky rightly observes, when comparing ships at the same speeds, the ship with the shortest natural pitching period will have the more favorable phase relationship to the waves. However, the period for Ship S cannot readily be attributable to form characteristics because as Professor Lewis points out she has finer load waterlines and the largest displacement-length ratio. It is more likely the result of the shorter gyradius due to the more favorable weight distribution on this ship.

On the basis of the foregoing it is not so clear that Ship S can necessarily be considered a "more sea-kindly form."

The Rayleigh Distribution

Dr. Jasper and Professor Pierson indicated some concern about our apparent repudiation of the Rayleigh distribution as a universal, one-parameter distribution, suitable for all conditions. Dr. Jasper reminds us of the successes he has had in the past with this distribution, and this is very proper. We had hoped that we would be equally successful. Unfortunately, this did not turn out to be the case, and for these data, at least, the distribution just did not work consistently.

Dr. Jasper is quite critical of our use of the parameter E_p to characterize a given response, rather than $E_J/4$, estimated from the mean square peak-to-peak variation. The latter is claimed to be the most efficient statistical estimate. This is true only when the sample comes from a Rayleigh distribution, and since we were beginning to doubt this assumption, we used E_p, which provides a forced fit in the region of greatest interest. We have carried out a comparison of E_p and $E_J/4$ for most of the runs listed in Table 22, using E(spectrum) as a standard. The results are a statistical "draw." E_p was a better estimate of E(spectrum) in 22 cases, and worse in 18 cases. On the other hand, the average "error" in E_J(spectrum) was 2 per cent, while the corresponding "error" for E_p for these runs was 4 per cent. The mean square error was slightly greater for $E_J/4$ than for E_p. The net conclusion then, is that there is little to choose between them. Still, it is admitted that the χ^2 test would be more meaningful if it has been based upon $E_J/4$.

The next point Dr. Jasper makes is much more significant, and we have gone to some effort to determine its effect. He observes that succeeding oscillations are not independent, and the χ^2 test assumes complete independence of the sample values. This is true, particularly for roll. This fact is noted in the paper, but Dr. Jasper feels that the effect might be very large. His suggested technique of adjusting the value of χ^2 by multiplying it by $1/m$ is of course not valid until a proper value of m is known. It is far from simple to estimate, and intuition is not a useful guide. We have reexamined a number of cases which failed the χ^2 test at the 0.1 per cent level. Correlations between oscillations at various intervals were computed. The coefficient was always significant between adjacent values, but became small after 2 to 5 half oscillations. The χ^2 test was applied to five cases in which alternate values were uncorrelated, using only independent readings to determine the distribution. There was some increase in the χ^2 probability: one case rose to 2 per cent two cases rose to 1 per cent, and the remaining cases remained at the 0.1 per cent level. Tests have also been made on four other records in which only every fourth or fifth reading is used. Here the adverse effect of the decrease in size of the sample is far more evident. Three rose to 20 per cent and one remained at 0.1 per cent. These cases were chosen on the basis of length of run, rather than expected behavior under the test. It is felt the conclusion can be drawn that there is a definite increase in probability when dependence between oscillations is taken into account, but the effect is by no means as extreme as Dr. Jasper implies. The probabilities remain well below 0.5, and our conclusion is not really affected, as an unusual number of events of 20 per cent probability can be significant. If one rolls a die 100 times, and a six appears 50 times, one begins to suspect the die, even if the odds against a six on one throw are only 5 to 1.

Incidentally, the decrease in effective size of sample will have an adverse effect also on the most probable extreme value. For 200 oscillations, taking m equal to 2 would result in a 7 per cent reduction in estimated maximum, and $m = 5$ would yield 17 per cent reduction. The estimated values based on the Rayleigh distribution are already lower than the observed, and hence the bias observed by Dr. Jasper would be significantly increased.

Truncation at the low end of the distribution for the computation of E hardly seems reasonable. Truncation at the upper end is necessary in estimating E_p, but this is restricted to only 1 to 2 per cent of the observations. A similar truncation at the lower end would have negligible effect upon both E_p and E_J. And a truncation of a significantly greater number of observations is a highly questionable procedure. For it to be justifiable, a clear rule must be stated which objectively establishes the points of truncation. If one is allowed an undefined amount of freedom in any given case, then obviously one can obtain as good a fit as one desires.

It is desired to remind Dr. Jasper that our argument is not based exclusively upon the χ^2 test, but rather upon the systematic deviations from the Rayleigh distributions which were observed. That such deviations exist is now proved more than ever, as otherwise the rise in probability of the χ^2 value with decreasing sample size would be much more spectacular.

We feel that a true statement of the value of the Rayleigh distribution, and of the significance of our findings, is as follows: It is well established that the asymptotic distribution of the double amplitudes of a narrow-band Gaussian process is the Rayleigh distribution. However, as Dr. Pierson suggests, and in fact it is easy to show, this distribution cannot be be expected to hold for all cases. The true distribution may be expected to deviate from the Rayleigh when the assumed conditions are not fulfilled. It may remain a useful approximation for certain purposes but because it is an approximation, it must not be given unqualified trust.

In the present case, both conditions are violated. For roll, the well-known nonlinearity of the phenomenon for large amplitudes affects the Gaussian assumption, and it is not remarkable that some of our distributions deviate from the asymptotic distribution, particularly in the tail. It is more surprising that there is at least a hint of irregularities at low amplitudes as well.

A narrow-band process is one in which the effective band width is very small in comparison with the center frequency. None of the variables considered, even the comparatively sharply-tuned roll, strictly satisfies this requirement, so, here also, some divergence from the asymptotic distribution is to be expected. In the absence of a valid theory, one must resort to empirical tests to establish the degree to which the limiting distribution is a useful approximation. In this paper, we have merely offered some evidence that the distribution does not always fit ship-motion data very well.

INITIAL DISTRIBUTION

Copies	
5	CHBUSHIPS 3 Tech Info Br (Code 335) 1 Captain Saunders (Code 106) 1 John Vasta (Code 442)
1	DIR, Towing Tank, USNA, Annapolis
1	DIR, Natl BuStand Attn: Hydrau Lab
10	CDR, ASTIA
1	DIR, Exper Nav Tank, Univ of Mich, Ann Arbor
1	DIR, Hydrau Model Basin, Univ of Calif, Richmond
2	Head, Dept NAME, MIT, Cambridge 1 Prof. M.A. Abkowitz
1	DIR, Robinson Model Basin, Webb Inst of Tech, Glen Cove
1	DIR, Fluid Mech Lab, Univ of Calif, Berkeley
1	DIR, Hydrau Lab, Colorado St Univ, Ft Collins
2	DIR, Iowa Inst of Hydrau Res, Iowa City 1 Dr. L. Landweber
1	DIR, St. Anthony Falls Hydrau Lab, Univ of Minnesota, Minneapolis
1	DIR, Scripps Inst of Oceanog, Univ of Calif, LaJolla
2	DIR, DL, SIT, Hoboken 1 Mr. Paul Spens
1	DIR, Towing Tank, Convair Div of Genl Dyn Corp, San Diego
2	NNS & DD Co, Newport News 1 DIR, Hydrau Lab
1	Math Reviews
1	CNO, Sci Analy, Op Eval Gr (Op 03EG) Att: Mr. Fendall Marbury, Jr.
1	O in C, PGSCOL, Webb Attn: Prof. E.V. Lewis
1	CO & DIR, USNMDL Attn: Dr. N.H. Jasper
1	Mr. Leo Tick, Res Div, New York Univ, N.Y.
1	Mr. Olswald J. Sibul, Dept of Engin, Inst of Engin Res, Univ of Calif, Berkeley
1	Dr. W.J. Pierson, Jr., Dept of Meteorology & Oceanography, New York Univ, N.Y.
1	Prof. Seizo Motora, Dept of Nav Arch, Univ of Mich, Ann Arbor
1	Dr. M. St. Denis, Inst for Def Analyses, WPNSYSEVALGRP
1	Prof. B.V. Korvin-Kroukovsky, Suffer, N.Y.

Copies	
1	Dr. Columbus O'D. Iselin, WHOI, Woods Hole
1	Mr. Vito L. Russo, Chief, Div of Prelim Des, Maritime Admin
1	Mr. J. Chadwick, Sperry Gyroscope, Mineola, N.Y.
1	Mr. S.M.Y. Lum, 1739 Delaware St, Berkeley 3, Calif
1	Dr. A. Andreoni, Tech Res Inst, Exper Tank, Sao Paulo, Brazil
1	Dir, Model Testing Basin, NRC, Ottawa, Canada
1	Dr. F.H. Todd, Supt of Ship Div, NPL, Teddington, England
1	Mr. E.C. Tupper, Admiralty Exper Works, Haslar, Gosport, Hampshire, England
1	Mr. Peter DuCane, Dir, Vosper Ltd, Portsmouth, England
8	ALUSNA, London, England
1	Dr. G. Weinblum, Institut fur Schiffbau Berlinertor 21, Hamburg 1, Germany
1	Attn: Dr. Ing. O. Grim, Dir Hamburgische Schiffbau-Versuchsanstalt, Bramfelderstrasse 164, Hamburg 33, Germany
1	Ir. G. Vossers, Nederlandsch Scheepsbouwkundig Proefstation, Haagsteeg 3, Wageningen, Holland
1	Chief, Meguro Model Basin, Transp Tech Res Inst, Tokyo, Japan
1	Dir, Exper Tank, Mitsubishi Shipbldg & Engin Co, Ltd, Nagasaki, Japan
1	Prof. H. Kato, Univ of Tokyo, Tokyo, Japan
1	Ir. W.H. Wamsinck, Steamship Co, "Nederland," Amsterdam, Netherlands
1	Mr. Egil Abrahamsen, Asst Ch, Dept for Ship and Mach Res, Det Norske Veritas P.O. Box 82, Oslo, Norway

David Taylor Model Basin. Report 1559.
SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS, by Margaret D. Bledsoe, Otto Bussemaker, and W.E. Cummins. Nov 1961. 101p. illus., photos., graphs, tables, refs. (Presented at Society of Naval Architects and Marine Engineers, Spring Meeting, Washington, D.C., May 26–28, 1960) UNCLASSIFIED

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not

1. Destroyers--Seaworthiness--Trials
2. Destroyers--Motion--Measurement
3. Seaworthiness
4. Destroyers--Netherlands

I. Bledsoe, Margaret D.
II. Bussemaker, Otto
III. Cummins, William E.

David Taylor Model Basin. Report 1559.
SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS, by Margaret D. Bledsoe, Otto Bussemaker, and W.E. Cummins. Nov 1961. 101p. illus., photos., graphs, tables, refs. (Presented at Society of Naval Architects and Marine Engineers, Spring Meeting, Washington, D.C., May 26–28, 1960) UNCLASSIFIED

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not

1. Destroyers--Seaworthiness--Trials
2. Destroyers--Motion--Measurement
3. Seaworthiness
4. Destroyers--Netherlands

I. Bledsoe, Margaret D.
II. Bussemaker, Otto
III. Cummins, William E.

David Taylor Model Basin. Report 1559.
SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS, by Margaret D. Bledsoe, Otto Bussemaker, and W.E. Cummins. Nov 1961. 101p. illus., photos., graphs, tables, refs. (Presented at Society of Naval Architects and Marine Engineers, Spring Meeting, Washington, D.C., May 26–28, 1960) UNCLASSIFIED

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not

1. Destroyers--Seaworthiness--Trials
2. Destroyers--Motion--Measurement
3. Seaworthiress
4. Destroyers--Netherlands

I. Bledsoe, Margaret D.
II. Bussemaker, Otto
III. Cummins, William E.

David Taylor Model Basin. Report 1559.
SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS, by Margaret D. Bledsoe, Otto Bussemaker, and W.E. Cummins. Nov 1961. 101p. illus., photos., graphs, tables, refs. (Presented at Society of Naval Architects and Marine Engineers, Spring Meeting, Washington, D.C., May 26–28, 1960) UNCLASSIFIED

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not

1. Destroyers--Seaworthiness--Trials
2. Destroyers--Motion--Measurement
3. Seaworthiness
4. Destroyers--Netherlands

I. Bledsoe, Margaret D.
II. Bussemaker, Otto
III. Cummins, William E.

recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.

recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.

recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.

recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.

David Taylor Model Basin. Report 1559.
SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS, by Margaret D. Bledsoe, Otto Bussemaker, and W.E. Cummins. Nov 1961. 101p. illus., photos., graphs, tables, refs. (Presented at Society of Naval Architects and Marine Engineers, Spring Meeting, Washington, D.C., May 26–28, 1960) UNCLASSIFIED

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not

1. Destroyers--Seaworthiness--Trials
2. Destroyers--Motion--Measurement
3. Seaworthiness
4. Destroyers--Netherlands
I. Bledsoe, Margaret D.
II. Bussemaker, Otto
III. Cummins, William E.

David Taylor Model Basin. Report 1559.
SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS, by Margaret D. Bledsoe, Otto Bussemaker, and W.E. Cummins. Nov 1961. 101p. illus., photos., graphs, tables, refs. (Presented at Society of Naval Architects and Marine Engineers, Spring Meeting, Washington, D.C., May 26–28, 1960) UNCLASSIFIED

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not

1. Destroyers--Seaworthiness--Trials
2. Destroyers--Motion--Measurement
3. Seaworthiness
4. Destroyers--Netherlands
I. Bledsoe, Margaret D.
II. Bussemaker, Otto
III. Cummins, William E.

David Taylor Model Basin. Report 1559.
SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS, by Margaret D. Bledsoe, Otto Bussemaker, and W.E. Cummins. Nov 1961. 101p. illus., photos., graphs, tables, refs. (Presented at Society of Naval Architects and Marine Engineers, Spring Meeting, Washington, D.C., May 26–28, 1960) UNCLASSIFIED

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not

1. Destroyers--Seaworthiness--Trials
2. Destroyers--Motion--Measurement
3. Seaworthiness
4. Destroyers--Netherlands
I. Bledsoe, Margaret D.
II. Bussemaker, Otto
III. Cummins, William E.

David Taylor Model Basin. Report 1559.
SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS, by Margaret D. Bledsoe, Otto Bussemaker, and W.E. Cummins. Nov 1961. 101p. illus., photos., graphs, tables, refs. (Presented at Society of Naval Architects and Marine Engineers, Spring Meeting, Washington, D.C., May 26–28, 1960) UNCLASSIFIED

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not

1. Destroyers--Seaworthiness--Trials
2. Destroyers--Motion--Measurement
3. Seaworthiness
4. Destroyers--Netherlands
I. Bledsoe, Margaret D.
II. Bussemaker, Otto
III. Cummins, William E.

recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.

recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.

recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.

recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.

David Taylor Model Basin. Report 1559.
SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS, by Margaret D. Bledsoe, Otto Bussemaker, and W.E. Cummins. Nov 1961. 101p. illus., photos., graphs, tables, refs. (Presented at Society of Naval Architects and Marine Engineers, Spring Meeting, Washington, D.C., May 26–28, 1960) UNCLASSIFIED

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not

1. Destroyers--Seaworthiness--Trials
2. Destroyers--Motion--Measurement
3. Seaworthiness
4. Destroyers--Netherlands
I. Bledsoe, Margaret D.
II. Bussemaker, Otto
III. Cummins, William E.

David Taylor Model Basin. Report 1559.
SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS, by Margaret D. Bledsoe, Otto Bussemaker, and W.E. Cummins. Nov 1961. 101p. illus., photos., graphs, tables, refs. (Presented at Society of Naval Architects and Marine Engineers, Spring Meeting, Washington, D.C., May 26–28, 1960) UNCLASSIFIED

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not

1. Destroyers--Seaworthiness--Trials
2. Destroyers--Motion--Measurement
3. Seaworthiness
4. Destroyers--Netherlands
I. Bledsoe, Margaret D.
II. Bussemaker, Otto
III. Cummins, William E.

David Taylor Model Basin. Report 1559.
SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS, by Margaret D. Bledsoe, Otto Bussemaker, and W.E. Cummins. Nov 1961. 101p. illus., photos., graphs, tables, refs. (Presented at Society of Naval Architects and Marine Engineers, Spring Meeting, Washington, D.C., May 26–28, 1960) UNCLASSIFIED

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not

1. Destroyers--Seaworthiness--Trials
2. Destroyers--Motion--Measurement
3. Seaworthiness
4. Destroyers--Netherlands
I. Bledsoe, Margaret D.
II. Bussemaker, Otto
III. Cummins, William E.

David Taylor Model Basin. Report 1559.
SEAKEEPING TRIALS ON THREE DUTCH DESTROYERS, by Margaret D. Bledsoe, Otto Bussemaker, and W.E. Cummins. Nov 1961. 101p. illus., photos., graphs, tables, refs. (Presented at Society of Naval Architects and Marine Engineers, Spring Meeting, Washington, D.C., May 26–28, 1960) UNCLASSIFIED

This paper presents the results of destroyer trials conducted under the joint sponsorship of the Royal Netherlands and United States Navies. Three destroyers of different types participated in the trials. The purpose was to obtain sufficient data for evaluating their relative seakeeping ability when operating parallel in the same seaway. Motions, stresses, accelerations, and slamming pressures were measured for a series of speeds and headings in two different sea conditions in order to obtain a representative picture of the behavior of the ships. While the sea was not

1. Destroyers--Seaworthiness--Trials
2. Destroyers--Motion--Measurement
3. Seaworthiness
4. Destroyers--Netherlands
I. Bledsoe, Margaret D.
II. Bussemaker, Otto
III. Cummins, William E.

recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.

recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.

recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.

recorded, photographs were taken and an attempt was made to reconstruct the sea state from hindcasts. With the exception of slamming, a statistical approach was used in the analysis of the data and the results are expressed in terms of a parameter E which defines the behavior of the double amplitudes. In the slamming analysis, pressures and the increased vibratory stresses are noted and an attempt was made to derive the impact pressure theoretically for one slamming condition. All results are presented in the form of plots and for completeness many are also recorded in tabular form.