Report 1336

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DEPARTMENT OF THE NAVY
DAVID TAYLOR MODEL BASIN

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METHODS OF ELASTIC ANALYSIS OF CIRCULAR BULKHEAD STIFFENING SYSTEMS

by

CDR S.R. Heller, Jr., USN, and Peter M. Palermo

[redacted]

STRUCTURAL MECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

November 1959

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DAVID TAYLOR MODEL BASIN
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3 Dec 1959

From: Commanding Officer and Director, David Taylor Model Basin
To: Chief, Bureau of Ships (335) (in duplicate)

Subj: Submarine bulkhead stiffening systems; methods of elastic analysis for

Ref: (a) DATMOBAS CONFIDENTIAL Report C-1005 of Aug 1959

Encl: (1) DATMOBAS Report 1336 entitled "Methods of Elastic Analysis of Circular Bulkhead Stiffening Systems" 3 copies

1. Deficiencies in the present procedure for designing internal bulkheads of submarines were pointed out in reference (a) and indicate the need for new analyses and new design procedures. In enclosure (1) the present Bureau of Ships design procedure for the main horizontal girder and vertical stiffener is modified to include the effects of shell deformation, end load, and variable end fixity. In addition, allowance is made for deformation of the vertical stiffeners at the intersection with the main horizontal girder. Calculations based on these elastic analyses agree with experimental evidence.

E. E. Johnson

E.E. JOHNSON
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NOTATION

A	Total area of section. Subscripts l, m, and u refer to lower, middle, and upper spans of vertical stiffeners, respectively
A_f	Gross area of flanges, $2bt_f$
A_w	Gross area of web, $2ht_w$
a	Arbitrary point
b	Flange width
E	Modulus of elasticity
e	Eccentricity of end load. Subscripts l and u refer to lower and upper ends of vertical stiffeners, respecitively
F	End load on vertical stiffener
f	Fixity factor
G	Shear modulus
h	Half-height of section
I	Moment of inertia of section. Subscripts l, m, and u refer to lower, middle, and upper spans of vertical stiffeners, respectively
k	Shear factor. Subscripts l, m, and u refer to lower, middle, and upper spans of vertical stiffeners, respectively
L	Span length. Subscripts l, m, and u refer to lower, middle, and upper spans of vertical stiffeners, respectively
M	Bending moment. Subscripts L and R refer to fixed end moments at left and right end of main girder, respectively; subscripts l and u refer to fixed end moments at lower and upper ends of vertical stiffeners, respectively
P	Concentrated load on main girder. Subscript n refers to position from end
p	Pressure
Q	Moment of area of section about axis through its centroid
Q_a	Moment of area of an elliptical segment about its chord
r	Radius of bulkhead
s	Stiffener spacing. Subscript n refers to position of stiffener from end
T	End load on main girder
t	Thickness. Subscripts f and w refer to flange and web, respectively

V	Vertical shearing force. Subscripts L and R refer to support reactions at left and right ends of main girder, respectively; subscripts b, g, l, and u refer to support reactions at second main girder, main girder, lower end, and upper end of vertical stiffeners, respectively
x	Distance along a member. Subscripts l and u refer to lower and upper spans of vertical stiffeners, respectively
y	Elastic deformation of a member. Subscripts l, m, and u refer to lower, middle, and upper, spans of vertical stiffeners, respectively. Subscript j refers to position of panel of main girder from end
z	Load distribution
α	Nondimensional distance, x/r
β	Square root of ratio of end load to flexural rigidity of beam. Subscripts l and u refer to lower and upper ends of vertical stiffeners, respectively
Γ	Ratio of moments of inertia of upper and lower spans of vertical stiffeners, I_l/I_u
γ	Slope at end of beam caused by vertical shear. Subscripts L and R refer to left and right ends of main girder, respectively; subscripts l and u refer to lower and upper ends of vertical stiffeners, respectively
δ	Elastic deflection of vertical stiffener at internal supports. Subscripts b and g refer to second main girder and main girder, respectively
θ	Slope at end of beam caused by bending. Subscripts L and R refer to left and right ends of main girder, respectively; subscripts l and u refer to lower and upper ends of vertical stiffener, respectively
τ	Shear stress at centroid, $k\tau_{ave}$
τ_{ave}	Average shear stress, P/A

ABSTRACT

Various methods of elastic analysis of circular bulkhead stiffening systems are presented. The present Bureau of Ships design procedure for the main horizontal girder and vertical stiffeners is modified to include the effects of shear deformation, end load, and variable end fixity. In addition, allowance is made for deflection of the vertical stiffeners at the intersection with the main horizontal girder. A grillage analysis of the entire stiffening system, including the effects of shear deformation, end load, and variable end fixity is presented. Formulas for moments, deflection, and the shear factor k are presented, and a numerical example is given.

A semigraphical, iterative design procedure that includes the advantages of the modified design procedure and the realism of the grillage analysis is presented.

INTRODUCTION

Weight is such an important characteristic and contributor to the cost of every structure that continual effort is made to eliminate unnecessary structural material. Refinement of analyses and resulting readjustment of design criteria and procedures are often profitable approaches. For these reasons submarine bulkheads were subjected to careful scrutiny.

The internal bulkheads of a submarine are installed primarily to limit the extent of flooding in the event of damage. It is apparent, therefore, that such bulkheads may be called upon to function in their primary capacity only occasionally. Safety is paramount, but deformation is secondary. To exploit the permissible deformation, an elasto-plastic or purely plastic design procedure appears to be the proper approach if weight-saving is to be effected without sacrifice of safety.

The present design procedure, outlined in Reference 1,* is not readily adaptable to include plastic effects. Moreover, the present design procedure is predicated on assumed loads which total more than the applied load. In addition, the assumed supports for some components are inconsistent with actual deflections of others. Furthermore, shear deformation and possible axial end loads are neglected. All these deficiences, previously pointed out in Reference 2, indicate the need for new analyses—and a new design procedure.

It is the purpose of this report to present elastic analyses which overcome the shortcomings of the design procedure now in use and which agree with experimental evidence.

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*References are listed on page 35.

GENERAL CONSIDERATIONS

The present design procedure[1] assumes a loading condition shown cross-hatched in Figure 1. The main girder is assumed to be fixed at the ends and loaded by an elliptically distributed load that is statically equivalent to one-half the total hydrostatic load on the bulkhead. This type of load distribution is realized only if the intersecting vertical stiffeners are bisected by the main girder, are completely fixed at the juncture with the pressure hull, have zero deflections at the intersection with the main girder, and are loaded with the uniformally distributed load shown in Figure 1. If all these conditions are met, the reaction force at the middle of the stiffeners will be one-half the total load on the stiffeners; these reaction forces are the applied loads on the main girder. Distributing each load over an area equivalent to a stiffener spacing results in a load distribution closely approximating the elliptical distribution shown.

The load distribution assumed for the vertical stiffeners is in keeping with standard practice for the design of roof and floor stringers. The present design procedure, however, assumes zero deflection of the vertical stiffeners at the intersection with the main girder. This assumption is incompatible with the assumption of flexure of the main girder.

The end load, as shown in Figure 1, is neglected in the present design procedure. However, the effect of the end load cannot be neglected if the pressure is applied to the face

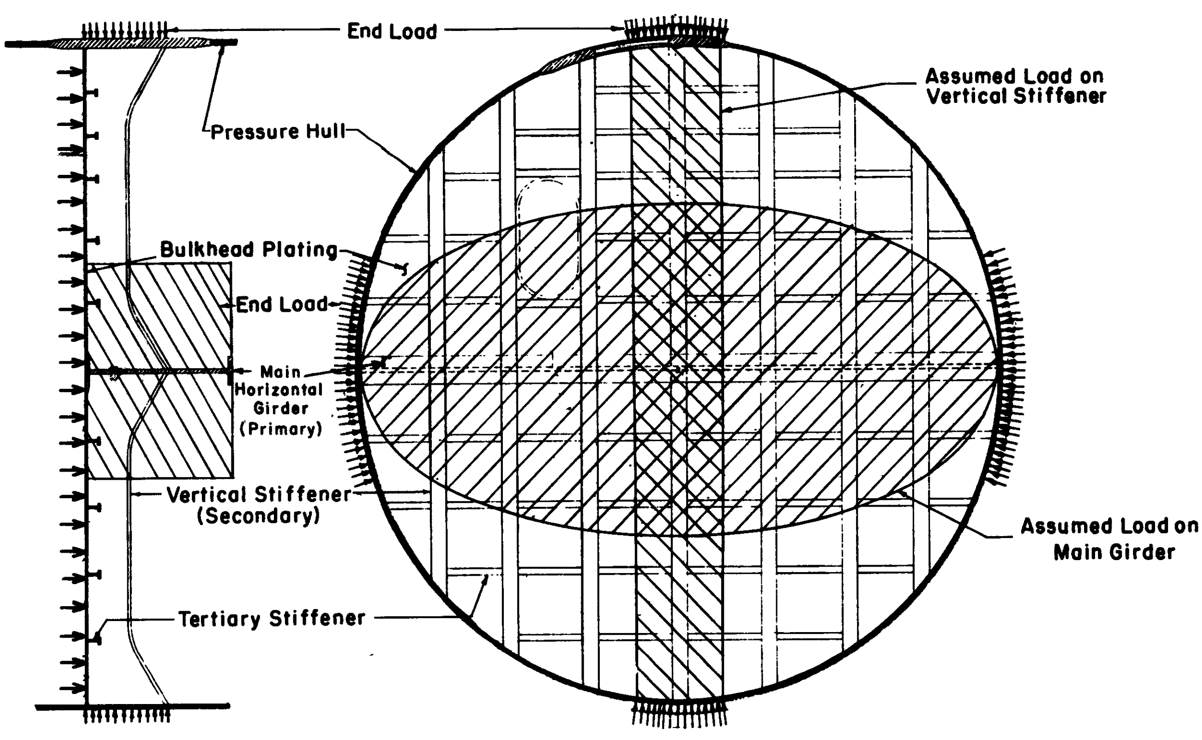
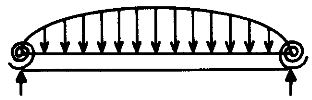
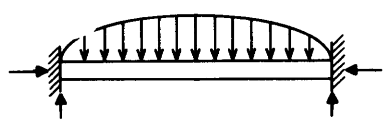
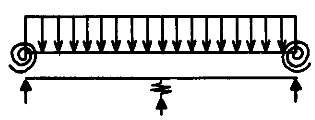
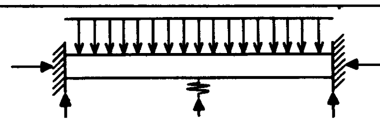
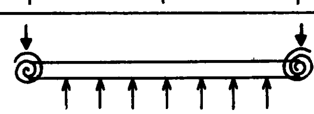
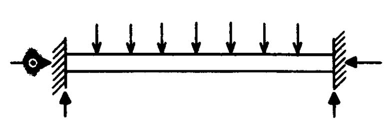


Figure 1 – Assumed Loading Configurations

of the bulkhead opposite to the stiffening-system face. This end load is assumed by the authors to be equal to the hydrostatic pressure acting over the area bounded by the length of the haunch and one-half the stiffener spacing on each side of the haunch.

Because of the relatively small depth-span ratio of the main horizontal girder, the deformation due to shear is of paramount importance. As shown in Reference 2, this effect is on the order of 130 percent of the deflection due to bending for contemporary submarine bulkheads. Therefore, the effect of the shear deformation is included in all subsequent calculations, even though it is neglected in the current design procedure.

Analysis of the results of the tests presented in Reference 2 indicates not only the necessity for including the effects of end load and shear distortion but also the apparent necessity for a grillage-type analysis. In this report the present design procedure is modified to allow for end load, shear deformation, variable end fixity, the use of a pseudo-grillage analysis* of the vertical stiffeners, and a grillage analysis of the vertical stiffeners and main girder. The cases considered are listed below.

Member	Case	Loading
Main Girder	I	 Elliptical load and variable end fixity
	II	 Elliptical load and end load for clamped beam
Vertical Stiffener (Pseudo-Grillage)	III	 Uniform load, variable fixity with elastic support at midspan
	IV	 Uniform load, end load, and elastic support at midspan for clamped beam
Main Girder and Vertical Stiffeners (Grillage)	V	 Variable fixity, loaded by midspan reactions of Case III
	VI	 Clamped beam with end load, loaded by midspan reactions of Case IV

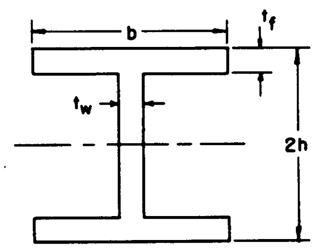
*Deflections at the intersection from Case I or Case II may be used as the deflection at the support for Case III or Case IV.

Formulas for moments and deflection are presented in this report, and a numerical example is given.

SHEAR FACTOR

As the ratio of span to depth of a beam in bending decreases, the effect of shear on the deflection becomes more important. This additional (shear) deflection is the result of the sliding of adjacent cross sections along each other. As a result of the nonuniform shear stress at any section, plane surfaces no longer remain plane; the centroids of the sections, however, remain vertical and slide along one another. Therefore, the deflection due to shear at any cross section is equal to the shear strain at the centroid of the cross section.

Consider the I-beam shown:



where $A_f = 2bt_f$

$A_w = 2ht_w$

$A_t = A_f + A_w - 2t_f t_w$

The moment of the area about the axis through the centroid is

$$Q = \frac{b}{2}\,[h^2 - (h-t_f)^2] + \frac{t_w}{2}\,[(h-t_f)^2] \qquad [1.0]$$

and the moment of inertia is

$$I = \frac{2}{3}\,[(h-t_f^3)\,t_w + bt_f^3 + 3bh^2\,t_f - 3bht_f^2] \qquad [1.1]$$

The shear stress at the centroid is

$$\tau = \frac{VQ}{It_w} \qquad [1.2]$$

or

$$\tau = \left(\frac{V}{A}\right)\frac{QA}{It_w} \qquad [1.2a]$$

Since $\dfrac{V}{A}$ is the average shear stress

$$\tau = k\,\tau_{ave}$$

and

$$k = \frac{QA}{It_w} = \frac{3 - \dfrac{A_f h^2}{I}}{2\left(1 - \dfrac{A_f}{A}\right)} \qquad [1.3]$$

In Equation [1.3] the total area of the section is used to define the average shear stress. The resulting k, for typical I-beams, is greater than 2. Normally, however, only the web area is considered to resist shear. Using the normal definition of average shear stress, V/A_w, the factor k is given by

$$k = \frac{2Qh}{I} = \frac{3h}{2(h - t_f)} - \frac{A_w h^2}{2I}\left(\frac{A_f}{A - A_f}\right) \qquad [1.4]$$

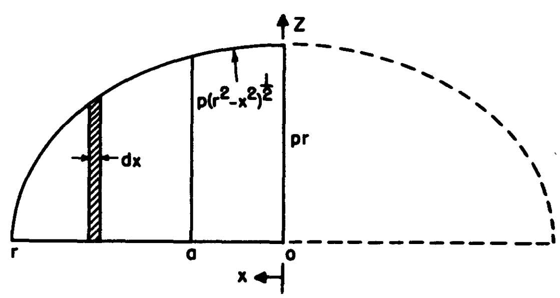
DETERMINATION OF DEFLECTIONS AND MOMENTS

In the following derivations of moments and deflections, all structural members are assumed to be of constant cross section. Although this assumption is an oversimplification of the general problem, any rigorous attempt to include the variation in inertia across the beam would result in very cumbersome expressions, with no real improvement in the computed deflections and stresses at midspan.

For general use by the designer, the equations presented in this section are easily solved and can be readily adapted to an electronic computer.

CASE I – UNIFORM BEAM WITH AN ELLIPTICALLY DISTRIBUTED LOAD AND VARIABLE END FIXITY

Consider a semiellipse the total area of which is one-half the total load on the bulkhead:



This load distribution is defined by

$$z = p\,(r^2 - x^2)^{1/2} \qquad [2.0]$$

The moment of area of any elliptical segment about its chord is

$$Q_a = \int_a^r z(x-a)\, dx \qquad [2.1]$$

Integration between the indicated limits yields

$$Q_a = \frac{pr^3}{3}\left[1-\left(\frac{a}{r}\right)^2\right]^{3/2} - \frac{\pi pr^3}{4}\left(\frac{a}{r}\right) + \frac{pr^3}{2}\left(\frac{a}{r}\right)^2\left[1-\left(\frac{a}{r}\right)^2\right]^{1/2} + \frac{pr^3}{2}\left(\frac{a}{r}\right)\sin^{-1}\left(\frac{a}{r}\right)$$

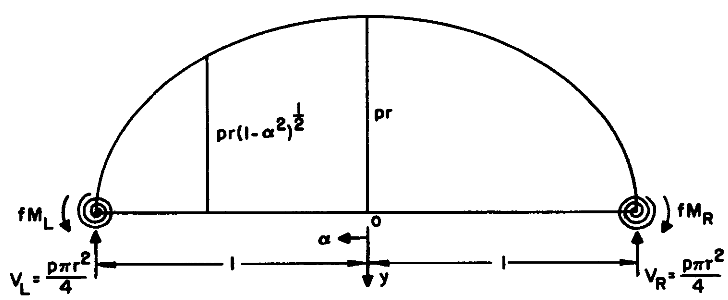
[2.2]

Since a was an arbitrary point, the general form of Equation [2.2] is obtained by substituting x (variable point) for a (fixed point):

$$Q_a = \frac{pr^3}{3}\left(1-\alpha^2\right)^{3/2} - \frac{\pi pr^3}{4}\alpha + \frac{pr^3\alpha^2}{2}(1-\alpha^2)^{1/2} + \frac{pr^3\alpha}{2}\sin^{-1}\alpha \qquad [2.2a]$$

where $\alpha = \dfrac{x}{r}$.

Now consider an elastically restrained beam subjected to such an elliptically distributed load:



The bending moment at any point α is

$$M = fM_L - V_L r(1-\alpha) + \frac{pr^3}{3}(1-\alpha^2)^{3/2} - \frac{p\pi r^3\alpha}{4} + \frac{pr^3\alpha^2}{2}(1-\alpha^2)^{1/2} + \frac{pr^3\alpha}{2}\sin^{-1}\alpha \qquad [2.3]$$

where M_L is the fixed end moment, and the differential equation of the elastic curve of the beam, considering the effect of shear distortion, may be written

$$EI\frac{d^2y}{dx^2} = \left[M - \frac{kEI}{AG}pr(1-\alpha^2)^{1/2}\right] \qquad [2.4]$$

Since

$$\alpha = \frac{x}{r}$$

$$\frac{dy}{dx} = \frac{dy}{d\alpha} \cdot \frac{d\alpha}{dx} = \frac{1}{r} y'$$

$$\frac{d^2y}{dx^2} = \frac{1}{r^2} y''$$

where primes indicate differentiation with respect to α. Therefore

$$\frac{EI}{r^2} y'' = fM_L - \frac{p\pi r^3}{4} + \frac{pr^3}{3} (1-\alpha^2)^{3/2} + \frac{pr^3\alpha^2}{2}(1-\alpha^2)^{1/2} + \frac{pr^3\alpha}{2} \sin^{-1}\alpha - \frac{kEI}{AG} pr (1-\alpha^2)^{1/2} \tag{2.5}$$

Upon integration

$$\frac{EI}{r} y' = fM_L r\alpha - \frac{p\pi r^4\alpha}{4} - \frac{pr^4\alpha}{24} (1-\alpha^2)^{3/2} + \frac{5pr^4\alpha}{16} (1-\alpha^2)^{1/2} + \frac{pr^4}{16} (1+4\alpha^2) \sin^{-1}\alpha$$
$$- \frac{kEI}{2AG} pr^2 [\alpha (1-\alpha^2)^{1/2} + \sin^{-1}\alpha] + C_1 \tag{2.6}$$

The pertinent boundary conditions are

when $\alpha = 0$, $y' = 0$ [Symmetry]

$\alpha = 1$, $\frac{1}{r} y' = (\gamma_L + \theta_L)$ [Variable fixity at support]

where γ_L is the slope due to shear

θ_L is the slope due to bending, and

$$\gamma_L = -\frac{kV_L}{AG} = -\frac{kp\pi r^2}{4AG}$$

Therefore

$$C_1 = 0$$

$$M_L = \frac{3p\pi r^3}{32}$$

$$\theta_L = \frac{M_L r}{EI} (1-f)$$

The second integration gives

$$EIy = fM_L r^2 \frac{\alpha^2}{2} - \frac{p\pi r^5 \alpha^2}{2} + \frac{pr^5}{120}(1-\alpha^2)^{5/2} - \frac{5pr^5}{48}(1-\alpha^2)^{3/2} + \frac{pr^5}{144}(17+4\alpha^2)(1-\alpha^2)^{1/2}$$

$$+ \frac{pr^5}{48}\alpha(3+4\alpha^2)\sin^{-1}\alpha - \frac{kEIpr^3}{2AG}\left[(1-\alpha^2)^{1/2} - \frac{1}{3}(1-\alpha^2)^{3/2} + \alpha\sin^{-1}\alpha\right] + C_2 \qquad [2.7]$$

The remaining boundary condition is

when $\alpha = 1$, $y = 0$ [Unyielding end support]

From which

$$C_2 = -fM_L\frac{r^2}{2} + \frac{5p\pi r^5}{96} + \frac{kEIp\pi r^3}{4AG}$$

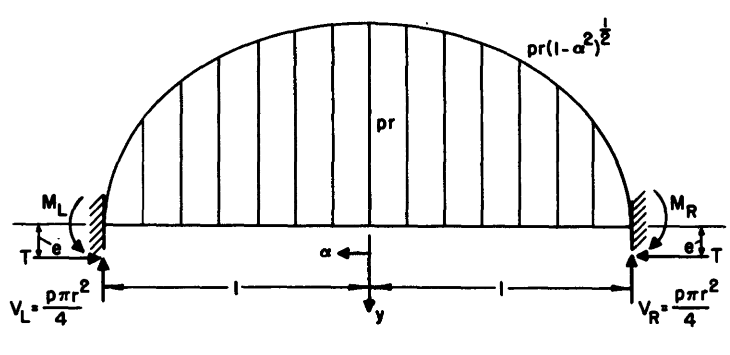
Therefore the equation of the elastic curve is

$$EIy = -\frac{fM_L r^2}{2}(1-\alpha^2) + \frac{pr^5\pi}{96}(5-12\alpha^2) + \frac{pr^5}{120}(1-\alpha^2)^{5/2} - \frac{5pr^5}{48}(1-\alpha^2)^{3/2} + \frac{pr^5}{144}(17+4\alpha^2)(1-\alpha^2)^{1/2}$$

$$+ \frac{pr^5\alpha}{48}(3+4\alpha^2)\sin^{-1}\alpha + \frac{kEIpr^3}{12AG}\left[3\pi + 2(1-\alpha^2)^{3/2} - 6(1-\alpha^2)^{1/2} - 6\alpha\sin^{-1}\alpha\right] \qquad [2.8]$$

CASE II – UNIFORM CLAMPED BEAM WITH AN ELLIPTICALLY DISTRIBUTED LOAD AND END LOAD

When Case I is extended to the condition where an end load is acting, the schematic arrangement is



The basic differential equation relating shear, curvature, and bending moment is

$$\frac{EI}{r^2}y'' + Ty = \left(M_L + Te - \frac{p\pi r^3}{4}\right) + \frac{pr^3}{3}(1-\alpha^2)^{3/2} + \frac{pr^3}{2}\alpha^2(1-\alpha^2)^{1/2} + \frac{pr^3\alpha}{2}\sin^{-1}\alpha - \frac{kEIpr}{AG}(1-\alpha^2)^{1/2} \qquad [3.0]$$

The boundary conditions are

$$\begin{aligned} &\text{when } \alpha = 1, && y = 0 && \text{[Unyielding end support]} \\ &\alpha = 1, && \frac{1}{r}y' = \gamma_L = -\frac{kp\pi r^2}{4AG} && \text{[Clamped end]} \qquad [3.1] \\ &\alpha = 0, && y' = 0 && \text{[Symmetry]} \end{aligned}$$

The complementary solution is

$$y = A\sin\beta\alpha + B\cos\beta\alpha \qquad [3.2]$$

where

$$\beta^2 = \frac{Tr^2}{EI}$$

The right-hand side must first be expressed as a power series in α. The following relations will be helpful:

$$(1-\alpha^2)^{1/2} = 1 - \sum_{n=0}^{\infty}\frac{(2n)!\,\alpha^{2(n+1)}}{2^{2n+1}n!(n+1)!}$$

$$\alpha^2(1-\alpha^2)^{1/2} = \alpha^2 - \sum_{n=1}^{\infty}\frac{(2n-2)!\,\alpha^{2(n+1)}}{2^{2n-1}n!(n-1)!} \qquad [3.3]$$

$$\alpha\sin^{-1}\alpha = \alpha^2 + \sum_{n=1}^{\infty}\frac{(2n-1)!\,\alpha^{2(n+1)}}{2^{2n-1}(2n+1)\,n!\,(n-1)!}$$

Substituting Equations [3.3] into [3.0] yields:

$$y'' + \beta^2 y = \frac{\beta^2 pr^3}{6T}\left[\frac{6}{pr^3}(M_L + Te) - \frac{3\pi}{2} + 2 - 3\sum_{n=0}^{\infty}\frac{(2n)!\,\alpha^{2(n+1)}}{(4n^2-1)\,2^{2n}\,n!(n+1)!}\right]$$

$$-\frac{kpr^3}{AG}\left[1 - \frac{1}{2}\sum_{n=0}^{\infty}\frac{(2n)!\,\alpha^{2(n+1)}}{2^{2n}\,n!(n+1)!}\right] \qquad [3.0a]$$

where the first term of the right-hand side expresses the effect of bending and end load and the second, the effect of shear deformation. To determine the particular solution of Equation [3.0a], let

$$y = C_0 + \sum_{n=0}^{\infty} C_{2(n+1)} \alpha^{2(n+1)}$$

$$y' = \sum_{n=0}^{\infty} (2n+2) C_{2(n+1)} \alpha^{2n+1} \qquad [3.4]$$

$$y'' = \sum_{n=0}^{\infty} (2n+2)(2n+1) C_{2(n+1)} \alpha^{2n}$$

Substituting Equations [3.4] into Equation [3.0a] and equating coefficients of like powers of α yields:

$$\beta^2 C_0 + 2C_2 = \frac{\beta^2 pr^3}{6T}\left[\frac{6}{pr^3}(M_L + Pe) - \frac{3\pi}{2} + 2\right] - \frac{kpr^3}{AG} \qquad [3.5]$$

$$C_{2(n+2)} = \frac{(-1)^n \beta^{2(n+1)}}{(2n+4)!}\left[\frac{pr^3}{T}\sum_{j=0}^{n}(-1)^{j+1}\frac{(2j-1)^2(2j-3)^2\cdots\cdot 5^2\cdot 3^2\cdot 1^2}{(2j-1)}\beta^{-2j} - 2C_2\right]$$

$$+\frac{kpr^3}{AG}\,\frac{(2n+1)(2n-1)\cdots\cdot 5\cdot 3\cdot 1\,(2n)!}{2^n\, n!\,(2n+4)!} \qquad [3.6]$$

Equation [3.6] is the recursion formula for the coefficients. The complete solution is normally obtained by adding the complementary and particular solutions:

$$y = A\sin\beta\alpha + B\cos\beta\alpha + C_0 + \sum_{n=0}^{\infty} C_{2(n+1)}\alpha^{2(n+1)} \qquad [3.7]$$

It is readily seen from the expansion of $\cos\alpha$

$$\cos\alpha = \sum_{n=0}^{\infty}\frac{(-1)^n\alpha^{2n}}{(2n)!} \qquad [3.8]$$

that $B\cos\beta\alpha$ is included in $C_0 + \sum_{n=0}^{\infty} C_{2(n+1)}\alpha^{2(n+1)}$. Hence $B\cos\beta\alpha$ is redundant. Therefore Equation [3.7] reduces to

$$y = A \sin \beta\alpha + C_0 + \sum_{n=0}^{\infty} C_{2(n+1)}\alpha^{2(n+1)} \quad [3.7a]$$

The use of the third of Equation [3.1] readily shows that

$$A = 0$$

Thus Equation [3.7a] becomes

$$y = C_0 + \sum_{n=0}^{\infty} C_{2(n+1)}\alpha^{2(n+1)} \quad [3.9]$$

Finally the use of the remaining two boundary equations together with Equation [3.5] produces the following system of simultaneous equations:

$$\beta^2 C_0 + 2C_2 = \frac{\beta^2 pr^3}{6\,T}\left[\frac{6}{pr^3}(M_L + Te) - \frac{3\pi}{2} + 2\right] - \frac{kpr^3}{AG}$$

$$C_0 + C_2 + \sum_{n=0}^{\infty} C_{2(n+2)} = 0 \quad [3.10]$$

$$2C_2 + \sum_{n=0}^{\infty}(2n+4)\,C_{2(n+2)} = -\frac{kp\pi r^3}{4\,AG}$$

From which

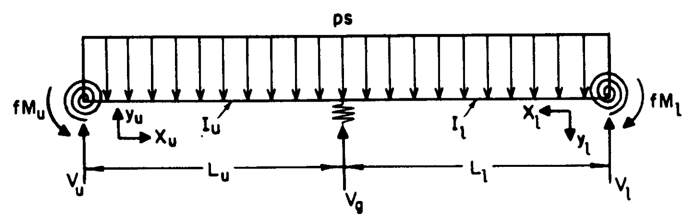
$$C_2 \doteq -\frac{\beta}{2\sin\beta}\cdot\frac{pr^3}{T}\left[\frac{kT}{AG} + \left(\frac{5\pi}{32} - \frac{1}{3}\right)\beta^2 - \left(\frac{21\pi}{768} - \frac{7}{90}\right)\beta^4 + \left(\frac{143\pi}{61{,}440} - \frac{269}{37{,}800}\right)\beta^6\right] \quad [3.11]$$

$$C_0 \doteq \frac{pr^3}{T}\left\{\frac{1-\cos\beta}{\beta\sin\beta}\left[\left(\frac{5\pi}{32} - \frac{1}{3}\right)\beta^2 - \left(\frac{21\pi}{768} - \frac{7}{90}\right)\beta^4 + \left(\frac{143\pi}{61{,}440} - \frac{269}{37{,}800}\right)\beta^6\right] - \left[\left(\frac{7\pi}{96} - \frac{17}{90}\right)\beta^2 - \left(\frac{11\pi}{1280} - \frac{323}{12{,}600}\right)\beta^4 + \left(\frac{143\pi}{258{,}048} - \frac{109}{63{,}504}\right)\beta^6\right]\right\} + \frac{kpr^3}{AG}\left(\frac{1-\cos\beta}{\beta\sin\beta} - \frac{5}{6} + \frac{\pi}{4}\right) \quad [3.12]$$

$$M_L \doteq -pr^3\left\{\frac{\cos\beta}{\beta\sin\beta}\left[\left(\frac{5\pi}{32} - \frac{1}{3}\right)\beta^2 - \left(\frac{21\pi}{768} - \frac{7}{90}\right)\beta^4 + \left(\frac{143\pi}{61{,}440} - \frac{269}{37{,}800}\right)\beta^6\right] + \left[\left(\frac{7\pi}{96} - \frac{17}{90}\right)\beta^2 - \left(\frac{11\pi}{1280} - \frac{323}{12{,}600}\right)\beta^4 + \left(\frac{143\pi}{258{,}048} - \frac{109}{63{,}504}\right)\beta^6\right] - \left(\frac{\pi}{4} - \frac{1}{3}\right)\right\} + \frac{kTpr^3}{AG}\left(\frac{1}{\beta^2} - \frac{\cos\beta}{\beta\sin\beta} - \frac{5}{6} + \frac{\pi}{4}\right) - Te \quad [3.13]$$

CASE III – UNIFORMLY LOADED BEAM WITH AN ELASTIC INTERNAL SUPPORT AND VARIABLE END FIXITY

For later use in developing a grillage analysis of the stiffener system, it will be advantageous to obtain the elastic curve for a beam elastically restrained at each end, continuous over an interior elastic support, and subjected to a uniformly distributed lateral load. Shear deformation will be included. The schematic arrangement is



where M_u and M_l are fixed end moments. The differential equation relating bending moment, effective shear deformation, and the elastic curve is

$$EI \frac{d^2y}{dx^2} = M + \frac{kEI}{AG} ps \qquad [4.0]$$

Then, for the upper span, this and the first two integrations are:

$$EI_u \frac{d^2y_u}{dx_u^2} = - fM_u + V_u x_u - \frac{psx_u^2}{2} + \frac{k_u EI_u}{A_u G} ps$$

$$EI_u \frac{dy_u}{dx_u} = - fM_u x_u + V_u \frac{x_u^2}{2} - \frac{psx_u^3}{6} + \frac{k_u EI_u}{A_u G} psx_u + A_1 \qquad [4.1]$$

$$EI_u y_u = - fM_u \frac{x_u^2}{2} + V_u \frac{x_u^3}{6} - \frac{psx_u^4}{24} + \frac{k_u EI_u}{2A_u G} psx_u^2 + A_1 x_u + A_2$$

The corresponding relations for the lower span are:

$$EI_l \frac{d^2y_l}{dx_l^2} = fM_l - V_l x_l + \frac{psx_l^2}{2} - \frac{k_l EI_l}{A_l G} ps$$

$$[4.2]$$

$$EI_l \frac{dy_l}{dx_l} = fM_l x_l - V_l \frac{x_l^2}{2} + \frac{psx_l^3}{6} - \frac{k_l EI_l}{A_l G} psx_l + B_1$$

$$EI_l y_l = fM_l \frac{x_l^2}{2} - V_l \frac{x_l^3}{6} + \frac{psx_l^4}{24} - \frac{k_l EI_l}{2A_l G} psx_l^2 + B_1 x_l + B_2 \qquad [4.2]$$

Use of the displacement boundary conditions at the exterior supports

$$x_u = 0, \quad y_u = 0$$

$$x_l = 0, \quad y_l = 0$$

leads to

$$A_2 = B_2 = 0$$

The displacement boundary condition at the interior support

$$x_u = L_u, \quad y_u = -\delta_g$$

$$x_l = L_l, \quad y_l = +\delta_g$$

leads to

$$A_1 = -\frac{EI_u \delta_g}{L_u} + \frac{fM_u L_u}{2} - \frac{V_u L_u^2}{6} + \frac{psL_u^3}{24} - \frac{k_u EI_u}{2A_u G} psL_u$$

$$M_u = \frac{2EI_u \delta_g}{L_u^2} + \frac{V_u L_u}{3} - \frac{psL_u^2}{12} + \frac{k_u EI_u}{A_u G}\left(ps - \frac{2V_u}{L_u}\right)$$

$$B_1 = \frac{EI_l \delta_g}{L_l} - \frac{fM_l L_l}{2} + \frac{V_l L_l^2}{6} - \frac{psL_l^3}{24} + \frac{k_l EI_l}{2A_l G} psL_l$$

$$M_l = \frac{2EI_l \delta_g}{L_l^2} + \frac{V_l L_l}{3} - \frac{psL_l^2}{12} + \frac{k_l EI_l}{A_l G}\left(ps - \frac{2V_l}{L_l}\right)$$

The slope boundary conditions at the exterior supports:

$$x_u = 0, \quad \frac{dy_u}{dx_u} = -\theta_u - \frac{k_u V_u}{A_u G}$$

$$x_l = 0, \quad \frac{dy_l}{dx_l} = \theta_l + \frac{k_l V_l}{A_l G}$$

where θ is slope due to bending alone, lead to:

$$\theta_u = \frac{M_u(1-f)L_u}{2EI_u}$$

$$\theta_l = -\frac{M_l(1-f)L_l}{2EI_l}$$

Continuity of slope at the interior support, i.e.,

$$\left.\frac{dy_u}{dx_u}\right|_{x_u=l_u} - \frac{k_u V_g}{A_u G} = \left.\frac{dy_l}{dx_l}\right|_{x_l=L_l}$$

yields

$$\Delta_1 V_l - \Delta_2 V_g + \Delta_3 V_u = \Delta_4 \tag{4.3}$$

where

$$\Delta_1 = \Gamma\left[\frac{L_l^2}{6}(3-2f) + \frac{k_l EI_l}{A_l G}(2f-1)\right]$$

$$\Delta_2 = \frac{k_u EI_u}{A_u G}$$

$$\Delta_3 = \frac{L_u^2}{6}(3-2f) + \frac{k_u EI_u}{A_u G}(2f-1)$$

$$\Delta_4 = \frac{ps}{12}(2-f)(L_u^3+\Gamma L_l^3) + 2fE\delta_g\left(\frac{I_u}{L_u} + \Gamma\frac{I_l}{L_l}\right) - E(2f-1)(I_u\theta_u + \Gamma I_l\theta_l)$$

$$- \frac{Eps(1-f)}{G}\left(\frac{k_u I_u L_u}{A_u} - \frac{k_l I_l L_l}{A_l}\right)$$

$$\Gamma = \frac{I_l}{I_u}$$

From the equations of static equilibrium are obtained:

$$\text{Force:} \quad V_l + V_g + V_u = \Delta_5 = ps(L_u + L_l) \tag{4.4}$$

$$\text{Moment:}\quad -\Delta_6 V_l + \Delta_7 V_u = \Delta_8 \qquad [4.5]$$

where

$$\Delta_6 = \frac{L_l}{3}(3-f) + \frac{2fk_l EI_l}{L_l A_l G}$$

$$\Delta_7 = \frac{L_u}{3}(3-f) + \frac{2fk_u EI_u}{L_u A_u G}$$

$$\Delta_8 = \frac{ps}{12}(6-f)(L_u^2 - L_l^2) + 2fE\delta_g\left(\frac{I_u}{L_u^2} - \frac{I_l}{L_l^2}\right) + \frac{fEps}{G}\left(\frac{k_u I_u}{A_u} - \frac{k_l I_l}{A_l}\right) + 2fE\left(\frac{I_l\theta_l}{L_l} - \frac{I_u\theta_u}{L_u}\right)$$

Equations [4.3], [4.4], and [4.5] may be solved simultaneously for V_l, V_g, and V_u in terms of δ:

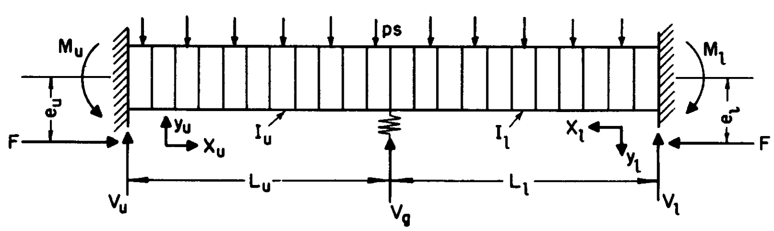
$$V_l = \frac{\Delta_7(\Delta_4 + \Delta_2\Delta_5) - \Delta_8(\Delta_2 + \Delta_3)}{\Delta_7(\Delta_1 + \Delta_2) + \Delta_6(\Delta_3 + \Delta_2)} \qquad [4.6]$$

$$V_g = \frac{\Delta_1(\Delta_5\Delta_7 - \Delta_8) + \Delta_3(\Delta_5\Delta_6 + \Delta_8) - \Delta_4(\Delta_7 + \Delta_6)}{\Delta_7(\Delta_1 + \Delta_2) + \Delta_6(\Delta_3 + \Delta_2)} \qquad [4.7]$$

$$V_u = \frac{\Delta_6(\Delta_4 + \Delta_2\Delta_5) + \Delta_8(\Delta_1 + \Delta_2)}{\Delta_7(\Delta_1 + \Delta_2) + \Delta_6(\Delta_3 + \Delta_2)} \qquad [4.8]$$

CASE IV – UNIFORMLY LOADED CLAMPED BEAM WITH AN ELASTIC INTERNAL SUPPORT AND END LOAD

In addition to the preceding relationship between the interior elastic support and its deflection, a similar relation for a similar beam but with end thrust added will be required. The schematic arrangement is



The basic differential equations relating bending moment, shear deformation, and curvature for the upper and lower spans are, respectively,

$$y_u'' + \beta_u^2 y_u = \left[-(M_u + Fe_u) + V_u x_u - \frac{psx_u^2}{2} + \frac{k_u EI_u ps}{A_u G} \right] \frac{1}{EI_u} \qquad [5.0]$$

$$y_l'' + \beta_l^2 y_l = \left[M_l + Fe_l - V_l x_l + \frac{psx_l^2}{2} - \frac{k_l EI_l ps}{A_l G} \right] \frac{1}{EI_l} \qquad [5.1]$$

where $\beta^2 = \frac{F}{EI}$.

The boundary conditions are:

$x_u = 0, \quad y_u = 0,$ [Unyielding end support]

$x_u = 0, \quad y_u' = \gamma_u = -\frac{k_u V_u}{A_u G}$ [Clamped end]

$x_u = L_u, \quad y_u = -\delta_g$ [Elastic internal support]

$x_l = 0, \quad y_l = 0,$ [Unyielding end support]

$x_l = 0, \quad y_l = \gamma_l = \frac{k_l V_l}{A_l G}$ [Clamped end]

$x_l = L_l, \quad y_l = \delta$ [Elastic internal support]

From which

$$y_u = C_1 \cos \beta_u x_u + C_2 \sin \beta_u x_u$$

$$- \frac{1}{F}\left(M_u + Fe_u - \frac{k_u EI_u}{A_u G} ps - \frac{psEI_u}{F} \right) + \frac{V_u x_u}{F} - \frac{psx_u^2}{2F} \qquad [5.2]$$

$$y_l = C_3 \cos \beta_l x_l + C_4 \sin \beta_l x_l$$

$$+ \frac{1}{F}\left(M_l + Fe_l - \frac{k_l EI_l}{A_l G} ps - \frac{psEI_u}{F} \right) - \frac{V_l x_l}{F} + \frac{psx_l^2}{2F} \qquad [5.3]$$

where

$$C_1 = \frac{1}{F}(M_u + Fe_u) - \frac{psEI_u}{F^2}\left(\frac{k_uF}{A_uG} + 1\right) \qquad C_3 = -\frac{1}{F}(M_l + Fe_l) + \frac{psEI_l}{F^2}\left(\frac{k_lF}{A_lG} + 1\right)$$

$$C_2 = -\frac{V_u}{\beta_u F}\left(\frac{k_uF}{A_uG} + 1\right) \qquad C_4 = \frac{V_l}{\beta_l F}\left(\frac{k_lF}{A_lG} + 1\right)$$

$$M_u = \frac{psEI_u}{F}\left(\frac{k_uF}{A_uG} + 1\right) + \frac{psL_u^2}{2(\cos\beta_u L_u - 1)} + \frac{V_u\left[(\sin\beta_u L_u)\left(\frac{k_uF}{A_uG} + 1\right) - \beta_u L_u\right]}{\beta_u(\cos\beta_u L_u - 1)}$$

$$- \frac{F\delta_g}{\cos\beta_u L_u - 1} - Fe_u \qquad [5.4]$$

$$M_l = \frac{psEI_l}{F}\left(\frac{k_lF}{A_lG} + 1\right) + \frac{psL_l^2}{2(\cos\beta_l L_l - 1)}$$

$$+ \frac{V_l\left[(\sin\beta_l L_l)\left(\frac{k_lF}{A_lG} + 1\right) - \beta_l L_l\right]}{\beta_l(\cos\beta_l L_l - 1)} - \frac{F\delta_g}{(\cos\beta_l L_l - 1)} - Fe_l \qquad [5.5]$$

Continuity of slope at the interior support, i.e.,

$$\left.\frac{dy_u}{dx_u}\right|_{x_u = L_u} - \frac{k_u V_g}{A_u G} = \left.\frac{dy_l}{dx_l}\right|_{x_l = L_l}$$

yields

$$\Delta_1 V_u + \Delta_2 V_g + \Delta_3 V_l = \Delta_4 \qquad [5.6]$$

where

$$\Delta_1 = \frac{\sin\beta_u L_u\left[\left(\frac{k_uF}{A_uG} + 1\right)\sin\beta_u L_u - \beta_u L_u\right]}{F(\cos\beta_u L_u - 1)} + \frac{\cos\beta_u L_u}{F}\left(\frac{k_uF}{A_uG} + 1\right) - \frac{1}{F}$$

$$\Delta_2 = \frac{k_u}{A_u G}$$

$$\Delta_3 = \frac{\sin\beta_l L_l \left[\left(\frac{k_l F}{A_l G} + 1\right)\sin\beta_l L_l - \beta_l L_l\right]}{F(\cos\beta_l L_l - 1)} + \frac{\cos\beta_l L_l}{F}\left(\frac{k_l F}{A_l G} + 1\right) - \frac{1}{F}$$

$$\Delta_4 = -\frac{ps}{F}(L_u + L_l) + \delta_g\left[\frac{\beta_u \sin\beta_u L_u}{(\cos\beta_u L_u - 1)} + \frac{\beta_l \sin\beta_l L_l}{(\cos\beta_l L_l - 1)}\right] - \frac{ps}{2F}\left[\frac{\beta_u L_u^2 \sin\beta_u L_u}{(\cos\beta_u L_u - 1)} + \frac{\beta_l L_l^2 \sin\beta_l L_l}{(\cos\beta_l L_l - 1)}\right]$$

From the equations of static equilibrium are obtained:

$$\text{Force: } V_u + V_g + V_l = \Delta_5 = ps(L_u + L_l) \qquad [5.7]$$

$$\text{Moment: } \Delta_6 V_u - \Delta_7 V_l = \Delta_8 \qquad [5.8]$$

where

$$\Delta_6 = L_u - \frac{\left[(\sin\beta_u L_u)\left(\frac{k_u F}{A_u G} + 1\right) - \beta_u L_u\right]}{\beta_u(\cos\beta_u L_u - 1)}$$

$$\Delta_7 = L_l - \frac{\left[(\sin\beta_l L_l)\left(\frac{k_l F}{A_l G} + 1\right) - \beta_l L_l\right]}{\beta_l(\cos\beta_l L_l - 1)}$$

$$\Delta_8 = \frac{ps}{2}\left[\frac{L_u^2}{(\cos\beta_u L_u - 1)} - \frac{L_l^2}{(\cos\beta_l L_l - 1)}\right] + F\delta_g\left[\frac{1}{(\cos\beta_l L_l - 1)} - \frac{1}{(\cos\beta_u L_u - 1)}\right]$$

$$+ \frac{psE}{F}\left[I_u\left(\frac{k_u F}{A_u G} + 1\right) - I_l\left(\frac{k_l F}{A_l G} + 1\right)\right] + ps(L_u^2 - L_l^2)$$

Equations [5.6], [5.7], and [5.8] may be solved simultaneously for V_u, V_g and V_l in terms of δ:

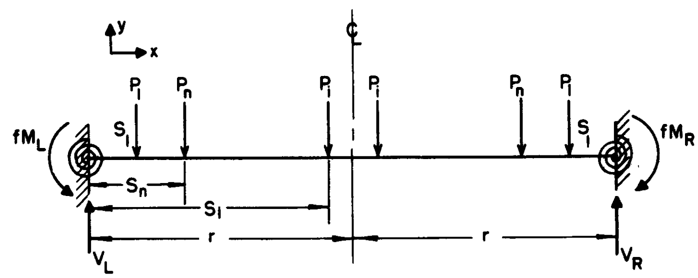
$$V_u = \frac{\Delta_7(\Delta_5\Delta_2 - \Delta_4) + \Delta_8(\Delta_2 - \Delta_3)}{\Delta_7(\Delta_2 - \Delta_1) + \Delta_6(\Delta_2 - \Delta_3)} \qquad [5.9]$$

$$V_g = \frac{\Delta_6(\Delta_4 - \Delta_3\Delta_5) + \Delta_7(\Delta_4 - \Delta_1\Delta_5) + \Delta_8(\Delta_3 - \Delta_1)}{\Delta_7(\Delta_2 - \Delta_1) + \Delta_6(\Delta_2 - \Delta_3)} \qquad [5.10]$$

$$V_l = \frac{\Delta_6(\Delta_2\Delta_5 - \Delta_4) + \Delta_8(\Delta_1 - \Delta_2)}{\Delta_7(\Delta_2 - \Delta_1) + \Delta_6(\Delta_2 - \Delta_3)} \qquad [5.11]$$

CASE V – ELASTICALLY RESTRAINED UNIFORM BEAM WITH CONCENTRATED LOADS SYMMETRICALLY SPACED ABOUT MIDSPAN

In the grillage approach to the analysis of bulkhead stiffening systems, it is assumed that the reactions at the elastic internal support of the previous cases are acting as concentrated loads on the main girder. In the beam shown below, the concentrated loads are the internal reaction forces of Case III.



The reactions are

$$V_L = V_R = \sum_{n=1}^{n=i} P_n \qquad [6.0]$$

The basic expression relating moment and curvature is

$$EIy_j'' = -fM_L + V_L x - \sum_{n=0}^{n=j-1} P_n (x - s_n) \qquad [6.1]$$

and the expressions for the slope and deflection are

$$EIy_j' = -fM_L x + V_L \frac{x^2}{2} - \sum_{n=0}^{n=j-1} P_n \left(\frac{x^2}{2} - xs_n\right) + A_j \qquad [6.2]$$

$$EIy_j = -fM_L \frac{x^2}{2} + V_L \frac{x^3}{6} - \sum_{n=0}^{n=j-1} P_n \left(\frac{x^3}{6} - \frac{x^2}{2} s_n\right) + A_j x + B_j \qquad [6.3]$$

when

$$s_{j-1} \lesseqgtr x \leq s_j$$

The boundary conditions are

$$(y_j')_{x=s_j} + \frac{kP_jEI}{AG} = (y_j')_{x=s_j} \qquad \text{[Continuity of slope]} \qquad [6.4]$$

$$(y_j)_{x=s_j} = (y_{j+1})_{x=s_j} \qquad \text{[Continuity of deflection]}$$

$$x = 0, \quad y = 0 \qquad \text{[Unyielding end support]}$$

$$x = 0, \quad y' = (\gamma_L + \theta_L) \qquad \text{[Variable end fixity]} \qquad [6.4]$$

where

$$\gamma_L = \frac{kV_L}{AG}$$

$$x = r, \quad y' = 0 \qquad \text{[Symmetry]}$$

Substituting the pertinent boundary conditions from Equations [6.4] into Equations [6.2] and [6.3], there results

$$A_1 = EI\,[\gamma_L + \theta_L]$$

$$A_j = A_1 - \sum_{n=0}^{n=j-1} P_n \left[\frac{s_n^2}{2} - \frac{kEI}{AG}\right]$$

$$B_1 = 0 \qquad [6.5]$$

$$B_j = B_1 + \sum_{n=0}^{n=j-1} P_n s_n \left[\frac{s_n^2}{6} - \frac{kEI}{AG}\right]$$

$$M_L = \sum_{n=0}^{n=i} P_n s_n \left[1 - \frac{s_n}{2r}\right]$$

$$\theta_L = -\frac{M_L}{2EI}(1-f)$$

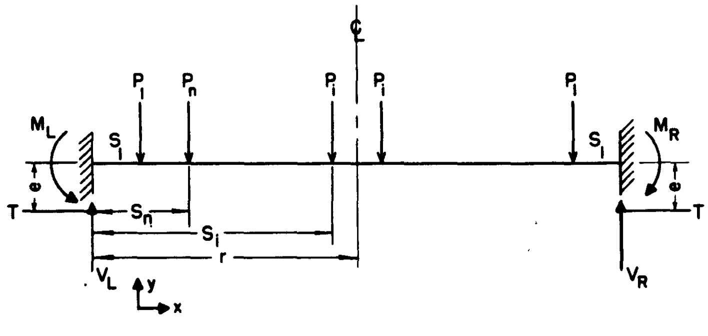
The equation for deflection is then

$$EIy_j = -fM_L \frac{x^2}{2} + \frac{x^3}{6} V_L - \frac{\sum_{n=0}^{n=j-1} P_n}{6} [x - s_n]^3$$

$$+ \frac{kEI}{AG} \sum_{n=0}^{n=j-1} P_n [x - s_n] - \frac{kEIx}{AG} \sum_{n=0}^{n=j} P_n + EIx\theta_L \qquad [6.6]$$

CASE VI – CLAMPED UNIFORM BEAM WITH END LOAD AND CONCENTRATED LOADS SYMMETRICALLY SPACED ABOUT MIDSPAN

Consider the beam shown, where the concentrated loads are the internal reaction forces of Case IV



By symmetry, the end reactions are

$$V_L = V_R = \sum_{n=1}^{n=i} P_n \qquad [7.0]$$

The basic expression relating moment and curvature for the j^{th} interval, $s_{j-1} \leq x \leq s_j$, is

$$EIy_j'' + Ty_j = -(M_L + Te) + V_L x - \sum_{n=0}^{n=j-1} P_n(x - s_n) \qquad [7.1]$$

Solving Equation [7.1] gives for the expression for deflection

$$y_j = A_j \cos\beta x + B_j \sin\beta x - \frac{1}{T}\left[M_L + Te - \sum_{n=0}^{n=j-1} P_n(s_n - x) - V_L x\right] \qquad [7.2]$$

and for slope

$$y_j' = -\beta A_j \sin\beta x + \beta B_j \cos\beta x + \frac{1}{T}\left(V_L - \sum_{n=0}^{n=j-1} P_n\right) \qquad [7.3]$$

where $\beta^2 = \dfrac{T}{EI}$. The boundary conditions are

$$(y_j')_{x=s_j} + \frac{kP_j}{AG} = (y_{j+1}')_{x=s_j} \qquad \text{[Continuity of slope]} \qquad [7.4]$$

$$(y_j)_{x=s_j} = (y_{j+1})_{x=s_j} \qquad \text{[Continuity of deflection]}$$

$$x = 0, \quad y = 0 \qquad \text{[Unyielding support]}$$

$$x = 0, \quad y' = \gamma_L = -\frac{kV_L}{AG} \qquad \text{[Clamped end]} \tag{7.4}$$

$$x = r, \quad y' = 0 \qquad \text{[Symmetry]}$$

from which

$$A_1 = \frac{1}{T}[M_L + Te]$$

$$A_j = A_1 - \sum_{n=0}^{n=j-1} \frac{P_n}{\beta T}\left[1 + \frac{kT}{AG}\right] \sin \beta s_n \tag{7.5}$$

$$B_1 = -\frac{V_L}{\beta T}\left(1 - \frac{kT}{AG}\right)$$

$$B_j = B_1 + \sum_{n=0}^{n=j-1} \frac{P_n}{\beta T}\left[1 + \frac{kT}{AG}\right] \cos \beta s_n$$

and

$$M_L + Te = \frac{1}{\beta}\left[1 + \frac{kT}{AG}\right]\left\{\sum_{n=0}^{n=i} P_n [\sin \beta s_n + \cos \beta s_n \cot \beta r] - V_L \cot \beta r\right\} \tag{7.6}$$

Substituting Equations [7.5] into Equation [7.2]

$$y_j = \frac{1}{\beta T}\left[1 + \frac{kT}{AG}\right](\cos \beta x - 1)\left\{\sum_{n=0}^{n=i} P_n \left[\frac{\cos \beta (r - s_n)}{\sin \beta r}\right] - V_L \cot \beta r\right\}$$

$$+ \sum_{n=0}^{n=j-1} \frac{P_n}{\beta T}\left\{\left[1 + \frac{kT}{AG}\right][\sin \beta (x - s_n)] + \beta (s_n - x)\right\}$$

$$+ \frac{V_L}{\beta T}\left\{\beta x - \left[1 + \frac{kT}{AG}\right] \sin \beta x\right\} \tag{7.7}$$

NUMERICAL EXAMPLE

The methods described herein for determining the deflections and stresses in the stiffening system of a bulkhead will be applied to a specific model tested at the Portsmouth Naval Shipyard, Portsmouth, New Hampshire. The details of the structure are shown in Figure 2. It is noted that in all calculations Equation [1.3] is used to determine the shear factor k.

For Case I, Equations [2.5] and [2.8] were used directly to obtain the moments and deflections of the main girder assuming full fixity. For Case II, Equation [3.9] was expanded to $n = 3$ for determining deflections.

For the bulkhead shown in Figure 2, Cases V and VI require the solution of eight simultaneous equations, four relating deflections of the vertical stiffeners at the interior supports to the interior support reactions of the vertical stiffeners, and four relating the deflection of the main girder at an intersection with a vertical stiffener to interior support reaction of that vertical stiffener.

For Case V, the equations assuming complete fixity at the supports, i.e., $f = 1$, $\theta_L = 0$, are

$$EIy_1 = - M_L \frac{s_1^2}{2} + \frac{s_1^3}{6}\left(P_1 + P_2 + P_3 + \frac{P_4}{2}\right) - \frac{kEIs_1}{AG}\left(P_1 + P_2 + P_3 + \frac{P_4}{2}\right)$$

$$EIy_2 = - M_L \frac{s_2^2}{2} + \frac{s_2^3}{6}\left(P_1 + P_2 + P_3 + \frac{P_4}{2}\right) - \frac{P_1}{6}(s_2 - s_1)^3 + \frac{kEIP_1}{AG}(s_2 - s_1)$$
$$- \frac{kEI}{AG} s_2 \left(P_1 + P_2 + P_3 + \frac{P_4}{2}\right)$$

$$EIy_3 = - M_L \frac{s_3^2}{2} + \frac{s_3^3}{6}\left(P_1 + P_2 + P_3 + \frac{P_4}{2}\right) - \frac{P_1}{6}(s_3 - s_1)^3 - \frac{P_2}{6}(s_3 - s_2)^3 + \frac{kEI}{AG}P_1(s_3 - s_1)$$
$$+ \frac{kEI}{AG}P_2(s_3 - s_2) - \frac{kEI}{AG} s_3\left(P_1 + P_2 + P_3 + \frac{P_4}{2}\right)$$

$$EIy_4 = - M_L \frac{r^2}{2} + \frac{r^3}{6}\left(P_1 + P_2 + P_3 + \frac{P_4}{2}\right) - \frac{P_1}{6}(r - s_1)^3 - \frac{P_2}{6}(r - s_2)^3 - \frac{P_3}{6}(r - s_3)^3$$
$$+ \frac{kEI}{AG}P_1(r - s_1) + \frac{kEI}{AG}P_2(r - s_2) + \frac{kEI}{AG}P_3(r - s_3) - \frac{kEIr}{AG}\left(P_1 + P_2 + P_3 + \frac{P_4}{2}\right)$$

$$EIy_1 = P_1\left(\frac{kEIL}{2AG} + \frac{L^3}{24}\right) - \frac{psL^4}{24} - \frac{kEIL^2ps}{2AG}$$

$$EIy_2 = P_2\left(\frac{kEIL}{2AG} + \frac{L^3}{24}\right) - \frac{psL^4}{24} - \frac{kEIL^2ps}{2AG}$$

$$EIy_3 = P_3\left(\frac{kEIL}{2AG} + \frac{L^3}{24}\right) - \frac{psL^4}{24} - \frac{kEIL^2ps}{2AG}$$

$$EIy_4 = P_4\left(\frac{kEIL}{2AG} + \frac{L^3}{24}\right) - \frac{psL^4}{24} - \frac{kEIL^2ps}{2AG}$$

where the first four equations are the expansions of Equation [6.6] for the main girder, and the last four equations are from Equation [4.7] with the *individual parameters obtained from each of the individual stiffeners*.

For Case VI, the equations are

$$y_1 = \frac{1}{\beta T}\left[1 + \frac{kT}{AG}\right](\cos\beta s_1 - 1)\left\{P_1\left[\frac{\cos\beta(r-s_1)}{\sin\beta r} - \cot\beta r\right] + P_2\left[\frac{\cos\beta(r-s_2)}{\sin\beta r} - \cot\beta r\right]\right.$$

$$\left. + P_3\left[\frac{\cos\beta(r-s_3)}{\sin\beta r} - \cot\beta r\right] + P_4\,\frac{(1-\cos\beta r)}{2\sin\beta r}\right\} + \left[P_1 + P_2 + P_3 + \frac{P_4}{2}\right]\left\{\frac{s_1}{T} - \left[1 + \frac{kT}{AG}\right]\frac{\sin\beta s_1}{\beta T}\right\}$$

$$y_2 = \frac{1}{\beta T}\left[1 + \frac{kT}{AG}\right](\cos\beta s_2 - 1)\left\{P_1\left[\frac{\cos\beta(r-s_1)}{\sin\beta r} - \cot\beta r\right] + P_2\left[\frac{\cos\beta(r-s_2)}{\sin\beta r} - \cot\beta r\right]\right.$$

$$\left. + P_3\left[\frac{\cos\beta(r-s_3)}{\sin\beta r} - \cot\beta r\right] + P_4\,\frac{(1-\cos\beta r)}{2\sin\beta r}\right\} + \frac{P_1}{\beta T}\left\{\left[1 + \frac{kT}{AG}\right][\sin\beta(s_2 - s_1)] + \beta(s_1 - s_2)\right\}$$

$$+ \left[P_1 + P_2 + P_3 + \frac{P_4}{2}\right]\left\{\frac{s_2}{T} - \left[1 + \frac{kT}{AG}\right]\frac{\sin\beta s_2}{\beta T}\right\}$$

$$y_3 = \frac{1}{\beta T}\left[1 + \frac{kT}{AG}\right](\cos\beta s_3 - 1)\left\{P_1\left[\frac{\cos\beta(r-s_1)}{\sin\beta r} - \cot\beta r\right] + P_2\left[\frac{\cos\beta(r-s_2)}{\sin\beta r} - \cot\beta r\right]\right.$$

$$\left. + P_3\left[\frac{\cos\beta(r-s_3)}{\sin\beta r} - \cot\beta r\right] + \frac{P_4(1-\cos\beta r)}{2\sin\beta r}\right\} + \frac{P_1}{\beta T}\left\{\left[1 + \frac{kT}{AG}\right][\sin\beta(s_3 - s_1)] + \beta(s_1 - s_3)\right\}$$

$$+ \frac{P_2}{\beta T}\left\{\left[1+\frac{kT}{AG}\right] \sin\beta(s_3-s_2) - \beta(s_2-s_3)\right\} + \left[P_1+P_2+P_3+\frac{P_4}{2}\right]\left\{\frac{s_3}{T} - \left[1+\frac{kT}{AG}\right]\frac{\sin\beta s_3}{\beta T}\right\}$$

$$y_4 = \frac{1}{\beta T}\left[1+\frac{kT}{AG}\right](\cos\beta r-1)\left\{P_1\left[\frac{\cos\beta(r-s_1)}{\sin\beta r} - \cot\beta r\right] + P_2\left[\frac{\cos\beta(r-s_2)}{\sin\beta r} - \cot\beta r\right]\right.$$

$$\left. + P_3\left[\frac{\cos\beta(r-s_3)}{\sin\beta r} - \cot\beta r\right] + P_4\,\frac{(1-\cos\beta r)}{2\sin\beta r}\right\} + \frac{P_1}{\beta T}\left\{\left[1+\frac{kT}{AG}\right]\sin\beta(r-s_1) + \beta(s_1-r)\right\}$$

$$+ \frac{P_2}{\beta T}\left\{\left[1+\frac{kT}{AG}\right]\sin\beta(r-s_2) + \beta(s_2-r)\right\} + \frac{P_3}{\beta T}\left\{\left[1+\frac{kT}{AG}\right]\sin\beta(r-s_3) + \beta(s_3-r)\right\}$$

$$+\left[P_1+P_2+P_3+\frac{P_4}{2}\right]\left\{\frac{r}{T} - \left[1+\frac{kT}{AG}\right]\frac{\sin\beta r}{\beta T}\right\}$$

$$y_1 = \frac{P_1}{2F}\left[\left(\frac{kF}{AG}+1\right)\frac{(\cos\beta l-1)}{\beta\sin\beta l} + l\right] - \frac{psl^2}{2F} - \left(\frac{kF}{AG}+1\right)\frac{\cos(\beta l-1)}{\beta F\sin\beta l}\,psl$$

$$y_2 = \frac{P_2}{2F}\left[\left(\frac{kF}{AG}+1\right)\frac{(\cos\beta l-1)}{\beta\sin\beta l} + l\right] - \frac{psl^2}{2F} - \left(\frac{kF}{AG}+1\right)\frac{(\cos\beta l-1)}{\beta F\sin\beta l}\,psl$$

$$y_3 = \frac{P_3}{2F}\left[\left(\frac{kF}{AG}+1\right)\frac{(\cos\beta l-1)}{\beta\sin\beta l} + l\right] - \frac{psl^2}{2F} - \left(\frac{kF}{AG}+1\right)\frac{(\cos\beta l-1)}{\beta F\sin\beta l}\,psl$$

$$y_4 = \frac{P_4}{2F}\left[\left(\frac{kF}{AG}+1\right)\frac{(\cos\beta l-1)}{\beta\sin\beta l} + l\right] - \frac{psl^2}{2F} - \left(\frac{kF}{AG}+1\right)\frac{(\cos\beta l-1)}{\beta F\sin\beta l}\,psl$$

where the first four equations are the expansions of Equation [7.7] for the main girder, and the last four equations are from Equation [5.10] with the *individual parameters obtained from each of the individual stiffeners.*

In Figures 3 and 4 the deflections and stresses computed from the foregoing equations are compared with actual experimental results for the main girder and centerline stiffener, respectively. In Table 1 the computed values are compared with average experimental results.

All the theoretical deflection curves for the main girder, Figure 3, agree well with experimental results. It is noted that the effect of the end load is negligible in computing deflections in the elastic range of the material for both the main girder and the vertical stiffener. The theoretical deflections for the assumed elliptical loading (Cases I and II) were

(Text continued on page 29.)

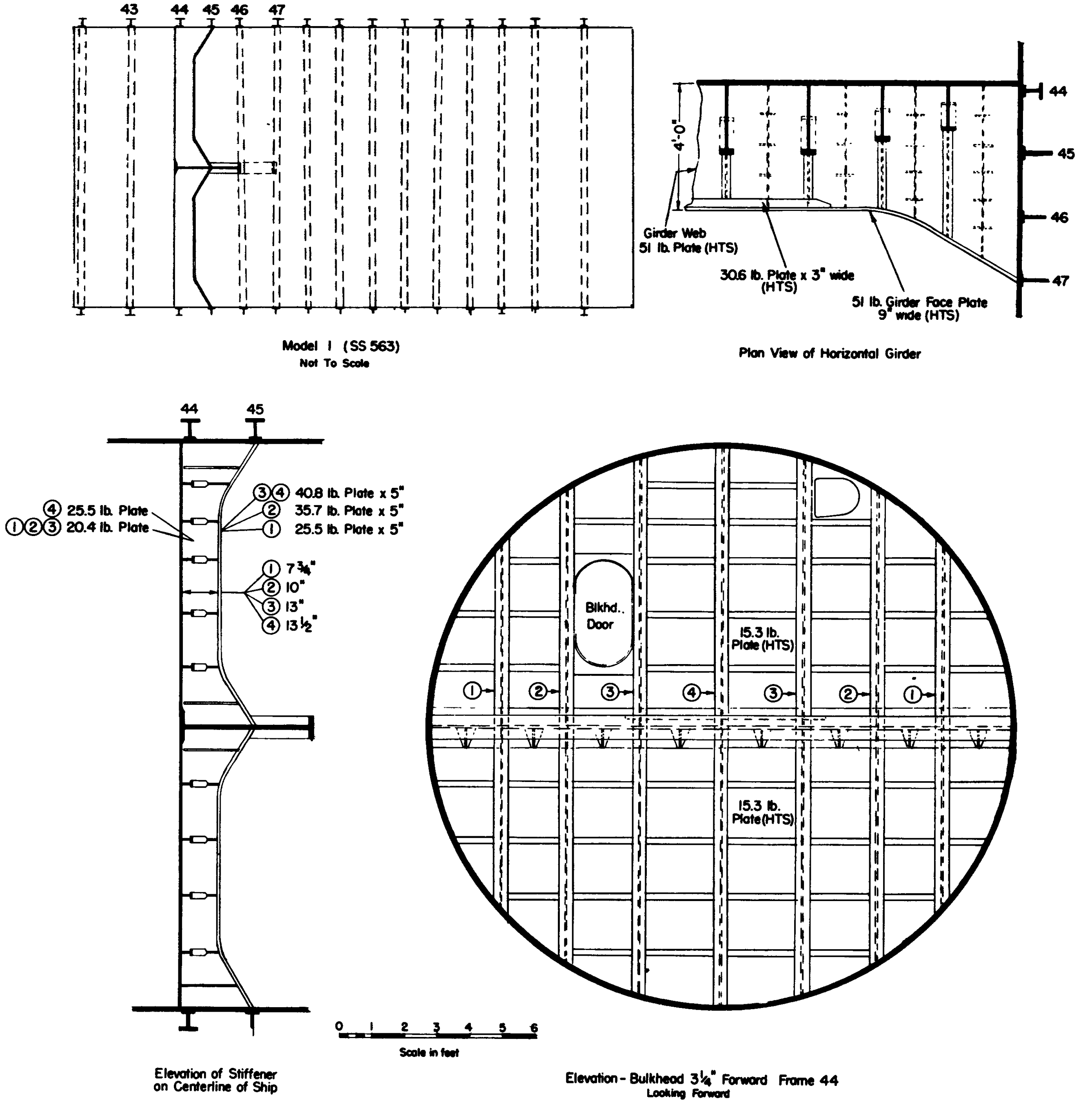


Figure 2 – Schematic Sketch of Bulkhead 1

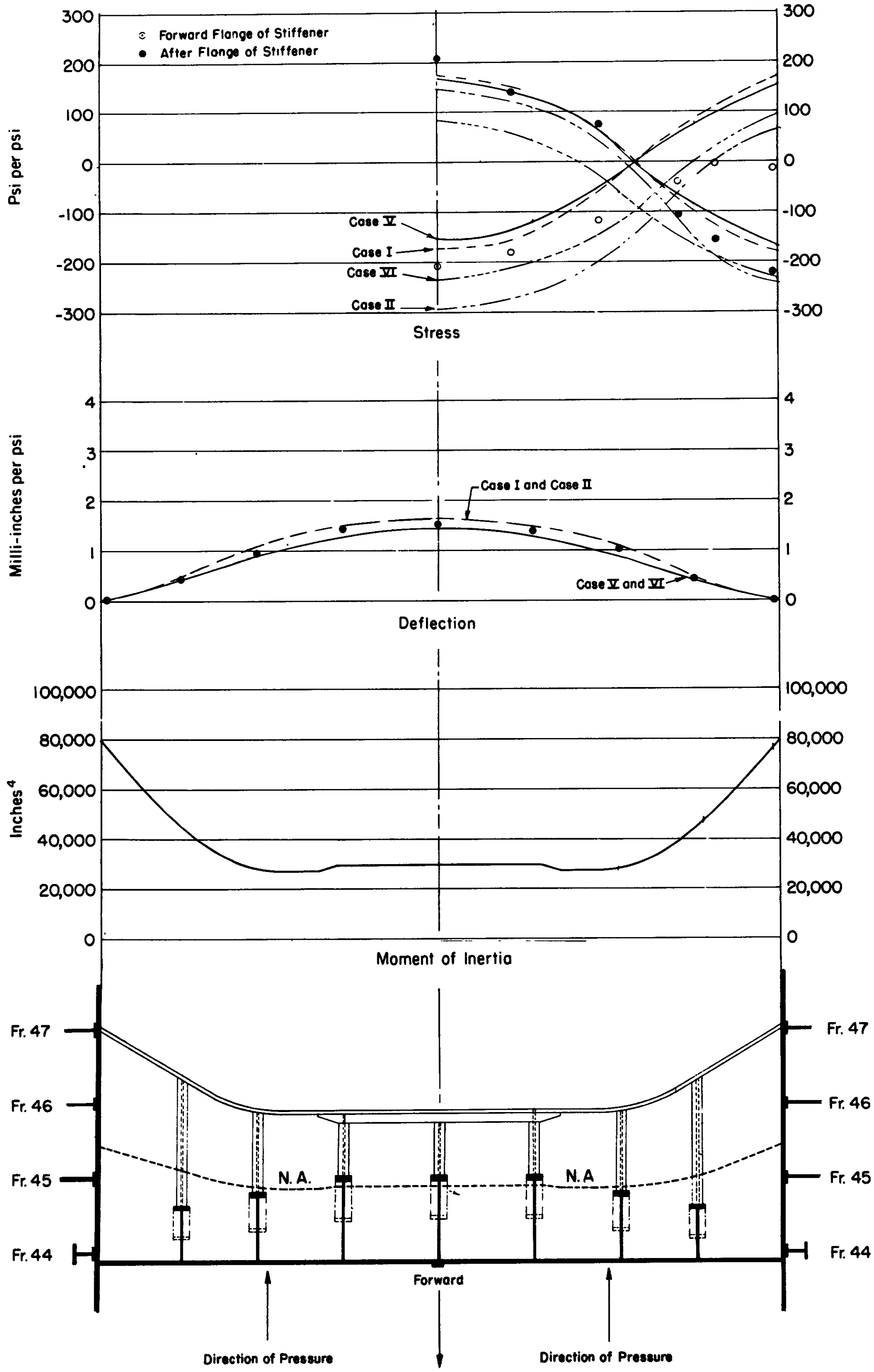


Figure 3 – Comparison of Theoretical and Experimental Deflections and Stresses for Main Girder

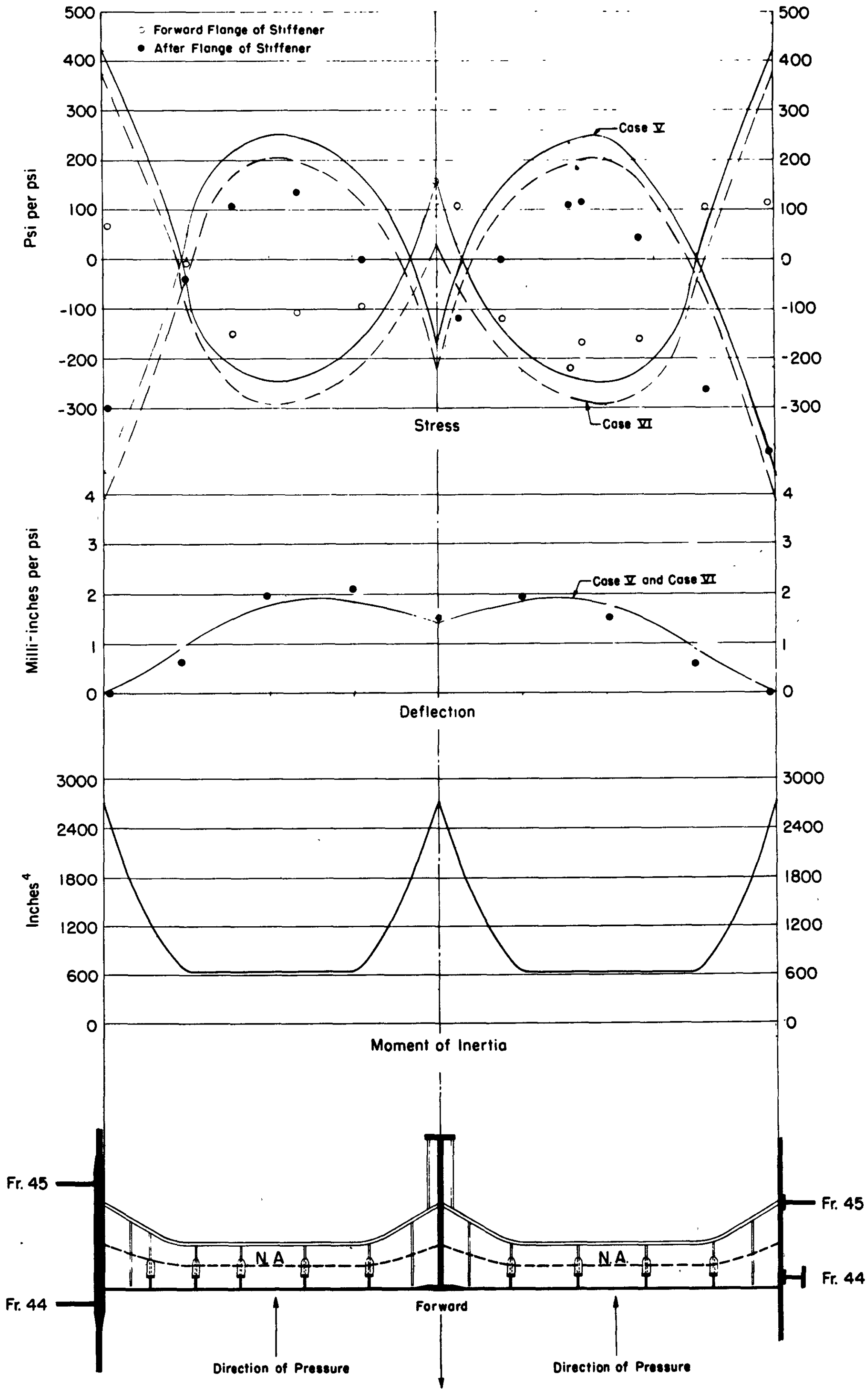


Figure 4 – Comparison of Theoretical and Experimental Deflections and Stresses for Vertical Stiffener

TABLE 1

Comparison of Average Measured Deflections and Stresses with Deflections and Stresses Computed by Various Theories

Member	Distance along Member, Measured from Hull x/r	Computed Deflection milli-in/psi				Average Measured Deflection milli-in/psi	Computed Stress in Outstanding Flange of Member psi/psi				Average Measured Stress in Outstanding Flange of Member psi/psi	Computed Stress in "Bulkhead Plating" Flange of Member psi/psi				Average Measured Stress in "Bulkhead Plating" Flange of Member psi/psi
		Case I	Case II	Case V	Case VI		Case I	Case II	Case* V	Case* VI		Case I	Case II	Case* V	Case* VI	
Main Girder	0	0.00	0.00	0.00	0.00	0.00	−180	−240	−170	−235	−220	+175	+ 60	+155	+ 95	− 20
	0.25	0.45	0.45	0.45	0.46	0.42	− 70	−140	− 85	−165	−110	+ 65	− 70	+ 80	0	− 40
	0.47	1.10	1.10	0.88	0.89	1.05	+ 70	+ 40	+ 30	− 60	+ 75	− 65	−200	− 25	−120	−115
	0.72	1.48	1.48	1.27	1.28	1.35	+150	+120	+130	+ 45	+140	−160	−270	−115	−200	−180
	1.0	1.68	1.68	1.42	1.43	1.52	+175	+150	+170	+ 85	+210	−175	−290	−150	−235	−210
Centerline Vertical Stiffener									(−430)	(−480)				(+420)	(+375)	
	0			0.00	0.00	0.00			−170	−205	−345			+170	+140	+ 85
	0.25			0.92	0.93	0.67			+ 50	0	−150			− 50	− 95	+ 40
	0.50			1.76	1.81	1.75			+255	+205	+120			−250	−295	−170
	0.75			1.85	1.87	2.00			+185	+135	0			−180	−225	−100
	1.0			1.42	1.43	1.52			(−160)	(−210)	−210			(+155)	(+110)	
									− 65	−100				+ 65	+ 30	+160

*Stresses in parentheses were computed assuming uniform beam, other stresses were determined by using the actual section moduli of the section in question.

approximately 17 percent greater than the theoretical deflections from the grillage analysis (Cases V and VI). This is expected, however, since the total assumed loads on the main girder for the elliptical loadings were 17 percent greater than the total assumed loads in the grillage analysis.

Agreement with stresses was not as good as expected. However, the grillage analysis, including the end load (Case VI), predicted stresses that agreed well with the measured *compressive* stresses in the middle portion of the girder. The grillage analysis neglecting the effect of the end load predicted stresses that agreed well with the measured *tensile* stresses in the middle portion of the beam. Much better agreement with stress can be obtained by adding the compressive stress due to the end load (T/A) to the compressive stress computed from the grillage analysis of Case V and retaining all tensile stresses as computed from Case V.

For the vertical stiffener, Figure 4, the deflections computed from the grillage analysis (Cases V and VI) agree well with the measured values. Agreement as to stress distribution is poor. The magnitude of computed maximum stress at midspan, however, does agree with the maximum measured there.

DESIGN PROCEDURE

Although the grillage-analysis approach is probably the most logical method of efficiently designing a bulkhead stiffening system, the method presently employed is not without merit. Some of the advantages and disadvantages of the grillage approach and the method currently used in design are:

	Present Design Method	Grillage Method
Calculations	Easy.	Very difficult without the use of an electronic computer.
Main Girder Deflections	Good, if effect of shear deformation is included or if the end restraints are relaxed.	Good.
Main Girder Stresses	Fair.	Good, if effect of end load is added to compressive bending stresses only.
Vertical Stiffener Deflections	Poor. Assumptions not compatible with assumptions for main girder.	Good.
Vertical Stiffener Stresses	Good at midspan; poor at ends.	Good agreement with maximum stresses at ends and at midspan. Poor agreement with distribution of stresses.

It is possible to devise a design procedure which will retain the basic ease of the present method and the greater realism and accuracy of the grillage analysis by making certain assumptions and simplifications.

ASSUMPTIONS

1. To reap the simplifying benefits of symmetry, the main girder is located on the horizontal diameter and the vertical stiffeners are symmetrically spaced about the vertical centerline.

2. The ratio of the inertia of the typical vertical stiffener to the inertia of the main girder is sufficiently small that the stiffening effect of the stiffener on the deflection of the girder can be neglected.

3. All members of the stiffening system are assumed to be of constant cross section. This is not realistic since the girder is haunched and the vertical stiffeners are bracketed at the ends to provide adequate area to resist shear and section modulus to resist bending moment.

SIMPLIFICATIONS

1. The length of the vertical stiffeners nearest the vertical centerline is taken as the hull radius. Only a very slight error is introduced.

2. The deflection due to axial end load is neglected. In the elastic range, the contribution is so slight as to be unworthy of the added complication.

3. Deflection due to shear deformation has been neglected for the main girder and compensated for by using a fixity factor of 70 percent. This value was previously determined in Reference 2 and is used in computing deflections but not stresses.

Before any design can be started, some basic parameters must be known by the engineer. In the case of bulkheads, these are usually the radius r of the bulkhead, the yield strength σ_y of the material, and the pressure p_y at which yielding is to start. With these parameters known the designer can tentatively space the vertical stiffeners so that the thickness of the bulkhead plating may be determined. The method outlined in Reference 1 for determining the thickness of the bulkhead plating is adequate. The bulkhead plating forms one flange of the vertical stiffeners, and a width of plating equal to 30 thicknesses, or the spacing of the stiffeners, whichever is less, is assumed to work with the stiffener in the calculation of the inertia of the stiffeners.

The nomographs of Figures 5 and 6 were developed to assist in determining the size of the main horizontal girder. Figure 5 was developed from Equation [2.5] for the midspan of the girder assuming a fixity factor of 0.7, and Figure 6 was developed from Equation [2.8] at midspan. The nomograph of Figure 7 is for the centerline vertical stiffener and was developed from Equation [4.1]. These three nomographs will tentatively give the size of the stiffening system. The methods of using the nomographs in design is as follows:

1. From Figure 5, for a given radius, yield strength and pressure, obtain a section modulus I/c.

2. From Figure 6, assuming an allowable midspan deflection on the order of 0.0015 in/psi, obtain a value for the inertia I. With I and c known, a tentative section may be chosen.

3. From Figure 7, for a given stiffener spacing s, a section modulus for the vertical stiffener may be obtained. One flange of the vertical stiffener is the bulkhead plating; the choice of the web and other flange is left to the discretion of the designer.

4. The section properties of the tentative members are then inserted into the grillage analysis of Case V, and a deflection at midspan of the main girder is obtained.

5. Using the deflection from Step 4 with Figure 6 for the given pressure and radius, a new inertia (I_2) is determined for the main girder. This new inertia is then used to determine the inertia of the beam (I) by means of the following relationship $\left(\frac{I_{orig}}{I_2}\right) I_{orig} = I$.

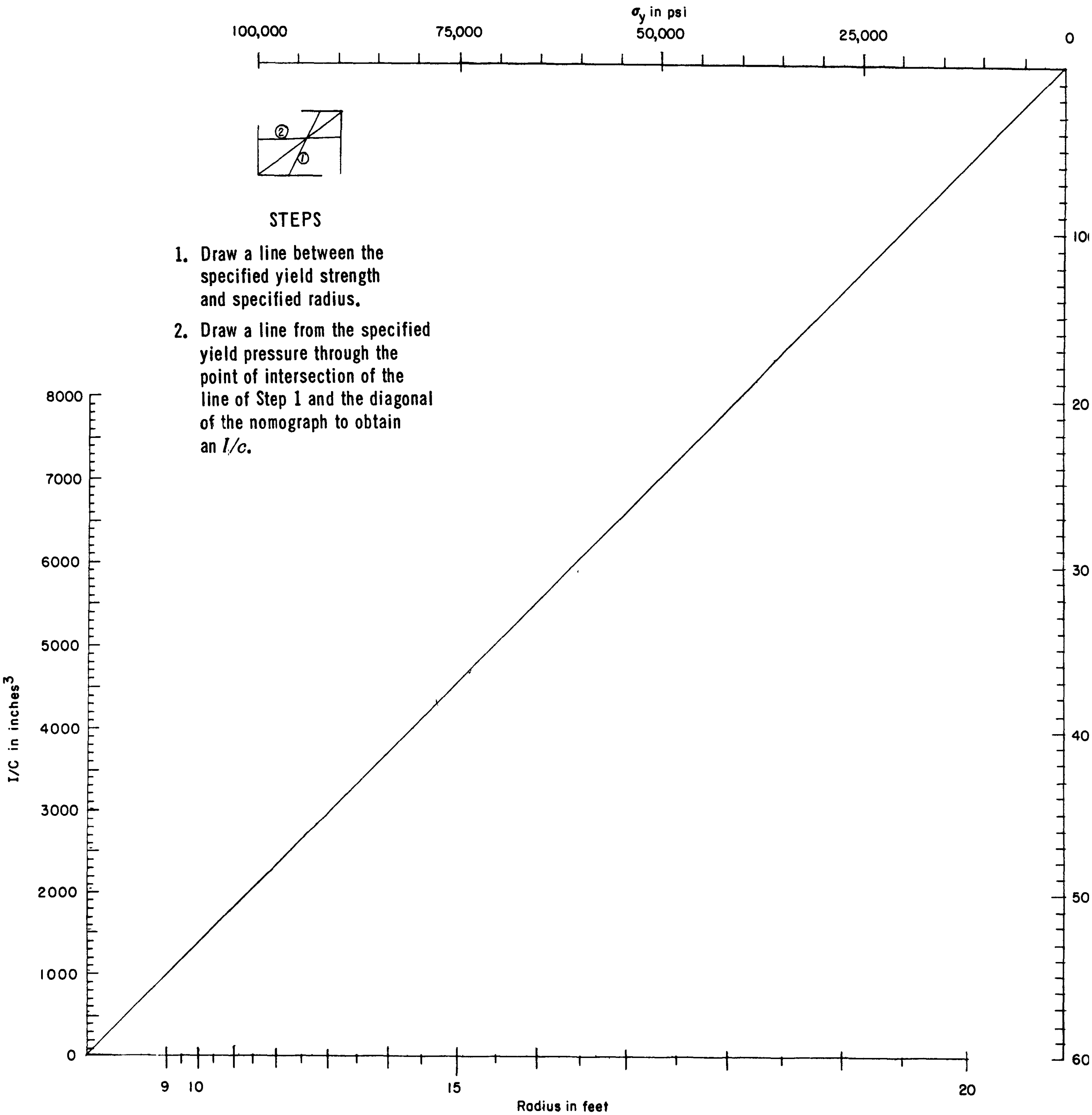


Figure 5 – Nomograph for Determining Section Modulus I/c of Main Horizontal Girder

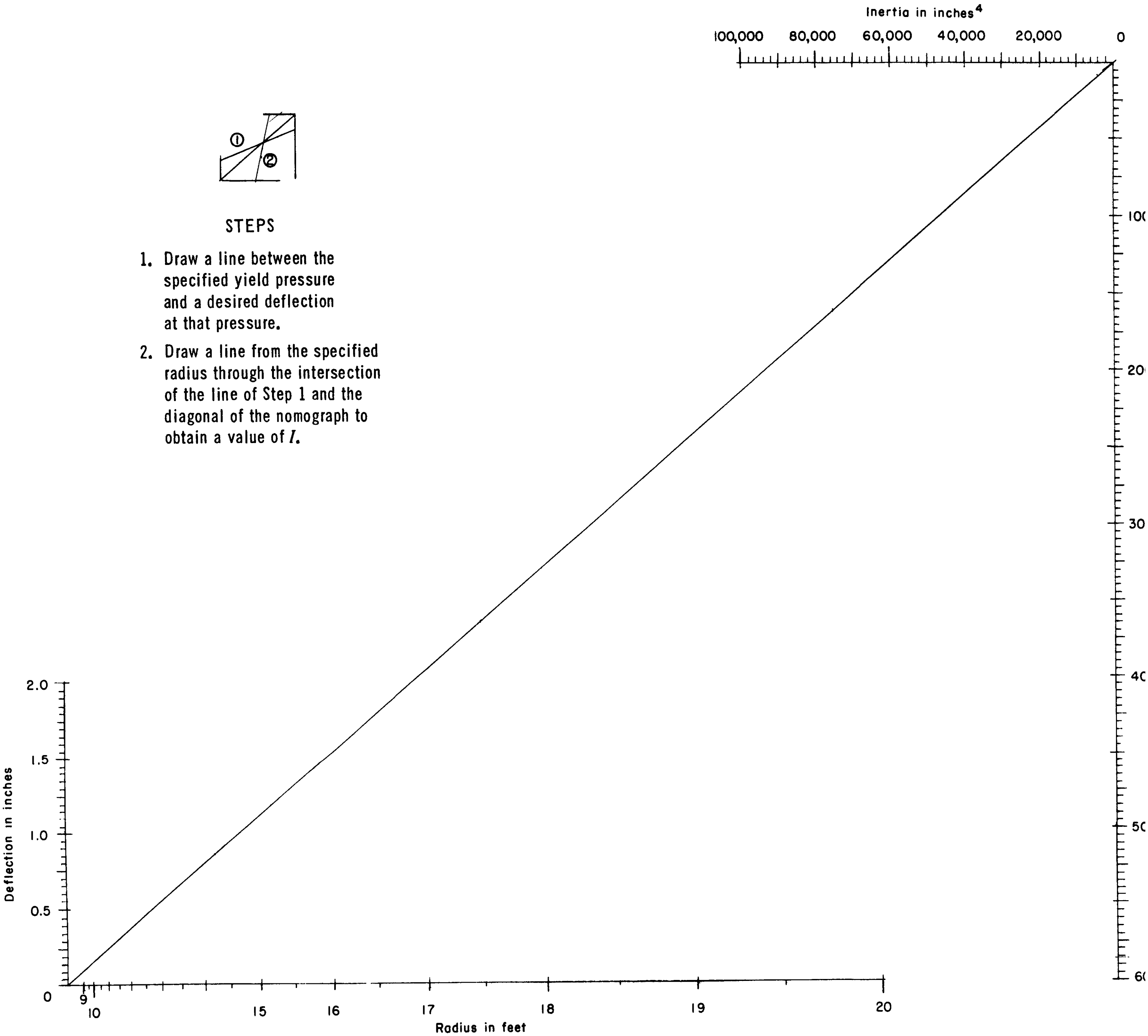


Figure 6 – Nomograph for Determining Inertia I of Main Horizontal Girder

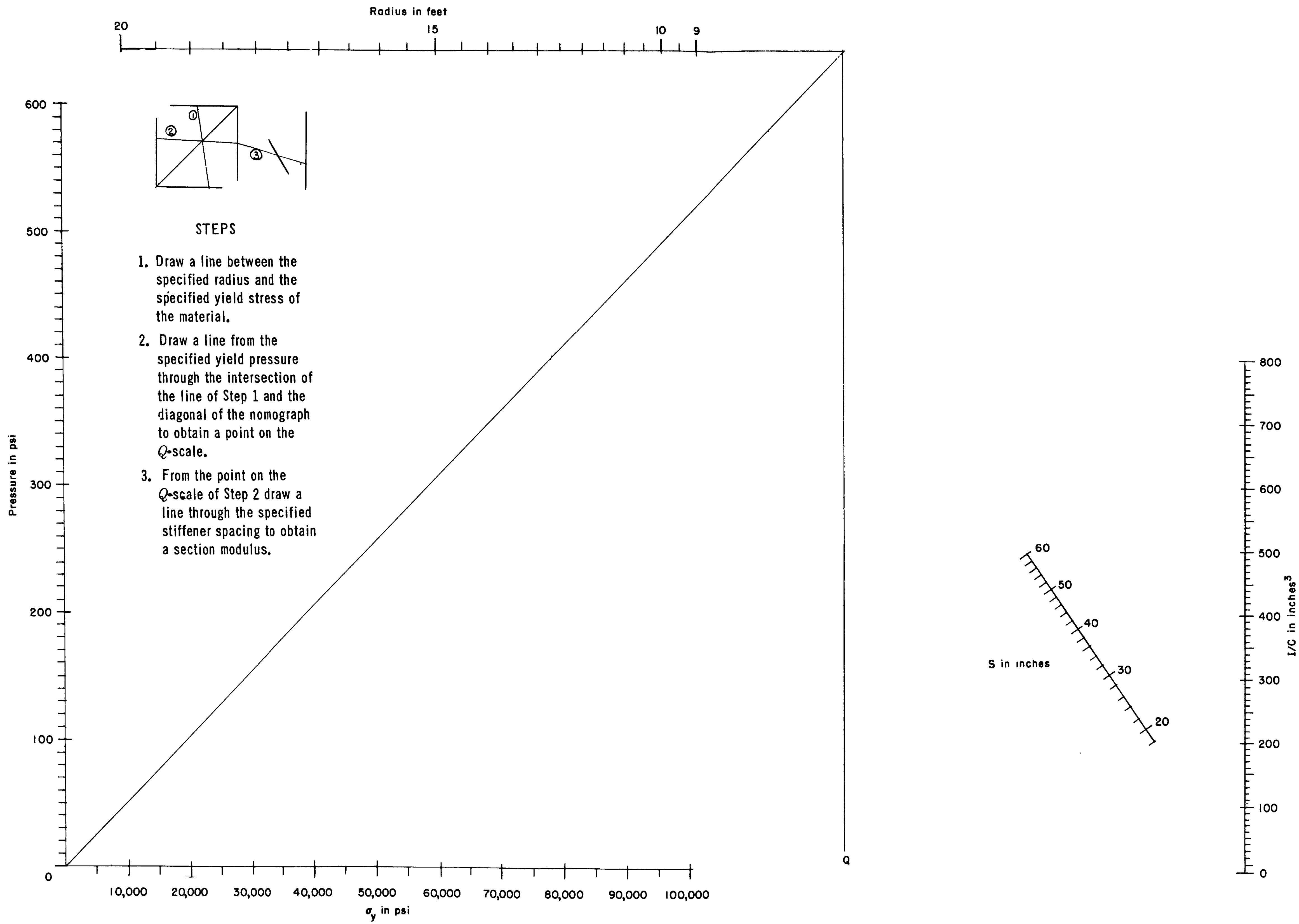


Figure 7 – Nomograph for Determining Section Modulus I/c of Vertical Stiffener

6. Using the new deflection from Step 4 with the pressure and inertia previously determined in Step 2, an *effective* radius is determined from Figure 6 for use in redetermining the size of the vertical stiffener.

7. Using the *effective* radius of Step 6 with the pressure, yield strength, and stiffener spacing, a new section modulus for the vertical stiffener may be determined from Figure 7.

8. Repeat Step 4 using the real lengths of the members.

9. Repeat Steps 4 through 8 until the deflection from Figure 6 agrees well with the deflection from the grillage analysis.

10. Further modifications, i.e., varying the size of each vertical stiffener, is left to the discretion of the designer.

To use the above procedure, the grillage analysis should be programmed on an electronic computer. This is currently being done at the David Taylor Model Basin

It is noted from the nomographs that radii greater than 14 ft entail the use of impractically large members if the deflections are not to be excessive. To overcome this, a grillage with two main girders will be necessary.

Because experimental data on bulkheads with two main girders are lacking, no attempt is made to develop a design procedure for this type bulkhead at the present time. However, equations for moment and deflection of the stiffeners of this type bulkhead are presented in the appendix.

ACKNOWLEDGMENTS

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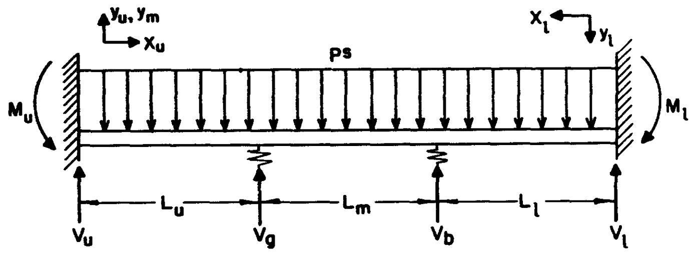
2. Palermo, P.M. and Heller, S.R., CDR, USN, "Hydrostatic Tests of Full-Scale Models of Holding Bulkheads of USS TANG (SS 563) and USS ALBACORE (AGSS 569)," David Taylor Model Basin Report C-1005 (Aug 1959) CONFIDENTIAL.

APPENDIX

UNIFORMLY LOADED, CLAMPED BEAM WITH TWO ELASTIC INTERNAL SUPPORTS

When the diameter is so great that two main horizontal girders are necessary, the vertical stiffeners may be considered as uniformly loaded, clamped beams with two elastic internal supports. Consider the beam shown below



where V_g and V_b are the reaction forces at the intersections with the two main girders.

The differential equation relating bending moments, shear deformation, and curvature is

$$EI\frac{d^2y}{dx^2} = M + \frac{kEI}{AG}ps \qquad [8.0]$$

Then, for the upper span, this and the first two integrations are

$$EI_u\frac{d^2y_u}{dx_u^2} = -M_u + V_u x_u - \frac{psx_u^2}{2} + \frac{k_u EI_u}{A_u G}ps$$

$$EI_u\frac{dy_u}{dx_u} = -M_u x_u + V_u\frac{x_u^2}{2} - \frac{psx_u^3}{6} + \frac{k_u EI_u psx_u}{A_u G} + A_1 \qquad [8.1]$$

$$EI_u y_u = -M_u\frac{x_u^2}{2} + V_u\frac{x_u^3}{6} - \frac{psx_u^4}{24} + \frac{k_u EI_u psx_u^2}{2A_u G} + A_1 x_u + B_1$$

The corresponding relations for the middle span are

$$EI_m\frac{d^2ym}{dx_u^2} = -M_u + V_u x_u + V_g(x_u - L_u) - \frac{psx_u^2}{2} + \frac{k_m EI_m ps}{A_m G} \qquad [8.2]$$

$$EI_m \frac{dy_m}{dx_u} = - M_u x_u + V_u \frac{x_u^{\,2}}{2} + V_g x_u \left(\frac{x_u}{2} - L_u\right) - \frac{psx_u^{\,3}}{6} + \frac{k_m EI_m psx_u}{A_m G} + A_2$$

[8.2]

$$EI_m y_m = - M_u \frac{x_u^{\,2}}{2} + V_u \frac{x_u^{\,3}}{6} + V_g \frac{x_u^{\,2}}{2} \left(\frac{x_u}{3} - L_u\right) - \frac{psx_u^{\,4}}{24} + \frac{k_m EI_m psx_u^{\,2}}{2A_m G} + A_2 x_u + B_2$$

The relationships for the lower span are

$$EI_l \frac{d^2 y_l}{dx_l^2} = M_l - V_l x_l + \frac{psx_l^2}{2} - \frac{k_l EI_l ps}{A_l G}$$

$$EI_l \frac{dy_l}{dx_l} = M_l x_l - V_l \frac{x_l^2}{2} + \frac{psx_l^3}{6} - \frac{k_l EI_l psx_l}{A_l G} + A_3 \qquad [8.3]$$

$$EI_l y_l = M_l \frac{x_l^2}{2} - V_l \frac{x_l^3}{6} + \frac{psx_l^4}{24} - \frac{k_l EI_l psx_l^2}{2A_l G} + A_3 x_l + B_3$$

Use of the displacement boundary conditions at the exterior supports

$$x_u = 0, \quad y_u = 0$$

$$x_1 = 0, \quad y_1 = 0$$

leads to

$$B_1 = B_3 = 0$$

The slope boundary conditions at the exterior supports

$$x_u = 0, \quad y_u' = - \frac{k_u V_u}{A_u G}$$

$$x_l = 0, \quad y_l' = - \frac{k_l V_l}{A_l G}$$

lead to

$$A_1 = -\frac{k_u E I_u V_u}{A_u G}$$

$$A_3 = -\frac{k_l E I_l V_l}{A_l G}$$

The displacement boundary conditions at the interior supports

$$x_u = L_u, \quad y_u = -\delta_g$$

$$x_1 = L_1, \quad y_1 = \delta_b$$

lead to

$$M_u = \frac{2EI_u\delta_g}{L_u^2} + V_u\frac{L_u}{3} - \frac{psL_u^2}{12} + \frac{k_u EI_u}{A_u G}\left(ps - \frac{2V_u}{L_u}\right)$$

[8.4]

$$M_l = \frac{2EI_l\delta_b}{L_l^2} + V_l\frac{L_l}{3} - \frac{psL_l^2}{12} + \frac{k_l EI_l}{A_l G}\left(ps - \frac{2V_l}{L_l}\right)$$

Continuity of slope at interior support g

$$\left.\frac{dy_u}{dx_u}\right|_{x_u=L_u} - \frac{k_u V_g}{A_u G} = \left.\frac{dy_m}{dx_u}\right|_{x_u=L_u}$$

leads to

$$A_2 = V_g\frac{L_u^2}{2} + \frac{k_u EI_m}{A_u G}\left[psL_u\left(1 - \frac{k_m A_u}{k_u A_m}\right) - V_u - V_g\right] + \left(1 - \frac{I_m}{I_u}\right)\left(M_u L_u - V_u\frac{L_u^2}{2} + \frac{psL_u^3}{6}\right)$$

Continuity of deflection at support g

$$y_u\big|_{x_u=L_u} = y_m\big|_{x_u=L_u}$$

leads to

$$B_2 = -V_g\frac{L_u^3}{6} - \frac{k_u EI_m L_u}{A_u G}\left[\frac{psL_u}{2}\left(1 - \frac{k_m A_u}{k_u A_m}\right) - V_g\right] - \left(1 - \frac{I_m}{I_u}\right)\left(M_u\frac{L_u^2}{2} - V_u\frac{L_u^3}{3} + \frac{psL_u^4}{8}\right)$$

From the equations of static equilibrum are obtained:

$$\text{Force: } V_u + V_g + V_b + V_l = psL_u\left(1 + \frac{L_m}{L_u} + \frac{L_l}{L_u}\right) = \Delta_1 \qquad [8.5]$$

$$\text{Moment: } -\Delta_2 V_u - \Delta_3 V_g - \Delta_4 V_b - \Delta_5 V_l = \Delta_6 \qquad [8.6]$$

where

$$\Delta_2 = \frac{L_u}{3} - \frac{2k_u EI_u}{L_u A_u G}$$

$$\Delta_3 = L_u$$

$$\Delta_4 = L_u\left(1 + \frac{L_m}{L_u}\right)$$

$$\Delta_5 = L_u\left(1 + \frac{L_m}{L_u} + \frac{2L_l}{3L_u}\right) + \frac{2k_l EI_l}{L_l A_l G}$$

$$\Delta_6 = \frac{2EI_u\delta_g}{L_u^2}\left[1 - \frac{\delta_b}{\delta_g}\cdot\frac{I_l}{I_u}\left(\frac{L_u}{L_l}\right)^2\right] + \frac{psL_u^2}{12}\left[\left(\frac{L_l}{L_u}\right)^2 - 1 - 6\left(1 + \frac{L_m}{L_u} + \frac{L_l}{L_u}\right)^2\right]$$

$$+ \frac{psk_u EI_u}{A_u G}\left(1 - \frac{k_l}{k_u}\cdot\frac{I_l}{I_u}\cdot\frac{A_u}{A_l}\right)$$

Continuity of slope at support b, i.e.,

$$y_m'\Big|_{x_u = L_u + L_m} - \frac{k_m V_b}{A_m G} = y_l'\Big|_{x_l = L_l}$$

yields

$$\Delta_7 V_u + \Delta_8 V_g - \Delta_9 V_b + \Delta_{10} V_l = \Delta_{11} \qquad [8.7]$$

where

$$\Delta_7 = \frac{I_l}{I_m}\left\{L_u^2\left[\frac{I_m}{6I_u} + \frac{2L_m}{3L_u} + \frac{1}{2}\left(\frac{L_m}{L_u}\right)^2\right] + \frac{k_u EI_u}{A_u G}\left(\frac{I_m}{I_u} + 2\frac{L_m}{L_u}\right)\right\}$$

$$\Delta_8 = \frac{I_l}{I_m}\left(\frac{L_m^2}{2} + \frac{I_m}{I_u}\cdot\frac{k_u E I_u}{A_u G}\right)$$

$$\Delta_9 = \frac{I_l}{I_m}\cdot\frac{k_m E I_m}{A_m G}$$

$$\Delta_{10} = \frac{L_l^2}{6} + \frac{k_l E I_l}{A_l G}$$

$$\Delta_{11} = \frac{psL_u^3}{12}\left\{\left(\frac{L_l}{L_u}\right)^3 - \frac{I_l}{I_m}\left[2 - \frac{I_m}{I_u} + \frac{L_m}{L_u} - 2\left(1 + \frac{L_m}{L_u}\right)^3\right]\right\} + \frac{psk_u E I_u}{A_u G}\, L_m\left(\frac{I_l}{I_m} - \frac{k_m}{k_u}\cdot\frac{I_l}{I_u}\cdot\frac{A_u}{A_m}\right)$$

$$+ \frac{2EI_u\delta_g}{L_u}\cdot\frac{I_l}{I_u}\left(\frac{\delta_b}{\delta_g}\cdot\frac{L_u}{L_l} + 1 + \frac{I_u}{I_m}\cdot\frac{L_m}{L_u}\right)$$

Continuity of deflection at support b, i.e.,

$$y_m\Big|_{x_u = L_u + L_m} = -\,y_l\Big|_{x_l = L_l}$$

gives

$$\Delta_{12}V_u + \Delta_{13}V_g = \Delta_{14} \qquad [8.8]$$

where

$$\Delta_{12} = \frac{I_l}{I_m}\left\{\frac{L_m^3}{6}\left[1 + 2\frac{L_u}{L_m} + \frac{I_m}{I_u}\left(\frac{L_u}{L_m}\right)^2\right] + \frac{k_u E I_u}{A_u G}\, L_m\left[\frac{L_m}{L_u} + \frac{I_m}{I_u}\right]\right\}$$

$$\Delta_{13} = \frac{I_l}{I_m}\left[L_m^3 - \frac{I_m}{I_u}\cdot\frac{k_u E I_u}{A_u G}\cdot L_m\right]$$

$$\Delta_{14} = \frac{I_l}{I_m}\left\{\frac{ps}{24}\, L_u^3 L_m\left[\left(\frac{L_m}{L_u}\right)^3 + 4\left(\frac{L_m}{L_u}\right)^2 + 5\frac{L_m}{L_u} + 2\frac{I_m}{I_u}\right] - \frac{psk_u E I_u}{2A_u G}\, L_m^2\left(1 - \frac{k_m}{k_u}\cdot\frac{A_u}{A_m}\cdot\frac{I_m}{I_u}\right)\right.$$

$$\left. + EI_u\delta_g\left[\left(\frac{L_m}{L_u}\right)^2 + \frac{I_m}{I_u}\left(1 + 2\frac{L_m}{L_u} - \frac{\delta_b}{\delta_g}\right)\right]\right\}$$

Equations [8.5] through [8.8] are then solved simultaneously for the vertical reactions in terms of the deflections

$$V_u = \frac{\Delta_{14}[\Delta_{10}(\Delta_4-\Delta_3) + \Delta_9(\Delta_5-\Delta_3) + \Delta_8(\Delta_5-\Delta_4)] - \Delta_{13}[\Delta_{11}(\Delta_5-\Delta_4) + \Delta_{10}(\Delta_1\Delta_4+\Delta_6) + \Delta_9(\Delta_1\Delta_5+\Delta_6)]}{\text{Denominator}} \qquad [8.9]$$

$$V_g = \frac{\Delta_{14}[\Delta_{10}(\Delta_2-\Delta_4) + \Delta_9(\Delta_2-\Delta_5) + \Delta_7(\Delta_4-\Delta_5)] + \Delta_{12}[\Delta_{11}(\Delta_5-\Delta_4) + \Delta_{10}(\Delta_1\Delta_4+\Delta_6) + \Delta_9(\Delta_1\Delta_5+\Delta_6)]}{\text{Denominator}} \qquad [8.10]$$

$$V_b = \frac{\Delta_{14}[\Delta_{10}(\Delta_3-\Delta_2) + \Delta_8(\Delta_2-\Delta_5) + \Delta_7(\Delta_5-\Delta_3)] + \Delta_{13}[\Delta_{11}(\Delta_5-\Delta_2) + \Delta_{10}(\Delta_1\Delta_2+\Delta_6) - \Delta_7(\Delta_1\Delta_5+\Delta_6)] - \Delta_{12}[\Delta_{11}(\Delta_5-\Delta_3) + \Delta_{10}(\Delta_1\Delta_3+\Delta_6) - \Delta_8(\Delta_1\Delta_5+\Delta_6)]}{\text{Denominator}} \qquad [8.11]$$

$$V_l = \frac{\Delta_{14}[\Delta_4(\Delta_8-\Delta_7) + \Delta_3(\Delta_7+\Delta_9) - \Delta_2(\Delta_8+\Delta_9)] + \Delta_{13}[-\Delta_{11}(\Delta_4-\Delta_2) + \Delta_9(\Delta_1\Delta_2+\Delta_6) + \Delta_7(\Delta_1\Delta_4+\Delta_6)] + \Delta_{12}[\Delta_{11}(\Delta_4-\Delta_3) - \Delta_9(\Delta_1\Delta_3+\Delta_6) - \Delta_8(\Delta_1\Delta_4+\Delta_6)]}{\text{Denominator}} \qquad [8.12]$$

where

$$\text{Denominator} = \Delta_{13}[\Delta_{10}(\Delta_2-\Delta_4) + \Delta_9(\Delta_2-\Delta_5) + \Delta_7(\Delta_4-\Delta_5)] + \Delta_{12}[\Delta_{10}(\Delta_4-\Delta_3) + \Delta_9(\Delta_5-\Delta_3) + \Delta_8(\Delta_5-\Delta_4)]$$

Equations [8.9] through [8.12] can be used in conjunction with Equation [6.6] for a grillage analysis of the bulkhead. Equation [6.6] can be used for both horizontal girders. The use of two girders doubles the numerical work. Eight simultaneous equations are replaced by sixteen.

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Various methods of elastic analysis of circular bulkhead stiffening systems are presented. The present Bureau of Ships design procedure for the main horizontal girder and vertical stiffeners is modified to include the effects of shear deformation, end load, and variable end fixity. In addition, allowance is made for deflection of the vertical stiffeners at the intersection with the main horizontal girder. A grillage analysis of the entire stiffening system, including the effects of shear deformation, end load, and variable end fixity is presented. Formulas for moments, deflection, and the shear factor k are presented, and a numerical example is given.

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