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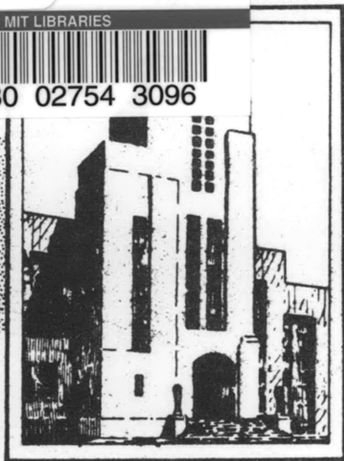

NAVY DEPARTMENT

DAVID TAYLOR MODEL BASIN

HYDROMECHANICS

AERODYNAMICS

STRUCTURAL MECHANICS

APPLIED MATHEMATICS

DESIGN FORMULAS FOR YIELDING SHOCK MOUNTS

by

[redacted]

Glenn D. Elmer

STRUCTURAL MECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

January 1959 Report 1287

PRNC-TMB-648 (11-56)

DEPARTMENT OF THE NAVY
DAVID TAYLOR MODEL BASIN
WASHINGTON 7, D.C.

IN REPLY REFER TO
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(710:MCC:pvd)
Ser 7-28
12 FEB 1959

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To: Chief, Bureau of Ships (312) (in duplicate)

Subj: Design of shock mounts; forwarding of report on

Encl: (1) David Taylor Model Basin Report 1287 entitled "Design Formulas for Yielding Shock Mounts" 23 copies

1. In enclosure (1) design formulas are given for both the elastic characteristics and the plastic limit loads for three different configurations of yielding shock mounts. The behavior of these mounts is discussed, and a sample design computation is carried out. The satisfactory performance of a mount of this type during a field test is discussed briefly. Although some requirements cannot be fulfilled with yielding mounts of the type discussed, yielding systems constructed of simple and inexpensive components do offer positive shock protection.

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DESIGN FORMULAS FOR YIELDING SHOCK MOUNTS

by

Glenn D. Elmer

January 1959

Report 1287
NS 711-111

ABSTRACT

Design formulas are given for both the elastic characteristics and the plastic limit loads for three different configurations of yielding shock mounts. In addition, the behavior of these mounts is discussed, and a sample design computation is carried out. The satisfactory performance of a mount of this type during a field test is also briefly cited.

INTRODUCTION

In Operation CROSSROADS the Taylor Model Basin test party used with success a simple spring shaped like the letter C to support a table for the recording instruments. More recently, a variant of the C-mount, consisting of a section of thin-walled cylinder, was used to provide shock isolation of recording instruments in a field test. These mounts were designed to yield under the shock loading, thus limiting the maximum acceleration of the supported mass to a prescribed low value.

A yielding mount of this type has some interesting and attractive characteristics, apart from its inherent cheapness and simplicity. For example, it is possible, by varying the geometry of the mount, to have an elastic system of relatively high natural frequency but which still will deform plastically at low accelerations. In view of the potential interest, it seems desirable to make more generally available, information on the properties of such a system. The purposes of this report are to present the formulas necessary to design a yielding mount of this type and to give some comments on its applicability.

DESIGN FORMULAS

The design formulas will be presented in tabular form for three different configurations of mounts: cylinders, half cylinders joined by straight portions (double C-mounts), and C-mounts. These configurations are shown in Figure 1. The base of the mount is assumed to be rigidly fastened to a structure much stiffer than the mount, so that no rotation of the base of the mount takes place. For the first two configurations, the top of the mount is also assumed to be rigidly fastened to the supported mass, so that the top is also clamped. For the C-mounts, formulas are given for both clamped and pin-ended conditions at the top of the mount. For simplicity it will be assumed that the mounts are all used in the vertical position as shown in Figure 1 and that the loading occurs at the top.

Almost all the elastic formulas in Table 1, 2, and 3 may be found in standard references, [1,2]* although sometimes in slightly different form. For the derivation of elastic formulas not given in the references, Castigliano's first theorem was used. A complete discussion of this method is given in Reference 1, beginning on page 79. The method used for deriving plastic limit loads is illustrated in the Appendix of this report.

The nomenclature used in Tables 1, 2, and 3 is as follows:

A_H Maximum horizontal acceleration of the mounted mass in inches per second squared

A_V Maximum vertical acceleration of the mounted mass in inches per second squared

E Modulus of elasticity of the mount material in pounds per inch squared

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*References are listed on page 14.

K_H	Effective spring constant for horizontal motions in pounds per inch
K_V	Effective spring constant for vertical motions in pounds per inch
ℓ	Half length of straight portion of double C-mounts and length of straight portion of single C-mounts in inches
L	Length of mount perpendicular to cross section in inches
M	Mass of supported load in pound-seconds squared per inch
$P_{H\ EL}$	Maximum horizontal elastic load in pounds
$P_{H\ LIM}$	Horizontal limit load at which the mount becomes fully plastic in pounds
$P_{V\ EL}$	Maximum vertical elastic load in pounds
$P_{V\ LIM}$	Vertical limit load at which the mount becomes fully plastic in pounds
R	Mean radius of circular portion of mount in inches
t	Thickness of mount in inches
σ_y	Yield stress of the mount material in pounds per inch squared

TABLE 1

Design Formulas for Cylindrical Mounts

Vertical Motions	Horizontal Motions
$K_V = \frac{ELt^3}{1.79\ R^3}$	$K_H = \frac{ELt^3}{3\pi R^3}$
$P_{V\ EL} = \frac{\pi}{6}\frac{\sigma_y Lt^2}{R}$	$P_{H\ EL} = \frac{\sigma_y Lt^2}{3R}$
$P_{V\ LIM} = \frac{\sigma_y Lt^2}{R}$	$P_{H\ LIM} = \frac{\sigma_y Lt^2}{2R}$
$A_V = \frac{\sigma_y Lt^2}{RM}$	$A_H = \frac{\sigma_y Lt^2}{2RM}$

TABLE 2

Design Formulas for Double C-Mounts and for C-Mounts with Top Clamped

For C-mounts with the top clamped divide all values by 2.

Vertical Motions
$K_V = \frac{1}{6} ELt^3 \frac{1}{\frac{2}{3}\ell^3 + \pi\ell^2 R + 4\ell R^2 + \frac{\pi}{2}R^3 - \frac{(\ell^2 + \pi\ell R + 2R^2)^2}{2\ell + \pi R}}$
$P_{V\ EL} = \frac{\sigma_y Lt^2}{3\left(\frac{\ell^2 + \pi\ell R + 2R^2}{2\ell + \pi R}\right)}$
$P_{V\ LIM} = \frac{\sigma_y Lt^2}{\ell + R}$
$A_V = \frac{\sigma_y Lt^2}{M(\ell + R)}$

Horizontal Motions
$K_H = \frac{1}{3} ELt^3 \frac{1}{3\pi R^3 - \frac{4\pi^2 R^4}{4\ell + 2\pi R}}$
$P_{H\ EL} = \frac{\sigma_y Lt^2}{3\left(\frac{4\ell R + \pi R^2}{2\ell + \pi R}\right)}$
$P_{H\ LIM} = \frac{\sigma_y Lt^2}{2R}$
$A_H = \frac{\sigma_y Lt^2}{2RM}$

TABLE 3

Design Formulas for C-Mounts with Top Free to Rotate

Vertical Motions
$$K_V = \frac{1}{12} ELt^3 \frac{1}{\frac{2}{3}\ell^3 + \pi\ell^2 R + 4\ell R^2 + \frac{\pi}{2}R^3}$$ $$P_{V\ EL} = \frac{\sigma_y Lt^2}{6(\ell + R)}$$ $$P_{V\ LIM} = \frac{\sigma_y Lt^2}{4(\ell + R)}$$ $$A_V = \frac{\sigma_y Lt^2}{4(\ell + R)M}$$
Horizontal Motions
$$K_H = \frac{Et^3 L}{18\pi R^3}$$ $$P_{H\ EL} = \frac{\sigma_y Lt^2}{12R}$$ $$P_{H\ LIM} = \frac{\sigma_y Lt^2}{8R}$$ $$A_H = \frac{\sigma_y Lt^2}{8RM}$$

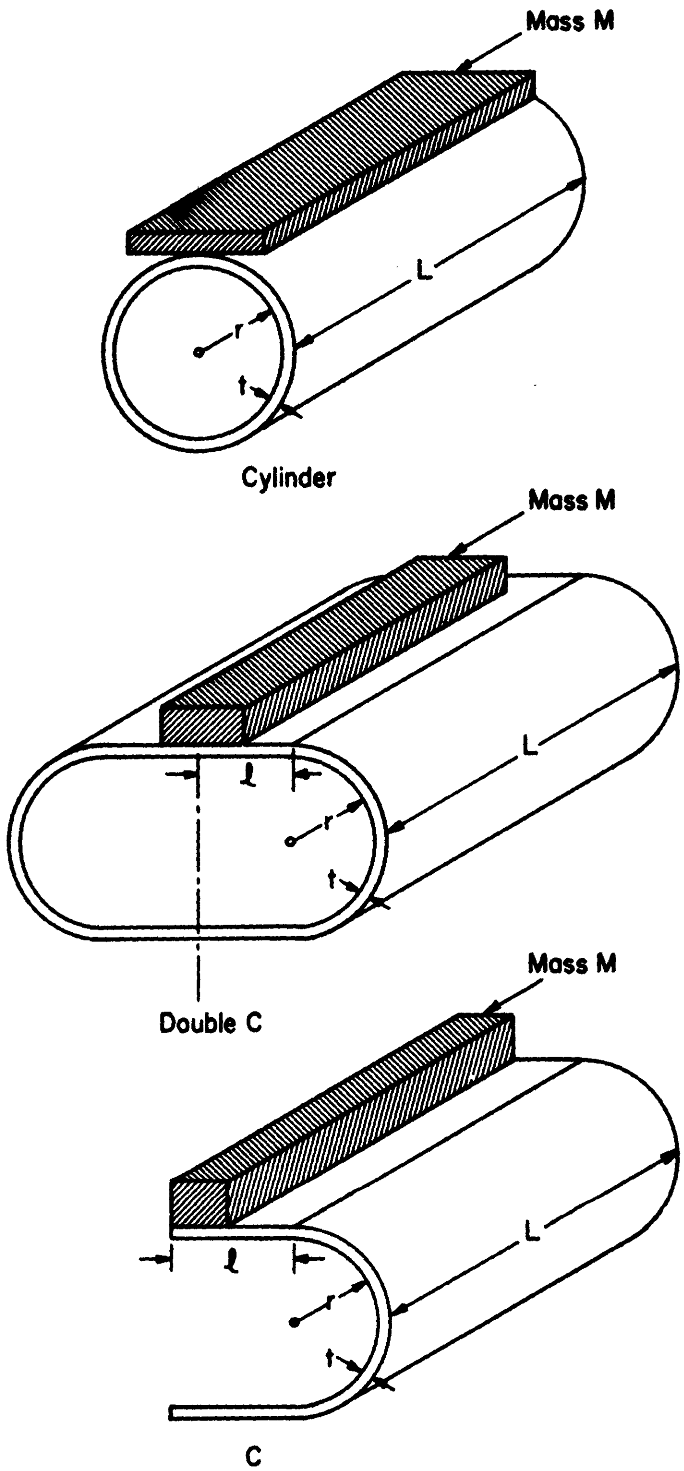

Figure 1 - Configurations of Mounts

DISCUSSION

Inspection of the design formulas shows that the elastic stiffness of the mounts varies as the cube of the thickness t and inversely as a cubic combination of ℓ and R. The plastic load varies as the square of the thickness and inversely as a linear combination of ℓ and R. Thus, by varying the relative magnitudes of t, ℓ, and R, it is possible, within limits, to design a mounting system with a particular natural frequency which will yield at a specified acceleration. This characteristic could be of use in some shipboard installations. The natural frequency of the mounting system could possibly be made high enough so that the system would be essentially rigid for most normal shipboard motions and yet would afford protection against excessive accelerations caused by shock. If the natural frequency is too low, there could be excessive motion under normal ship motions in heavy seas.

Mounts of the types shown in Figure 1 provide protection in only two directions, since they are inherently rigid along the length of the mount. A mount which would provide protection in all three directions could be made by using two short mounts in series, (one above the other) with the axes displaced by 90 deg.

The details of attachment at the top and bottom of the mounts influence the stiffness of the mounts much more than they do the limit load. For example, if the bottom of a cylindrical mount is not fastened exactly on the vertical axis as assumed but at two points displaced 30 deg either side of vertical, the vertical stiffness will increase by about 25 percent, while the vertical limit load will increase by only about 4 percent. The same general behavior would hold for the other mount configurations.

Since the value of the mounts as shock attenuators depends primarily upon the possibility of large deformations at low forces, the mounts should be made of a material with good ductility. Low-carbon steel is one possible material; cylindrical mounts of this material have been tested to static deformations of over one mount radius without any sign of fracture.

Yielding mounts are essentially irreversible and thus are particularly suited for situations requiring protection against only one shock in any one direction. It is possible, however, to have protection against multiple shocks, if the deformation after the initial shock is not too great since a slightly deformed mount would still afford about the same measure of protection. Also, if the original configuration of the mount can be restored before another shock, the protection afforded should be essentially the same.

SAMPLE CALCULATION

To illustrate a possible design procedure, a sample calculation for a cylindrical mount will be carried out. The requirements of the mounting system are as follows:

1. The maximum vertical acceleration of a 1000-lb load should not exceed 4 g.
2. The mount should be able to deform at least 6 in.
3. The mount may be up to 30 in. long.
4. The vertical natural frequency of the elastic system should be at least 10 cps.

The first requirement dictates that the vertical limit load should not exceed 5000 lb. To meet the second requirement, the minimum radius of the cylinder is 6 in. if deformations as as large as one radius are allowed. For the fourth requirement, the vertical stiffness must not be less than 10,200 lb/in. The material is assumed to be low-carbon steel with a yield stress of 35,000 psi.

From Table 1 we see that

$$P_{V\ LIM} = \frac{\sigma_y L t^2}{R} \leq 5000 \text{ lb}$$

and

$$K_V = \frac{ELt^3}{1.79\ R^3} \geq 10{,}200 \text{ lb/in.}$$

Substituting the values for σ_y and E, and assuming L = 30 in.

$$\frac{t^2}{R} \leq 4.8 \times 10^{-3}$$

$$\frac{t^3}{R^3} \geq 2.1 \times 10^{-5}$$

Using the equalities, these relations are solved to find t = 0.172 in. and R = 6.2 in. This exceeds the desired minimum radius of 6 in., so the two relations are compatible. Using a standard stock thickness, say 3/16 in., the requirement on the limit load is met if R = 7.4 in. With this value for the radius the natural frequency of the elastic system is about 9 cps, slightly less than specified. However, as was stated before, the details of any probable method of attachment would tend to increase the natural frequency more than the limit load, so that this compromise geometry may meet the specifications.

PERFORMANCE OF YIELDING MOUNTS

The design and performance of one typical mount application in the field will be briefly discussed. It was desired to isolate an instrument table weighing 10,000 lb so that the maximum vertical acceleration would be about 2 g and deformations of at least 12 in. would be possible.

For this application the table was mounted on two cylinders of low-carbon steel with a yield stress of 50,000 psi. Each cylinder was 60 in. long and 24 in. in diameter, and had a 0.250-in. wall thickness. With these parameters the design maximum vertical acceleration was about 2.1 g, and the vertical natural frequency was about 4 cps.

The mounts performed well under severe shock loadings. In one instance a very rapid displacement of the supporting structure of about 8 in. resulted in the expected plastic deformation of about 8 in. with no sign of instrument malfunction. Figure 2 shows the instrument table after the shock. After this loading the mounts were restored to the original circular cross section and protected the instruments equally as well during a subsequent shock of about the same magnitude.

Inspection of the oscillograph records after these tests showed that some high-frequency oscillations of small amplitude were transmitted through the steel mounts. These high-frequency components may be attenuated by inserting a pad of felt or soft rubber under the recording instruments. The soft pad would have no effect on the gross performance of the mounts.

Some difficulty was encountered with the mounts during installation on a ship because of the large compliance of the mounts in the horizontal direction. This difficulty was partially remedied by welding gussets to the pipe which materially raised the stiffness but had a much smaller effect on the limit load. In addition, the table was fastened down with rods and turnbuckles to prevent large oscillation during normal shipboard motions. The bracing was needed because of the extremely low level acceleration which was desired and the accompanying low natural frequency in the horizontal direction. If a higher level of acceleration could have been tolerated, it might have been possible to eliminate the bracing when underway.

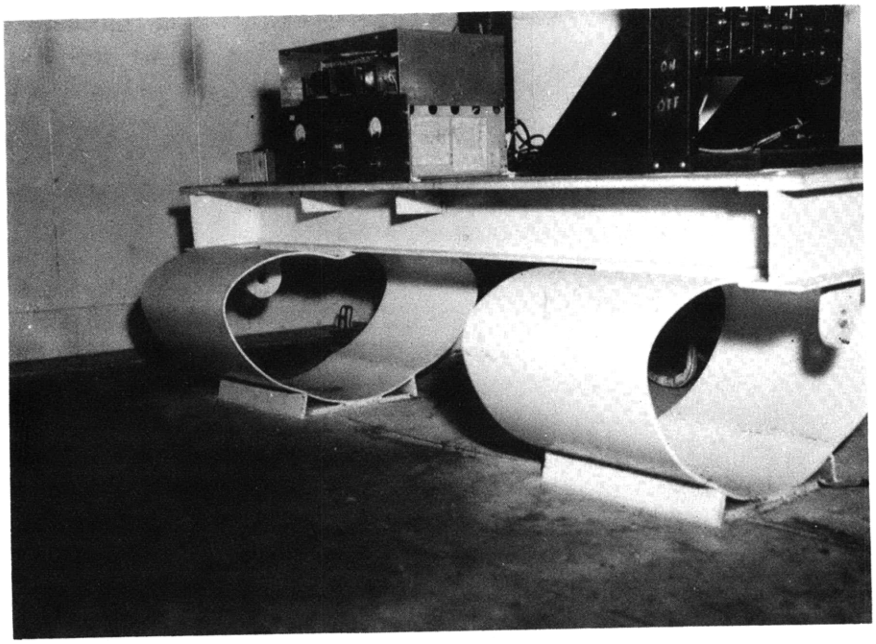

Figure 2 - Instrument Table after Shock Loading

Note the large deformation of the cylindrical mounts.

In conclusion, it should be pointed out that there are requirements which cannot be fulfilled with yielding mounts of the type discussed. For many applications, however, yielding systems constructed of simple and inexpensive components offer positive shock protection.

APPENDIX

The principle of virtual displacements may be used to derive the limit loads. As an example, consider the cylindrical mount under vertical loading. For this case symmetry requires that four plastic hinges form, two on the vertical axis and two on the horizontal axis. Then, if the point directly under the load is allowed a small displacement δ, it is easy to see that the angle through which each of the four hinges will rotate is given by $\frac{\delta}{R}$. Equating the work done by the load to that absorbed by the structure, we have

$$P_{V\ LIM} \cdot \delta = \frac{\sigma_y t^2 L}{4} \cdot 4\frac{\delta}{R}$$

and

$$P_{V\ LIM} = \frac{\sigma_y t^2 L}{R}$$

The other limit loads may be derived similarly.

All equations, both elastic and plastic, were derived on the assumption of small changes in geometry. Also, for the plastic limit loads, an elastic-perfectly-plastic stress-strain curve was assumed. For very large deformations, the formulas will be more approximate since both the changes in geometry and the work-hardening characteristics of the material will influence the limit load. The limit load in practice may vary at large displacements.

REFERENCES

1. Timoshenko, S., "Strength of Materials, Part II," D. Van Nostrand Company, Inc., New York (1930).

2. Roark, Raymond J., "Formulas for Stress and Strain," Third Edition, McGraw-Hill Book Company, Inc., New York (1954).

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I. Elmer, Glenn D.
II. NS711-111

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