MEASUREMENT OF TRANSITION ON A SPHERE
AT HIGH REYNOLDS NUMBERS

by

E.Y. Hsu and M.S. Macovsky

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ABSTRACT

A hot-wire survey was conducted on a smooth 18-inch sphere to determine the position of boundary layer transition from laminar to turbulent flow. Tests run both in the towing basin and in the transonic wind tunnel, at Reynolds numbers from 2.2 to $3.5 \times 10^6$ (based on diameter), indicated transition in a region 90 degrees from stagnation. This result agrees with predictions based on neutral stability theory.

INTRODUCTION

The David Taylor Model Basin was recently requested\textsuperscript{1} to investigate experimentally the position of transition from laminar to turbulent flow on a sphere at a Reynolds number of approximately $3 \times 10^6$ (based on diameter). With the exception of the work of Fage,\textsuperscript{2} the position of transition on a sphere has not been considered to any great extent, and even in this referenced work, the transition points were not determined by direct measurement.

This investigation at the Model Basin was conducted both in the transonic wind tunnel and in the towing basin. In both instances, the hot wire technique was employed to detect transition and, in both experiments, transition occurred in the region of 85 to 90 degrees from the stagnation point. Computation of the neutral stability region at the test Reynolds numbers substantiated the test results.

This report will primarily summarize the theoretical computations and basin test techniques and results. The wind tunnel data will also be given.

EXPERIMENTAL PROCEDURES

The investigation in both the transonic wind tunnel and the towing basin was conducted on an existing hollow, highly-polished, brass sphere whose dimensions were precisely held to $\pm 0.001$ inch on any diameter. For tests in the towing basin, the sphere was mounted at one end of a tapered sting approximately 2 1/2 feet in length. The other end of the sting was connected to an existing tapered, streamlined towing strut. The assembly, as shown in Figure 1a, was mounted on the carriage to permit towing the sphere at a depth of 9 feet to its center, which was sufficient to render free-surface effects negligible and to insure freedom from cavitation.

Four hot-wire probes were mounted on the sphere at four quadrants, namely, at 70, 80, 90, and 100 degrees respectively from the stagnation

\textsuperscript{1}References are listed on page 5.
point. When the sphere was towed through the desired speed range, at least one of the hot-wire probes was expected to detect the transition phenomenon. The probes, shown in Figure 1b, were designed for operation near the spherical surface with a minimum of interference and were fastened rigidly to the sphere to eliminate the possibility of probe vibration. The lucite portion of the probe was slotted to permit a maximum adjustment of 5 degrees. Since only a qualitative distinction between laminar and turbulent flow is required, insofar as the location of the transition point is concerned, the wires were operated by a simple constant-current circuit without compensation; see Figure 2. The amplified voltage fluctuations corresponding to velocity fluctuations were recorded on a 4-channel Sanborn recorder. The onset of transition, as shown in Figure 3, is marked by intermittent peaks of the output voltage signal.

In the wind-tunnel experiments, a constant-temperature hot-wire circuit was used to measure the root-mean-square of the fluctuating velocity component in the direction of the local streamline. Here, a sudden increase in the rms value was taken as being indicative of the onset of transition. In addition, a visual display of the hot-wire signal on an oscilloscope was employed as an independent detector of transition. The appearance of the scope, in this instance, was comparable to the records taken in the basin tests. Both of these wind-tunnel techniques gave consistent results.

During the towing tests considerable vibration of the sphere and towing strut was present in the high-speed range. Consequently, spurious vibration signals were superimposed on some of the hot-wire signals, making the interpretation of these records difficult. The thickness of the boundary layer caused another problem. Since the boundary layer is extremely thin at these high Reynolds numbers (≈ 0.01-0.02 in.) and since the hot wire must of necessity be imbedded deeply within the boundary layer, the use of a conducting sphere resulted in the unforeseen possibility of short circuiting. This happened occasionally.

In spite of these difficulties, useful information concerning the location of the transition has been obtained from oscillograph records similar to those of Figure 3. Illustrated in this figure are sample records depicting the onset of transition, and the evolution of the boundary layer to fully developed turbulence. Analysis of the data indicates that transition occurs in a region 85 to 90 degrees from the stagnation point at Reynolds numbers between $2.2 \times 10^6$ and $2.3 \times 10^6$. This result is further substantiated by the tests conducted in the transonic wind tunnel at a higher Reynolds number, and by the neutral stability point analysis based on small oscillation theory as mentioned earlier. This analysis will be presented in the following section.
From a consideration of the stability of laminar boundary layers, one can determine the existence of a point of neutral stability in whose immediate neighborhood and upstream thereof disturbances of all frequencies are damped out. Mangler derived the criteria which specify the neutral stability point in terms of the critical value \( R_0 = \frac{U_0}{\nu} \), as a function of a pressure gradient parameter

\[
\frac{\beta}{\nu} \frac{dU}{ds}
\]

where

- \( U \) is the velocity at the outer edge of the boundary layer,
- \( \theta \) is the momentum thickness,
- \( s \) is the arc length along a meridian profile, and
- \( \nu \) is the kinematic viscosity.

The solid curve shown in Figure 4, which is based on the stability characteristics of Pohlhausen-type velocity profiles as calculated by Schlichting and Ulrich is applicable to both two-dimensional and axisymmetric boundary layers.

In order to make the neutral stability point analysis, it is necessary to calculate the momentum thickness distribution along a meridian of the sphere. Although the series solution of the velocity distribution in the laminar boundary layer of a sphere is known, it is simpler to use an approximate method to evaluate the momentum thickness distribution directly, using both the momentum equation and the potential-flow solution for the velocity distribution on the sphere. The integrated momentum equation for laminar boundary layer expressed in nondimensional form can then be expressed as

\[
\left( \frac{r_w}{D^2} \right)^2 = 4 \left( \frac{1}{RD} \right) \frac{1}{U_{\infty}} \int_0^{1/D} \left( \frac{r_w}{D} \right)^2 \left( \frac{U}{U_{\infty}} \right)^5 \sec \alpha \ d \left( \frac{1}{D} \right)
\]

where \( r_w \) is the transverse radial distance from the centerline to surface of the body

- \( D \) is the diameter of the sphere,
- \( U_{\infty} \) is the undisturbed free-stream velocity,
- \( RD \) is the Reynolds number based on the diameter of the sphere and the undisturbed free-stream velocity,
- \( l \) is the axial distance along the body measured from the stagnation point, and
- \( \alpha \) is the angle between a tangent to the surface and the longitudinal axis of body.

Figure 5 illustrates these geometric parameters.

The potential-flow solution for velocity distribution along the sphere is
\[
\frac{U}{U_\infty} = \frac{3}{2} \sin \phi = \frac{3}{2} \frac{r_w}{R}
\]  \[2\]

where \(\phi\) is the angle at the center of the sphere between the point in consideration and the stagnation point, and \(R\) is the radius of the sphere. Substituting Equation [2] in Equation [1], there is obtained

\[
\left(\frac{r_w \theta}{D^2}\right)^2 = \frac{162}{3} \frac{R_D}{U_\infty} \frac{1}{U_\infty} \frac{1}{6} \int_0^{l/D} \left(\frac{r_w}{D}\right)^6 d\left(\frac{l}{D}\right)
\]  \[3\]

Since

\[
\frac{r_w}{D} = \left[\frac{l}{D} - \left(\frac{l}{D}\right)^2\right]^{3/2}
\]

Equation [3] may then be reduced to

\[
\left(\frac{r_w \theta}{D^2}\right)^2 = \frac{162}{3} \frac{R_D}{U_\infty} \frac{1}{U_\infty} \frac{1}{6} \int_0^{l/D} \left[\frac{l}{D} - \left(\frac{l}{D}\right)^2\right]^3 d\left(\frac{l}{D}\right)
\]

or

\[
\left(\frac{\theta}{D}\right)^2 = \frac{2}{27} \left(\frac{l}{D}\right)^8 \frac{1}{R_D} \left[\frac{1}{5} - \frac{3}{5} \left(\frac{l}{D}\right) + \frac{1}{2} \left(\frac{l}{D}\right)^2 - \frac{1}{7} \left(\frac{l}{D}\right)^3\right]^{1/4}
\]  \[4\]

Equation [4] represents the momentum thickness distribution along the sphere for different Reynolds numbers \(R_D\). Figure 6 shows the nondimensional momentum thickness distribution as a function of angle \(\phi\) for various Reynolds numbers \(R_D\).

The two parameters involved in the neutral stability point calculation may be reduced simply in terms of \(R_D\), \(\theta/D\), and the angle \(\phi\) as follows:

\[R_\theta = R_D \left(\frac{\theta}{D}\right)^{3/2} \sin \phi
\]

and

\[
\frac{\theta^2}{\nu} \frac{du}{ds} = 3R_D \left(\frac{\theta}{D}\right)^2 \cos \phi
\]  \[5\]

The location of the neutral stability point may be computed easily by using Equations [5] and Figure 6. The results of these computations are plotted as the dotted lines in Figure 4 for various Reynolds numbers; the points of intersection represent the critical values \(R_\theta\), \(N\) for neutral stability. Figure 7 represents the angular locations of the neutral stability points as a function of Reynolds number \(R_D\). It can be seen that the location of neutral stability varies from 87 degrees to 81 degrees for Reynolds numbers from 1 x 10^6 to 6 x 10^6 respectively. The small variation in the location of neutral stability as Reynolds number increases may be explained by the strong favorable pressure gradient over the front portion of the sphere which contributes a stabilizing effect tending to maintain the laminar boundary layer.
CONCLUSION

The neutral stability points as calculated represent the theoretical forward limiting location of natural (unstimulated) transition. In the Reynolds number range of these tests, therefore, transition should be expected to occur at values of \( \phi \) greater than 84 degrees. The observed transition points, as well as Fage's results, shown in Figure 7, are consistent with this prediction.

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REFERENCES


Figure 1a - Sphere and Tapered Sting Assembly

Figure 1b - Hot-wire Probe Mounted on the Sphere

Figure 1 - Sphere Assembly and Hot-Wire Probe

Figure 2 - Constant-Current Hot-Wire Circuit Diagram
Onset of Transition

\[ R_D = 2.33 \times 10^6 \]

\[ R_D = 2.39 \times 10^6 \]

\[ R_D = 2.45 \times 10^6 \]

\[ R_D = 2.51 \times 10^6 \] Fully Turbulent

Figure 3 - Typical Oscillograph Record of Hot-Wire Signal

Record shows development of turbulent boundary layer at a point on the sphere 90 degrees from stagnation. Paper speed was 50 mm/sec.
Figure 4 - Determination of Neutral Stability Point for Different Reynolds Numbers.

Figure 5 - Geometric Parameters of the Sphere
Figure 6 - Momentum Thickness Distribution along the Sphere for Different Values of $R_D$

Figure 7 - Location of Neutral Stability Point as Function of $R_D$
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