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APPLIED MATHEMATICS

MATHEMATICAL SHIP SURFACE

by

P. C. Pien

HYDROMECHANICS LABORATORY

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A	A matrix used in Equation (22)
a_{ij}	The elements of the A matrix
B	A matrix used in fairing the offsets of a waterline
B'	The transpose of the matrix B
C	A matrix, each column of which represents the coefficient of a waterline of the $\overline{f}_2(x,z)$ surface
c_{mi}	The general coefficient of the mth waterline equation of the $\overline{f}_2(x,z)$ surface
D	A matrix the elements of which represent the offsets of the $\overline{f}_2(x,z)$ surface
D'	The transpose of the matrix D
d_{mj}	The elements of the D matrix
E	A matrix, E = C F
e_{mi}	The elements of the E matrix
F	A matrix used to generate additional waterlines of the $\overline{f}_2(x,z)$ surface
$f(x)$	An expression for a waterline
$f_m(x)$	An expression for the mth waterline
$f(x,z)$	An expression for the given hull surface
$\overline{f}(x,z)$	An expression for the modified hull surface
$\overline{f}_1(x,z)$	An expression for an auxiliary surface
$\overline{f}_2(x,z)$	An expression for the final modified surface
G	A matrix used in the fairing of various waterlines
i	The general power of x
j	The general power of z
M	An arbitrary integral

N	An arbitrary integral
P	The length of a waterline
P (z)	The length of various waterlines
X, Y, Z	Coordinates
x, y, z	Dimensionless coordinates

ABSTRACT

For practical application, the problem of mathematical ship surface has been subdivided into two portions: (1) To approximate a given hull form mathematically by a surface equation; and (2) to modify a ship surface equation to obtain the desired changes in hull characteristics from one design to another. For the case of a single-screw, flat-bottom, merchant-ship hull form, a procedure of obtaining a surface equation to fair and to interpolate a given table of offsets has been developed. This equation will give not only the waterlines and stations but also the end profile. The numerical computation of such a procedure has been programmed into a high-speed electronic computer. From a given table of offsets, any number of waterlines and stations may be computed. The second portion of this work, that is, the use of the surface equation for design purpose, will be dealt with in the future. The whole work will then be extended to cover other types of ship hull forms.

INTRODUCTION

The attempt to express a ship hull surface mathematically is an old problem. An historical account of this problem is given in Reference 1.* D.W. Taylor[2] has shown that a 5th-degree polynomial can be used to express ship waterlines or sectional area curves. For transverse sections, he uses 4th-degree parabolas for fine sections and hyperbolas for full sections. In the example given in Reference 2, he uses the deadrise, flare, and area of a given section as the parameters in his equation for sections. To ensure the fairness of the waterlines, the parameters of the sections are faired graphically. Frank W. Benson[3] has taken a different approach. He expresses the waterlines mathematically, instead of the sections, by using Taylor's 5th-degree polynomial. To ensure the fairness of the sections, the parameters used in the equation for waterlines (namely, slope of tangent at end, area coefficient, and curvature amidships) are faired graphically. The mathematics used in both of these approaches is essentially two-dimensional. The third dimension is brought in graphically. G.P. Weinblum has contributed a great deal to this problem. He has used a three-dimensional surface equation to express simple ship hull forms.[4] However, for the normal hull form, it is a far more difficult problem, as indicated in the case of a Series 57 Model.[5]

In recent years, no additional publication of significance has been found. In view of the importance of this problem, a research

*References are listed on page 30

project was initiated at David Taylor Model Basin to further explore this problem. As a starting point, this work is based on the single-screw merchant-ship hull form. In the future this work will be extended to cover the military types of ships.

SCOPE OF WORK

When consideration is first given to writing a mathematical surface equation for a ship hull form, the initial inclination is that such an equation should be expressed in terms of hull characteristics. On the other hand, it is realized that a normal ship hull form is a rather complicated surface; an equation to express it would involve a few dozen parameters. It is extremely difficult for anyone to write down a set of compatible parameters so that a surface of desired form would be obtained. In practice, a naval architect usually develops his new design based on a "parent ship." From these considerations, the surface equation described below is based on a given table of offsets. The type of ship is limited to the single-screw, flat-bottom merchant ship.

A system of axes is assumed, as shown in Figure 1. The XY plane coincides with the top waterline; the XZ plane is the centerline or plane of symmetry, and the YZ plane coincides with the midship section. The positive direction of Z is downward. Let x, y, and z be the dimensionless coordinates defined as follows:

$$x = \frac{X}{\frac{1}{2} L}$$

$$y = \frac{Y}{\frac{1}{2} B}$$

$$z = \frac{Z}{D}$$

where L, B, and D are the length, the beam, and the depth, respectively, of a given ship.

For convenience, the forebody and the afterbody are treated separately. The X value is positive toward the end for both forebody and afterbody.

Assume $y = f(x,z)$ to be the required surface equation. (1)
Then we must have

$$y = f(x, z_m) \text{ to be the mth waterline} \tag{2}$$

$$y = f(x_n, z) \text{ to be the nth station} \tag{3}$$

$$y = f[P(z), z] = 0 \tag{4}$$

where $x = P(z)$ is the end profile,
x_n is the fixed x value at the nth station, and
z_m is the fixed z value at the mth waterline.

In principle, a continuous function $f(x,z)$ with prescribed boundaries can be approximated with any degree of accuracy by a polynomial in x and z. In the case of a single-screw, flat-bottom, merchant-ship hull surface, the stations are tangent to the baseline. An extremely large number of terms in z, in such a polynomial, would be required. Furthermore, the end profile, especially the stern profile, will require a large number of terms in both x and z, in order to obtain a good approximation.

The approach used in this report is to make a number of modifications to the original ship hull surface, before a polynomial approximation is applied, so that a reasonable approximation can be obtained with a relatively small number of terms in such a polynomial. To simplify the treatment, only one-half of the hull is considered at a time. In this analysis, the modifications to the hull have been made in three steps. The first modification removes the complexity due to the end profile by introducing an end condition factor which will be described below, and establishing stations 1/2 or 19 1/2 as the end of the modified ship. The second step consists of establishing a new hull which passes through three arbitrarily chosen stations of the hull derived from the first modification. This new hull has the general form of the first modified form but different offsets at all but the three control stations. The final surface derived is represented by the differences between the two hulls derived above; thus a much simplified surface without extreme changes of curvature is determined. This new surface can be easily approximated by a polynomial of comparatively few terms. Then the surface equation of the last modification is transformed backward to obtain the surface equation for the original hull.

As mentioned above, the first modification is made to remove the difficulty due to the end profile. Let us consider a single waterline first

$$y = f(x) \tag{5}$$

with

$$y = f(P) = 0 \tag{6}$$

where P is the length of the waterline.

Write $$y = \bar{f}(x)\left[1 - \left(\frac{x}{P}\right)^2\right]^{\frac{1}{M}} \tag{7}$$

where M is an arbitrary constant. If the values of y obtained from Equations (5) and (7) are the same for $0 \leq x < P$, Equations (5) and (6) can be replaced by Equation (7). This suggests that if each of the offsets of the original waterline is divided by the end condition factor

$$\left[1 - \left(\frac{x}{P}\right)^2\right]^{\frac{1}{M}},$$

and if $\bar{f}(x)$ is used to express the modified offsets for $0 \leq x < P$, Equation (7) will express the original waterline with correct end condition. Likewise, if each of the original offsets of a given hull is divided by the appropriate value of the end condition factor,

$$\left\{1 - \left[\frac{x}{P(z)}\right]^2\right\}^{\frac{1}{M}}, \text{ and } \bar{f}(x,z)$$

is used to express the modified offsets for $0 \leq x < P$, then

$$y = f(x,z) = \bar{f}(x,z) \quad \left\{1 - \left[\frac{x}{P(z)}\right]^2\right\}^{\frac{1}{M}} \tag{8}$$

where P(z) is the expression for the end profile. This will express the original hull surface with correct end profile. Assume the end profile is beyond $x = 0.95$; then $\bar{f}(x,z)$ is defined for $0 \leq x = 0.95$. It has a vertical end profile, $x = 0.95$.

Figure 2 shows the extent of this modification, the details of which will be given later when a numerical example is discussed. Even though the $f(x,z)$ surface has a vertical end profile, the station curves are still tangent to the baseline.

To remove the difficulty arising from the tangency of station curves to the baseline, we introduce an auxiliary surface $\bar{f}_1(x, z)$ as follows:

$$\bar{f}_1(x,z) = \phi\left[x, \bar{f}(0,z), \bar{f}(0.6,z), \bar{f}(0.95,z)\right] \tag{9}$$

The surface, $\bar{f}_1(x,z)$, is obtained by interpolating between the three stations of $\bar{f}(x,z)$ at $x = 0$, 0.6, and 0.95. The form of this equation should be so chosen that the difference between $\bar{f}(x, z)$ and $\bar{f}_1(x, z)$ would be small for the range $x = 0$ to 0.95. Let

$$\bar{f}_2(x,z) = \bar{f}(x,z) - \bar{f}_1(x,z) \tag{10}$$

The surface $\bar{f}_2(x, z)$ is a very simple one which has zero offsets at both ends and at $x = 0.6$ and has a vertical end profile. The station curves of $\bar{f}_2(x,z)$ are no longer tangent to the baseline as shown in Figure 4a. For such a simple surface, it can be easily approximated by a polynomial as follows:

$$\bar{f}_2(x, z) = \sum_i \sum_j a_{ij} x^i z^j \tag{11}$$

Enough powers of x should be included in Equation (11) so that a good approximation for all waterlines can be obtained. Also enough powers of z should be included so that all of the station curves can be closely approximated. The values of the coefficients, a_{ij}, are determined from the offsets of $\bar{f}_2(x,z)$ surface, which in turn are obtained from the given offsets table of f(x,z) by using Equations (8), (9), and (10). The method of obtaining numerical values of a_{ij} will be discussed later when a numerical example is given.
Finally, from Equations (8), (9), (10), and (11), we obtain the following equation we set out to find.

$$\begin{aligned} y = f(x,z) &= \bar{f}(x,z) \left\{1 - \left[\frac{x}{P(z)}\right]^2\right\}^{\frac{1}{M}} \\ &= \left\{\phi\left[x,\ \bar{f}(0,z),\ \bar{f}(.6,z),\ \bar{f}(.95,z)\right] + a_{ij}x^i z^j\right\} \left\{1 - \left[\frac{x}{P(z)}\right]^2\right\}^{\frac{1}{M}} \end{aligned} \tag{12}$$

It is noted that there are four two-dimensional curves involved in this equation: the stations of the first modified surface at x = 0, 0.6, and 0.95, respectively, and the end profile of the original hull surface. Since they are all two-dimensional curves, no difficulty should be encountered in obtaining proper expressions to approximate them. If one wishes, they can be approximated piecewise. In certain applications they can even be tabulated numerically against z values. The arbitrary constant M should be fairly large, so that the effect of the end condition factor would be localized near the end.

ILLUSTRATIVE EXAMPLE

A computing program to obtain a surface equation from a given table of offsets and to compute the offsets of any number of waterlines or stations from this surface equation has been accomplished, based on the procedure mentioned above. A numerical example is given here for the purpose of explaining this procedure in more detail.

Table 1 gives the offsets of an afterbody of a typical Series 60 model, Model 4210. These values are taken from Reference 6. The second line gives the x_n values at which station offsets are given. The second column indicates the z_m values at which waterline offsets are given. The last column gives the stern profile. Since the stern profile is beyond x = 0.95, the $\bar{f}_2(x,z)$ is defined for the range $0 \leq x \leq 0.95$. The first step is to modify the given offsets within this range by dividing each offset by the appropriate factor

$$\left\{1 - \left[\frac{x}{P(z)}\right]^2\right\}^{\frac{1}{M}}.$$

The value of M is chosen to be 10 so that the extent of this modification is localized near the end. Table 2 gives the modified offsets.

In this program, Equation (9) takes the following form

$$\begin{aligned} \bar{f}_1(x,z) = {} & \bar{f}(0,z)\,(1 - 3.8858x^2 + 3.0779x^4) \\ & + \bar{f}(0.6,z)\,(4.6211x^2 - 5.12033x^4) \\ & + f(0.95,z)\,(- 0.73529x^2 + 2.04246x^4) \end{aligned} \qquad (13)$$

Equation (13) is chosen arbitrarily. The values of $\bar{f}(0,z)$, $\bar{f}(0.6,z)$, and $\bar{f}(0.95, z)$ are given in columns 3, 8, and 13, respectively, of Table 2. These columns are the same for Table 3, the offsets table of $\bar{f}_1(x,z)$ surface. The remaining columns of Table 3 are computed by Equation (13). The values of Tables 2 and 3 are plotted in Figure 3.

The differences between the corresponding values in Tables 2 and 3 give the offsets of the $\bar{f}_2(x,z)$ surface. They are plotted in Figure 4, and are listed in Table 4. For later convenience, a matrix D is defined, the elements of which are the offsets of $\bar{f}_2(x,z)$. In this case the matrix D has eleven columns and eight rows, as shown in Table 4, after omitting the first zero column. Now we are ready to determine the polynomial

$$\bar{f}_2(x, z) = \sum_i \sum_j a_{ij} z^j x^i \qquad (14)$$

to approximate the elements of matrix D. The following values of i and j have been tried and found to give good results:

i = 2, 3, 4, 5, 6, 7, 8, 9

j = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

The values of a_{ij} may be determined in one step by the method of least squares from the values of the elements of the matrix D. However, this will lead to a system of eighty simultaneous equations with eighty unknowns, the solution of which requires a computer with a large memory capacity. The accuracy of this solution will be poor even if such a computer is available, because of the large system of equations involved. This difficulty is avoided by using two steps instead of one.

For each set of waterline offsets, each row of the matrix D, a waterline equation

$$f_m(x) = \sum_i c_{mi} x^i \qquad (15)$$

if first obtained.

There are only eight unknowns in Equation (15). This equation cannot be forced to go through the eleven points. The coefficients c_{mi} are determined by the method of least square. The following equation, derived in the Appendix, gives the required values of c_{mi}.

$$\left[c_{mi}\right] = B \cdot \left[d_{mj}\right] \tag{16}$$

where $\left[d_{mj}\right]$ is a column vector with the mth row of D as the elements, and $\left[c_{mi}\right]$ is a column vector, the elements of which give the coefficients of Equation (15). Applying Equation (16) to all the rows of the matrix D, we have

$$C = B \cdot D' \tag{17}$$

where D' is the transpose of D.
The matrix C has eight rows and eight columns, each column of which corresponds to a waterline for $\bar{f}_2(x,z)$ surface. Altogether, there are eight waterlines available, corresponding to the eight waterlines in the given table of offsets, Table 1.

To have a better control of the shape of the station curves, (to avoid the surface inflections between these eight waterlines), a total of twenty-five waterlines is obtained by interpolating adjacent waterlines as follows

$$E = F \cdot C' \tag{18}$$

where C' is the transpose of C, and the elements of F are derived from the interpolating polynomials. These interpolating polynomials can be chosen arbitrarily as long as they give the desired local section shape. At the joints, the adjacent polynomials have the same tangent value so that the interpolated station offsets will be along a smooth curve. The matrix E has eight columns and twenty-five rows, each of which corresponds to a waterline. Now let

$$a_{ij} z_m^j = e_{mi} \tag{19}$$

The right-hand side of Equation (19), e_{mi}, represents the various elements of a column of E, corresponding to the coefficients of the same power of x. This equation gives a system of twenty-five equations with ten unknowns. It is again solved by the method of least squares as follows:

$$A = \left[a_{ij}\right] = G \cdot E \tag{20}$$

The derivation of matrix G is essentially the same as that of matrix B of Equation (17). The matrix A gives the required coefficients in Equation (14).

$$
F = \left|\begin{array}{cccccccc}
1.00000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.58789 & 0.65625 & -0.33984 & 0.10938 & -0.01367 & 0 & 0 & 0 \\
0.29688 & 1.00000 & -0.40625 & 0.12500 & -0.01562 & 0 & 0 & 0 \\
0.10742 & 1.09375 & -0.26953 & 0.07812 & -0.00976 & 0 & 0 & 0 \\
0 & 1.00000 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.04492 & 0.78125 & 0.33203 & -0.07812 & 0.00976 & 0 & 0 & 0 \\
-0.04688 & 0.50000 & 0.65625 & -0.12500 & 0.01562 & 0 & 0 & 0 \\
-0.02539 & 0.21875 & 0.90234 & -0.10938 & 0.01367 & 0 & 0 & 0 \\
0 & 0 & 1.00000 & 0 & 0 & 0 & 0 & 0 \\
0.00684 & -0.07715 & 0.84180 & 0.27539 & -0.05176 & 0.00488 & 0 & 0 \\
0.00781 & -0.08594 & 0.57812 & 0.57812 & -0.08594 & 0.00781 & 0 & 0 \\
0.00488 & -0.05176 & 0.27539 & 0.84180 & -0.07715 & 0.00684 & 0 & 0 \\
0 & 0 & 0 & 1.00000 & 0 & 0 & 0 & 0 \\
0 & 0.01367 & -0.10938 & 0.90234 & 0.21875 & -0.02539 & 0 & 0 \\
0 & 0.01562 & -0.12500 & 0.65625 & 0.50000 & -0.04688 & 0 & 0 \\
0 & 0.00976 & -0.07812 & 0.33203 & 0.78125 & -0.04492 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.04261 & 0.76278 & 0.41315 & -0.20292 & 0.06960 \\
0 & 0 & 0 & -0.04138 & 0.47045 & 0.78604 & -0.32468 & 0.11136 \\
0 & 0 & 0 & -0.02216 & 0.19289 & 1.01591 & -0.28409 & 0.09744 \\
0 & 0 & 0 & 0 & 0 & 1.00000 & 0 & 0 \\
0 & 0 & 0 & 0.00767 & -0.06136 & 0.69144 & 0.52780 & -0.16534 \\
0 & 0 & 0 & 0.00454 & -0.03636 & 0.26688 & 0.97402 & -0.20909 \\
0 & 0 & 0 & -0.00085 & 0.00682 & -0.04111 & 0.93344 & 0.10170 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00000
\end{array}\right|
$$

From Equations (17), (18), and (20),

$$A = G \cdot E = G \cdot F \cdot C' = G \cdot F \cdot D \cdot B' = W \cdot D \cdot B' \tag{21}$$

where B' is the transpose of B, and $W = G \cdot F$. The matrix W and B', once derived, can be used to obtain the values of a_{ij} required in Equation (14) if the given offsets table conforms with Table 1. The approximation expression for $f_2(x,z)$ becomes a simple matrix multiplication as follows:

$$\bar{f}_2(x,z) = [x^i] \cdot A \cdot [z^j] \tag{22}$$

where $[x^i]$ is a row vector with x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, as elements, $[z^j]$ is a column vector with z^0, z^1, z^2, z^3, z^4, z^5, z^6, z^7, z^8, z^9 as elements, and A is given by Equation (21).

There are four expressions yet to be found to approximate the four two-dimensional curves; namely, $\bar{f}(0,z)$, $\bar{f}(0.6,z)$, $\bar{f}(0.95,z)$, and P(z). There is no difficulty in finding these expressions. However, the forms of the expressions may differ from one hull form to another, depending upon the nature and shape of these curves. To make the computing program more flexible, the tabulated values of these curves are used to compute the faired offsets.

The final computed offsets of the present example are given in Table 5. This result, together with that of the forebody similarly obtained, is plotted in Figures 5, 6a, and 6b. Figure 7 shows the comparison of the computed body plan and that faired by a draftsman. Table 6 gives the differences as shown in Figure 7.

HULL FORM WITH A BULBOUS BOW

In the case of hull form with a bulbous bow, Equation (4) can be written as

$$y = f(1,z) = B\ (z) \tag{4a}$$

where B(z) is the imaginary station at F.P. Let

$$f(x,z) = f_b(x,z) - x^n B(z) \tag{23}$$

or

$$f_b(x,z) = f(x,z) + x^n B(Z) \tag{24}$$

where $f_b(x,z)$ is the surface with a bulbous bow, and n is an integer.

The surface with a bulbous bow is first changed to a surface without a bulbous bow by Equation (23). Then the required surface equation of the latter is obtained by Equation (24) after that of the former is found. The rounding off of the end of the waterlines can be made mathematically. However, this has not been done in the existing computing program. Figure 8 shows the results obtained by using the existing computing program for the case of the forebody of MARINER.

DISCUSSION AND CONCLUSION

From the numerical examples shown and other examples not shown here, it can be said that the procedure of obtaining a surface equation to approximate a given table of offsets, as suggested in this report, is quite satisfactory. In Equation (14) as many terms as we wish can be taken to obtain the desired accuracy of approximation. The additional waterlines or stations required for the least square method of fairing can be obtained by interpolating the adjacent waterlines or stations, as is done by Equation (18), with appropriate F matrix. A polynomial is used for its simplicity. However, any other expressions may be chosen for the same purpose. Any unfairness among the original given offsets is removed by the process of least square fairing. It is quite possible that the station curves or waterlines of the $\overline{f}_2(x,z)$ surface may have some degree of inflection. Since the offsets of this surface are very small and with enough locally interpolated points, these possible inflections can be limited to a tolerable degree. The values of a_{ij} in Equation (15) are determined by the offsets of f(x,z) within the range of $0 \leq x = 0.95$. The stations obtained from Equation (12) beyond x = 0.95, in a sense, are extrapolated. These stations may not conform closely with the original given form, especially when the stern overhang is large. This difficulty can be overcome by the following procedure:

For the waterlines, the end of which are considerably beyond x = 0.95, the offsets beyond x = 0.95 should be included in obtaining the waterline equation, Equation (15), to approximate the modified waterline. The offsets of the original waterline are computed by Equation (7). Then the offsets of that waterline in the original table of offsets are replaced by the computed ones. By doing this, the extrapolation beyond x = 0.95 is avoided.

The existing computing program is believed to be quite useful in many applications, such as reducing the mold lofting work in shipyards, in obtaining various hydrostatic characteristics, or in obtaining the dynamic characteristics of a given hull form.

In certain applications it may be desirable to have one surface equation to cover the whole ship rather than treat the forebody and afterbody separately. This can be done by decomposing the hull form into two parts: a symmetrical part, and an asymmetrical part. These two parts are then treated separately. Using only even powers of x for the symmetrical part and only odd powers of x for the asymmetrical part, the sum of the two results will give the required single equation to cover the whole hull form.

ACKNOWLEDGMENT

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APPENDIX

DETERMINATION OF WATERLINE EQUATION FROM THE GIVEN WATERLINE OFFSETS

The waterline equation:

$$f_m(x) = \Sigma\, c_{mi}\, x^{i} \tag{15}$$

contains eight unknowns. There are eleven offsets given at x = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.85, 0.9, and 0.95. The coefficients c_{mi} are determined so that Equation (15) would give the best approximation to the eleven given offsets by the method of least square.
Let

$$E = \sum_K \left[\sum_i c_{mi} x_k^{i} - d_{mk}\right]^2 \tag{25}$$

where d_{mk} is the offset of mth waterline at $x = x_k$

For $\quad \dfrac{\partial E}{\partial c_{m2}} = 0, \quad \dfrac{\partial E}{\partial c_{m3}} = 0, \; \ldots\ldots \quad \dfrac{\partial E}{\partial c_{m9}} = 0$

we have

$$\left.\begin{aligned}
&\sum_i \left[\sum_K x_K^2 \cdot c_{mi}\, x_K^i\right] = \sum_K x_K^2\, d_{mK} \\
&\sum_i \left[\sum_K x_K^3 \cdot c_{mi}\, x_K^i\right] = \sum_K x_K^3\, d_{mK} \\
&-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,- \\
&-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,- \\
&\sum_i \left[\sum_K x_K^9 \cdot c_{mi}\, x_K^i\right] = \sum_K x_K^9\, d_{mK}
\end{aligned}\right\} \tag{26}$$

Equation (26) may be writted as

$$\mathbf{X}\, C_m = \bar{\bar{\mathbf{X}}}\, D_m \tag{27}$$

where

$$\mathbf{X} = \begin{vmatrix} \Sigma x_K^4 & \Sigma x_K^5 & --- & \Sigma x_K^{11} \\ \Sigma x_K^5 & \Sigma x_K^6 & --- & \Sigma x_K^{12} \\ - & - & - & - \\ - & - & - & - \\ \Sigma x_K^{11} & \Sigma x_K^{12} & --- & \Sigma x_K^{18} \end{vmatrix} \qquad \bar{\bar{\mathbf{X}}} = \begin{vmatrix} x_1^2 & x_2^2 & --- & x_{11}^2 \\ x_1^3 & x_2^3 & --- & x_{11}^3 \\ - & - & - & - \\ - & - & - & - \\ x_1^9 & x_2^9 & --- & x_{11}^9 \end{vmatrix} \qquad C_m = \begin{vmatrix} C_{m2} \\ C_{m3} \\ -- \\ -- \\ C_{m9} \end{vmatrix} \qquad D_m = \begin{vmatrix} d_{m1} \\ d_{m2} \\ -- \\ -- \\ d_{m11} \end{vmatrix}$$

From Equation (27)

$$C_m = (\mathbf{X}^{-1} \bar{\bar{\mathbf{X}}}) \cdot D_m = B \cdot D_m \tag{28}$$

where $\mathbf{X}^{-1}$ is the inverse of $\mathbf{X}$, and B is a matrix of eight rows and eleven columns. With x_k values as 0.1, 0.2, ... 0.95, the B matrix has the following form:

$$
B = \left| \begin{array}{rrrrr}
269.5370 & -23.4839 & -28.9817 & .6681 & 17.3541 \\
-2371.9026 & 395.2705 & 450.7874 & -48.7642 & -293.8034 \\
7557.7722 & -1798.3281 & -1897.8240 & 513.9972 & 1459.4094 \\
-9683.5903 & 2929.0356 & 3001.7992 & -1316.0556 & -2713.0846 \\
349.4406 & 21.8994 & -663.3482 & 225.5148 & 1026.1900 \\
11457.3461 & -5337.8077 & -3236.9327 & 3112.2829 & 2715.7317 \\
-10665.5961 & 5704.3971 & 3442.7158 & -3957.5081 & -3312.8730 \\
3086.4798 & -1891.8066 & -1068.0595 & 1471.1682 & 1100.8949
\end{array} \right.
$$

$$
\left. \begin{array}{rrrrrr}
-1.7108 & -12.8010 & 7.7875 & 6.7835 & -10.0292 & 2.8080 \\
53.5999 & 220.2862 & -151.4914 & -119.4357 & 190.2080 & -54.8237 \\
-467.2707 & -1139.9932 & 932.7990 & 644.9741 & -1141.2051 & 341.6758 \\
1294.8808 & 2289.2184 & -2232.5372 & -1379.8828 & 2712.3758 & -843.3487 \\
-558.7867 & -1165.0731 & 1298.7141 & 812.7417 & -1709.5153 & 556.1032 \\
-2603.5199 & -1903.4251 & 2843.0222 & 1149.0487 & -3050.6573 & 1001.2967 \\
3784.2333 & 2599.9290 & -4536.6064 & -1766.0829 & 5133.7879 & -1755.7754 \\
-1503.2728 & -877.3768 & 1840.8568 & 651.1536 & -2129.3818 & 756.0784
\end{array} \right|
$$

TABLE 1

Afterbody Offsets of Model 4210

WL	Sta. / z_m \ x_n	10	11	12	13	14	15	16	17	18	18½	19	19½	P(z)
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9	0.95	
1.50	0	1.000	1.000	1.000	1.000	0.999	0.994	0.975	0.924	0.834	0.769	0.686	0.579	1.0575
1.25	0.16667	1.000	1.000	1.000	0.997	0.990	0.977	0.933	0.844	0.709	0.626	0.530	0.418	1.0500
1.00	0.33333	1.000	1.000	1.000	0.994	0.975	0.937	0.857	0.725	0.536	0.425	0.308	0.193	1.03375
0.75	0.50000	1.000	1.000	1.000	0.987	0.943	0.857	0.728	0.541	0.321	0.216	0.116	0.033	0.9658
0.50	0.66667	1.000	1.000	0.994	0.962	0.884	0.754	0.592	0.413	0.236	0.156	0.085	0.022	0.9617
0.25	0.75000	0.985	0.975	0.944	0.879	0.769	0.629	0.476	0.325	0.190	0.128	0.075	0.020	0.9617
0.075	0.95000	0.886	0.870	0.817	0.732	0.621	0.496	0.366	0.236	0.134	0.090	0.051	0.018	0.9617
0	1.00000	0.710	0.685	0.626	0.544	0.442	0.329	0.219	0.119	0.046	0.023	0.010	0.007	0.9617

TABLE 2

Modified Afterbody Offsets of Model 4210

WL \ Sta.	10	11	12	13	14	15	16	17	18	18½	19	19½
1.50	1.000	1.0009	1.0036	1.0084	1.0145	1.0195	1.0136	0.9788	0.9080	0.8532	0.7803	0.6825
1.25	1.000	1.0009	1.0037	1.0055	1.0057	1.0025	0.9707	0.8951	0.7733	0.6963	0.6052	0.4958
1.00	1.000	1.0009	1.0038	1.0028	0.9910	0.9623	0.8929	0.7709	0.5873	0.4757	0.3550	0.2325
0.75	1.000	1.0011	1.0044	0.9971	0.9609	0.8841	0.7644	0.5828	0.3604	0.2507	0.1421	0.0465
0.50	1.000	1.0011	0.9984	0.9719	0.9010	0.7781	0.6219	0.4454	0.2655	0.1816	0.1047	0.0319
0.25	0.985	0.9761	0.9482	0.8880	0.7838	0.6491	0.5000	0.3505	0.2138	0.1490	0.0924	0.0290
0.075	0.886	0.8710	0.8206	0.7395	0.6329	0.5119	0.3845	0.2545	0.1503	0.1048	0.0628	0.0261
0	0.710	0.6858	0.6288	0.5501	0.4505	0.3395	0.2300	0.1283	0.0518	0.0268	0.0123	0.0102

TABLE 3

Afterbody Offsets of the $\bar{f}_1$ Surface

z_m \ x_n	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9	0.95
0	1.000	1.0056	1.0107	1.0208	1.0290	1.0292	1.0136	0.9728	0.8956	0.8392	0.7689	0.6825
0.16667	1.000	1.0023	1.0080	1.0140	1.0151	1.0038	0.9707	0.9041	0.7903	0.7108	0.6134	0.4958
0.33333	1.000	1.0006	1.0011	0.9980	0.9850	0.9536	0.8929	0.7893	0.6270	0.5181	0.3876	0.2325
0.50000	1.000	0.9960	0.9833	0.9591	0.9190	0.8568	0.7644	0.6321	0.4484	0.3330	0.1998	0.0465
0.66667	1.000	0.9896	0.9585	0.9065	0.8333	0.7386	0.6219	0.4827	0.3205	0.2304	0.1342	0.0319
0.75000	0.985	0.9697	0.9243	0.8508	0.7525	0.6336	0.5000	0.3587	0.2179	0.1507	0.0872	0.0290
0.95000	0.886	0.8692	0.8199	0.7409	0.6371	0.5154	0.3845	0.2551	0.1399	0.0921	0.0534	0.0261
1.00000	0.710	0.6931	0.6432	0.5650	0.4638	0.3484	0.2300	0.1222	0.0410	0.0161	0.0049	0.0102

$$\bar{\bar{f}}_1(x,z) = \bar{f}(0,z)\,(1-3.8858x^2 + 3.0779x^4) + \bar{f}(0.6,Z)(4.6211x^2-5.12033x^4) +$$

$$\bar{f}(0.95,Z)\,(-0.73529x^2+2.04246x^4)$$

TABLE 4

Afterbody Offsets of the $\bar{f}_2$ Surface

z_m \ x_n	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9	0.95
0	0	-0.0047	-0.0071	-0.0124	-0.0145	-0.0097	0	0.0060	0.0124	0.0140	0.0114	0
0.16667	0	-0.0014	-0.0043	-0.0085	-0.0094	-0.0013	0	-0.0090	-0.0170	-0.0145	-0.0028	0
0.33333	0	0.0003	0.0027	0.0048	0.0060	0.0087	0	-0.0184	-0.1397	-0.0424	-0.0326	0
0.50000	0	0.0051	0.0211	0.0380	0.0419	0.0273	0	-0.0493	-0.0880	-0.0823	-0.0577	0
0.66667	0	0.0115	0.0399	0.0654	0.0677	0.0395	0	-0.0373	-0.0550	-0.0488	-0.0295	0
0.75000	0	0.0064	0.0239	0.0372	0.0313	0.0155	0	-0.0082	-0.0041	-0.0017	0.0052	0
0.95000	0	0.0018	0.0007	-0.0014	-0.0042	-0.0035	0	-0.0006	0.0109	0.0127	0.0094	0
1.00000	0	-0.0073	-0.0144	-0.0149	-0.0133	-0.0089	0	0.0061	0.0108	0.0107	0.0074	0

$$\bar{f}_2(x,z) = \bar{f}\,(x,z) - \bar{f}_1(x,z)$$

Computed Offsets Table 5

STA.	TAN. WL	0.075 WL	0.25 WL	0.50 WL	0.75 WL	1.00 WL	1.25 WL	1.50 WL
½	0.0014	0.0251	0.0403	0.0407	0.0423	0.0509	0.0760	0.1193
1	0.0015	0.0544	0.0799	0.0860	0.0895	0.1017	0.1324	0.1972
1½	0.0070	0.0833	0.1250	0.1409	0.1459	0.1597	0.1960	0.2762
2	0.0139	0.1111	0.1748	0.2032	0.2117	0.2278	0.2696	0.3588
3	0.0364	0.1766	0.2902	0.3451	0.3661	0.3902	0.4389	0.5285
4	0.0931	0.2770	0.4284	0.5022	0.5332	0.5631	0.6090	0.6824
5	0.1974	0.4140	0.5806	0.6602	0.6903	0.7176	0.7528	0.8021
6	0.3341	0.5626	0.7254	0.8003	0.8216	0.8397	0.8611	0.8870
7	0.4719	0.6923	0.8406	0.9060	0.9186	0.9270	0.9365	0.9458
8	0.5869	0.7885	0.9176	0.9695	0.9777	0.9798	0.9822	0.9836
9	0.6711	0.8550	0.9643	0.9956	1.0000	1.0000	0.9999	0.9999
10	0.7100	0.8866	0.9850	1.0000	1.0000	1.0000	1.0000	1.0000
11	0.6850	0.8706	0.9754	0.9996	1.0000	1.0000	1.0000	1.0000
12	0.6254	0.8178	0.9435	0.9931	1.0000	0.9991	0.9987	0.9999
13	0.5420	0.7318	0.8764	0.9593	0.9869	0.9935	0.9963	0.9999
14	0.4401	0.6212	0.7692	0.8808	0.9443	0.9764	0.9917	0.9999
15	0.3277	0.4946	0.6302	0.7547	0.8611	0.9363	0.9762	0.9953
16	0.2162	0.3621	0.4758	0.5919	0.7279	0.8570	0.9327	0.9754
17	0.1180	0.2374	0.3241	0.4118	0.5435	0.7236	0.8448	0.9263
18	0.0458	0.1343	0.1903	0.2370	0.3268	0.5338	0.7093	0.8359
18½	0.0234	0.0923	0.1313	0.1575	0.2187	0.4230	0.6256	0.7710
19	0.0115	0.0547	0.0751	0.0850	0.1186	0.3074	0.5304	0.6885
19½	0.0100	0.0204	0.0213	0.0230	0.0343	0.1928	0.4179	0.5812

Differences Between The Given And Computed Cffsets Table 6

STA.	TAN. WL	0.075 WL	0.25 WL	0.50 WL	0.75 WL	1.00 WL	1.25 WL	1.50 WL
½	-0.0050	-0.0033	-0.0011	-0.0003	-0.0007	-0.0001	0	-0.0007
1	-0.0077	-0.0023	-0.0009	-0.0010	-0.0005	-0.0003	-0.0006	-0.0008
1½	-0.0065	-0.0009	0.0009	-0.0001	-0.0021	-0.0003	0.0010	-0.0018
2	-0.0031	-0.0015	-0.0005	-0.0008	-0.0014	-0.0002	-0.0004	-0.0012
3	-0.0026	0.0028	0.0006	-0.0009	-0.0017	-0.0008	-0.0011	-0.0015
4	-0.0020	-0.0014	-0.0011	0.0002	-0.0018	0.0011	0.0020	-0.0006
5	0.0022	0.0008	0.0004	0.0002	-0.0007	-0.0004	-0.0012	-0.0019
6	0.0011	0.0040	0.0034	-0.0017	-0.0024	-0.0013	-0.0009	-0.0020
7	-0.0010	0.0016	-0.0006	0	0.0016	0.0010	0.0005	-0.0002
8	-0.0031	-0.0077	-0.0034	-0.0015	0.0007	0.0008	0.0012	0.0016
9	0.0001	0.0003	0	-0.0004	0	0	-0.0001	-0.0001
10	0	0	0	0	0	0	0	0
11	0	0	0.0002	-0.0004	0	0	0	0
12	-0.0008	0.0004	-0.0001	-0.0009	0	-0.0009	-0.0013	-0.0001
13	-0.0026	-0.0005	-0.0022	-0.0027	-0.0001	-0.0005	-0.0007	-0.0001
14	-0.0015	-0.0003	-0.0001	-0.0032	0.0013	0.0014	0.0017	0.0009
15	-0.0010	-0.0019	0.0008	0.0007	0.0041	-0.0013	-0.0008	0.0013
16	-0.0032	-0.0041	0	-0.0001	-0.0001	0	-0.0003	0.0004
17	-0.0013	0.0007	-0.0009	-0.0012	0.0025	-0.0014	0.0008	0.0023
18	-0.0004	-0.0005	0.0002	0.0010	0.0058	-0.0022	0.0003	0.0019
18½	0.0007	0.0018	0.0033	0.0015	0.0027	-0.0020	-0.0004	0.0020
19	0.0016	0.0033	0.0002	0	0.0026	-0.0006	0.0004	0.0025
19½	0.0029	0.0027	0.0016	0.0010	0.0010	-0.0002	-0.0001	0.0022

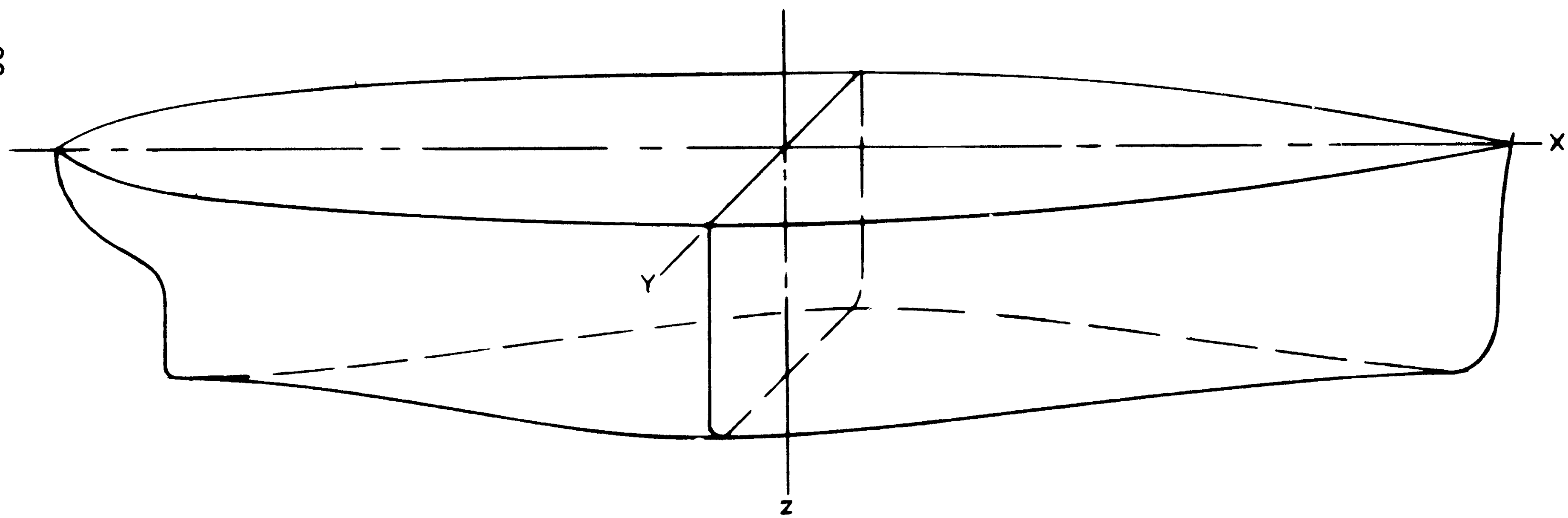


Figure 1 - Axes of Reference

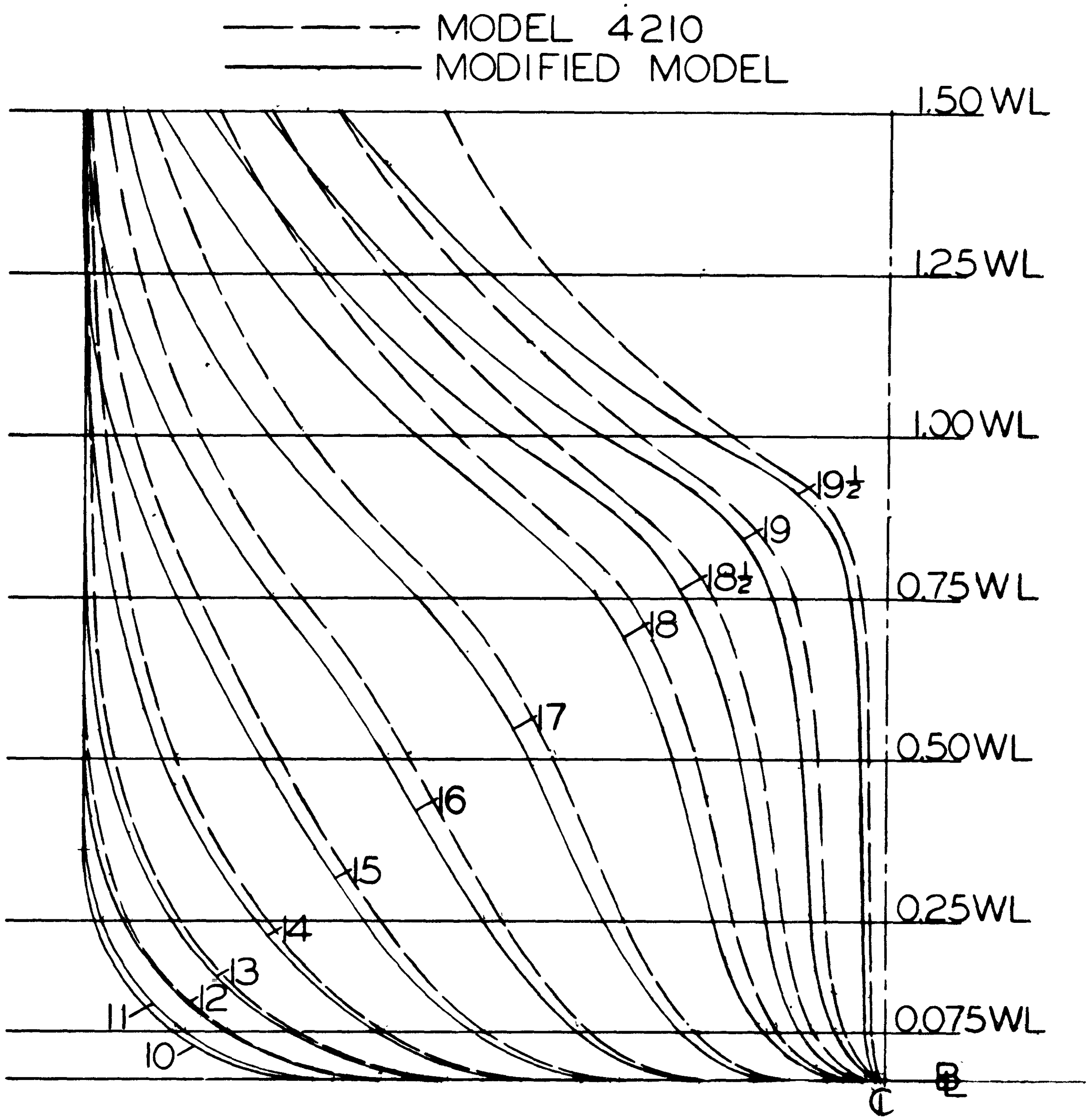


Figure 2 - Comparison of Afterbody Plans of Model 4210 And of Modified Model, $\bar{f}(x,z)$ Surface

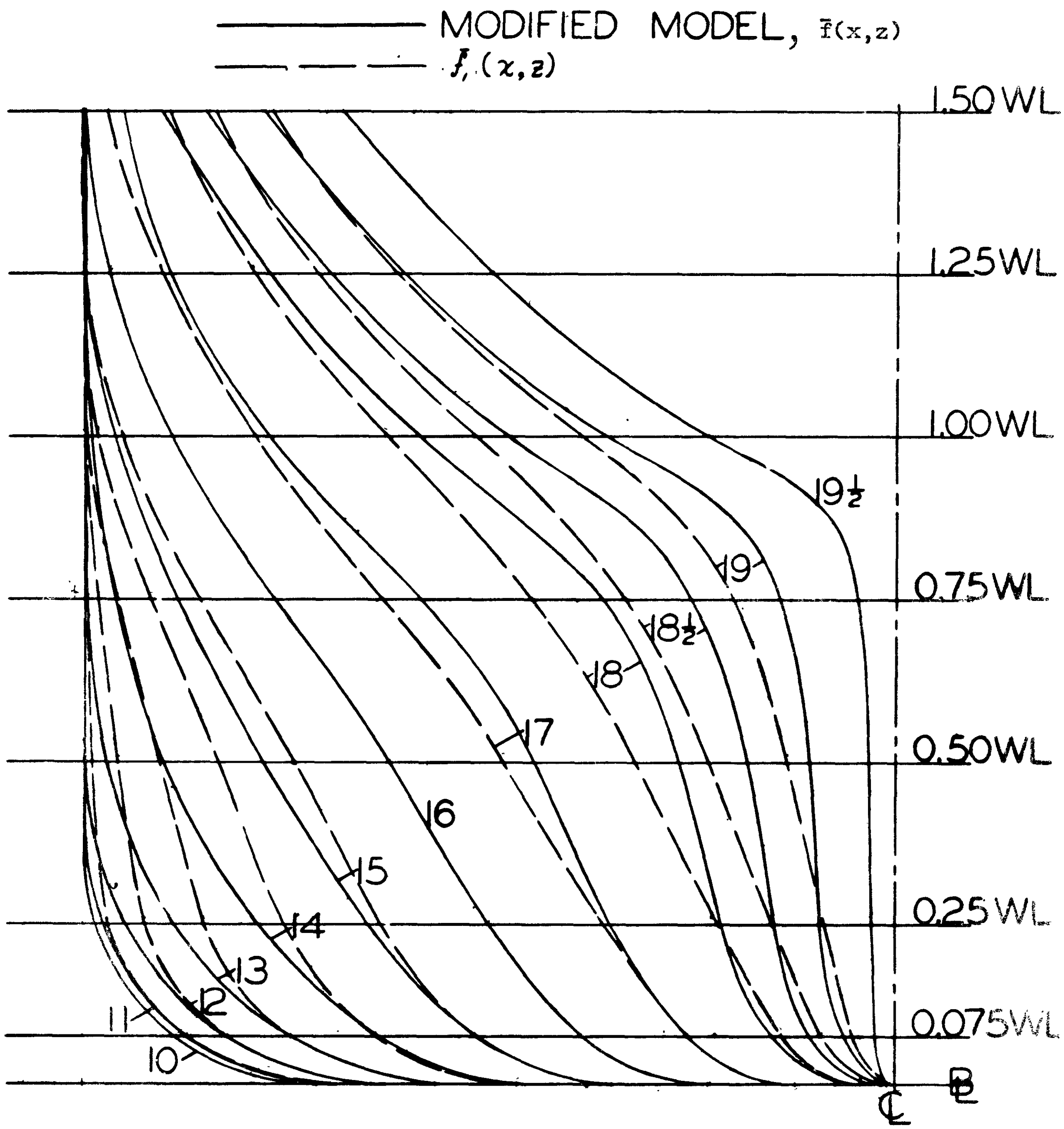
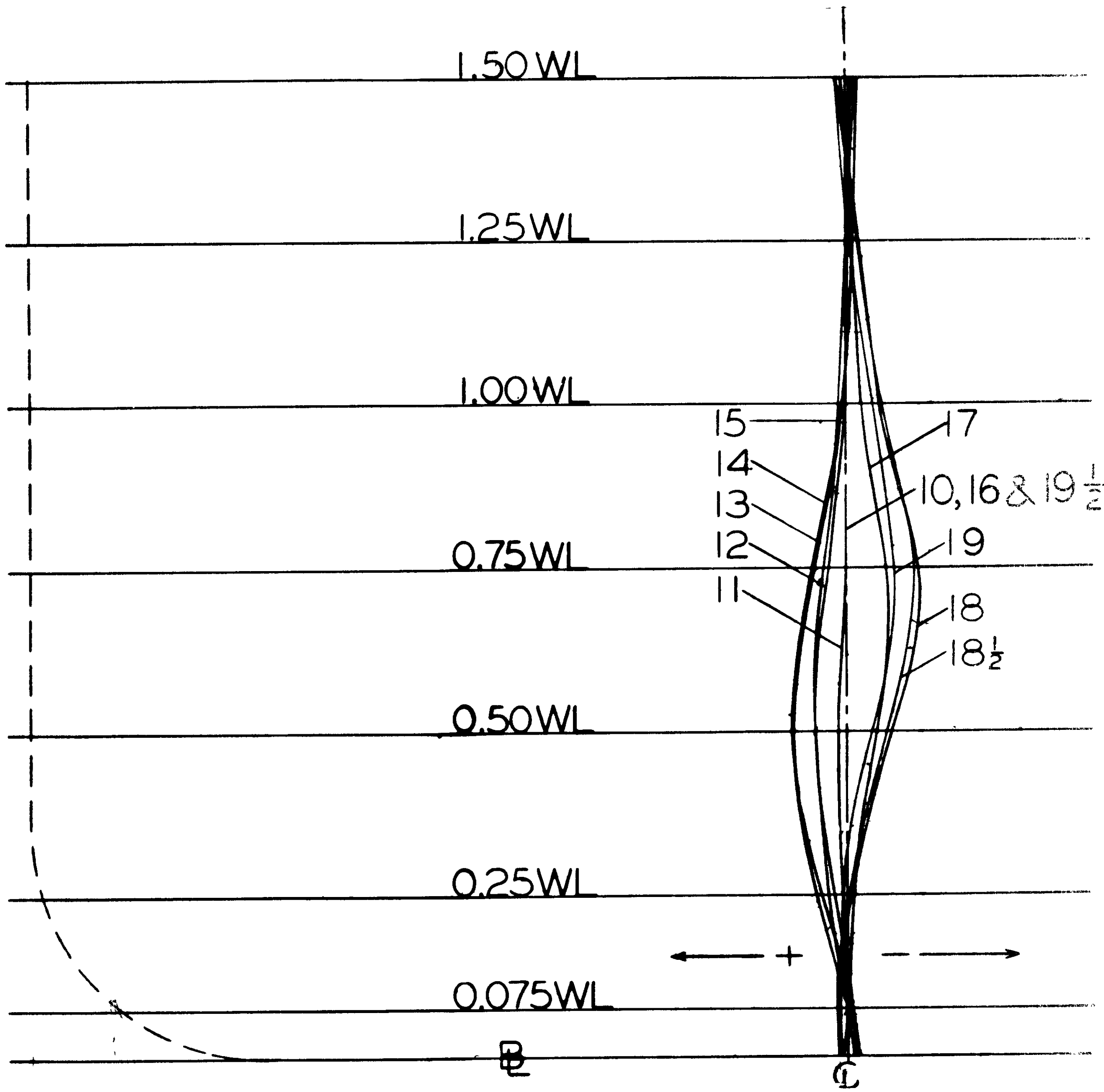
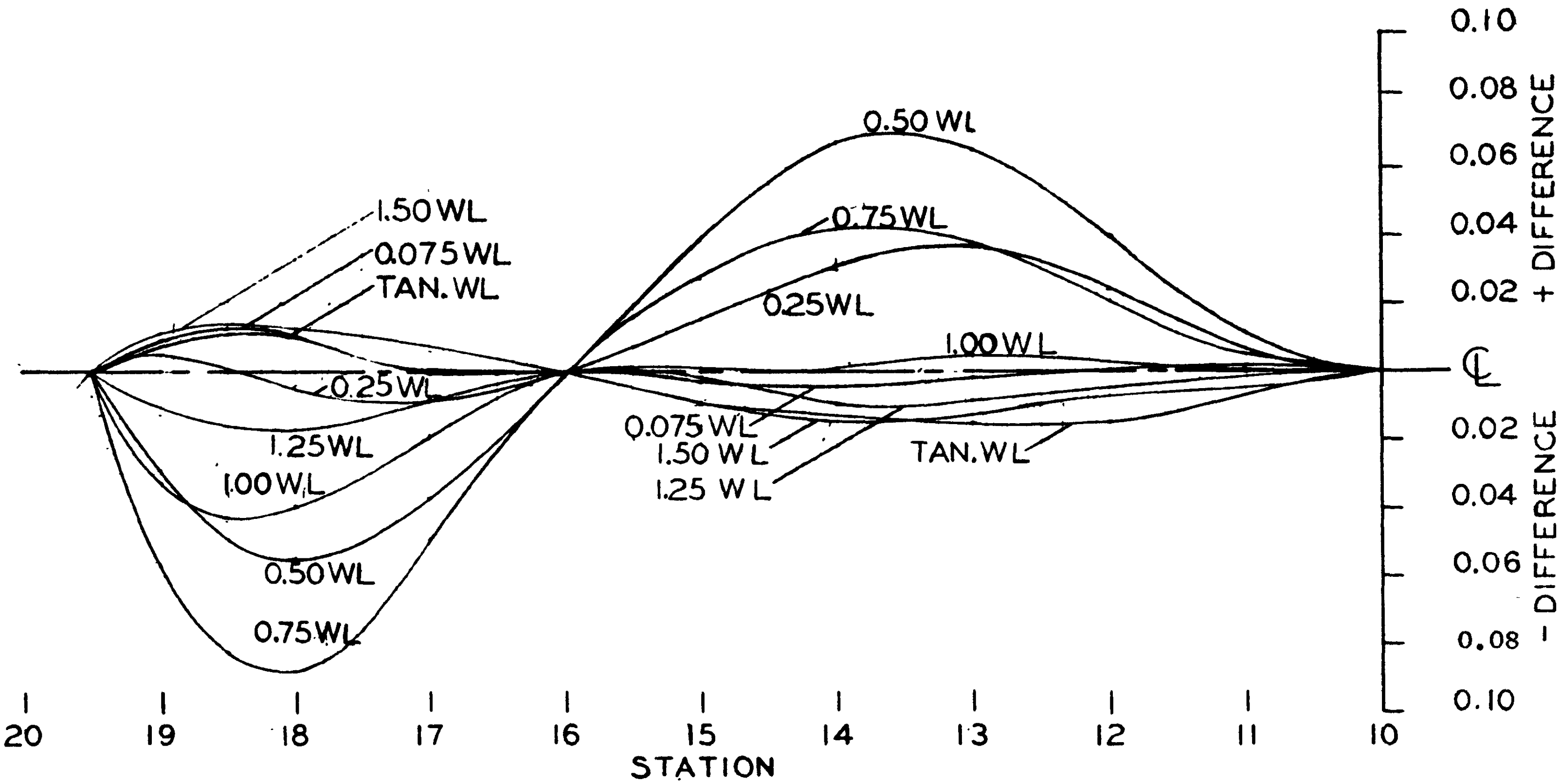


Figure 3 - Comparison of Afterbody Plans of Modified Model and of $\bar{f}_1(x,z)$ Surface



1.50 WL
1.25 WL
1.00 WL
0.75 WL
0.50 WL
0.25 WL
0.075 WL
15
14
13
12
11
17
10, 16 & 19 1/2
19
18
18 1/2
+
BL
CL

(a) - Body Plan



(b) - Waterlines

Figure 4 - Body Plan and Waterlines of $\bar{f}_2(x.z)$ Surface

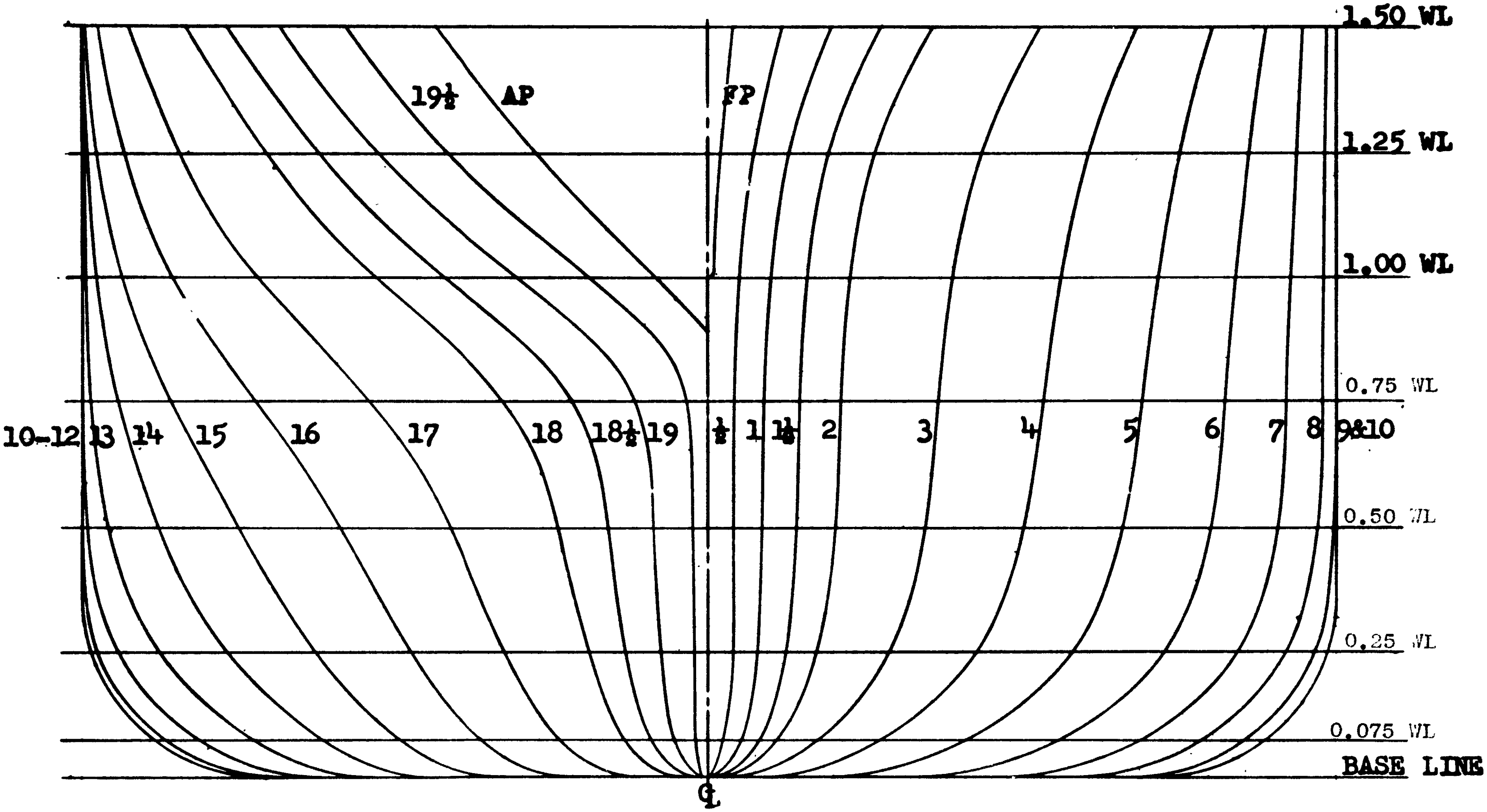
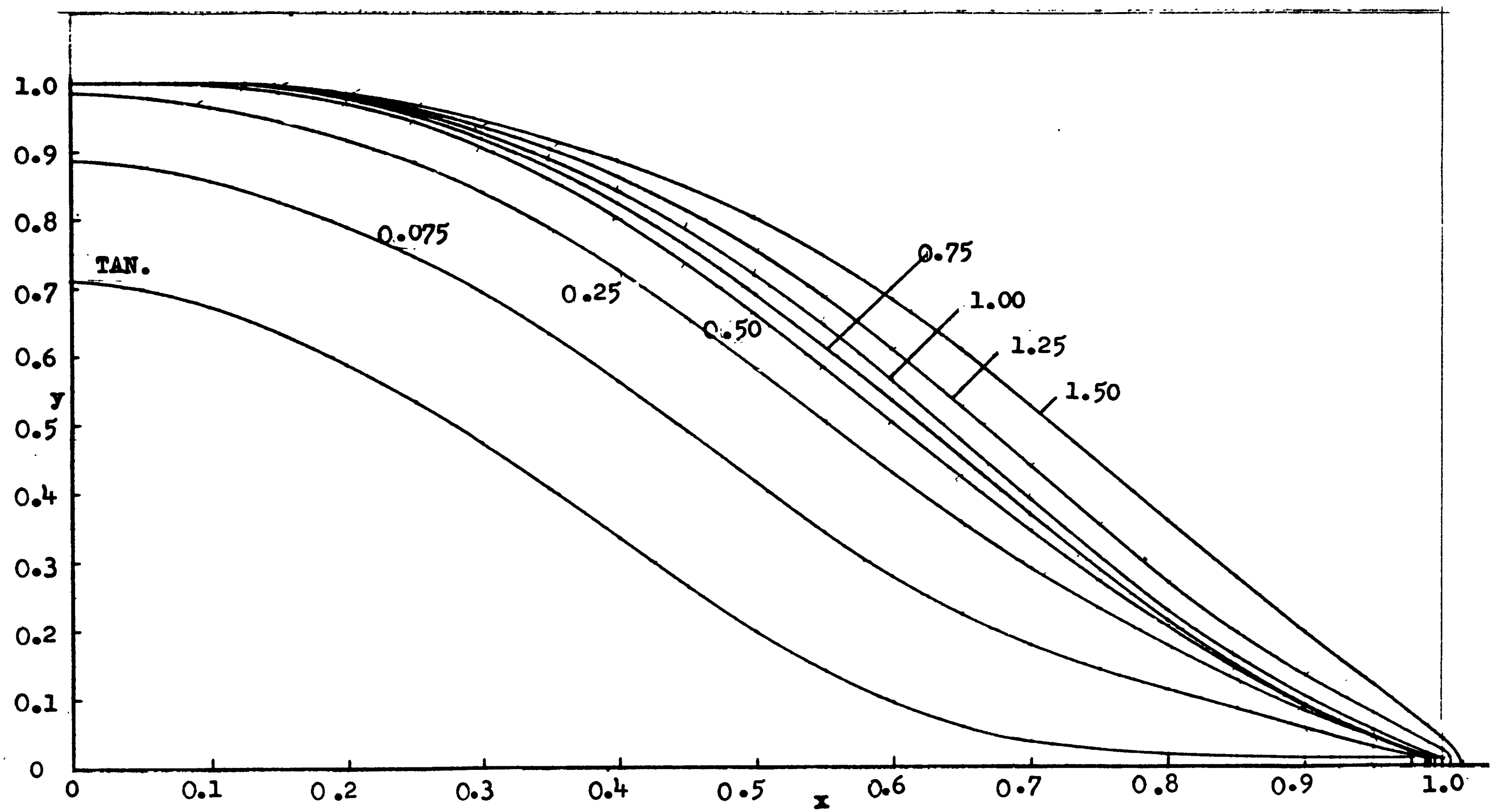
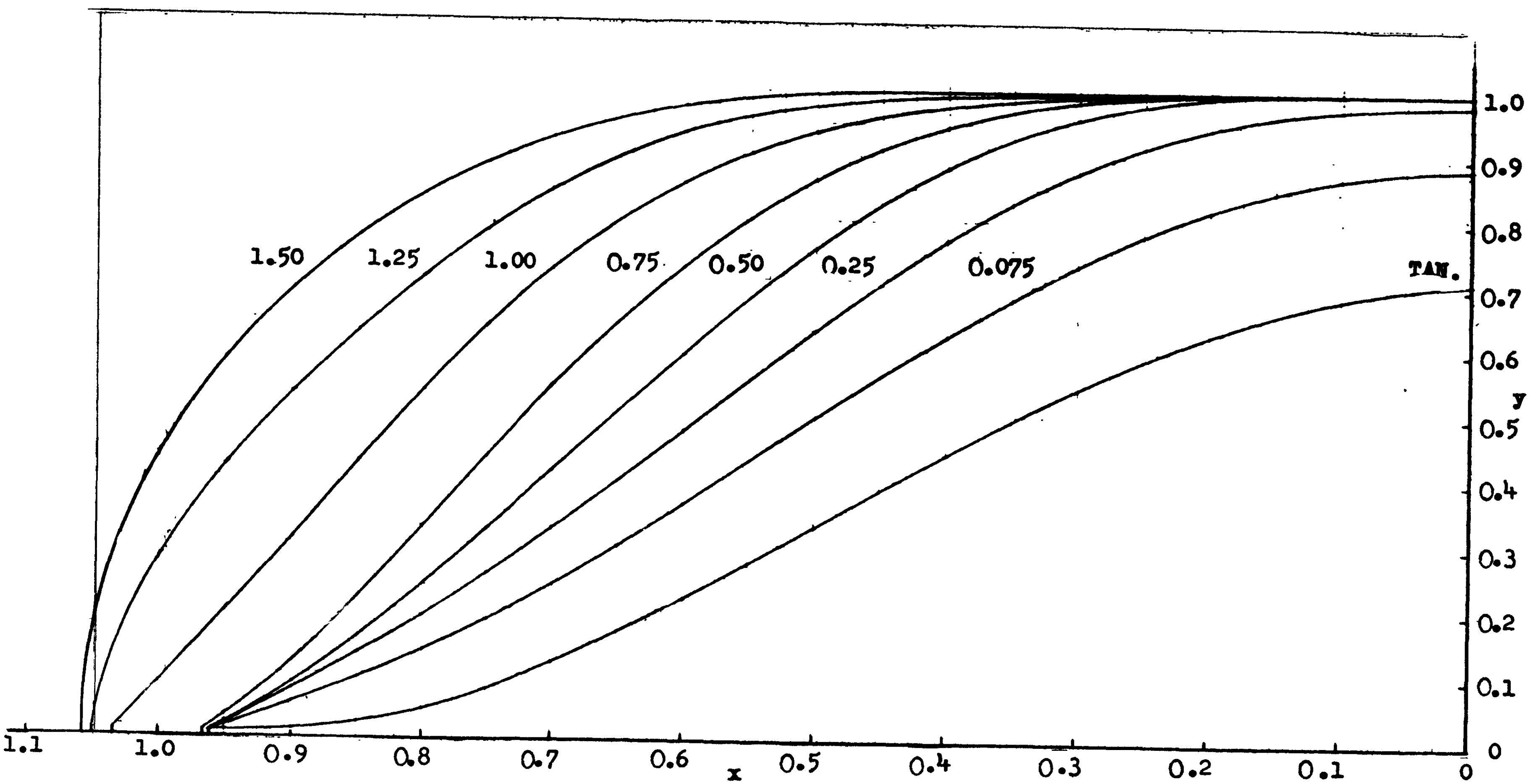


Figure 5 - Computed Body Plan, Model 4210



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x
TAN.
0.075
0.25
0.50
0.75
1.00
1.25
1.50

(a) - Forebody



(b) - Afterbody

Figure 6 - Computed Forebody and Afterbody Waterlines of Model 4210

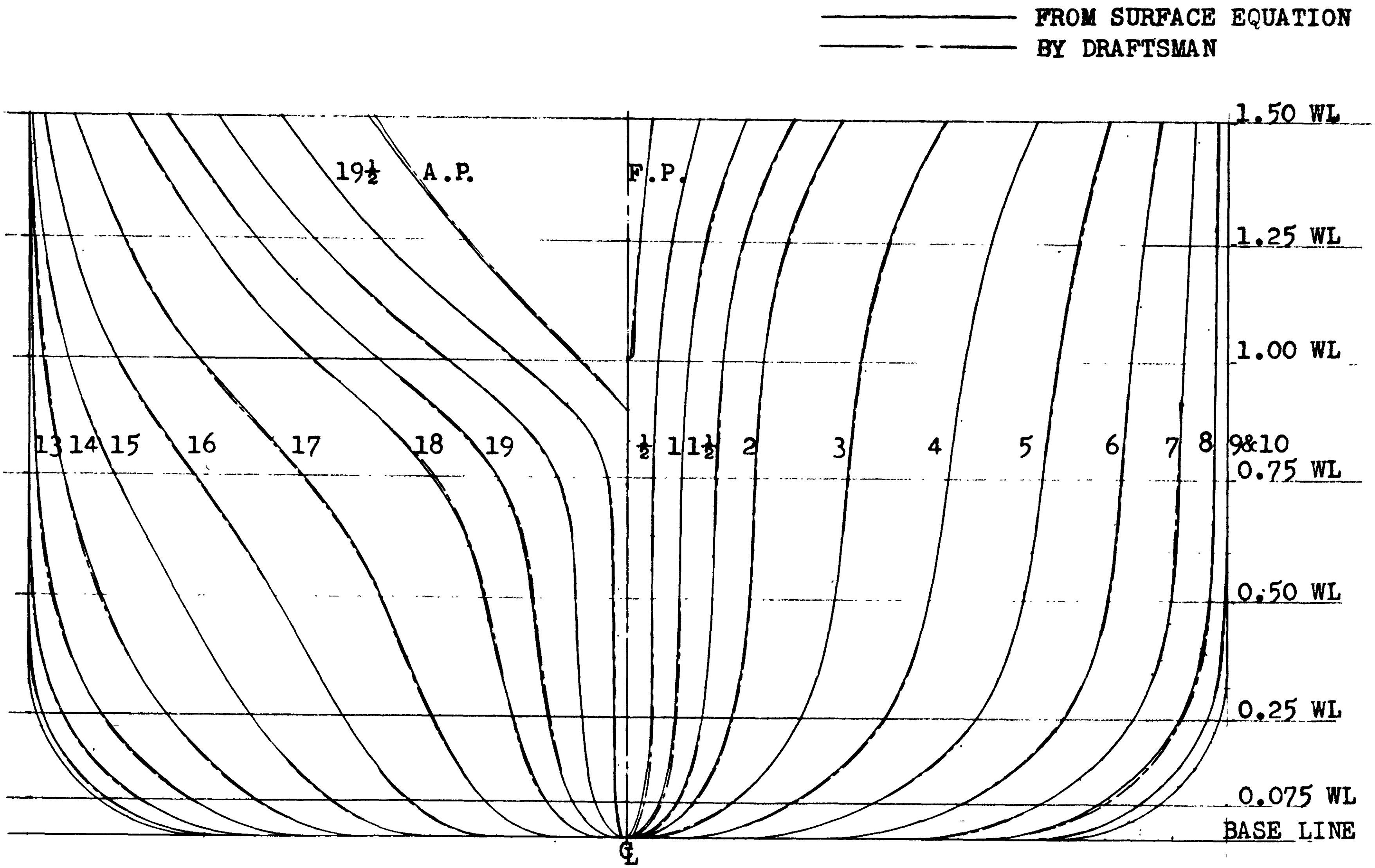


Figure 7 - Body Plan Comparison, Model 4210

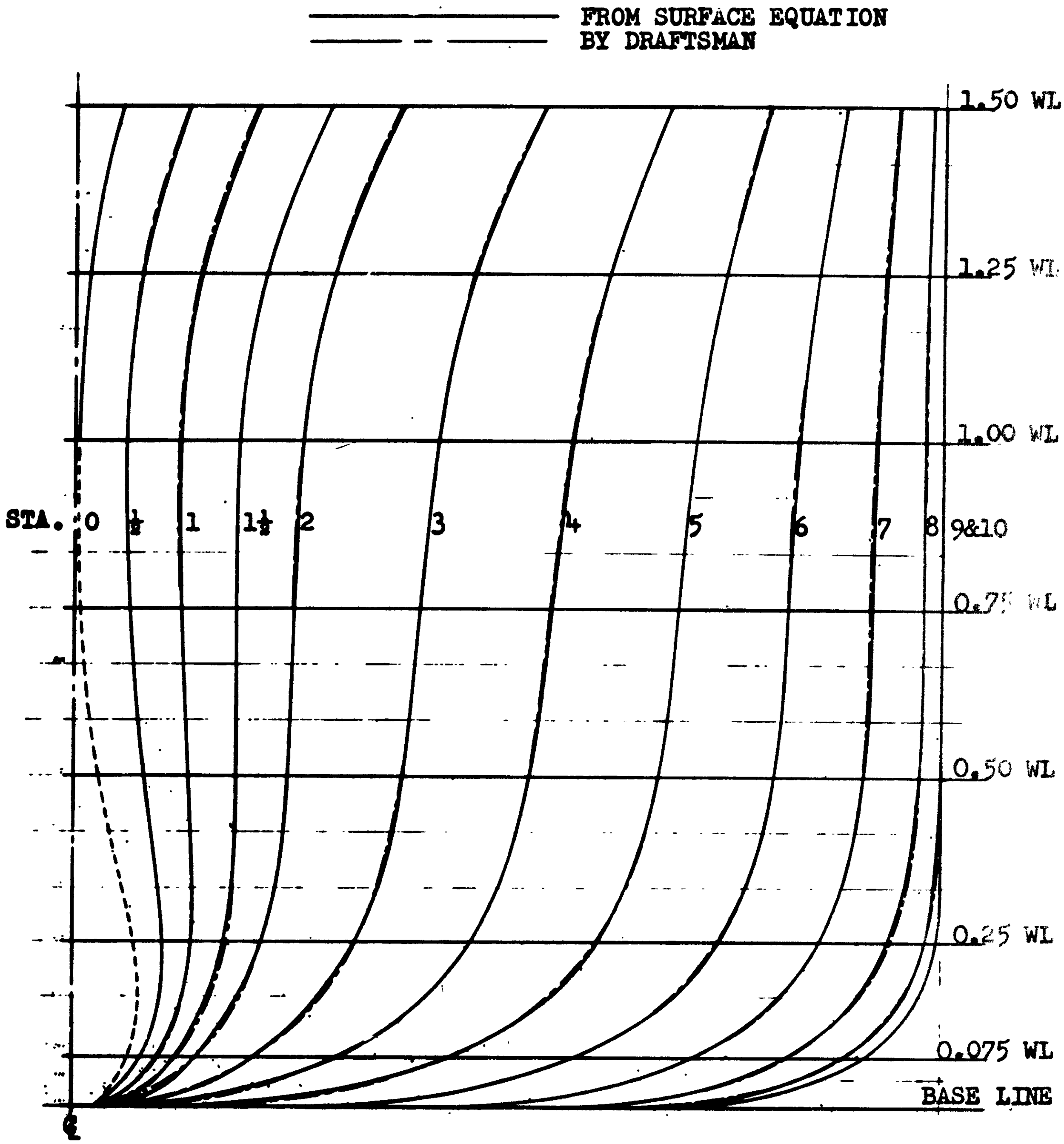


Figure 8 - Forebody Half of Body Plan, Model 4144

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