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NATURAL FREQUENCIES OF SHAFT STRUTS

by

Commander S. R. Heller, Jr., USN

RESEARCH AND DEVELOPMENT REPORT

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NATURAL FREQUENCIES OF SHAFT STRUTS

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an Engineering Duty Officer of the United States Navy, was graduated from the University of Michigan with degrees in Naval Architecture and Marine Engineering and in Mathematics. Following graduation he was employed by the New York Shipbuilding Corporation. On entry into the U.S. Navy he was ordered to the Norfolk Naval Shipyard and assigned to carrier construction. After World War II he was ordered to the Massachusetts Institute of Technology for postgraduate instruction leading to the degrees of Naval Engineer and Doctor of Science in Naval Architecture. Since that time he has been Assistant for Aircraft Carriers in the Hull Design Branch of the Bureau of Ships, Maintenance Officer on the Staff of Commander Amphibious Group ONE, and is now Structural Mechanics Officer at David Taylor Model Basin.

INTRODUCTION

During the design of *Forrestal* (CVA59) the rather long unsupported lengths of the main propeller-shaft strut arms seemed a potential vibration hazard. Although there had been no prior history of vibratory troubles with these components, the importance and financial investment of *Forrestal* required assurance of safety. Accordingly, New York Naval Shipyard (Material Laboratory) conducted a vibration generator survey of the main shaft struts of *Bennington* (CVA20) to determine the natural frequencies of the strut arms, both in air and submerged. The findings were never made generally available.

Unpublished memoranda prepared in the Bureau of Ships generalized these data into design information. The constraint conditions of the strut arms were likened to those for a fixed-hinged beam; the mass of entrained water was treated as about one-half the mass of the strut arm. Somewhat later the shaft struts of both *Forrestal* and *Saratoga* (CVA60) were surveyed, and the results reported, (1) and (2).* Reference (1) is not generally applicable to the problem since the strut construction in *Forrestal* is markedly different from that in *Bennington*. Reference (2), however, is quite pertinent since strut construction is similar, but the numerical results from *Saratoga* did not seem to agree directly with those from *Bennington* either as to constraint conditions or entrained water.

It appeared appropriate to develop an analytic solution to the problem which might reconcile the differences between *Bennington* and *Saratoga*. It is this aspect with which this paper is concerned.

* Numbers in parentheses refer to the Bibliography at the end of this paper.

THEORETICAL DEVELOPMENT

When shear deflection, rotary energy, and longitudinal forces are neglected, the amplitude equation of lateral vibration of a straight beam of constant cross section is:

$$EIy^{IV} - \mu\omega^2 y = 0 \quad \ldots\ldots (1)$$

with the solution (3):

$$y = C_1(\cos kx + \cosh kx) + C_2(\cos kx - \cosh kx) + C_3(\sin kx + \sinh kx) + C_4(\sin kx - \sinh kx) \quad \ldots\ldots (2)$$

where E is Young's modulus

I is moment of inertia

$k^4 = \frac{\mu\omega^2}{EI}$

μ is mass per unit length

ω is natural frequency which depends on the boundary conditions.

Figure 1 shows the typical shaft strut construction. Figure 2a shows an idealized form of the strut which will be treated analytically. The hinged support which replaces the strut barrel and shaft was chosen for simplicity. It is realized that this represents a stiffer support than the axial stiffness of the strut arms, but the extremely small amplitudes at the barrel reported in (2) lend validity to the assumption. The fixed ends in Figure 2a must be stiffer

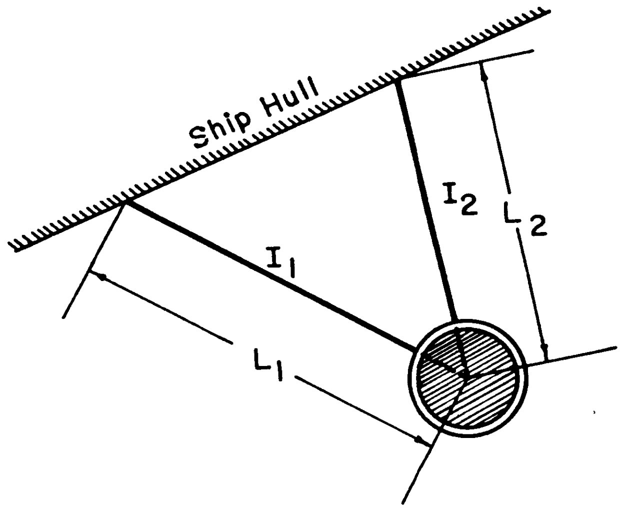


Figure 1. Typical Shaft Strut.

than the actual construction; also the rigid connection between arms must be stiffer than the strut barrel. The combination of these assumptions must yield frequencies somewhat higher than actual because of increased stiffness.

It is also of interest to note that the idealized system of Figure 2a is the equivalent of the system of Figure 2b: a beam fixed at either end and continuous over an external simple support. The solution given herein is also applicable to structures of the type shown in Figure 2b.

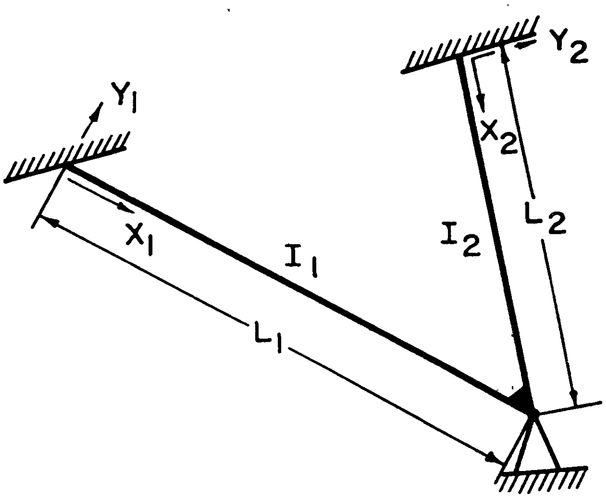


Figure 2a. Idealized Shaft Strut System.

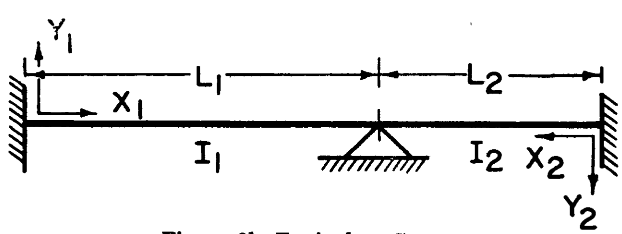


Figure 2b. Equivalent System.

The evaluation of the constants in Equation [2] depends on the boundary conditions imposed. The boundary conditions corresponding to both Figures 2a and 2b are:

$$\left.\begin{array}{ll} \text{(a)} & y_1 = 0 \text{ at } x_1 = 0 \\ \text{(b)} & y_2 = 0 \text{ at } x_2 = 0 \\ \text{(c)} & y_1' = 0 \text{ at } x_1 = 0 \\ \text{(d)} & y_2' = 0 \text{ at } x_2 = 0 \end{array}\right\} \text{Fixed ends}$$

$$\left.\begin{array}{ll} \text{(e)} & y_1 = 0 \text{ at } x_1 = L_1 \\ \text{(f)} & y_2 = 0 \text{ at } x_2 = L_2 \end{array}\right\} \text{No displacement at joint}$$

$$\text{(g)} \quad y_1' \Big|_{x_1 = L_1} = y_2' \Big|_{x_2 = L_2} \qquad \text{Rigid connection at joint}$$

$$\text{(h)} \quad y_1'' \Big|_{x_1 = L_1} = -y_2'' \Big|_{x_2 = L_2} \qquad \text{Simple support, no external moment}$$

where the prime symbol (') indicates differentiation with respect to x. From boundary conditions (a) and (b), $C_{1_1} = C_{1_2} = 0$. Also, from boundary conditions (c) and (d), $C_{3_1} = C_{3_2} = 0$. Thus, for a beam with one end fixed, the amplitude equation becomes:

$$y = C_2(\cos kx - \cosh kx) + C_4(\sin kx - \sinh kx) \qquad \text{(2a)}$$

Application of boundary conditions (e) and (f) yields:

$$-\frac{C_{2_1}}{C_{4_1}} = \frac{\sin k_1L_1 - \sinh k_1L_1}{\cos k_1L_1 - \cosh k_1L_1}$$
$$-\frac{C_{2_2}}{C_{4_2}} = \frac{\sin k_2L_2 - \sinh k_2L_2}{\cos k_2L_2 - \cosh k_2L_2} \qquad \text{(3)}$$

Next, the application of boundary condition (g) and Equations [3] gives:

$$\frac{1 - \cos k_1L_1 \cosh k_1L_1}{\cos k_1L_1 - \cosh k_1L_1} = \frac{C_{4_2}}{C_{4_1}} \bullet \frac{k_2}{k_1} \bullet \frac{1 - \cos k_2L_2 \cosh k_2L_2}{\cos k_2L_2 - \cosh k_2L_2} \qquad \text{(4)}$$

Finally, from boundary condition (h) and Equation [4], the characteristic or natural-frequency equation is:

$$\frac{\sin k_1L_1 \cosh k_1L_1 - \cos k_1L_1 \sinh k_1L_1}{1 - \cos k_1L_1 \cosh k_1L_1} =$$
$$-\frac{k_2}{k_1} \bullet \frac{\sin k_2L_2 \cosh k_2L_2 - \cos k_2L_2 \sinh k_2L_2}{1 - \cos k_2L_2 \cosh k_2L_2} \qquad \text{(5)}$$

The k's are related by the frequencies. Since a common frequency is sought, $\omega_1 = \omega_2$. Then:

$$k_1^2\sqrt{\frac{E_1I_1}{\mu_1}} = k_2^2\sqrt{\frac{E_2I_2}{\mu_2}} \qquad \text{(6)}$$

The ratio $\frac{k_2}{k_1}$, used in Equation (5), has the following forms:

a. Different materials and different cross sections:

$$\frac{k_2}{k_1} = \sqrt[4]{\frac{E_1}{E_2} \bullet \frac{\gamma_2}{\gamma_1} \bullet \frac{r_1^2}{r_2^2}} \qquad \text{(6a)}$$

where r is the radius of gyration of the cross section and γ is the specific weight of the material.

b. Same material but different cross sections:

$$\frac{k_2}{k_1} = \sqrt{\frac{r_1}{r_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6b)$$

c. Same material and same cross section:

$$\frac{k_2}{k_1} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6c)$$

To obtain a solution of Equation [5] it is convenient to employ an artifice:

$$\text{Let } \phi = \frac{\sin kL \cosh kL - \cos kL \sinh kL}{1 - \cos kL \cosh kL}$$

Then Equation [5] becomes:

$$\phi_1 = -\frac{k_2}{k_1}\,\phi_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5a)$$

In Figure 3, ϕ is plotted against kL with the negative branches also plotted positive in the dashed form. The solution is obtained by entering Figure 3 with a given value of ϕ and finding a corresponding value of k_1L_1 from a positive branch. Associated with this is a modified negative value of $\phi \left(-\frac{k_2}{k_1}\,\phi \right)$ and a corresponding value of k_2L_2 from a negative branch. Figure 4 represents a number of such solutions for $\frac{k_2}{k_1} = 1$. These solutions are plotted as $\frac{L_2}{L_1}$ against kL_1. Figure 5 represents the first two modes for the most representative cases for which $\frac{L_2}{L_1}$ is in the range from 0.7 to 1.0.

CHARACTERISTICS OF SOLUTIONS

Some of the characteristics of ϕ deserve further discussion. The numerator of ϕ and hence ϕ itself have zeros when:

$$\sin kL \cosh kL - \cos kL \sinh kL = 0 \ldots\ldots (7a)$$

This corresponds to:

$$\tan kL = \tanh kL \quad \ldots\ldots\ldots\ldots (7b)$$

or the frequency equation for a uniform beam fixed at one end and simply supported at the other. The denominator has zeros, and hence ϕ becomes infinite when:

$$\cos kL \cosh kL = 1 \ldots\ldots\ldots\ldots (7c)$$

or the frequency equation for a uniform fixed-fixed beam.

Special significance is attached to solutions of Equation (5) obtained at zeros of ϕ from branches adjacent to the zeros. These solutions are for con-

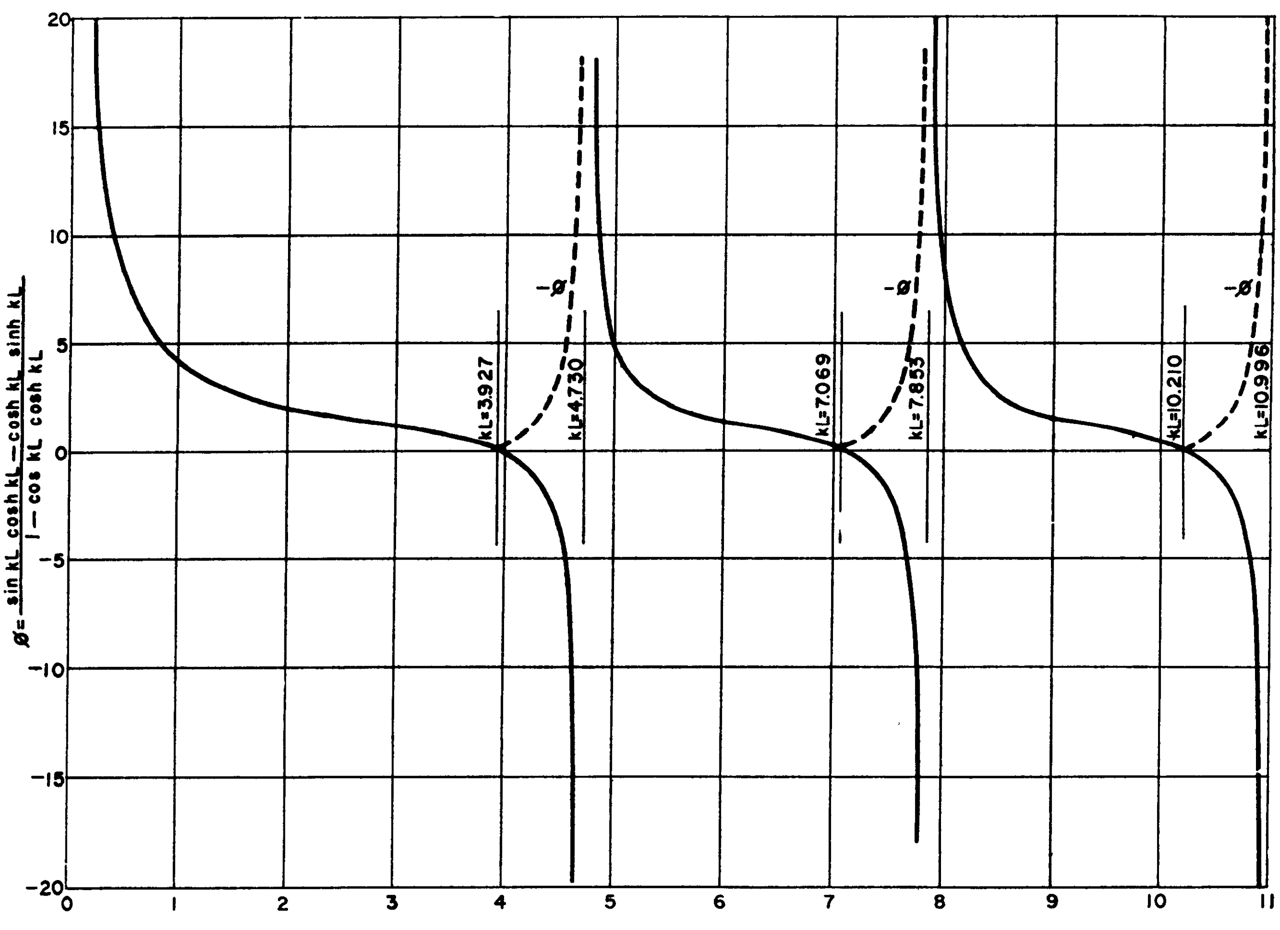


Figure 3. ϕ vs kL—An Aid to the Solution of the Frequency Equation.

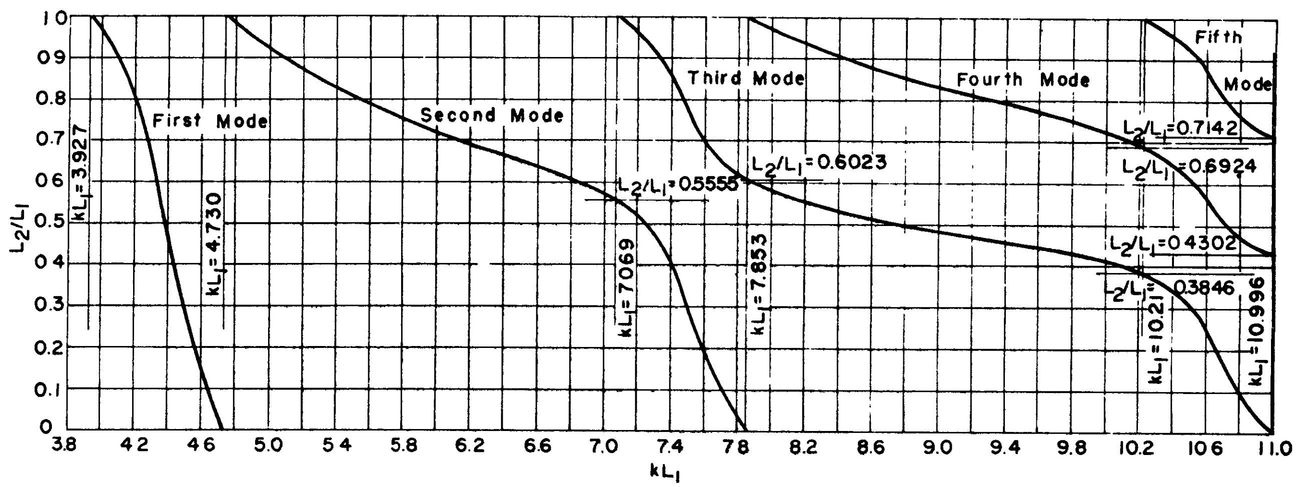


Figure 4. Solutions of the Frequency Equation for $k_1 = k_2$.

secutive *odd* modes for equal length arms (or equal length spans) regardless of the value of $\frac{k_2}{k_1}$, whereas consecutive solutions of Equation (7b) are for the consecutive modes of a uniform beam fixed at one end and simply supported at the other. Similarly, there is special significance for solutions of Equation (5) obtained at asymptotes from the branches adjacent to the asymptote. These solutions are for consecutive *even* modes for equal length struts regardless of the value of $\frac{k_2}{k_1}$, whereas consecutive solutions of Equation (7c) are for the consecutive modes of a uniform fixed-fixed beam.

For the special case of equal length arms the shapes of the amplitude curves for the two arms are identical. As the ratio of arm lengths decreases from unity, corresponding to ϕ increasing from zero (odd modes) or decreasing from infinity (even modes), the amplitude in the shorter arm decreases. When a transition to two new branches of Figure 3 occurs, the characteristic shape of the amplitude curve in the shorter arm also changes in such a way that the

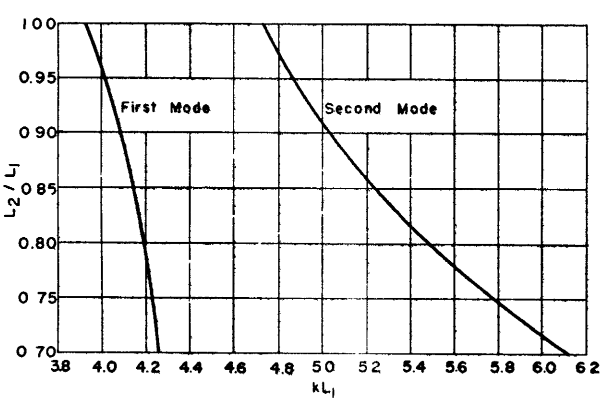


Figure 5. Expansion of the Most Representative Portion of Figure 4.

number of points of zero slope of the tangent is decreased by one. Obviously, the number of possible shapes of the amplitude curve for the shorter arm corresponds to the mode number. Table 1 summarizes the ranges and shapes for $\frac{k_2}{k_1} = 1$.

Solutions for $\frac{L_2}{L_1} = 0$, regardless of the value of $\frac{k_2}{k_1}$, always have frequencies and mode shapes corresponding to a uniform fixed-fixed beam. There appears to be a physical inconsistency here for a single-arm strut, but this is explainable by boundary condition (d). The actual physical system being analyzed is shown in Figure 2b. When the simple support is moved to $x_2 = 0$, the simple support is replaced by the fixed support and the beam must behave as fixed-fixed. The physical system of a single-arm strut is probably more like a uniform beam fixed at one end and simply supported at the other, the solution for which is given by Equation (7b) and hence the zeros in Figure 3 or the values of $\frac{L_2}{L_1} = 1$ for odd modes in Figure 4.

APPLICATION

In order to use the solutions presented in Figures 4 and 5, a relationship must be established involving the pertinent parameters and dimensions.

Originally k was defined as:

$$k^4 = \frac{\mu\omega^2}{EI} \qquad (8a)$$

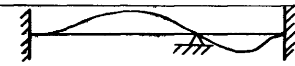
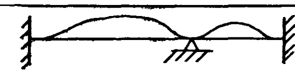
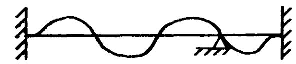
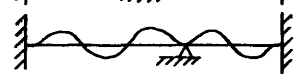
Frequency is then determined by solving for ω:

$$\omega = k^2\sqrt{\frac{EI}{\mu}} \qquad (8b)$$

which can be further simplified to:

$$\omega = k^2 r\sqrt{\frac{Eg}{\gamma}} \qquad (8c)$$

TABLE 1—*Regions and Mode Shapes for System with $k_1 = k_2$*

Mode	kL of Branches: Positive	kL of Branches: Negative	Range of $\frac{L_2}{L_1}$	Mode Shape
1	0 to 3.927	3.927 to 4.730	0 to 1.0	
2	4.730 to 7.069	3.927 to 4.730	0.5555 to 1.0	
	0 to 3.927	7.069 to 7.853	0 to 0.5555	
3	4.730 to 7.069	7.069 to 7.853	0.6023 to 1.0	
	7.853 to 10.21	3.927 to 4.730	0.3846 to 0.6023	
	0 to 3.927	10.21 to 10.996	0 to 0.3846	
4	7.853 to 10.21	7.069 to 7.853	0.6924 to 1.0	
	4.730 to 7.069	10.21 to 10.996	0.4302 to 0.6924	
5	7.853 to 10.21	10.21 to 10.996	0.7142 to 1.0	

Since we are primarily concerned with steel and the English system of units, Equation [8c] is put in its most useful form:

$$f = 193 \frac{(kL_1)^2 r}{\left(\frac{L_1}{100}\right)^2} \quad \text{(8d)}$$

where f is frequency in air, cpm
L_1 is length of longer arm, in.
kL_1 is the value taken from Figure 4 or 5 for the appropriate value of $\frac{L_2}{L_1}$.

The available experimental frequencies in air and the corresponding theoretical frequencies are summarized in Table 2.

EFFECT OF ENTRAINED WATER

It has long been known that a system vibrating in water moves with it a certain mass of water which effectively increases the mass of the system and consequently reduces the frequency under that in air. The shaft-strut system is no exception.

Although the precise distribution of the entrained water is unknown, the effect can be approximated by considering a cylinder of water surrounding the arm of diameter equal to the maximum dimension of the cross section. The ratio of the masses can be expressed by:

$$\frac{\mu_w}{\mu_a} = \frac{\pi D^2}{4A} \bullet \frac{\gamma_w}{\gamma} - \frac{\gamma_w}{\gamma} + 1 \quad \text{(9a)}$$

where A is area of cross section

TABLE 2—*Comparison of Theoretical and Experimental Frequencies in Air*

Ship	L_1, in.	L_2, in.	r, in.	$\frac{L_2}{L_1}$	kL_1	Frequency, cpm: Theory	Frequency, cpm: Exp.	Percent Dif.
Bennington	164	122	1.295	0.744	4.235	1666	1725	− 3.42
	140	136	1.295	0.972	3.978	2018	1875	+ 7.63
Saratoga	160	127	1.533	0.794	4.197	2035	1980	+ 2.78
	278	237	1.962	0.853	4.143	842	890	− 5.39
	278	237	1.962	0.853	5.228*	1341	1500	−10.6
	128	115	1.533	0.899	4.09	3019	2640	+14.35
	273	254	2.166	0.931	4.047	918	870	+ 5.52

* Second mode.

D is diameter of the cylinder

γ_w is specific weight of water, and

the subscripts "a" and "w" refer to air and water respectively.

For shaft struts we are usually concerned with sea water and steel. Equation (9a) then reduces to:

$$\frac{\mu_w}{\mu_a} = 0.8694 + 0.1026\frac{D^2}{A} \quad \text{(9b)}$$

It has long been naval practice to use struts of airfoil section with a chord-thickness ratio of six. With this additional condition Equation (9b) reduces to:

$$\frac{\mu_w}{\mu_a} = 0.8694 + \frac{0.6155}{K} \quad \text{(9c)}$$

where K is a factor depending on the shape of the strut. For the standard strut section (4), K=0.701; for the Taylor Model Basin "Ellipse-Parabola-Hyberbola" (EPH) section, K=0.747. Consequently, for the two most frequently used cross sections, Equation (9c) becomes:

$$\left.\begin{aligned} \frac{\mu_w}{\mu_a} &= 1.6474 \text{ for "Standard Strut"} \\ \frac{\mu_w}{\mu_a} &= 1.6934 \text{ for EPH} \end{aligned}\right\} \quad \text{(9d)}$$

Equation (8b) indicates that frequency is inversely proportional to the square root of mass per unit length. Since the constraint conditions do not change with immersion in water (only the mass per unit length), the following relation must exist:

$$\frac{f_a}{f_w} = \sqrt{\frac{\mu_w}{\mu_a}} \quad \text{(10)}$$

where the subscripts "a" and "w" refer to air and water respectively. Combining Equations (9d) and (10) yields

$$\left.\begin{aligned} \frac{f_a}{f_w} &= 1.2835 \text{ for "Standard Strut"} \\ \frac{f_a}{f_w} &= 1.3013 \text{ for EPH} \end{aligned}\right\} \quad \text{(11)}$$

Scant experimental evidence is available for comparison. Measurements on one pair of struts in water are reported in (2). Theoretical and experimental frequencies for *Saratoga* are compared in Table 3.

COMPARISON OF THEORY AND EXPERIMENT

Tables 2 and 3 summarize available experimental data in air and water respectively. It can be seen from Table 2 that the theory developed herein generally predicts a higher frequency in air than observed. This was to be expected because all idealizations made to facilitate analysis were in the direction of increased stiffness. It is unfortunate that the scant data available on vibration immersed should be for the one pair of struts for which theory predicted a lower frequency than observed. It is gratifying to note in Table 3, however, that the theoretical frequency in water differs from the observed by the same relative amount as did the frequency in air. This lends credence to the assumption of the cylinder whose diameter is equal to maximum dimension of the cross section. It is also interesting to note that for the first mode the mass of entrained water is slightly overestimated whereas for the second mode it is slightly underestimated.

It is extremely gratifying to see that the frequencies observed with the ship underway in free route, a realistic condition, agree even better with the theoretical frequencies than did those determined under the artificial condition of a ship in a flooded drydock.

GENERAL COMMENTS

The consideration of the strut system as a single arm fixed at one end and simply supported at the other, the design basis developed from the *Bennington* data, is conservative in that it predicts resonance at frequencies somewhat lower than observed. Its chief disadvantage lies in predicting different frequencies for the two arms, which fact is not confirmed by observation. On the other hand, the analysis developed herein is predicted on a single frequency per mode for the system as a whole. The solution given in this paper is somewhat more complicated than that previously used, but its application is equally simple if Figure 5 and Equation (8d) are used together.

The designer is interested in natural frequencies only when excitation exists. Careful orientation of the strut arms eliminates the possibility of flow excitation. Thus, the designer is faced with the problem of avoiding blade frequency resonance. If such a resonance cannot be avoided throughout the operating range, it should be accepted only at small fractions of rated power. That this is acceptable is verified by the extremely small amplitudes and stresses measured at resonances at the lower end of the operating range (2).

TABLE 3—*Comparison of Theoretical and Experimental Frequencies in Water for Saratoga*

Mode	$\frac{L_2}{L_1}$	Strut Section	f_a, cpm		Percent Dif.	f_w, cpm		Percent Dif.	f_w, cpm	Percent Dif.
			Theory	Exp.*		Theory	Exp.*		Exp.**	
1	0.853	EPH	842	890	− 5.39	647	690	−6.23	660	−1.97
2	0.853	EPH	1341	1500	−10.6	1031	1140	−9.57	1130	−9.52

* Vibration generator test in flooded drydock.
** Underway in free route.

CONCLUSIONS

1. Equation (5) predicts the natural frequencies in air of the shaft strut system with satisfactory engineering accuracy. The predicted frequencies are usually slightly higher than those observed.

2. Equation (8d) when used in conjunction with Figure 5 is a rapid way of predicting natural frequencies of the strut system.

3. For vibration normal to the plane of the strut arm, the mass of entrained water may be taken as that of a cylinder whose diameter is equal to the maximum dimension of the cross section.

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