THE RADIAL DISTRIBUTION OF PROPELLER THRUST
FROM MODEL WAKE MEASUREMENTS

by

John L. Beveridge

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### NOTATION

#### Dimensions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Dimensions in Mass-Length-Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>Propeller Diameter</td>
<td>$L$</td>
</tr>
<tr>
<td>$n$</td>
<td>Propeller rate of revolution</td>
<td>$T^{-1}$</td>
</tr>
<tr>
<td>$R$</td>
<td>Propeller tip radius</td>
<td>$L$</td>
</tr>
<tr>
<td>$R'$</td>
<td>Radius of slip stream in plane of wake measurement</td>
<td>$L$</td>
</tr>
<tr>
<td>$r$</td>
<td>Local radius</td>
<td>$L$</td>
</tr>
<tr>
<td>$T$</td>
<td>Propeller thrust</td>
<td>$MLT^{-2}$</td>
</tr>
<tr>
<td>$v$</td>
<td>Axial velocity of propeller or pitot tube through the water at radius fraction $x$</td>
<td>$LT^{-1}$</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Ship speed</td>
<td>$LT^{-1}$</td>
</tr>
<tr>
<td>$\Delta v$</td>
<td>Axial velocity change at radius fraction $x$</td>
<td>$LT^{-1}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Mass density</td>
<td>$ML^{-3}$</td>
</tr>
</tbody>
</table>

#### Coefficients and Ratios

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{TS}$</td>
<td>$\frac{T}{\frac{1}{2} \rho \pi R^2 v_s^2}$</td>
<td>Propeller thrust-load coefficient</td>
</tr>
<tr>
<td>$J$</td>
<td>$\frac{v}{nd}$</td>
<td>Propeller advance coefficient</td>
</tr>
<tr>
<td>$J_a$</td>
<td>$\frac{v_s}{nd}$</td>
<td>Propeller apparent advance coefficient</td>
</tr>
<tr>
<td>$K_T$</td>
<td>$\frac{T}{\rho n^2 d^4}$</td>
<td>Propeller thrust coefficient</td>
</tr>
<tr>
<td>$W_x$</td>
<td>$1 - \frac{v}{v_s}$</td>
<td>Taylor wake fraction at radius fraction $x$</td>
</tr>
<tr>
<td>$x$</td>
<td>$\frac{r}{R}$ or $\frac{r}{R'}$</td>
<td>Radius fraction</td>
</tr>
</tbody>
</table>
ABSTRACT

The radial distribution of propeller thrust-load coefficient is calculated from experimental wake data obtained from towed and self-propulsion tests on a submerged body of revolution. The experimentally obtained thrust distribution is in excellent agreement with a theoretically calculated distribution. Numerical integration yields a total thrust-load coefficient which is in good agreement with propulsion test results.

INTRODUCTION

Past efforts to determine the radial distribution of propeller thrust have been mostly by theoretical means. Information is especially needed for a propeller operating behind a submerged body rather than for the open-water propeller condition.

In this report, a radial distribution of propeller thrust obtained theoretically is compared with one obtained experimentally. The experimentally determined radial wake distributions were obtained from tests on a submerged body of revolution. The radial distribution of propeller thrust, or more conveniently the propeller thrust-load coefficient, $C_{TS}$ is easily obtained from such wake data. The results are presented as curves of wake fraction and thrust coefficients. The calculated total thrust-load coefficient is compared to the thrust-load coefficient obtained from propulsion tests.

EXPERIMENTAL WAKE DISTRIBUTIONS

The wake curves of Figure 1 were obtained from measurements made a short distance (0.227 d) abaft the propeller rotational plane on both a towed and self-propelled model. The wake survey was made on both the port and starboard sides of the model by means of pitot rakes. The method of mounting the wake survey rakes in the upper quadrants and the general arrangement of the port and starboard wake survey assemblies on the model are shown in Figure 2. The axes of the pitot static tubes of the rakes were set parallel to the average slope (tan 14 deg) of the afterbody and lie in a 45-deg meridian plane. For the purpose of computing thrust, a cosine correction was applied to the measured velocities. The effect of slip stream rotation on the experimentally derived thrust distribution as obtained by the pitot survey was mathematically investigated. For this case, the correction as calculated by the method of Pankhurst\textsuperscript{1} was found to be negligibly small.

\textsuperscript{1}References are listed on page 8.
Figure 1 – Radial Distribution of Wake Fraction With and Without a Propeller as Determined from Tests on a Submerged Body of Revolution

RADIAL THRUST DISTRIBUTIONS

The axial momentum equation functionally relates the wake fraction curves from towed and self-propulsion tests (Figure 1) to the incremental thrust as follows:

\[ dT = 2\pi \rho \left( v + \frac{\Delta v}{2} \right) \Delta v r \delta r \]  

where \( \Delta v \) represents the velocity change between the towed (without propeller) and the propelled condition. \( \Delta v \) and \( v \) are expressed nondimensionally (Figure 3) as \( \Delta \frac{v}{v_s} \) and \( (1 - W_x) \), respectively. The curves shown in Figure 3 were derived from the curves of Figure 1 by applying the appropriate cosine corrections (mentioned previously) and by correcting for the contraction of the propeller slip stream. In computing \( z \) for the propelled condition, \( R' \) was estimated to be equal to 0.968 \( R \) based on data derived by Theodorsen.\(^2\)

Equation [1] can also be conveniently written in nondimensional terms. Let \( z = \frac{r}{R} \) and divide by \( (\frac{1}{2} \rho v_s^2 \pi R^2) \) then the thrust distribution is given by

\[ \frac{dC_T S}{dz} = 4z \left[ \frac{\Delta v \cdot \phi}{v_s^2} + \frac{1}{2} \left( \frac{\Delta v}{v_s} \right)^2 \right] \]  

\[ [2] \]
Figure 2a – Plan View

Figure 2b – Elevation

Figure 2 – View of Wake Survey Rakes
Figure 3 – Radial Distribution of $(1-W_x)$ and Axial Velocity Change from Towed and Self-Propelled Tests

Figure 4 – Thrust-Load Coefficients for Propeller 3273
The experimental thrust distribution $dC_{TS}/dz$ obtained from Equation [2] and a theoretically obtained distribution are plotted in Figure 4. The agreement between the experimental curve and the theoretical curve is excellent. Recent developments in the circulation theory \(^3,4,5\) and the manner in which the experimental distribution was obtained, both make necessary an explanation of the curves shown in Figure 4. The shape of the distributions towards the propeller hub and propeller tip is of particular interest. At the propeller tip, the experimental curve exhibits a finite value of $dC_{TS}/dz$ (solid line) which fairs into real experimental distribution at $x=0.9$. Why the diameter of the slipstream is larger than the propeller diameter is not completely understood. A number of factors such as flow separation on the appendages; interaction between the hull, appendages, and propeller; or the appendage arrangement itself could be contributing factors. This virtual $\Delta v$ does not, however, produce any additional propeller thrust due to the lack of a point of application beyond the propeller disk. Equation [2] does not consider this fact; therefore, the real experimental distribution is faired to zero at $x=1.0$. In contrast to the experimental curve just discussed, the theoretical thrust distribution falls to zero at both the propeller hub and propeller tip. The accuracy of theoretical thrust distributions was greatly increased by Tachmindji's formulation and solution of the potential problem for the circulation distribution of a propeller with a finite hub.\(^3\) Prior to the publication of the data contained in References 4 and 5, theoretical thrust distributions were less accurate near the extremities. The circulation distribution as calculated assuming zero hub diameter would yield a finite value for $dC_{TS}/dz$ at the propeller hub. This is caused by the extension of the vortex sheets to the propeller axis.

**TOTAL THRUST**

Using Simpson's rule, the total thrust-load coefficient was calculated from the experimental distribution shown in Figure 4. A $C_{TS}$ value of 0.595 results from the integration. The total thrust obtained from a model self-propulsion test is estimated from Figure 5. The characteristic curves for Propeller 3273 are entered on the abscissa at a $J$ of 0.605 as determined from a model self-propulsion test. The corresponding $K_T/J^2$ value is found, by interpolation, to be 0.545. By calculation $K_T/J \alpha^2 = 0.545 (1 - \omega)^2$ is found to be equal to 0.2163. Therefore, the value of $C_{TS}$ obtained from the propulsion test is $8/\pi \cdot 0.2163 = 0.551$. The agreement between the $C_{TS}$ values obtained by the two methods is very good.

**CONCLUSIONS**

1. For the hull-propeller combination considered, the experimental and theoretical thrust distributions are in excellent agreement.

2. An integration of the experimental thrust distribution yields a total thrust which is in good agreement with the results obtained from submerged model propulsion tests.
Propeller 3273
TMB Drawing C-3273

<table>
<thead>
<tr>
<th>Number of Blades</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. Area Ratio</td>
<td>0.611</td>
</tr>
<tr>
<td>MWR</td>
<td>0.242</td>
</tr>
<tr>
<td>BTF</td>
<td>Var.</td>
</tr>
<tr>
<td>p/d</td>
<td>0.951</td>
</tr>
<tr>
<td>Diameter</td>
<td>9.900 in.</td>
</tr>
<tr>
<td>Pitch</td>
<td>9.415 in.</td>
</tr>
<tr>
<td>Rotation</td>
<td>RH</td>
</tr>
<tr>
<td>Test RPM</td>
<td>858</td>
</tr>
<tr>
<td>Test $V_0$</td>
<td>2.5 - 7.0 knots</td>
</tr>
</tbody>
</table>

Reynolds Number,
$$R_e = \frac{b_0.7 \sqrt{V_0^2 + (0.7 \pi n d)^2}}{\nu}$$

Thrust Coefficient,
$$K_t = \frac{T}{\rho n^2 d^4}$$

Torque Coefficient,
$$K_q = \frac{Q}{\rho n^2 d^5}$$

Speed Coefficient,
$$J = \frac{V_0}{n d}$$

Efficiency,
$$e = \frac{T V_0}{2 \pi Q n} = \frac{K_t}{K_q} \times \frac{J}{2 \pi}$$

$T$ = Thrust
$Q$ = Torque
$n$ = Revolutions per Unit Time
$V_0$ = Speed of Advance
$\nu$ = Kinematic Viscosity
$d$ = Diameter
$p$ = Pitch
$\rho$ = Density of Water
Figure 5 – Plans and Open-Water Characteristic Curves for Propeller 3273
ACKNOWLEDGMENTS

The author wishes to express his appreciation to Mr. A.J. Tachmindji whose critical review and suggestions have greatly enhanced the value of the report, and to Mr. W.B. Morgan for his contribution to the programming of the propeller design on the Univac.

REFERENCES


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