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ON THE IN-VACUO VIBRATIONS OF SIMPLY SUPPORTED, RING-STIFFENED CYLINDRICAL SHELLS

by

Gerard D. Galletly

STRUCTURAL MECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

February 1958

Report 1195

PRNC-TMB-648 (11-56)

DEPARTMENT OF THE NAVY
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Subj: Vibrations of ring-stiffened cylindrical shells; forwarding of report on

Encl: (1) TMB Report 1195 entitled "On the In-vacuo Vibrations of Simply Supported, Ring-stiffened Cylindrical Shells" (12 copies)

1. The David Taylor Model Basin is conducting studies of the strength characteristics of stiffened cylindrical shells under Project NS731-038. In enclosure (1) is presented an analytical solution for the problem of determining the in-vacuo frequencies of vibration of a simply supported thin cylindrical shell reinforced by equally spaced, equal strength ring stiffeners. The numerical values obtained by this solution are compared with those calculated by approximate methods and are found to agree within ten percent.

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ON THE IN-VACUO VIBRATIONS OF SIMPLY SUPPORTED RING-STIFFENED CYLINDRICAL SHELLS

by

Gerard D. Galletly

Reprinted from the
Proceedings of the Second U.S. National Congress
of Applied Mechanics
Held at University of Michigan, Ann Arbor, Michigan
June 14-18, 1954

February 1958

Report 1195

ON THE IN-VACUO VIBRATIONS OF SIMPLY SUPPORTED, RING-STIFFENED CYLINDRICAL SHELLS

Gerard D. Galletly

Head, Plates and Shells Section,
David Taylor Model Basin, Washington 7, D. C.

An analytical solution is presented for the problem of determining the in-vacuo frequencies of vibration of a simply supported thin cylindrical shell, which is reinforced by equally spaced, equal strength circular ring stiffeners. The displacement configuration assumed permits inter-ring deformation of the shell. Numerical calculations were made for a cylindrical shell with various sizes of stiffening rings and curves were plotted which show the effect of the rings upon the frequency of vibration. The numerical values obtained were also compared with those calculated by approximate methods and were found to agree within ± 10 per cent.

NOMENCLATURE

The following nomenclature is used in the paper:

a = mean radius of shell

e = distance between middle surface of shell and centroid of ring cross-section

h = thickness of shell

g = acceleration due to gravity

s = coordinate in circumferential direction

u, v, w = displacements of a point on the shell in the direction of the x-, y- and z- axes

x, y, z = rectangular coordinates — see Fig. 1 for orientation

A_F = cross-sectional area of stiffening ring

C_T = St. Venant torsion constant

E = Young's modulus

G = shear modulus

I_p, I_{XG}, I_{ZG} = polar and ordinary moments of inertia of ring cross section about axes passing through the centroid of the cross section

L = distance between supports

L_F = distance between ring stiffeners

N = total number of ring stiffeners

β = angle of rotation of the ring cross-section about the y-axis

ϵ_s = circumferential strain

γ = density of shell material (lb per unit volume)

ν = Poisson's ratio

χ_1, χ_2 = changes of curvature of ring stiffeners

θ = angular coordinate in circumferential direction

ω = circular frequency

Φ = angle of twist per unit circumferential length of ring stiffener

Other symbols used are defined where they first appear.

INTRODUCTION

The object of the present paper is the prediction of the in-vacuo vibration characteristics of the simply supported ring-stiffened thin cylindrical shell shown in Fig. 1. Arnold and Warburton [1]* have studied this problem for the case of the unstiffened cylinder. For their displacement configuration they assumed only the first terms of the double Fourier series and obtained very good agreement between theory and experiment. In a recent paper by Junger [2] the vibrations of ring-reinforced cylindrical shells in a vacuum and in a fluid are discussed. However, for the problem under consideration the displacement pattern assumed by him does not appear appropriate. Bleich, also, in some unpublished notes [3], has studied the vibrations of ring-stiffened cylinders. His procedure was to assume a displacement configuration and then use Rayleigh's principle. He succeeded in obtaining formulae for the reinforced cylinder which are similar in form to those obtained by Lamb and Southwell for the vibrations of a spinning disk [4].

In the present paper strain energy expressions are used for the rings which are more complete than those used by previous authors and the vibration shape assumed permits inter-ring deformation of the shell. The method of solution is similar to that used by Arnold and Warburton and consists in evaluating the strain and kinetic energies of the shell-ring combination and utilizing Lagrange's equations to derive the dynamic equations of the system.

*Numbers in brackets refer to the bibliography at the end of the paper.

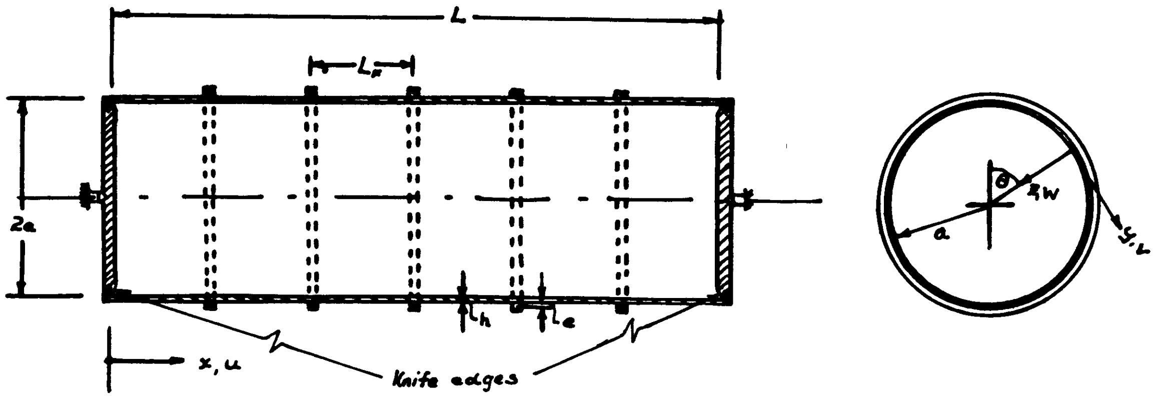

FIGURE 1. SIMPLY SUPPORTED CYLINDRICAL SHELL REINFORCED WITH EQUALLY SPACED RINGS.

STRAIN AND KINETIC ENERGY EXPRESSIONS

For the cylinder the strain-energy expression derived by Bleich and DiMaggio [5] will be used, viz:

$$U_s = \frac{Eab}{2(1-\nu^2)} \int_0^{2\pi} \int_0^L \left[u_x^2 + \left(\frac{v_\theta - w}{a}\right)^2 + 2\nu u_x \left(\frac{v_\theta - w}{a}\right) + \left(\frac{1-\nu}{2}\right)\left(v_x + \frac{u_\theta}{a}\right)^2 \right] d\theta\, dx \, +$$

$$\frac{Eb^3}{24a(1-\nu^2)} \int_0^{2\pi} \int_0^L \left[a^2 w_{xx}^2 + \left(\frac{w_{\theta\theta} + w}{a}\right)^2 + 2\nu w_{xx}\,(w_{\theta\theta} + w) + \left(\frac{1-\nu}{2}\right)\left(w_{x\theta} - \frac{u_\theta}{a}\right)^2 + 3\left(\frac{1-\nu}{2}\right)(v_x + w_{x\theta})^2 + 2a u_x w_{xx} \right] d\theta\, dx \qquad (1)$$

The kinetic energy of the cylinder is given by:

$$T_s = \frac{ab\gamma}{2g} \int_0^{2\pi} \int_0^L [\dot{u}^2 + \dot{v}^2 + \dot{w}^2]\, d\theta\, dx \qquad \cdot \equiv \frac{d}{dt} \qquad (2)$$

Since the depth of the ring stiffener is small in comparison with the radius of the cylinder, it is permissible to use the Bemoulli-Euler theory for evaluation of the strain-energy expressions for the rings. In terms of the changes of curvature and twist per unit length about the centroid of the section, the strain energy in the ring stiffener shown in Fig. 2 is given by:

$$U_R = \frac{EI_{zG}}{2} \int_0^s \chi_1^2\, ds + \frac{EI_{xG}}{2} \int_0^s \chi_2^2\, ds + \frac{EA_F}{2} \int_0^s \epsilon_s^2\, ds + \frac{GC_T}{2} \int_0^s \Phi^2\, ds \qquad (3)$$

The strain energy associated with restricted warping has been neglected in equation (3). For open-section stiffeners, however, non-uniform torsion may be a significant factor and the strain energy associated with it should be included.

From Ref. [6], section 54, there follows for a ring of radius $(a + e)$:

$$\left.\begin{aligned} \chi_1 &= -\frac{1}{(a+e)}\,[\beta + u_{G\theta\theta}] \\ \chi_2 &= \frac{1}{(a+e)^2}\,[w_G + w_{G\theta\theta}] \\ \Phi &= \frac{1}{(a+e)}\,[-\beta_\theta + \frac{u_{G\theta}}{a+e}] \\ \epsilon_s &= \frac{1}{(a+e)}\,[v_{G\theta} - w_G] \end{aligned}\right\} \qquad (4)$$

It will be observed from Fig. 2 that the positive direction of β is opposite to that of Ref. [6]. This accounts for the sign changes in equation (4). It can also be seen from Fig. 2 that the following relationships hold between the displacements of the centroid of the ring section G and the displacements of a point O, directly below G, on the middle surface of the shell:

$$\left.\begin{aligned} u_G &= u + e\beta \\ v_G &= v\,(1 + \frac{e}{a}) + \frac{e}{a}\, w_\theta \\ w_G &= w \\ \beta &= w_x \end{aligned}\right\} \qquad (5)$$

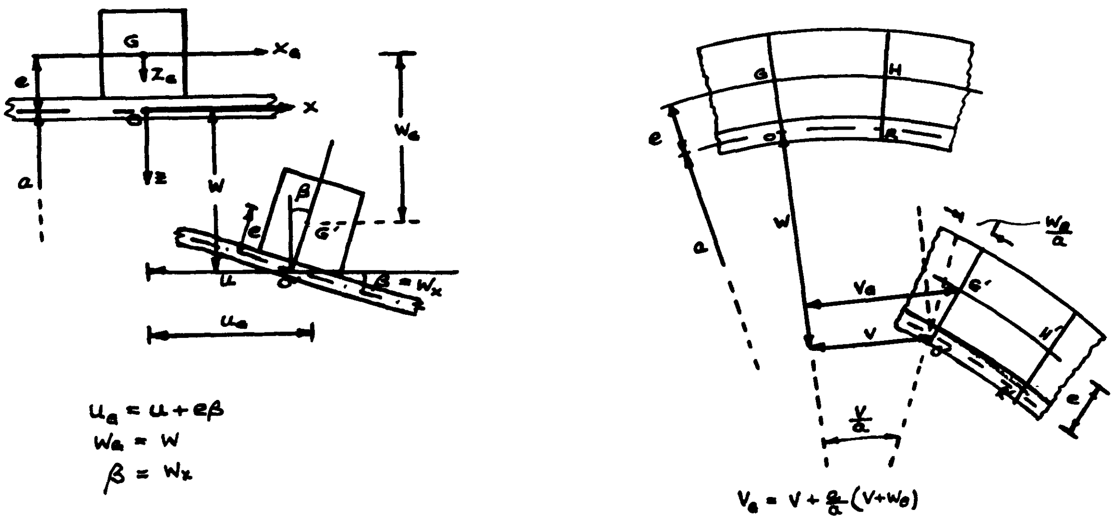

FIGURE 2. RELATION BETWEEN DISPLACEMENTS OF RING AND THOSE OF CYLINDER.

The relations (5) are correct to the first order only.

Substitution of equations (4) and (5) into (3) yields the following expression for the strain energy of one ring:

$$U_R = \frac{1}{(a+e)^3}\Bigg[\frac{EI_{XG}}{2}\int_0^{2\pi}(w + w_{\theta\theta})^2\, d\theta \;+$$

$$\frac{EI_{ZG}\cdot a^2}{2}\int_0^{2\pi}\left\{\left(\frac{u_{\theta\theta}}{a} + w_x\right) + \frac{e}{a}\,(w_x + w_{x\theta\theta})\right\}^2 d\theta \;+$$

$$\frac{EA_F(a+e)^2}{2}\int_0^{2\pi}\left\{(v_\theta - w) + \frac{e}{a}\,(v_\theta + w_{\theta\theta})\right\}^2 d\theta \;+$$

$$\frac{GC_T a^2}{2}\int_0^{2\pi}\left(\frac{u_\theta}{a} - w_{x\theta}\right)^2 d\theta\Bigg] \qquad (6)$$

The kinetic energy of one ring is:

$$T_R = \frac{\gamma}{2g}\left[A_F\int_0^{s}(\dot{u}_G^2 + \dot{v}_G^2 + \dot{w}_G^2)\, ds \;+\; I_P\int_0^{s}\dot{\beta}^2\, ds\right] \qquad \cdot \equiv \frac{d}{dt} \qquad (7)$$

Substitution of the relations (5) into (7) gives:

$$T_R = \frac{\gamma(a+e)}{2g}\left[A_F\int_0^{2\pi}\left\{(\dot{u} + e\dot{w}_x)^2 + \left(\dot{v} + \frac{e}{a}\,[\dot{v} + \dot{w}_\theta]\right)^2 + \dot{w}^2\right\} d\theta + I_P\int_0^{2\pi}\dot{w}_x^2\, d\theta\right] \qquad \cdot \equiv \frac{d}{dt} \qquad (8)$$

It is to be noted that in the calculation of U_R and T_R the displacement quantities must be inserted with the values that prevail at $x = rL_F$, where $r = 1, 2, \ldots\ldots N$.

The total strain energy U and the total kinetic energy T of the combined system is then:

$$\left.\begin{aligned} U &= U_s + \sum_{r=1}^{N}[U_R]_{x=rL_F} \\ T &= T_s + \sum_{r=1}^{N}[T_R]_{x=rL_F} \end{aligned}\right\} \qquad (9)$$

ASSUMED DISPLACEMENT PATTERN

The following pattern is assumed for the displacements:

$$\left.\begin{aligned} u &= A\cos n\theta\cos\lambda x \\ v &= \sin n\theta[B\sin\lambda x + C\{\sin(\lambda+\lambda_1)x + \sin(\lambda-\lambda_1)x\}] \\ w &= \cos n\theta[D\sin\lambda x + E\{\sin(\lambda+\lambda_1)x + \sin(\lambda-\lambda_1)x\}] \end{aligned}\right\} \qquad (10)$$

where:

$$\lambda = \frac{m\pi}{L}$$

$$\lambda_1 = \frac{2\pi}{L_F}$$

n = number of waves in the circumferential direction

m = number of half-waves in the axial direction

A, B, C, D and E, are simple harmonic functions of time only, with a common circular frequency ω.

At the supports ($x = 0, L$) this pattern does not permit radial or tangential displacements but does allow axial displacement and slope (i. e., w_x). Inter-ring deformation of the shell is permitted by the inclusion of the functions C and E. The nature of this latter deformation can be seen by setting $C = -\frac{C'}{2}$, $B = B' + C'$ i.e. it is represented by $\sin \frac{m\pi x}{L}\left(1 - \cos \frac{2\pi x}{L_F}\right)$.
Fig. 3 shows the pattern for w, given by equation (10), when $m = 1$.

It is, of course, possible to add another term for the u-displacement, similar to those added to the v- and w-displacements. However, it has been found for buckling problems that additional u-terms influence the numerical results in a very minor way [7] and, consequently, they have not been retained herein.

THE FREQUENCY EQUATION

For the problem being considered the Lagrange equations are:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} = -\frac{\partial U}{\partial q} \qquad (11)$$

where q is A, B, C, D or E.

Substitution of equations (9) and (10) into equation (11) yields the following matrix equation:

$$([a] - [b]\,\psi)\,[x] = 0 \qquad (12)$$

where

$$\psi = \frac{(1-\nu^2)\,\gamma}{Eg} \cdot \omega^2 \qquad (13)$$

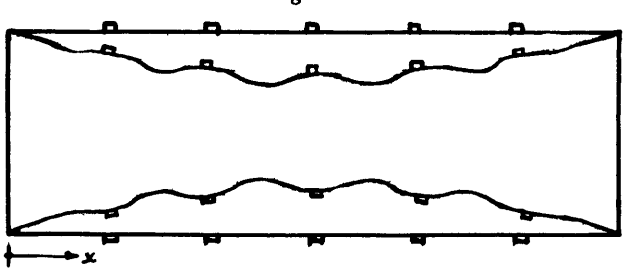

FIGURE 3. ASSUMED w-DISPLACEMENT PATTERN FOR $m = 1$.

and the elements of the a- and b-matrices are given by:

$$a_{11} = \lambda^2 + (1+k)\left(\frac{1-\nu}{2}\right)\frac{n^2}{a^2} + \delta n^4 (M + Q)$$

$$a_{12} = -\frac{n\lambda}{2a}(1+\nu)$$

$$a_{13} = 0$$

$$a_{14} = \frac{\nu\lambda}{a} + k\left[a\lambda^3 - \left(\frac{1-\nu}{2}\right)\frac{n^2\lambda}{a}\right] + \frac{1}{2}a_{15}$$

$$a_{15} = -2\delta a\lambda n^2 M\,[1 - (n^2-1)S]$$

$$a_{22} = \frac{n^2}{a^2} + \lambda^2\left(\frac{1-\nu}{2}\right)(1+3k) + \frac{1}{2}a_{23}$$

$$a_{23} = 2\alpha n^2 P\,(1+S)^2$$

$$a_{24} = -\left[\frac{n}{a^2} + 3k\left(\frac{1-\nu}{2}\right)n\lambda^2\right] + \frac{1}{2}a_{25}$$

$$a_{25} = -2\alpha n P\,[1 + (n^2+1)S + n^2S^2]$$

$$a_{33} = \frac{2n^2}{a^2} + (1-\nu)(\lambda^2+\lambda_1^2)(1+3k) + 4\alpha n^2 P\,(1+S)^2$$

$$a_{34} = a_{25}$$

$$a_{35} = -\left[\frac{2n}{a^2} + 3k(1-\nu)\,n(\lambda^2+\lambda_1^2)\right] + 2a_{25}$$

$$a_{44} = \frac{1}{a^2} + k\left[a^2\lambda^4 + \frac{(n^2-1)^2}{a^2} + 2\lambda^2(n^2-\nu)\right] + \frac{1}{2}a_{45}$$

$$a_{45} = 2\alpha[J + P\,(1+n^2S)^2] + 2\delta a^2\lambda^2[M\{1-(n^2-1)S\}^2 + n^2Q]$$

$$a_{55} = \frac{2}{a^2} + 2k\left[a^2(\lambda^4 + 6\lambda^2\lambda_1^2 + \lambda_1^4) + \frac{(n^2-1)^2}{a^2} + 2(\lambda^2+\lambda_1^2)(n^2-\nu)\right] + 2a_{45}$$

$$b_{11} = 1 + \delta F$$

$$b_{12} = b_{13} = 0$$

$$b_{14} = \delta a S\lambda F$$

$$b_{15} = 2b_{14}$$

$$b_{22} = 1 + \alpha(1+S)^2 F$$

(1

$$b_{23} = 2(b_{22} - 1)$$

$$b_{24} = -\alpha n S(1+S)F$$

$$b_{25} = 2b_{24}$$

$$b_{33} = 4b_{22} - 2$$

$$b_{34} = b_{25}$$

$$b_{35} = 4b_{24}$$

$$b_{44} = 1 + \delta\lambda^2(a^2S^2F + H) + \alpha(1 + n^2S^2)F$$

$$b_{45} = 2b_{44} - 2$$

$$b_{55} = 4b_{44} - 2$$

$$[x] = \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix}$$

$$k = \frac{b^2}{12a^2}$$

$$F = \frac{2(1+S)A_F}{bL}$$

$$H = \frac{2(1+S)I_P}{bL}$$

$$J = \frac{2(1-\nu^2)I_{XG}}{a^4bL(1+S)^3} \cdot (n^2-1)^2$$

$$M = \frac{2(1-\nu^2)I_{ZG}}{a^4bL(1+S)^3}$$

$$P = \frac{2(1-\nu^2)A_F}{a^2bL(1+S)}$$

$$Q = \frac{(1-\nu)C_T}{a^4bL(1+S)^3}$$

$$S = \frac{e}{a}$$

$$\alpha = \frac{N+1}{2} \quad \text{when } \frac{m}{N+1} \neq \text{integer}$$

$$= 0 \quad \text{when } \frac{m}{N+1} = \text{integer}$$

$$\delta = \frac{N-1}{2} \quad \text{when } \frac{m}{N+1} \neq \text{integer}$$

$$= N \quad \text{when } \frac{m}{N+1} = \text{integer}$$

(14) *contd.*

The frequency equation is then obtained from the determinantal relation:

$$|a - b\psi| = 0 \tag{15}$$

However, as usually only the smallest value of ψ (and hence ω) is required, it is more expedient to work with equation (12) itself. The minimum value of ψ is found, in the usual manner by computing $[a]^{-1}$, premultiplying equation (12) by $[a]^{-1}$, setting $\Omega = 1/\psi$, and then solving the resulting matrix equation for the largest value of Ω.

The numerical computation is, of course, greatly facilitated if the inter-ring deformation of the shell is neglected. The matrix equation for this case is obtained from equation (12) by omitting all matrix elements with subscript 3 or 5 and setting $C = E = 0$. Expanding the determinant, and linearizing with respect to ψ, yields the following equation:

$$\psi = \frac{N}{D} \tag{16}$$

where:

$$N = a_{11}a_{22}a_{44} + 2a_{12}a_{14}a_{24} - a_{11}a_{24}^2 - a_{22}a_{14}^2 - a_{44}a_{12}^2$$

$$D = a_{11}a_{22}b_{44} + a_{11}a_{44}b_{22} + a_{22}a_{44}b_{11} - a_{24}^2b_{11} - a_{14}^2b_{22} - a_{12}^2b_{44} - 2a_{11}a_{24}b_{24} + 2a_{12}a_{14}b_{24} + 2a_{12}a_{24}b_{14} - 2a_{14}a_{22}b_{14} \tag{17}$$

As it is intended to utilize Bleich's method in the numerical example which follows, a brief resume of his procedure is given below, due to the fact that it [3] is not generally available. He gives, for the frequency of in-vacuo vibration of stiffened cylinders,

$$\omega^2 = \omega_S^2 + \omega_I^2 \tag{18}$$

where: ω_S = frequency of a shell of

$$\text{mass } \bar{m}_S = \frac{\alpha_I}{\bar{\alpha}_I} \cdot m_S + \frac{m_I}{\bar{\alpha}_I \cdot L_F}$$

ω_I = frequency of a ring of

mass $m_r = \overline{\alpha}_I \cdot L_F \cdot \overline{m}_S$

m_S = mass of shell per unit of area

m_I = mass of ring frame per unit length

L_F = frame spacing.

The parameters α_I and $\overline{\alpha}_I$ are defined by the relations:

$$\overline{\alpha_I} = [1 + \frac{e}{a}(n^2 - 1)]^2$$

$$\alpha_I = \frac{2\int_0^{\frac{L_F}{2}} \left[\left\{1 + \frac{e}{a}(n^2-1) - \frac{e}{a}(n^2-1)\varphi(x)\right\}^2 + \frac{1}{n^2}\right] dx}{L_F\,(1 + \frac{1}{n^2})}$$

$$\varphi(x) = e^{-\zeta x}[\cos \zeta x + \sin \zeta x]$$

$$\zeta = \sqrt[4]{\frac{3(1-\nu^2)}{a^2 b^2}} \tag{19}$$

For the case when e/a is very small and $4 < \zeta L_F < 5$ Bleich gives a simplified expression for α_I. As, however, the second criterion is not quite met in the numerical example which follows, the complete expression for α_I given above was used in the calculations. The ring frequencies ω_I are calculated from the equation:

$$\omega_I^2 = \frac{n^2(n^2-1)^2}{n^2+1} \cdot \frac{1}{a^4} \cdot \frac{EI}{m_r} \qquad (20$$

The I in equation (20) refers to the second moment of inertia of the ring frame section plus a length of shell plating equal to $2/\zeta$.

The shell frequencies ω_S are calculated from the expression:

$$\omega_S^2 = \frac{K^2}{a^2} \cdot \frac{Eb}{2(1-\nu)\overline{m}_S} \cdot \left[1 + \frac{\overline{K}^2}{K^2} \cdot \frac{b^2}{a^2}\right] \tag{21}$$

The factors K and $\frac{\overline{K}}{K}^2$ are tabulated in reference 8.

NUMERICAL EXAMPLE

Calculations were performed using equations (12), (16) and (18) for a stiffened cylindrical shell with the following characteristics:

L = 18.540 in., b = 0.047 in., a = 4.082 in.,
L_F = 1.236 in., N = 14, E = 30×10^6 lb/sq. in.
ν = 0.3, m = 1.
b = width of ring stiffener = 0.086 in.
d = depth of ring stiffener = 0.1145 in., 0.2290 in., 0.3435 in.

The results of the calculations are shown in Table 1.

TABLE I

FREQUENCIES OF VIBRATION OF THE STIFFENED CYLINDERS, IN RAD./SEC. CALCULATED BY THE DIFFERENT METHODS

n	d = 0.3435″			d = 0.2290″			d = 0.1145″		
2	4750	4855	5260	4425	4450	4790	4450	4470	4600
3	8590	9500	8815		6325	6155	3580	3655	3885
4	16310	18010	15005		11790	10330	5675	5950	5895
5	23685	25570	21750		19020	15365	8985	9510	9005
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)

N. B. Columns (a) were calculated using equation (12)
Columns (b) were calculated using equation (16)
Columns (c) were calculated using equation (18)

The frequencies of the unstiffened shell were also calculated and are tabulated in Table II.

TABLE II

FREQUENCIES OF VIBRATION OF THE UNSTIFFENED CYLINDER IN RAD./SEC. CALCULATED BY THE DIFFERENT METHODS

n	Equation (16)	Equation (21)	Arnold & Warburton
2	4670	4690	4705
3	2735	2730	2730
4	2935	2930	2945
5	4230	4230	4230

Figures 4 and 5 show graphically how the frequency of vibration varies with the depth of the ring stiffener. The values plotted are those given in columns (c) of Table I.

DISCUSSION OF RESULTS AND CONCLUSIONS

Reference to columns (c) (Bleich's method) and (a) in Table I shows that the values therein differ by ± 10% maximum. The same can also be said of the values in columns (b) (inter-ring deformation of the shell neglected) and (a). The values for the unstiffened cylinder listed in Table II check quite closely, as would be expected due to the similarity of the energy expressions used for the cylindrical shell. Figure 4 shows the anomaly that has been previously pointed out by Arnold and Warburton for unstiffened cylinders and by Bleich for stiffened cylinders, i.e., an initial decrease in the frequency of vibration with an increase in the number of circumferential lobes. This anomaly only occurs for certain cylinder

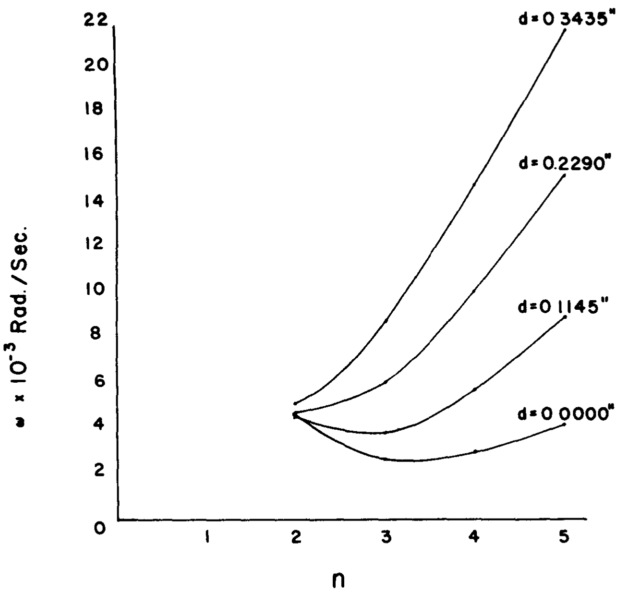

FIGURE 4. VARIATION OF FREQUENCY WITH *n*, THE NUMBER OF CIRCUMFERENTIAL WAVES, FOR DIFFERENT DEPTHS OF STIFFENERS.

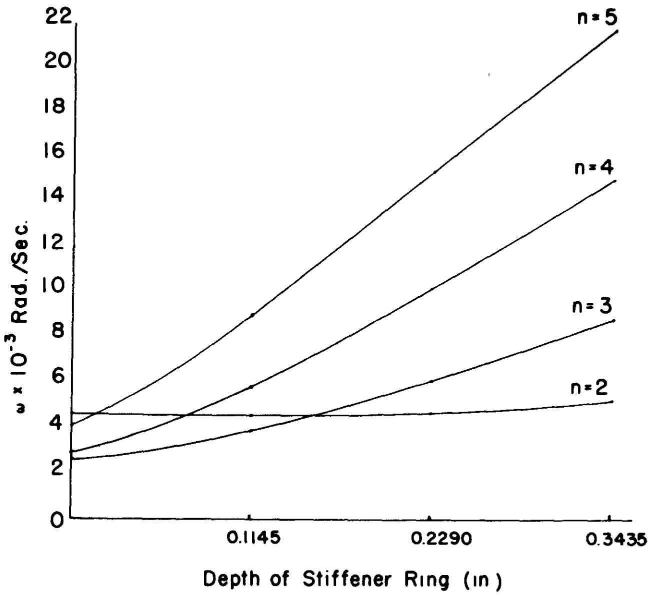

FIGURE 5. VARIATION OF FREQUENCY WITH DEPTH OF STIFFENER FOR DIFFERENT VALUES OF *n*.

geometries, however. Figure 5 also indicates that, for the particular cylinder considered, a decrease in frequency is obtained in the $n = 2$ mode by the addition of stiffeners of small depth.

In conclusion, it would appear that Bleich's method yields values for the vibration frequencies which should be accurate enough for general engineering purposes. The analysis presented in this paper, however, gives more accurate results and can also be applied to cylindrical shells with small length-radius ratios.

ACKNOWLEDGMENT

The author would like to express his appreciation to Messrs. R. Bart and T. E. Reynolds, of the David Taylor Model Basin staff, for checking the mathematical analysis occurring in this paper and for assistance in performing the numerical calculations.

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ON THE IN-VACUO VIBRATIONS OF SIMPLY SUPPORTED, RING-STIFFENED CYLINDRICAL SHELLS, by Gerard D. Galletly. Feb 1958. p. 1-9. diagrs., refs. (Reprinted from the Proceedings of the Second U.S. National Congress of Applied Mechanics held at University of Michigan, Ann Arbor, Michigan, June 14-18, 1954) UNCLASSIFIED

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