NAVY DEPARTMENT
THE DAVID W. TAYLOR MODEL BASIN
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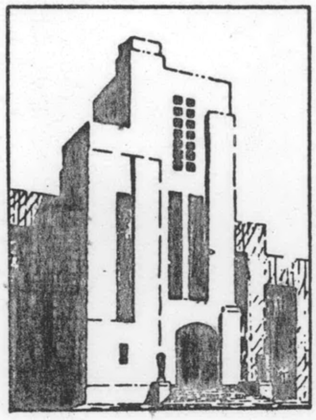
SKIN FRICTION FORMULATIONS

by

Paul S. Granville

Paper to be presented to
The American Towing Tank Conference, 1956



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Report 1063

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LINEARIZED RESISTANCE DIAGRAM

A linearized diagram of total coefficient of resistance C_t in terms of coefficient of flat-plate resistance C_f is proposed for studying the viscous resistance of ship hulls. By the basic Froude hypothesis

$$C_t(R,F) = C_v(R) + C_w(F)$$

where C_v is the coefficient of viscous resistance, C_w the coefficient of wave-making resistance, R the Reynolds number and F the Froude number. C_v may be related to C_f at the same Reynolds number. The difference, $C_v - C_f$, known as the coefficient of form resistance is usually assumed independent of Reynolds number and in recent proposals $C_v - C_f$ is made directly proportional to C_f. A more general relation due to Landweber[1] is

$$C_v - C_f = n\,C_f + j$$

where n and j are independent of Reynolds number and only a function of the ship geometry.

C_t can be written then

$$C_t = (1+n)\,C_f + j + C_w$$

This permits a linearized diagram when C_t is plotted against C_f for a family of geometrically similar models as shown in the Figure. A series of parallel straight lines results for contours of constant Froude number, the slope being $1 + n$ and the intercepts $j + C_w$. (C_t is actually plotted against $-C_f$ to retain the usual orientation of model results to the left of full-scale results.)

The linearized diagram is very useful for fairing experimental points of families of geometrically similar models. For the case of a single model with

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[1] References are listed on page 5.

j assumed zero, the linearized diagram is very convenient for determining C_v at the condition $F \rightarrow 0$ ($C_t \rightarrow C_v$). This just requires that a straight line be drawn from the point $C_f = 0$.

For specifying the results of different sized models of a family it is necessary to use a non-dimensional length Λ where

$$\Lambda = \frac{R}{F} = \frac{L^{\frac{3}{2}} g^{\frac{1}{2}}}{\nu}$$

as shown in the Figure. Here L is the vessel length, g the acceleration due to gravity and ν the kinematic viscosity of the water. Then

$$C_t = f(R, \Lambda)$$

The linearized diagram is in principle the same as that advocated by Telfer for many years. Instead of C_f as the linearizing function, Telfer uses $R^{-\frac{1}{3}}$ which has long been a controversial procedure.

GENERAL RESISTANCE FORMULA FOR FLAT PLATES

The determination of the frictional resistance of flat plates can be formulated from the similarity laws of boundary-layer flow: the inner and outer laws of the mean-velocity profile. It is possible to derive a general relation in series form

$$\frac{A}{\sqrt{2}} \ln R = \frac{1}{\sqrt{C_f}} - \frac{A}{\sqrt{2}} \ln C_f - \frac{A}{2}\left(\frac{A}{2} + \frac{\gamma}{\beta}\right)\sqrt{C_f} + \cdots \quad + \text{const.}$$

where A, β and γ are boundary-layer constants. Terms of higher order than $\sqrt{C_f}$ are negligible. The term involving $\sqrt{C_f}$ is small and can be linearized in terms of $\frac{1}{\sqrt{C_f}}$ or

$$\sqrt{C_f} = \text{const.} + \frac{\text{const.}}{\sqrt{C_f}}$$

The result is

$$\frac{1}{\sqrt{C_f}} = A_1 \log(C_f R) + B_1$$

where A_1 and B_1 are constants. This is the Kármán-Schoenherr formula which was originally derived by Schoenherr[2] by inserting a power-law relation into von Kármán's asymptotic formula for local skin friction.

If arbitrarily one-half of the term containing $\ln C_f$ is also linearized or

$$\frac{1}{2}\ln C_f = \text{const.} + \frac{\text{const.}}{\sqrt{C_f}}$$

and the other half of the term retained, the result is the Lap-Troost formula[3] for flat plates

$$\frac{1}{\sqrt{C_f}} = A_2 \log(\sqrt{C_f}\, R) + B_2$$

where A_2 and B_2 are constants. This is a more rational derivation than the original one of Lap and Troost which involved various arbitrary assumptions.

If the whole term containing $\ln C_f$ is linearized or

$$\ln C_f = \text{const.} + \frac{\text{const.}}{\sqrt{C_f}}$$

then the interpolation formula results

$$\frac{1}{\sqrt{C_f}} = A_3 \log R + B_3$$

or

$$C_f = \frac{\frac{1}{A_3^2}}{\left(\log R + \frac{B_3}{A_3}\right)^2}$$

This relation which has the advantage of expressing C_f explicitly as a function of R has been obtained empirically by a number of investigators: Hama[4] to fit the Schoenherr formula, Troost[5] to fit Schultz-Grunows' data and Hughes[6] to fit his own test data.

If the term containing ln C_f is retained and the $\frac{1}{\sqrt{C_f}}$ – term linearized with respect to ln C_f instead or

$$\frac{1}{\sqrt{C_f}} = \text{const.} + \text{const.} \ln C_f$$

the power-law formula then results

$$C_f = \frac{B_4}{R^{A_4}}$$

where A_4 and B_4 are constants.

Empirically all the formulas with the exception of the power-law formula have been found to be good fits to resistance data for the practical range of Reynolds numbers.

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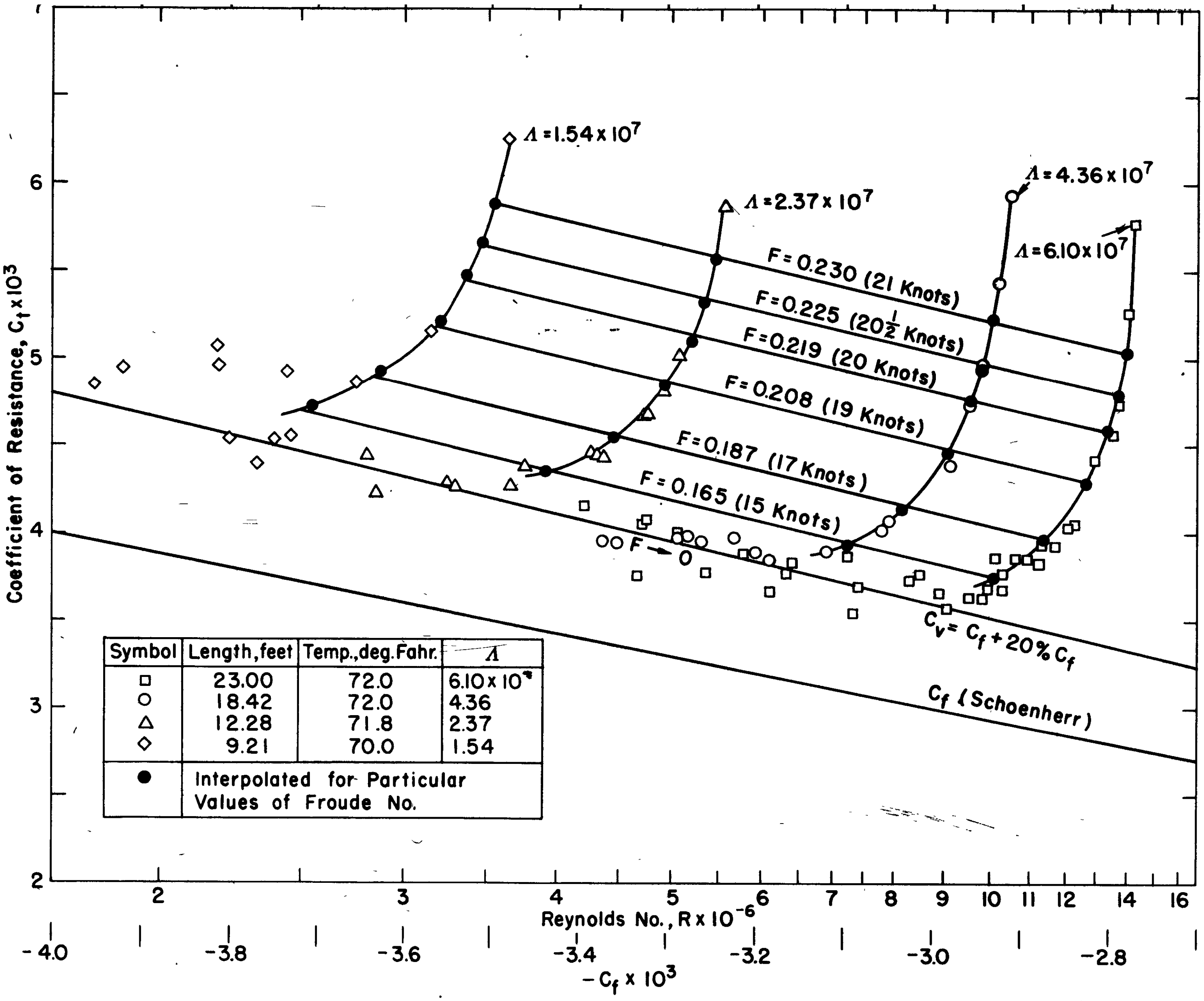

Linearized resistance diagram for family of models of the supertanker TINA ONASSIS

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