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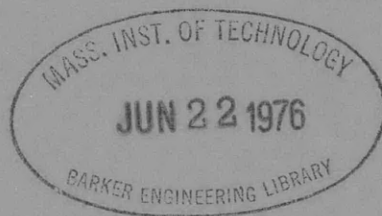
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APPLIED
MATHEMATICS

SPLASHES AS SOURCES OF SOUND IN LIQUIDS

by

G.J. Franz



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Splashes as Sources of Sound in Liquids*

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(Received July 14, 1958)

The mechanisms of sound production by the splashes made by the gas-to-liquid entry of objects are discussed. The sound from the splash is considered to be associated with acoustic multipoles of all orders, the main ones being simple sources, dipoles, and quadrupoles. The orders of the multipoles that predominate during the various phases of the splash are estimated from the flow and boundary conditions. The sounds radiated into the water by the low-velocity vertical entry of single water droplets, sprays of water droplets, and various other objects, such as spheres, cones, and wedges, have been measured and found to have the characteristics of acoustic dipoles. The extensive experimental data on the spectrum of the underwater sound from the splashes of droplets and sprays and the scaling laws for dipoles are used to estimate the spectrum levels of the underwater sound from the splashing of rain on the surface of a sea in terms of the rate of rainfall.

I. INTRODUCTION

THE splash made by the gas-to-liquid entry of an object radiates sound both into the gas and into the liquid. Probably the most common example of this is the sound produced by the splashes of water droplets on a water surface. The appearance of the splashes seems to have received more attention in the literature than the sounds produced by the splashes. Worthington,¹ for instance, made an excellent photographic study of the splashes of droplets and spheres in about 1900. His early studies were repeated and extended with more modern equipment by Richardson,² May,³ Hobbs,⁴ and many others. Their studies were concerned mainly with the scaling of cavities formed in the liquid and with the determination of the drag of the object entering the free surface.

Since about 1920, general observations of the sounds produced by single splashes have been reported by a number of authors. The early investigators studied the air-borne sound produced by the splashes of water droplets and spheres falling on water. Since the sounds in air were observed to have a musical character, they were attributed to the vibrations of the cavity formed behind the object⁵ or to entrained bubbles.⁶ The air-borne sounds from the splashes made by water droplets were also measured⁷ and recorded.^{8,9} These sounds were found to vary unsystematically with variations in height of fall of the droplets.

About 1933, Minnaert¹⁰ explained the generation of

underwater sound by bubbles, and suggested that the sounds of running water are associated with these resonant bubbles. His main contribution was the determination of the resonant frequency of a bubble in volume pulsation.

In the course of their study on flesh wounds made by high-speed missiles, McMillen^{11,12} and Harvey¹³ obtained some excellent shadowgrams of the shock waves produced by the high-speed vertical entry of spheres into water. The shock waves radiated into the water from the point of impact are strongest directly ahead of the sphere and weakest to the side, roughly according to a cosine law. A large number of secondary waves, which seem to originate from the sphere as it penetrates the water, appear behind the impact wave. These secondary waves were attributed to spheroidal vibrations of the sphere excited at impact.

Richardson¹⁴ observed both the air-borne and water-borne sound from the splashes of spheres entering at low velocities, and correlated the sound with motion pictures of the cavity formed on entry. He concluded that most of the sound was produced by closed resonant cavities, or bubbles, behind the spheres. A rather rapidly damped high-frequency underwater sound of low amplitude was detected immediately after the instant of impact. This he attributed to natural vibrations of the solid sphere excited on impact. He also gives a good list of possible splash-noise sources.

The underwater sound from the impact of rain on a water surface has been observed in a number of instances.^{15,16} Unfortunately, no quantitative measurement of the rate of rainfall was taken. In general the

* Research undertaken as part of the Fundamental Hydro-mechanics Research Program (NS 715-102).

¹ A. M. Worthington, *A Study of Splashes* (Longman's Green, and Company, Inc., New York, 1908).

² E. G. Richardson, *Proc. Phys. Soc. (London)* **61**, 352 (1948).

³ A. May *et al.*, *J. Appl. Phys.* **19**, 1109 (1948).

⁴ V. Hobbs *et al.*, "Oblique entry of spheres into water," *Natl. Bur. Standards Rept. No. 2788*, August 16, 1954.

⁵ A. Mallock, *Proc. Roy. Soc. (London)* **A95**, 138 (1919).

⁶ W. Bragg, *The World of Sound* (G. Bell and Sons, Ltd., London, 1920).

⁷ A. T. Jones, *Science* **52**, 295 (1920).

⁸ C. V. Raman and A. Dey, *Phil. Mag.* **39**, 145 (1920).

⁹ A. L. Narayan, *Phil. Mag.* **42**, 773 (1921).

¹⁰ M. Minnaert, *Phil. Mag.* **16**, 235 (1933).

¹¹ J. H. McMillen, *Phys. Rev.* **68**, 198 (1945).

¹² J. H. McMillen and E. N. Harvey, *J. Appl. Phys.* **17**, 541 (1946).

¹³ E. N. Harvey *et al.*, *War Med.* **8**, 91 (1945).

¹⁴ E. G. Richardson, *Proc. Phys. Soc. (London)* **68**, 541 (1955).

¹⁵ V. O. Knudsen *et al.*, "Survey of underwater sound," *Rept. No. 3*, "Ambient Noise" Office of Scientific Research and Development, No. 4333, September 26, 1944.

¹⁶ Heindsmann, Smith, and Arneson, *J. Acoust. Soc. Am.* **27**, 378 (1955).

sound spectra are relatively flat, with broad peaks near the middle of the audible frequency range.

Surface disturbances are also an important source of ambient noise in the sea. This noise has a well-known correlation with sea state and is considered to be caused mainly by waves and white caps. Present opinion is that ambient noise in the open sea in the frequency range from 0.1 to 50 kc is predominantly from the sea surface. Some of the directional properties of deep-water ambient noise were discussed by Urick.¹⁷ He obtained theoretical expressions for the variation of the ambient noise with depth, assuming absorption and different directional properties for the noise from the sea surface but assuming nothing about the generating mechanism. Available experimental data obtained with directional hydrophones are not conclusive in determining the directional properties of this sound.

Splashing around ships under way also makes noise. Folsom, Howe, and O'Brien,¹⁸ for instance, observed a sharp increase in the sound intensity in the water as they increased the speed of a model hull to the point that the bow wave broke.

The present study produced information on (1) the instantaneous sound pressure and the spectrum of the sound energy radiated into the water by the vertical impact of single water droplets, (2) the root-mean-square sound pressure and the spectrum of the sound energy radiated into the water by the vertical impact of a spray of water droplets, and (3) the instantaneous sound pressure radiated into the water by the vertical impact of spheres, cones, and wedges. The principal sources of sound in the water from the splashes made by the air-to-water entry of these objects at low velocities may be summarized as (1) impact and passage of the object through the free surface, (2) natural vibrations of the object if it has rigidity, and (3) volume pulsations of any air-filled cavities formed in the water.

II. MECHANISMS OF SOUND PRODUCTION BY SPLASHES

1. General Considerations

The splash made by the gas-to-liquid entry of an object is a complex phenomenon. Most of the motions are too rapid to follow with the naked eye, but with the aid of high-speed motion pictures most phases of the splash may be examined at leisure. An examination of motion pictures of the splashes produced by the air-to-water entry of objects of various sizes, shapes, and velocities reveals a number of possible sources of sound.

Air-borne sound radiated directly into the air may originate from the transient passage of the object

through the free surface, from vibrations of the object, from the spray thrown up from the splash, from secondary splashes of water droplets, from resonant vibrations of cavities open to the atmosphere, and from bubbles breaking on the free surface. Sound may also be transmitted into the air, with attenuation, from sources in the water. Some of these possible sources are the transient introduction of the object into the water, vibrations of the object, and oscillations of air bubbles.

Possible direct sources of underwater sound are the transient introduction of the object into the water, vibrations of the body, secondary splashes of water droplets that are thrown up by the entry, oscillations of air bubbles and of cavities open to the atmosphere, and secondary slaps of the afterbody against the side of the cavity behind the object. Sounds originating in the air, mainly from vibrations of the object, may also be transmitted, with attenuation, into the water.

Most of these possible sources can be grouped into three general phases: (1) the flow establishment phase, (2) the body vibration phase, and (3) the cavity and bubble phase. Before examining the various aspects of these phases in more detail, a few useful concepts will be given.

The sound radiated by the gas-to-liquid entry of an object is greatly influenced by the presence of the free surface and is, of necessity, a transient. For free-field conditions, the theory of sound radiation by the natural vibrations of bodies and by the pulsations of resonant cavities¹⁹ is well understood. This theory can be modified to take account of the free surface. If, for simplicity, the surface is assumed to remain plane during the splash, its influence on the sound pressure can be approximated by use of the method of images. Thus, for calculating the sound pressure in the gas associated with a source above the free surface, the free surface is replaced by an image source of the same polarity; for calculating the sound pressure in the liquid associated with a source below the free surface, the free surface is replaced by an image source of opposite polarity. In effect, if a multipole source of some order is well within a wavelength of the free surface, the presence of the free surface either doubles the strength of the multipole or increases the order of the multipole by one. This makes it convenient to consider the sound from the splash to be generated by a sum of acoustic multipoles of all orders, and to use a knowledge of the flow and boundary conditions to estimate what order and orientation of multipole would predominate in the radiated sound field in the gas and in the liquid during the various phases of the splash.

Three of the orders of acoustic multipoles that may be expected to be associated with splashes are (1) the simple source, (2) the dipole source having a vertical or horizontal axis, and (3) the lateral quadrupole source having vertical and horizontal axes. If the sources are well within a wavelength of the free surface, certain

¹⁷ R. J. Urick, "Some directional properties of deep water ambient noise," U. S. Naval Research Laboratory Rept. No. 3796, January 16, 1951.

¹⁸ Folsom, Howe, and O'Brien, "Notes on hydraulic noise," Office of Scientific Research and Development Rept. No. 949, June 11, 1942.

¹⁹ M. Strasberg, *J. Acoust. Soc. Am.* **28**, 20 (1956).

multipoles such as simple sources in the water, vertical dipoles in the air, and horizontal dipoles in the water would not be expected to contribute to the sound field because of symmetry conditions introduced by the presence of the free surface.

The sound pressure and the sound energy radiated by the multipole sources are determined by the respective source strengths which in turn are determined by the flow and boundary conditions. The general expressions for the instantaneous sound pressure p in terms of the time derivatives of the source strengths μ_s , μ_d , and μ_q , of the simple source, dipole, and lateral quadrupole, respectively, can be obtained by taking the density ρ of the fluid times the time derivative of the respective velocity potentials. These expressions, which unfortunately must be gleaned from a number of references,^{20,21} are†

$$p = \rho(\partial\mu_s/\partial t)/r, \quad (1)$$

$$p = \rho \cos\theta [(\partial\mu_d/\partial t)/r^2 + (\partial^2\mu_d/\partial t^2)/rc], \quad (2)$$

$$p = \rho \cos\theta \sin\theta \cos\phi \left[\frac{3}{r^3} \frac{\partial\mu_q}{\partial t} + \frac{3}{r^2c} \frac{\partial^2\mu_q}{\partial t^2} + \frac{1}{rc^2} \frac{\partial^3\mu_q}{\partial t^3} \right], \quad (3)$$

where the source strengths are functions of retarded time $t-r/c$; c is the velocity of sound in the fluid; and r , θ , and ϕ are spherical polar coordinates with origin at the multipole. The simple source strength is equal to $1/4\pi$ times the time rate of introducing volume into the fluid. The dipole source strength is then equal to this simple source strength times the distance between the simple source and sink that constitute the dipole. The lateral quadrupole strength, in turn, is obtained by multiplying this dipole strength by the distance between the two dipoles that constitute the lateral quadrupole. In the dipole case, the angle θ is measured from the axis of the dipole. In the lateral quadrupole case, the angle θ is measured from one of the axes of the quadrupole and the angle ϕ is measured from the other axis of the quadrupole in a plane that is perpendicular to the axis from which the angle θ is measured. In each case the term that is proportional to $1/r$ is the far-field or radiation-field term and the others are near-field or "induction-field" terms. The sound energy radiated into the gas or into the liquid is obtained from these transient sound pressures by integrating $p^2/\rho c$ over time and over the surface area of a large hemispherical surface surrounding the sound sources. The near-field terms do not contribute to the radiated sound energy but are associated with unradiated kinetic energy of the fluid.^{22,23}

²⁰ M. Abraham and R. Becker, *Classical Electricity and Magnetism* (Blackie and Son, Ltd., London, 1932), p. 223.

²¹ M. J. Lighthill, *Proc. Roy. Soc. (London)* **211**, 564 (1952).

† The sound pressure of the dipole component is also related to the more conventionally measured vertical drag force F on the entering body by the expression $p = (1/2\pi)\cos\theta[F/r^2 + (\partial F/\partial t)/rc]$.

²² R. H. Randall, *An Introduction to Acoustics* (Addison-Wesley Press, Inc., Cambridge, 1951), p. 107.

²³ H. H. Skilling, *Fundamentals of Electric Waves* (John Wiley & Sons, Inc., New York, 1942), p. 138.

Estimates of the strengths of the predominant multipoles can be made in some cases. These will be made as part of the discussion of the three general phases of the splash produced by the air-to-water entry of an object. The discussion, of course, also holds for other gases and liquids.

2. Flow Establishment Phase

Any supersonic flow of water in the vicinity of the initial contact point of blunt bodies entering the water may produce shock waves in the water and in the air. The duration of this supersonic flow is very short, being approximately equal to the time during which the rate of growth of the wetted area is supersonic ($aV/2c^2$ for a sphere of radius a and vertical impact velocity V).²⁴ Since, for this supersonic flow regime, the sound radiated into the water has a wavelength that is short compared to the characteristic dimension of the wetted portion of the body, the entering body should act as a simple sound source in the water. In other words, out-of-phase reflections from the free surface are not effective in cancelling the simple source sound radiated into the water by the wetted portion of the body. The source strength might be taken to be approximately equal to that of a piston having the size of the wetted area and the velocity of the entering body (μ_s equals the cross-sectional area of the body at the waterline times $V/2\pi$). If spray, rise of the free surface, and attenuation are all neglected and constant entry velocity is assumed, the instantaneous sound pressure at points that are many sphere diameters directly below a sphere entering vertically would then be given, for a very short time after the arrival of the pulse, approximately by

$$p = (\rho a V^2/r)[1 - (t-r/c)V/a], \quad (4)$$

where t is taken to be zero at the instant the sphere first contacts the water.

As shown by McMillen,^{11,12} shock waves are also produced in the air by the high-speed water entry of objects. The shock waves, which seem to be produced in the air by spray droplets moving at supersonic velocities, might be determined from the size, shape, and velocity of the spray droplets. The secondary splashes of these droplets may be treated as new splashes, as far as sound radiation is concerned.

When the underwater sound radiated during the subsonic flow regime of the flow establishment phase is considered, the presence of the free surface can no longer be neglected. Considering a vertically entering object to be a distributed source of fluid just below the free surface, it is apparent that if this source is separated from its image in the free surface by much less than a wavelength, the resultant simple source strength is negligible and the combination constitutes a dipole source of sound with a vertical axis.

²⁴ P. Dergarabedian, "Compressibility effects during water entry," NAVORD Rept. 3523, June 23, 1955.

To a first approximation, the rise of the free surface may be neglected and the dipole strength may be taken as that of a fully submerged, rapidly growing, rigid body moving with a vertical velocity V in a fluid fixed at infinity and consisting of the submerged portion of the real body and the image of this submerged portion in the free surface.²⁵ In turn, this pseudobody moving with a vertical velocity V may be replaced by a fixed pseudobody having a surface distribution of simple sources given at each incremental area on the surface of the pseudobody by $1/4\pi$ times the dot product of the velocity and the outwardly directed incremental area vector.²⁶ The pseudobody itself may also be replaced by the images in the pseudobody of the distribution of simple sources on the surface of the pseudobody.²⁷ For purposes of estimating the sound radiated into the water, the approximate dipole strength of the pseudobody having a vertical velocity V may then be obtained by integrating over the submerged portion of the real body the incremental dipole strengths obtained from the product of (1) the incremental simple source $VdA/4\pi$ on the submerged portion of the real body as well as the image of this incremental simple source in the pseudobody and (2) the distances from these simple sources to their negative images in the free surface. The incremental area dA is the horizontal projection of the incremental area on the submerged surface of the body. If the images in the pseudobody of the sources on the surface of the pseudobody are not known, the effect of the presence of the pseudobody on the dipole strength might be approximated by using some average distance around the pseudobody for the distance between the source $VdA/4\pi$ and its image in the free surface, instead of using the distance through the pseudobody. In effect, the presence of the pseudobody increases the effective distance between pairs of sources and sinks.

The velocity potential obtained from this dipole strength, of course, does not accurately describe the field in the vicinity of the entering body but describes the field of an equivalent sphere having the same dipole strength in this region. For this reason, such a velocity potential cannot be used to get the pressure distribution on the body or the velocity distribution in the immediate vicinity of the body, but it might give a fair estimate of the induced mass of the entering body.

Since the image system in the pseudobody of the source distribution on the surface of the pseudobody is not known in general,²⁵ a further approximation is made by neglecting the images in the pseudobody of the sources on the surface of the pseudobody. For this case,

the dipole strength is then given approximately by

$$\mu_d(t) \cong (1/2\pi) \int_0^z (z-s)(dz/dt)(dA/ds)ds, \quad (5)$$

where $z=z(t)$ is the depth of penetration of the body into the liquid and $A=A(s)$ is the horizontal cross-sectional area of the body as a function of the vertical distance s from the nose of the body.[‡]

The seriousness of neglecting the images in the pseudobody of the sources on the surface of the pseudobody can be estimated from the known error introduced by neglecting these images in certain limiting bodies. For long slender pseudobodies having cylindrical midsections, neglecting the images in the pseudobody introduces negligible error. The image system in a rigid sphere of a source on the surface of the sphere consists of an equal positive source on the inner surface of the sphere and an equal negative source distributed between the point on the sphere and the center of the sphere.²⁷ The image systems in the sphere then contribute a dipole strength of the same sign and equal to one-half the strength of the dipole contributed by the surface distribution itself. The image in an infinite rigid plane surface of a source on this surface consists of an equal positive image on the inner surface of the rigid surface. The image system in the surface of very blunt bodies might then be expected to contribute a dipole strength of the same sign and essentially equal to the strength of the dipole contributed by the surface distribution itself. Unfortunately, this is not a good approximation. A better approximation is obtained by considering the sources on the lower part of the pseudobody to be effectively separated from the sinks on the upper part of the pseudobody by a distance equal to the distance around the pseudobody rather than by the distance through the pseudobody. Thus a fair approximation to the dipole strength of blunt pseudobodies is obtained by multiplying the results of Eq. (5) by a "bluntness factor," such as the ratio b of the average distance from the nose to the tail around the surface of the pseudobody to the distance from the nose to the tail through the pseudobody. The dipole strengths given by Eq. (5) should then be multiplied roughly by the following factors: 1 for long slender pseudobodies, $\frac{3}{2}$ for near-spherical pseudobodies, and b for blunt pseudobodies.

In general, the shape of the pseudobody changes as the real body enters the free surface. In the case of a sphere, for instance, the pseudobody is initially blunt, then becomes spherical at half-submergence, and, if a cavity forms behind the entering sphere, eventually can be considered long and slender.[§] In the case of a cone,

[‡] In the case of the vertical entry of a sphere of radius a , for example, $A = \pi(2as - s^2)$. Therefore, $\mu_d(t) = z^2(\partial z/\partial t)(3a-z)/6$ and $\partial^2[\mu_d(t)]/\partial t^2 = (\partial z/\partial t)^3(a-z) + 3z(\partial z/\partial t)(\partial^2 z/\partial t^2)(2a-z)/2 + z(\partial^3 z/\partial t^3)(3a-z)/6$.

[§] If no cavity is formed, the pseudobody eventually becomes just two spheres.

²⁵ M. Schiffman and D. Spencer, "The force of impact on a sphere striking a water surface," Office of Scientific Research and Development Rept. 42.1R, February 1945.

²⁶ H. Föttinger, *Jahrb. schiffbautech. Ges.* **25**, 295 (1924) (David Taylor Model Basin translation 48, p. 16).

²⁷ L. M. Milne-Thomson, *Theoretical Hydrodynamics* (The Macmillan Company, New York, 1950), pp. 207, 417.

the pseudobody has the same shape but grows in size as the cone enters, point first. The dipole strength given by Eq. (5) should then be multiplied by a constant factor that depends on the bluntness of the cone. This consideration also holds for a wedge if end effects are neglected. It should be emphasized that since the radiated sound pressure is proportional to the second time derivative of the dipole strength, small errors in the time dependence of the dipole strength may become large errors in the radiated sound pressure.

Bodies entering a free surface at angles other than vertical would be expected to produce additional quadrupole radiation in the water, especially at high entry velocities. The additional quadrupole strength attributable to the horizontal component of the velocity of the body may be approximated as that of the submerged portion of the body moving parallel to the free surface and an image body in the free surface moving in the opposite direction. At large distances, the sound pressure due to this combination would have the characteristics of a lateral quadrupole with one axis vertical and the other horizontal and in the direction of the horizontal component of the impact velocity.

For purposes of estimating the sound pressure, the approximate lateral quadrupole strength of this pseudobody may be obtained by combining the incremental quadrupole strengths obtained from the product of (1) the incremental simple source strength $V_h dA_h/4\pi$ on the forward part of the submerged portion of the real body, (2) the distance from this incremental simple source to its image in the free surface, and (3) the distance from this incremental simple source on the forward part of the body to the incremental simple source directly behind it on the rear part of the body. The velocity V_h is the horizontal component of the impact velocity, and dA_h is the projection of the incremental area on the surface of the body on a vertical plane perpendicular to the horizontal component of the impact velocity. For simplicity, images in the pseudobody have been neglected. The strength of the quadrupole changes mainly through the action of the vertical component of the impact velocity of the body.

3. Body Vibration Phase

The large transient forces exerted on an elastic body entering the water may excite vibrations of the body. Transient forces may also act on the body at other stages of the water entry, say, by the impingement of a re-entrant jet, by the alternate shedding of vortices, or by the slap of the after part of long bodies against the side of the cavity. The excited body will then radiate sound to any compressible fluid in contact with the surface of the body. If the dimensions of the vibrating body are small compared with a wavelength of the sound radiated, the body will behave like a spherical source and the sound may be analyzed in terms of its multipole source components. The sound radiated directly into the

air would have the properties of an assemblage of multipole sources near a rigid plane baffle.

Probably the most important mode of vibration excited in a rigid solid sphere falling vertically and striking the water is the spheroidal one in which the sphere is distorted into an ellipsoid of revolution. The frequency of this mode of vibration is given by $f = 0.3c_s/a(1+\sigma)^{1/2}$ where σ is Poisson's ratio for the sphere, a is the radius of the sphere, and c_s is the velocity of sound in the sphere.²⁸ Initially the motion of the wetted surface that is due to the spheroidal vibrations is in phase at all points on the wetted surface, and the sound radiated into the water from the vibrations is predominantly dipole in character. Even if the wetted surface includes out-of-phase portions, higher order multipole radiation will not be present in the water. Higher order vibrations of the sphere are required to produce higher order multipole radiation. The amplitude of the sound would be expected to vary rapidly because the wetted area and its distance from the free surface change rapidly in the flow establishment phase. An approximate dipole strength might be obtained by adding to dz/dt in Eq. (5) a velocity $v(s)$ which varies sinusoidally at the frequency of the spheroidal vibrations. If the vibrations are not highly damped, the vibrating sphere may continue to radiate while in the cavity phase of the splash.

4. Cavity and Bubble Phase

The air-to-water entry of a body usually forms a cavity behind the body and may entrain air bubbles of various sizes. If the cavity is open to the atmosphere, it may act as a Helmholtz resonator. The cavity can be excited by the rapid changes in cavity volume and surface constriction. In the air the cavity would appear to be a simple source of sound.

When air is entrained in the water as bubbles, sound may be radiated into the water. Bubbles radiate sound mainly by volume pulsations of a frequency given by the expression due to Minnaert,¹⁰ $f = (3\gamma P_0/\rho)^{1/2}/2\pi a_b$ where a_b is the radius of the bubble, γ is the ratio of specific heats of the air in the bubble, P_0 is the environmental pressure, and ρ is again the density of the water. Bubble volume oscillations can be excited when there are changes in the environmental pressure or when the bubbles are formed. When a bubble is formed within a wavelength of the free surface, most of the sound radiated by the bubble is cancelled by a negative image source in the free surface. The bubble source and its negative image then constitute a dipole source of sound. In effect, to obtain the dipole sound pressure, the sound pressure expected from the bubble itself in a free field is multiplied by the factor $(2\pi f d \cos\theta)/c$, where d is the distance between source and image. Except when the bubble is within a few bubble diameters of the free sur-

²⁸ A. E. H. Love, *Mathematical Theory of Elasticity* (Dover Publications, New York, 1944), p. 186.

face, the frequency of the dipole sound is essentially the same as for the bubble in a free field.²⁹

If the cavity behind the body is sealed off at an early stage of the splash, the maximum volume attained by the cavity may be many times its equilibrium volume. If this occurs, subsequent volume oscillations of the cavity will be nonlinear and sound akin to cavitation noise may be radiated into the water. The shape of the cavity is usually far from spherical so it would be difficult to estimate the magnitude of the sound to be expected. Under certain conditions, cavitation bubbles may also appear in the boundary layer of the body itself.

When bubbles rise to the free surface and break, sound may be produced. Because of the action of surface tension, the pressure just inside the bubble is higher than the pressure just outside the bubble. Thus when the bubble breaks, a sound pressure wave is radiated both into the gas and into the liquid. The close proximity of the free surface would be expected to reduce somewhat the sound effectively radiated into the liquid. The amount of the reduction would be difficult to determine because the wavelength of the sound is not long compared with the diameter of the original bubble. The peak value of the sound radiated into the gas, however, would be expected to be approximately equal, at the bubble surface, to the surface tension pressure in the bubble.¹⁹ Since the surface tension pressure is inversely proportional to the radius of the bubble and the sound pressure is inversely proportional to the distance from the source, the sound pressure at a distance is essentially independent of the size of the original bubble. The duration of the pulse, however, would depend on the size of the bubble and would be expected to be of the order of the time required for sound to travel a distance equal to the diameter of the original bubble.³⁰ According to this reasoning, the initial peak of the sound pressure pulse in air from the breaking of an air bubble on a water surface would be expected to be approximately 1 d/cm² at 1 m from the bubble. The pulse from a bubble with a diameter of 0.15 cm would have a duration of approximately 1 μ sec. Such a pulse would have most of its sound energy in the ultrasonic frequency range, but it might be heard as a sharp click at short distances.

III. EXPERIMENTAL OBSERVATIONS

1. Splashes of Single Water Droplets

(a) *Experimental Apparatus*

In order to substantiate some of the speculations about how splashes produce sound, an experiment was devised to determine the underwater sound radiated from the splashes of single water droplets. A general examination of the underwater sound from the individ-

ual splashes indicated that the sound (1) was a transient and (2) had the general characteristics of a dipole source of sound. The instantaneous sound pressure at a point in the water and the spectrum of the underwater sound energy radiated by the splash were determined for various sizes and velocities of water droplets. Motion pictures were also taken in order to correlate the development of the splash with the instantaneous sound pressure radiated by the splash.

The splashes were produced on the surface of the water in a glass-walled tank 3 ft sq in plan and 5 ft high. The tank was nearly filled with filtered tap water at room temperature. The water bottle and droppers of various sizes and shapes used to produce the water droplets were supported above the tank at various heights, to give the desired impact velocity. The vertical velocity of the water droplets at impact was estimated from curves of the velocity of fall as a function of height of fall and droplet size.^{31,32}

The underwater sound from the splash was received by a Massa Model M115 hydrophone placed at an effective distance of 7 cm directly below the splash. The effects of sound reflections from the walls of the tank were determined to be negligible at this distance.

To obtain sound traces, the signals associated with the transient pressure pulses were presented on the screen of a cathode-ray oscillograph and photographed on continuously moving film. Since the hydrophone was in the near field of the dipole for all frequencies below about 25 kc, a compensator was inserted in the electrical circuit in order to obtain sound pulses as they would appear in the far field. The compensator consisted of a simple *RC* high-pass filter, the input being across a series combination of resistance R_E and capacitance C_E , and the output being across the resistance alone. By comparing the frequency dependence of the filter with the frequency dependence of the near field sound pressure of a dipole, given by the factor $[(c/2\pi fr)^2 + 1]^{1/2}$, it can be shown that the compensator just cancels the near-field effect if the time constant R_EC_E is chosen so that $R_EC_E = r/c$ for the particular distance r at which the hydrophone is located. This compensating network was used only when photographing sound traces.

In order to obtain a spectrum analysis of the sound energy radiated by the splashes, the electrical signals from the hydrophone were recorded on magnetic tape and subsequently played back through half-octave filters, a squaring circuit, and an integrating circuit. The squaring device utilized the nonlinear properties of selected copper-oxide rectifiers shunted by linear resistors. The time constant of the integrating circuit was 1 sec, which is long compared to the duration of the transient sounds. Calibration was accomplished by introducing short pulses of sine waves into the crystal circuit of the

²⁹ M. Strasberg, J. Acoust. Soc. Am. **25**, 537 (1953).

³⁰ R. G. Campbell, J. Appl. Phys. **29**, 55 (1958).

³¹ A. F. Spilhaus, J. Meteorol. **5**, 108 (June 1948).

³² J. O. Laws, Hydrology papers, Trans. Am. Geophys. Union **22**, 709 (1941).

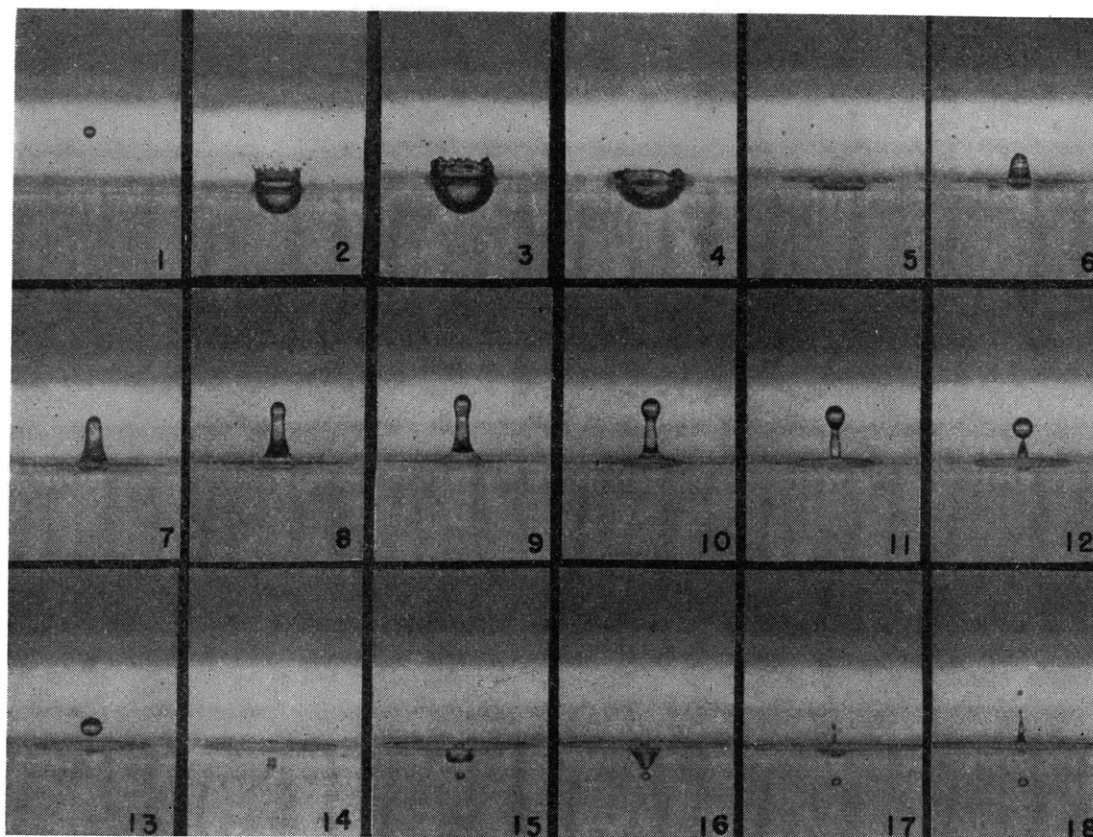


FIG. 1. Photographs of the splash of a 56-mg water droplet (0.24 cm radius) having a vertical impact velocity of 350 cm/sec. The frames are spaced at 13-msec intervals.

hydrophone. The output of the integrating circuit was amplified and recorded with a recording gavanometer. The peak indication of the recorder after the passage of a pulse of sound was taken as a measure of the time integral of the square of the sound pressure passed by the half-octave filter in the circuit. Since $1/\rho c$ times the average mean-square sound pressure on a large hemisphere in the water gives the average radiated sound energy per unit time and per unit area on this hemisphere, the half-octave sound energy E_i radiated into the water is given by the expression

$$E_i = \frac{2\pi r^2}{3\rho c \cos^2\theta [1 + (c/2\pi f r)^2]} \int_{-\infty}^{\infty} p_i^2 dt, \quad (6)$$

where f is taken as the geometric-mean frequency of the half-octave band, r is the radial distance from the splash to the hydrophone, θ is the angle between the vertical and the line from splash to hydrophone, and p_i is the sound pressure in the half-octave band without using the compensating circuit.

(b) Qualitative Results

Motion pictures of the splashes of droplets were taken with the camera directed horizontally at the water surface, thus giving a simultaneous view of the motions in

the air and in the water. These motion pictures show a number of phenomena pertinent to the generation of sound, the more important of which are shown in Figs. 1, 2, and 3. There are only two main sources of underwater sound from these splashes. The initial entry of the droplet always causes a characteristic peaked pulse of sound to be radiated. Under some conditions, an air bubble can be entrained in the water by the later development of the splash. Any bubble so entrained usually radiated a damped sine-wave pulse of sound. Aural observations indicate that the sound in the air is predominantly bubble sound that is transmitted through the free surface and the walls of the container. Sound traces of the underwater sounds from splashes similar to those shown are given in Fig. 4.

It is apparent in Fig. 1 that the impacting droplet imparts an impulse motion to the water during the flow-establishment phase of the entry. The hemispherical cavity closes most rapidly from the bottom, thus forming an emergent jet. Because of the action of surface tension, the emergent jet breaks up into droplets which, when they fall, are so timed as to trap a small bubble of air beneath the surface. The impact sound is produced at times between frames 1 and 2, and the bubble sound is produced at times between frames 13 and 15. The impact sound is very consistent and reproducible. The

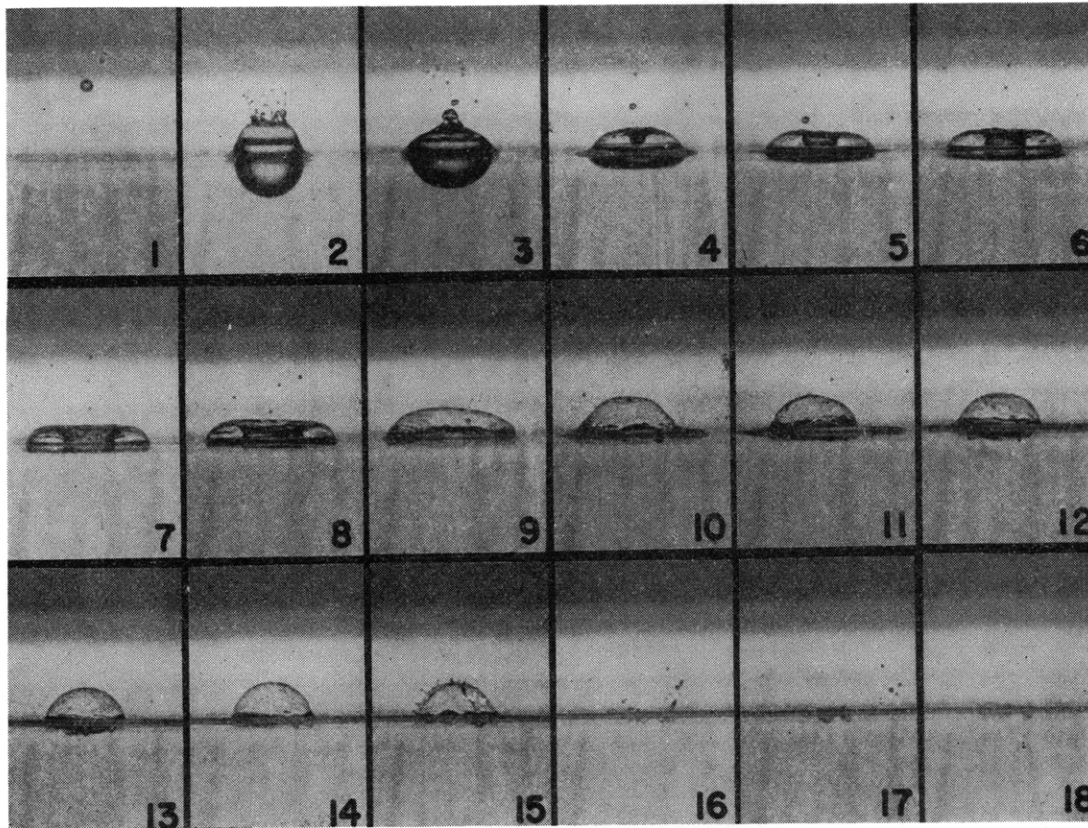


FIG. 2. Photographs of the splash of a 56-mg water droplet (0.24 cm radius) having a vertical impact velocity of 550 cm/sec. The frames are spaced at 20-msec intervals.

bubble sound, however, varies considerably in frequency and amplitude and is sometimes nonexistent because the entrainment of air is very sensitive to minor variations in the condition of the surface, in the shape of the droplet, and in the angle of impact of the droplet; all of which influence the spacing and orientation of the secondary droplets. Water droplets falling vertically at velocities between, roughly, 2 and 5 m/sec, or falling from 1 to 5 ft generally produce the type of splash shown in Fig. 1.

Figure 2 shows a splash typical of those produced by large water droplets falling at velocities between about 5 m/sec and terminal velocity (which is about 9 m/sec for large droplets). The closing of the "canopy" prevents the mechanism observed in Fig. 1 from forming bubbles, so appreciable underwater sound is produced only at times between frames 1 and 2. A typical far-field sound trace for this type of splash is given by the second oscillogram in Fig. 4. The breaking of the canopy seems to produce no measurable sound in the water.

The splash shown in Fig. 3 is the splash produced by a large water droplet followed by three small water droplets. These small droplets, sometimes called "Plateau's spherules," are formed behind the large droplets after the large droplet necks off at the dropper. Large bubbles can be entrained if the small following droplets hit in

the bottom of the hemispherical cavity made by the impact of the large droplet. A typical sound trace for this type of splash is given by the third oscillogram in Fig. 4. The impact sounds of all three small droplets appear just before the bubble sounds.

(c) Analysis of the Underwater Sound

The frequency spectrum and the amplitude of the sound radiated into the water by water droplets splashing on a water surface may depend on many variables. The variables of primary importance may be assumed to be the size, shape, and velocity of the droplet and the density and acoustic properties of the water. Other variables which would also be expected to influence the sound produced are the surface tension and viscosity of the water, and the density, compressibility, pressure, and viscosity of the air above the water. However, only the influence of the droplet size and velocity was investigated experimentally.

The underwater sound pressure p resulting from the impact of water droplets varies in amplitude and duration in a systematic manner with changes in impact velocity and droplet size. A trace of one of the typical sound pulses, as it appears in the far field of the dipole, is presented in dimensionless form in Fig. 5, using the droplet radius a as a characteristic length and the verti-

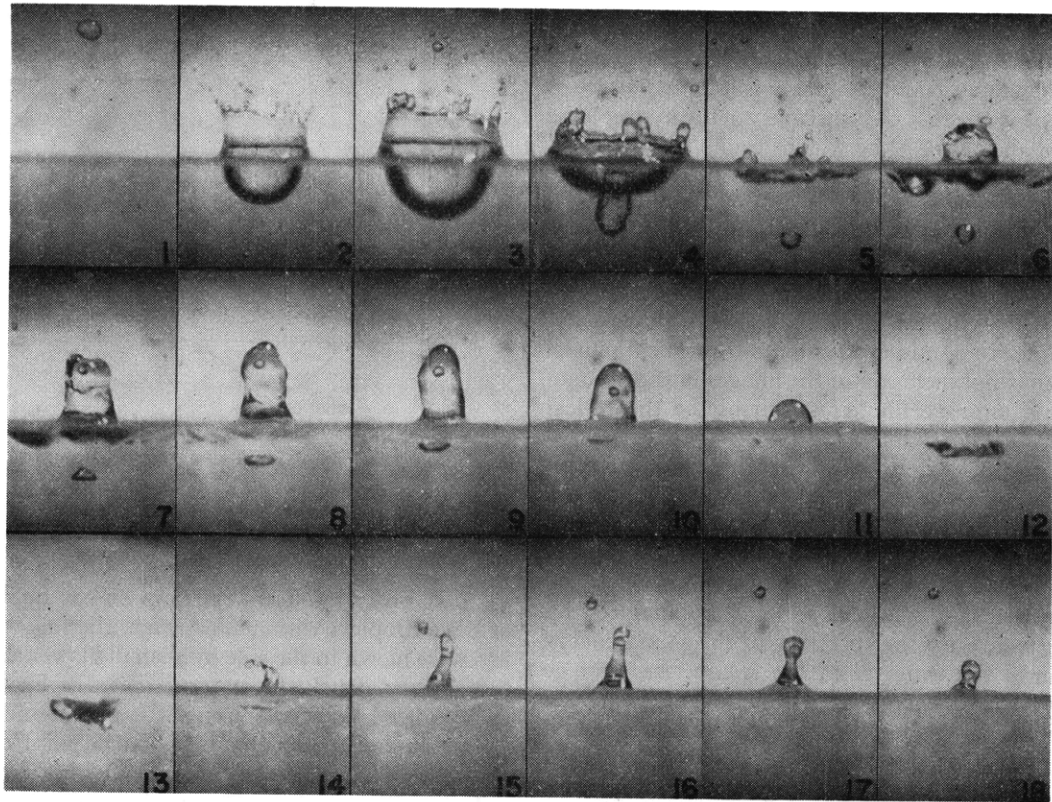


FIG. 3. Photographs of the splash of a 182-mg water droplet (0.35-cm radius) having a vertical impact velocity of 400 cm/sec and with three closely following water droplets (frame 3) having a total mass of 50 mg. The frames are spaced at 21-msec intervals.

cal impact velocity V as a characteristic velocity, and where t is taken to be zero at the instant the droplet touches the free surface. The peak of an uncompensated trace of the sound pressure, as it appears at a distance 7 cm below the splash, is blunt, and the pressure does not recross the zero pressure axis until a dimensionless

retarded time of about 2 is reached. These near-field traces are much richer in low-frequency components but still have the same high-frequency components as the far-field traces, because of the fact that the near-field and far-field traces have the same rapid rise to the initial peak. Within a range of roughly 50% in amplitude and

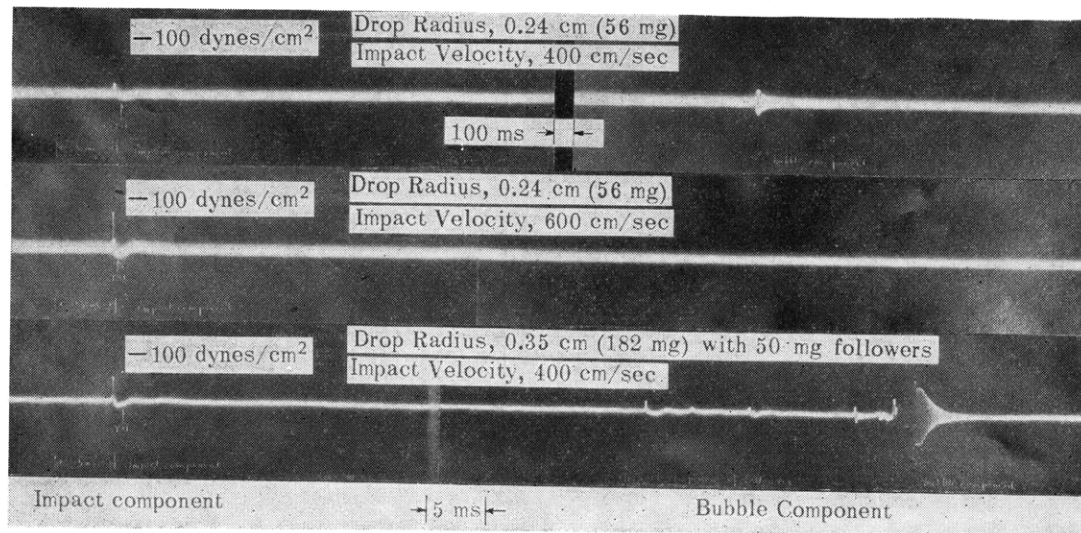


FIG. 4. Oscillograms of the instantaneous underwater sound pressure from the splashes of water droplets similar to those shown in Figs. 1-3. The sound pressure was measured through the filter band extending over a range corresponding to dimensionless frequencies fa/V from 0.03 to 6 and at a distance of 7 cm below the splashes. The near-field sound pressure has been removed by a compensator.

10% in time, this trace is typical for water droplets in the range 10 to 180 mg and velocity range 2 to 7 m/sec. There is some uncertainty in the absolute amplitude of the sharp peak because the sensitivity of the hydrophone has a resonance near 70 kc which results in an error if an average sensitivity is used. It is of interest to note that the observed pulses are much like those obtained by using Eqs. (2) and (5), both in amplitude and shape. This result somewhat justifies ignoring the simple source contribution and using the drastic idealization used to obtain Eq. (5), namely, ignoring images in the pseudobody in order to obtain approximate theoretical pulses. It is not known if inclusion of the images in the pseudobody would improve the agreement of the theoretical and experimental results.

The sound pulses from entrained air bubbles are typically damped sine waves, but their amplitude and frequency cannot be predicted, except in very general terms, from a knowledge of the droplet size and velocity.

As was indicated previously, the sound produced by a single splash is a transient and can be analyzed by measuring indirectly the spectrum of the sound energy radiated into the water from the splash. An average half-octave sound energy as a function of frequency was obtained, for each of a number of droplet size and velocity conditions, by averaging the data obtained from analyzing the sound from a large number of splashes under each condition. The average spectra are presented in Figs. 6-8. Since a relatively long silence separates the transient sounds produced by the impact and by the bubbles, it was possible to obtain separate spectra for

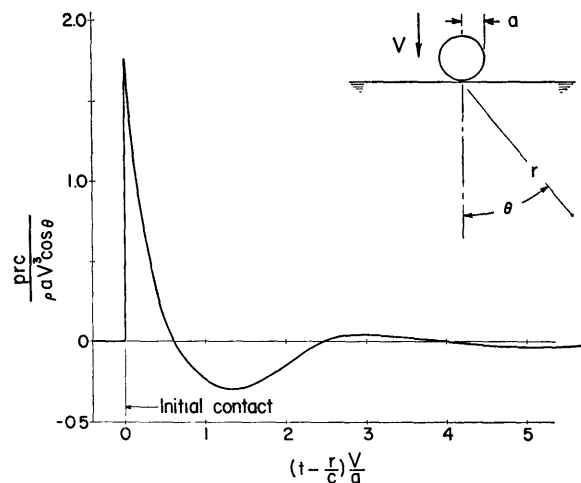


FIG. 5. The typical shape of the sound pressure pulse radiated into the water by the vertical impact of a water droplet.

each. In order to obtain the data on the bubble sound of large droplets without followers, the following droplets were blown to the side by a small blower and caught before they struck the water surface. In Figs. 6 and 7 each plotted point represents the arithmetic mean of 40 measurements, and in Fig. 8 each point represents the arithmetic mean of 20 measurements.

It is apparent from Fig. 6 that the sound energy radiated into the water by the initial entry of the droplet increases systematically with increase in droplet size and velocity. The scatter of the points is believed to be

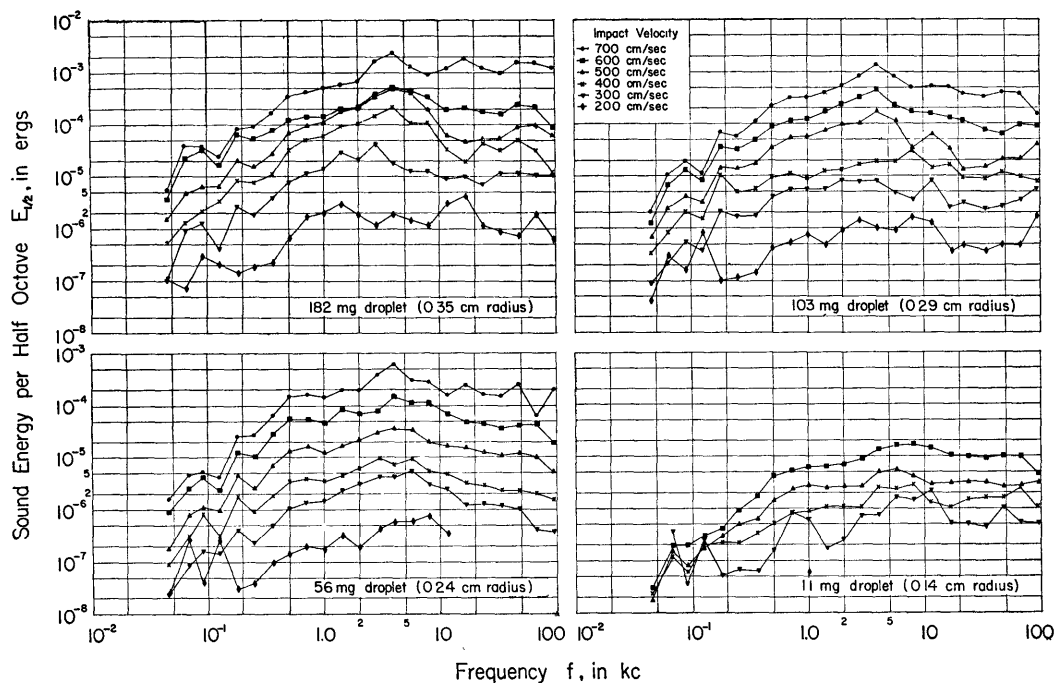


FIG. 6. The half-octave frequency spectra of the impact part of the sound energy radiated into the water by the splashes of single water droplets.

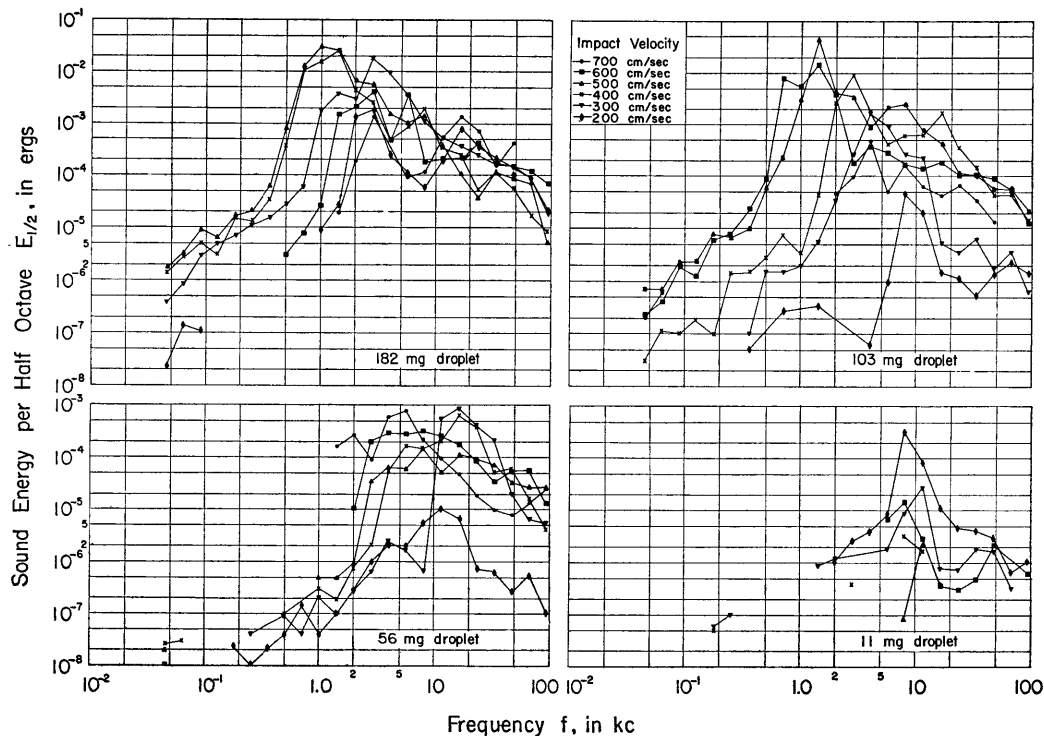


FIG. 7. The half-octave frequency spectra of the bubble part of the sound energy radiated into the water by the splashes of single water droplets.

due mostly to inaccuracies in the acoustic measurements and to variations in droplet shape caused by capillary vibrations and air currents.

The bubble component of the underwater sound energy from the splash of a water droplet is very erratic under most conditions. The data in Fig. 7 indicate, however, that certain ranges of droplet size and velocity are more conducive to the production of this bubble component than are others. In general, the droplets at intermediate velocities produce, on the average, more

bubble sound than the droplets at either high or low velocities, and the small droplets trap smaller bubbles than do the large droplets and thus produce sound of higher frequency. The bubble sound made by the large droplets with followers, as shown in Fig. 8, on the contrary, is quite repeatable and depends very little on the impact velocity of the droplet.

The total sound energy radiated by an individual bubble was often greater than the sound energy radiated by the impact of the water droplet, and the sound energy in the particular half-octave band in which the bubble was radiating most of its energy was usually much greater than the sound energy in the same band due to impact. However, since the frequency at which the bubbles radiated energy varied from splash to splash over several octaves, and since quite often the bubbles were not formed at all, the sound energy per half-octave averaged over many droplets, is about equal for the bubble and impact components, over the frequency range in which the bubble sound is appreciable. Also, since the bubble sound energy decreases, if anything, with increase in impact velocity whereas the impact sound energy is proportional to about the fifth power of the impact velocity, the impact sound becomes more predominant at the higher velocities.

For the bubbles, the peak in the average sound energy per half octave usually occurs at a frequency between 1 and 20 kc with a sharp decline below 1 kc and a more gradual decline above 20 kc. The sharp decline below

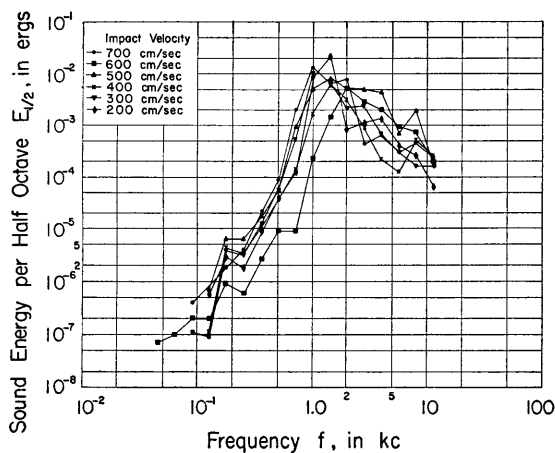


FIG. 8. The half-octave frequency spectra of the bubble part of the sound energy radiated into the water by the splashes of single 182-mg water droplets with closely following water droplets having a total mass of 50 mg.

1 kc is due to the almost complete absence of bubbles larger than about 0.3 cm in radius, but the gradual decline at the higher frequencies is due more to a decrease in the radiated sound energy per bubble than to a decrease in the prevalence of bubbles. In general, a sharp peak in the average sound spectrum indicates a condition in which bubbles are formed uniformly of the same size, in contrast to the usual condition in which the bubbles vary considerably in size from splash to splash.

Some of the data points are missing from the half-octave spectra. These energy values could not be obtained because they are below the limits of resolution of the instrumentation used in the experiment.

The data of Fig. 6 are given in dimensionless form in Fig. 9, where T is the kinetic energy of the droplet at impact times the ratio of the density of the liquid to the density of the droplet and $M = V/c$ is the Mach number of the droplet based on the velocity of impact. In general, the scatter of the points is quite random, but the data on the largest droplets at the highest velocities have a disproportionate amount of sound energy in the high-frequency bands. This can be explained by recognizing that the large droplets at high velocities are flattened by air currents at the time of impact on the water surface. The sound pressure pulse from the impact of such a droplet has a sharper and higher peak than it would have if the droplet were spherical, because less time is required for the droplet to pass through the free surface and the apparent radius of the droplet at the initial point of contact is larger than for the spherical case. The total sound energy E radiated into the water is approximately equal to $2TM^3$. It is apparent that the efficiency of conversion of kinetic energy of the droplet to sound energy radiated into the water, represented by $\eta = 2M^3$, is very low at low Mach numbers. It is also apparent that this expression cannot hold for high Mach numbers.

2. Splashes of a Spray of Water Droplets

(a) Experimental Apparatus

The characteristics of the underwater sound radiated by the surface splashes due to a spray of water droplets have also been investigated in the laboratory. Photo-

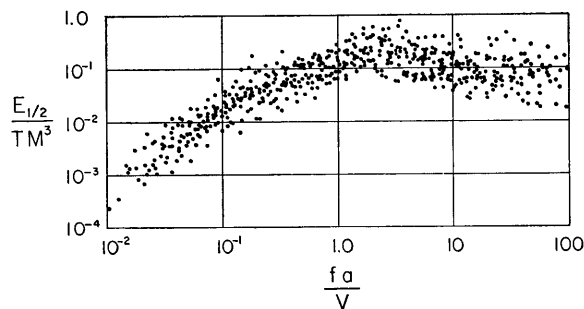


FIG. 9. The half-octave dimensionless frequency spectrum of the sound energy radiated into the water by the impact of single droplets of water.

graphs of the splashing were taken, the instantaneous sound pressure in the water was observed, and the rms sound pressure in the water was analyzed. Only the analysis of the rms sound pressure will be presented here.

The tests were performed in the deep-water basin of the David Taylor Model Basin with the spray area about 12 ft from a wooden bulkhead. The deep-water basin is 51 ft wide and 22 ft deep. The spray apparatus was suspended above the center of the basin at various heights by a carriage and hoist arrangement attached to the side of an aluminum bridge. The shower head, consisting of a closed shallow plastic tank with 205 brass tubes projecting down from the bottom of the tank at 1-in. intervals, produced a nominal circular area of random splashing of about 250 sq in. To control the rate of formation of the droplets, a short length of plastic tubing with a very small bore was inserted in the upper end of each brass tube. The $\frac{1}{8}$ -in. o.d. tubes used in this study produced 50-mg droplets (0.23-cm radius). For the flow rates used (0.1 to 0.4 gal/min), droplets fell at approximately 1-sec intervals from each tube. Tap water was piped into the top of the plastic tank by way of a control valve, a flowmeter, and a length of garden hose.

The underwater sound from the splashing on the free surface was measured by means of a hydrophone (Massa Model M115) placed 18 in. directly below the center of the spray area. The signal from the hydrophone consisted of a series of sharp peaks and occasional damped sine waves due to the impact of the individual droplets and to entrained air bubbles, respectively. Because this signal had a high peak-to-rms ratio, some difficulty was encountered in determining true rms values. The rms sound pressure in various half-octave bands was determined satisfactorily by using a vacuum-tube wattmeter (John Fluke Engineering Company VAW Meter Model 101) to measure the power dissipated in a known resistance.

The variation of the sound pressure with depth was determined by measuring the rms sound pressure as a function of depth directly below the spray area. The variation of the sound pressure with angle was determined by measuring the rms sound pressure as a function of angle for a fixed distance of 30 in. from the center of the spray area. The variation with depth and angle were determined only in the half-octave band centered at 11.4 kc, at a spray rate of 28 in./hr, and at a droplet impact velocity of 800 cm/sec.

(b) Analysis of the Underwater Sound

In order to describe the sound produced by a continuing sequence of surface splashes which are random in time and space in the area of splashing, a statistical description of the pressure pulses radiated by the individual surface splashes, such as the rms sound pressure, is used. If the rms sound pressure at a point in the liquid from a sequence of surface splashes is known, an average of the rate of producing sound energy at the free surface

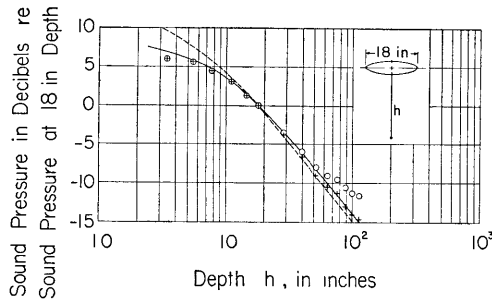


FIG. 10. The relative rms sound pressure in the half-octave band centered at 11.4 kc as a function of depth below a circular spray area 18 in. in diam.

may be obtained, or conversely, if the rate of producing sound energy is known, the rms sound pressure may be obtained. An expression for the mean-square sound pressure \bar{p}^2 in the water due to an area of spraying on the free surface, using Eq. (6), is

$$\bar{p}^2 = \int (3\rho c I / 2\pi r^2) (1 + c^2 / 4\pi^2 f^2 r^2) \cos^2 \theta dS, \quad (7)$$

where I is the average water-borne dipole sound energy per unit of time and per unit of free surface area produced by the splashing on the free surface; dS is an element of area on the free surface described by the coordinates r , θ , and ϕ ; and a homogeneous nonabsorbing medium has been assumed. The expression for the mean-square sound pressure directly below a circular spray area is then

$$\bar{p}^2 = (3\rho c I / 2) [(1 - \cos^2 \alpha) + c^2 (1 - \cos^4 \alpha) / 8\pi^2 f^2 h^2], \quad (8)$$

where α is the half-angle subtended at a depth h by the spray area. Areas in the shape of circular rings and segments of rings can be accommodated with slight changes in Eq. (8). Since the rate at which sound energy is produced in the water is proportional to the rate at which kinetic energy of the droplets is intercepted by the free surface, I may be expressed in terms of the

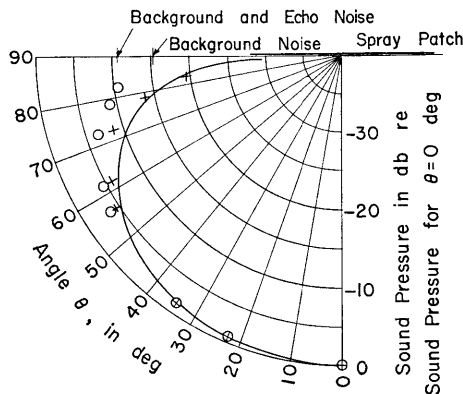


FIG. 11. The relative rms sound pressure in the half-octave band centered at 11.4 kc as a function of angle at a distance of 30 in. from the center of a circular area of splashing 18 in. in diam.

spray rate R as

$$I = (1/2) \rho R V^2 M^3 (E / T M^3), \quad (9)$$

where R is the volume of the droplets impinging on the free surface per unit of time and per unit of free surface area.

In order to check the depth and angular variations of the underwater sound field produced by a patch of spray on the free surface, a survey was made of the sound field below a small spray area. The observed and theoretical variations with depth h and with angle θ are given in Figs. 10 and 11. In Fig. 10 the data for small h are slightly lower than predicted. This may be attributed to the fact that the hydrophone is slightly less sensitive to that part of the sound coming from directions other than perpendicular to the axis of the hydrophone. The drop in the solid theoretical curve at shallow depths is a result of the influence of what might be called the "geometric near field." This is due to the fact that, at shallow depths, the distance to the outer reaches of the spray area is significantly greater than the distance h to

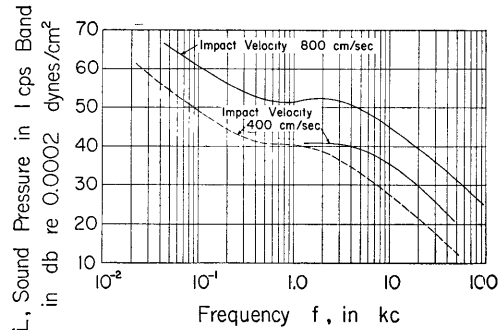


FIG. 12. Typical sound-pressure-spectrum levels of the underwater sound 18 in. below a spray of 50-mg water droplets on a circular spray area of 250 sq in. at a spray rate of 22.4 in./hr.

the center of the spray area. The high frequency of the half-octave band used was deliberately chosen so as to avoid the effects of the "induction near field" at depths greater than a few inches. The data for large h are higher than predicted because of the presence of background and echo noise in the basin. The depth at which the measured value is 3 db higher than the predicted value may be taken as the depth at which the sound pressure due to echoes and background noise just equals the direct sound. When the measured values are corrected for this echo and background noise, the agreement with the predicted values is very good. The dashed curve gives the predicted variation with depth if simple sources are assumed instead of dipole sources. The open circles indicate the observed points and the crosses the corrected points in both Fig. 10 and Fig. 11.

The solid curve in Fig. 11 gives the theoretical variation of the sound pressure for a constant distance of 30 in. from a spray area, under the assumption that the diameter of the spray area is small compared with this distance. The echo and background level would be ex-

pected to be lower at distances less than a wavelength from the free surface, and as a consequence, the corrected data for θ near 90° are not too reliable. In general, the agreement between the corrected-measured values and the values predicted on the assumption of dipole sound sources is good.

In order to indicate the character of the pressure-spectrum level of the sound in the water for the particular spray conditions used, a pair of pressure-spectrum levels calculated from measurements of the sound pressure in half-octave bands is given in Fig. 12. The experimental points lie within 2 db of the solid curves shown. The levels at the low frequencies are high because of the presence of near-field sound pressure at the particular depth used. The slope of the spectrum at low frequencies is 6 db per octave lower than it would be in the absence of near-field sound. The dashed curve indicates the calculated level for 400 cm/sec if the data for 800 cm/sec are used, and if it is assumed that the level is proportional to the square of the impact velocity. The difference between the calculated curve and the measured

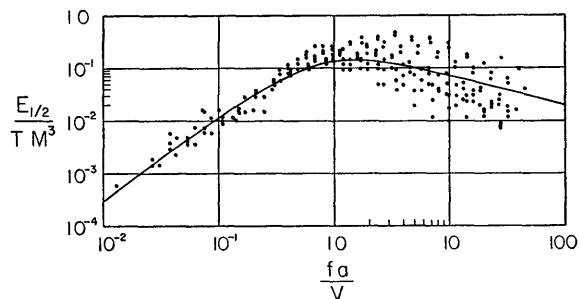


FIG. 13. The half-octave dimensionless frequency spectrum of the sound energy radiated into the water by the splashes of a spray of water droplets.

curve for 400 cm/sec is attributed mainly to sound from entrained air bubbles. The contribution of entrained air bubbles to the sound for 400 cm/sec, however, is not as great as would be indicated by the single-splash data previously discussed. This difference may be attributed to the fact that in the case of sprays, the rough surface disrupts somewhat the production of air bubbles. Furthermore, if the spray is not falling vertically, even less bubble sound would be expected.

The rms sound pressure in half-octave bands from 45 cps to 95 kc was measured below sprays of 50-mg droplets at velocities of 4, 5, 6, 7, and 8 m/sec and at spray rates of 5.6, 11.2, and 22.4 in./hr. These data were converted to dimensionless sound energy per half octave by using Eqs. (8) and (9), and are plotted in Fig. 13. The solid curve is the average of the data on single splashes in Fig. 9. The points from the different velocities and spray rates show little systematic variation from the average curve, except that at the high frequencies the points from the 4- and 5-m/sec data are somewhat higher than the points from the 6-, 7-, and 8-m/sec data. Oscillogram traces of the sound pressure

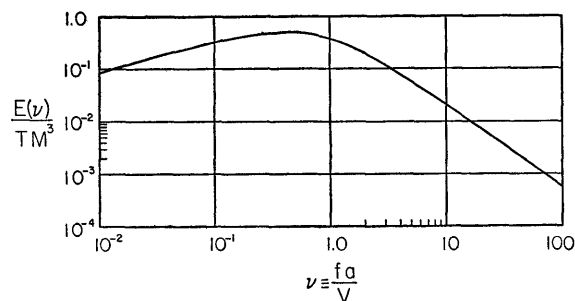


FIG. 14. The dimensionless spectral density of the sound energy radiated into the water by the impact of single droplets of water or by the splashes of a spray of water droplets.

for velocities of 4 and 5 m/sec indicated the presence of a number of damped sine waves due to entrained air bubbles. Under most conditions, the curve in Fig. 13 may be used as a representative spectrum of the dimensionless half-octave sound energy for water droplet or spray splashes.

The general result of Fig. 13 can be used to estimate the sound pressure levels to be expected in the liquid from the large-scale splashing of water droplets. The dimensionless function in terms of half-octave bands may be reduced to a more useful form by introducing the sound energy spectral density $E(v)$ where $E(v)dv$ is the sound energy in the dimensionless band $dv \equiv (a/V)df$. This spectral density is given in dimensionless form in Fig. 14.

For points directly below a circular spray area the conventional sound-pressure-spectrum level L , defined as the sound pressure in a 1-cps band in decibels relative to 0.0002 d/cm², is then given by

$$L = 74 + 10 \log \left\{ \frac{3\rho^2 a V^4 R E(v)}{4c^2 T M^3} \times \left[c^2(1 - \cos^4 \alpha) + \frac{c^2(1 - \cos 4\alpha)}{8\pi^2 f^2 h^2} \right] \right\}, \quad (10)$$

where centimeter-gram-second units are to be used. The sound-pressure-spectrum level may be given for other spray configurations on the free surface. For instance, if the linear size of the spray area on the free surface is small compared to the distance from the area, and if the distance from the area is much greater than a wavelength, the sound-pressure-spectrum level is

$$L = 74 + 10 \log \left[\frac{3\rho^2 a V^4 R E(v) S \cos^4 \theta}{4c^2 T M^3 \pi h^2} \right], \quad (11)$$

where S is the area of splashing on the surface and θ is the angle between the vertical and the line to the area. In terms of the solid angle Ω subtended at the measuring point by this area, the sound-pressure-spectrum level is

$$L = 74 + 10 \log \left[\frac{3\rho^2 a V^4 R E(v) \Omega \cos \theta}{4c^2 T M^3 \pi} \right]. \quad (12)$$

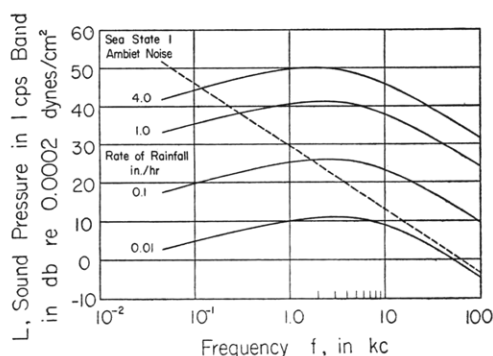


FIG. 15. The estimated sound-pressure-spectrum levels of the underwater sound from the impact of rain on the surface, at depths greater than a wavelength from the surface.

In order to obtain the levels for most other spray configurations, an integration over the area is required, in accordance with Eq. (7).

(c) Estimated Underwater Sound from Rain

Sprays of water droplets may be described by giving the size distribution of the droplets, the velocities of these droplets, and the rate of spraying. These quantities may vary considerably under actual conditions. In the case of rainfall, available data indicate that the size and velocity distributions of the droplets are functions primarily of the rate of rainfall, so that a fair description of this type of spray is available if the rate of rainfall is known.³³ By using this data and Eq. (10), the estimated pressure-spectrum levels for four rates of rainfall were computed and the results are shown in Fig. 15. Weighted averages of the droplet sizes and terminal velocities were used instead of using the full spectrum of droplet sizes in the rain. The weighted-average droplet size was taken as the median size of the size range of droplets contributing the largest portion of the sound energy. A small amount of flattening of the spectrum curves would result if the full distribution of droplet sizes were to be used. It is seen that even light rain (less than 0.1 in./hr) would be expected to raise the noise level above the ambient level for sea state 1 at the higher frequencies. The levels shown are independent of depth for depths greater than a wavelength from the free surface. For lesser depths, the factor $10 \log(1 + c^2/8\pi^2 f^2 h^2)$ should be added to these levels. Ambient noise from the sea surface should also exhibit this near-field dependence. If the ambient noise decreases instead of increases upon approaching within a wavelength of the free surface, the source of the ambient noise is in the body of the liquid rather than at the free surface. The influence of absorption of sound has been neglected, but it can be taken into account by using the general theory developed by Urick.¹⁷

Two wide-band pressure levels in db relative to

³³ J. O. Laws and D. A. Parsons, Hydrology reports and papers, Am. Geophys. Union, Twenty-Fourth Annual Meeting, April 23 and 24, 1943.

0.0002 d/cm² are given in terms of the rate of rainfall R in in./hr by the approximate formulas

$$B(0.1-10 \text{ kc}) \cong 80 + 14.5 \log R \quad (13)$$

$$B(0.1-100 \text{ kc}) \cong 83 + 14.5 \log R. \quad (14)$$

3. Splashes of Spheres, Cones, and Wedges

(a) Experimental Apparatus

Some preliminary data on the instantaneous underwater sound from the splashes of rigid spheres, cones, and wedges have also been obtained. The bodies were dropped from various heights over the basin at the same location used to study the underwater sound from sprays. The cones and wedges entered the water, point first, and were stabilized in flight by a tail shaft and fin.

Only oscillograms of the instantaneous underwater sound pressure were obtained, using the compensating network previously described to correct to far-field conditions. The hydrophone (Massa Model M115) was placed 13.5 cm below and 13.5 cm to the side of the point of impact.

(b) Description of the Underwater Sound

A trace from the vertical impact of a sponge-rubber sphere is shown in Fig. 16. Salient features are the sharp rise due to first contact, the damped oscillations due to spheroidal vibrations of the sphere, and the damped sine waves due to resonant bubbles entrained by the splash. The initial peak pressure is approximately that expected on the basis of peak values observed for the impact of water droplets and shown in Fig. 5. The

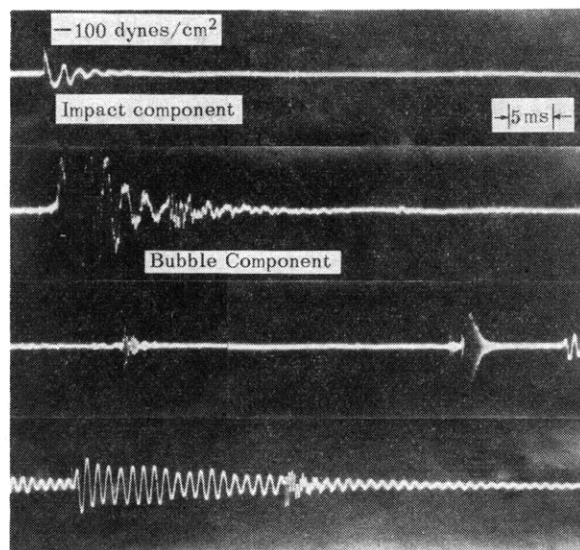


FIG. 16. Oscillogram of the instantaneous far-field sound pressure in the water at a radial distance of 19 cm and at a dipole angle of 45 deg from the vertical entry of a sponge-rubber ball having a radius of 1.7 cm, an impact velocity of 400 cm/sec, and a specific gravity of 0.66. The sharp rise in the impact trace has been retouched to make it legible.

frequency of the oscillations that occur after the initial peak corresponds well with the calculated frequency of the spheroidal vibrations of the sphere. In this case, several oscillations occur before the sphere is half submerged. For more rigid spheres, such as steel spheres, or for spheres of the same size and material but impacting at lower velocities, many more oscillations may occur before half submergence. If these frequencies are sufficiently high, they might be filtered out so as to obtain a trace representative of the entry itself. The first bubble pulse is believed to be produced at the time the initial cavity closes behind the sphere, and subsequent bubble pulses are produced as this cavity loses or "detrains" air in the form of smaller bubbles.

Shadowgrams of the shock wave radiated by the entry of spheres at high speeds, obtained by McMillen,¹¹ furnish further evidence that the sound radiated into the liquid from splashes is dipole in character. He observed that the $\cos\theta$ variation of the sound field below the splash was obtained for bodies of different shapes, thus deprecating any geometric explanation for this variation. The initial peak pressures, which he determined approximately from the width of the dark band on the shadowgrams and from the eccentricity of the shock front, agree in order of magnitude with the peak pressures expected if the peak values for the low-velocity entry of spheres are extrapolated to higher velocities on the assumption that the splash is a dipole source of sound. Although the pressures agree in order of magnitude over the relatively small range of velocities and dimensions of McMillen's measurements, his results indicate variations with velocity and size according to the simple source theory rather than the variations predicted by the dipole theory. Some simple source radiation would be expected at these high velocities because flow velocities greater than the velocity of sound occur during a significant portion of the splash and the sound does not always originate well within a wavelength of the free surface. The radiation from the spheroidal vibrations of the sphere indicate this fact because they do not always exhibit the $\cos\theta$ variation apparent for the initial peak. This apparent difference, however, could be explained by the fact that McMillen computed the effective pressure from the width of the dark band, under the assumption that the pressure is nearly constant over the region immediately behind the shock front. He recognized, however, that the peaks were sharp and that the pressures he computed should be considered to be effective or average pressures. Under his experimental conditions the width of the peak, according to the dipole theory, is approximately equal to the width of the dark band on the shadowgrams. Thus the average pressure indicated by the width of the dark band should be proportional to the area under the initial pressure peak in Fig. 5. This area is indeed proportional to the square of the impact velocity and to the cross-sectional area of the sphere as observed by McMillen.

A few traces obtained from the vertical impact of cones and wedges entering the water, point first, indicate that underwater sound from high-frequency resonances in these bodies masks most of the sound from the entry itself. Traces from the low-velocity entry of brass cones were filtered through a low-pass filter to remove most of the high-frequency sound due to resonant vibrations of the cones. These filtered traces indicate that the basic trace from the entry itself is triangular in shape with a gradual rise in pressure until the cone is terminated, then a gradual fall in pressure. The initial rise in pressure is roughly proportional to the depth of immersion. Similar traces obtained from the entry of wedges indicate that the basic trace from the entry itself is very much like that for a sphere except that the fall-off behind the initial sharp peak is more gradual. The shape, amplitude, and duration of these basic pulses are in essential agreement with those computed by using Eqs. (2) and (5).

IV. SUMMARY

The splashes produced by the air-to-water entry of objects produce sound both in the air and in the water. The main sources of underwater sound from such a splash have been found to be (1) impact and passage of the body through the water surface, (2) resonance vibrations of the body, and (3) volume pulsations of closed cavities of air in the water. The underwater sounds from the impact and entry of the body and from resonant vibrations of the body, if it has rigidity, predominate at high velocities of entry. The underwater sound from entrained air bubbles, if they are formed, predominates at low velocities of entry.

The underwater sound from the splashes of water droplets exhibits the properties of a dipole source of sound in that (1) a near-field sound is present at distances less than a wavelength from the splashes, (2) the magnitude of the sound pressure from the impact is proportional to the radius of the droplet and to the cube of the impact velocity, and (3) the sound pressure below the splashes varies as $\cos\theta$ where θ is the angle with the vertical. The instantaneous underwater sound pressure and the spectrum of the sound energy from the impact of single water droplets as well as the rms sound pressure under a spray of water droplets are amenable to presentation in dimensionless form using the radius of the droplet as a characteristic length and the impact velocity as a characteristic velocity. The sound from entrained air bubbles is neither directly related to the radius nor to the velocity of the impacting droplet but is, however, also dipole in character because of the presence of the free surface. The sound in the water under the splashing of a spray of water droplets consists of a number of randomly spaced transient sounds from the individual splashes. Occasional damped sine waves from entrained air bubbles are interspersed among the sharp spikes caused by the impacts.

The underwater sound from large-scale splashing, such as that due to breaking waves or rain, may be estimated by using the dimensionless spectrum of the sound from sprays and information on the rate of spraying, the sizes and velocities of fall of the droplets in the spray, and the orientation of the disturbed surface with respect to the point at which the sound is received. Such estimates of the underwater sound from rains agree in general with available data on this source of sound.

A few observations of the instantaneous sound pressure in the water produced by the vertical entry of spheres, cones, and wedges, indicate that the sound is compatible with the dipole assumption and similar to that observed for the splashes of water droplets, except that resonant vibrations of the bodies modify the characteristic impact sound considerably. Sounds from the oblique entry of objects have not been studied experimentally but have been discussed in general terms.

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2. Underwater sound - Spectrum
3. Underwater sound - Propagation
4. Drops - Water entry
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- II. NS 715-102

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