TAYLOR MODEL BASIN
WASHINGTON D.C.

DEPARTMENT OF THE NAVY

HYDROMECHANICS

AERODYNAMICS

STRUCTURAL MECHANICS

APPLIED MATHEMATICS

PLASTIC GENERAL-INSTABILITY PRESSURE OF RING-STIFFENED CYLINDRICAL SHELLS

by

M. E. Lunchick, Ph.D.

STRUCTURAL MECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

September 1963 Report 1587

DEPARTMENT OF THE NAVY
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WASHINGTON, D.C. 20007

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Subj: Investigation of the stability of ring-stiffened cylinders in the strain-hardening range; forwarding of report on

Encl: (1) DATMOBAS Report 1587 entitled, "Plastic General-Instability Pressure of Ring-Stiffened Cylindrical Shells" 2 copies

1. Recent studies have indicated the possibility of utilizing strain hardening materials such as aluminum alloys and very high-strength steels in the pressure hulls of oceanographic vehicles operating at great depths. It is therefore important that the effects of strain hardening on collapse strength be understood. Enclosure (1) presents the results of a theoretical investigation of these effects on the general instability of ring-stiffened cylinders under external pressure.

2. A buckling solution is developed which accounts for strain hardening in terms of the tangent and secant moduli. In the elastic range the solution reduces to Bryant's equation for elastic general instability.

E. E. Johnson

E. E. JOHNSON
By direction

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FOREWORD

This work was conducted when the author was employed at the David Taylor Model Basin. It has since been edited by members of the Model Basin staff.

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LIST OF FIGURES

NOTATION

A_f	Cross-sectional area of frame
A_1, A_2, A_{12}, etc.	Plasticity coefficients
B	Axial rigidity
b	Faying width of frame
C	Coefficient in p_{sp} term
D	Flexural rigidity
E	Young's modulus
E_s	Secant modulus
E_t	Tangent modulus
F_1	$\dfrac{L_e}{L_f - b}$
h	Shell thickness
I	Moment of inertia of frame and one frame space of shell plating
I_e	Moment of inertia of frame and effective width L_e of shell plating
L	Length of cylinder
L_e	Effective width of shell plating
L_f	Center to center frame spacing
M	Moment
N	Force
n	Number of circumferential buckle lobes
P	Column load per unit width

p	Pressure
p_{cr}	General-instability pressure
p_f	Frame term
p_s	Shell term
q	External pressure per unit length of frame centerline
R	Mean shell radius
u, v, w	Displacement in the axial, circumferential, and radial directions, respectively
x, y, z	Coordinates in the axial, circumferential, and radial directions, respectively
α	$\dfrac{h^2}{12R^2}$
β	$\dfrac{3(1 - E_t/E_s)}{\sigma_i^2}$
γ	$\dfrac{pR^2}{2Eh^2} \sqrt{3(1 - \nu^2)}$
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Variational axial membrane, circumferential membrane, and shearing strains, respectively
η_1	$\frac{1}{2}\sqrt{1 - \gamma}$
η_2	$\frac{1}{2}\sqrt{1 + \gamma}$
θ	Angular coordinate
λ	$\dfrac{\pi R}{L}$

ν	Poisson's ratio
σ	Normal stress
σ_i	Stress intensity
τ	Shear stress
ϕ	$\dfrac{\left[3(1-\nu^2)\right]^{1/4}\left[L_f - b\right]}{\sqrt{Rh}}$
ϕ_1	$\dfrac{3pR}{4E_s h}$
ϕ_2	$-\dfrac{3N_x}{4E_s h}$
χ_1, χ_2, χ_3	Variational axial, circumferential, and twisting curvatures, respectively

Subscripts

p	Plastic
x, y	Axial and circumferential directions, respectively
xy or yx	Twisting

ABSTRACT

A cylindrical shell stiffened by uniformly spaced transverse rings and subjected to external hydrostatic pressure is analyzed for collapse by plastic general instability. In this analysis, strain-hardening of the material is taken into account. The plastic general-instability equation reduces to the elastic equation of Bryant when the secant and tangent moduli are set equal to Young's modulus.

INTRODUCTION

Recent efforts in the development of oceanographic vehicles for operation at great depths have led to the consideration of strain-hardening materials, such as aluminum alloys and steel alloys, with yield strengths above 125,000 psi. When these materials are used in conventional submarine structure, the ring-stiffened cylindrical shell, the influence of strain-hardening on collapse strength should be examined. Theory has already been developed to account for strain-hardening when failure occurs by instability of the shell between stiffeners. Reynolds[1] has presented a solution for the plastic buckling of the shell in the asymmetric mode and Lunchick[2] a solution for plastic buckling in the axisymmetric mode. A third mode of failure in the plastic domain is possible by plastic general instability involving collapse of both stiffeners and

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1 References are listed on page 26.

shell. To round out the theoretical plasticity research, a solution will be presented for the plastic general-instability pressure of a ring-stiffened cylindrical shell of strain-hardening material. In conjunction with the two theories applicable to shell failure, this theory should lead to more adequate design procedures for ring-stiffened cylindrical portions of deep-diving vehicles made from strain-hardening materials.

BACKGROUND

The David Taylor Model Basin has examined a number of solutions of the general-instability problem when failure is in the elastic range. Test results[3] indicate that the solution of Kendrick, Part III,[4] is substantially correct. Bryant[5] presented an approximate solution which agrees closely with another solution, Part I,[6] by Kendrick; both these solutions neglect interframe deformations. Bryant's formula offers the advantages of considerable simplicity over Kendrick's work and consequently is extensively used by the submarine designer. Krenzke[7] has shown that when Bryant's formula is modified to include an effective width of shell acting with the stiffener, the calculated pressures agree closely with the more exact Part III solution[4] which theoretically takes interframe deformations into account.

Since Bryant's formula is relatively simple to use and, when modified for an effective width concept, agrees substantially with test results, the most promising approach to the problem was to extend Bryant's elastic theory to plastic behavior.

ANALYSIS

Bryant's elastic theory will be modified to include plastic behavior. Bryant's formula is as follows:

$$p_{cr} = p_s + p_f \qquad [1]$$

where

$$p_s = \frac{Eh}{R} \bullet \frac{\lambda^4}{\left(n^2 + \frac{\lambda^2}{2} - 1\right)\left(n^2 + \lambda^2\right)^2} \qquad [2]$$

and

$$p_f = \frac{(n^2 - 1)EI}{R^3 L_f} \qquad [3]$$

Quoting Bryant,[5] the shell term p_s represents essentially "the pressure at which the same cylinder without ring frames and supported by its end bulkhead only, would buckle into one half-longitudinal wave and n circumferential waves." Again quoting Bryant, on the other hand, the frame term p_f "is the pressure at which a simple ring composed of one frame space of plating together with the ring frame would buckle into circumferential waves." Thus, Bryant has presented a simple expression showing the contribution of the shell and the framing as separate factors.

Frame term:

The elastic frame term p_f is found from the solution of the following differential equation:[8]

$$\frac{d^2w}{d\theta^2} + w\left(1 + \frac{qR^3}{EI}\right) = \frac{-M_A R^2 + qR^3 w_A}{EI} \qquad [4]$$

For $R = \infty$ and $M_A = w_A = 0$ (see Figure 1), the ring frame is transformed

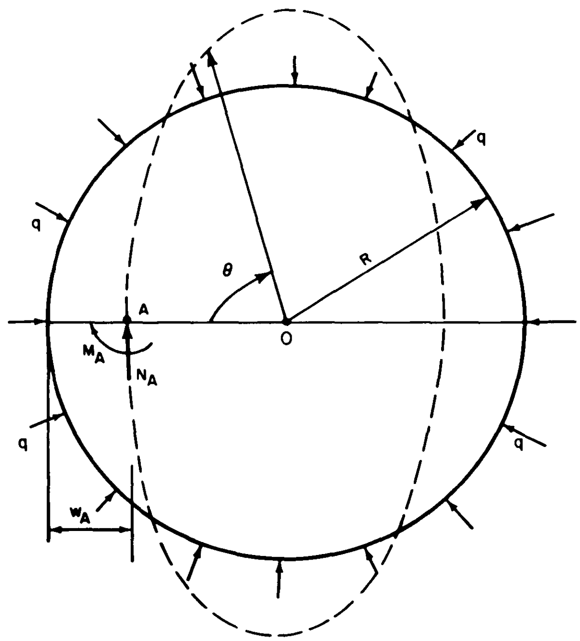


Figure 1 - Notation for Buckling of a Circular Ring under External Pressure

into a straight column with simple supports. Also, the quantity qR, which is the circumferential force in the ring, can be equated to P, the load per unit width on the straight column. Thus, for $R = \infty$, $M_A = w_A = 0$, $qR = P$, Equation [4] reduces to the equilibrium equation for the straight column.

$$EI\frac{d^2w}{dy^2} + Pw = 0 \qquad [5]$$

For buckling by pure bending, Engesser[9] and Shanley[10] have shown that Equation [5] is extended to the plastic case by the simple substitution of the tangent modulus E_t for Young's modulus E; thus:

$$E_t I \frac{d^2 w}{dy^2} + Pw = 0 \qquad [6]$$

Similarly, if buckling of the ring frame is by pure bending (no twisting), the same reasoning used for the straight column leads to the following modification of Equation [4] to apply to the plastic case:

$$\frac{d^2 w}{d\theta^2} + w\left(1 + \frac{qR^3}{E_t I}\right) = \frac{-M_A R^2 + qR^3 w_A}{E_t I} \qquad [7]$$

The solution of Equation [7] is analogous to that for the elastic case; thus:

$$q = \frac{(n^2 - 1) E_t I}{R^3} \qquad [8]$$

For a given width L_f, $q = p_{fp} L_f$; therefore the expression for the <u>plastic</u> frame term is:

$$p_{fp} = \frac{(n^2 - 1) E_t I}{R^3 L_f} \qquad [9a]$$

or dividing Equation [9a] by Equation [3]

$$p_{fp} = \frac{E_t}{E} p_f \qquad [9b]$$

Shell term:

Timoshenko[11] presents the equilibrium equations for a cylindrical shell in terms of moments and forces when the loading is by uniform lateral pressure or by uniform axial pressure. Although he treats the case of a cylindrical shell under combined lateral and axial pressure, he does not state explicitly comparable equilibrium expressions (in terms of moments and forces) for the case of hydrostatic pressure loading. Since the expressions for the case of hydrostatic pressure are combinations of those expressions for the other two types of loading and are derived in similar fashion, they will be presented without any lengthy derivation.

$$\frac{R\partial N_x'}{\partial x} + \frac{\partial N_{yx}'}{\partial \theta} - N_y \left(\frac{\partial^2 v}{\partial x \partial \theta} - \frac{\partial w}{\partial x} \right) = 0$$

$$\frac{\partial N_y'}{\partial \theta} + R\,\frac{\partial N_{xy}'}{\partial x} - \frac{\partial M_y'}{R\partial \theta} + \frac{\partial M_{xy}'}{\partial x} + RN_x \frac{\partial^2 v}{\partial x^2} = 0 \qquad [10]$$

$$\frac{\partial^2 M_{yx}'}{\partial x \partial \theta} + R\,\frac{\partial^2 M_x'}{\partial x^2} + \frac{\partial^2 M_y'}{\partial \theta^2} \quad \frac{\partial^2 M_{xy}'}{\partial x \partial \theta}$$

$$+ RN_x \frac{\partial^2 w}{\partial x^2} + N_y' - p\left(w + \frac{\partial^2 w}{\partial \theta^2} \right) = 0$$

The primed terms in Equation [10] denote variations arising during the buckling process. Also, the terms u, v, and w are variational displacements but are not primed for the sake of convenience. The notation applied to the displacement, forces, and moments is shown in

Figure 2. Equations [10] account for the stretching of the middle surface of the shell during buckling.

The variational moments and forces of Equation [10] can be expressed in terms of shell deformations during buckling. Gerard[12] presents expressions for the variational forces and moments for fully plastic states of buckling as follows:

$$
\begin{aligned}
N'_x &= B_p \left[A_1\varepsilon_1 + \frac{1}{2}A_{12}\varepsilon_2 - \frac{1}{2}A_{13}\varepsilon_3\right] \\
N'_y &= B_p \left[A_2\varepsilon_2 + \frac{1}{2}A_{12}\varepsilon_1 - \frac{1}{2}A_{23}\varepsilon_3\right] \\
N'_{xy} = N'_{yx} &= \frac{B_p}{2} \left[A_3\varepsilon_3 - \frac{1}{2}A_{31}\varepsilon_1 - \frac{1}{2}A_{32}\varepsilon_2\right] \\
M'_x &= -D_p \left[A_1\chi_1 + \frac{1}{2}A_{12}\chi_2 - \frac{1}{2}A_{13}\chi_3\right] \\
M'_y &= -D_p \left[A_2\chi_2 + \frac{1}{2}A_{21}\chi_1 - \frac{1}{2}A_{23}\chi_3\right] \\
M'_{yx} = -M'_{xy} &= -\frac{D_p}{2} \left[A_3\chi_3 - \frac{1}{2}A_{31}\chi_1 - \frac{1}{2}A_{32}\chi_2\right]
\end{aligned}
\qquad [11]
$$

where

$$B_p = \frac{4E_s h}{3} \qquad [12]$$

$$D_p = \frac{E_s h^3}{9} \qquad [13]$$

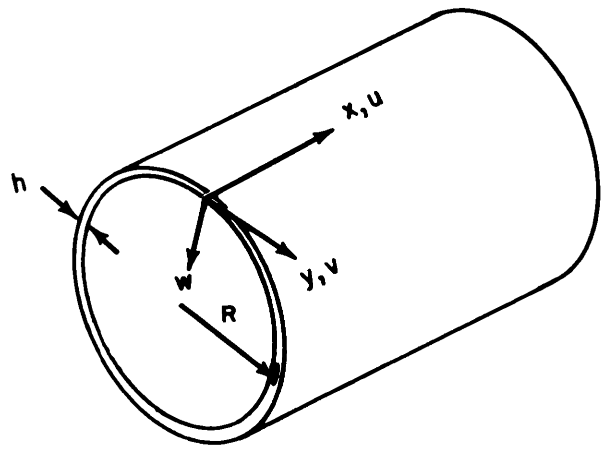


Figure 2a - Coordinates and Components of Displacement

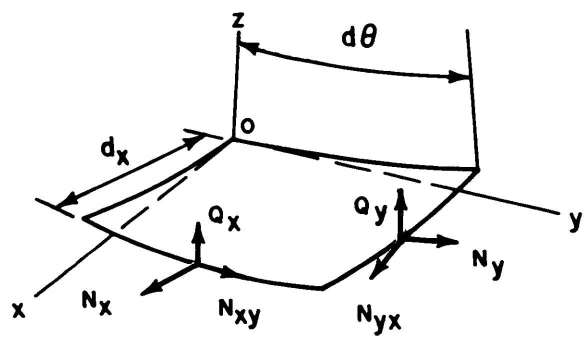


Figure 2b - Forces

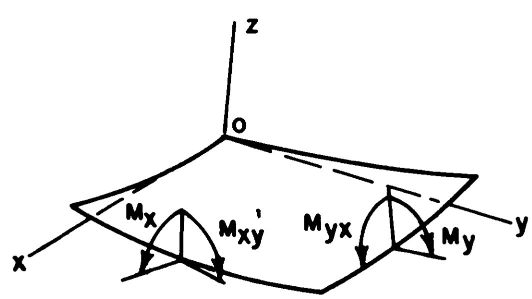


Figure 2c - Moments

Figure 2 - Notation for Buckling of a Cylindrical Shell under External Pressure

$$A_1 = 1 - \frac{\beta\sigma_x^2}{4}$$

$$A_2 = 1 - \frac{\beta\sigma_y^2}{4}$$

$$A_3 = 1 - \beta\tau^2$$

$$A_{21} = A_{12} = 1 - \frac{\beta\sigma_x\sigma_y}{2}$$

$$A_{31} = A_{13} = \beta\sigma_x\tau \qquad [14]$$

$$A_{32} = A_{23} = \beta\sigma_y\tau$$

$$\beta = \frac{3}{\sigma_i^2}\left(1 - E_{t/E_s}\right)$$

$$\sigma_i^2 = \sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y$$

For hydrostatic pressure loading $\tau = 0$, therefore:

$$A_{31} = A_{13} = A_{32} = A_{23} = 0$$

and

$$A_3 = 1$$

Consequently, Equations [13] simplify to

$$
\begin{aligned}
N_x' &= B_p\left[A_1\varepsilon_1 + \frac{1}{2}A_{12}\varepsilon_2\right] \\
N_y' &= B_p\left[A_2\varepsilon_2 + \frac{1}{2}A_{12}\varepsilon_1\right] \\
N_{xy}' &= N_{yx}' = \frac{B_p}{2}A_3\varepsilon_3 \\
M_x' &= -D_p\left[A_1\chi_1 + \frac{1}{2}A_{12}\chi_2\right] \\
M_y' &= -D_p\left[A_2\chi_2 + \frac{1}{2}A_{12}\chi_1\right] \\
M_{yx}' &= -M_{xy}' = -\frac{D_p}{2}\chi_3
\end{aligned}
\qquad [15]
$$

The variational strains and curvatures can be expressed in terms of displacements, thus:[11]

$$
\begin{aligned}
&\varepsilon_1 = \frac{\partial u}{\partial x} \qquad \varepsilon_2 = \frac{\partial v}{R\partial\theta} - \frac{w}{R} \qquad \varepsilon_3 = \frac{1}{2}\left(\frac{\partial u}{R\partial\theta} + \frac{\partial v}{\partial x}\right) \\
&\chi_1 = \frac{\partial^2 w}{\partial x^2} \qquad \chi_2 = \frac{1}{R^2}\left(\frac{\partial v}{\partial\theta} + \frac{\partial^2 w}{\partial\theta^2}\right) \qquad \chi_3 = \frac{1}{R}\left(\frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x\partial\theta}\right)
\end{aligned}
\qquad [16]
$$

When Equations [15] and [16] are substituted into Equations [10], equilibrium equations in terms of displacements and the plasticity coefficients (A's) result:

$$A_1 R^2 \frac{\partial^2 u}{\partial x^2} + \left(\frac{A_{12} + {}^1/2}{2}\right) R \frac{\partial^2 v}{\partial x \partial \theta} - \frac{A_{12}}{2} R \frac{\partial w}{\partial x}$$

$$+ R_1 \phi_1 \left(\frac{\partial^2 v}{\partial x \partial \theta} - \frac{\partial w}{\partial x}\right) + \frac{1}{4} \frac{\partial^2 u}{\partial \theta^2} = 0$$

$$\left(\frac{A_{12} + {}^1/2}{2}\right) R \frac{\partial^2 u}{\partial x \partial \theta} + \frac{1}{4} R^2 \frac{\partial^2 v}{\partial x^2} + A_2 \frac{\partial^2 v}{\partial \theta^2} - A_2 \frac{\partial w}{\partial \theta}$$

$$+ \alpha \left[A_2 \frac{\partial^2 v}{\partial \theta^2} + A_2 \frac{\partial^3 w}{\partial \theta^3} + \left(\frac{A_{12} + 1}{2}\right) R^2 \frac{\partial^3 w}{\partial x^2 \partial \theta} + \frac{R^2}{2} \frac{\partial^2 v}{\partial x^2} \right]$$

$$- R^2 \phi_2 \frac{\partial^2 v}{\partial x^2} = 0 \qquad [17]$$

$$\frac{A_{12}}{2} R \frac{\partial u}{\partial x} + A_2 \frac{\partial v}{\partial \theta} - A_2 w$$

$$- \alpha \left[A_2 \frac{\partial^3 v}{\partial \theta^3} + \left(\frac{2 + A_{12}}{2}\right) R^2 \frac{\partial^3 v}{\partial x^2 \partial \theta} + A_1 R^4 \frac{\partial^4 w}{\partial x^4} \right.$$

$$\left. + A_2 \frac{\partial^4 w}{\partial x^4} + (1 + A_{12}) R^2 \frac{\partial^4 w}{\partial x^2 \partial \theta^2} \right] =$$

$$\phi_1 \left(w + \frac{\partial^2 w}{\partial \theta^2} \right) + \phi_2 R^2 \frac{\partial^2 w}{\partial x^2}$$

where

$$\alpha = \frac{h^2}{12R^2} \qquad [18]$$

$$\phi_1 = \frac{3pR}{4E_s h} \qquad [19]$$

$$\phi_2 = -\frac{3N_x}{4E_s h} \qquad [20]$$

Equations [17] reduce to those in Reference 11 for the elastic case when $E_t = E_s = E$ and Poisson's ratio $\nu = 1/2$.

Displacement functions are chosen, assuming that the edges of the shell are simply supported during buckling and that only one half-wave of buckling occurs in the axial direction.

$$u = A \sin n\theta \cos \frac{\pi x}{L}$$

$$v = B \cos n\theta \sin \frac{\pi x}{L} \qquad [21]$$

$$w = C \sin n\theta \sin \frac{\pi x}{L}$$

Substituting Equations [21] into Equation [17], one obtains for A_1, B, and C three homogeneous linear equations

$$\left(A_1\lambda^2 + \frac{n^2}{4}\right)A + \left[\left(\frac{2A_{12} + 1}{4}\right) + \phi_1\right](n\lambda)B$$

$$+ \left(\frac{A_{12}}{2} + \phi_1\right)\lambda C = 0$$

$$\left(\frac{2A_{12} + 1}{4}\right)(n\lambda)A + \left[\frac{\lambda^2}{4} + A_2 n^2 - \lambda^2\phi_2 + \left(A_2 n^2 + \frac{\lambda^2}{2}\right)\alpha\right]B$$

$$+ \left[A_2 + \left(A_2 n^2 + \frac{A_{12} + 1}{2} \bullet \lambda^2\right)\alpha\right](n)C = 0 \qquad [22]$$

$$\left(\frac{A_{12}\lambda}{2}\right)A + \left[A_2 + \left(A_2 n^2 + \frac{2 + A_{12}}{2} \bullet \lambda^2\right)\alpha\right](n)B$$

$$+ \left\{A_2 + (1 - n^2)\phi_1 - \lambda^2\phi_2 + \left[A_2 n^4 + (1 + A_{12})n^2\lambda^2 + A_1\lambda^4\right]\alpha\right\}C = 0$$

where $\lambda = \frac{\pi R}{L}$.

Setting the determinant of Equation [22] equal to zero, one obtains an expression containing the buckling pressure. Thus

$$C_1 + C_2\alpha = C_3\phi_1 + C_4\phi_2 \qquad [23]$$

As in the derivation of the elastic Bryant term p_s, the term $C_2\alpha$ can be neglected if n is small ($n \leq 5$) and λ is large ($\lambda > 1.35$). Also for a long cylinder under hydrostatic pressure

$$N_x = -\frac{pR}{2} \quad \text{so} \quad \phi_2 = \frac{1}{2}\phi_1$$

Equation [23] can be simplified to:

$$C_1 = \left(C_3 + \frac{C_4}{2}\right)\phi_1 \qquad [24]$$

in which

$$C_1 = \left[A_1 - \frac{A_{12}^2}{4A_2}\right]\lambda^4$$

$$C_3 = (n^2 - 1)\left[\frac{A_1}{A_2}\lambda^4 + \frac{4A_1A_2 - A_{12}^2 - A_{12}}{A_2}(n^2\lambda^2) + n^4\right] + \frac{A_{12}}{2A_2}\lambda^4 \qquad [25]$$

$$C_4 = \lambda^2\left[\frac{A_1}{A_2}\lambda^4 + \frac{4A_1A_2 - A_{12}^2 - A_{12}}{A_2}(n^2\lambda^2) + n^4\right] + \lambda^2 n^2 + \left(4A_1 - \frac{A_{12}^2}{A_2}\right)\lambda^4$$

The plasticity coefficients of Equation [14] for a long cylinder under hydrostatic pressure are given by Reynolds as:

$$A_1 = \frac{3}{4} + \frac{1}{4}\frac{E_t}{E_s}$$

$$A_2 = A_{12} = \frac{E_t}{E_s} \qquad [26]$$

Substituting Equations [25], [26], and [19] into Equation [24], one obtains, after neglecting small order terms, the following expression for the plastic shell term:

$$p_{sp} = E_t\left(\frac{h}{R}\right)\frac{\lambda^4}{\left(n^2 + \frac{\lambda^2}{2} - 1\right)\left(n^2 + \lambda^2\right)^2\left[\frac{E_t}{E_s} + \frac{3}{4}\left(1 - \frac{E_t}{E_s}\right)\frac{\lambda^4}{\left(n^2 + \lambda^2\right)^2}\right]} \qquad [27]$$

Dividing Equation [27] by Equation [2] gives an expression for p_{sp} in terms of the elastic term p_s.

$$p_{sp} = C\,\frac{E_s}{E}\,p_s \qquad [28]$$

where

$$C = \frac{E_t/E_s}{\left[\frac{E_t}{E_s} + \frac{3}{4}\left(1 - \frac{E_t}{E_s}\right)\frac{\lambda^4}{\left(n^2 + \lambda^2\right)^2}\right]} \qquad [29]$$

Equation [29] is presented in graphical form in Figure 3 so that the coefficient C can be rapidly obtained.

Figure 3 - Coefficient for Term p_{sp}

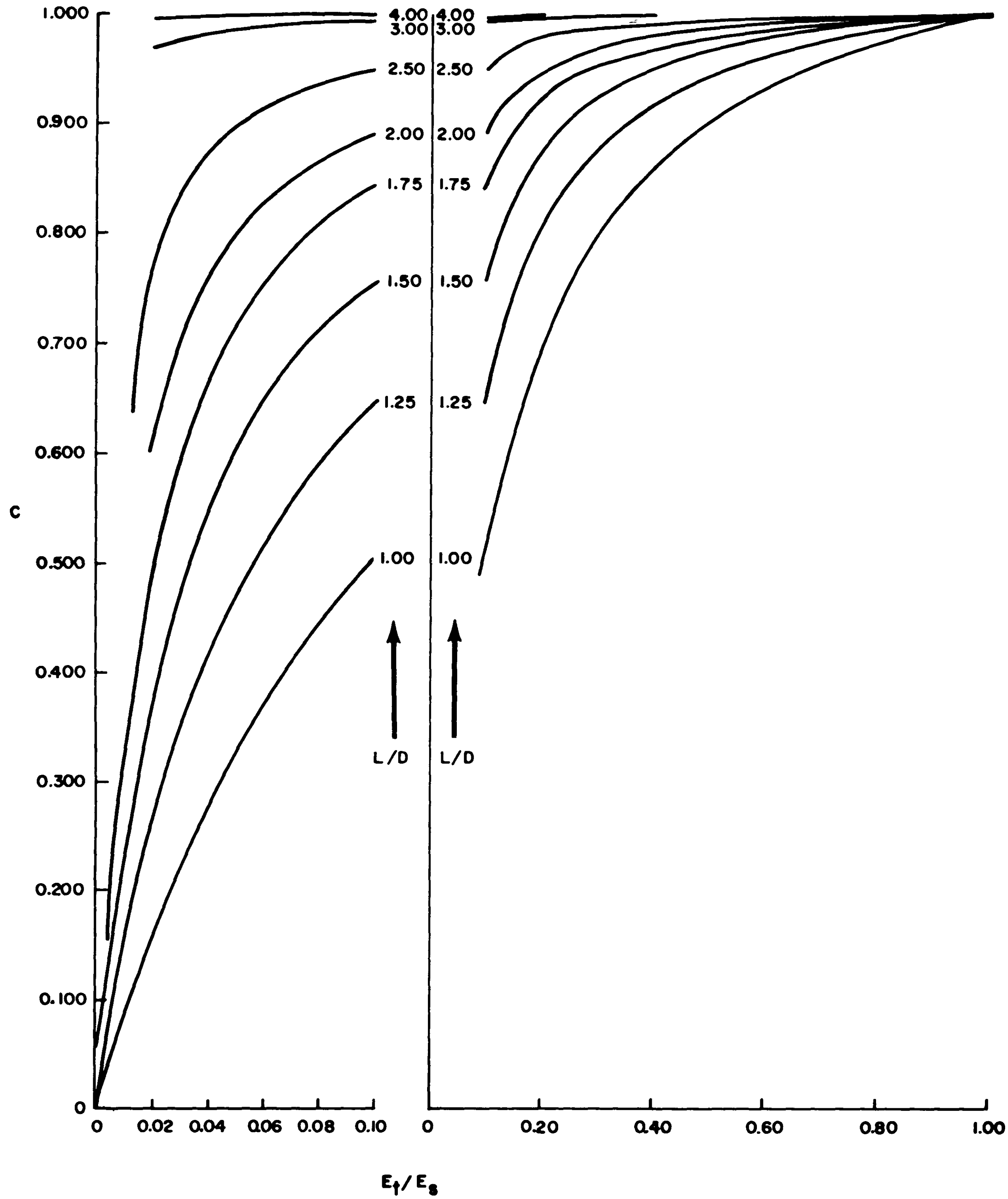
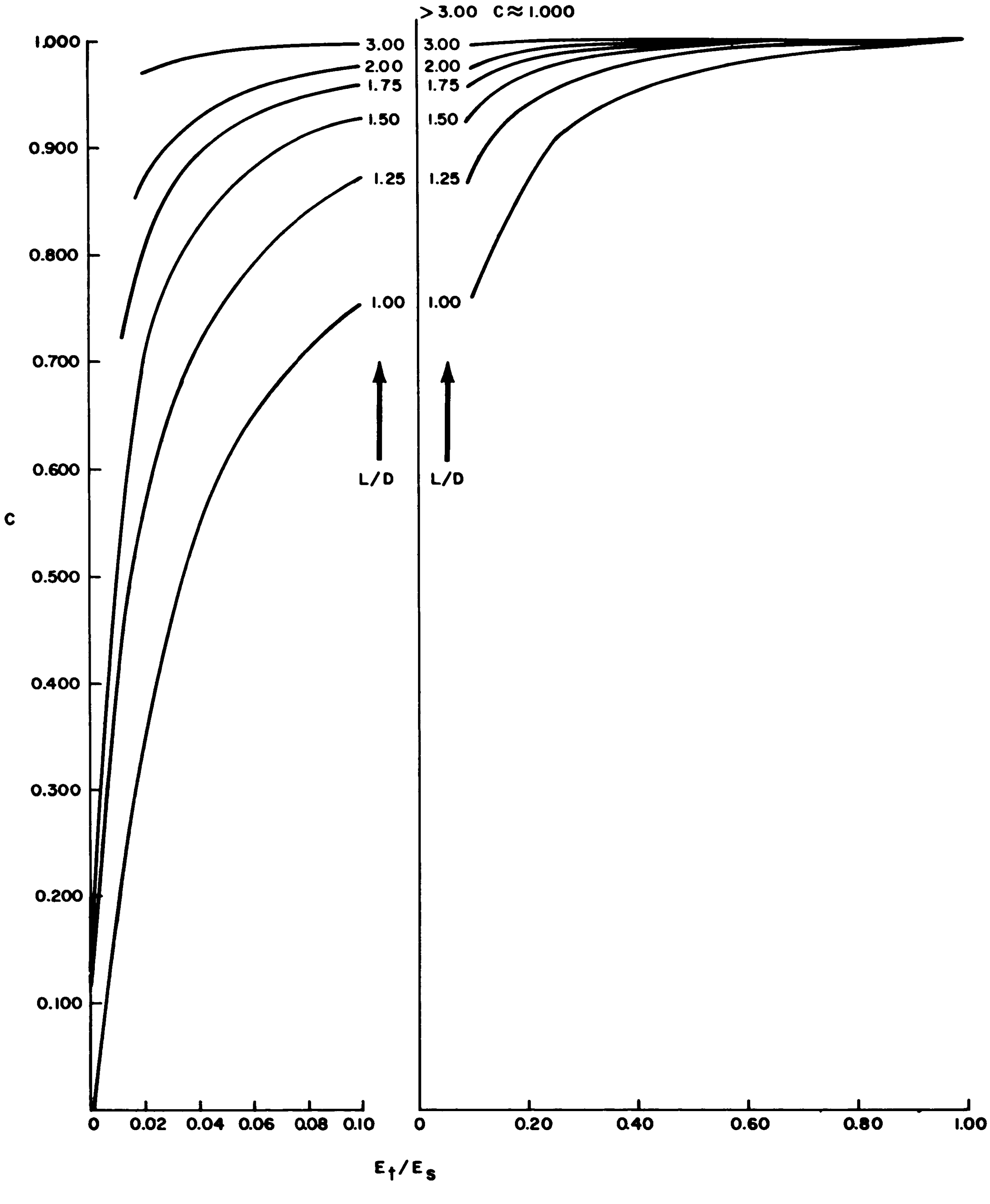


Figure 3a - n=2



> 3.00 C ≈ 1.000
1.000
0.900
0.800
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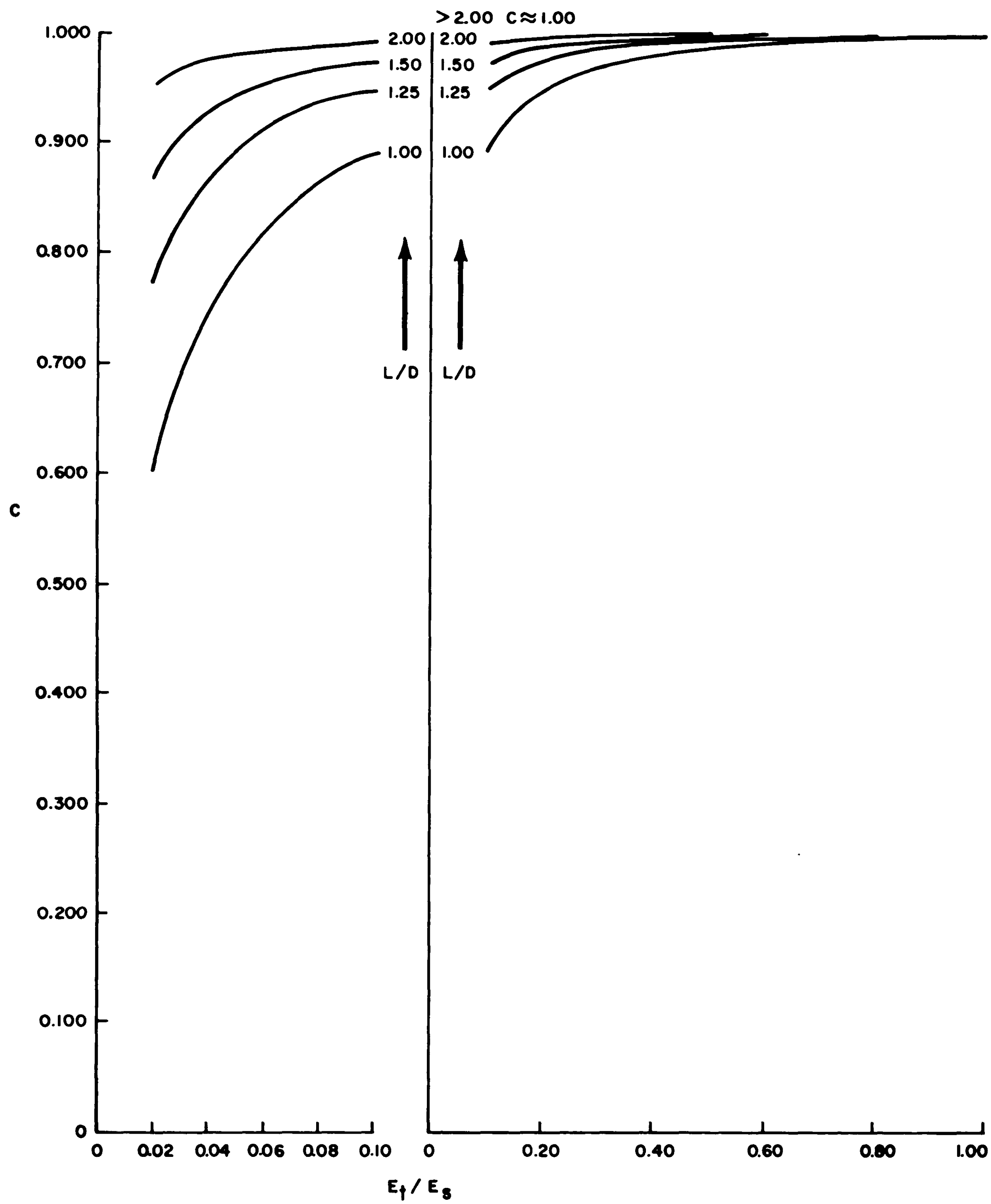
0.600
0.500
0.400
0.300
0.200
0.100
C
3.00
2.00
1.75
1.50
1.25
1.00
L/D
L/D
0
0.02
0.04
0.06
0.08
0.10
0.20
0.40
0.60
0.80
1.00
Et/Es

Figure 3b - n=3



>2.00 C≈1.00
1.000
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C
2.00
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L/D
0
0.02
0.04
0.06
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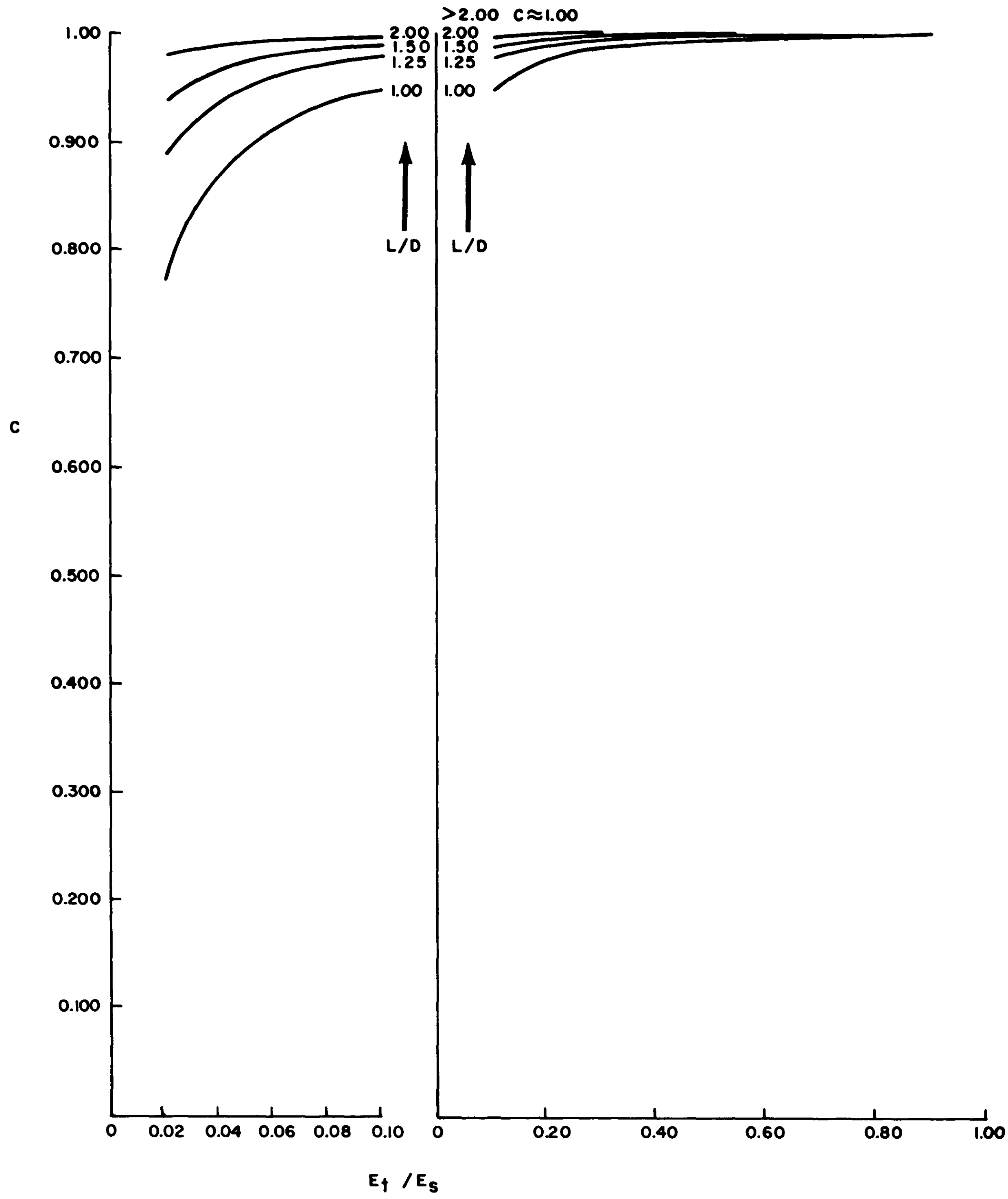
0.40
0.60
0.80
1.00
E_t / E_s

Figure 3c - n=4


>2.00 C≈1.00
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0.08
0.10
0.20
0.40
0.60
0.80
1.00
Et /Es

Figure 3d - n=5

Summing Equations [9b] and [28], one obtains the expression for the plastic general-instability pressure:

$$p_p = p_{sp} + p_{fp} \qquad [30a]$$

or

$$p_p = C \frac{E_s}{E} p_s + \frac{E_t}{E} p_f \qquad [30b]$$

For $E_s = E_t = E$, Equation [30b] reduces to the elastic expression of Bryant.

PROCEDURES FOR DETERMINING PLASTIC GENERAL-INSTABILITY PRESSURE

The procedure for determining the plastic general-instability pressure is analogous to those presented[1,2] for the two modes of plastic shell buckling. Essentially, one must ascertain that pressure at which the prebuckling equilibrium of the cylinder and the instability expression, Equation [30b], are satisfied simultaneously. The procedure is to plot p-σ_i curves for both the prebuckling and buckling states; the intersection of the two curves is the critical pressure.

The expression describing the prebuckling state is derived by assuming that the stiffened cylinder is stressed uniformly. In reality, stresses vary along the length of the cylinder and also through the shell thickness. Hence, the moduli of Equation [30b] vary from point to point. However, in the derivation of Equation [30b], the term p_{fp} was based on a uniformly stressed frame-shell combination and the term p_{sp}

was based on a uniformly stressed long cylindrical shell. Thus, neglecting bending, the assumption of uniform stresses results in the following when the internal stresses are equilibrated to the external hydrostatic pressure loads:

$$\sigma_x = \frac{pR}{2h}$$

$$\sigma_y = \frac{pR^*}{h\left(1 + \frac{A_f}{L_f h}\right)} \qquad [31]$$

The expression for stress intensity is[1,2]

$$\sigma_i = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y} \qquad [32]$$

When Equations [31] are substituted into Equation [32], the prebuckling equilibrium expression results:

$$\sigma_i = \frac{pR}{2h\left(1 + \frac{A_f}{L_f h}\right)} \cdot \sqrt{4 - 2\left(1 + \frac{A_f}{L_f h}\right) + \left(1 + \frac{A_f}{L_f h}\right)^2} \qquad [33]$$

For a given geometry (R, h, L_f, A_f), Equation [33] can be used to plot p against σ_i.

---

* Although the value for σ_y given here is inconsistent with Equations [19] and [26] where it was defined as equal to $\frac{pR}{h}$, it is preferable at this point not to make such a simplification. By retaining the variability of σ_y with A_f, as expressed in Equation [31], a more realistic value of the stress level is obtained.

From the uniaxial compressive stress-strain curve of the material, a table can be obtained of σ_i, E_s and E_t. The modulus E_s is computed as simply total stress divided by total strain and the modulus E_t is the slope of the stress-strain curve at a particular stress, taken equal to σ_i. For different σ_i, p_p can be computed from Equation [30b]. A plot of p_p against σ_i can then be obtained.

The pertinent buckling pressure p_{cr} is obtained at the intersection of the p - σ_i curves plotted from Equations [30b] and [33]. The procedure is clearly shown in Figure 4.

For an elastic, perfectly plastic material or one that exhibits <u>no</u> strain-hardening, the foregoing procedure would result in the general-instability pressure being equal to that value at which σ_i by Equation [33] is equal to the yield strength of the material. Or in other words, plastic general instability occurs when frames and shell are uniformly stressed to the yield point.

DISCUSSION

The theoretical expression for the plastic general-instability pressure, Equation [30b], has a number of advantages. It reduces to the elastic Bryant equation. Like the Bryant equation, the plastic equation is short, easy to use, and separates the strength contributions of the stiffening rings from that of the shell. The plastic equation accounts for material strain-hardening and is based on the actual shape of stress-strain curve and not on some approximate shape, e.g.,linear, piece-wise curve.

P_e = ELASTIC BUCKLING PRESSURE

P_{cr} = PLASTIC BUCKLING PRESSURE

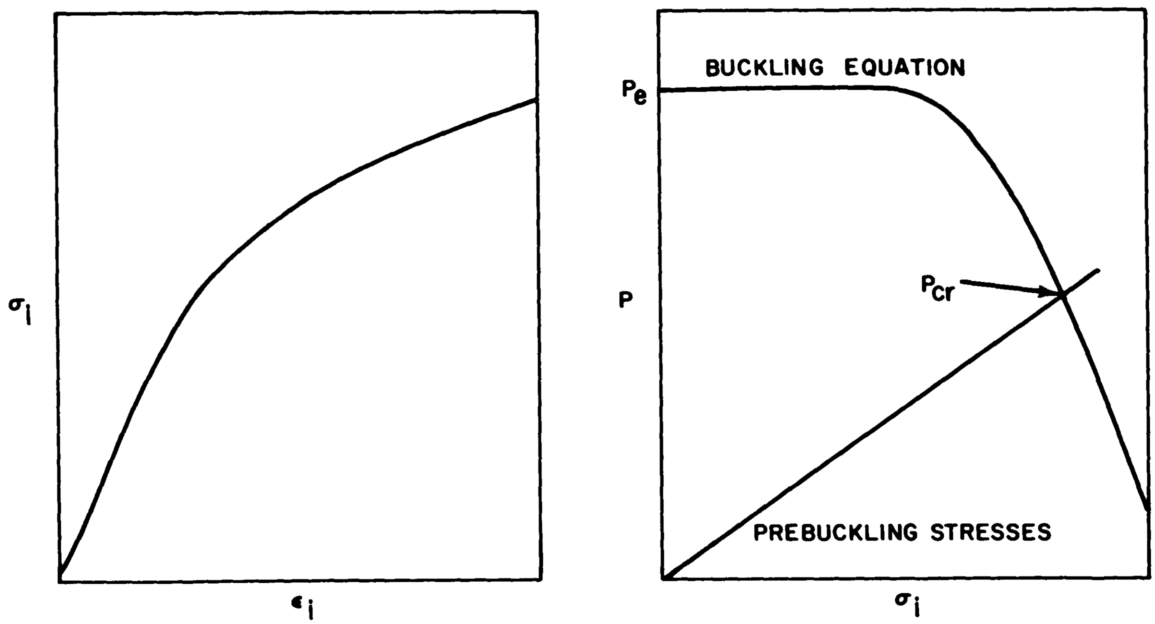

Figure 4a - Strain-Hardening Material

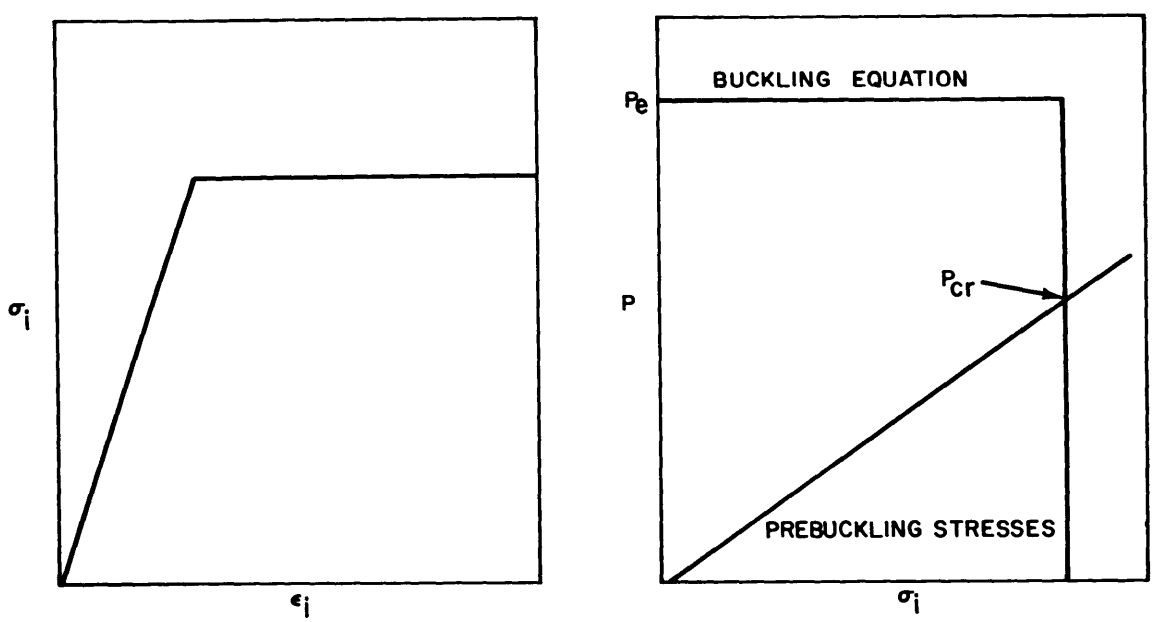

Figure 4b - Elastic-Perfectly Plastic Material

Figure 4 - Graphical Determination of Plastic Buckling Pressure

The theoretical expression, of course, has certain limitations. It has all the limitations of the Bryant equation, which is an approximation. Some of these limitations should be overcome, as in the elastic case, by using an effective width of shell. It is recommended that the following expression for L_e presented by Krenzke[7] be used.

$$L_e = (L_f - b)F_1 \tag{34}$$

where

$$F_1 = \frac{4}{\theta}\left[\frac{\cosh^2\eta_1\theta - \cos^2\eta_2\theta}{\dfrac{\cosh\eta_1\theta\sinh\eta_1\theta}{\eta_1} + \dfrac{\cos\eta_2\theta\sin\eta_2\theta}{\eta_2}}\right]$$

$$\eta_1 = \frac{1}{2}\sqrt{1-\gamma} \quad , \quad \eta_2 = \frac{1}{2}\sqrt{1+\gamma}$$

$$\gamma = \frac{pR^2}{2Eh^2}\sqrt{3(1-\nu^2)}$$

$$\theta = \frac{\left[3(1-\nu^2)\right]^{1/4}}{\sqrt{Rh}} \cdot \left[L_f - b\right]$$

F_1 may be determined graphically from Reference 13. Admittedly, this effective width expression is derived from elastic considerations, but it is offered in lieu of a more exact plastic expression, which is not available.

Instead of the more rigorous incremental theory of plasticity, the analysis is based on the deformation theory of plasticity with its limitations. As with the buckling of columns and plates, the usefulness of the deformation theory has to be assessed in the light of experimental results. It is suspected, however, that deformation theory gives a lower value of the frame term p_f than would incremental theory. The tangent modulus is used as in the case of column buckling where the tangent modulus load has been shown by Shanley[10] to be a lower limit.

The plasticity coefficients (A_1, A_2, A_{12}) in the final buckling equation were derived for a Poisson's ratio of 1/2 whereas, in reality, Poisson's ratio generally increases with stress intensity σ_i from an elastic value (e.g., $\nu = 0.3$) to an upper limit of 1/2 for an isotropic, incompressible material.

Finally, another important point is that the plastic general-instability equation does not account for geometrical imperfections and residual stresses. These influences should warrant further study.

In design, a plastic general-instability expression should prove useful. The general-instability pressure should be computed on the basis of the plastic theory instead of the elastic theory. New margins of safety should be used with the plastic pressure instead of taking the design collapse pressure arbitrarily as 1/2 or 1/3 the elastic general-instability pressure.

RECOMMENDATIONS

1. The plastic theory requires experimental confirmation. As in the elastic case, tests should be conducted of both internally framed and externally framed cylinders so that failure by twisting of internal frames

can be assessed. In addition, both welded and machined-stiffened cylinders should be tested to evaluate the role of geometrical imperfections and residual stresses.

2. Expressions for effective width L_e should be derived for plastic behavior.

3. Theory should be developed to account for the influence of out-of-roundness on the plastic general-instability pressure.

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Mr. R. D. Short provided valuable service in the development and review of the theoretical analysis.

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