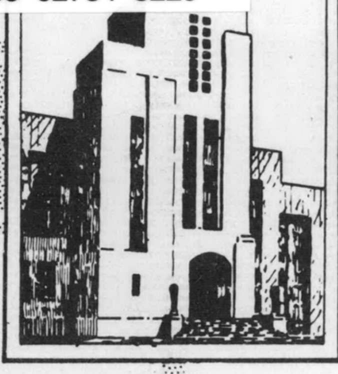

DEPARTMENT OF THE NAVY

DAVID TAYLOR MODEL BASIN

HYDROMECHANICS

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AERODYNAMICS

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STRUCTURAL MECHANICS

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APPLIED MATHEMATICS

A NOMENCLATURE FOR STABILITY AND CONTROL

by

Frederick H. Imlay

[redacted]

HYDROMECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

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ABSTRACT

A general purpose nomenclature is presented for dealing with problems in stability and control of surface ships, submarines, torpedoes, and towed bodies. The nomenclature is essentially an extension of the nomenclature for submerged bodies that was adopted by the American Towing Tank Conference in 1948. The need for the revision has been brought about by the numerous new techniques that have been introduced in the interim, and by the desire to develop a uniform nomenclature to cover all classes of stability and control problems dealt with by the Stability and Control Division at the David Taylor Model Basin.

INTRODUCTION

The nomenclature adopted by the American Towing Tank Conference has been used in reporting the work of the Stability and Control Division at the David Taylor Model Basin for many years.[1] This nomenclature has been very satisfactory because the symbols have a meaningful connotation with the quantities represented and most of the symbols can be produced on a standard typewriter.

A marked broadening of the work of the Stability and Control Division has taken place since the adoption of the nomenclature of Reference 1. Problems dealing with surface ships, submarines, torpedoes, and towed bodies are explored by means of full-scale trials, model tests, analog simulator studies, and theoretical analyses. Techniques from servomechanism theory, human engineering, statistical methods, and network theory are routinely employed in the solution of these problems.

As a consequence of the expanded scope of the work of the Division a need has developed for a more comprehensive nomenclature. In any revision of the nomenclature it was desirable that the past policy of reporting all work of the Division in a consistent nomenclature be continued. Consequently a committee was appointed by the Division Chief, Mr. Morton Gertler, to explore the Division's needs and make recommendations for a revised nomenclature. Members of the committee were:

F. H. Imlay, Division Specialist, Chairman
A. Goodman, Head of Prediction Branch
S. C. Gover, Head of Evaluation Branch
W. M. Ellsworth, Head of Towing Problems Branch

The nomenclature presented in this report is largely based on the work of this committee. In addition to Reference 1, References 2, 3, and 4 were consulted during the preparation of the nomenclature.

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[1] References are listed on page 60

Objectives considered desirable by the committee in selecting the nomenclature were:

Adherence insofar as practicable to the nomenclature adopted by the American Towing Tank Conference.[1]

Extension of the notation to be based on principles established in Reference 1, so that the advantage of meaningful connotation would be retained.

Restriction of the notation to symbols that are readily obtainable for office typewriters.

Elimination of any cases where two different symbols define the same quantity.

Avoidance of the use of superscripts.

Location of subscripts all on one level, i.e., no subscripts on subscripts.

The nomenclature is presented in two separate lists. The first list groups the symbols functionally, thus providing a ready reference for suitable symbols if one knows the kind of quantity to be designated. The second list is alphabetical, grouped in the order: English alphabet, Greek alphabet, numerical, other.

Much of the existing work on cables towed in water has been reported in a nomenclature different in concept from that of Reference 1. This specialized nomenclature will continue to be of interest as long as the papers that employ it remain the best available references on the subject; consequently it has been included in a section following the Alphabetical List of Symbols. The specialized nomenclature for towed cables was obtained from Reference 5. As future work is done in this field by the Stability and Control Division, the basic nomenclature for stability and control will be extended to include towed cables.

Because the broad general needs of various groups working on problems related to surface ships, submerged bodies, and towed bodies have been kept in mind, the definitions of the various symbols have been kept as general as possible. It is hoped that this revised nomenclature will prove sufficiently adjustable to current needs, therefore, to insure its wide official adoption after a reasonable period of refinement.

GENERAL NOTES

The following general remarks about the nomenclature may be made: The quantities defined in the Functional List of Symbols are for the most part dimensional. Nondimensional forms for these quantities are given in the Alphabetical List of Symbols. The primary method of nondimensionalizing is that used in Reference 1, wherein a characteristic length ℓ of the body, half the mass of a cube of fluid of dimension ℓ on each edge, and the time for the body to travel a distance ℓ are employed. These quantities, raised to appropriate powers, are used as divisors to convert a dimensional symbol to a nondimensional one. The resultant nondimensional symbol is indicated by adding a prime after the symbol.

Some of the quantities to be nondimensionalized are functions of absolute velocity whereas others are functions of the velocity relative to the fluid. It would be desirable to nondimensionalize the latter on the basis of the relative velocity. If such a course is followed, however, inconsistencies develop, as for example, when time derivatives of such quantities are in turn nondimensionalized. Consequently, the nondimensional quantities have all been based on absolute velocity. Data obtained in the usual towing-basin tests can readily be reduced to the prescribed nondimensional form because in such tests the relative velocity is also the absolute velocity. When such nondimensional results are applied to a problem where the free-stream fluid is not at rest, however, the ratio of the relative to the absolute velocity will also be involved.

All quantities involving angles or their derivatives are in radian measure unless an author specifically defines them otherwise in a given case.

The use of a bar above a quantity has been reserved to indicate a vector.

An effort has been made to avoid the use of superscripts. With the exception of the established use of a prime to indicate a nondimensional quantity, and a few symbols in the nature of mathematical operators placed above a character, all modifications are made by the use of subscripts. As a result, the use of superscript notations within a paper to indicate footnotes or references will not lead to confusion if the notation happens to follow a symbol.

In the formation of compound subscripts, if the symbol indicating an axis is involved, this subscript usually is placed at the end. The exception occurs when the basic quantity is a symbol with an axis subscript and the secondary subscript is a modifier of the basic symbol with axis subscript combination, e.g., the maximum value of an x_0 distance is designated x_{0m}.

Subscripts defining coordinates of a specific point are indicated by capital letters.

All of the needed subscript variations of a given quantity may not necessarily be listed. In forming additional variations in such cases, one can be guided by the given examples. For further guidance, a list of subscripts with their definitions is given after the Functional List of Symbols.

Variations in the definition of a given quantity are indicated by subscript modification. A somewhat related problem is that requiring the designation of different numerical values of the same physical quantity. Three mechanisms are available for making such distinctions. In the first, an index consisting of a numerical subscript modifier is added, e.g., σ_i for the different stability roots. The second method consists of enclosing the affected quantity in parentheses and placing a subscript outside. When the difference is spatial the subscript is a capital letter, e.g., $(T)_P$ and $(T)_S$ for port and starboard thrust. The third method is employed when more than one parameter is necessary to fix the value of a quantity. In this case, the values of the parameters are enclosed in parentheses after the quantity, e.g., $M_{qw}(q_1, w_1)$ specifies the value of $\partial^2 M/\partial_q \partial_w$ when $q = q_1$ and $w = w_1$.

Subscript modifications need not be employed when their omission cannot introduce confusion. For example, if only fixed axes are needed in a particular case, it is permissible to designate them x, y, z instead of x_0, y_0, z_0. Whenever such subscripts are omitted, however, it is the author's responsibility to completely define the departure from the normal meaning of the symbol.

FUNCTIONAL LIST OF SYMBOLS

REFERENCE FRAMES

Axes

x_0, y_0, z_0 — a right-hand orthogonal system of fixed axes (sometimes referred to as "inertial" axes). No reference is of known absolute fixity, but unless otherwise specified, the x_0, y_0, z_0 system is fixed relative to the surface of the earth, the z_0 axis is vertically down, the x_0z_0 plane contains the initial direction of motion of the body whose motion is under study, and the x_0 axis lies in the general direction of the initial motion of the body.

x, y, z — a right-hand orthogonal system of moving axes, fixed in the body. If not otherwise specified, the z axis is directed toward the bottom of the body, the xz plane lies in the vertical plane of symmetry of the body, the origin O is located at the center of mass of the body, and the x axis is forward and parallel to the reference or base line used to establish the body's shape. (See Figure 1.) If only one set of moving axes is required, this set shall be used, with sufficient qualifying specifications to completely establish any exceptions from the description just given.

x_a, y_a, z_a — a system of axes similar to x, y, z except that the axes lie along the principal axes of inertia through the origin O_a of the system. If not otherwise specified, the remaining features of the system correspond to the usual definition for the x, y, z system. Specifically the subscript a denotes that the reference frame is aligned with principal axes of inertia.

x_b, y_b, z_b — a translating system of axes, whose respective axes remain parallel to the x_0, y_0, z_0 system, but whose origin O_b maintains coincidence with the position of the origin of the moving x, y, z system or other specified moving reference frame

x_c, y_c, z_c — a right-hand orthogonal system of moving axes fixed in a control surface with specified orientation

x_f, y_f, z_f — a right-hand orthogonal system of moving axes fixed in the fluid with specified location and orientation

x_g, y_g, z_g — a right-hand orthogonal system of moving axes whose origin O_g is located at the center of mass or some other specified geometric point. The orientation of the system shall be specified.

x_s, y_s, z_s — a supplemental right-hand orthogonal system of moving axes for a second body or similar uses

Origins

O	origin of the x, y, z reference frame
O_a	origin of the x_a, y_a, z_a reference frame
O_b	origin of the x_b, y_b, z_b reference frame
O_c	origin of the x_c, y_c, z_c reference frame
O_f	origin of the x_f, y_f, z_f reference frame
O_g	origin of the x_g, y_g, z_g reference frame
O_s	origin of the x_s, y_s, z_s reference frame
O_0	origin of the x_0, y_0, z_0 reference frame

Orientation of Axes

Euler angles

ψ, θ, ϕ — angles of heading, pitch, and roll, respectively. These angles, taken in the order given here, define the angular orientation of the x, y, z axes relative to the x_0, y_0, z_0 axes. Thus, referring to Figure 2, and starting from initial coincidence of the two systems, the x, y, z axes are rotated in a clockwise sense through the angle ψ about the z_0 axis. The x, y, z axes thus acquire the new position x_1, y_1, z_0. The x, y, z axes are next rotated in a clockwise sense through the angle θ about the y_1 axis to acquire the second intermediate position x, y_1, z_1. Finally the x, y, z axes are rotated in a clockwise sense through the angle ϕ about the x axis and thus acquire the desired position x, y, z.

ψ_a, θ_a, ϕ_a	angles defining the angular orientation of the x_a, y_a, z_a axes relative to the x_0, y_0, z_0 axes
ψ_f, θ_f, ϕ_f	angles defining the angular orientation of the x_f, y_f, z_f axes relative to the x_0, y_0, z_0 axes. When x_f has the direction of the instantaneous linear velocity of the origin O of the body relative to the fluid, θ_f is the flight-path angle.

Direction cosines

ℓ_x, ℓ_y, ℓ_z	direction cosines giving the component, in the direction of the x, y, or z axis, of a unit vector along the x_0 axis. The quantities ℓ_x, ℓ_y, ℓ_z are the cosines of the angles $\alpha_x, \alpha_y, \alpha_z$ in Figure 3.
m_x, m_y, m_z	direction cosines giving the component, in the direction of the x, y, or z axis, of a unit vector along the y_0 axis
n_x, n_y, n_z	direction cosines giving the component, in the direction of the x, y, or z axis, of a unit vector along the z_0 axis

GEOMETRIC CHARACTERISTICS OF BODY

Bodies In General

Linear dimensions

λ	linear ratio, full-scale to model size
ℓ	length; when not otherwise defined, the overall physical length. This symbol should be reserved for some characteristic length.
b	beam or breadth
h	height
t	thickness
d	diameter
R	radius
H	draft

x, y, z	a length or coordinate in the direction of the $x, y,$ or z axis
x_a, y_a, z_a	a length or coordinate in the direction of the $x_a, y_a,$ or z_a axis
x_{Fi}, y_{Fi}, z_{Fi}	$x, y,$ or z coordinate of a fin, hydrofoil, etc.; $i = 1, 2, \cdots$ (i is omitted if only one item is involved)
x_P, y_P, z_P	$x, y,$ or z coordinate of the center of pressure or of a typical point P
x_T, y_T, z_T	$x, y,$ or z coordinate of the towpoint
LBP	length between perpendiculars
LCB	longitudinal distance of center of buoyancy from forward perpendicular
LCG	longitudinal distance of center of mass from forward perpendicular
LOA	length overall
LWL	length at design waterline

Specific locations

AP	after perpendicular
FP	forward perpendicular
CB	center of buoyancy
CG	center of mass
CM	metacenter
CP	center of pressure
CS	static center; the center of the resultant of weight and buoyancy
P	a typical point
TP	towpoint

Areas

A	area or projected area
A_f, A_h, A_ℓ	projected frontal area, horizontal area, or lateral area of a body onto the yz, zx, or xy plane
A_x, A_y, A_z	projected area of the submerged part of a body onto the yz, zx, or xy plane
C_w	waterline coefficient; $C_w = A_z/b\ell$
C_x	maximum section coefficient; $C_x = A_x/bH$
S	wetted-surface area

Volumes

V	volume
C_B	block coefficient; $C_B = V/Hb\ell$
C_P	prismatic coefficient; $C_P = V/A_x\ell$
C_{Pv}	vertical prismatic coefficient; $C_{Pv} = V/A_zH$

Lifting Surfaces

b	span
c	chord
c_g	mean geometric chord
c_r	root chord
c_t	tip chord
t	thickness
R	radius
a	aspect ratio; $a = b^2/A$ (area to be specified by appropriate subscript)
λ	taper ratio; $\lambda = c_t/c_r$
Λ	sweepback angle of quarter-chord line

Γ	dihedral angle
i	angle of incidence
A	area
A_b, A_d, A_f, A_r, A_s	projected area of bow plane, dorsal rudder, flap, rudder, or stern plane (planform area)
A_{bx}, A_{by}, A_{bz}	projection of bowplane area onto the yz, zx, or xy plane
$A_{bxa}, A_{bya}, A_{bza}$	projection of bowplane area onto the $y_a z_a$, $z_a x_a$, or $x_a y_a$ plane
S	wetted-surface area
δ	angular deflection of a control surface, positive for clockwise rotation of control when viewed from port side, from above, or from astern
$\delta_b, \delta_d, \delta_f, \delta_r, \delta_s$	deflection of bow plane, dorsal rudder, flap, rudder, or stern plane (see δ for positive sense)
δ_{ft}	deflection of tab on a doubly-movable flap (see δ for positive sense)

HYDROSTATIC CHARACTERISTICS OF BODY

B	buoyancy, positive upward
Δ	displacement, positive upward
H	draft
CB	center of buoyancy
x_B, y_B, z_B	x, y, or z coordinate of the center of buoyancy
x_{Ba}, y_{Ba}, z_{Ba}	x_a, y_a, or z_a coordinate of the center of buoyancy
CS	static center; the center of the resultant of weight and buoyancy
x_S, y_S, z_S	x, y, or z coordinate of the static center
CM	metacenter
x_M, y_M, z_M	x, y, or z coordinate of the metacenter
GM	metacentric height

K_ϕ	rate of change of rolling moment with roll angle, namely, the partial derivative $\partial K/\partial \phi$
$K_{\phi a}$	partial derivative $\partial K_a/\partial \phi_a$
M_θ	partial derivative $\partial M/\partial \theta$
X_θ	partial derivative $\partial X/\partial \theta$

MASS CHARACTERISTICS OF BODY

m	mass
W	weight, positive down; $W = mg$
ΔW	incremental change in ballast, positive when weight is added
Δ	displacement, positive upward
CG	center of mass
x_G, y_G, z_G	x, y, or z coordinate of center of mass
x_{Ga}, y_{Ga}, z_{Ga}	x_a, y_a, or z_a coordinate of center of mass
I	moment of inertia
I_x, I_y, I_z	moment of inertia about x, y, or z axis
I_{xa}, I_{ya}, I_{za}	moment of inertia about x_a, y_a, or z_a axis
k	radius of gyration
k_x, k_y, k_z	radius of gyration about x, y, or z axis
k_{xa}, k_{ya}, k_{za}	radius of gyration about x_a, y_a, or z_a axis
I_{xy}, I_{yz}, I_{zx}	product of inertia about xy, yz, or zx axes
$I_{xy0}, I_{yz0}, I_{zx0}$	product of inertia about x_0y_0, y_0z_0, or z_0x_0 axes

KINEMATIC QUANTITIES

Linear Displacements

x, y, z	displacement in direction of x, y, or z axis
x_a, y_a, z_a	displacement in direction of x_a, y_a, or z_a axis

Angular Displacements

ψ, θ, ϕ	displacement about the z_0, y_1, or x axis (see Figure 2)
δ	angular deflection of a control surface, positive for clockwise deflection of control when viewed from port side, from above, or from astern
$\delta_b, \delta_d, \delta_f, \delta_r, \delta_s$	angular deflection of bow plane, dorsal rudder, flap, rudder, or stern plane (see δ for positive sense)
δ_{ft}	deflection of tab on a doubly-movable flap (see δ for positive sense)
γ	angle between x_0y_0 plane and tangent to towline
ϵ	downwash angle

Linear Velocities

U	absolute velocity of origin of x, y, z axes
U_e	velocity, relative to fluid, of origin of x, y, z axes
V_k	absolute velocity of origin of x, y, z axes, in knots
V_{ke}	velocity, relative to fluid, of origin of x, y, z axes, in knots
u, v, w	component of U in the direction of the x, y, or z axis
u_a, v_a, w_a	component of U in the direction of the x_a, y_a, or z_a axis

u_e, v_e, w_e	component of U_e in the direction of the x, y, or z axis
u_{ea}, v_{ea}, w_{ea}	component of U_e in the direction of the x_a, y_a, or z_a axis
c	wave velocity

Angular Velocities

Ω	absolute angular velocity of origin of x, y, z axes
Ω_e	angular velocity, relative to fluid, of origin of x, y, z axes
p, q, r	component of Ω about x, y, or z axis
p_a, q_a, r_a	component of Ω about x_a, y_a, or z_a axis
p_e, q_e, r_e	component of Ω_e about x, y, or z axis
p_{ea}, q_{ea}, r_{ea}	component of Ω_e about x_a, y_a or z_a axis
$\dot{\psi}, \dot{\theta}, \dot{\phi}$	angular velocity about z_0, y_1, or x axis (see Figure 2). Because these axes are not mutually perpendicular, the corresponding angular velocities are not equivalent to p, q, and r.
$\dot{\delta}$	rate of deflection of control surface, positive for clockwise rotation of control when viewed from port side, from above, or from astern
$\dot{\delta}_b, \dot{\delta}_d, \dot{\delta}_f, \dot{\delta}_r, \dot{\delta}_s$	rate of deflection of bow plane, dorsal rudder, flap, rudder, or stern plane
ω	frequency in radians per second
ω_n	natural frequency in radians per second
f	frequency in cycles per second
n	revolutions per second
CR	center of rotation

Linear Accelerations

a	linear acceleration
a_x, a_y, a_z	component of linear acceleration along x, y, or z axis
a_{xa}, a_{ya}, a_{za}	component of linear acceleration along x_a, y_a, or z_a axis
a_{x0}, a_{y0}, a_{z0}	component of linear acceleration along x_0, y_0, or z_0 axis
$\dot{U}$	absolute linear acceleration of origin of x, y, z axes
g	acceleration of gravity

Pseudo Linear Accelerations

$\dot{u}, \dot{v}, \dot{w}$	time rate of change of u, v, or w in direction of x, y, or z axis
$\dot{U}_e$	time rate of change of linear velocity, relative to fluid, of origin of x, y, z axes
$\dot{u}_e, \dot{v}_e, \dot{w}_e$	time rate of change of u_e, v_e, or w_e in direction of x, y, or z axis

Angular Accelerations

$\dot{\Omega}$	absolute angular acceleration of origin of x, y, z axes
$\dot{p}, \dot{q}, \dot{r}$	component of $\dot{\Omega}$ about x, y, or z axis
$\dot{p}_a, \dot{q}_a, \dot{r}_a$	component of $\dot{\Omega}$ about x_a, y_a, or z_a axis
$\dot{p}_0, \dot{q}_0, \dot{r}_0$	component of $\dot{\Omega}$ about x_0, y_0, or z_0 axis

Pseudo Angular Accelerations

$\dot{\Omega}_e$	time rate of change of angular velocity, relative to fluid, of origin of x, y, z axes
$\dot{p}_e, \dot{q}_e, \dot{r}_e$	time rate of change of p_e, q_e, or r_e
$\dot{p}_{ea}, \dot{q}_{ea}, \dot{r}_{ea}$	time rate of change of p_{ea}, q_{ea}, or r_{ea}

Momentum and Moment of Momentum

P	momentum
P_x, P_y, P_z	momentum along x, y, or z axis
H	moment of momentum (angular momentum)
H_x, H_y, H_z	moment of momentum about x, y, or z axis
H_{xa}, H_{ya}, H_{za}	moment of momentum about x_a, y_a, or z_a axis

Time, Frequency, and Stability Roots

t	time; in seconds, unless specified otherwise
t_1	time at initiation of a maneuver
t_s	time to reach steady-state conditions of motion
$t_{\frac{1}{2}}$	time for an oscillation to decay to half amplitude
t_{90}	time to reach 90-degree change of heading in a turn
t_{180}	time to reach 180-degree change of heading in a a turn
t_x	time for origin of x, y, z axes to reach maximum forward point in a turn
t_y	time for origin of x, y, z axes to reach maximum lateral displacement in a horizontal maneuver
t_ψ	time to reach maximum heading angle in an overshoot or zig-zag maneuver
t_*	time to regain initial heading in an overshoot or zig-zag maneuver
t_θ	time to reach maximum pitch angle in an overshoot or zig-zag maneuver
t_i	time at i th execute; $i = 1, 2, \cdots$
$\dot{A}$	derivative of the quantity A with respect to time, dA/dt
f	frequency in cycles per second

T	period of oscillation in seconds
ω	frequency in radians per second
ω_n	undamped natural frequency in radians per second
Υ_i	phase angle; $i = 1, 2, \cdots$ or $\theta, \phi, \cdots$ to distinguish various phases, when needed
λ	wave length
n	revolutions per second
σ_i	roots of stability equation; $i = a$ for real root; $i = b, c$ for conjugate complex pair of roots; $a = 1, 2, \cdots$, $b = 1, 2, \cdots$, $c = 1, 2, \cdots$, $a \neq b \neq c$
σ_{ih}	roots of stability equation for horizontal motion
σ_{iv}	roots of stability equation for vertical motion
$Re\sigma_i$	real part of the complex root σ_i; or a real root σ_i
$Im\sigma_i$	imaginary part of the complex root σ_i

FORCES

F	total external force
F_x, F_y, F_z	component of F in the $x, y,$ or z direction
F_{xa}, F_{ya}, F_{za}	component of F in the direction of the $x_a, y_a,$ or z_a axis
X, Y, Z	component of the hydrodynamic force in the direction of the $x, y,$ or z axis; also referred to as longitudinal, lateral, or vertical force
X_a, Y_a, Z_a	component of the hydrodynamic force in the direction of the $x_a, y_a,$ or z_a axis
X_*, Y_*, Z_*	$x, y,$ or z component of hydrodynamic force at zero angle of attack and zero angle of drift
L, D, C	component of the hydrodynamic force in the lift, drag, or cross-force direction (see Figure 4 for direction and positive sense of these components)

D_f	friction drag
D_r	residuary drag
T	thrust, or towline tension
T_x, T_y, T_z	component of T in the direction of the x, y, or z axis
B	buoyancy, positive upward
W	weight, positive downward; $W = mg$
ΔW	incremental change in ballast, positive when weight is added
Δ	displacement, positive upward

MOMENTS

Q	moment of total external force
Q_x, Q_y, Q_z	total moment about x, y, or z axis
Q_{xa}, Q_{ya}, Q_{za}	total moment about x_a, y_a, or z_a axis
K, M, N	moment of hydrodynamic force about x, y, or z axis; also referred to as rolling, pitching, or yawing moment
K_a, M_a, N_a	moment of hydrodynamic force about x_a, y_a, or z_a axis
K_*, M_*, N_*	moment of hydrodynamic force about x, y, or z axis at zero angle of attack and zero angle of drift
Q_c	moment of total external force about control stock
$Q_{cb}, Q_{cd}, Q_{cf}, Q_{cr}, Q_{cs}$	total moment about stock of bow plane, dorsal rudder, flap, rudder, or stern plane
Q_T	moment produced by thrust

H_c — moment about control stock of hydrodynamic force on control

$H_{cb}, H_{cd}, H_{cf}, H_{cr}, H_{cs}$ — moment about control stock of hydrodynamic force on bow plane, dorsal rudder, flap, rudder, or stern plane

STABILITY DERIVATIVES

Because the form and meaning of each stability derivative in a given set is similar, only the first derivatives of each set will be defined. Likewise, the formation of related derivatives for other systems of reference axes is so obvious that only a few examples will be given at the end under a "miscellaneous" category. A few samples of higher order derivatives are also given in this category.

Static Derivatives

X_u, X_v, X_w — rate of change of X hydrodynamic force with u_e, v_e, or w_e; the partial derivative $\partial X/\partial u_e$, $\partial X/\partial v_e$, or $\partial X/\partial w_e$

Y_u, Y_v, Y_w

Z_u, Z_v, Z_w

K_u, K_v, K_w

M_u, M_v, M_w

N_u, N_v, N_w

Rotary Derivatives

X_p, X_q, X_r — rate of change of X hydrodynamic force with p_e, q_e, or v_e; the partial derivative $\partial X/\partial p_e$, $\partial X/\partial q_e$, or $\partial X/\partial r_e$

Y_p, Y_q, Y_r

Z_p, Z_q, Z_r

K_p, K_q, K_r

M_p, M_q, M_r

N_p, N_q, N_r

Added Mass Derivatives

These derivatives are measures of the apparent additional mass that must be added to the actual mass to represent the fluid that is accelerated by the accelerating body.

$X_{\dot{u}}, X_{\dot{v}}, X_{\dot{w}}$	rate of change of X hydrodynamic force with $\dot{u}_e, \dot{v}_e$, or $\dot{w}_e$; the partial derivative $\partial X/\partial \dot{u}_e$, $\partial X/\partial \dot{v}_e$, or $\partial X/\partial \dot{w}_e$
$Y_{\dot{u}}, Y_{\dot{v}}, Y_{\dot{w}}$	
$Z_{\dot{u}}, Z_{\dot{v}}, Z_{\dot{w}}$	
$X_{\dot{p}}, X_{\dot{q}}, X_{\dot{r}}$	
$Y_{\dot{p}}, Y_{\dot{q}}, Y_{\dot{r}}$	
$Z_{\dot{p}}, Z_{\dot{q}}, Z_{\dot{r}}$	

Added Moment of Inertia Derivatives

$K_{\dot{u}}, K_{\dot{v}}, K_{\dot{w}}$	rate of change of K hydrodynamic moment with $\dot{u}_e, \dot{v}_e$, or $\dot{w}_e$; the partial derivative $\partial K/\partial \dot{u}_e$, $\partial K/\partial \dot{v}_e$, or $\partial K/\partial \dot{w}_e$
$M_{\dot{u}}, M_{\dot{v}}, M_{\dot{w}}$	
$N_{\dot{u}}, N_{\dot{v}}, N_{\dot{w}}$	
$K_{\dot{p}}, K_{\dot{q}}, K_{\dot{r}}$	
$M_{\dot{p}}, M_{\dot{q}}, M_{\dot{r}}$	
$N_{\dot{p}}, N_{\dot{q}}, N_{\dot{r}}$	

Metacentric Derivatives

$X_{\psi}, X_{\theta}, X_{\phi}$	rate of change of X force with ψ, θ, or ϕ; the partial derivative $\partial X/\partial \psi$, $\partial X/\partial \theta$, or $\partial X/\partial \phi$
$Y_{\psi}, Y_{\theta}, Y_{\phi}$	
$Z_{\psi}, Z_{\theta}, Z_{\phi}$	
$K_{\psi}, K_{\theta}, K_{\phi}$	
$M_{\psi}, M_{\theta}, M_{\phi}$	
$N_{\psi}, N_{\theta}, N_{\phi}$	

Control Derivatives

X_δ	rate of change of X hydrodynamic force with control deflection δ; the partial derivative $\partial X/\partial \delta$

$X_{\delta b}, X_{\delta d}, X_{\delta f}, X_{\delta r}, X_{\delta s}$
$Y_{\delta b}, Y_{\delta d}, Y_{\delta f}, Y_{\delta r}, Y_{\delta s}$
$Z_{\delta b}, Z_{\delta d}, Z_{\delta f}, Z_{\delta r}, Z_{\delta s}$
$K_{\delta b}, K_{\delta d}, K_{\delta f}, K_{\delta r}, K_{\delta s}$
$M_{\delta b}, M_{\delta d}, M_{\delta f}, M_{\delta r}, M_{\delta s}$
$N_{\delta b}, N_{\delta d}, N_{\delta f}, N_{\delta r}, N_{\delta s}$

Miscellaneous Derivatives

X_{u0}	rate of change of X_0 hydrodynamic force with u_{e0}; the partial derivative $\partial X_0/\partial u_{e0}$
X_{ua}	rate of change of X_a hydrodynamic force with u_{ea}; the partial derivative $\partial X_a/\partial u_{ea}$
$X_{\delta a}$	rate of change of X_a hydrodynamic force with control deflection δ; the partial derivative $\partial X_a/\partial \delta$
K_{uu}	second partial derivative $\partial^2 K/\partial u_e^2$
X_{uu}	second partial derivative $\partial^2 X/\partial u_e^2$
X_{uv}	second partial derivative $\partial^2 X/\partial u_e \partial v_e$
X_{uuu}	third partial derivative $\partial^3 X/\partial u_e^3$
X_{uuv}	third partial derivative $\partial^3 X/\partial u_e^2 \partial v_e$
X_{uvw}	third partial derivative $\partial^3 X/\partial u_e \partial v_e \partial w_e$

PROPERTIES OF FLUID ENVIRONMENT

ρ	mass density
μ	coefficient of viscosity
ν	kinematic viscosity, μ/ρ
P	pressure
Q	dynamic pressure, $\frac{1}{2}\rho U_e^2$
λ	wave length
c	wave velocity
σ	cavitation number

FLIGHT PATH GEOMETRY

θ_f	flight path angle; the angle from the horizontal plane to U_e, positive when the motion is directed above the horizontal plane
α	angle of attack. To measure the angle, project the line of motion relative to the fluid onto the x-positive portion of the xz plane. The angle of attack is the angle from this projection to the x axis, positive in the positive sense of rotation about the y axis. (See Figure 5.)
β	angle of drift; the angle from U_e to the x-positive portion of the xz plane, positive in the positive sense of rotation about the z axis. (See Figure 5.) The two angles α and β are sufficient to completely define the orientation of the x, y, z axes with respect to the direction of motion of the body relative to the fluid.
execute	the time at which a control movement is commenced for the purpose of initiating or modifying a maneuver
ψ_m, θ_m, ϕ_m	maximum value of heading, pitch, or roll angle, measured from value at first execute (see Figure 6)

x_{0s}, y_{0s}, z_{0s}	maximum displacement experienced by any point on body during a maneuver (usually same point on the stern); measured from position of origin O at first execute, in the direction of x_0, y_0, or z_0 when the x_0, y_0, z_0 axes are so chosen that the direction of the initial motion lies in the x_0-positive portion of the x_0z_0 plane and z_0 is vertically down
x_{0m}, y_{0m}, z_{0m}	maximum displacement of origin 0; measured from the position of the origin at the first execute, in the direction of x_0, y_0, or z_0 when the x_0, y_0, z_0 axes are chosen as for the preceding symbol (see Figure 7)
z_{0d}	ordered depth at origin O
z_{0e}	depth error; $z_{0e} = z_0 - z_{0d}$
h	altitude

Turning Maneuvers

TD	tactical diameter; measured at point where heading has changed 180 degrees (see Figure 7)
AD	advance; measured at point where heading has changed 90 degrees (see Figure 7)
TR	transfer; measurement same as for AD
D	steady-turning diameter (see Figure 7)
CR	center of rotation

Spiral Maneuvers

$\dot{\psi}_*$	rate of change of heading at zero rudder angle (see Figure 8)
$\dot{\psi}_b$	rate of change of heading at branch point (see Figure 8)

Overshoot and Zig-Zag Maneuvers

O_ψ, O_θ	overshoot in heading or pitch angle, measured from value at second execute (see Figure 6)
O_d	overshoot in depth at origin O, measured from value at second execute
δ_i	average control deflection angle during period shortly prior to i th execute; $i = 1, 2, 3, \cdots$. Thus, for an overshoot, δ_1 is the neutral angle, δ_2 is the initial disturbing angle, and δ_3 is the checking angle.
t_i	time at the i th execute; $i = 1, 2, 3, \cdots$. Normally time will be measured from the first execute so only t_2, t_3, $\cdots$ will be significant.
ψ_i, θ_i	heading or pitch execute angle at i th execute, measured from value at first execute; $i = 2, 3, \cdots$
$\dot{\psi}_i$	rate of change of heading at i th execute; $i = 2, 3, \cdots$
x_{0i}, y_{0i}, z_{0i}	displacement of origin O at i th execute; measured from the position of the origin at the first execute, in the direction of x_0, y_0, or z_0 when the x_0, y_0, z_0 axes are so chosen that the direction of the initial motion lies in the x_0-positive portion of the x_0z_0 plane and z_0 is vertically down; $i = 2, 3, \cdots$
x_{0*}, y_{0*}, z_{0*}	displacement of origin O at time initial heading is first regained; measured from the position of the origin at the first execute, in the direction of x_0, y_0, or z_0 when the x_0, y_0, z_0 axes are so chosen that the direction of the initial motion lies in the x_0-positive portion of the x_0z_0 plane and z_0 is vertically down

MISCELLANEOUS INDICES

F	Froude number; $F = U_e/\sqrt{g\ell}$
R	Reynolds number; $R = U_e\ell/\nu$
R_d, R_x, R_δ	Reynolds number based on the distance d, x, or δ
S	Strouhal number; $S = n\ell/U_e$
S_d	Strouhal number based on diameter
σ	cavitation number
σ_i	roots of stability equation; i = a for a real root, i = b, c for a conjugate complex pair of roots, where a = 1, 2, $\cdots$ b = 1, 2, $\cdots$ c = 1, 2, $\cdots$ $a \neq b \neq c$
σ_{ih}	roots of stability equation for horizontal motion
σ_{iv}	roots of stability equation for vertical motion
$\mathrm{Re}\ \sigma_i$	real part of complex root σ_i or real root σ_i; also referred to as stability index
$\mathrm{Im}\ \sigma_i$	imaginary part of complex root σ_i
c/c_c	damping ratio
α_δ	flap effectiveness parameter at constant C_L
$C_{L\alpha}$	slope of curve of C_L versus α

MATHEMATICAL SYMBOLS

$\approx$	approximately equal to
$\sim$	asymptotically equal to
$n!$	factorial n
$\lvert A \rvert$	absolute value of A
$\lvert A \rvert$	the determinant A

$\|A\|$	the matrix A
$\|\bar{A}\|$	adjoint of matrix A
Σ	sum of
Π	product of
$\bar{A}$	the vector A
$\bar{i}, \bar{j}, \bar{k}$	unit vector along the x, y, or z axis
$\hat{A}$	the tensor A
$\bar{A} \cdot \bar{B}$	dot product of vectors A and B
$\bar{A} x \bar{B}$	cross product of vectors A and B
Re σ_i	real part of complex quantity σ_i, or the real quantity σ_i
Im σ_i	imaginary part of complex quantity σ_i
ln	natural logarithm
c	damping constant
c_c	critical damping constant
Δ	incremental change
$\dot{A}$	derivative of A with respect to time, dA/dt
D	operator d/dt
D'	operator d/dt'
1(t)	unit step function
$\delta(t)$	Dirac impulse function
$\mathcal{L}$	Laplace transformation operator
s	complex variable in Laplace transform

LIST OF SUBSCRIPTS

a	principal axis
B	block (coefficient)
B	center of buoyancy
b	bow plane
b	translating axis
C	cross force
c	control
c	critical (damping)
D	drag
d	depth
d	diameter (R_d, S_d)
d	dorsal
d	drag (section coefficient)
d	ordered (z_{0d})
e	effective (relative to fluid)
e	error (z_{0e})
f	flap
f	fluid axis
f	friction (drag)
f	frontal (area)
G	center of mass
g	geometric axis
g	mean geometric (chord)
h	horizontal
i	initial condition

i	index
j	index
K	rolling moment
k	knots
L	lift
ℓ	lateral (area)
ℓ	lift (section coefficient)
M	metacenter
M	pitching moment
m	maximum
m	pitching moment (section coefficient)
m	value for model (affected quantity to be enclosed in parentheses)
N	yawing moment
n	natural (ω_n)
n	normal force
P	center of pressure
P	prismatic (coefficient)
P	typical point
P	port (affected quantity to be enclosed in parentheses)
p	rate of change with respect to p_e
$\dot{p}$	rate of change with respect to $\dot{p}_e$
q	rate of change with respect to q_e
$\dot{q}$	rate of change with respect to $\dot{q}_e$
r	rate of change with respect to r_e
r	residuary (drag)

r	root (chord)
r	rudder
$\dot{r}$	rate of change with respect to $\dot{r}_e$
S	starboard (affected quantity to be enclosed in parentheses)
s	maximum displacement (x_{0s})
s	steady-state
s	stern plane
s	supplemental axis
s	value for full-scale (affected quantity to be enclosed in parentheses)
T	towpoint
t	tab
t	tangential
t	tip (chord)
t	total (drag)
u	rate of change with respect to u_e
$\dot{u}$	rate of change with respect to $\dot{u}_e$
v	rate of change with respect to v_e
v	vertical
$\dot{v}$	rate of change with respect to $\dot{v}_e$
w	rate of change with respect to w_e
$\dot{w}$	rate of change with respect to $\dot{w}_e$
X, Y, Z	force in direction of x, y, or z axis
x, y, z	associated with x, y, or z axis
x, y, z	based on characteristic length in direction of x, y, or z axis (R_x)
x, y, z	projected onto yz, zx, or xy plane

xy, yz, za	with respect to xy, yz, or za pair of axes (I_{xy})
α	rate of change with respect to α ($C_{L\alpha}$)
δ	based on boundary layer thickness (R_δ)
δ	rate of change with respect to δ
θ	maximum pitch angle
θ	rate of change with respect to θ
ϕ	rate of change with respect to ϕ
ψ	maximum heading angle
ψ	rate of change with respect to ψ
0	fixed (inertial) axis
$\frac{1}{2}$	time to damp to half amplitude
1	first execute (start of maneuver)
90	associated with 90 degree change of heading
180	associated with 180 degree change of heading
*	associated with zero value of a parameter

ALPHABETICAL LIST OF SYMBOLS

A	$A' = \frac{A}{\ell^2}$	area, or projected area
A_b, A_d	$A_b' = \frac{A_b}{\ell^2}$	projected area of bow plane or dorsal rudder (planform area)
A_{bx}, A_{by}, A_{bz}	$A_{bx}' = \frac{A_{bx}}{\ell^2}$	projection of bowplane area onto yz, zx, or xy plane
$A_{bxa}, A_{bya}, A_{bza}$	$A_{bxa}' = \frac{A_{bxa}}{\ell^2}$	projection of bowplane area onto $y_a z_a$, $z_a x_a$, or $x_a y_a$ plane
A_f, A_h, A_ℓ	$A_f' = \frac{A_f}{\ell^2}$	projected frontal area, horizontal area, or lateral area of body onto yz, zx, or xy plane
A_f, A_r, A_s	$A_r' = \frac{A_r}{\ell^2}$	projected area of flap, rudder, or stern plane (planform area)
A_x, A_y, A_z	$A_x' = \frac{A_x}{\ell^2}$	projected area of submerged part of body onto yz, zx, or xy plane
AD	$AD' = \frac{AD}{\ell}$	advance
AP		after perpendicular
a	$a = \frac{b^2}{A}$	aspect ratio (area to be specified by appropriate subscript)
a	$a' = \frac{a\ell}{U^2}$	linear acceleration
a_x, a_y, a_z	$a_x' = \frac{a_x \ell}{U^2}$	component of linear acceleration along x, y, or z axis
a_{xa}, a_{ya}, a_{za}	$a_{xa}' = \frac{a_{xa}\ell}{U^2}$	component of linear acceleration along x_a, y_a, or z_a axis
a_{x0}, a_{y0}, a_{z0}	$a_{x0}' = \frac{a_{x0}\ell}{U^2}$	component of linear acceleration along x_0, y_0, or z_0 axis
B	$B' = \frac{B}{\frac{1}{2}\rho U^2 \ell^2}$	buoyancy force
b	$b' = \frac{b}{\ell}$	beam, or span

C	$C' = \dfrac{C}{\frac{1}{2}\rho U^2 \ell^2}$	cross force component of hydrodynamic force
	$C_C = \dfrac{C}{\frac{1}{2}\rho U^2 A}$	cross force coefficient (area to be specified)
	$c_C = \dfrac{C}{\frac{1}{2}\rho U^2 V^{\frac{2}{3}}}$	
C_B	$C_B = \dfrac{V}{Hb\ell}$	block coefficient
C_C		see C
C_D		see D
C_K		see K
C_L		see L
$C_{L\alpha}$	$C_{L\alpha} = \dfrac{\partial C_L}{\partial \alpha}$	slope of curve of C_L versus α
C_M		see M
C_N		see N
C_P	$C_P = \dfrac{V}{A_x \ell}$	prismatic coefficient
C_{Pv}	$C_{Pv} = \dfrac{V}{A_z H}$	vertical prismatic coefficient
C_w	$C_w = \dfrac{A_z}{b\ell}$	waterline coefficient
C_X		see X
C_x	$C_x = \dfrac{A_x}{bH}$	maximum section coefficient
C_Y		see Y
C_Z		see Z
C_f		see D_f
C_r		see D_r
C_t		see D

CB		center of buoyancy
CG		center of mass
CM		metacenter
CP		center of pressure
CR		center of rotation
CS		static center
c	$c' = \frac{c}{\ell}$	chord
c		damping constant
c		wave velocity
c_C		see C
c_D		see D
c_K		see K
c_L		see L
c_M		see M
c_N		see N
c_X		see X
c_Y		see Y
c_Z		see Z
c_c		critical damping constant
c_d		see D
c_g		mean geometric chord
c_ℓ		see L
c_m		see M
c_r		root chord
c_t		tip chord
$\frac{c}{c_c}$		damping ratio

D	$D' = \frac{D}{\frac{1}{2}\rho U^2 \ell^2}$	drag component of hydrodynamic force
	$C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$	drag coefficient (area to be specified)
	$C_t = \frac{D}{\frac{1}{2}\rho U^2 S}$	
	$c_D = \frac{D}{\frac{1}{2}\rho U^2 V^{\frac{2}{3}}}$	
	$c_d = \frac{\text{drag per unit span at infinite aspect ratio}}{\frac{1}{2}\rho U^2 c}$ = section drag coefficient	
D	$D' = \frac{D\ell}{\ell}$	operator $\frac{d}{dt}$
D	$D' = \frac{D}{\ell}$	steady-turning diameter
D_f	$C_f = \frac{D_f}{\frac{1}{2}\rho U^2 S}$	friction drag
D_r	$C_r = \frac{D_r}{\frac{1}{2}\rho U^2 S}$	residuary drag; $C_r = C_t - C_f$
d	$d' = \frac{d}{\ell}$	diameter
execute	the time at which a control movement is commenced for the purpose of initiating or modifying a maneuver.	
F	$F = \frac{U_e}{\sqrt{g\ell}}$	Froude number
F	$F' = \frac{F}{\frac{1}{2}\rho U^2 \ell^2}$	external force
F_x, F_y, F_z	$F_x' = \frac{F_x}{\frac{1}{2}\rho U^2 \ell^2}$	component of total external force along x, y, or z axis
F_{xa}, F_{ya}, F_{za}	$F_{xa}' = \frac{F_{xa}}{\frac{1}{2}\rho U^2 \ell^2}$	component of total external force along x_a, y_a, or z_a axis

FP		forward perpendicular
f		frequency in cycles per second
GM	$GM' = \frac{GM}{\ell}$	metacentric height
g	$g' = \frac{g\ell}{U^2}$	acceleration of gravity
H	$H' = \frac{H}{\ell}$	draft
H	$H' = \frac{H}{\frac{1}{2}\rho U \ell^4}$	moment of momentum (angular momentum)
H_c	$H_c' = \frac{H_c}{\frac{1}{2}\rho U^2 \ell^3}$	moment about control stock of hydrodynamic force on control
$H_{cb}, H_{cd}, H_{cf}, H_{cr}, H_{cs}$	$H_{cb}' = \frac{H_{cb}}{\frac{1}{2}\rho U^2 \ell^3}$	moment about control stock of hydrodynamic force on bow plane, dorsal rudder, flap, rudder, or stern plane
H_x, H_y, H_z	$H_x' = \frac{H_x}{\frac{1}{2}\rho U \ell^4}$	moment of momentum about x, y, or z axis
H_{xa}, H_{ya}, H_{za}	$H_{xa}' = \frac{H_{xa}}{\frac{1}{2}\rho U \ell^4}$	moment of momentum about x_a, y_a, or z_a axis
h	$h' = \frac{h}{\ell}$	height, or altitude
I	$I' = \frac{I}{\frac{1}{2}\rho \ell^5}$	moment of inertia
I_x, I_y, I_z	$I_x' = \frac{I_x}{\frac{1}{2}\rho \ell^5}$	moment of inertia about x, y, or z axis
I_{xa}, I_{ya}, I_{za}	$I_{xa}' = \frac{I_{xa}}{\frac{1}{2}\rho \ell^5}$	moment of inertia about x_a, y_a, or z_a axis
I_{xy}, I_{yz}, I_{zx}	$I_{xy}' = \frac{I_{xy}}{\frac{1}{2}\rho \ell^5}$	product of inertia about xy, yz, or zx axes
$I_{xya}, I_{yza}, I_{zxa}$	$I_{xya}' = \frac{I_{xya}}{\frac{1}{2}\rho \ell^5}$	product of inertia about $x_a y_a$, $y_a z_a$, or $z_a x_a$ axes
$\mathrm{Im}\,\sigma_i$	$\mathrm{Im}\,\sigma_i' = \frac{\mathrm{Im}\,\sigma_i \ell}{U}$	imaginary part of complex root σ_i

i		angle of incidence
$\bar{i}$		unit vector along x axis
$\bar{j}$		unit vector along y axis
K	$K' = \dfrac{K}{\frac{1}{2}\rho U^2 \ell^3}$	hydrodynamic rolling moment about x axis
	$C_K = \dfrac{K}{\frac{1}{2}\rho U^2 A \ell}$	rolling-moment coefficient (area to be specified)
	$c_K = \dfrac{K}{\frac{1}{2}\rho U^2 V}$	
K_a	$K_a' = \dfrac{K_a}{\frac{1}{2}\rho U^2 \ell^3}$	hydrodynamic moment about x_a axis
K_p	$K_p' = \dfrac{K_p}{\frac{1}{2}\rho U \ell^4}$	partial derivative $\dfrac{\partial K}{\partial p_e}$
K_{pa}	$K_{pa}' = \dfrac{K_{pa}}{\frac{1}{2}\rho U \ell^4}$	partial derivative $\dfrac{\partial K_a}{\partial p_{ea}}$
K_{pp}	$K_{pp}' = \dfrac{K_{pp}}{\frac{1}{2}\rho \ell^5}$	partial derivative $\dfrac{\partial^2 K}{\partial p_e^{\,2}}$
K_{ppp}	$K_{ppp}' = \dfrac{K_{ppp} U}{\frac{1}{2}\rho \ell^6}$	partial derivative $\dfrac{\partial^3 K}{\partial p_e^{\,3}}$
K_{ppu}	$K_{ppu}' = \dfrac{K_{ppu} U}{\frac{1}{2}\rho \ell^5}$	partial derivative $\dfrac{\partial^3 K}{\partial p_e^{\,2} \partial u_e}$
K_{pu}	$K_{pu}' = \dfrac{K_{pu}}{\frac{1}{2}\rho \ell^4}$	partial derivative $\dfrac{\partial^2 K}{\partial p_e \partial u_e}$
K_{puu}	$K_{puu}' = \dfrac{K_{puu} U}{\frac{1}{2}\rho \ell^4}$	partial derivative $\dfrac{\partial^3 K}{\partial p_e \partial u_e^{\,2}}$
$K_{\dot{p}}$	$K_{\dot{p}}' = \dfrac{K_{\dot{p}}}{\frac{1}{2}\rho \ell^5}$	partial derivative $\dfrac{\partial K}{\partial \dot{p}_e}$
$K_{\dot{p}a}$	$K_{\dot{p}a}' = \dfrac{K_{\dot{p}a}}{\frac{1}{2}\rho \ell^5}$	partial derivative $\dfrac{\partial K_a}{\partial \dot{p}_{ea}}$
K_r	$K_r' = \dfrac{K_r}{\frac{1}{2}\rho U \ell^4}$	partial derivative $\dfrac{\partial K}{\partial r_e}$
K_{ra}	$K_{ra}' = \dfrac{K_{ra}}{\frac{1}{2}\rho U \ell^4}$	partial derivative $\dfrac{\partial K_a}{\partial r_{ea}}$

$K_{\dot r}$	$K_{\dot r}' = \dfrac{K_{\dot r}}{\frac{1}{2}\rho \ell^5}$	partial derivative $\dfrac{\partial K}{\partial \dot r_e}$
$K_{\dot r a}$	$K_{\dot r a}' = \dfrac{K_{\dot r a}}{\frac{1}{2}\rho \ell^5}$	partial derivative $\dfrac{\partial K_a}{\partial \dot r_{ea}}$
K_u	$K_u' = \dfrac{K_u}{\frac{1}{2}\rho U \ell^3}$	partial derivative $\dfrac{\partial K}{\partial u_e}$
K_{ua}	$K_{ua}' = \dfrac{K_{ua}}{\frac{1}{2}\rho U \ell^3}$	partial derivative $\dfrac{\partial K_a}{\partial u_{ea}}$
K_{uu}	$K_{uu}' = \dfrac{K_{uu}}{\frac{1}{2}\rho \ell^3}$	partial derivative $\dfrac{\partial^2 K}{\partial u_e^2}$
K_{uuu}	$K_{uuu}' = \dfrac{K_{uuu} U}{\frac{1}{2}\rho \ell^3}$	partial derivative $\dfrac{\partial^3 K}{\partial u_e^3}$
K_{uv}	$K_{uv}' = \dfrac{K_{uv}}{\frac{1}{2}\rho \ell^3}$	partial derivative $\dfrac{\partial^2 K}{\partial u_e \partial v_e}$
$K_{\dot u}$	$K_{\dot u}' = \dfrac{K_{\dot u}}{\frac{1}{2}\rho \ell^4}$	partial derivative $\dfrac{\partial K}{\partial \dot u_e}$
$K_{\dot u a}$	$K_{\dot u a}' = \dfrac{K_{\dot u a}}{\frac{1}{2}\rho \ell^4}$	partial derivative $\dfrac{\partial K_a}{\partial \dot u_{ea}}$
K_δ	$K_\delta' = \dfrac{K_\delta}{\frac{1}{2}\rho U^2 \ell^3}$	partial derivative $\dfrac{\partial K}{\partial \delta}$
$K_{\delta d}$	$K_{\delta d}' = \dfrac{K_{\delta d}}{\frac{1}{2}\rho U^2 \ell^3}$	partial derivative $\dfrac{\partial K}{\partial \delta_d}$
$K_{\delta da}$	$K_{\delta da}' = \dfrac{K_{\delta da}}{\frac{1}{2}\rho U^2 \ell^3}$	partial derivative $\dfrac{\partial K_a}{\partial \delta_d}$
K_ϕ	$K_\phi' = \dfrac{K_\phi}{\frac{1}{2}\rho U^2 \ell^3}$	partial derivative $\dfrac{\partial K}{\partial \phi}$
$K_{\phi a}$	$K_{\phi a}' = \dfrac{K_{\phi a}}{\frac{1}{2}\rho U^2 \ell^3}$	partial derivative $\dfrac{\partial K_a}{\partial \phi_a}$
k	$k' = \dfrac{k}{\ell}$	radius of gyration
k_x, k_y, k_z	$k_x' = \dfrac{k_x}{\ell}$	radius of gyration about x, y, or z axis
k_{xa}, k_{ya}, k_{za}	$k_{xa}' = \dfrac{k_{xa}}{\ell}$	radius of gyration about x_a, y_a, or z_a axis
$\bar k$		unit vector along z axis

L	$L' = \frac{L}{\frac{1}{2}\rho U^2 \ell^2}$	lift component of hydrodynamic force
	$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$	lift coefficient (area to be specified)
	$c_L = \frac{L}{\frac{1}{2}\rho U^2 V^{\frac{2}{3}}}$	

$$c_\ell = \frac{\text{lift per unit span at infinite aspect ratio}}{\frac{1}{2}\rho U^2 c}$$

= section lift coefficient

LBP		length between perpendiculars
LCB		longitudinal distance of center of buoyancy from FP
LCG		longitudinal distance of center of mass from FP
LOA		length overall
LWL		length at design waterline
ℓ	$\ell' = \frac{\ell}{\ell}$	characteristic length
$\mathcal{L}$.		Laplace transformation operator
ln		natural logarithm
ℓ_x, ℓ_y, ℓ_z		direction cosine of x_0 relative to x, y, or z axis
M	$M' = \frac{M}{\frac{1}{2}\rho U^2 \ell^3}$	hydrodynamic pitching moment about y axis
	$C_M = \frac{M}{\frac{1}{2}\rho U^2 A \ell}$	pitching moment coefficient (area to be specified)
	$c_M = \frac{M}{\frac{1}{2}\rho U^2 V}$	

$$c_m = \frac{\text{pitching moment per unit span at infinite aspect ratio}}{\frac{1}{2}\rho U^2 c^2}$$

= section pitching-moment coefficient

See subscript variations under K for analogous variants of M.

M_q	$M_q' = \dfrac{M_q}{\frac{1}{2}\rho U \ell^4}$	partial derivative $\dfrac{\partial M}{\partial q_e}$
$M_{\dot{q}}$	$M_{\dot{q}}' = \dfrac{M_{\dot{q}}}{\frac{1}{2}\rho \ell^5}$	partial derivative $\dfrac{\partial M}{\partial \dot{q}_e}$
$M_{\delta b}, M_{\delta s}$	$M_{\delta b}' = \dfrac{M_{\delta b}}{\frac{1}{2}\rho U^2 \ell^3}$	partial derivative $\dfrac{\partial M}{\partial \delta_b}$ or $\dfrac{\partial M}{\partial \delta_s}$
M_θ	$M_\theta' = \dfrac{M_\theta}{\frac{1}{2}\rho U^2 \ell^3}$	partial derivative $\dfrac{\partial M}{\partial \theta}$
M_*	$M_*' = \dfrac{M_*}{\frac{1}{2}\rho U^2 \ell^3}$	hydrodynamic pitching moment about y axis at zero angle of attack
m	$m' = \dfrac{m}{\frac{1}{2}\rho \ell^3}$	mass
m_x, m_y, m_z		direction cosine of y_0 relative to x, y, or z axis
N	$N' = \dfrac{N}{\frac{1}{2}\rho U^2 \ell^3}$	hydrodynamic yawing moment about z axis
	$C_N = \dfrac{N}{\frac{1}{2}\rho U^2 A \ell}$	yawing moment coefficient (area to be specified)
	$c_N = \dfrac{N}{\frac{1}{2}\rho U^2 V}$	

See subscript variations under K for analogous variants of N.

$N_{\delta r}$	$N_{\delta r}' = \dfrac{N_{\delta r}}{\frac{1}{2}\rho U^2 \ell^3}$	partial derivative $\dfrac{\partial N}{\partial \delta_r}$
n	$n' = \dfrac{n\ell}{U}$	revolutions per second
n_x, n_y, n_z		direction cosine of z_0 relative to x, y, or z axis
O		origin of x, y, z axes
O_a		origin of x_a, y_a, z_a axes
O_b		origin of x_b, y_b, z_b axes

O_c		origin of x_c, y_c, z_c axes
O_d	$O_d' = \frac{O_d}{\ell}$	overshoot in depth, measured from value at second execute
O_f		origin of x_f, y_f, z_f axes
O_g		origin of x_g, y_g, z_g axes
O_s		origin of x_s, y_s, z_s axes
O_0		origin of x_0, y_0, z_0 axes
O_θ		overshoot in pitch, measured from value at second execute
O_ψ		overshoot in heading, measured from value at second execute
P	$P' = \frac{P}{\frac{1}{2}\rho U \ell^3}$	momentum
P_x, P_y, P_z	$P_x' = \frac{P_x}{\frac{1}{2}\rho U \ell^3}$	momentum along x, y, or z axis
P		a typical point
P	$P' = \frac{P}{\frac{1}{2}\rho U^2}$	pressure
p	$p' = \frac{p\ell}{U}$	absolute angular velocity about x axis
p_a	$p_a' = \frac{p_a \ell}{U}$	absolute angular velocity about x_a axis
p_e	$p_e' = \frac{p_e \ell}{U}$	angular velocity about x axis relative to fluid
p_{ea}	$p_{ea}' = \frac{p_{ea} \ell}{U}$	angular velocity about x_a axis relative to fluid
$\dot{p}$	$\dot{p}' = \frac{\dot{p}\ell^2}{U^2}$	absolute angular acceleration about x axis

See subscript variations under p for analogous variants of $\dot{p}$.

Q		dynamic pressure, $\frac{1}{2}\rho U_e^2$

Q	$Q' = \dfrac{Q}{\frac{1}{2}\rho U^2 \ell^3}$	moment of total external force
Q_c	$Q_c' = \dfrac{Q_c}{\frac{1}{2}\rho U^2 \ell^3}$	moment of total external force about control stock
$Q_{cb}, Q_{cd}, Q_{cf}, Q_{cr}, Q_{cs}$	$Q_{cb}' = \dfrac{Q_{cb}}{\frac{1}{2}\rho U^2 \ell^3}$	total moment about stock of bow plane, dorsal rudder, rudder, or stern plane
Q_T	$Q_T' = \dfrac{Q_T}{\frac{1}{2}\rho U^2 \ell^3}$	moment produced by thrust
Q_x, Q_y, Q_z	$Q_x' = \dfrac{Q_x}{\frac{1}{2}\rho U^2 \ell^3}$	total moment about x, y, or z axis
Q_{xa}, Q_{ya}, Q_{za}	$Q_{xa}' = \dfrac{Q_{xa}}{\frac{1}{2}\rho U^2 \ell^3}$	total moment about x_a, y_a, or z_a axis
q	$q' = \dfrac{q\ell}{U}$	absolute angular velocity about y axis

See variations under p for analogous variants of q.

R	$R' = \dfrac{R}{\ell}$	radius
R		Reynolds number $\dfrac{U_e \ell}{\nu}$
R_d, R_x, R_δ		Reynolds number $\dfrac{U_e d}{\nu}$, $\dfrac{U_e x}{\nu}$, or $\dfrac{U_e \delta}{\nu}$
$\mathrm{Re}\ \sigma_i$	$\mathrm{Re}\ \sigma_i' = \dfrac{\mathrm{Re}\ \sigma_i \ell}{U}$	real part of complex root σ_i or real root σ_i
r	$r' = \dfrac{r\ell}{U}$	absolute angular velocity about z axis

See variations under p for analogous variants of r.

S		Strouhal number $\dfrac{n\ell}{U_e}$
S_d		Strouhal number $\dfrac{nd}{U_e}$

S	$S' = \frac{S}{\ell^2}$	wetted surface area
s		complex variable in Laplace transform
T	$T' = \frac{TU}{\ell}$	period of oscillation
T	$T' = \frac{T}{\frac{1}{2}\rho U^2 \ell^2}$	thrust, or towline tension
T_x, T_y, T_z	$T_x' = \frac{T_x}{\frac{1}{2}\rho U^2 \ell^2}$	component of T along x, y, or z axis
TD	$TD' = \frac{TD}{\ell}$	tactical diameter
TP		towpoint
TR	$TR' = \frac{TR}{\ell}$	transfer
t	$t' = \frac{t}{\ell}$	thickness
t	$t' = \frac{tU}{\ell}$	time
t_s	$t_s' = \frac{t_s U}{\ell}$	time to reach steady-state conditions of motion
t_x	$t_x' = \frac{t_x U}{\ell}$	time to reach maximum forward point in a turn
t_y	$t_y' = \frac{t_y U}{\ell}$	time to reach maximum lateral displacement in a horizontal maneuver
t_θ	$t_\theta' = \frac{t_\theta U}{\ell}$	time to reach maximum pitch angle in an overshoot or zig-zag maneuver
t_ψ	$t_\psi' = \frac{t_\psi U}{\ell}$	time to reach maximum heading angle in an overshoot or zig-zag maneuver
$t_{\frac{1}{2}}$	$t_{\frac{1}{2}}' = \frac{t_{\frac{1}{2}} U}{\ell}$	time for an oscillation to decay to half amplitude

t_i	$t_i' = \frac{t_i U}{\ell}$	time at i th execute; i = 1, 2, · · ·
t_1	$t_1' = \frac{t_1 U}{\ell}$	time at initiation of a maneuver
t_{90}	$t_{90}' = \frac{t_{90} U}{\ell}$.	time to reach 90° change of heading in a turn
t_{180}	$t_{180}' = \frac{t_{180} U}{\ell}$	time to reach 180° change of heading in a turn
t_*	$t_*' = \frac{t_* U}{\ell}$	time to regain initial heading in an overshoot or zig-zag maneuver
U	$U' = \frac{U}{U}$	absolute linear velocity of origin of x, y, z axes
U_e	$U_e' = \frac{U_e}{U}$	linear velocity of origin of x, y, z axes relative to fluid
$\dot{U}$	$\dot{U}' = \frac{\dot{U}\ell}{U^2}$	linear acceleration of origin of x, y, z axes

See subscript variations under O for corresponding variants of U, U_e, and $\dot{U}$.

u	$u' = \frac{u}{U}$	component of absolute linear velocity U along x axis

See subscript variations under p for analogous variants of u.

$\dot{u}$	$\dot{u}' = \frac{\dot{u}\ell}{U^2}$	time rate of change of u in direction of x axis (not equivalent to a_x)

See subscript variations under p for analogous variants of $\dot{u}$.

V	$V' = \frac{V}{\ell^3}$	volume
V_k		absolute speed in knots
V_{ke}		speed relative to fluid in knots
v	$v' = \frac{v}{U}$	component of absolute linear velocity U along y axis

See u and $\dot{u}$ for notations that also apply to v by analogy.

W	$W' = \frac{W}{\frac{1}{2}\rho U^2 \ell^2}$	weight; $W = mg$
w	$w' = \frac{w}{U}$	component of absolute linear velocity U along z axis

See u and $\dot{u}$ for notations that also apply to w by analogy.

X	$X' = \frac{X}{\frac{1}{2}\rho U^2 \ell^2}$	component of hydrodynamic force along x axis; also referred to as longitudinal force

See subscript variations of K and M for analogous variants of X. Some typical examples follow for the purpose of indicating the nondimensional form.

X_q	$X_q' = \frac{X_q}{\frac{1}{2}\rho U \ell^3}$	partial derivative $\frac{\partial X}{\partial q_e}$
X_{qa}	$X_{qa}' = \frac{X_{qa}}{\frac{1}{2}\rho U \ell^3}$	partial derivative $\frac{\partial X_a}{\partial q_{ea}}$
X_{qq}	$X_{qq}' = \frac{X_{qq}}{\frac{1}{2}\rho \ell^4}$	partial derivative $\frac{\partial^2 X}{\partial q_e^2}$
X_{qqq}	$X_{qqq}' = \frac{X_{qqq} U}{\frac{1}{2}\rho \ell^5}$	partial derivative $\frac{\partial^3 X}{\partial q_e^3}$
X_{qqu}	$X_{qqu}' = \frac{X_{qqu} U}{\frac{1}{2}\rho \ell^4}$	partial derivative $\frac{\partial^3 X}{\partial q_e^2 \partial u_e}$
X_{qu}	$X_{qu}' = \frac{X_{qu}}{\frac{1}{2}\rho \ell^3}$	partial derivative $\frac{\partial^2 X}{\partial q_e \partial u_e}$
X_{quu}	$X_{quu}' = \frac{X_{quu} U}{\frac{1}{2}\rho \ell^3}$	partial derivative $\frac{\partial^3 X}{\partial q_e \partial u_e^2}$
$X_{\dot{q}}$	$X_{\dot{q}}' = \frac{X_{\dot{q}}}{\frac{1}{2}\rho \ell^4}$	partial derivative $\frac{\partial X}{\partial \dot{q}_e}$
X_u	$X_u' = \frac{X_u}{\frac{1}{2}\rho U \ell^2}$	partial derivative $\frac{\partial X}{\partial u_e}$
X_{uu}	$X_{uu}' = \frac{X_{uu}}{\frac{1}{2}\rho \ell^2}$	partial derivative $\frac{\partial^2 X}{\partial u_e^2}$
X_{uuu}	$X_{uuu}' = \frac{X_{uuu} U}{\frac{1}{2}\rho \ell^2}$	partial derivative $\frac{\partial^3 X}{\partial u_e^3}$
$X_{\dot{u}}$	$X_{\dot{u}}' = \frac{X_{\dot{u}}}{\frac{1}{2}\rho \ell^3}$	partial derivative $\frac{\partial X}{\partial \dot{u}_e}$

X_δ	$X_\delta' = \dfrac{X_\delta}{\frac{1}{2}\rho U^2 \ell^2}$	partial derivative $\dfrac{\partial X}{\partial \delta}$
X_θ	$X_\theta' = \dfrac{X_\theta}{\frac{1}{2}\rho U^2 \ell^2}$	partial derivative $\dfrac{\partial X}{\partial \theta}$
x, y, z		moving axes fixed in body
x_a, y_a, z_a		principal moving body axes
x_b, y_b, z_b		translating axes, parallel to x_0, y_0, z_0 axes, and moving with origin of x, y, z
x_c, y_c, z_c		moving axes fixed in control surface
x_f, y_f, z_f		axes moving with fluid
x_g, y_g, z_g		special moving axes related to geometric properties, center of mass, etc.
x_s, y_s, z_s		supplemental moving axes for a second body or similar uses
x_0, y_0, z_0		inertial axes fixed in space
x	$x' = \dfrac{x}{\ell}$	displacement in direction of x axis

See list of axes, immediately preceding, for subscript variants of x.

x_B	$x_B' = \dfrac{x_B}{\ell}$	x coordinate of center of buoyancy
x_{Ba}	$x_{Ba}' = \dfrac{x_{Ba}}{\ell}$	x_a coordinate of center of buoyancy
x_{Fi}	$x_{Fi}' = \dfrac{x_{Fi}}{\ell}$	x coordinate of i th fin, hydrofoil, etc.; $i = 1, 2, \cdots$ (i omitted if only one item is involved)
x_G	$x_G' = \dfrac{x_G}{\ell}$	x coordinate of center of mass
x_{Ga}	$x_{Ga}' = \dfrac{x_{Ga}}{\ell}$	x_a coordinate of center of mass
x_M	$x_M' = \dfrac{x_M}{\ell}$	x coordinate of metacenter

x_P	$x_P' = \frac{x_P}{\ell}$	x coordinate of center of pressure or of typical point P
x_S	$x_S' = \frac{x_S}{\ell}$	x coordinate of static center
x_T	$x_T' = \frac{x_T}{\ell}$	x coordinate of towpoint
x_{0m}	$x_{0m}' = \frac{x_{0m}}{\ell}$	maximum distance, in direction of approach path, reached by origin O in a turn
x_{0i}	$x_{0i}' = \frac{x_{0i}}{\ell}$	displacement of origin O, in direction of approach path, measured from first execute to i th execute; i = 2, 3, $\cdots$
x_{0s}	$x_{0s}' = \frac{x_{0s}}{\ell}$	maximum distance, in direction of approach path, achieved by any point on body during a turn, measured from position of origin O at start of turn
x_{0*}	$x_{0*}' = \frac{x_{0*}}{\ell}$	displacement of origin O, in direction of approach path, at time initial heading has been regained in an overshoot or zig-zag maneuver
Y	$Y' = \frac{Y}{\frac{1}{2}\rho U^2 \ell^2}$	component of hydrodynamic force along y axis; also referred to as lateral force

See subscript variations of K and N for analogous variants of Y.

y	$y' = \frac{y}{\ell}$	displacement in direction of y axis

See subscript variations of x for analogous variants of y.

y_{0s}	$y_{0s}' = \frac{y_{0s}}{\ell}$	maximum lateral displacement from approach path achieved by any point on body during a maneuver, measured from position of origin O at start of maneuver
Z	$Z' = \frac{Z}{\frac{1}{2}\rho U^2 \ell^2}$	component of hydrodynamic force along z axis; also referred to as vertical force

See subscript variations of K and M for analogous variants of Z.

z	$z' = \frac{z}{\ell}$	displacement in direction of z axis

See subscript variations of x for analogous variants of z.

z_{0d}	$z_{0d}' = \frac{z_{0d}}{\ell}$	ordered depth
z_{0e}	$z_{0e}' = \frac{z_{0e}}{\ell}$	depth error
z_{0s}	$z_{0s}' = \frac{z_{0s}}{\ell}$	maximum depth achieved by any point on body during a maneuver, measured from position of origin O at start of maneuver
α		angle of attack
α_δ		flap effectiveness parameter at constant C_L
β		angle of drift
Γ		dihedral angle
γ		angle between x_0y_0 plane and tangent to towline
Δ	$\Delta' = \frac{\Delta}{\frac{1}{2}\rho U^2 \ell^2}$	displacement
Δ		incremental change
ΔW	$\Delta W' = \frac{\Delta W}{\frac{1}{2}\rho U^2 \ell^2}$	incremental change in ballast
δ		angular deflection of control surface
$\delta_b, \delta_d, \delta_f, \delta_r, \delta_s$		deflection of bow plane, dorsal rudder, flap, rudder, or stern plane
δ_{ft}		deflection of tab on doubly-movable flap
δ_i		average control deflection angle during period shortly prior to i th execute; $i = 1, 2, 3, \cdots$
$\delta(t)$		Dirac impulse function

$\dot{\delta}$	$\dot{\delta}' = \frac{\dot{\delta}\ell}{U}$	control deflection rate
$\dot{\delta}_b, \dot{\delta}_d, \dot{\delta}_f, \dot{\delta}_r, \dot{\delta}_s$	$\dot{\delta}_b' = \frac{\dot{\delta}_b \ell}{U}$	rate of deflection of bow plane, dorsal rudder, flap, rudder, or stern plane
ϵ		downwash angle
θ		pitch angle referred to x, y, z axes
θ_a		pitch angle referred to x_a, y_a, z_a axes
θ_f		flight-path angle
θ_m		maximum pitch angle in a maneuver, measured from value at first execute
θ_i		pitch execute angle at i th execute measured from value at first execute; $i = 2, 3, \cdots$
Λ		sweepback angle of quarter-chord line
λ		linear ratio, full-scale to model size
λ		taper ratio, tip chord to root chord
λ	$\lambda' = \frac{\lambda}{\ell}$	wave length
μ	$\mu' = \frac{\mu}{\frac{1}{2}\rho U \ell}$	coefficient of viscosity
ν	$\nu' = \frac{\nu}{U\ell}$	kinematic viscosity, μ/ρ
Π		product of
ρ	$\rho' = 2$	mass density of water
Σ		sum of
σ		cavitation number

σ_i	$\sigma_i' = \frac{\sigma_i \ell}{U}$	roots of stability equation; i = a for real root; i = b, c for conjugate complex pair of roots; $a = 1, 2, \cdots$ $b = 1, 2, \cdots$ $c = 1, 2, \cdots$ $a \neq b \neq c$
σ_{ih}	$\sigma_{ih}' = \frac{\sigma_{ih} \ell}{U}$	roots of stability equation for horizontal motion
σ_{iv}	$\sigma_{iv}' = \frac{\sigma_{iv} \ell}{U}$	roots of stability equation for vertical motion
Υ_i		phase angle; $i = 1, 2, \cdots$ or $\theta, \phi, \cdots$ to distinguish various phases, when needed
ϕ		roll angle referred to x, y, z axes
ϕ_a		roll angle referred to x_a, y_a, z_a axes
ϕ_m		maximum roll angle in a maneuver
ψ		heading angle referred to x, y, z axes
ψ_a		heading angle referred to x_a, y_a, z_a axes
ψ_m		maximum heading angle, measured from value at first execute
ψ_i		heading execute angle at i th execute, measured from value at first execute; $i = 2, 3, \cdots$
$\dot{\psi}$	$\dot{\psi}' = \frac{\dot{\psi} \ell}{U}$	rate of change of heading
$\dot{\psi}_b$	$\dot{\psi}_b' = \frac{\dot{\psi}_b \ell}{U}$	rate of change of heading at branch point for spiral tests
$\dot{\psi}_i$	$\dot{\psi}_i' = \frac{\dot{\psi}_i \ell}{U}$	rate of change of heading at i th execute; $i = 2, 3, \cdots$
$\dot{\psi}_*$	$\dot{\psi}_*' = \frac{\dot{\psi}_* \ell}{U}$	rate of change of heading at zero rudder for spiral tests
Ω	$\Omega' = \frac{\Omega \ell}{U}$	absolute angular velocity of origin of x, y, z axes
Ω_e	$\Omega_e' = \frac{\Omega_e \ell}{U}$	angular velocity of origin of x, y, z axes relative to fluid

$\dot{\Omega}$	$\dot{\Omega}' = \frac{\dot{\Omega}\ell^2}{U^2}$	absolute angular acceleration of origin of x, y, z axes
ω	$\omega' = \frac{\omega\ell}{U}$	frequency in radians per second
ω_n	$\omega_n{}' = \frac{\omega_n\ell}{U}$	natural frequency in radians per second
$1(t)$		unit step function
$\lvert A \rvert$		absolute value of A
$\lvert A \rvert$		determinant
$\lVert A \rVert$		matrix
$\bar{A}$		vector
$\hat{A}$		tensor
$\bar{A} \times \bar{B}$		cross product of vectors A and B
$\dot{A}$		derivative of A with respect to time, dA/dt
$\bar{A} \cdot \bar{B}$		dot product of vectors A and B
$n!$		factorial n
$\lVert \tilde{A} \rVert$		adjoint of matrix A
$\approx$		approximately equal to
$\sim$		asymptotically equal to

INTERIM NOMENCLATURE FOR TOWED CABLES

F	drag per unit length of cable when cable is parallel to stream
f	ratio F/R
P	component of external force acting on element of cable in direction of element
p	ratio P/R
Q	component of external force acting on element of cable in direction 90° counterclockwise from direction of element
q	ratio Q/R
R	drag per unit length of cable when cable is normal to stream
s	distance along cable measured positively in sense of positive progression along cable (see Figure 9)
T	tension in cable at arbitrarily chosen point
T_0	tension in cable at point chosen as origin of coordinate system
W	weight in water of unit length of cable
w	ratio W/R
x, y	rectangular coordinates of arbitrarily chosen point on cable (see Figure 9)
ξ, η	nondimensional rectangular coordinates; $\xi = Rx/T_0$, $\eta = Ry/T_0$
σ	nondimensional distance along cable; $\sigma = Rs/T_0$
τ	nondimensional tension; $\tau = T/T_0$
ϕ	angle from direction of motion to direction of tangent to cable at arbitrarily chosen point, the direction of the tangent being taken in the sense of increasing s (see Figure 9)
ϕ_c	critical angle of cable; the value of ϕ when the cable is freely trailed
ϕ_0	value of ϕ at point chosen as origin of coordinate system

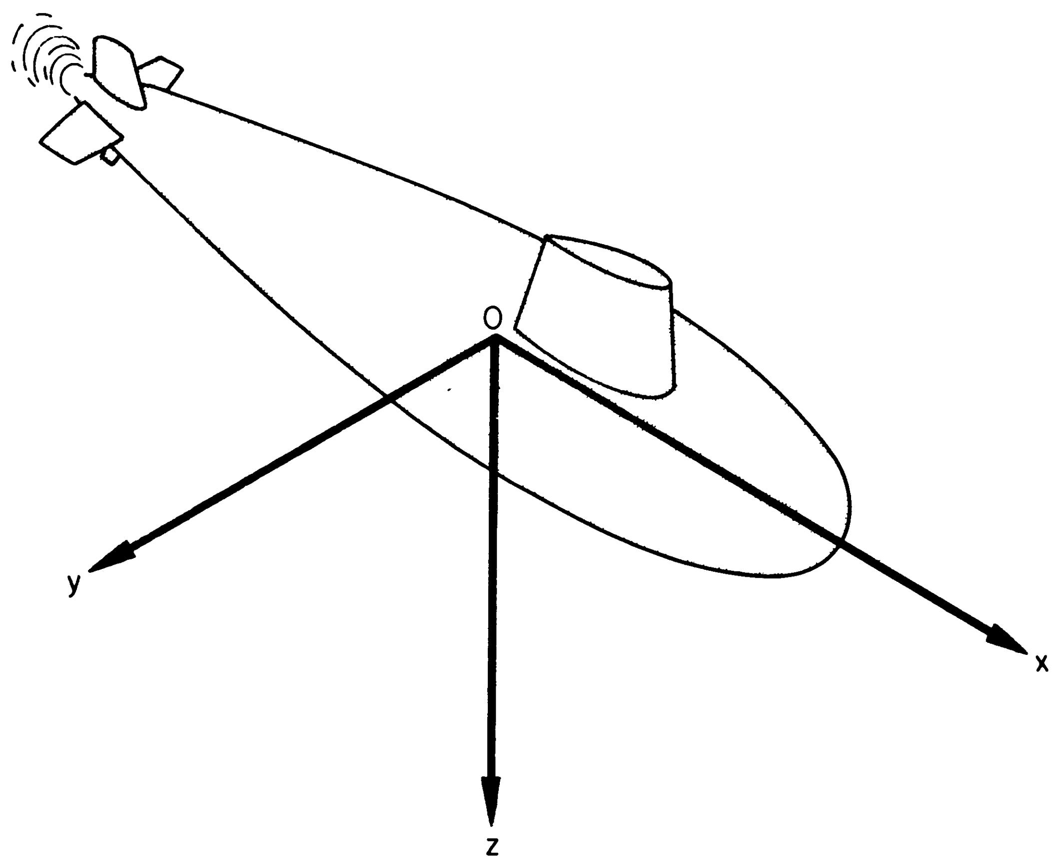


Figure 1 - Alignment of Moving Axes that are Fixed in a Body

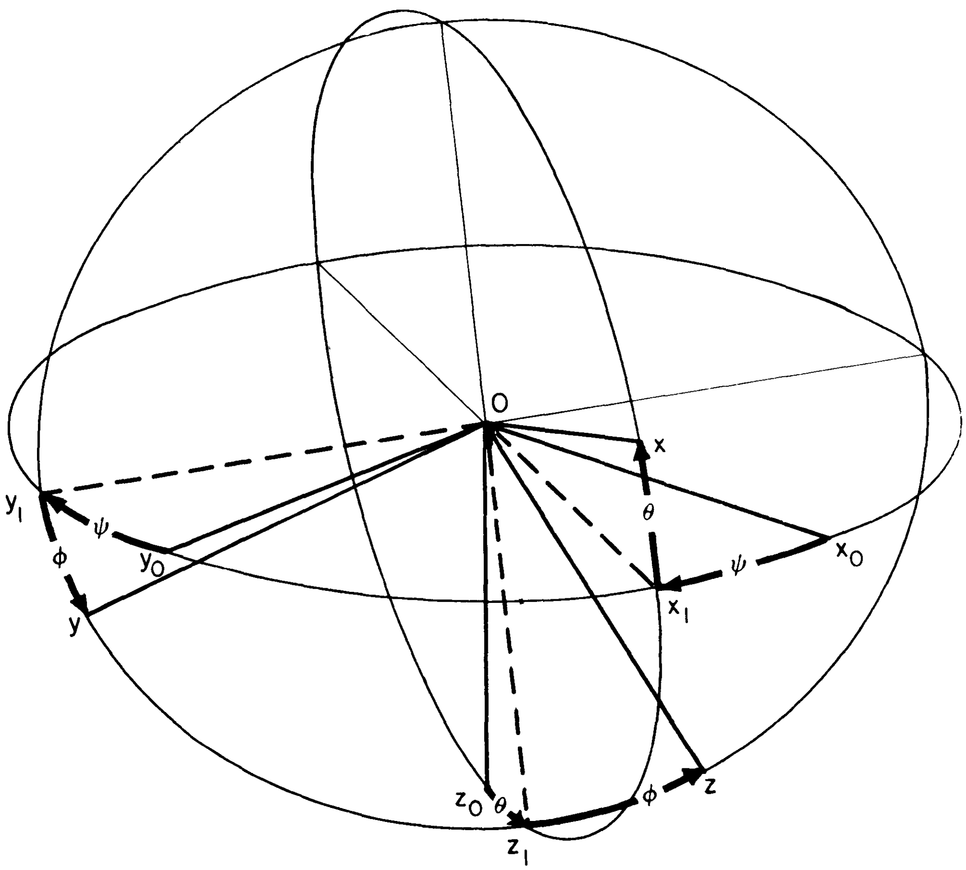


Figure 2 - Euler Angles Defining Orientation of x, y, z Axes Relative to x_0, y_0, z_0 Axes

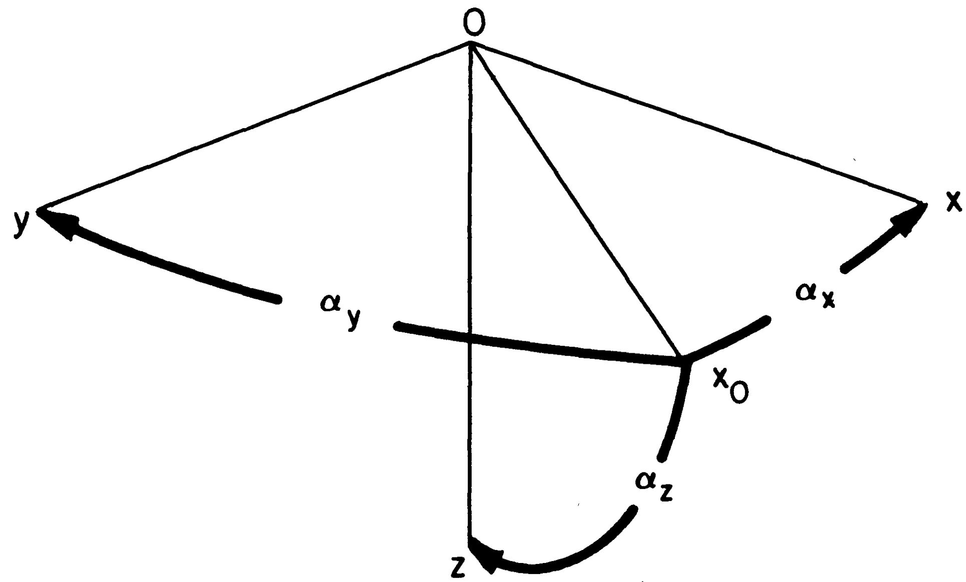
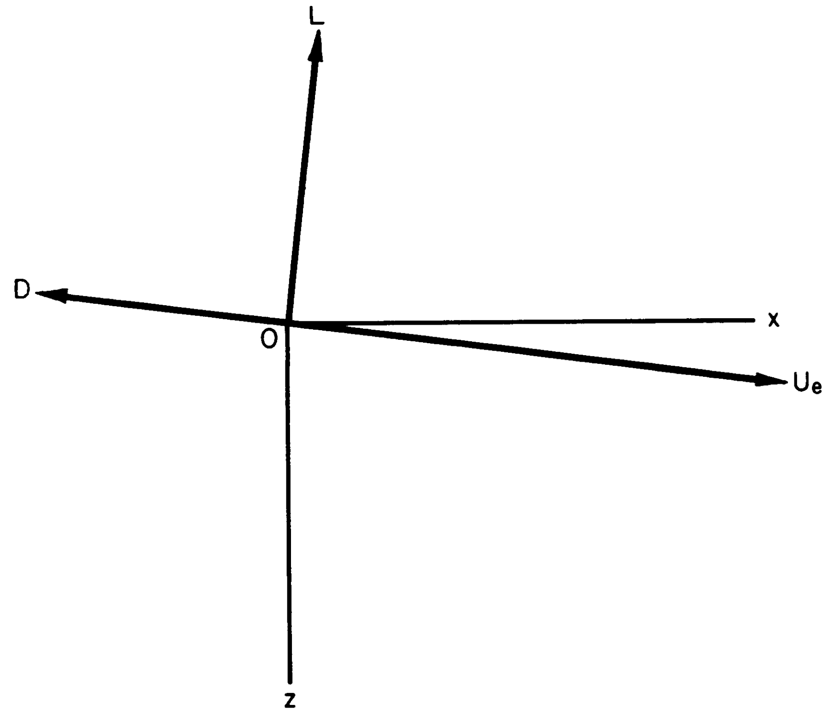
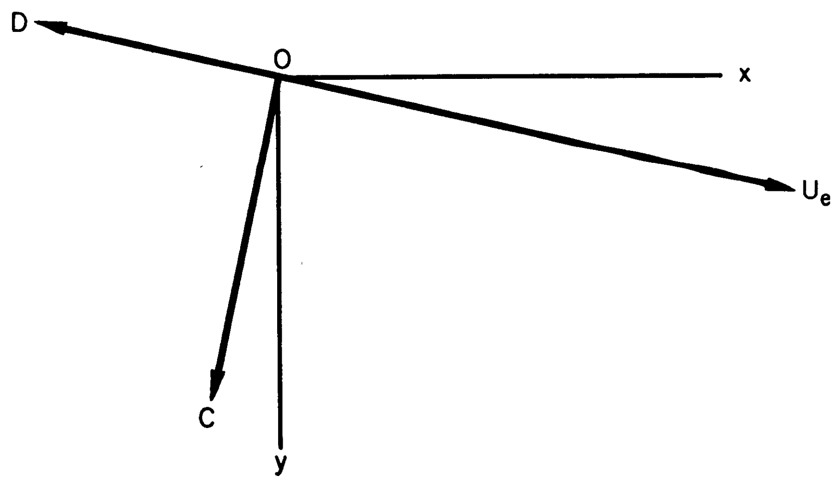


Figure 3 - Angles that Furnish the Direction Cosines Relating the x_0 Axis to the x, y, z Axes



a. View Looking Along Negative Direction of y Axis



b. View Looking Along z Axis

Figure 4 - Positive Directions of Lift, Drag, and Cross Force Components of Hydrodynamic Force

Component L lies in the xz plane and is normal to U_e; component D is directed opposite to U_e and hence is displaced from the xz plane by the amount of the angle β (see Figure 5). Neither D nor C lie in the xy plane. Component C is normal to L and D; thus L, D, and C form a right-hand orthogonal system.

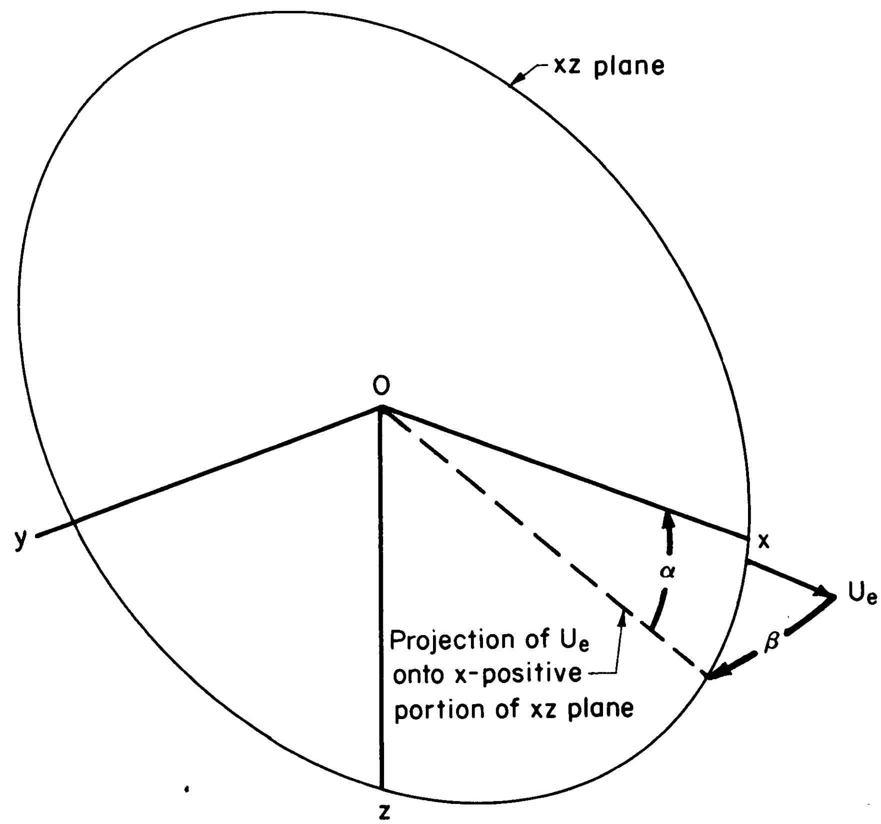


Figure 5 - Orientation of x, y, z Axes with Respect to Relative Motion of Body

Positive senses of α and β are shown.

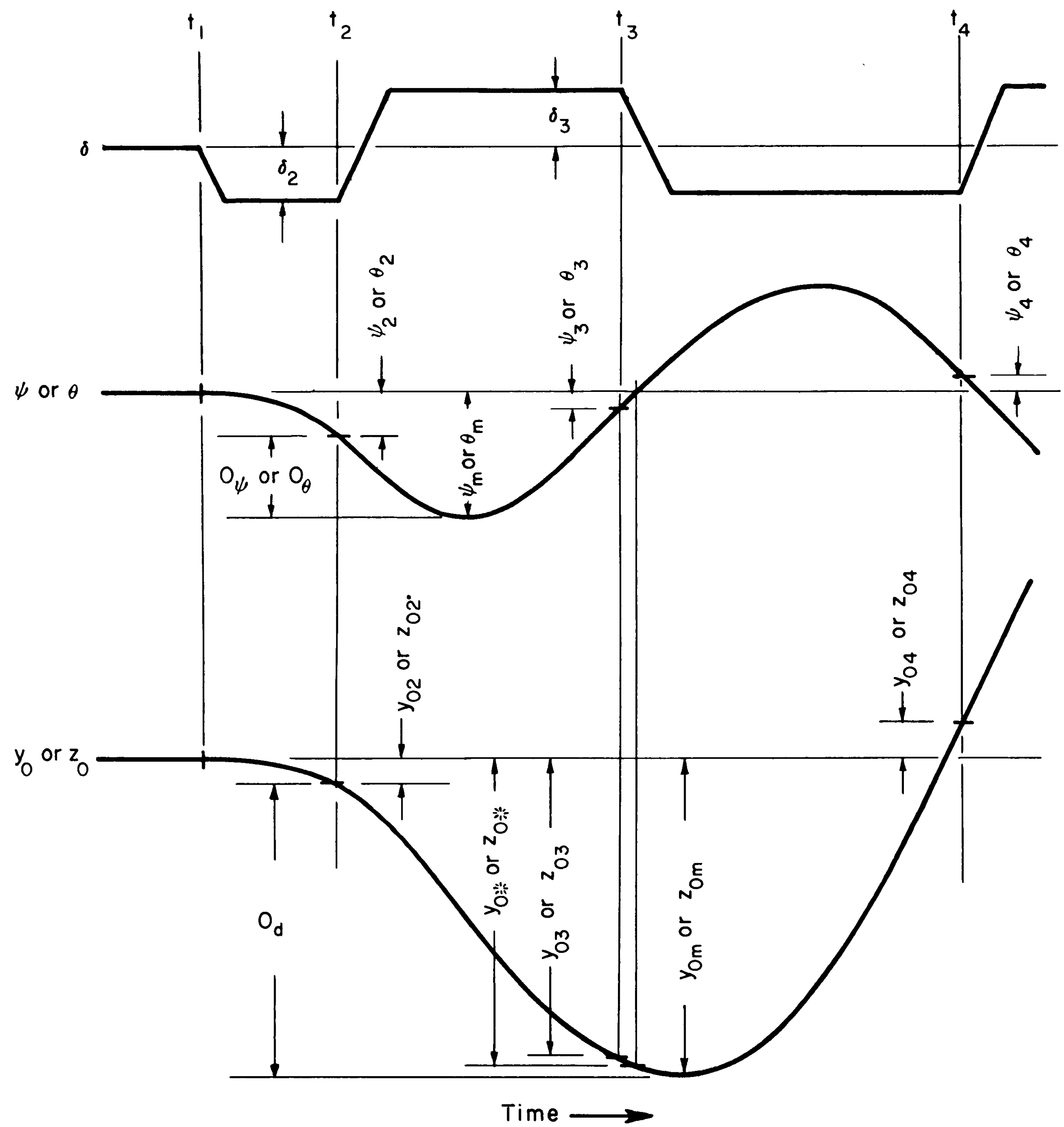


Figure 6 - Parameters for Overshoot and Zig-Zag Maneuvers

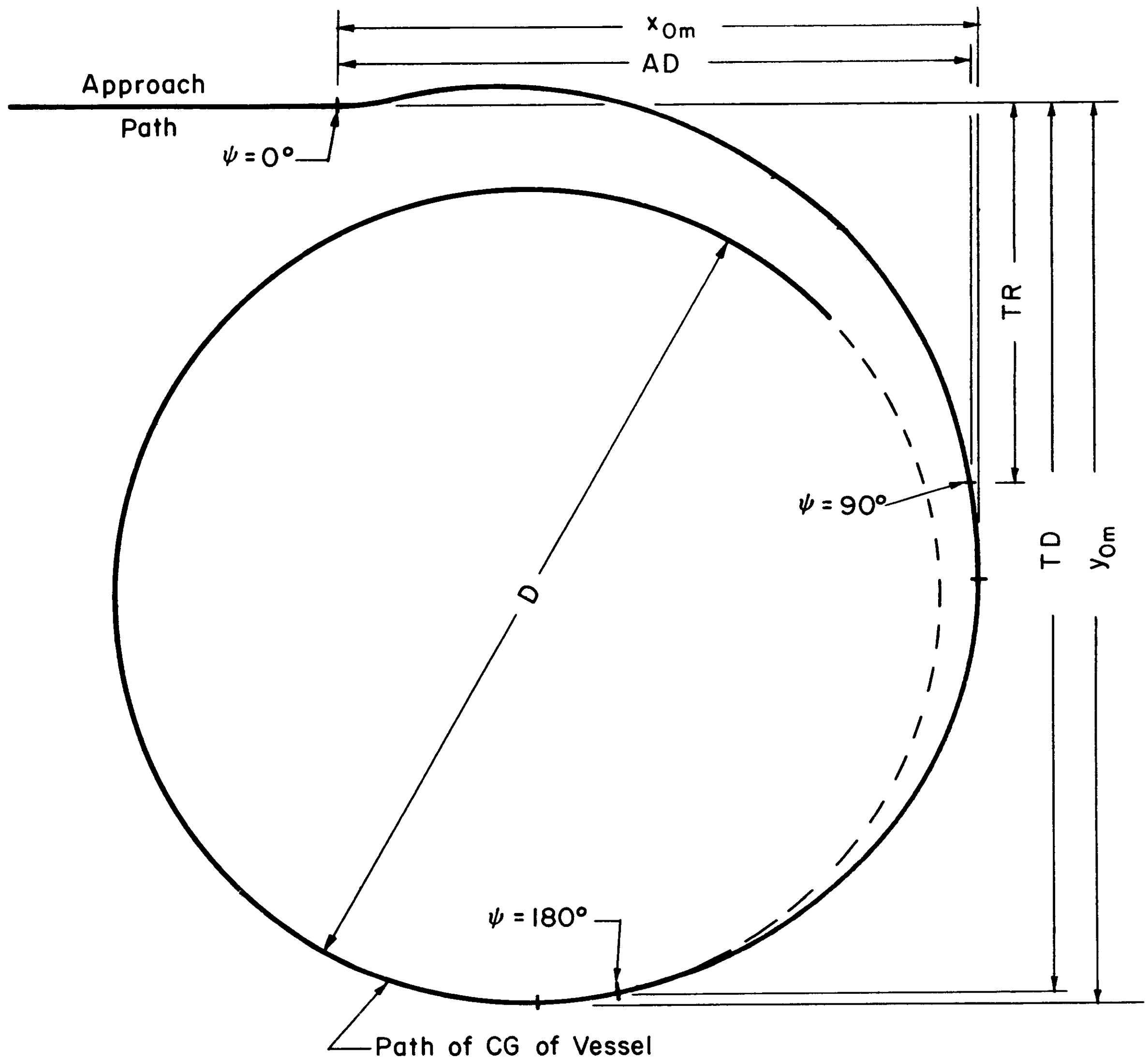


Figure 7 - Parameters for Turning Maneuvers

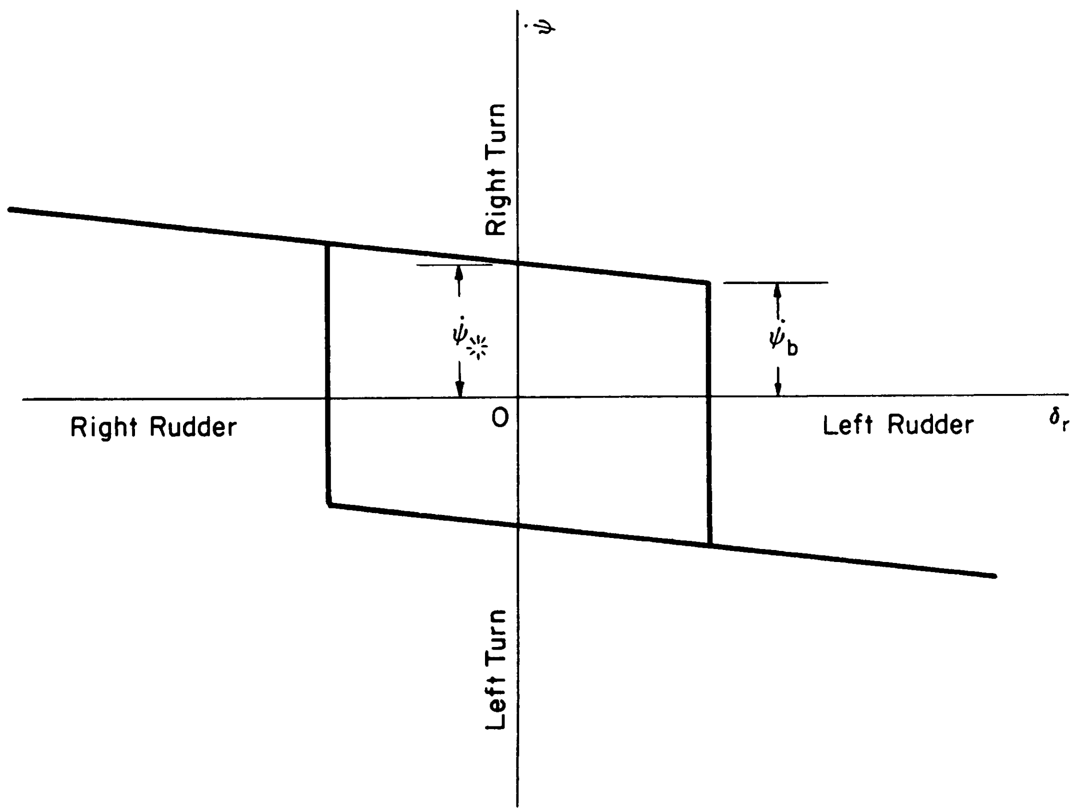


Figure 8 - Turning Rate Versus Rudder Deflection in a Dieudonné Spiral Maneuver[6]

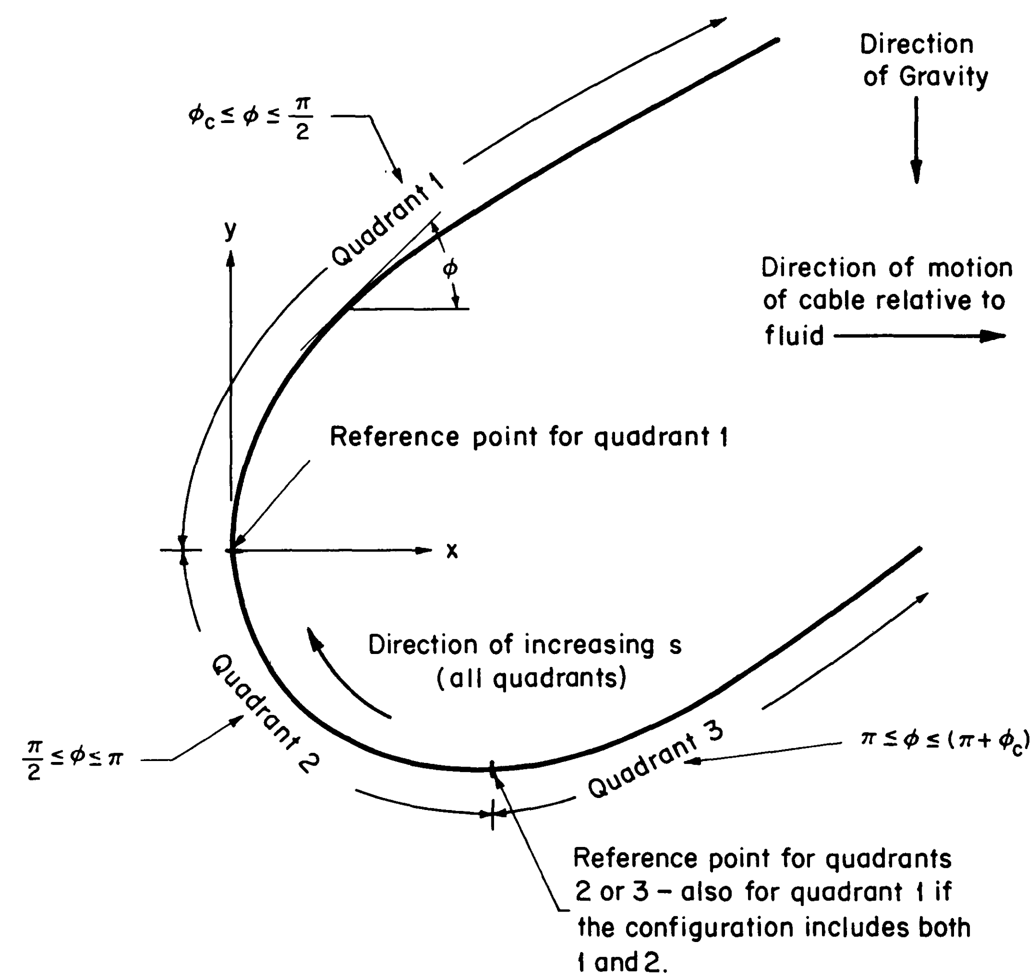


Figure 9 - Configuration of a Cable in a Uniform Stream

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