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DEPARTMENT OF THE NAVY

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A TMB MEAN LINE SERIES TO DELAY CAVITATION INCEPTION

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by

Richard J. Wirth

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HYDROMECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATION

a	Velocity distribution parameter defined in Figure 2
b	Velocity distribution parameter defined in Figure 2
c	Chord length
C_{L_i}	Ideal lift coefficient
$C_{m_{c/4}}$	Quarter chord moment coefficient
m	Velocity distribution parameter defined in Figure 2 $(0 \leq m \leq 1)$
q	Dynamic pressure $(1/2 \rho V^2)$
V	Free stream velocity
V_u	Velocity over upper side of mean line
V_l	Velocity over lower side of mean line
ξ	Variable of integration in Equation 2
y	Mean line ordinate
y_b	Ordinate of mean line at ideal angle of attack
α_i	Ideal angle of attack
α_{oL}	Angle of zero lift
γ	Velocity difference over mean line $(V_u - V_l)$
γ_o	Maximum velocity difference
Γ	Circulation
ρ	Density of fluid

ABSTRACT

The derivation of a generalized mean line series, designated as the TMB "c" mean lines is presented. This series is designed to delay cavitation inception on propeller blades. Included are figures showing the relationship between the ideal angle of attack and the parameters which compose the mean line equations, and examples of various "c" mean lines.

INTRODUCTION

Propeller cavitation, which can be described as the formation of water vapor bubbles in low pressure regions, is generally undesirable. These bubbles which alternately form and collapse, cause metal erosion and create a decrease in thrust and torque which results in a decrease of efficiency. One cause of cavitation inception is the formation of large low-pressure peaks near the leading edge of the blade. In order to delay cavitation inception, a mean line series was developed which reduces this peak low-pressure region.

Figure 1 shows a typical pressure distribution over a blade section. The pressure distribution on a blade can be considered to be composed of three separate and independent components:

1. The distribution due to the velocity over the basic thickness form at zero angle of attack.
2. The distribution due to the mean line shape at ideal angle of attack.
3. The distribution due to additional angle of attack.

Of these three components only the distribution due to mean line shape will be considered here. Since the pressure and velocity over the mean line are related by Bernoulli's equation, the leading edge low-pressure peak can be reduced by decreasing the velocity over the leading edge. Considering this, the velocity distribution of Figure 2 is used as a basis for the mean line derivation. This figure represents the velocity distribution due only to camber and does not represent the total section distribution. The method of mean line derivation used here is similar to that outlined in Reference 1 which is based on thin airfoil theory.[1]

Similar derivations have been developed in which mean lines were obtained from predetermined velocity distributions. For instance the NACA "a" mean lines[2] were obtained from the distribution of Figure 3. The use of NACA "a" mean lines creates large leading edge low-pressure peaks at small angles of attack. The TMB "b" mean lines[3], for which the ideal angle of attack is always negative, were obtained from the distribution of Figure 4. These characteristics are undesirable and we shall derive new mean lines to minimize them. Since the leading edge low-pressure peak can be reduced by merely lowering the velocity over the leading edge, a comparison between Figures 2 and 3 shows that the TMB "c" mean lines will have lower peaks than the NACA "a" mean lines. Also, the location of the velocity fall-off point near the trailing edge has an appreciable effect on the magnitude of the ideal angle of attack. Because this point is non-existant on the TMB "b" mean line velocity distribution, the mean lines are all of negative angle of attack. Therefore the TMB "c" mean lines are derived from a velocity distribution containing such a point which is variable so as to represent the most general case.

DERIVATION OF THE TMB "c" MEAN LINES

From Reference 1 we obtain the following equation for the mean line

$$y = y_b + x\alpha_i \quad * \qquad (1)$$

where y_b represents the ordinates of the mean line at ideal angle of attack whose trailing edge (unless the ideal angle of attack is zero) falls either above or below the x-axis and $x\alpha_i$ is the term which shifts the

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1 References are listed on page 6

* All linear dimensions are nondimensionalized by chord length c.

axes so that the trailing edge lies on the x-axis. A relation for the slope of the mean line whose ordinates are y_b is also given by Reference 1:

$$\frac{dy_b}{dX} = \frac{1}{\pi}\int_0^1 \frac{\frac{2\gamma(x)}{V}\, d\xi}{4(\xi - x)} \tag{2}$$

where x is a point on the mean line and ξ is the variable of integration. $\gamma(x)$ in this equation is the mean line velocity distribution shown in Figure 2. Now we wish to express $\gamma(x)$ in terms of lift coefficient and the parameters a, b and m. Using Kutta-Joukowski's theorem, the ideal lift coefficient becomes

$$C_{L_i} = \frac{2\Gamma}{V} \tag{3}$$

where Γ is the circulation around the mean line and is expressed by

$$\Gamma = \int_0^1 \gamma(x)\, dX$$

Refering to Figure 2, the circulation becomes

$$\Gamma = \int_0^b \gamma_0 \left[\frac{x(1-m) + mb}{b}\right] dX + \int_b^a \gamma_0\, dX + \int_a^1 \gamma_0 \left(\frac{1-x}{1-a}\right) dX$$

or finally

$$\Gamma = \frac{\gamma_0}{2}\left[b(m-1) + a + 1\right]$$

Substituting Γ into Equation 3, an expression is obtained for the maximum velocity difference γ_0. Using this expression, the velocity distribution can now be written in terms of the lift coefficient and parameters a, b and m. Therefore

$$\gamma(x) = \frac{C_{L_i} V}{b(m-1)+a+1}\left[\frac{x(1-m)+mb}{b}\right] \qquad \text{for } 0 \le X \le b$$

$$\gamma(x) = \frac{C_{L_i} V}{b(m-1)+a+1} \qquad \text{for } b \le X \le a$$

$$\gamma(x) = \frac{C_{L_i} V}{b(m-1)+a+1}\left(\frac{1-x}{1-a}\right) \qquad \text{for } a \le X \le 1$$

Substituting the values for $\gamma(x)$ into Equation 2 and integrating first with respect to ξ and then with respect to x we obtain an expression for y_b.

$$y_b = \frac{C_{Li}}{4\pi[b(m-1)+a+1]}\left\{\frac{1}{1-a}\left[\frac{(1-x)^2}{2} - (1-x)^2\ln(1-x) + (a-x)^2\ln|a-x| - \frac{(a-x)^2}{2}\right]\right.$$
$$\left. + \frac{1-m}{b}\left[(b-x)^2\ln|b-x| + \frac{2bx-b^2}{2} - \left(\frac{(1-m)x^2+2mbx}{1-m}\right)\ln(x)\right]\right\} + \text{constant}$$

For small values, the ideal angle of attack is given by

$$\alpha_i = (y_b)_{x=0} - (y_b)_{x=1}$$

Therefore
$$\alpha_i = \frac{C_{Li}}{4\pi[b(m-1)+a+1]}\left\{\frac{1}{1-a}\left[a^2\ln(a) - (a-1)^2\ln(1-a) + (1-a)\right]\right.$$
$$\left. + \frac{1-m}{b}\left[b^2\ln(b) - (b-1)^2\ln(1-b) - b\right]\right\} \qquad (4)$$

Substituting y_b and α_i into Equation 1 and using the boundary conditions y = 0 at x = 0 and x = 1, we find

$$y = \frac{C_{Li}}{4\pi[b(m-1)+a+1]}\left\{\frac{1}{1-a}\left[(a-x)^2\ln|a-x| - (1-x)^2\ln(1-x) - x(a-1)^2\ln(1-a)\right.\right.$$
$$\left. + a^2(x-1)\ln(a)\right] + \frac{1-m}{b}\left[(b-x)^2\ln|b-x| - \left(\frac{(1-m)x^2+2mbx}{1-m}\right)\ln(x) - x(b-1)^2\ln(1-b)\right.$$
$$\left.\left. + b^2(x-1)\ln(b)\right]\right\} \qquad (5)$$

Equation 5 gives the ordinates of the mean line series that will yield the velocity distribution of Figure 2. This series is designated the TMB "c" mean lines.

Equations for the mean line slope, angle of zero lift and quarter chord moment coefficient are given below:

Mean line slope: $$\frac{dy}{dx} = \frac{C_{Li}}{4\pi[b(m-1)+a+1]}\left\{\frac{1}{1-a}\left[-2(a-x)\ln|a-x| - (a-1) + 2(1-x)\ln(1-x)\right.\right.$$
$$\left. -(a-1)^2\ln(1-a) + a^2\ln(a)\right] + \frac{1-m}{b}\left[-2(b-x)\ln|b-x| - b\left(\frac{1+m}{1-m}\right) - 2\left(\frac{(1+m)x+mb}{1-m}\right)\ln(x) - (b-1)^2\ln(1-b) + b^2\ln(b)\right]\left.\right\} \qquad (6)$$

Angle of zero lift in radians: $$\alpha_{o\ell} = \frac{C_{Li}}{4\pi[b(m-1)+a+1]}\left\{\frac{1}{1-a}\left[a^2\ln(a) - (a-1)^2\ln(1-a)\right.\right.$$
$$\left. + a(2a-1) - 1\right] + \frac{1-m}{b}\left[b^2\ln(b) - (b-1)^2\ln(1-b) + 2b^2\left(\frac{1-m}{m}\right) - b\right]\left.\right\} \qquad (7)$$

Quarter chord moment coefficient:

$$C_{m_{c/4}} = \frac{C_{L_i}}{(m-1)b+a+1}\left\{\frac{b\,(1-m)}{12}(4b-3)+\frac{1}{1-a}\left[\frac{4a^3-3a^2-1}{12}\right]\right\} \tag{8}$$

RESULTS

Examples of the TMB "c" mean lines have been worked out for the case when m = 0.5 and the results are tabulated in tables 1, 2 and 3. Figure 5 shows the effect of parameters a and b on the magnitude of the ideal angle of attack for m = 0.5. Three examples of the TMB "c" mean lines are shown in Figure 6.

Figure 7 shows the relation between a, b and m at zero ideal angle of attack. This figure gives the maximum values of a, b and m for a non-negative ideal angle of attack, i.e., a, b or m for the selected mean line must be less than or equal to the values given by the curves of Figure 7 in order to insure a non-negative ideal angle of attack.

Figure 8 shows the variation of velocity difference with m at various positions along the chord for the case when α_i is zero. Figures of this type (i.e., for other α_i's) will allow the propeller designer to choose a value of m such that the velocity due to camber will be a minimum at a desired position along the chord.

The figures and examples included in this report are for a few values of the pertinent parameters. Additional figures and tables for other values of the parameters can easily be obtained by use of Equations 4, 5, 6, 7, and 8. The TMB "c" mean line series given in this report represents all cases of the so-called "flat top" and " roof top" velocity distributions. When m = 0 and a = 1 the equations reduce to the TMB "b" mean lines and when m = 1, they reduce to the NACA "a" mean lines.

REFERENCES

1. Allen, H. Julian, "General Theory of Airfoil Sections Having Arbitrary Shape or Pressure Distribution," NACA Report No. 833 (1945)

2. Abbott, Ira H. and Von Doenhoff, Albert E., "Theory of Wing Sections," Dover Publications, Inc., New York (1959)

3. Morgan, W.B. "Derivation of a Meanline to Delay Cavitation Inception," TMB Technical Note No. 26 (1958)

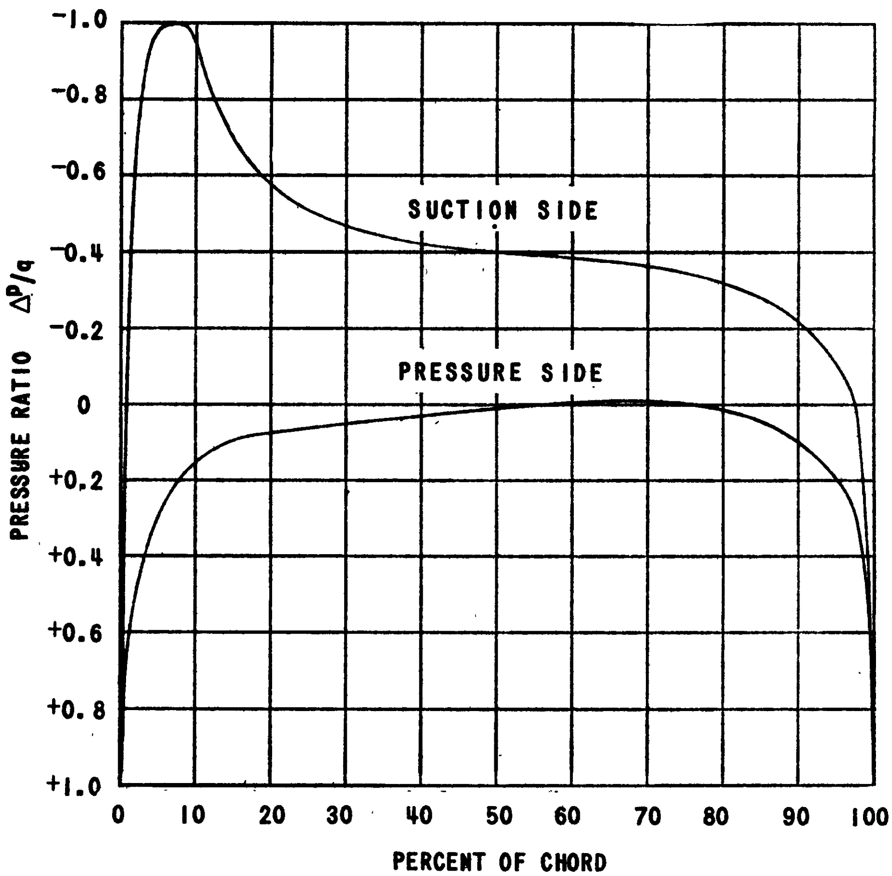

Figure 1 - Typical Pressure Distribution Over a Blade Section

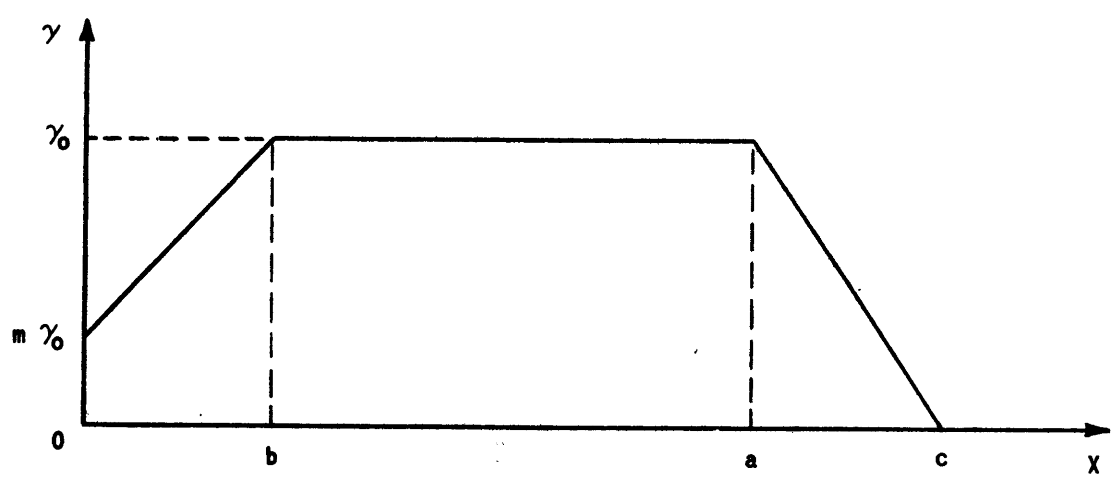

Figure 2 - Velocity Distribution for the TMB "c" Mean Lines

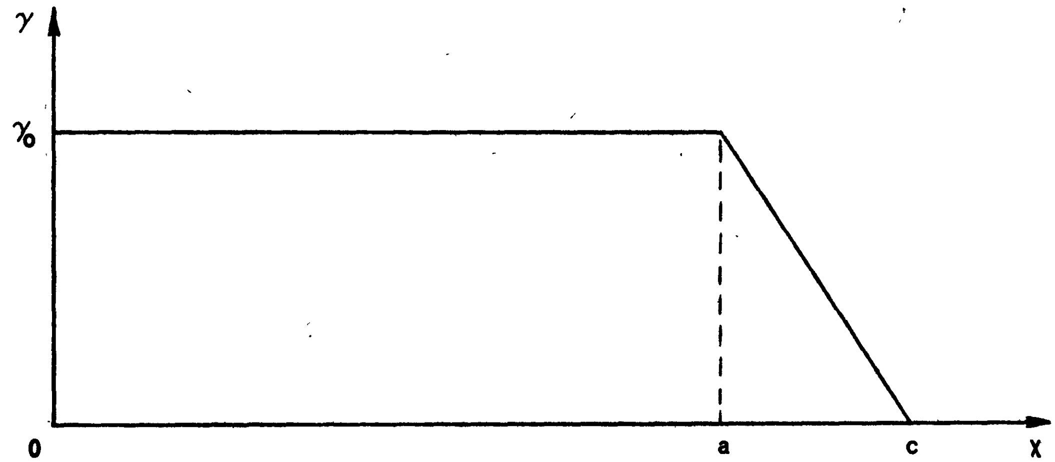

Figure 3 - Velocity Distribution for the NACA "a" Mean Lines

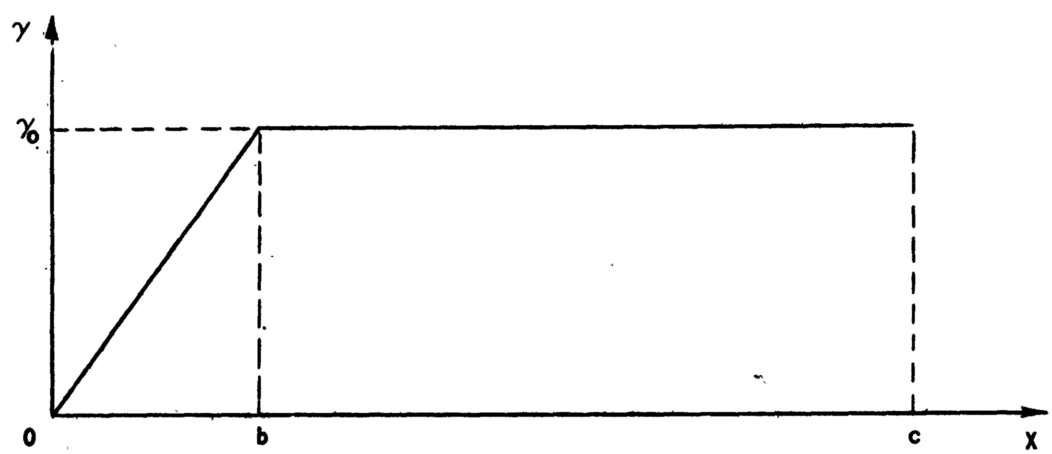

Figure 4 - Velocity Distribution for the TMB "b" Mean Lines

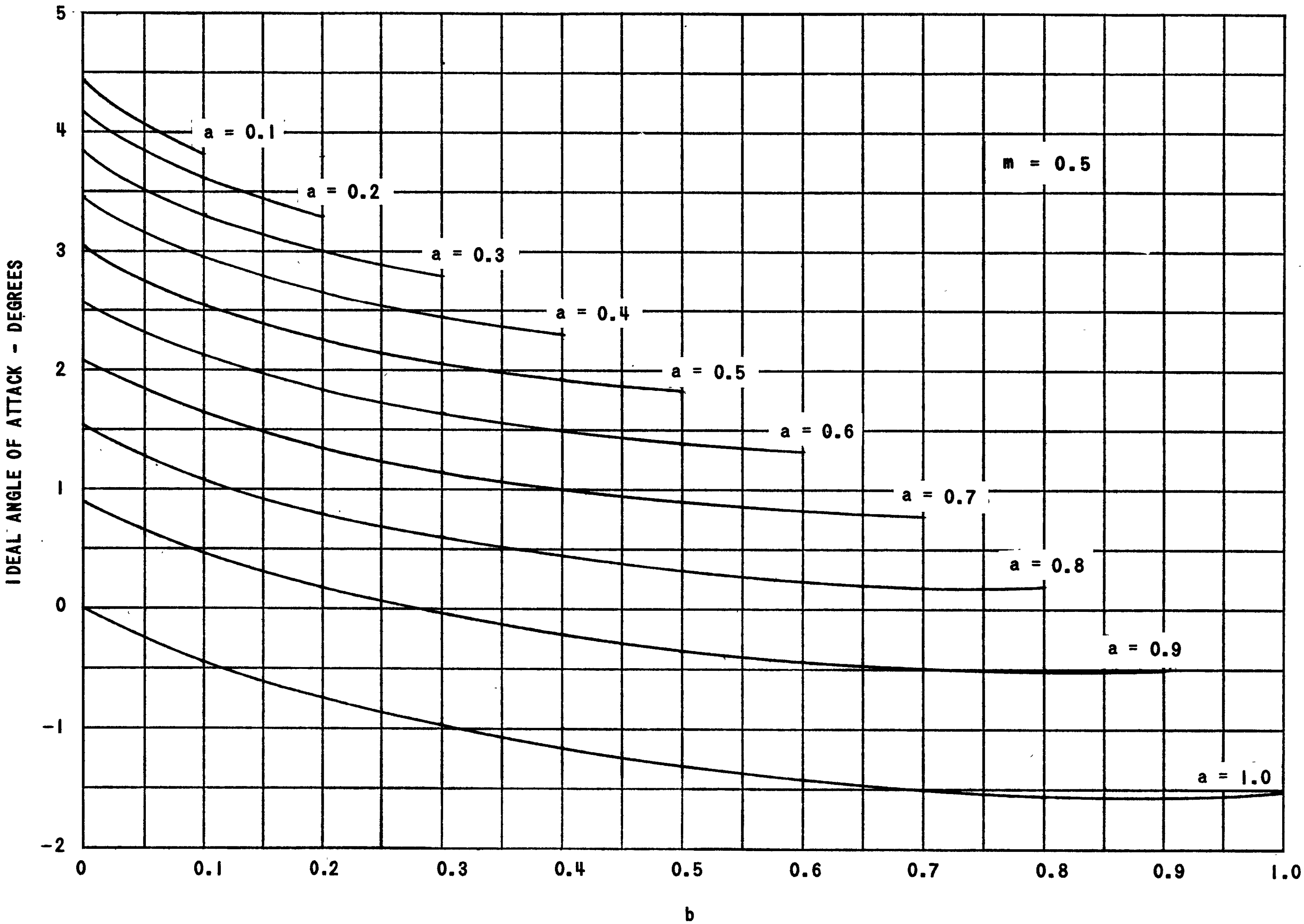

Figure 5 - Variation of Ideal Angle of Attack with Parameters a and b at m = 0.5

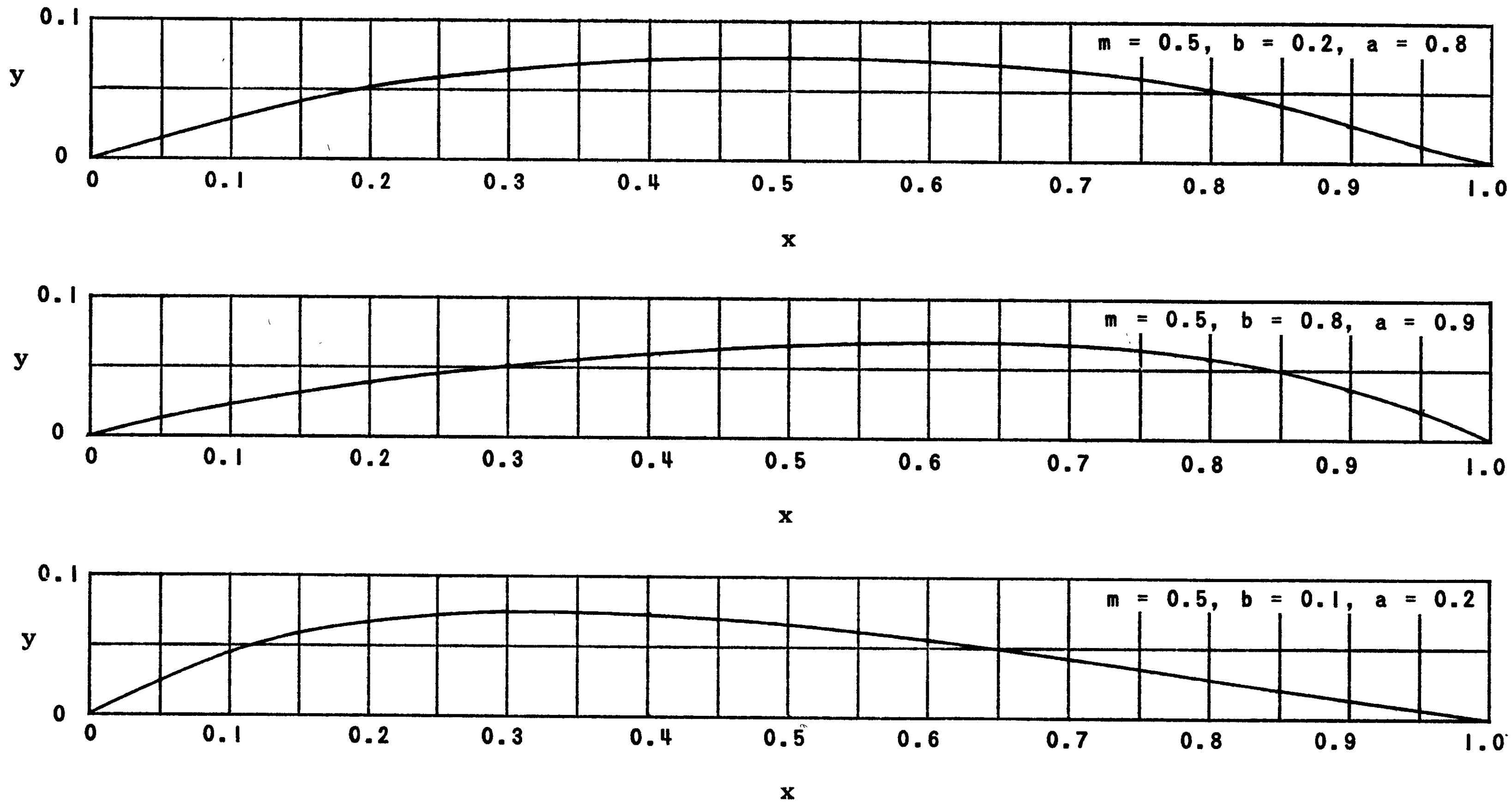

Figure 6 - Examples of TMB "c" Mean Lines

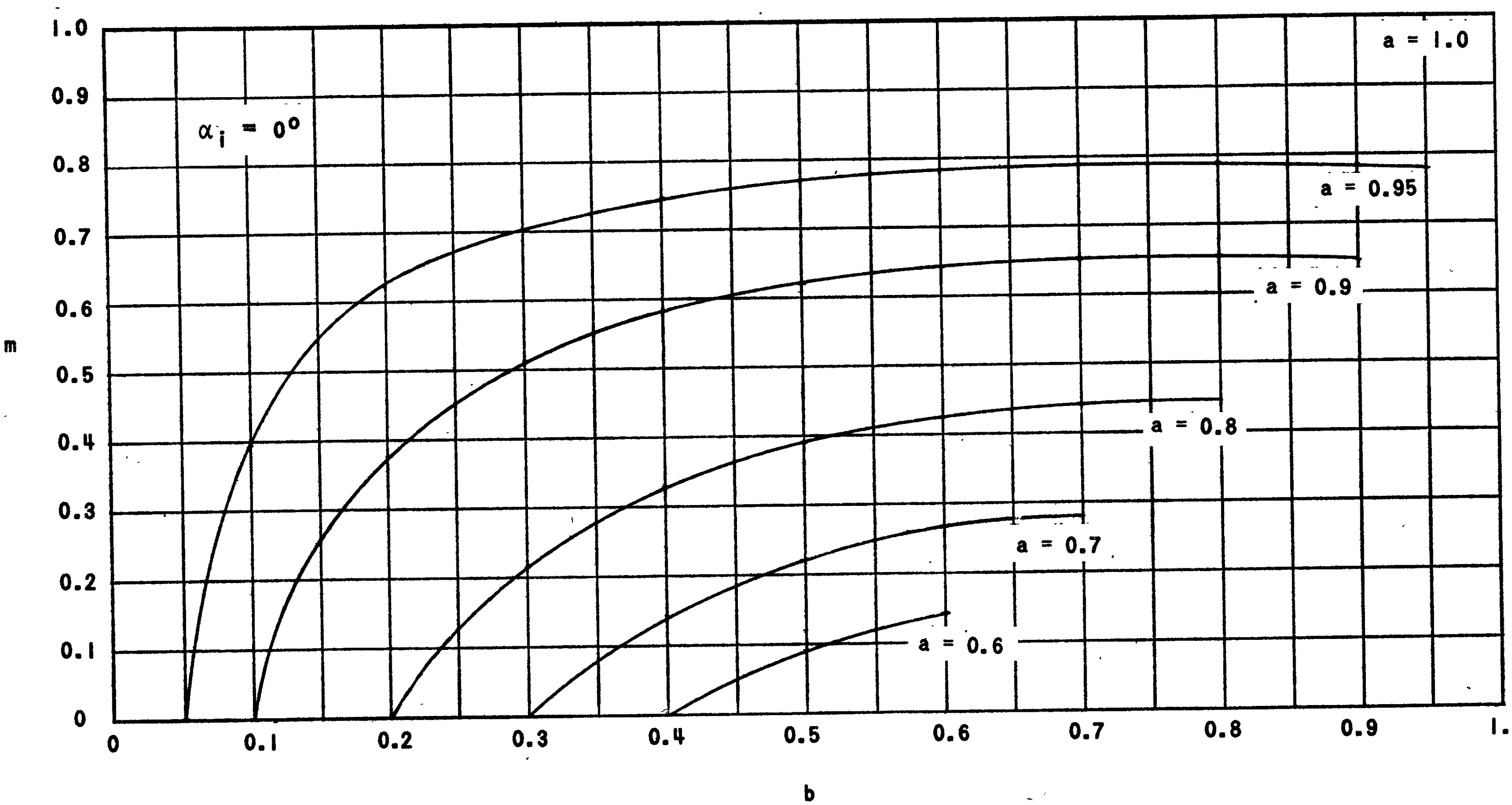

Figure 7 - Relationship Between Parameters a, b and m for Zero Ideal Angle of Attack

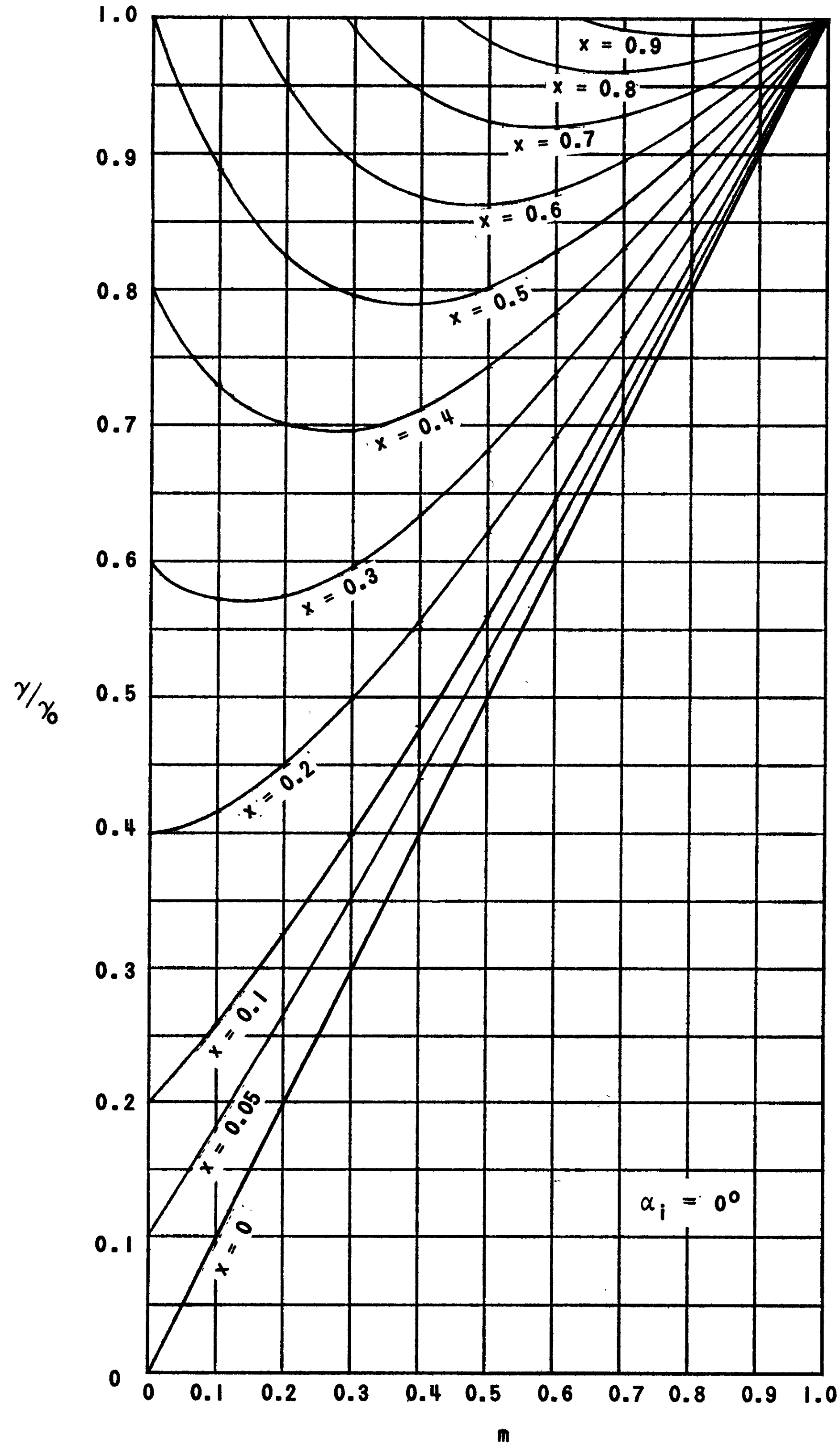

Figure 8 - Variation of Velocity Difference with m at Various Positions along the Chord

TABLE 1

TMB "c" MEAN LINE

$m = 0.5 \quad b = 0.05 \quad C_{L_i} = 1$

x (Percent c)	y (percent c)			
	a = 0.60	a = 0.70	a = 0.80	a = 0.90
0.000	0	0	0	0
0.760	0.401	0.377	0.354	0.331
3.015	1.420	1.336	1.254	1.172
6.699	2.774	2.611	2.451	2.287
11.698	4.095	3.859	3.621	3.372
17.861	5.323	5.024	4.713	4.381
25.000	6.369	6.028	5.658	5.250
32.899	7.156	6.801	6.393	5.924
41.318	7.615	7.287	6.871	6.361
50.000	7.681	7.441	7.057	6.536
58.682	7.269	7.225	6.928	6.435
67.101	6.198	6.591	6.475	6.059
75.000	4.810	5.381	5.684	5.423
82.139	3.405	3.889	4.466	4.550
88.302	2.161	2.494	2.952	3.458
93.301	1.179	1.368	1.639	2.085
96.985	0.501	0.582	0.699	0.904
99.240	0.119	0.139	0.166	0.213
100.000	0	0	0	0
α_i (degrees)	2.30	1.81	1.27	0.64
$C_{m_{c/4}}$	- .165	- .187	- .208	- .227
α_{0L} (degrees)	-6.82	-7.32	-7.85	-3.47

TABLE 2

TMB "c" MEAN LINE

m = 0.5 b = 0.1 C_{L_i} = 1

x (percent c)	y (percent c)			
	a = 0.60	a = 0.70	a = 0.80	a = 0.90
0.000	0	0	0	0
0.760	0.374	0.351	0.329	0.308
3.015	1.301	1.223	1.147	1.070
6.699	2.652	2.495	2.339	2.179
11.698	4.161	3.917	3.672	3.417
17.861	5.473	5.160	4.837	4.494
25.000	6.566	6.207	5.822	5.399
32.899	7.378	7.003	6.578	6.092
41.318	7.844	7.498	7.064	6.537
50.000	7.905	7.648	7.246	6.708
58.682	7.475	7.418	7.106	6.596
67.101	6.373	6.761	6.633	6.204
75.000	4.946	5.518	5.818	5.546
82.139	3.504	3.988	4.568	4.648
88.302	2.225	2.559	3.020	3.529
93.301	1.215	1.405	1.677	2.127
96.985	0.517	0.598	0.716	0.923
99.240	0.123	0.142	0.170	0.218
100.000	0	0	0	0
α_i (degrees)	2.11	1.63	1.09	0.46
$C_{m_{c/4}}$	- .172	-.190	- .215	- .235
α_{oL} (degrees)	- 7.01	- 7.49	- 8.03	- 8.65

TABLE 3

TMB "c" MEAN LINE

m = 0.5 b = 0.2 C_{L_i} = 1

x (percent c)	y (percent c)			
	a = 0.60	a = 0.70	a = 0.80	a = 0.90
0.000	0	0	0	0
0.760	0.351	0.329	0.308	0.287
3.015	1.193	1.120	1.047	0.973
6.699	2.407	2.260	2.113	1.962
11.698	3.885	3.651	3.414	3.167
17.861	5.453	5.133	4.802	4.451
25.000	6.758	6.376	5.970	5.527
32.899	7.667	7.264	6.811	6.298
41.318	8.183	7.805	7.340	6.783
50.000	8.258	7.972	7.539	6.970
58.682	7.813	7.734	7.394	6.854
67.101	6.666	7.047	6.900	6.443
75.000	5.177	5.752	6.047	5.755
82.139	3.671	4.160	4.747	4.819
88.302	2.334	2.672	3.139	3.655
93.301	1.276	1.468	1.744	2.203
96.985	0.543	0.626	0.745	0.956
99.240	0.130	0.149	0.177	0.226
100.000	0	0	0	0
α_i (degrees)	1.83	1.35	0.82	0.18
$C_{m_{c/4}}$	-.183	-.202	-.224	-.245
α_{α} (degrees)	-7.29	-7.77	-8.30	-8.93

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