Report 1106

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A GRAPHICAL METHOD FOR DETERMINING THE GENERAL-INSTABILITY STRENGTH OF STIFFENED CYLINDRICAL SHELLS

by

Thomas E. Reynolds

STRUCTURAL MECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

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NAVY DEPARTMENT
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Subj: NS731-038; General-instability strength of stiffened shells; forwarding of report on

Encl: (1) TMB Report 1106 entitled "A Graphical Method for Determining the General-Instability Strength of Stiffened Cylindrical Shells" 11 copies

1. Under Project NS731-038 various theories for predicting the general-instability strength of stiffened cylindrical shells have been examined at the Taylor Model Basin. The second solution of Kendrick (NCRE Report R.244) gives collapse pressures which correspond most closely with experimental pressures but requires lengthy calculations. To obtain a short method of approximating these pressures an extensive program of numerical solutions over a wide range of geometries was carried out with the aid of Univac. The results are summarized in graphical form in enclosure (1). From these graphs the general-instability strength of a structure and also an estimate of how variations in scantlings affect this strength can be readily determined.

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A GRAPHICAL METHOD FOR DETERMINING THE GENERAL-INSTABILITY STRENGTH OF STIFFENED CYLINDRICAL SHELLS

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

ABSTRACT

The results of numerical calculations of the general-instability strength of ring-stiffened circular cylinders are presented in graphical form. The calculations are based on Kendrick's "second solution" which is published in Naval Construction Research Establishment Report No. 244 (Part III). The collapse pressures from these graphs agree within 10 percent with those computed by Kendrick's theory throughout the normal range of submarine geometries.

INTRODUCTION

One consideration in the structural design of submarine pressure hulls is the possibility of collapse by general instability, i.e., large deformations of frames and shell between holding bulkheads. It is also recognized that the elastic general-instability pressure governs the extent to which imperfections reduce the load-carrying capacity of the frames. For this reason, an accurate determination of this pressure can be useful in frame design even though it is far greater than the pressure encountered at normal operating depth.

The most reliable of several theoretical investigations of the general-instability problem appears to be the recent work of Kendrick conducted at the Naval Construction Research Establishment, Rosyth, Scotland. This was published in the form of two separate analyses.[1,2] Both employ the same general approach, but Reference 2 (Kendrick, Part III) is a more general treatment than Reference 1 (Kendrick, Part I) and always gives a lower collapse pressure.

While Kendrick's analysis is a significant advance in the study of stiffened cylinders, its application to submarine design is difficult because of the lengthy calculations required. Recently Bryant,[3] also of the Naval Construction Research Establishment, developed an approximation which agrees closely with Kendrick's Part I solution. Because of its simplicity, Bryant's formula is a valuable aid in design calculations.

These solutions have been examined at the Taylor Model Basin, and several tests[4] have been conducted with machined models to provide experimental evaluation. In general, agreement between experiment and theory was good, the failure pressures usually being slightly lower than the prediction of Kendrick Part I or Bryant, and slightly higher than that of Kendrick Part III.* The difference in the pressures given by these solutions is insignificant for cylinders with light frames but becomes much larger for heavy frames such as used on submarines. It was found, in fact, that for contemporary submarine geometries, Bryant's formula

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[1] References are listed on page 15.

* It should be noted that this analysis contains two solutions. The first represents a physically impossible buckling configuration but is presented because it gives a lower collapse pressure. In this report, all mention of Kendrick Part III will be confined to the second solution of that analysis.

and Kendrick's Part I solution give pressures as much as 35 percent higher than the Part III solution. In such cases use of the more exact solution in design work is desirable, but it is impractical because of the extensive calculations required.

In view of these difficulties, it seemed worthwhile to look for a short method of approximating Kendrick's Part III solution. Accordingly, an extensive program was begun at the Model Basin to obtain numerical solutions over a wide range of geometries, the objective being to summarize the results in some graphical form which would be of practical use to the designer. Such a presentation would not only provide a quicker and more accurate means of determining the general-instability strength of a structure but would present a better picture of how variations in the scantlings affect this strength. Because of the very large amount of computation involved, this program would have been virtually impossible without the aid of the high-speed computer UNIVAC.*

The results of the calculations are summarized in this report in the form of graphs which relate general-instability strength to variations in frame size, frame spacing, shell radius and thickness, and compartment length. All calculations were for externally framed steel cylinders with a Young's modulus of 30×10^6 psi. Since the elastic general-instability pressure is directly proportional to the modulus, these results are readily applicable to other elastic materials having Poisson's ratio $\nu = 0.3$. Moreover, since internal frames theoretically provide slightly higher general-instability strength than external frames of the same dimensions, the results can be safely applied to internally framed cylinders. The accuracy of the graphical results is demonstrated by a comparison with numerical solutions of Kendrick's Part III theory for a wide range of geometry. The use of the graphs is illustrated in Appendix A by a numerical example. In Appendix B, approximate formulas are given whereby a frame strength parameter can be determined, and several numerical examples are provided to demonstrate the accuracy of the formulas.

METHOD

Since Kendrick's Part III analysis cannot be reduced to a simple algebraic expression, the problem of presenting the theory in graphical form must be approached somewhat indirectly. The method adopted consisted of plotting the results of many numerical calculations against various geometrical parameters until, by a process of trial and error, a system of coordinates was found in which the points followed closely a set of single-valued curves.

In attempting to define general-instability strength in terms of the shape and size of the structure, at least five quantities must be considered, i.e., shell radius, shell thickness, compartment length, frame spacing, and some measure of frame size. This situation is further complicated by the variability of frame shape and the fact that the number of circumferential

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* Numerical results for more than 200 different geometries were obtained on UNIVAC through the solution of a fifth-order matrix by iteration methods.

waves into which the cylinder buckles (also a function of the relative dimensions) is a vital factor in the determination of general-instability strength. Thus it is clear that the problem would be greatly simplified if a few fundamental parameters could be obtained by combining some of the dimensions in a rational manner.

It is shown in Bryant's paper[3] that the general-instability pressure can be treated approximately as the sum of two terms, one involving the strength of the shell and the other the strength of frame per unit length of shell. This latter term was defined as the moment of inertia about the centroid of a section comprising one frame plus a length of shell equal to one frame spacing. Numerical results indicate that Kendrick's second solution in Reference 2 can be broken down fairly successfully in this way. However, a slightly different parameter used by Bijlaard[5] in treating the same problem lent itself better to a graphical presentation and gave less scatter in the results. Bijlaard expresses frame strength as the moment of inertia I_e about the centroid of the combined section of a frame plus an effective length L_e of shell. L_e is taken to be $1.57\sqrt{Rh}$ so long as the frame spacing L_f exceeds $2\sqrt{Rh}$. For smaller values of L_f, L_e can be determined from Table 46 of Reference 6. The quantity I_e can be written

$$I_e = \frac{A_f e^2}{1+\dfrac{A_f}{L_e h}} + I_f + \frac{L_e h^3}{12} \qquad [1]$$

L_e

Where R is the radius to the median surface of the shell,

h is the shell thickness,

A_f is the frame area,

I_f is the moment of inertia of the frame about its own centroid, and

e is the distance from the median surface of the shell to the centroid of the frame.

With the parameters as defined above, the general-instability pressure p_{cr} of a stiffened cylinder can be expressed as the sum of two terms:

$$p_{cr} = p_s\left(\frac{L_b}{R}, \frac{h}{R}, n\right) + p_f\left(\frac{I_e}{L_f R^3}, n\right) \qquad [2]$$

where n is the number of circumferential waves,

L_b is the bulkhead spacing, and

p_s is a linear function of h/R.

The quantity p_s can be determined readily from Figure 1 in which $p_s \cdot \frac{R}{100h}$ is plotted against L_b/R for values of n of 2, 3, 4, and 5. Similarly, Figure 2 shows the variation of p_f with $I_e/L_f R^3$ for the different values of n. These curves were drawn to fit a large number of calculated points.* The same information is presented more concisely in Figure 3 where the results from Figures 1 and 2 are combined in one graph having the coordinates

*Since only values for p_{cr} were obtained from the calculations, p_{cr} was plotted against $I_e/L_f R^3$, and p_s and p_f were determined by extrapolation to $I_e/L_f R^3 = 0$.

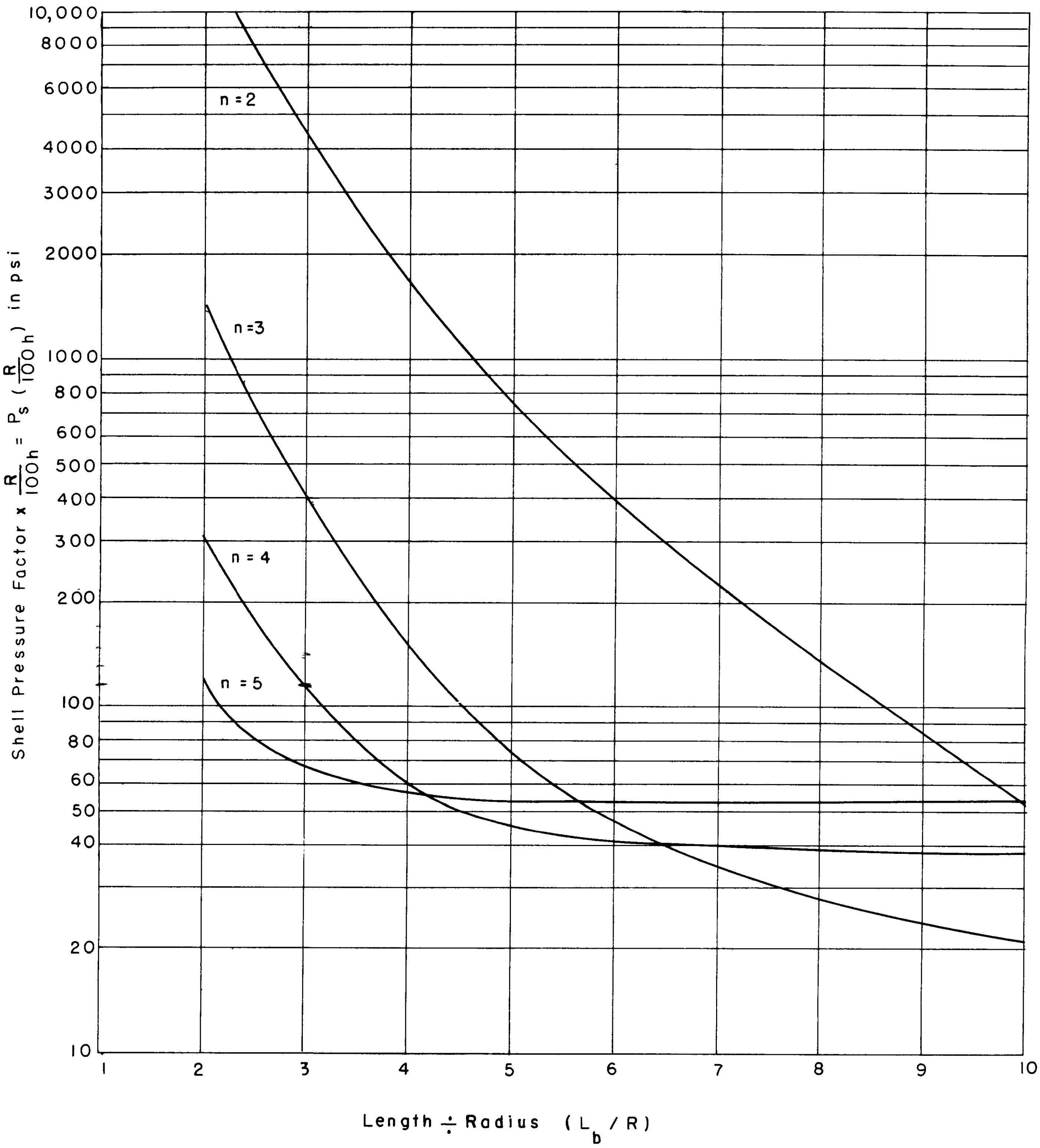

Figure 1 - Shell Pressure Factor (p_s) as a Function of Bulkhead Spacing (L_b/R)

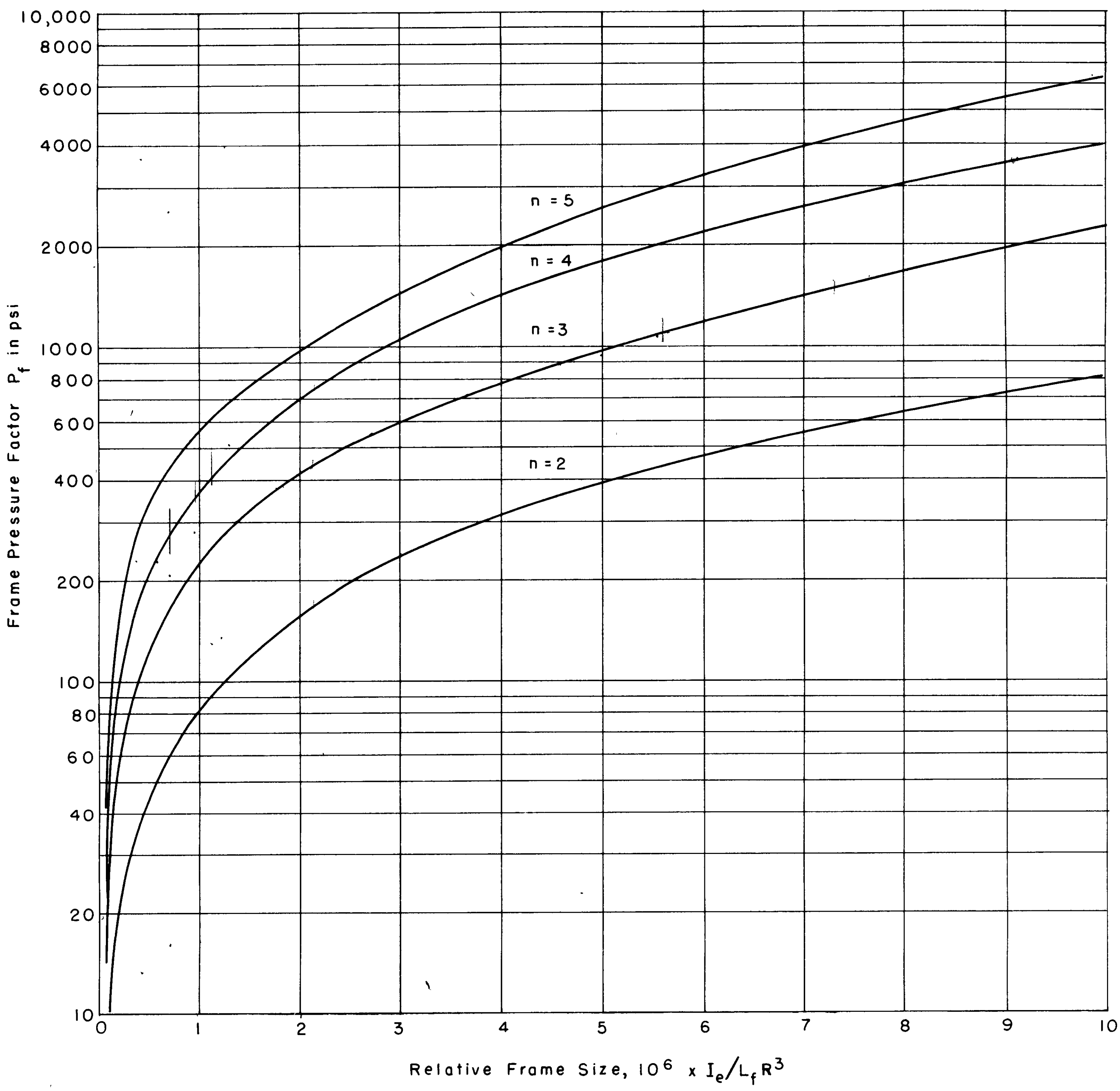

Figure 2 - Frame Pressure Factor (p_f) as a Function of Frame Stiffness ($I_e/L_f R^3$)

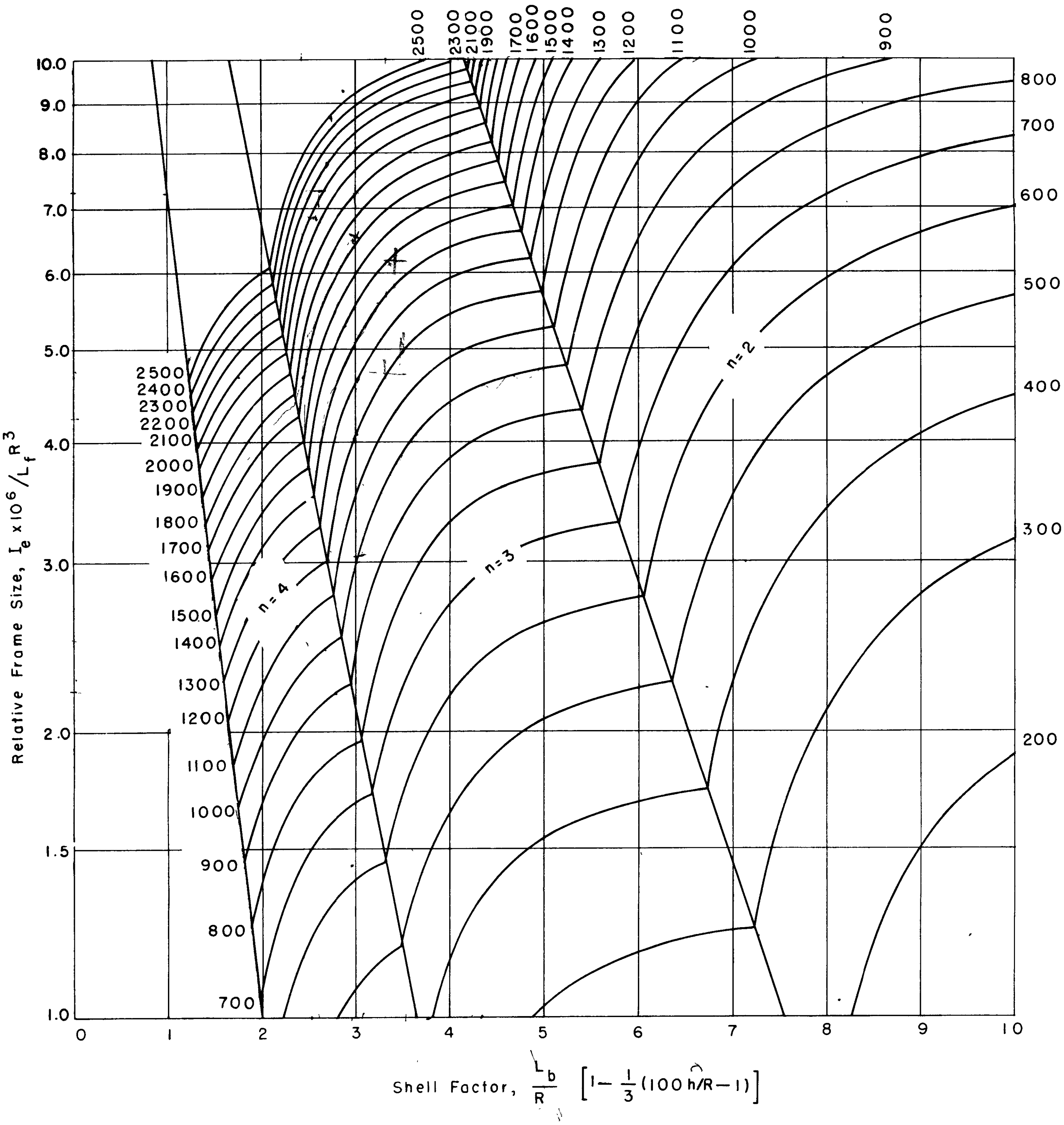

Figure 3 - Curves of Constant Pressure

$\left[1 - \frac{1}{3}(100\,h/R - 1)\right] L_b/R$ and $I_e/L_f R^3$. Here plots of constant values for p_{cr} appear as families of curves intersecting along straight lines. These lines divide the graph into regions of n = 2, 3, 4, and 5. A close estimate of the collapse pressure can thus be obtained directly from this figure once the coordinates are known. For a more precise determination of p_{cr} Figures 1 and 2 should be used after the appropriate value of n has been found from Figure 3.

ACCURACY OF RESULTS

In the construction of Figures 1 through 3, some scatter in the calculated points was unavoidable, as might be expected. However, in only a few isolated instances was the difference between the graphical results and those from Kendrick Part III as high as 15 percent. In the great majority of cases, it was less than 10 percent. A representative set of results obtained from Figures 1 to 3 is presented for comparison with Kendrick's theory in Table 1.

ACKNOWLEDGMENT

The author is indebted to Dr. E. Wenk, Jr., at whose suggestion this project was initiated, and to the members of the Applied Mathematics Laboratory for their valuable assistance in programming the numerous calculations on UNIVAC.

TABLE 1

Comparison of Numerical Results with Kendrick's (Part III) Theory

Relative Thickness $100h/R$	Relative Length L_b/R	Relative Frame Stiffness I_e/L_fR^3	Collapse Pressure p_{cr}		Mode of Collapse n	
			From Figures 1 and 2	From Kendrick's Theory	From Figure 3	From Kendrick's Theory
0.6	2.194	4.93×10^{-6}	1632	1558	3	3
0.8		5.99×10^{-6}	2096	2036	3	3
1.0		6.62×10^{-6}	2475	2409	3	3
1.22		7.15×10^{-6}	2850	2766	3	3
0.6	4.539	4.93×10^{-6}	1015	959	3*	2
0.8		5.99×10^{-6}	1260	1218	3*	2
1.0		6.62×10^{-6}	1430	1464	3*	2
1.22		7.15×10^{-6}	1572	1671	3	3
0.6	6.884	4.93×10^{-6}	529	520	2	2
0.8		5.99×10^{-6}	662	623	2	2
1.0		6.62×10^{-6}	757	715	2	2
1.22		7.15×10^{-6}	853	804	2	2
0.6	9.229	4.93×10^{-6}	430	455	2	2
0.8		5.99×10^{-6}	530	534	2	2
1.0		6.62×10^{-6}	592	599	2	2
1.22		7.15×10^{-6}	651	659	2	2
0.6	2.194	1.145×10^{-6}	561	539	4	4
0.8		1.385×10^{-6}	703	660	4	4
1.0		1.581×10^{-6}	810	781	4	4
1.22		1.773×10^{-6}	937	923	4	4
0.6	4.539	1.145×10^{-6}	310	293	3	3
0.8		1.385×10^{-6}	377	347	3	3
1.0		1.581×10^{-6}	435	401	3	3
1.22		1.773×10^{-6}	492	464	3	3
0.6	6.884	1.145×10^{-6}	235	223	2	2
0.8		1.385×10^{-6}	290	281	2	2
1.0		1.581×10^{-6}	363	338	2	2
1.22		1.773×10^{-6}	413	391	3	3
0.6	9.229	1.145×10^{-6}	136	136	2	2
0.8		1.385×10^{-6}	168	164	2	2
1.0		1.581×10^{-6}	198	191	2	2
1.22		1.773×10^{-6}	229	223	2	2

*For these three geometries, collapse pressures for $n = 2$ were nearly the same as for $n = 3$.

APPENDIX A
NUMERICAL EXAMPLE

A numerical example is provided to illustrate the use of the curves presented herein. A typical case has been chosen in which the dimensions could be those of a submarine pressure hull with external frames. The pertinent scantlings are:

$$R = 96 \text{ in.} \qquad A_f = 9.63 \text{ in.}^2$$
$$h = 0.768 \text{ in.} \qquad I_f = 80.48 \text{ in.}^4$$
$$L_f = 30 \text{ in.} \qquad e = 4.49 \text{ in.}$$
$$L_b = 384 \text{ in.}$$

The cylinder has 13 frames with one short bay at each end. First the value of the effective length L_e is found to be 13.48 in. Then

$$\frac{I_e}{L_f R^3} \text{ is } 6.85 \times 10^{-6}$$

and

$$\left[1 - \frac{1}{3}(100\,h/R - 1)\right]\frac{L_b}{R} \text{ is } 4.26$$

From Figure 3, p_{cr} is found to be approximately 1500 psi corresponding to the mode $n = 3$. Using the ratio $L_b/R = 4$ in Figure 1, $p_s \cdot R/100\,h$ is 150 psi. This is multiplied by 0.80, the value of $100\,h/R$, to give $p_s = 120$ psi. From Figure 2, the value p_f for $n = 3$ is 1370 psi. Adding these two pressures, p_{cr} is found to be 1490 psi as compared with 1500 psi found directly from Figure 3.

Table 2 compares the values of p_{cr} for this example as determined by Kendrick Part I, Kendrick Part III, Bryant, and this graphical method. While the pressure obtained from the graphs agrees closely with Kendrick Part III, it is interesting to note that both the Kendrick Part I and Bryant values are considerably higher and predict a different number of lobes.

TABLE 2

Example of Results Given by Various Methods

	Bryant	Kendrick Part I	Graphical Method	Kendrick Part III
p_{cr} (psi)	2045	1855	1490	1409
n	2	2	3	3

APPENDIX B

APPROXIMATE FORMULA FOR I_e

Considerable computation is required to determine the parameter I_e, which is a measure of frame strength. To simplify this work, an approximate formula has been derived which is applicable to frames of various cross-sectional shapes.

As previously defined, I_e is the moment of inertia of the combination of a frame plus an effective length of shell L_e about the centroid of the combined section. Figure 4 is a diagram of the most general frame shape in practical use, i.e., the builtup H-section with unequal flanges. Other shapes, such as T- or I-sections, are special cases of this shape. Referring to Figure 4, the quantity e of Equation [1] on page 3 can be written:

$$e = \frac{1}{A_f}\left[bt\left(d-\frac{t}{2}\right) + \frac{as}{2}(d+u-t) + \frac{cu^2}{2}\right] + \frac{h}{2} \qquad [3]$$

where a, b, c, d, s, t, and u are defined in Figure 4. Using the relationship

$$A_f = as + bt + cu \qquad [4]$$

Equation [3] can be rearranged so that

$$e = \frac{1}{A_f}\left[(d-t)\left(bt + \frac{as}{2}\right) + \frac{uA_f}{2} + \frac{bt}{2}(t-u)\right] + \frac{h}{2} \qquad [5]$$

If the small quantity $t-u$ is neglected, Equation [5] becomes

$$e = \frac{1}{2}\left\{(d-t)\left[1 + \frac{bt-cu}{A_f}\right] + u + h\right\} \qquad [6]$$

and $A_f e^2$ is written

$$A_f e^2 = \frac{A_f}{4}(d-t)^2\left[1 + \frac{bt-cu}{A_f} + \frac{u+h}{d-t}\right]^2 \qquad [7]$$

The quantity I_f can also be simplified if a few small terms are neglected. The exact expression for I_f is

$$I_f = \frac{1}{12}(bt^3 + a^3 s + cu^3) + bt\left[d - \frac{t}{2} - e + \frac{h}{2}\right]^2 \qquad [8]$$

$$+ as\left[\frac{1}{2}(d-t+u+h) - e\right]^2 + cu\left[\frac{1}{2}(u+h) - e\right]^2$$

Using the expression for e of Equation [6] the above becomes

$$I_f = \frac{1}{12}\left(bt^3 + a^3s + cu^3\right) + \frac{bt}{4}\left[d - u - (d-t)\frac{(bt-cu)}{A_f}\right]^2 \qquad [9]$$

$$+ \frac{as}{4}\left[(d-t)\frac{(bt-cu)}{A_f}\right]^2 + \frac{cu}{4}(d-t)^2\left[1 + \frac{bt-cu}{A_f}\right]^2$$

With the approximation $u = t$, Equation [9] becomes

$$I_f = \frac{1}{12}\left(bt^3 + a^3s + cu^3\right) + \frac{bt}{4}(d-t)^2\left[1 - \frac{bt-cu}{A_f}\right]^2 \qquad [10]$$

$$+ \frac{as}{4}\left[(d-t)\frac{(bt-cu)}{A_f}\right]^2 + \frac{cu}{4}(d-t)^2\left[1 + \frac{bt-cu}{A_f}\right]^2$$

or,

$$I_f = \frac{1}{12}\left(bt^3 + a^3s + cu^3\right) + \frac{(d-t)^2}{4}\left[bt + cu - \frac{(bt-cu)^2}{A_f}\right] \qquad [11]$$

With a further simplification that the first term of Equation [11] is approximately $\frac{as}{12}(d-t)^2$, the resulting expression for I_f is

$$I_f = \frac{(d-t)^2}{12} A_f \left[1 + 2\frac{(bt+cu)}{A_f} - \frac{3}{A_f^2}(bt-cu)^2\right] \qquad [12]$$

In calculating I_e from Equation [1], the term $L_e h^3/12$ is usually so small that it can be neglected. With the substitution of Equations [7] and [12] into Equation [1], the terms can be rearranged so that

$$I_e = \frac{A_f(d-t)^2}{4}\left\{\frac{1}{1+\frac{A_f}{L_e h}}\left[1 + \frac{bt-cu}{A_f} + \frac{h+u}{d-t}\right]^2 \right. \qquad [13]$$

$$\left. + \frac{1}{3}\left[1 + 2\frac{(bt+cu)}{A_f} - 3\left(\frac{bt-cu}{A_f}\right)^2\right]\right\}$$

For an I-beam a simpler expression is obtained. Since $b = c$ and $u = t$ Equation [3] becomes

$$e = \frac{d + h}{2} \tag{14}$$

while the Equation [12] for I_f reduces to

$$I_f = \frac{(d - t)^2}{12} A_f \left(1 + \frac{4bt}{A_f} \right) \tag{15}$$

The expression for I_e is

$$I_e = \frac{A_f}{4} \left[\frac{(d + h)^2}{1 + \frac{A_f}{L_e h}} + \frac{(d - t)^2}{3} \left(1 + \frac{4bt}{A_f} \right) \right] \tag{16}$$

For a T-section, the quantities c and u are zero and Equation [5] can be written

$$e = \frac{1}{A_f} \left[(d - t) \frac{(bt + A_f)}{2} + \frac{bt^2}{2} \right] + \frac{h}{2} \tag{17}$$

which reduces to

$$e = \frac{d}{2} \left(1 + \frac{bt}{A_f} - \frac{t}{d} \right) + \frac{h}{2} \tag{18}$$

Substituting this expression in Equation [8]

$$I_f = \frac{1}{12} (bt^3 + a^3 s) + \frac{d^2}{4} bt \left(1 - \frac{bt}{A_f} \right) \tag{19}$$

or

$$I_f = \frac{d^2 A_f}{12} \left[\left(1 - \frac{2t}{d}\right) \left(1 - \frac{bt}{A_f}\right) + \frac{t^2}{d^2} \right] + \frac{d^2}{4} bt \left(1 - \frac{bt}{A_f}\right) \tag{20}$$

If the small quantity t^2/d^2 is neglected, this can be reduced to

$$I_f = \frac{d^2 A_f}{12}\left(1 - \frac{bt}{A_f}\right)\left(1 - \frac{2t}{d} + \frac{3bt}{A_f}\right) \qquad [21]$$

and

$$I_e = \frac{A_f d^2}{12}\left[\frac{3}{1 + \dfrac{A_f}{L_e h}}\left(1 + \frac{bt}{A_f} + \frac{h-t}{d}\right)^2 + \left(1 - \frac{bt}{A_f}\right)\left(1 - \frac{2t}{d} + \frac{3bt}{A_f}\right)\right] \qquad [22]$$

To demonstrate the accuracy of these frame formulas some numerical examples are provided. Table 3 lists the dimensions of several frames representing shapes and sizes which might be used in submarine construction. Type 1 is an H-section with unequal flanges which was used as the example in Appendix A. Types 2 - 5 are I- and T-sections taken from the steel construction manual of the American Institute of Steel Construction.[7] Table 4 lists shell dimensions for each of these frames and compares the values of I_f and I_e calculated from the simplified formulas with the exact values from Reference 7. The error in each case is found to be less than 2 percent.

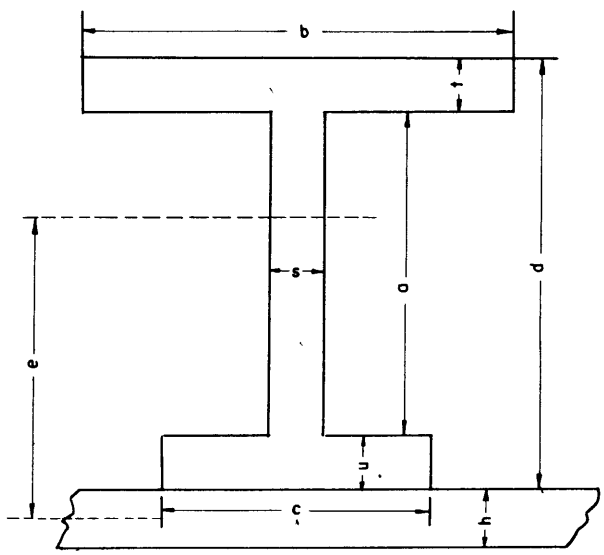

Figure 4 - Identification of Symbols Used in Calculating I_e

TABLE 3

Dimensions of Typical Frame Cross Sections

Frame Type	Shape	Frame Dimensions in inches						
		A_f^*	d	b	c	s	t	u
1	H	9.63	7.28	6.06	4.34	0.555	0.633	0.555
2	I (6×3 3/8)	5.02	6.00	3.565	3.565	0.465	0.359	0.359
3	I (6 × 6)	5.90	6.20	6.018	6.018	0.258	0.367	0.367
4	ST 13 WF	13.83	13.45	9.99	–	0.490	0.747	–
5	ST 61	4.63	6.00	5.00	–	0.350	0.544	–

***Values of A_f in inches^2 for Frames 2-5 are those given in the A.I.S.C. Handbook.[7] In some cases, these include fillets not listed in this table.**

TABLE 4

Comparison of Approximate and Exact Values for I_f and I_e.

Frame Type	I_f in in.^{4}		R, Shell Radius in inches	h, Shell Thickness in inches	I_e in in.^{4}	
	Exact*	Approximate			Exact	Approximate
1	80.5	78.9	96	0.768	181.5	178.5
2	26.0	26.9	96	1.125	77.94	76.9
3	41.7	41.8	96	1.125	103.5	101.7
4	238.5	241.0	168	2.00	1618	1589
5	14.90	14.80	96	1.125	111.3	109.3

***Exact values of I_f for Frames 2-5 are those given in Reference 7.**

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