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CENTROID AND MOMENT OF INERTIA OF A SUPERCAVITATING SECTION

by

W. B. Morgan

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HYDROMECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

August 1957 Report No. 1193

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CENTROID AND MOMENT OF INTERTIA OF A SUPERCAVITATING SECTION

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W. B. Morgan

August 1957 Report No. 1193

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NOTATION

A	Area of section
B'	Coefficient for determining pressure face ordinates
C_L	Section design lift coefficient
E'	Coefficient for determining pressure face ordinates
F and F'	Coefficients for determining section thickness
I_{x_o}, I_{y_o}	Section area moments of inertia about x_o and y_o axes
k and k_2	Camber correction coefficients from Reference 6
ℓ	Section chord length
M_{x_o}, M_{y_o}	Bending moments on section about x_o and y_o axes
N and N'	Coefficients for determining section thickness
t	Section thickness perpendicular to nose-tail line
x	Abscissa of face or back of section with reference to leading edge
$x_{y_{max}}$	Abscissa of y_{max} with reference to leading edge
$\bar{x}$	Abscissa of center of gravity with reference to leading edge
x_o	Horizontal axis through centroid parallel to nose-tail line
x_1	Abscissa of leading edge with reference to axis through centroid

x_2	Abscissa of trailing edge on face with reference to axis through centroid
x_3	Abscissa of y_{max} with reference to axis through centroid
y	Ordinate of face or back of section with reference to nose-tail line
y_f	Ordinate of face with reference to nose-tail line
y_{max}	Maximum back ordinate with reference to nose-tail line
$\bar{y}$	Ordinate of centroid with reference to nose-tail line
y_o	Vertical axis through centroid perpendicular to nose-tail line
y_1	Ordinate of leading edge with reference to axis through centroid
y_2	Ordinate of trailing edge with reference to axis through centroid
y_3	Ordinate of y_{max} with reference to axis through centroid
α	Section design angle of attack

ABSTRACT

Equations are derived by numerical integration for the ordinate of the centroid and moment of inertia about the x_o-axis of Tulin's SC section with the TMB modified thickness distribution. These equations are evaluated for a flat face section and for a section with a given relationship between design lift coefficient and design angle of attack. The results are plotted in a form convenient for propeller blade stress analysis.

INTRODUCTION

The increasing interest in the use of supercavitating (SC) sections for high-speed operation has made it desirable to simplify some of the basic calculations. One of the more laborious computations is the determination of the nominal stress distribution in the section. Since the thickness of a SC section is restricted to the cavity thickness[1*] a number of calculations must be performed to obtain a section with the proper strength and minimum drag.

Previously, the stress problem consisted mainly of determining, by numerical integration, the centroid and moment of inertia of a SC section. By integrating

*References are listed on page 11

the equations describing the section shape, these geometric properties can be obtained directly. This report describes the method of calculation for a family of SC sections[2] and gives the results so that they can be used for propeller design purposes. A discussion of the hydrodynamic optimum SC section will be found in Reference 3.

GENERAL CONSIDERATIONS

Nominal stresses in propeller blades are normally calculated by application of the simple beam theory in combination with the radial load distribution on the blade and the geometric properties of the section being investigated. Although simplified methods[4,5] have been developed for obtaining the geometric properties of a number of sections used for propellers, the normal procedure for their calculation is numerical integration. Determination of the geometric properties of a section requires solution of the following integrals:

for cross-sectional area

$$A = \iint dydx, \tag{1}$$

for horizontal distance from leading edge of the section to the centroid

$$\bar{x} = \frac{\iint xdydx}{A}, \tag{2}$$

for vertical distance from the nose-tail line to the centroid

$$\bar{y} = \frac{\iint y\,dy\,dx}{A}, \tag{3}$$

for the moment of inertia about an axis parallel to the nose-tail line and through the centroid

$$I_{x_o} = \left(\iint y^2 dy\,dx\right) - A\bar{y}^2, \tag{4}$$

and for the moment of inertia about an axis perpendicular to the nose-tail line and through the centroid

$$I_{y_o} = \left(\iint x^2 dy\,dx\right) - A\bar{x}^2, \tag{5}$$

where x is the abscissa measured from the leading edge parallel to the nose-tail line and y is the ordinate measured from the nose-tail line.

As shown in Figure 1, the abscissas x_1, x_2, and x_3 and the ordinates y_1, y_2, and y_3 are used to denote the abscissas and ordinates of the leading edge, trailing edge on face, and point of maximum back ordinate, respectively, where the center of the coordinate system is at the centroid of the section. Alsc, from Figure 1 it can be seen that the abscissas have the following relationships with $\bar{x}$:

$$x_1 = \bar{x} \tag{6}$$

$$x_2 = \bar{x} - \ell \tag{7}$$

$$x_3 = \bar{x} - x_{y_{max}} \tag{8}$$

and that the ordinates have the following relationships with $\bar{y}$:

$$y_1 = y_2 = -\bar{y} \tag{9}$$

$$y_3 = y_{max} - \bar{y} \tag{10}$$

where y_{max} is the maximum ordinate on the back (suction side) of the section measured from the nose-tail line and ℓ is the section length.

Using simple beam theory the stresses at the above three points on the section are given by:

$$\text{stress at leading edge} = -\frac{y_1 M_{x_o}}{I_{x_o}} - \frac{x_1 M_{y_o}}{I_{y_o}}, \tag{11}$$

$$\text{stress at trailing edge (on face)} = -\frac{y_2 M_{x_o}}{I_{x_o}} - \frac{x_2 M_{y_o}}{I_{y_o}} \tag{12}$$

$$\text{and the stress at } y_{max} = -\frac{y_3 M_{x_o}}{I_{x_o}} - \frac{x_3 M_{y_o}}{I_{y_o}} \tag{13}$$

The moments M_{x_o} and M_{y_o} are calculated by knowing the load distribution along the hydrofoil or the propeller blade[4]. In these equations a positive stress denotes tension and a negative stress denotes compression.

The centroid and moment of inertia of a given section can be obtained in terms of the design lift coefficient, the maximum thickness, and section length provided that the section shape can be described mathematically. For an SC section, the section thickness distribution is assumed to be equal to the cavity shape and therefore is a function of the design lift coefficient (C_L), the design angle of attack (α), and the section length (ℓ). The equation for the thickness distribution (t) of Tulin's SC section can be simplified[1] and expressed as

$$t/\ell \quad = \quad FC_L + N\alpha \qquad (14)$$

where F and N are coefficients from Table 1.

Because of the thinness and resulting vibration at the leading edge, the thickness given by Equation (14) has been increased toward the leading edge and has also been decreased toward the trailing edge to prevent overstressing at the point of maximum thickness. The thickness distribution is, therefore, taken to be

$$t/\ell \quad = \quad F'C_L + N'\alpha \qquad (15)$$

where F' and N' are coefficients from Table 1. A section having such a thickness distribution will be referred to as the TMB modified section.

The section camber (y_f) can also be described in terms of the design lift coefficient and design angle of attack and is equal to the ordinate of the face (pressure side) of the section. From References 1 and 2, Tulin's camber line has been simplified to:

$$\frac{y_f}{\ell} \quad = \quad B'C_L - E'\alpha \qquad (16)$$

where B' and E' are coefficients from Table 1.

Equation (16) represents the camber of a two-dimensional SC hydrofoil. However, for propeller design this camber must be corrected for the curvature of flow over the section. Using the correction given in Reference (6), the face ordinates become

$$\frac{y_f}{\ell} = (B'C_L - E'\alpha)\ k_1\ k_2 \tag{17}$$

where k_1 and k_2 are camber correction coefficients.

Equations (15) and (17) can be integrated to give the centroid and moments of inertia in terms of the lift coefficient and angle of attack.

CALCULATIONS

In calculating the stresses at any point on the section, the term $\frac{x\ M_{y_o}}{I_{y_o}}$ is small compared to the term $\frac{y\ M_{x_o}}{I_{x_o}}$ when the section is thin and the moments M_{x_o} and M_{y_o} are of the same order of magnitude. Comparison of calculations for a number of SC sections has shown that the $\frac{x\ M_{y_o}}{I_{y_o}}$ term contributes a maximum of 2 percent to the stress. This is negligible compared to the accuracy of the assumption that the propeller blade can be assumed to act as a simple beam. As a first approximation, therefore, this term can be neglected. The calculation for the stress is then essentially reduced to the calculation of the ordinate of the centroid ($\bar{y}$) and the moment of inertia about the x_o-axis (I_{x_o}).

The integration for $\bar{y}$ and I_{x_o} was carried out by numerical integration using Equations (1), (3), and (4) where the ordinate y is a function of the face ordinate (y_f) and the thickness (t). The coefficients B', E', F', and N' were integrated while keeping C_L, α , and k_1 k_2 as constants. The solution for $\bar{y}$ is

$$\bar{y} = \left[\frac{(6.072-7.285k_1k_2)C_L^2-(0.644-4.959k_1k_2)C_L\alpha+(0.485-0.131k_1k_2)\alpha^2}{42.18\,\alpha - 33.31C_L}\right]\ell \qquad (18)$$

and the solution for I_{x_o} is

$$\begin{aligned} I_{x_o} = {} & (0.2809C_L^3 + 1.3530C_L^2\alpha - 0.0707C_L\alpha^2 + 0.0384\alpha^3)\ell^4 \cdot 10^{-4} \\ & + (5.2853C_L^3 - 1.7890C_L^2\alpha + 0.6165C_L\alpha^2 - 0.0157\alpha^3)k_1k_2\ell^4 \cdot 10^{-4} \\ & - (5.5233C_L^3 - 3.3610C_L^2\alpha + 0.1719C_L\alpha^2 - 0.0023\alpha^3)k_1k_2^2\ell^4 \cdot 10^{-4} \\ & + (7.2853C_L^2 - 4.9579C_L\alpha + 0.1305\alpha^2)k_1k_2\,\bar{y}\ell^3 \cdot 10^{-3} \\ & - (6.0724C_L^2 - 0.6441C_L\alpha + 0.4855\alpha^2)\,\bar{y}\ell^3 \cdot 10^{-3} \\ & - (1.6655C_L - 2.1092\alpha)\,\bar{y}^2\ell^2 \cdot 10^{-2} \end{aligned} \qquad (19)$$

Equations (18) and (19) can be evaluated for any combination of C_L, α and k_1k_2. For a flat face SC section, where $\alpha = 36.5C_L$, the terms involving k_1k_2 become zero and the above equations reduce to

$$\bar{y} = 0.417\ C_L\,\ell, \tag{20}$$

and

$$I_{x_o} = 0.0509\ C_L^{\,3}\,\ell^{4} \tag{21}$$

From Equations (20) and (21), the stresses in a flat face section are obtained in terms of the moment, section length, and design coefficient of the section.

$$\text{stress at leading edge} \doteq -\frac{y_1 M_{x_o}}{I_{x_o}} = \frac{8.2M_{x_o}}{\ell^3 C_L^{\,2}} \tag{22}$$

$$\begin{array}{l}\text{stress at trailing}\\ \text{edge (on face)}\end{array} \doteq -\frac{y_2 M_{x_o}}{I_{x_o}} = \frac{8.2M_{x_o}}{\ell^3 C_L^{\,2}} \tag{23}$$

$$\text{and stress at } y_{max} \doteq -\frac{y_3 M_{x_o}}{I_{x_o}} = -\frac{10.6M_{x_o}}{\ell^3 C_L^{\,2}} \tag{24}$$

These equations have been plotted as coefficients of M_{x_o} in the form of non-dimensional coefficients $\frac{\ell^3 y_1}{I_{x_o}}$, $\frac{\ell^3 y_2}{I_{x_o}}$, and $\frac{\ell^3 y_3}{I_{x_o}}$ for various values of C_L, Figure 2.

The evaluation of Equations (18) and (19) was also made for the recommended section of Reference (7). This section had the following relations between α and C_L.

$$\alpha = 36.5 C_L \text{ for } 0 < C_L \leq 0.0548 \tag{25}$$

$$\alpha = 2^{\circ} \text{ for } 0.0548 \leq C_L \leq 0.2 \tag{26}$$

$$\alpha = 10 C_L \text{ for } 0.2 \leq C_L \tag{27}$$

Evaluation of $\bar{y}$ and I_{x_o} was made for a range of camber corrections ($k_1 k_2$) from 0.6 to 2.2. As for the flat face section, the results were combined to form the non-dimensional coefficients $\frac{\ell^3 y_1}{I_{x_o}}$, $\frac{\ell^3 y_2}{I_{x_o}}$, and $\frac{\ell^3 y_3}{I_{x_o}}$ and are plotted in Figures 3 and 4 against C_L. From these plots the stresses in the recommended section can be evaluated directly by knowing C_L, ℓ, and the moment on the section. It should be noted that below a C_L of 0.0548 the recommended section is a flat face section and the stresses are given by Equations (22), (23), and (24).

CONCLUSIONS

For thin sections, such as the supercavitating section, it can be assumed that the stress term arising from the moment about the y_o-axis is negligible. With this assumption stress in the section is a function of the vertical location of the centroid, the moment of inertia about the x_0-axis and the moment about the x_0-axis.

Equations are derived for the ordinate of the centroid and the moment of inertia about the x_0-axis, for Tulin's SC section with the TMB modified thickness distribution. Propeller camber corrections are also included in the equations. These equations are evaluated for a flat face section and for a section with the recommended relationship between the design lift coefficient and the design angle of attack. The results are plotted in a form convenient for propeller blade stress analysis.

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2. Tulin, M., and Burkhart, M., "Linearized Theory for Flows About Lifting Foils at Zero Cavitation Number," David Taylor Model Basin Report C-638 (Feb 1955)

3. Morgan, W., David Taylor Model Basin Report C-856 (Aug 1957) CONFIDENTIAL

4. Morgan, W., "An Approximate Method of Obtaining Stress in a Propeller Blade," David Taylor Model Basin Report 919 (Oct 1954)

5. Muckle, W., "Stresses in Propeller Blades," The Shipbuilder and Marine Engine - Builder, Vol. XLVII, No. 388, pp. 336-341 (Nov 1941)

6. Eckhardt, M. and Morgan, W., "A Propeller Design Method," Transactions of the Society of Naval Architects and Marine Engineers (1955)

TABLE I

Coefficients for Obtaining Ordinates of SC Section

x/ℓ	B'	E'	F	N	F'	N'
0	0	0	0	0	0	0
0.0075	0.00292	0.000080	-0.00066	0.000626	-0.00133	0.00125
0.0125	0.00503	0.000138	-0.00129	0.000935	-0.00257	0.00187
0.05	0.02203	0.000604	-0.00841	0.002852	-0.01682	0.00570
0.10	0.04524	0.001240	-0.01982	0.005052	-0.03952	0.01007
0.20	0.08792	0.002410	-0.04069	0.008943	-0.07894	0.01735
0.30	0.12178	0.003339	-0.05419	0.012339	-0.09760	0.02222
0.40	0.14463	0.003965	-0.05765	0.015299	-0.09287	0.02465
0.50	0.15526	0.004257	-0.04981	0.017842	-0.07178	0.02571
0.60	0.15265	0.004185	-0.02913	0.019948	-0.03775	0.02585
0.70	0.13628	0.003736	0.00502	0.021626	0.00594	0.02561
0.80	0.10561	0.002895	0.05276	0.022876	0.05688	0.02466
0.90	0.06032	0.001654	0.1152	0.023687	0.11209	0.02305
0.95	0.03207	0.000879	0.1516	0.023939	0.13962	0.02205
1.00	0	0	0.1919	0.024066	0.16695	0.02094

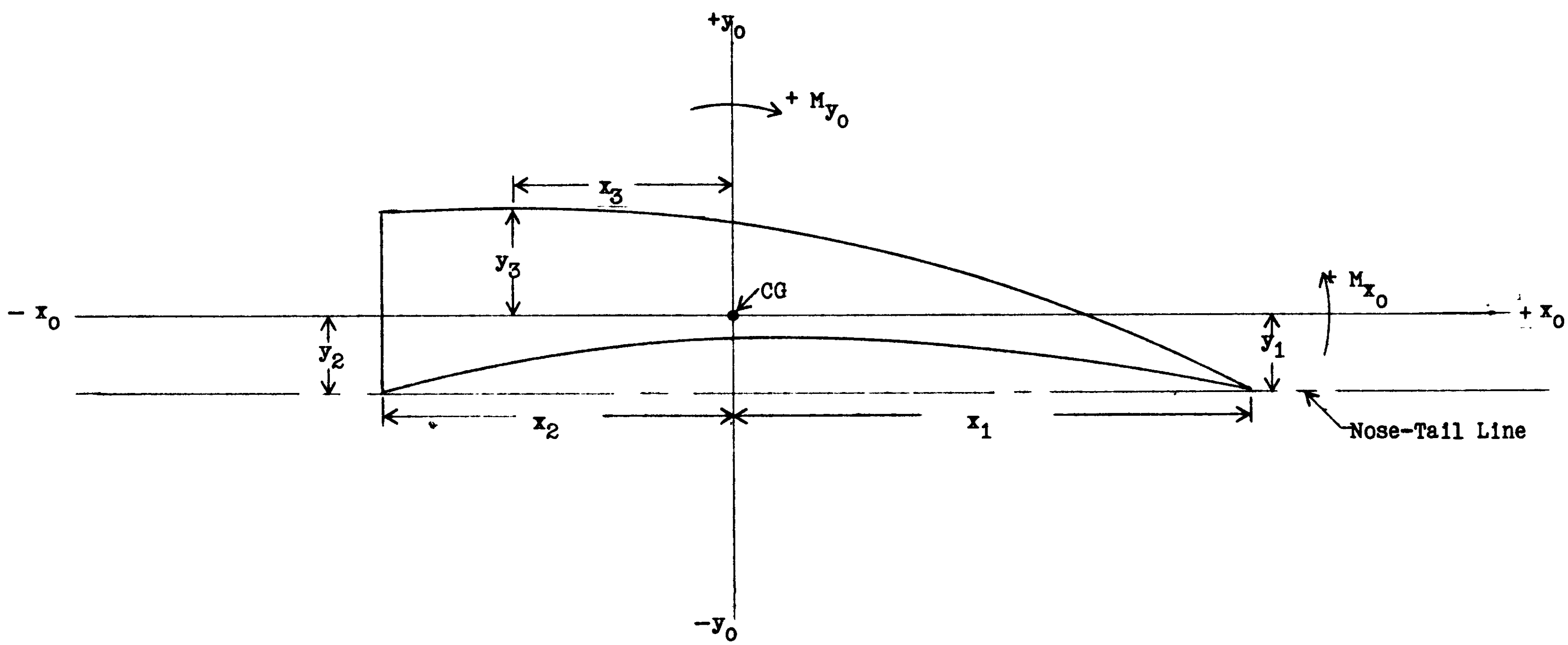

Figure 1 Geometric Properties of a SC Section

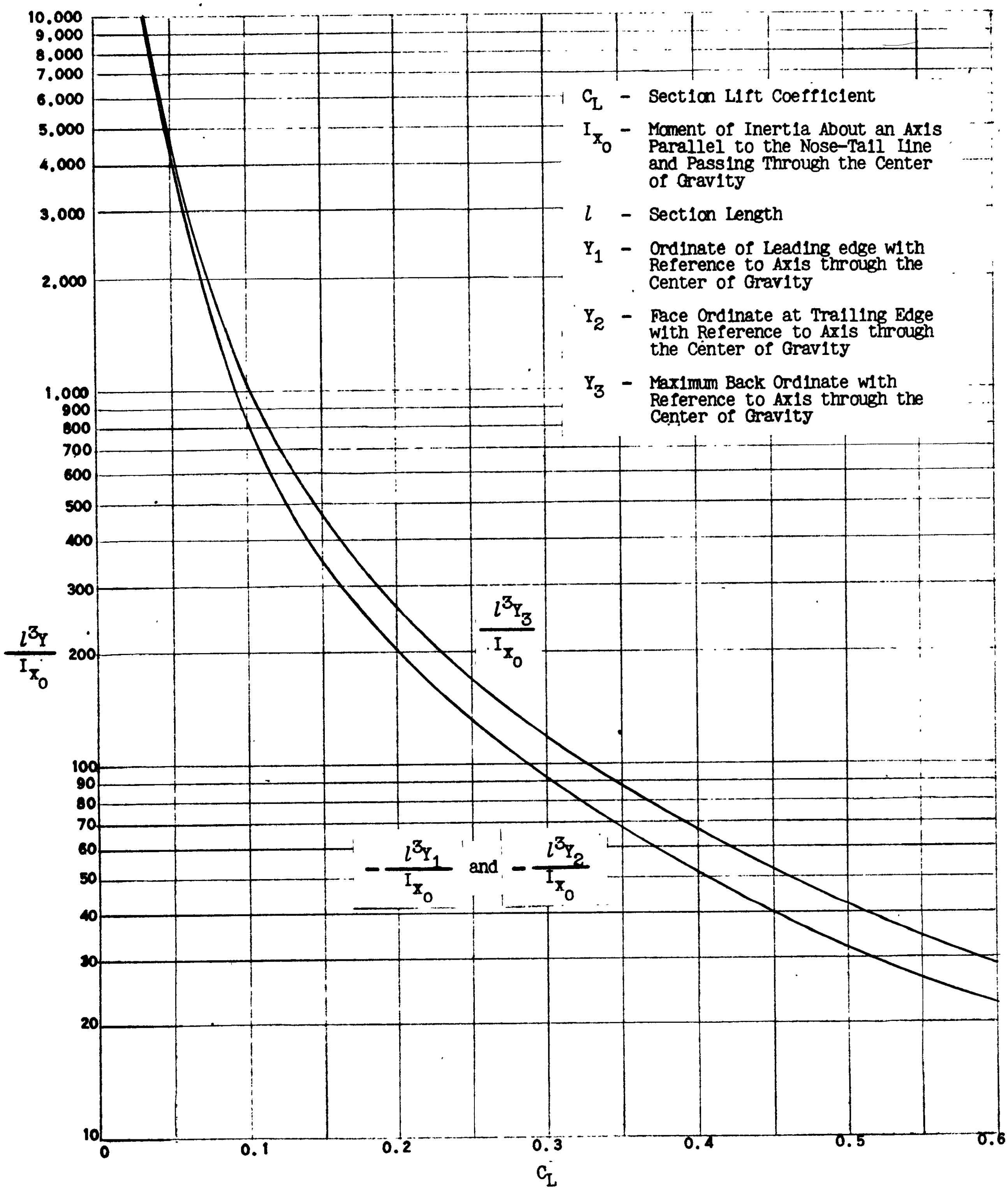

Figure 2 Coefficient for Determining Stress of a Flat Face Section with TMB Modification of Tulin's Thickness Distribution, $\alpha = 36.48\ C_L$

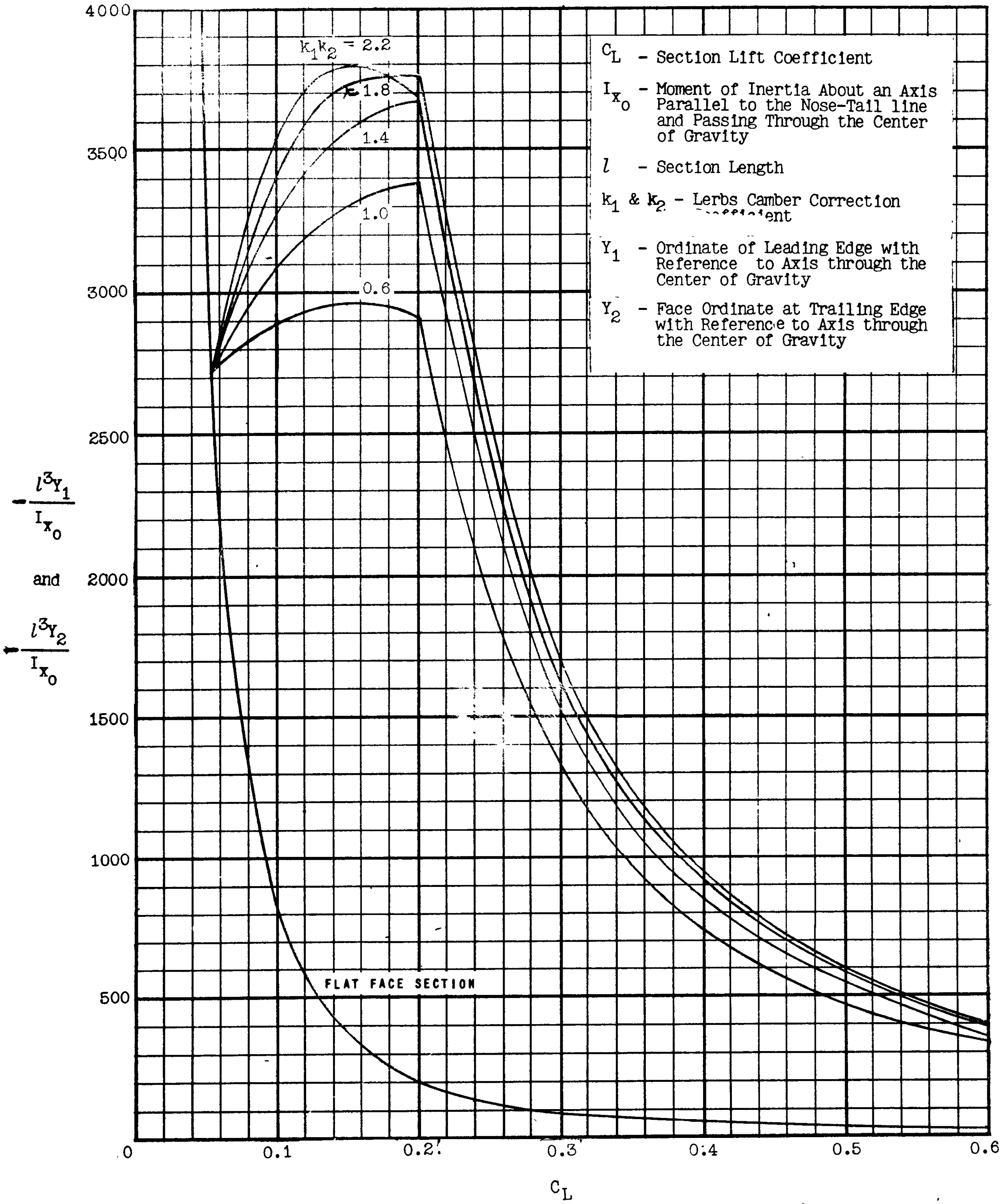

Figure 3 Coefficient for Determining the Stress of the TMB Modified Supercavitating Section, Where α = 2 degrees for $0.05482 \leq C_L \leq 0.2$, $\alpha = 10\ C_L$ for $0.2 \leq C_L$, and for a Flat Face Section $\alpha = 36.48\ C_L$.

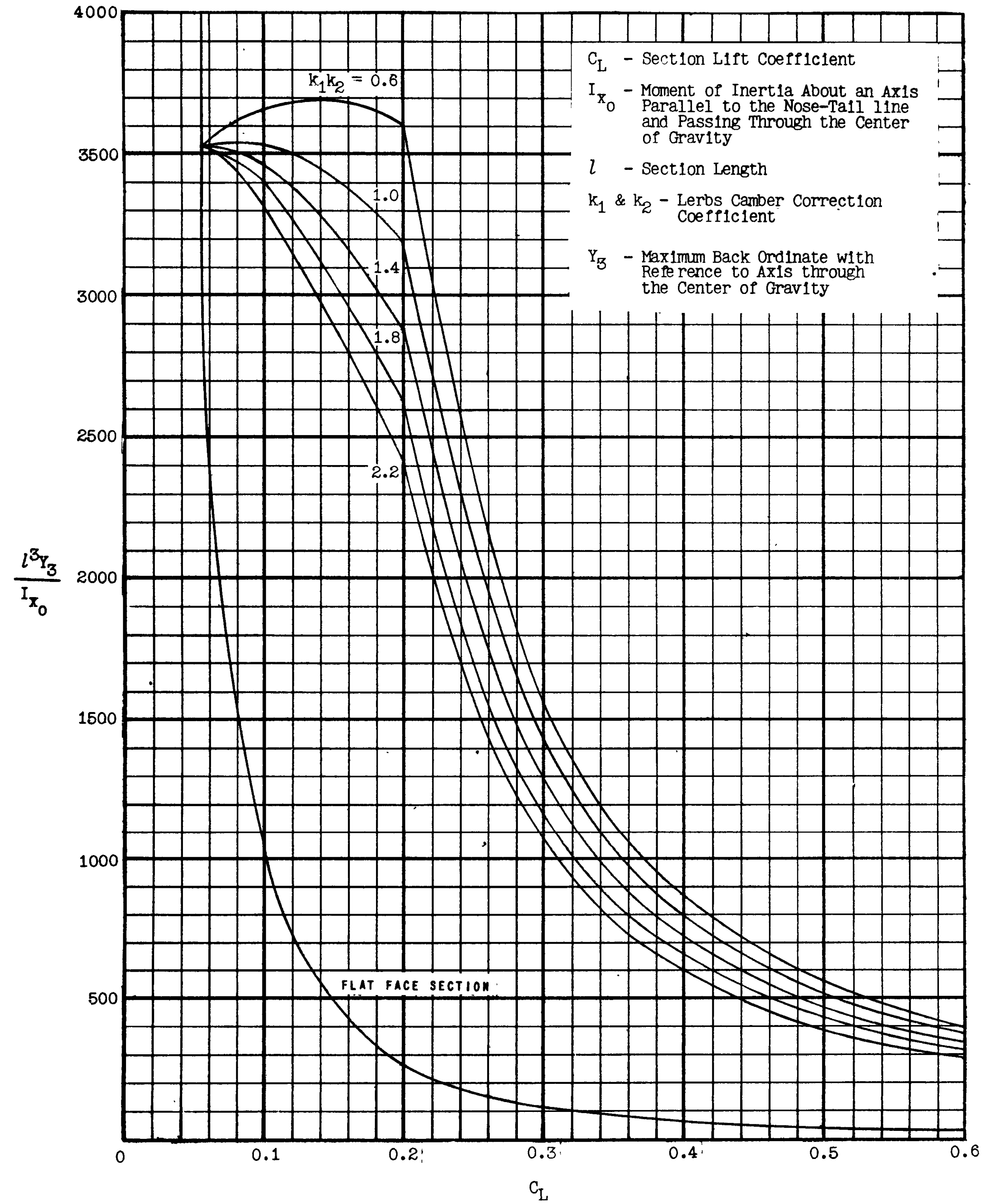

Figure 4 Coefficient for Determining the Stress of the TMB Modified Supercavitating Section, Where α = 2 degrees for $0.05482 \leqq C_L \leqq 0.2$, $\alpha = 10\ C_L$ for $0.2 \leqq C_L$, and for a Flat Face Section $\alpha = 36.48\ C_L$.

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