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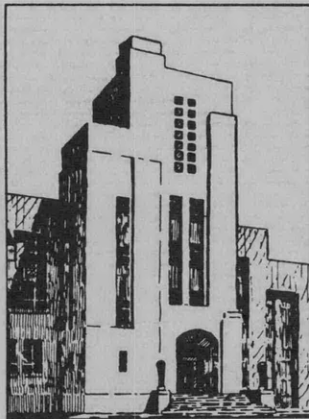## THE DAVID W. TAYLOR MODEL BASIN
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THE WEAKENING EFFECT OF INITIAL TILT AND LATERAL BUCKLING
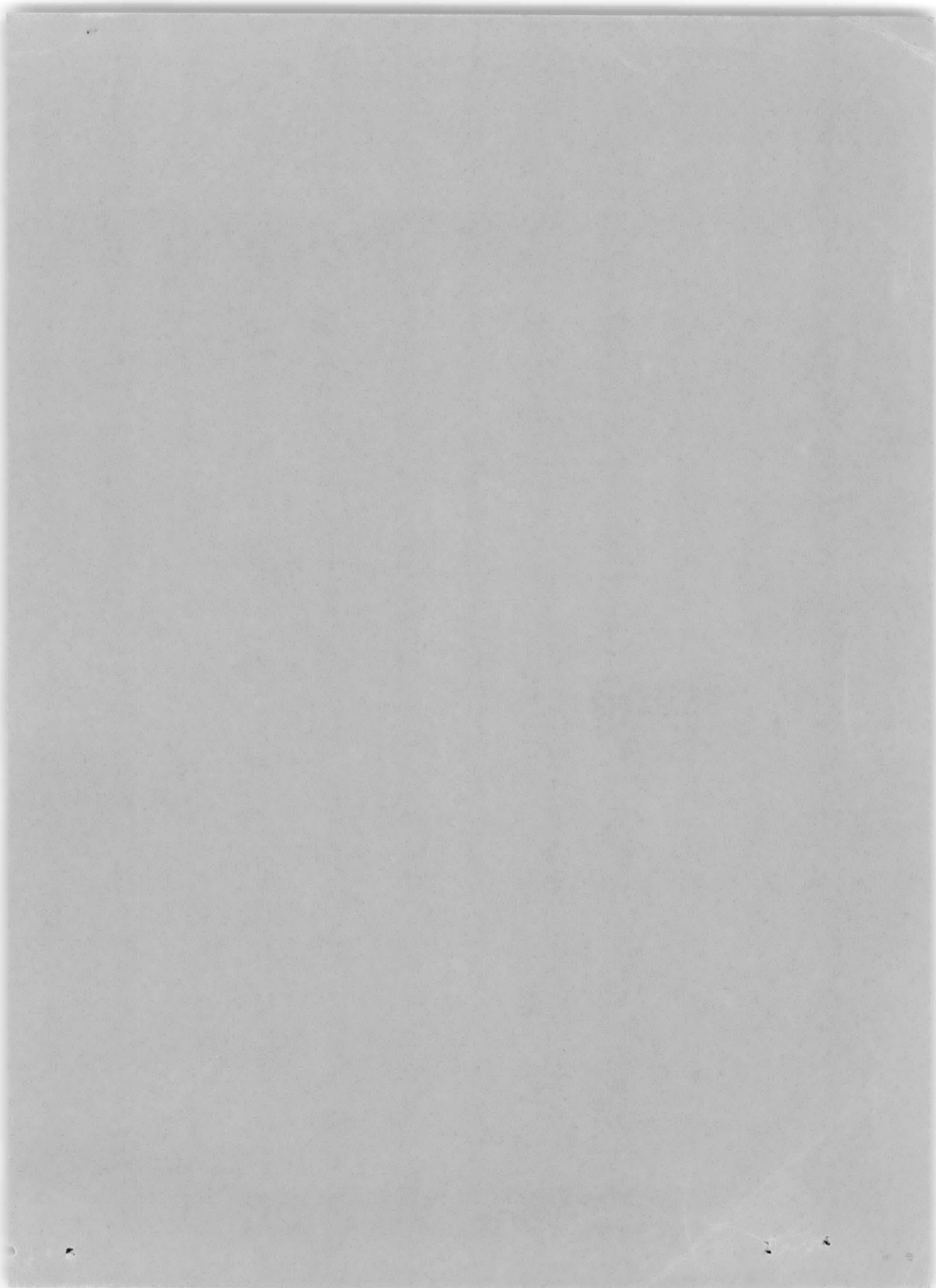## OF RING STIFFENERS ON CYLINDRICAL PRESSURE VESSELS

by

E. Wenk, Jr. and E.H. Kennard

RESEARCH AND DEVELOPMENT REPORT

December 1956   Report 1073

From: Commanding Officer and Director
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Subj: NS731-038, Weakening effect of initial tilt of
       stiffeners; forwarding of report on

Encl: (1) TMB Report 1073 entitled "The Weakening Effect
           of Initial Tilt and Lateral Buckling of Ring
           Stiffeners on Cylindrical Pressure Vessels" -
           11 copies.

1. Although the general-instability collapse of ring-stiffened
cylindrical shells under external pressure has frequently been
accompanied by twisting of the stiffener, it has not been
determined whether failure was actually precipitated by twist-
ing weakness of the stiffener. Also, there has been no accept-
able theory for the twisting strength of rings. While twisting
strongly suggests a form of lateral instability of the ring,
the same mode of failure might result from yielding in the
stiffener induced by initial axisymmetric tilt. A preliminary
analysis confirmed the possibility of this behavior. These
studies were subsequently refined and extended to include both
axisymmetric and nonsymmetric buckling of the nontilted stiff-
ener. Results of the symmetrical case and a discussion of their
practical application are contained in enclosure (1). Results
of the nonsymmetrical case will be published in TMB Report 1079
which is now in preparation.

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THE WEAKENING EFFECT OF INITIAL TILT AND LATERAL BUCKLING
OF RING STIFFENERS ON CYLINDRICAL PRESSURE VESSELS

by

E. Wenk, Jr. and E.H. Kennard

TABLE OF CONTENTS

NOTATION

A_f	Area of T-stiffener flange
A_w	Area of T-stiffener web
A_b	Area of faying flange of H- or I-stiffener
A, B, C, D	Arbitrary constants
b	Faying width of stiffener (length of contact between shell and stiffener)
d	Depth of web
e	Eccentricity of radial loads on stiffener, due to initial tilt
E	Young's modulus
E_s	Effective modulus
F	Radial force per unit circumference imposed on stiffener and associated shell plating, positive inward
$\overline{F}$	Radial force per unit circumference imposed on T-stiffener alone
$\overline{\overline{F}}$	Radial force per unit circumference acting in web at distance z from shell
F_{cr}	Value of F that causes buckling of internal stiffener
h	Thickness of shell
I_w	Moment of inertia of web per unit circumferential length about axis normal to radius and to cylinder axis
I_{xx}	Moment of inertia of stiffener cross section about radial axis
I_f	Moment of inertia of flange of T about radial axis
m	See Equation [39]
$M(z), M_t$	Twisting moment per unit circumference tending to rotate elements of stiffener
M_d	Twisting moment per unit circumference at juncture of web and flange
M_0	Twisting moment per unit circumference at juncture of web and shell
p	Pressure acting on cylindrical shell
p_{cr}	Pressure at which axisymmetric buckling occurs in internal stiffeners
Q_0	Radial shear force at stiffener per unit pressure

r	Stiffness ratio, defined in Equation [48]
R	Radius of cylinder measured to median of plating
R_f	Radial distance of flange; see figure
t	Thickness of web
$u(z)$	Additional elastic axial displacement of web
$u_0(z)$	Initial axial displacement of web
w	Width of outstanding flange
y	Distance from xx (radial) axis to any circumferential fiber in stiffener
z	Radial distance, positive inward, measured from inner surface of shell
$\alpha, \beta, \gamma, \delta$	Functions of A_f, A_w, bh, etc.
θ	Angle of initial tilt
ϕ	Rotation of flange
ν	Poisson's ratio
σ_b	Circumferential stress in stiffener due to in-plane bending
σ_c	Membrane stress in stiffener
σ_f	Maximum circumferential bending stress in flange
σ_r	Uniform radial stress in web
σ_w	Maximum radial bending stress in web
σ_y	Yield stress of material

ABSTRACT

The lateral stability of a T stiffening ring on a cylinder under external pressure is investigated analytically for the case of axisymmetric deformation only. Both failure by yield due to initial tilt of the T and lateral buckling of the ideal T are considered. The practical application of the results is discussed.

INTRODUCTION

Although the general-instability collapse of ring-stiffened cylindrical shells under external pressure has frequently been accompanied by a twisting or tripping action of the stiffener, it has not been possible to determine whether the failure was actually precipitated by twisting weakness of the stiffener. Moreover, any such evaluation has thus far been without benefit of an acceptable theory for the twisting strength of rings. It was suggested by the senior author that, while the tripping strongly suggested a form of lateral instability of the ring, the same mode of failure might possibly result from yielding in the stiffener induced by initial, axisymmetric tilt. A preliminary analysis following a general line of inquiry into weakening effects of imperfect shape of submarine hulls confirmed the possibility of this behavior. These studies have subsequently been refined and extended to include both axisymmetric and nonsymmetric buckling of the nontilted stiffener. Results of the symmetrical case are presented in this report; results of the nonsymmetrical case will be presented in Reference 1.*

From these analyses, it appears that tripping of stiffeners by purely elastic action is unlikely in practical cases because prior failure of the structure in some other mode would intervene at lower pressures. However, the initial tilt can definitely introduce additional circumferential stresses in the flange of T-stiffeners and radial stresses in the web at its attachment to the cylinder. Thus, if these stresses combined with membrane stresses and with local effects of initial out-of-roundness became excessive, the yielding which results may serve as a trigger for ring tripping accompanied by a general-instability mode of collapse of the entire cylinder.

It has also been found by this analysis that the resistance of rings to such premature tripping failure can be improved most effectively by increasing the web thickness or flange breadth.

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*References are listed on page 21.

FIRST PRINCIPLES

We shall consider first the manner in which ring stiffeners on cylindrical pressure vessels are loaded and how initial tilt may influence their behavior. Under external hydrostatic pressure a perfect cylindrical shell develops a membrane deformation homogeneous along its length. Ring stiffeners used to improve the buckling strength interrupt this uniform deformation and develop local bending and shear stresses in the shell, by which, through mutual interaction, the stiffener itself is loaded radially by the shell.

If the stiffener is perfectly symmetrical, the radial load squeezes it uniformly to a smaller radius, with no bending in or out of its plane. If, however, the ring stiffener is initially tilted, the plane of action of the radial load forces does not pass through the centroids of the ring cross sections. Since the load force F^* on an element is balanced by a component of the compressive circumferential force in the curved ring and since this latter force may, to a first approximation, be assumed to act at the centroid, by virtue of this eccentricity e (as shown in Figure 1) there will be a twisting moment M_t per circumferential inch tending to rotate the elements of the ring. The forces F can be determined directly from von Sanden and Gunther's analysis of ring-stiffened cylinders.[3]

If the ring were simply supported at its junction with the shell, its elements would rotate through an angle ϕ without appreciable distortion under the action of the moment M_t.

Figure 1 - Twisting of Stiffeners Induced by Initial Tilt

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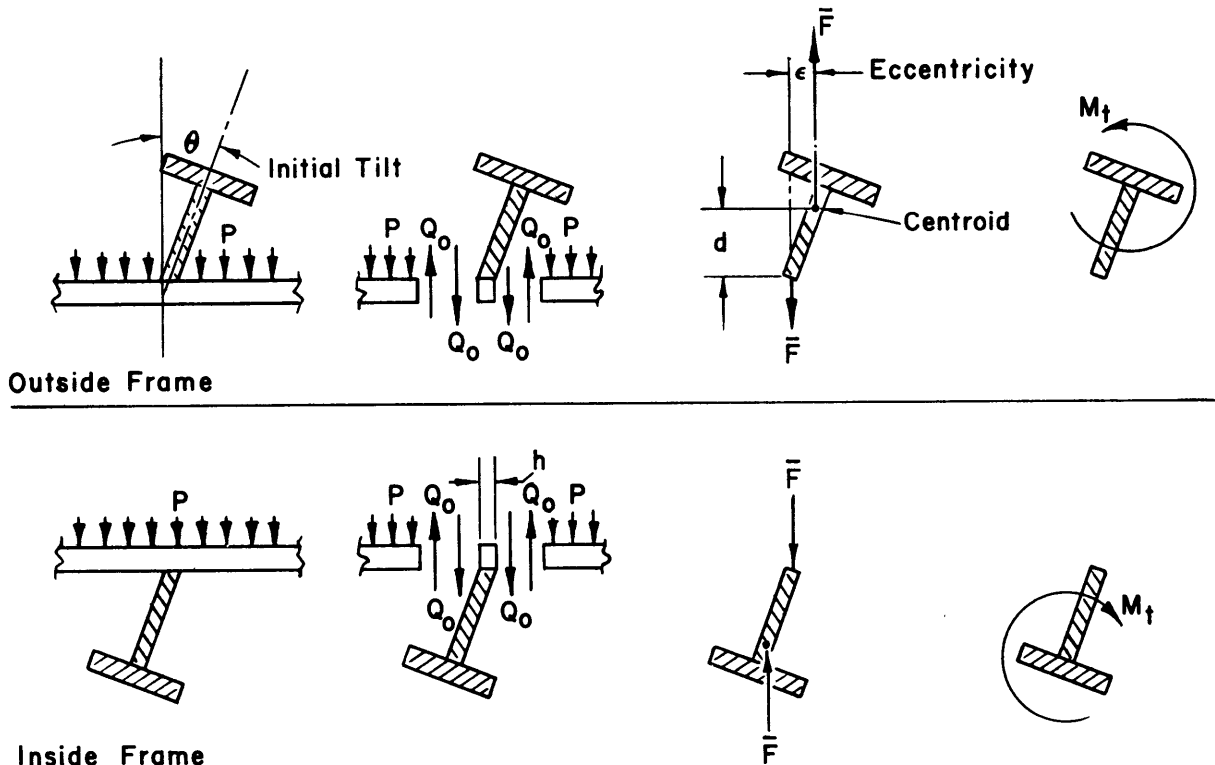
*F in this report is the same as q in Equation [8a] in Reference 2.

Also, circumferential stresses σ_f are induced which are proportional to the distance from the centroid and which may be tensile or compressive depending on whether the fiber is displaced by the rotation to a larger or smaller radius; see Figure 2.

This behavior is described by Timoshenko[4] as:

$$\phi = \frac{M_t R^2}{E I_{xx}} \quad \text{and} \quad \sigma_f = \frac{M_t R y}{I_{xx}}$$

[1a, b]

where R is the centroidal radius of the ring,

E is Young's modulus,

I_{xx} is the areal moment of inertia of the cross-section of the ring about a radial axis (or the axis of symmetry of the cross section), and

y is the distance from the xx-axis to any circumferential fiber.

The analysis shows that the torsional rigidity of the ring plays no role; the elements of the ring are actually bent in planes parallel to the axis of the cylinder.

Because the stiffener is attached to the shell, however, free rotation is obviously prevented. Analytical estimates indicate that the cylinder is relatively very stiff; hence, to a first approximation, the web may be considered to be clamped at its junction with the shell. The web must therefore bend somewhat as a cantilever, and significant bending stresses are thereby produced in the web; see Figure 3. An adequate analysis must include analysis of the bending of the web.
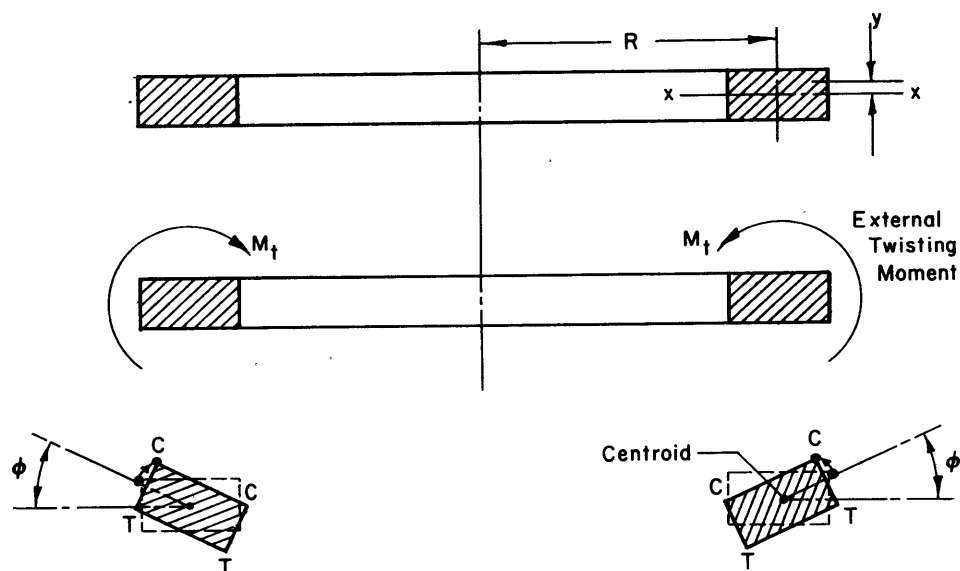

Figure 2 - Elastic Behavior of Circular Rings Subjected to
Uniform Symmetrical Twisting Moments

Under action of moment M_t the ring cross section rotates through angle ϕ and circumferential fibers undergo tensile or compressive strains as indicated by letters T and C.
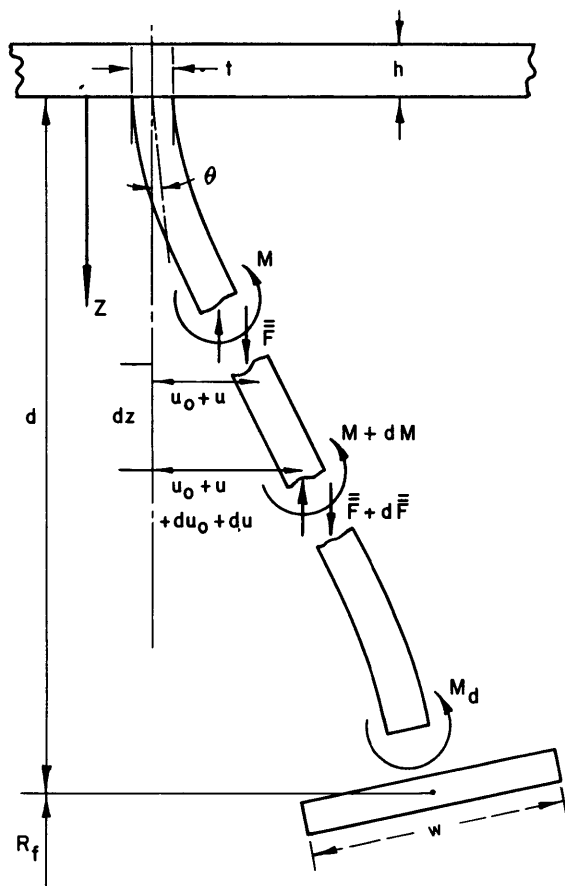
Figure 3 - Free-Body Diagram Showing
Forces and Moments Acting on an Element
of a T-Stiffener

Only axisymmetric deformations will be treated in this report. If the web were thin enough, it could buckle "locally" under sufficient load, with only a relatively small rotation of the flange. It will be assumed that such local buckling is prevented by giving sufficient thickness to the web and, also, that a similar local buckling of the flange itself is prevented by sufficient thickness of the flange.

Quite apart from the rotation imparted by initial imperfections, a similar twisting action can occur even in a perfect ring by a quasi-torsional instability of the stiffener. In some respects, this is similar to torsional or lateral buckling of laterally loaded beams. It is important, incidentally, not only as a mode of failure by itself but also with respect to the initial tilt problem, because the approach to buckling pressure p_{cr} may give rise to an amplification of tilt-induced stresses somewhat according to the relationship

$$\frac{1}{1-\dfrac{p}{p_{cr}}} \qquad [2]$$

This buckling of the entire ring may be either in a symmetric or nonsymmetric mode, although with outside stiffeners only the nonsymmetric may occur.

It is also possible for the flange or web to undergo purely local buckling with or without axial symmetry. For purposes of engineering analysis, these elements may be considered as straight plates of length $2\pi R$, carrying axial compressive thrust of magnitudes given by Equations [8a,b]. The half flange could be treated approximately by assuming its long edges respectively clamped and free, and the web treated with both edges clamped. No further elaboration of this mode of buckling is given here, except in the numerical examples at the end of the paper.

LOADS ON THE T-STIFFENER

We first determine the radial loads on the T-stiffener and the distribution of internal thrust and radial loads which are correspondingly evoked. Consider a T-shaped stiffening ring on the inside of a cylindrical shell of inside radius R; see Figure 3. External pressure

on the shell will give rise to a radial load on the ring at the point of attachment to the shell of F pounds per circumferential inch. Here

$$F = (2Q_0 + b)p \qquad [3]$$

where Q_0 is the radial shear force per unit circumferential length per unit pressure and b is the faying width of the stiffener. The quantity F may be computed directly from equations of von Sanden and Gunther.[3]

Note, however, that the cross section considered here includes both the stiffener and a portion of the shell plating of a width equal to the faying width, which for the tee is the web thickness t, i.e., $b = t$. Assuming that the radial force is resisted by circumferential thrust RF developed uniformly over the cross section, that portion $\overline{F}$ of radial load taken by the T-stiffener alone is

$$\overline{F} = \delta\,F \qquad [4]$$

where

$$\delta = \frac{A_f + A_w}{A_f + A_w + bh} \qquad [5]$$

A_f is the area of the T-stiffener flange,

A_w is the cross-sectional area of the web, and

h is the thickness of the shell.

This assumption regarding uniform distribution of circumferential thrust over the cross section is believed valid if the analysis is confined to relatively thin rings where the curvature of all circumferential fibers is approximately the same. By equilibrium considerations, this requires a uniform rate of decrease of the radial force along the web, from cylinder to flange.

The total circumferential thrust on the web and flange, respectively, can then be written as

$$\alpha\,R\overline{F} \text{ and } (1-\alpha)\,RF \qquad [6a, b]$$

where

$$\alpha = \frac{A_w}{A_f + A_w} \qquad [7]$$

Further calculation is simplified by relating the thrust to the quantity F rather than $\overline{F}$, the components in web and flange then being respectively

$$\beta\,RF \text{ and } \gamma\,RF \qquad [8a, b]$$

5

where

$$\beta = \frac{A_w}{A_f + A_w + bh}$$ [9a]

and

$$\gamma = \frac{A_f}{A_f + A_w + bh}$$ [9b]

At any point on the web at distance z from the shell, the thrust on the entire portion of the ring that lies beyond this point can be regarded as evoked by a radial force, of magnitude $\overline{\overline{F}}(z)$ pounds per circumferential inch, that acts at that point in the web. $\overline{\overline{F}}$ must be approximately a linear function of z, and from equilibrium considerations $\overline{\overline{F}}$ then varies from a value of δF at $z = 0$ to γF at $z = d$ at the flange; or from the relationships [5] and [9]

$$\overline{\overline{F}} = F(\delta - \beta \frac{z}{d})$$ [10]

TWISTING FROM INITIAL TILT, FIRST APPROXIMATION

If the T-ring is assumed to be "perfect," true buckling instability may occur. If it is imperfect, however, the radial loading F develops through the initial tilt a deformation of the sort shown in Figure 3. Because of the simplicity of an approximate analysis we treat the imperfect case first; this case is incidentally found to be of the greater practical significance. The initial tilt of the web is assumed to be through an axisymmetric angle θ; other imperfections of shape are neglected. For purposes of analysis, the ring is taken of T-section, on the inside of the cylinder, but modifications required for other shapes, or when it is on the outside, will be indicated.

Referring to a single typical transverse cross section, Figure 3, let u_0 be the initial axial displacement and u an additional elastic displacement at any point in the web, both u_0 and u being functions of z. In addition to the radial force $\overline{\overline{F}}(z)$, there will be a moment of M inch-pounds per inch circumference. The assumed positive senses of M, $\overline{\overline{F}}$, u_0, and u are shown in the figure.

From rotational equilibrium of an element dz

$$dM + \overline{\overline{F}} \frac{d}{dz} (u_0 + u) \, dz = 0$$ [11a]

or

$$\frac{dM}{dz} + \overline{\overline{F}} \frac{d}{dz}(u_0 + u) = 0$$ [11b]

6

Here

$$M = E_s I_w \frac{d^2 u}{dz^2} \tag{12}$$

$$E_s = \frac{E}{1-\nu^2} \tag{13}$$

$$I_w = t^3/12 \tag{14}$$

where E is Young's modulus,

ν is Poisson's ratio, and

t is the web thickness.

Equation [12] implies a treatment of the web by the theory of thin plates, an assumption believed of sufficient accuracy.

If desired, equilibrium of the differential element in a radial direction may be supposed to be preserved by fictitious radial load forces distributed over the element.

From Equations [10], [11], and [12], we obtain a differential equation for u:

$$E_s I_w \frac{d^3 u}{dz^3} + F \left(\delta - \beta \frac{z}{d} \right) \left(\frac{du_0}{dz} + \frac{du}{dz} \right) = 0 \tag{15}$$

This equation for u can be integrated, at least numerically, when the initial deflection u_0 is known. In this report, however, only the simple case will be considered in which the web is straight and initially inclined to the radius by the small angle θ so that

$$u_0 = \theta z \tag{16a}$$

$$\frac{du_0}{dz} = \theta \tag{16b}$$

In practical cases, the result should be conservative if θ is taken large enough to include all points in the web.

Two boundary conditions are that, at $z = 0$ where the web is clamped to the shell,

$$u = du/dz = 0 \tag{17}$$

7

A third condition is furnished by the continuity of web and flange. Let M_d denote the value of M at $z = d$. Then a moment equal to $-M_d$ will act symmetrically on the flange and must suffice to rotate the flange through an angle equal to du/dz at $z = d$. (In Figure 3 the positive direction for M_d is indicated; the actual direction is opposite.) Recalling Equation [1] for the twisting of rings subjected to uniformly distributed couples, we have at the boundary

$$\left(\frac{du}{dz}\right)_{z\,=\,d} = -\frac{M_d R_f^{\,2}}{E\,I_f}$$

[18]

Here I_f is the moment of inertia of the flange cross section about a radial axis.

To simplify analysis, the elastic deflection u will be assumed in this section to be small compared with u_0. This is shown in Appendix A to be valid if $6d^2F/Et^3$ is small relative to unity. Then du/dz can be dropped from [15] in comparison with du_0/dz or θ, and the equation can be readily integrated. Integrating twice,

$$E_s I_w \frac{du}{dz} + \theta F\left(\frac{\delta z^2}{2} - \frac{\beta z^3}{6d}\right) = M_0\,z + \overline{M}_0$$

[19]

where M_0 and $\overline{M}_0$ represent the first and second constants of integration.

From the conditions [12] and [17] at the boundary $z = 0$ we find

$$\overline{M}_0 = 0 \text{ and } M_0 = M_{z\,=\,0}$$

[20]

Actually, from [12]

$$M = E_s I_w \frac{d^2 u}{dz^2} = M_0 - \theta F\left(\delta z - \frac{\beta z^2}{2d}\right)$$

[21]

which at the flange $z = d$ has the value M_d such that

$$M_d = M_0 - \theta dF\left(\delta - \frac{\beta}{2}\right)$$

[22]

By substituting this expression for M_d in [18] and using [19] with $z = d$, it is found that

$$M_0 = \tfrac{1}{2}\,\theta\,dF\,\frac{2\,(\delta - \beta/2)\,R_f^{\,2}I_w + (\delta - \beta/3)\,(1 - \nu^2)\,dI_f}{R_f^{\,2}I_w + (1 - \nu^2)\,dI_f}$$

[23]

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and

$$M_d = -\tfrac{1}{2}\,\theta\,d^2 F\,\frac{(\delta - 2\,\beta/3)\,(1 - \nu^2)\,I_f}{R_f^2\,I_w + (1 - \nu^2)\,d\,I_f} \qquad [24]$$

From Equation [21] we note that

$$\frac{dM}{dz} = -\,\theta\,F\left(\delta - \frac{\beta z}{d}\right) \qquad [25]$$

so that with $\delta > \beta$ and $z < d$, dM/dz is always negative. Hence, the variation of M with z is monotonic and the greater numerical bending moment M_{max} is the greater of $|M_0|$ and $|M_d|$.

The maximum radial bending stress in the *web* is thus

$$\sigma_w = \frac{t}{2}\,\frac{M_{max}}{I_w} = \frac{6M_{max}}{t^2} \qquad [26]$$

The maximum circumferential bending stress at one corner of the flange, σ_f, is

$$\sigma_f = \tfrac{1}{2}\frac{w}{I_f}\,R_f\,\big|M_d\big| \qquad [27]$$

where w is the flange width.

SIMPLIFIED FORMULAS, FIRST APPROXIMATION

For an engineering type of analysis, simplified formulas may be obtained by noting that in [23] and [24] $(1 - \nu^2)$ differs only slightly from unity and that in most cases δ is almost unity and considerably larger than β, so that $(1 - \nu^2)$, $(\delta - 2\beta/3)$, $(\delta - \beta/2)$, and $(\delta - \beta/3)$ may be replaced by unity. Then, for simple T-stiffeners,

$$M_0 = \tfrac{1}{2}\,\theta dF\,\frac{2R_f^2 I_w + d I_f}{R_f^2 I_w + d I_f} = \theta dF\,\frac{R_f^2 t^3 + 6\,d I_f}{R_f^2 t^3 + 12\,d I_f} \qquad [23a]$$

and

$$M_d = -\tfrac{1}{2}\,\theta d^2 F\,\frac{I_f}{R_f^2 I_w + d I_f} = -\theta d^2 F\,\frac{6\,I_f}{R_f^2 t^3 + 12\,d I_f} \qquad [24a]$$

By inspection, $|M_0| > |M_d|$; thus the maximum radial bending stress in the web, Equation [26], becomes

$$\sigma_w = \frac{6 \theta d F}{t^2} \frac{R_f^2 t^3 + 6 d I_f}{R_f^2 t^3 + 12 d I_f}$$

[26a]

and the maximum circumferential stress in the flange becomes

$$\sigma_f = \frac{3 \theta d^2 F R_f w}{R_f^2 t^3 + 12 d I_f}$$

[27a]

These expressions agree closely with the results of preliminary analysis[5] used up to now in the evaluation of submarine model strength at the Taylor Model Basin.

STIFFENING RINGS ON THE OUTSIDE OR OF OTHER SHAPES

For rings *outside* of the cylinder, the derivation may remain unchanged except that F is replaced by $-F$ in all equations where it occurs. The actual directions of M_0 and M_d are thereby reversed from their directions with inside rings, the assumed positive directions of these moments remaining as for inside rings.

For rings of *rectangular* cross section without flanges, the equations can be used with sufficient accuracy by substituting $A_f = 0$ and $I_f = 0$. Then, from Equations [5], [9a], and [23],

$$M_0 = \frac{\theta d F}{2} \frac{A_w}{A_w + h}$$

[28]

A certain error, however, is present in this formula because rectangular rings are relatively much thicker than the web of a T. The change of the R of outer fibers, which is the cause of the resisting moment in the flange, has a similar effect in the web; this is negligible in ordinary webs but considerably larger in a rectangular ring.

For rings of *H or I cross section*, the results of Equations [23], [24], [26], and [27] may be applied to the outstanding flange and web, regarded together as a T-section. In this case, however, the faying flange participates in resisting the thrust produced by the radial forces F; this action, for obvious reasons, may be accommodated simply by defining, instead of with Equations [5] and [9a]

$$\delta = \frac{A_f + A_w}{A_f + A_w + A_b + bh}$$

[29]

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and

$$\beta = \frac{A_w}{A_f + A_w + A_b + bh}$$ [30]

where A_b is the area of the faying flange.

The simplified formulas [26a] and [27a] may be adapted to this case by adding a factor δ on the right side of those equations.

WEAKENING EFFECTS OF INITIAL TILT

In practical cases, the stresses computed by the foregoing analysis are seldom found to be very high, even at the design collapse pressure of the stiffened cylinder. Nevertheless they may play an important role in the failure process.

The senior author and S. Kendrick at the Naval Construction Research Establishment, Rosyth, Scotland, independently had pointed out that the *combination* of stresses in the ring might be important in considering the weakening effects of out-of-roundness in cylinders because the yield stress would thus be reached sooner in the stiffener. Similar effects will occur in tripping.

The total stress at any point in the flange can be written

$$\sigma = \sigma_c + \sigma_b + \sigma_f$$ [31]

where σ_c is membrane stress, σ_b is in-plane bending stress, and σ_f is tilt-induced stress; all are in the circumferential direction computed for the stiffener flange, at pressure p. As a failure criterion for the stiffener flange, it is suggested that at $p = p_{\text{design max}}$

$$\sigma \gtreqless \sigma_y$$ [32]

where σ_y is the yield strength of the material. Here, σ_f is computed from Equation [27] and

$$\sigma_c = \frac{RF}{A_f + A_w + A_b + bh}$$ [33]

neglecting the Lamé thick-ring effects. This latter quantity is readily computed in terms of geometry alone, whereas both σ_f and σ_b depend on the magnitude of initial imperfections either actually present or assumed to exist to the degree permitted by pressure vessel codes.

It may be argued that the design criterion proposed above, [31] and [32], is unduly conservative in that failure is implied as occurring at the pressure at which the stress at one corner of the flange reaches yield. Probably no such extreme consequence would develop.

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At only slightly higher pressures, however, the area of yielding would propagate into the flange until plastic buckling of the flange as a plate would precipitate stiffener tripping. No further elaboration on this aspect of failure is believed warranted until some experimental verification is obtained of the purely elastic phenomena associated with initial tilt. Application is left to the judgment of the designer.

For the stiffener web, consideration must be given to both radial and circumferential stress. In the radial direction, there is a bending stress σ_w given by Equation [26] and a compressive stress σ_r due to axisymmetric deformation from effects of pressure where

$$\sigma_r = \frac{\overline{\overline{F}}}{t} \qquad [34]$$

The compressive circumferential stress σ_c is given by [33].

With this biaxial state of stress, for the side of the web on which the bending stress is tensile, the von Mises criterion for yielding becomes

$$(\sigma_w - \sigma_r)^2 + \sigma_c^{\;2} + \sigma_c\,(\sigma_w - \sigma_r) \leqq \sigma_y^{\;2} \qquad [35]$$

As before, this expression only defines the condition when yielding first begins; it is thus a crude device for predicting failure pressure and is certainly subject to refinement.

TWISTING FROM INITIAL TILT, SECOND APPROXIMATION

To investigate purely elastic buckling, the assumption that u can be neglected compared with u_0 must be discarded and a better approximation made. If u is not omitted, Equation [15] can only be solved numerically because of the term $\beta z/d$. To obtain an approximate solution in terms of ordinary functions, but keeping u, the factor $(\delta - \beta z/d)$ will be replaced by a constant. As z increases in value from 0 to d, the term $\beta z/d$ averages $\beta/2$; this will be taken as a satisfactory effective value.

Equation [15] is then replaced by

$$E_s I_w \frac{d^3 u}{dz^3} + F(\delta - \beta/2)\left(\frac{du_0}{dz} + \frac{du}{dz}\right) = 0 \qquad [36]$$

As before, it will be assumed that $u_0 = \theta z$. Then the differential equation takes the form

$$B \frac{d^3 u}{dz^3} + C \frac{du}{dz} + D = 0 \qquad [37]$$

where $B = E_s I_w$, $C = F(\delta - \beta/2)$, $D = F\theta(\delta - \beta/2)$. The solution takes the form

$$u = a_1 + a_2 \cos mz + a_3 \sin mz + a_4 z \qquad [38]$$

where a_1, a_2, and a_3 are arbitrary constants of integration, $a_4 = -\dfrac{D}{C}$, and

$$m = \sqrt{\frac{C}{B}} = \sqrt{\frac{F(\delta - \beta/2)}{E_s I_w}} \qquad [39]$$

After satisfying the two boundary conditions at $z = 0$, the solution takes the form:

$$u = -\theta\left(z - \frac{\sin mz}{m}\right) - a_1(\cos mz - 1) \qquad [40]$$

Then

$$\frac{du}{dz} = -\theta(1 - \cos mz) + a_1(m \sin mz) \qquad [41]$$

and

$$M = E_s I_w \frac{d^2 u}{dz^2} = -E_s I_w m (\theta \sin mz - ma_1 \cos mz) \qquad [42]$$

Proceeding with the use of [18] and [41], we find

$$a_1 = \frac{\theta - \theta \cos md - M_d R_f^2 / EI_f}{m \sin md} \qquad [43]$$

Then

$$M_d = -\theta EI_w m \frac{(1 - \cos md) I_f}{(1 - \nu^2) I_f \sin md + mR_f^2 I_w \cos md} \qquad [44]$$

and

$$M_0 = \theta E_s I_w m \frac{(1 - \nu^2) I_f (1 - \cos md) + mR_f^2 I_w \sin md}{(1 - \nu^2) I_f \sin md + mR_f^2 I_w \cos md} \qquad [45]$$

Values of σ_{max} can be calculated as before from these values of M_0 and M_d.

For rings on the *outside* of the cylinder, F is replaced by $-F$ in Equation [36].

It is convenient, however, to let F continue to stand for the numerical value of F in Equation [39], so that m still represents a real number. Then in the solution, $\sin md$ and

cos md are found to be replaced by corresponding hyperbolic functions so that*

$$M_d = \theta E I_w m \frac{(\cosh\ md - 1)\ I_f}{(1 - \nu^2)\ I_f \sinh\ md + mR_f^2\ I_w \cosh\ md} \qquad [46]$$

and

$$M_0 = -\theta E_s I_w m \frac{(1 - \nu^2)\ I_f (\cosh\ md - 1) + mR_f^2 I_w \sinh\ md}{(1 - \nu^2)\ I_f \sinh\ md + mR_f^2\ I_w \cosh\ md} \qquad [47]$$

It will be noted that for inside rings M_d is negative and M_0 is positive, whereas for outside rings the signs are reversed.

More accurate values for the effects of initial tilt can be obtained by using Equations [44] to [47] for the moments M_d and M_0 instead of Equations [23] and [24]. Possible nonlinear effects are thereby included.

AXISYMMETRIC ELASTIC BUCKLING

According to Equations [44] and [45] M_d and M_0 became infinite when

$$- \frac{\tan\ md}{md} = \frac{R_f^2\ I_w}{(1 - \nu^2) d\ I_f} = r \qquad [48]$$

This is the axisymmetric buckling condition for a perfect ring where $\theta = 0$. Having solved Equation [48] for md, the critical value of F, i.e. F_{cr}, may be found by substituting this value of md in Equation [39], or

$$F_{cr} = \frac{m^2\ E_s I_w}{(\delta - \beta/2)} = \frac{E_s I_w\ (md)^2}{(\delta - \beta/2)\ d^2} \qquad [49]$$

---

*Equations [46] and [47] can be obtained either by repeating the analysis with the rings outside or by assuming F to be negative and substituting in Equations [44] and [45] $m = i|m|$ where $i = \sqrt{-1}$ and

$$|m| = \sqrt{\frac{|F|\ (\delta - \beta/2)}{E_s I_w}}$$

Note that $\sin i|m|d = i \sinh|m|d$, $\cos i|m|d = \cosh\ md$. Then, for simplicity, $|m|$ and $|F|$ are replaced by m and F with the understanding that these symbols stand for numerical values in all formulas.
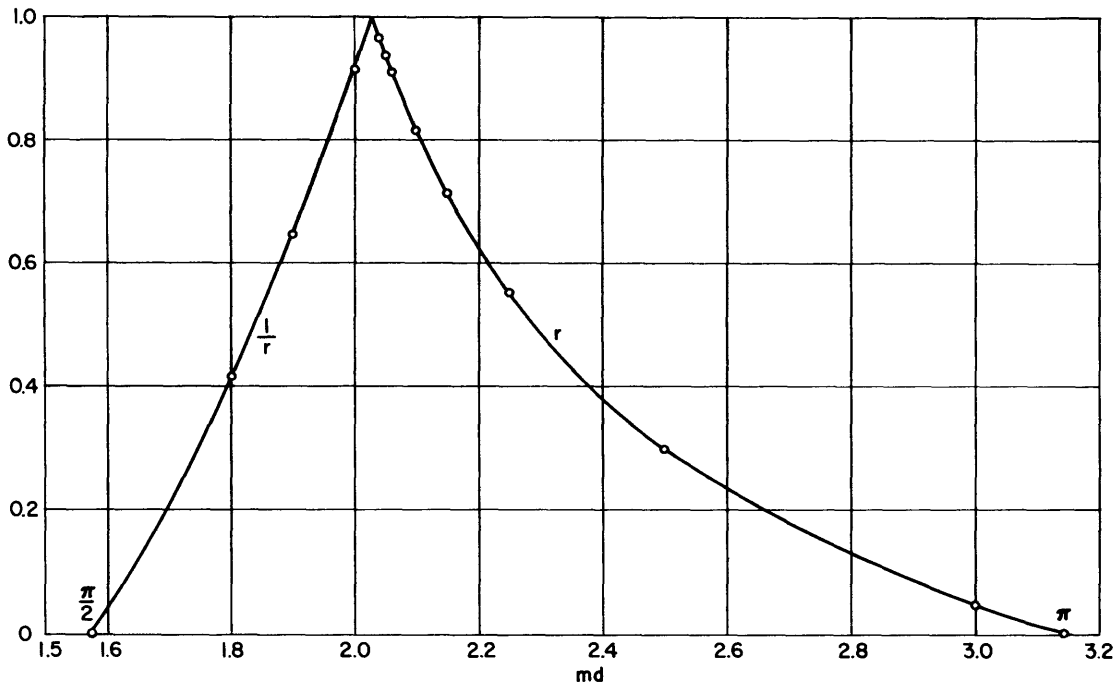
Figure 4 - Plot of *md* for Buckling as a Function of *r*

For convenience in obtaining quick solutions, *md* is plotted as a function of the quantity *r* in Figure 4.

When the rings are *outside*, axisymmetric buckling cannot occur.

From numerical examples, it is found that F_{cr} is always considerably higher than the value of F associated with the design collapse pressure. However, as noted in Equation [2], the approach to a buckling condition causes in an imperfect structure an exaggeration of the tripping effect, with the concomitant nonlinear increase in twist-induced stresses. Thus, with the amplification of effect of either ring, web, or flange instability, the stresses given by earlier equations are likely to be low. In one regard though, this may be offset by the overly conservative nature of the failure criterion in terms of stress, the two errors possibly compensating.

STRENGTH OF CYLINDERS WITH INSIDE VERSUS OUTSIDE STIFFENERS

Where tests on cylindrical shells with external pressure have indicated some superiority of outside stiffening, there is a cogent question concerning relevant effects of initial tilt. The differences in analysis for the inside versus outside stiffeners have been noted where they occur in the report and, in general, would not seem to account for the observed differences in strength. It is seen, however, that with outside frames, the radial forces impart a twist that reduces initial tilt, whereas with inside frames the corresponding action increases the tilt. Furthermore, the nonlinear effects included in the second approximation serve to exaggerate the tilt for inside but not for outside stiffeners.

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APPLICATION TO STIFFENERS ON CONICAL SHELLS

In the case of cylindrical shells, the discontinuity effects of shear and moment at the juncture of shell and stiffener are symmetrical so that no net twisting moment is developed if there is no initial tilt.

With conical shells, however, the symmetry disappears, and neither shear nor moments are the same on the two sides of the stiffener. A resultant moment thus acts to twist the stiffener in somewhat the same fashion as the tilt-induced moment M; its value may be readily computed using published results.[6]

EVALUATION

Without experimental verification, it may be premature to apply the results directly to stiffener design. However, it is believed that this derivation shows the relative significance of the web and flange dimensions which heretofore have been lumped into gross properties of area and moment of inertia as required by other considerations of strength. The numerical solutions of Tables 1, 2, and 3 show the level of stress which 3 degrees of tilt may induce in a stiffener designed by other considerations, not including effects of initial tilt. An example of failure in a stiffened cylinder due to inadequate stiffeners is shown in Figure 5.
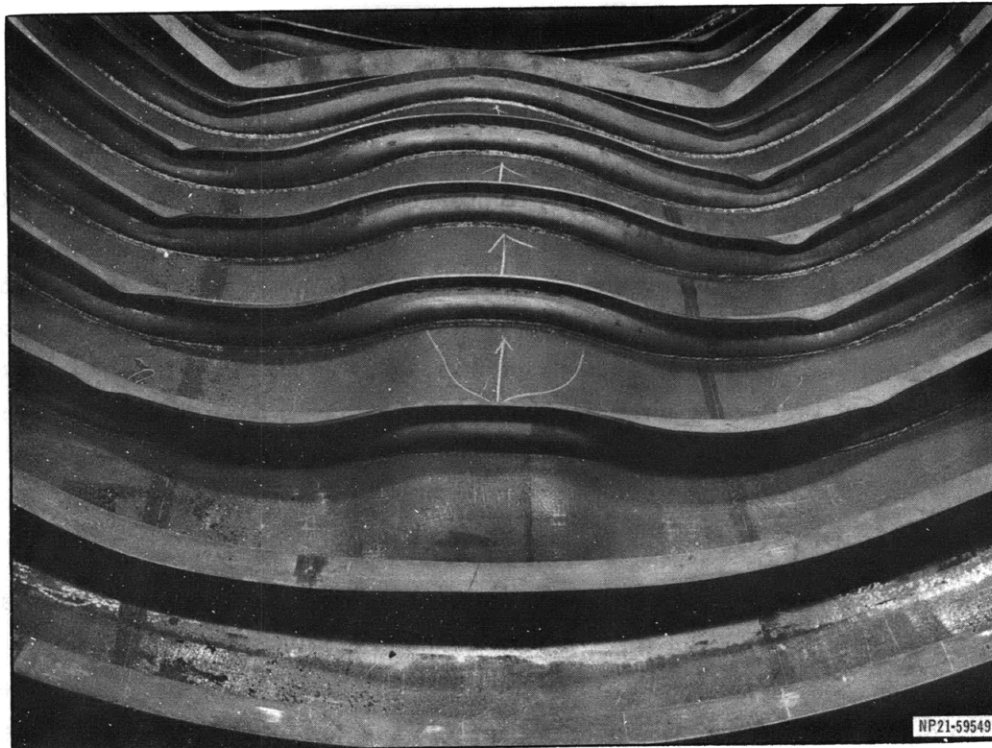

Figure 5 - Inside View of Cylinder with Inside T-Stiffeners Showing Mode of
Failure Probably Due to Effects of Initial Tilt

TABLE 1

Examples of Stresses in Internal T-Stiffeners Due to 3-Degree Initial Tilt

Dimensions are given in Figure 6a.

	p psi	F lb/in.	M_0 in.-lb	M_d in. lb	σ_w psi	σ_f psi
First Approximation	1	1.58	0.1041	− 0.0322	39.34	− 9.43
Simplified First Approximation	1	1.58	0.1321	− 0.0489	49.93	−14.34
Second Approximation	1	1.58	0.1012	− 0.0354	38.26	−10.37
First Approximation	241.7*	381.7	25.16	− 7.78	9,508	−2,278
Simplified First Approximation	241.7	381.7	31.92	−11.82	12,065	−3,465
Second Approximation	241.7	381.7**	25.64	− 9.07	9,691	−2,656

*Value computed to cause hoop yield in material with σ_y = 36,000 psi.

**F_{cr} = 2,834, i.e., p_{cr} > 1,790 psi.

h = 0.281"

R = 41.859"

R_f = 39.532"

t = b = 0.126"
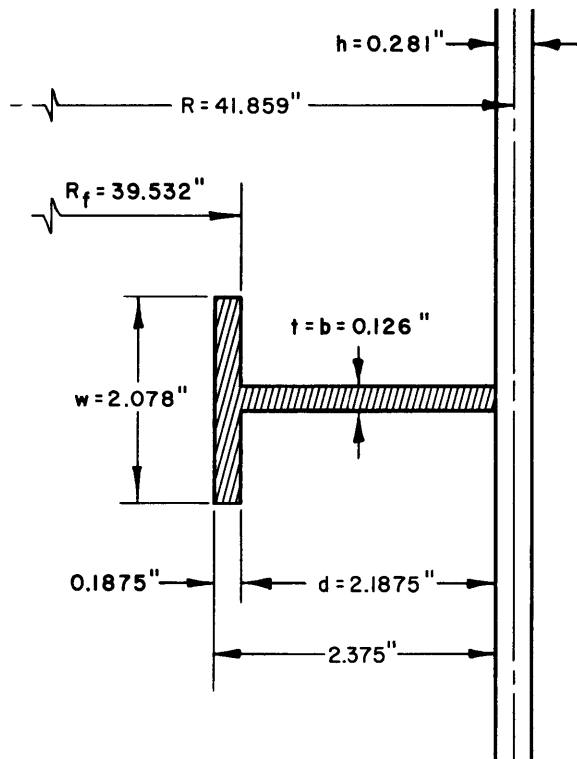
w = 2.078"

0.1875"

d = 2.1875"

2.375"

Figure 6a - Dimensions of Internal T-Stiffeners

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TABLE 2

Examples of Stresses in Internal H-Stiffeners Due to 3-Degree Initial Tilt

Dimensions are given in Figure 6b.

	p psi	F lb/in.	M_0 in.-lb	M_d in.-lb	σ_w psi	σ_f psi
First Approximation	1	9	0.5744	− 0.1634	24.51	− 6.53
Simplified First Approximation*	1	9	0.7139	− 0.2485	30.46	− 9.93
Second Approximation	1	9	0.5539	− 0.1795	23.63	− 7.18
First Approximation	376.3**	3386	216.12	−61.48	9,222	−2,457
Simplified First Approximation*	376.3	3386	268.61	−93.50	11,460	−3,738
Second Approximation	376.3	3386†	217.97	−72.09	9,300	−2,882

*Includes multiplication by δ.

**Value computed to cause hoop yield in material with $\sigma_y = 42,000$ psi.

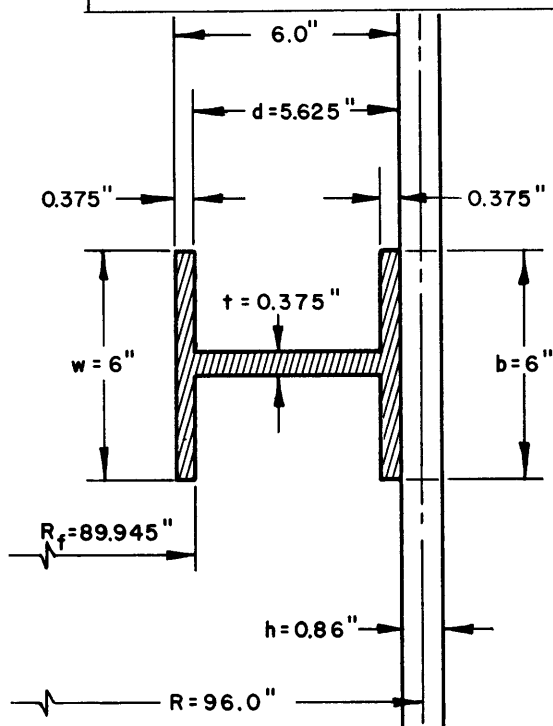† $F_{cr} = 11,298$, i.e. $p_{cr} > 1255$ psi.

Figure 6b - Dimensions of Internal H-Stiffeners

TABLE 3

Examples of Stresses in External H-Stiffeners Due to 3-Degree Initial Tilt

Dimensions are given in Figure 6c.

	p psi	F lb/in.	M_0 in.-lb	M_d in.-lb	σ_w psi	σ_f psi
First Approximation	1	− 9	− 0.5952	0.1426	− 25.40	6.468
Simplified First Approximation*	1	− 9	− 0.7442	0.2182	− 31.75	9.897
Second Approximation	1	− 9	− 0.5764	0.1565	− 24.59	7.096
First Approximation	376.3**	−3386	−223.94	53.653	−9555.0	2433.6
Simplified First Approximation*	376.3	−3386	−280.01	82.093	−11947.4	3723.6
Second Approximation	376.3	−3386	−208.81	56.520	−8909.4	2563.6

*Includes multiplication by δ.

**Value computed to cause hoop yield in material with σ_y = 42,000 psi.
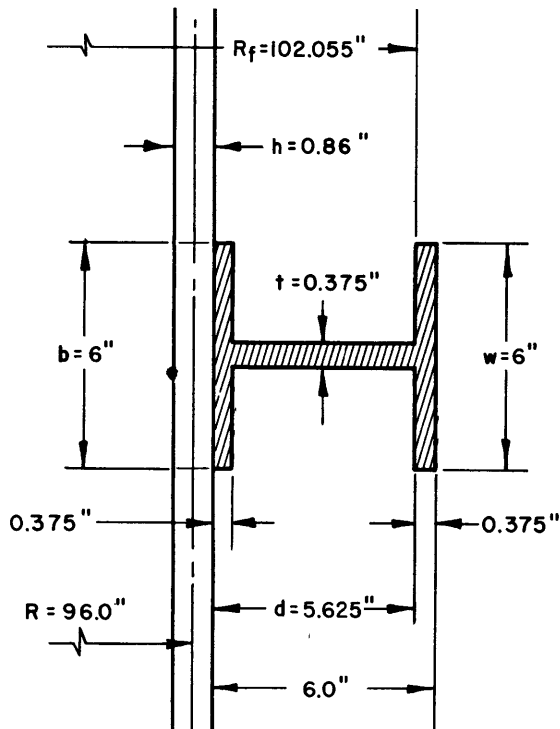

Figure 6c - Dimensions of External H-Stiffeners

Figure 6 - Dimensions of Stiffened Cylinders Used in Numerical Examples

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APPENDIX A

VALIDITY OF FIRST APPROXIMATION

To compare the maximum value of the elastic tilt du/dz with the initial tilt du_0/dz or θ, note first that, by Equation [21], the derivative of du/dz or d^2u/dz^2 vanishes when z has such a value that

$$M_0 = \theta F \left(\delta z - \frac{\beta z^2}{2d} \right)$$

Substituting this value for M_0 in [19]

$$E_s I_w \frac{du}{dz} = \theta F z^2 \left(\frac{\delta}{2} - \frac{1}{3} \frac{\beta z}{d} \right)$$

This gives the maximum value of du/dz, which starts from zero at $z = 0$. Since $\beta < \delta < 1$ and $z \leqq d$, it follows that

$$\left(\frac{du}{dz} \right)_{max} < \frac{\theta F d^2}{2 E_s I_w}$$

Inserting $I_w = t^3/12$ and substituting E for E_s for simplicity,

$$\frac{1}{\theta} \left(\frac{du}{dz} \right)_{max} < \frac{6 d^2 F}{E t^3}$$

ACKNOWLEDGMENT

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