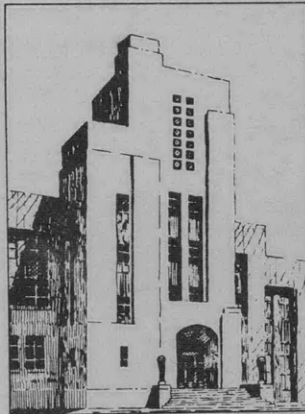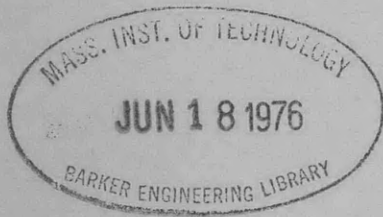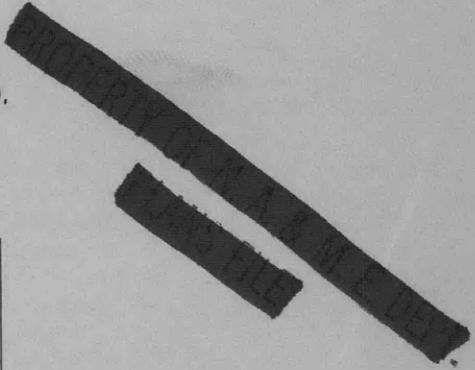# NAVY DEPARTMENT
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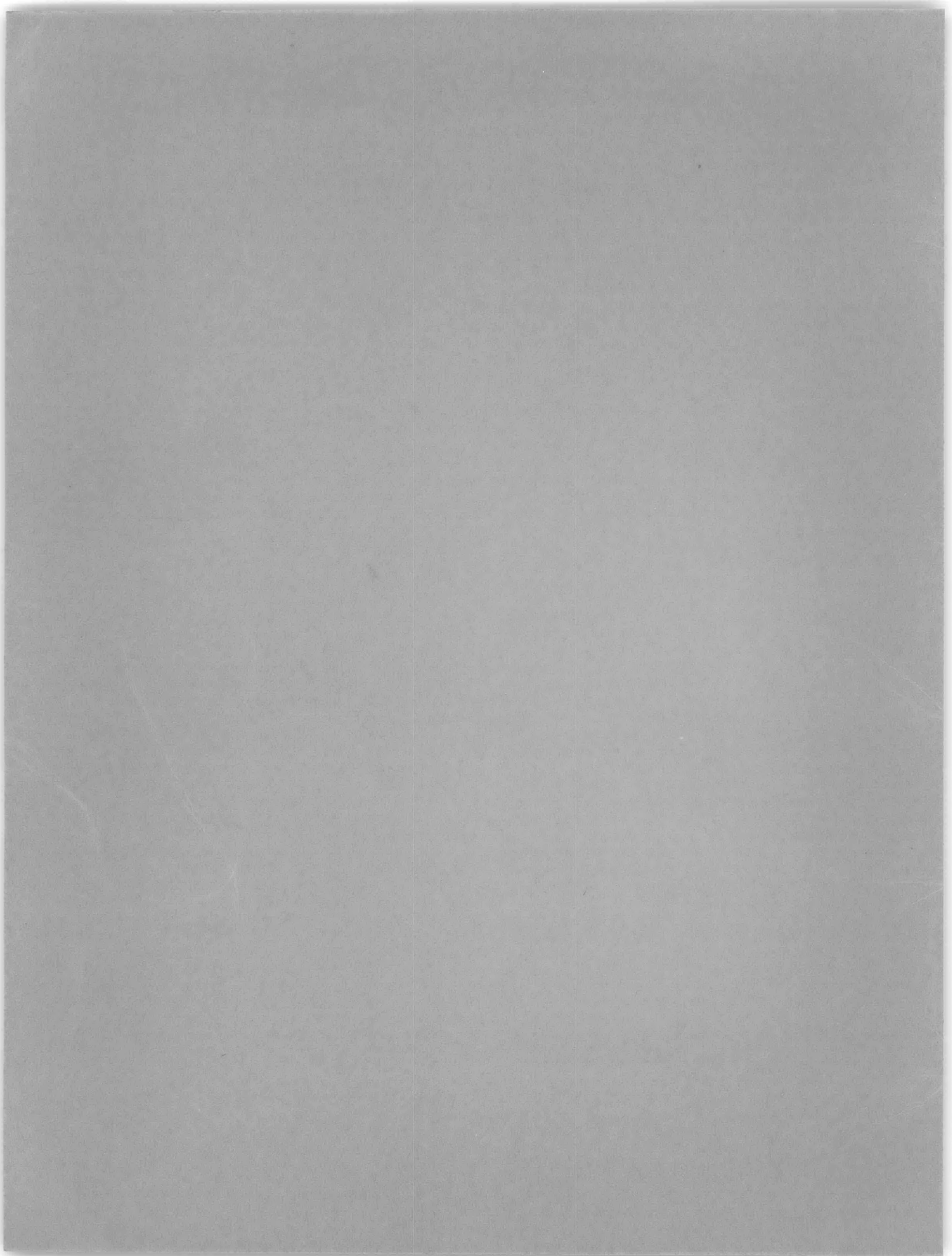
THE DIFFUSION OF AIR
INTO A PULSATING CAVITATION BUBBLE

by

Leonard Pode, Ph.D.

March 1955

Report 804

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NOTATION

A A nondimensional constant defined by $A = e + \dfrac{1}{\gamma - 1} + 1$

B A nondimensional constant defined by $B = \dfrac{1}{\gamma - 1}$

C The instantaneous concentration of air in the water at the boundary of the bubble, expressed in molecules per unit volume

C_0 The concentration of air in water of temperature T_0 that is in equilibrium with respect to diffusion with air of temperature T_0 and pressure P_0, expressed in molecules per unit volume

C_1 The concentration of air in water of temperature T_0 that is in equilibrium with air of temperature T_1 and pressure P_0, expressed in molecules per unit volume

C_∞ The concentration of air in the water at infinity, expressed in molecules per unit volume

D The diffusion coefficient for air in water at temperature T_0

$\dfrac{dN_G}{dt}$ The gross influx of molecules into the bubble

$\dfrac{dN_L}{dt}$ The gross efflux of molecules from the bubble

E The energy of the motion of the bubble

e The ratio of the energy of motion to the internal energy of the bubble when the bubble is at its equilibrium size, i.e., $e = E/P_0 V_1$

K The ratio Γ_a / Γ_w

KE Kinetic energy

k The Boltzmann gas constant

m The molecular mass for air

N The instantaneous number of molecules in the bubble

N_1 The number of molecules in the bubble when the bubble is at its equilibrium volume

n The instantaneous molecular density of the air in the bubble

n_0 The density of air at temperature T_0 and pressure P_0

n_1 The density of air at temperature T_1 and pressure P_0

P_0 The pressure in the water at infinity

R The instantaneous radius of the bubble

$\dot{R}$ The instantaneous radial velocity of the bubble

R_1 The radius of the bubble when the pressure in the bubble is P_0, i.e., the equilibrium radius

R_{max} The maximum radius of the bubble

R_{min} The minimum radius of the bubble

r The distance from the center of the bubble to an arbitrary point in the water

$\dot{r}$ The radial velocity of the water at any point distant r from the center of the bubble

T The instantaneous Kelvin temperature of the air in the bubble

T_0 The Kelvin temperature in the water at infinity

T_1 The Kelvin temperature of the air in the bubble when the pressure in the bubble is P_0

t Time

V The instantaneous volume of the bubble

V_1 The volume of the bubble when the air in the bubble is at pressure P_0, i.e., the equilibrium volume

β The value arising in Kinetic Theory defined by the equation $\beta^2 = m/2kT$; see Equation [8]

β_0 The value of β for $T = T_0$

β_1 The value of β for $T = T_1$

γ The ratio of specific heats c_p/c_v

Γ_a The rate at which molecules hit a unit surface from one side in air at pressure P_0 and temperature T_0; i.e., $\Gamma_a = n_0/(2\sqrt{\pi}\beta_0)$

Γ_w The net flow of molecules diffusing through a unit surface in water of temperature T_0 when the gradient of concentration is C_0/δ, i.e., $\Gamma_w = C_0 D/\delta$

δ The instantaneous thickness of the diffusion layer

δ_1 The thickness of the diffusion layer when the bubble is at its equilibrium volume

η The relative number of molecules in the bubble; $\eta = N/N_1$

ξ The ratio of the instantaneous radius of the bubble R to the equilibrium radius R_1, i.e., $\xi = R/R_1$

ξ_{max} The ratio of the maximum radius of the bubble R_{max} to the equilibrium radius

ξ_{min} The ratio of the minimum radius of the bubble to the equilibrium radius

ρ_a The mass density of air at pressure P_0 and temperature T_1

ρ_w The mass density of water at temperature T_0

τ A nondimensional time scale defined by $\tau = t/\beta_1 R_1$

THE DIFFUSION OF AIR INTO A PULSATING CAVITATION BUBBLE*

by

Leonard Pode

ABSTRACT

A calculation is made of the growth of a cavitation bubble through the diffusion of air induced by the spherically symmetrical mechanical motion of the pulsating bubble. The equation of motion of the boundary is derived from energy considerations. Kinetic theory is applied to determine the boundary conditions for the diffusion equation. The diffusion equation is treated by means of the approximation that the diffusion takes place in a very small layer of water about the boundary. Numerical results are obtained for one case of a strongly pulsing bubble. It is found that the growth per period is relatively small unless the bubble is extremely tiny.

INTRODUCTION

Although the phenomenon of cavitation has been of interest to hydraulic engineers for some time it is only in recent years that this subject has gained the attention of physicists to any great extent. The concern of the engineers arises from the attendant destructive action on hydraulic structures and the object of their research has been to find ways of avoiding cavitation and means of reducing damage when cavitation cannot be avoided.[1] The approach has been mainly empirical, involving the close observation of the phenomena and study of the character of the destructive action.

Various theories have been advanced to account for the extraordinary severity of cavitation damage. The theory that chemical corrosion is involved has been pretty well discredited by the evidence of damage to noncorrosive surfaces such as glass and concrete. It is generally accepted now that the destructive action is of a mechanical nature due either to direct blows of water or the occurrence of high pressures in the neighborhood of collapsing cavities. There is, however, at present no theory that adequately describes the mechanism. Indeed, the problem of explaining the creation and subsequent pulsation of a cavitation bubble involves the complicated interaction of so many factors that only a rough approximation can be attempted by theoretical investigation. In fact, the fundamental laws governing some of the factors

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involved are not yet fully understood. The properties of inertia, compressibility, and viscosity of water are involved; also, vaporization and condensation of water, expansion and contraction of the gas-filled cavity, heat conduction, surface tension, and finally, diffusion of air into the cavity. It is in the last mentioned factor, namely the diffusion of air into the cavity, that the interest of this paper lies.

Water free from impurities and gas nuclei can support considerable negative pressure without rupture. The inception of cavitation at low positive pressures, i.e. the vapor tension, indicates the presence of gas nuclei in the water. Experiments indicate that the noise attendant upon the pulsations of bubbles decreases with increasing air content.[2] The bubbles which contain the least air make the most noise. In general, the acoustical properties of bubbles in water are related to the volume of air contained. It also appears likely that the damage caused by the collapse of a cavitation bubble is similarly related to the air content. Therefore, although the process of diffusion of air into the cavity probably has little effect upon the motion of the boundary of the cavity, the increase of the air content during the pulsation of a bubble is of interest and it is the purpose of this paper to evaluate the extent to which a pulsing bubble can grow.

GENERAL DISCUSSION

Of all the factors that affect the motion of the boundary of the cavity, the inertia of the water is undoubtedly the most important. The first attempts to describe the motion considered this factor exclusively. Lord Rayleigh[3] (and others) wrote the equation of motion for the collapse of a spherical vacuum surrounded by an infinite volume of water that is assumed incompressible. The result probably describes the true motion fairly well[5] except that it leads to complete collapse with the attainment of infinite velocities instead of the ultimate stopping of the motion at some minimum size of the cavity and the subsequent reversal of the motion which in fact occurs. Rayleigh also wrote the equation of motion for the case that the cavity is gas-filled and is expanded and compressed isothermally. The cavity does then attain some minimum size and the motion is reversed. However, in view of the rapidity of the motion it seems very likely that the expansion and compression of the gas-filled cavity is adiabatic rather than isothermal. With this alteration the equation becomes identical with that used to describe the pulsation of the gas globe following an underwater explosion. The behavior of the latter is so closely analogous to that of the cavitation bubble that a qualitative description of this phenomenon is in order.[4]

The underwater explosion instantaneously releases a considerable amount of free energy which appears partly in a shock wave that travels away and partly in the very high pressure attained by the gases which are formed by the explosion. A radial motion of the water away from the center of the explosion then follows. Because of the momentum acquired by the fluid, the gas-filled cavity if overexpanded so that by the time the motion is arrested quite low pressure obtains in the cavity. The water then rushes back toward the center of the explosion compressing the gases with such rapidity that high pressures are again reached and

another shock wave is generated. When the gases are contracted to a minimum volume the motion is again reversed and another pulse started. Energy is dissipated by turbulence and viscous effects in addition to that radiated away in the shock wave so that each succeeding pulse is less violent. When the energy of the motion is largely spent the gaseous globe settles down to "sonic" oscillations, i.e., small amplitude vibrations. The sonic motion is essentially different from the initial pulsations in that the sonic motion is approximately simple harmonic whereas the pulsing motion is characterized by high radial velocity when the globe is small and slow radial velocity when the globe is large.

The motion of the boundary of the cavitation bubble appears to be quite similar.[5] Of course the two phenomena differ in the origin of the energy involved. The energy released by an underwater explosion is known from the chemical nature of the explosives used. The energy of the oscillation of a cavitation bubble is not so simply found. It seems probable that in addition to the passage through low-pressure and then high-pressure areas the effects of surface tension play a part. However, the action of surface tension especially under dynamic conditions is not yet adequately understood. For our purpose we shall ignore the surface tension when considering the motion of the boundary and simply assume the energy of motion to be known.

The question we wish to discuss is: How does the motion of the boundary affect the diffusion of air into the bubble? Although it appears unlikely that the diffusion that would take place in the time of a single pulse if the boundary of the bubble were stationary could be appreciable, the evidence seems to indicate that the motion of the boundary does cause an increase in the amount of air within the cavity by diffusion. For example, this would help to explain the growth of bubbles when a container of water is vigorously struck.[6] One may see how the motion of the boundary might cause a resultant one-sided diffusion into the cavity. It is clear that the direction of the diffusion, i.e., whether air will move into the water from the cavity or out of the water into the cavity depends upon the traffic of air molecules at the boundary. Because of the asymmetrical nature of the motion of the boundary, the bubble will be larger than its equilibrium size for a longer time than it is smaller. When the bubble is large the streaming of molecules into the cavity from the water increases because of the greater surface area, whereas the return of molecules from the cavity to the water is not increased to the same extent since the density of the air in the cavity is reduced. The reverse effect that occurs when the bubble is compressed is of less influence because of both the smaller surface area and the shorter time involved.[7*]

The calculation may be divided into three parts:

1. The equation of motion of the boundary

2. The traffic of molecules at the boundary

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*Harvey has suggested that an oscillating bubble may grow by diffusion because of the larger surface area during the positive cycle. The present calculation includes this effect as well as the additional effect produced by the asymmetry of the motion.

3. The diffusion of air through the water

To simplify the calculation it will be assumed that the period of time being considered is so short that the air content of the bubble changes only by a small fraction so that for many purposes the air content may be taken as constant. Thus, for example, when considering the motion of the boundary the effects of the change in the air content upon the volume of the bubble, upon the energy of the expansion of the gas, and upon the internal energy of the gas in the bubble will be ignored. Also, when the traffic of molecules at the boundary is calcula-ted, the density of the gas in the bubble will be taken as inversely proportional to the volume just as though the air content of the bubble were constant. As a consequence of this treatment of the change in the air content a detailed time history of the diffusion process can not be sought but instead the average rate of diffusion for the time considered is found.

THE EQUATION OF MOTION

The equation of motion of the boundary and the water surrounding the bubble has been presented by many writers. However, in order to obtain the equation in the form presently desired we shall derive the equation anew. The water is assumed to be incompressible. Dissi-pation of energy is ignored and also the effect of diffusion upon the motion. The air in the cavity is treated as a homogeneous, chemically uniform gas and the presence of water vapor is disregarded.

Consider an infinite volume of water at undisturbed pressure P_0 and temperature (Kelvin) T_0, i.e., the pressure and temperature of the water at infinity is fixed at P_0 and T_0 respectively. Let there be a spherical cavity of volume V in the water occupied by such a quantity of air that at equilibrium conditions it would occupy a volume V_1. Let R and R_1 represent the radii of spheres of volumes V and V_1 respectively.

Care must be exercised in defining "equilibrium" conditions. Reference is now being made to mechanical equilibrium. This requires only that the pressure in the bubble be equal to the pressure in the water. Since the motion is assumed to be adiabatic, when the bubble reaches the volume V_1 and pressure P_0 the temperature T_1 in the bubble will not in general be equal to the temperature T_0 of the water. In other words the position of mechanical equilibrium does not necessarily coincide with the position of thermal equilibrium. In fact progress towards thermal equilibrium requires the exchange of heat energy between the air in the bubble and the water and this is expressly excluded by the assumption that the motion is adiabatic.

Assume that as the cavity expands and contracts the flow always retains spherical symmetry so that at any instant the cavity has a radius R and a radial velocity $\dot{R}$. Let r represent the distance from the center of the cavity to an arbitrary point in the water and $\dot{r}$ the radial velocity of the water at such a point. Incompressibility requires that the radial velocity of the water vary inversely as the square of the radius so that:

$$\dot{r} = \frac{R^2}{r^2} \dot{R}$$ [1]

It is seen that the motion of the entire volume of water is known when the motion of the boundary is known.

The simplest way to determine the equation of motion of the boundary is, after Rayleigh, to calculate the energy involved. The kinetic energy KE of the water is given by

$$KE = \int_{R}^{\infty} \rho_w \frac{4\pi r^2 \dot{r}^2}{2} \, dr$$ [2]

where ρ_w is the density of the water. Hence

$$KE = 2\pi \rho_w \dot{R}^2 R^3$$ [3]

The potential energy due to expansion of the cavity against the pressure of the water is given by $4/3 \, \pi P_0 \, [R^3 - R_1^3]$ and the internal energy due to adiabatic compression of the gas, if the effect of diffusion of gas into the cavity is neglected, is

$$\frac{P_0 \, 4/3 \, \pi R_1^3}{\gamma - 1} \left[\left(\frac{R_1}{R}\right)^{3(\gamma - 1)} - 1 \right]$$

where γ is the ratio of specific heats for air c_p/c_v. Therefore the total energy E may be written

$$E = 2\pi \rho_w \dot{R}^2 R^3 + \frac{4}{3}\pi P_0 \, [R^3 - R_1^3] + \frac{\frac{4}{3}\pi P_0 R_1^3}{\gamma - 1} \left[\left(\frac{R_1}{R}\right)^{3(\gamma - 1)} - 1 \right]$$ [4]

or upon solving for $\dot{R}^2$

$$\dot{R}^2 = \frac{E}{2\pi \rho_w R^3} + \frac{2P_0}{3\rho_w} \left(\frac{R_1^3}{R_3} - 1\right) - \frac{2P_0}{3(\gamma - 1)\rho_w} \left(\frac{R_1^{3\gamma}}{R^{3\gamma}} - \frac{R_1^3}{R^3}\right)$$ [5]

and multiplying this equation by $\rho_a/2P_0$ where ρ_a is the density of air at pressure P_0 and temperature T_1

$$\frac{\rho_a \dot{R}^2}{2P_0} = \frac{\rho_a}{3\rho_w} \left\{ \frac{E}{\frac{4}{3}\pi P_0 R_1^3} \left(\frac{R_1}{R}\right)^3 + \left(\frac{R_1}{R}\right)^3 - 1 - \frac{1}{\gamma - 1} \left(\frac{R_1^{3\gamma}}{R^{3\gamma}} - \frac{R_1^3}{R^3}\right) \right\}$$ [6]

So that

$$\beta_1^2 \dot{R}^2 = \frac{\rho_a}{3\rho_w} \left\{ \frac{A}{\xi^3} - \frac{B}{\xi^{3\gamma}} - 1 \right\}$$ [7]

where $A = \dfrac{E}{\frac{4}{3}\pi P_0 R_1^{\,3}} + \dfrac{1}{\gamma - 1} + 1 = \dfrac{E}{P_0 V_1} + \dfrac{1}{\gamma - 1} + 1,$

$B = \dfrac{1}{\gamma - 1},$

$\xi = R/R_1,$ and

$\beta_1 = \sqrt{\rho_a / 2 P_0}$

The quantity β_1 is related to the value β that arises in kinetic theory defined by

$$\beta^2 = \frac{m}{2kT}$$

where k is the Boltzmann constant,

$\qquad m$ is the molecular mass, and

$\qquad T$ is the temperature (Kelvin) of the gas.

For the air in the bubble

$$\beta^2 = \frac{m}{2kT} \frac{N_1 T_1}{N_1 T_1} = \frac{M_1 T_1}{2 P_0 V_1 T} = \frac{\rho_a}{2 P_0} \left(\frac{R}{R_1}\right)^{3(\gamma - 1)} = \beta_1^{\,2} \, \xi^{3(\gamma - 1)} \qquad [8]$$

where N_1 and M_1 are the number of molecules and the mass of air in the cavity respectively. It is clear that β_1 is the value of β that applies to air at temperature T_1. Since β_1 has the dimensions of the reciprocal of velocity, $\beta_1^{\,2}\dot{R}^2$ will be nondimensional. The equation of motion can be expressed entirely in nondimensional terms by suitable adjustment of the time scale. Thus let $\tau = t/\beta_1 R_1$.* Then the equation of motion becomes

$$\frac{d\xi}{d\tau} = \left(\frac{\rho_a}{3\rho_w}\right)\left(\frac{A}{\xi^3} - \frac{B}{\xi^{3\gamma}} - 1\right)^{1/2} \qquad [9]$$

The motion of the boundary is defined by the nondimensional parameters ρ_a/ρ_w, γ and $e = E/(P_0 V_1)$. The ratio of densities ρ_a/ρ_w enters only as a scale factor. The parameter e is $1/(\gamma - 1)$ times the ratio of the energy of motion to the internal energy of the air in the bubble when the air is at pressure P_0 and temperature T_1 and is the only parameter that varies from case to case. The maximum and minimum values of ξ occur when $d\xi/d\tau = 0$ and may be found by solving the equation $A/\xi^3 - B/\xi^{3\gamma} - 1 = 0$. Conversely from this equation the value of e can be found if either ξ_{max} or ξ_{min} is known. Often the value of ξ_{max} is more readily available than the value of e. For example, suppose that a bubble of radius R_i is at rest in water at pressure P_i and the pressure of the water is suddenly increased to P. Then $R_i = R_{max}$ and $P_i (R^3_{max})^\gamma = P_0 R_1^{\,3\gamma}$ where R_1 is the equilibrium radius for the subsequent motion. Hence $\xi_{max} = R_{max}/R_1 = (P_0/P_i)^{1/3\gamma}$.

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*Since the natural frequency of the bubble for sonic oscillations is given by $1/2\pi R_1 \sqrt{(3\gamma P_0)/\rho_w}$ the time interval $\beta_1 R_1$ is equal to $1/2\pi \sqrt{3\gamma \rho_a/2\rho_w}$ times the natural period.

THE TRAFFIC OF MOLECULES AT THE BOUNDARY

The traffic of molecules at the boundary of the bubble is studied by considering the rate dN_L/dt at which molecules leave the cavity and enter the water and then the rate dN_G/dt at which molecules stream away from the water surface and enter the cavity, thus obtaining the net rate of transfer of molecules $dN/dt = dN_G/dt - dN_L/dt$. Because of spherical symmetry it is only necessary to find the rate of transfer of molecules per unit area of the boundary $(dN/dt)/4\pi R^2$. This rate may be determined by considering just a small section of the boundary. If the mean free path is sufficiently small compared to the radius of the bubble a section of the spherical surface may be taken small enough to be practically a plane area. From kinetic theory the flow of molecules per unit area through a plane surface in a gas is given by $n/2\sqrt{\pi}\beta$ where n is the molecular density of the gas and as before $\beta^2 = m/2kT$. Although some of the molecules that strike the water surface rebound into the gas, it appears reasonable to assume that almost all pass into the water. Then the specific rate of loss is

$$\frac{1}{4\pi R^2} \frac{dN_L}{dt} = \frac{n}{2\sqrt{\pi}\beta}$$

where the values of n and β are those appropriate to the air in the cavity.

The rate at which molecules stream into the cavity from the water surface depends upon the concentration of the dissolved gas and the temperature of the water at the boundary since these factors determine the state of gas with which the water is in equilibrium. (We now refer to "equilibrium" in connection with the process of diffusion.) The density of air in equilibrium with water when both are at the same fixed temperature is proportional to the concentration of dissolved gas in accordance with Henry's Law. The effect of temperature is to vary the constant of proportionality. Since the conduction of heat is neglected the temperature of the water T_0 is everywhere constant. Let C_0 be the concentration of air in the water when the water is in equilibrium with air at pressure P_0 and temperature T_0. When the concentration at the boundary is C_0 the rate at which molecules leave a unit area of the boundary is given by $n_0/(2\sqrt{\pi}\beta_0)$ where n_0 is the molecular density of air at pressure P_0 and temperature T_0, and β_0 is the appropriate value of β, i.e., $\beta_0^2 = m/2kT_0$. Let C be the actual concentration at the boundary so that the ratio to the concentration C_0 is C/C_0 and the water is in equilibrium with air at temperature T_0 and molecular density $(C/C_0)n_0$. The rate at which molecules stream away from a unit area of the boundary is also in the same proportion; hence

$$\frac{1}{4\pi R^2} \frac{dN_G}{dt} = \frac{n_0}{2\sqrt{\pi}\beta_0} \frac{C}{C_0} \quad \text{and}$$

$$\frac{1}{4\pi R^2} \frac{dN}{dt} = \frac{1}{2\sqrt{\pi}}\left[\frac{n_0 C}{\beta_0 C_0} - \frac{n}{\beta}\right] = \frac{n_1}{2\sqrt{\pi}\beta_1}\left[\frac{n_0\beta_1 C}{n_1\beta_0 C_0} - \frac{n\beta_1}{n_1\beta}\right] = \frac{n_1}{2\sqrt{\pi}\beta_1}\left[\frac{C}{C_1} - \frac{n\beta_1}{n_1\beta}\right] \quad [10]$$

where n_1 is the molecular density of air at pressure P_0 and temperature T_1, and $C_1 = C_0 (n_1\beta_0)/(n_0\beta_1)$ is the concentration in water of temperature T_0 which is in equilibrium with air at pressure P_0 and temperature T_1.

If it is assumed that during the interval of time being considered the number of molecules in the bubble changes only by a small amount, the molecular density n is approximately inversely proportional to the volume, i.e., $n/n_1 = R_1^3/R^3 = 1/\xi^3$ where n is the molecular density when the bubble is at its equilibrium volume. From Equation [8] $\beta/\beta_1 = \xi^{3(\gamma-1)/2}$ so that the rate of change of the number of molecules in the bubble may be written

$$\frac{dN}{dt} = \frac{3N_1}{2\sqrt{\pi}\beta_1 R_1} \left[\frac{C}{C_1} \xi^2 - \xi^{\frac{-3\gamma+1}{2}} \right] \qquad [11]$$

where $N_1 = \dfrac{4\pi R_1^3 n_1}{3}$.

THE DIFFUSION OF AIR THROUGH THE WATER

The calculation of the diffusion into a bubble whose radius moves in accordance with the equation of motion [9] subject to the condition expressed by Equation [11] is difficult not only because of the nature of the boundary conditions and the variety of initial conditions that are possible but also because of the necessity of evaluating the effect of turbulence upon the diffusion process in the water. The turbulence that is to be expected in the water surrounding a cavitation bubble does not bear the same relationship to the bubble as the turbulence that arises in the parallel flow about a sphere bears to the sphere because in the case of the cavitation bubble the turbulence is created by the flow about an extraneous body. The situation is as though there were a number of mechanical stirrers agitating the water about the bubble. As a result of such a mixing process it may be expected that the gradient of the concentration of air in the water would be reduced. If it can be assumed that the turbulence is sufficiently effective in this respect so that the concentration of air in the water at infinity C_∞ is maintained to within a small distance δ from the boundary of the bubble the calculation of the diffusion is considerably simplified.

When the concentrations C and C_∞ are expressed in terms of molecules per unit volume, the rate at which molecules pass through the thin diffusion layer of thickness δ at the boundary of the bubble is given by

$$\frac{dN}{dt} = \frac{4\pi R^2 D}{\delta}(C_\infty - C) = \frac{4\pi R_1^2 D}{\delta}(C_\infty - C)\xi^2 \qquad [12]$$

where D is the diffusion coefficient for air in water. Equations [11] and [12] may be regarded as two simultaneous equations in the unknowns dN/dt and C. The solution of these equations is

$$C = \frac{C_\infty + K\,\xi^{-3(\gamma+1)/2}\,C_1}{1+K} \qquad [13]$$

and

$$\frac{dN}{dt} = \frac{\dfrac{3N_1}{2\sqrt{\pi}\,\beta_1 R_1}}{1 + K}\left[\frac{C_\infty}{C_1}\,\xi^2 - \xi^{\frac{-3\gamma+1}{2}}\right] \qquad [14]$$

where

$$K = \frac{3N_1}{2\sqrt{\pi}\,\beta_1 R_1 C_1} \cdot \frac{\delta}{4\pi DR_1^2} = \frac{n_1}{2\sqrt{\pi}\,\beta_1 C_1 D}\frac{\delta}{1} = \frac{n_0}{2\sqrt{\pi}\,\beta_0} \cdot \frac{\delta}{C_0 D} = \frac{\Gamma_a}{\Gamma_w} \qquad [15]$$

Here $\Gamma_a = n_0/(2\sqrt{\pi}\,\beta_0)$ is the rate at which molecules hit a unit surface from one side in air at pressure P_0 and temperature T_0 and $\Gamma_w = (C_0 D)/\delta$ is the net flow of air molecules through a unit surface in water where the gradient of the concentration is C_0/δ. Since Γ_a is very much larger than Γ_w, K will be a very large number so that the factor $1/(1 + K)$ may be replaced by $1/K$. Equation [14] may be put entirely in nondimensional terms by expressing the rate of change of the relative number of molecules $\eta = N/N_1$ in terms of the nondimensional time $\tau = t/\beta_1 R_1$; thus

$$\frac{d\eta}{d\tau} = \frac{3}{2\sqrt{\pi}\,K}\left[\frac{C_\infty}{C_1}\,\xi^2 - \xi^{\frac{-3\gamma+1}{2}}\right] \qquad [15a]$$

As mentioned before, because of the neglect of the effect of the change in numbers of molecules upon the motion of the boundary as well as neglect of this effect of the diffusion upon itself this evaluation of the rate of change of the air content applies legitimately only to time intervals so short that the change in air content of the bubble is small relative to the initial air content. Consequently, rather than the instantaneous variation of the rate $d\eta/d\tau$, what is wanted is the average value of the rate $\overline{d\eta/d\tau}$ for the time interval considered. For the present purpose we may consider the time interval of one pulse or assuming the energy of motion to be fixed the average may be taken over a half pulse since in this case the contraction of the bubble is exactly reverse to the expansion. If it is also assumed that the thickness of the diffusion layer δ is constant then

$$\frac{\overline{d\eta}}{d\tau} = \frac{3}{2\sqrt{\pi}K}\left[\frac{C_\infty}{C_1}\,\overline{\xi^2} - \overline{\xi^{\frac{-3\gamma+1}{2}}}\right] \qquad [16]$$

The average value of any power of ξ may be expressed by quadratures. For example, the average value of $\overline{\xi^n}$ is given by

$$\overline{\xi^n} = \int_{\xi_{min}}^{\xi_{max}} \xi^n\,\frac{d\xi}{\left(\dfrac{d\xi}{d\tau}\right)} \div \int_{\xi_{min}}^{\xi_{max}} \frac{d\xi}{\left(\dfrac{d\xi}{d\tau}\right)} \qquad [17]$$

where $d\xi/d\tau$ is given as a function of ξ by Equation [9].

Similarly the average value of C could be computed from

$$\overline{C} = \frac{C_\infty + K\ \overline{\xi^{-3/2(\gamma+1)}}C_1}{1 + K} = \overline{\xi^{\pm 3(\gamma+1)/2}}\ C_1 \qquad [18]$$

where the second expression for $\overline{C}$ may be used when C_∞ and C_1 are of the same order of magnitude. This average however is found to be a poor representative value for the "effective" concentration. The question is: What constant value of C would give the correct value of $\overline{d\eta}/d\tau$ when substituted in the diffusion Equation [12]? It is readily verified that such a value is given by

$$\overline{C}_{\text{eff}} = \frac{\overline{\xi^{\frac{-3\gamma+1}{2}}}}{\overline{\xi^2}}\ C_1 \qquad [19]$$

Since it is not certain that the thickness of the diffusion layer may be taken as constant it is of interest to determine the modifications that must be made when the thickness of the diffusion layer is assumed to vary directly with the radius of the bubble, i.e. $\delta = \delta_1 R/R_1 = \delta_1 \xi$ where δ_1 is the thickness of the diffusion layer when the radius of the bubble is at its equilibrium value. The following results are then obtained

$$\overline{\frac{d\eta}{d\tau}} = \frac{3}{2\sqrt{\pi}\ K}\left[\frac{C_\infty}{C_1}\ \overline{\xi} - \overline{\xi^{\frac{-3\gamma-1}{2}}}\right] \qquad [20]$$

$$\overline{C}_{\text{eff}} = \frac{\overline{\xi^{\frac{-3\gamma-1}{2}}}}{\overline{\xi}}\ C_1 \qquad [21]$$

where $K = \dfrac{\Gamma_a}{\Gamma_w}$ as before but now $\Gamma_w = \dfrac{DC_0}{\delta_1}$.

Although we have spoken of an air bubble in water, it is clear that the foregoing theory applies just as well to a bubble of almost any gas in any liquid. The average value of any power of ξ and hence the ratio $\overline{C}_{\text{eff}}/C_1$ depends only upon the ratio of specific heats γ and the ratio of the maximum radius to the equilibrium radius, $\xi_{\max}$. Since γ is substantially the same for many gases the ratio $\overline{C}_{\text{eff}}/C_1$ is characteristic mainly of the geometry of the motion.

NUMERICAL CALCULATION

In order to illustrate the use of the formulas and obtain an evaluation of the magnitude of the effect of the motion of the boundary of the bubble upon the diffusion process, a numerical computation has been carried through for the case when the maximum radius is two times the equilibrium radius, i.e., $\xi_{\max} = 2$. This motion is comparable to that resulting from a

rather small explosion. For air γ is about 1.4 but little error is introduced if for the convenience of computation γ is taken as $4/3$. The equation of motion [9] then becomes

$$\frac{d\xi}{d\tau} = \sqrt{\left(\frac{\rho_a}{3\rho_w}\right)\left(\frac{9.5}{\xi^3} - \frac{3}{\xi^4} - 1\right)}$$

$$\xi_{max} = 2 \quad \gamma = 4/3 \quad l = 4.5 \quad \xi_{min} = 0.317.$$

Through numerical integration the following values were obtained:

$$\overline{\xi^2} = 2.736 \quad \overline{\xi} = 1.599 \quad \overline{\xi^{-3/2}} = 0.667 \quad \overline{\xi^{-5/2}} = 0.755.$$

For constant thickness of the diffusion layer the effective average concentration is from Equation [19]

$$\overline{C}_{eff} = \frac{\overline{\xi^{-3/2}}}{\overline{\xi^2}} \ C_1 = \frac{0.667}{2.74} \ C_1 = 0.243 \ C_1$$

When the thickness of the diffusion layer is proportional to the radius, the effective average concentration is from Equation [21]

$$\overline{C}_{eff} = \frac{\overline{\xi^{-5/2}}}{\overline{\xi}} = \frac{0.755}{1.60} \ C_1 = 0.472 \ C_1$$

Assuming that $P_0 = 760$ mm Hg, $T_0 = T_1 = 20°$ C and that the concentration in the water at infinity is such that in the absence of the motion of the boundary there would be no diffusion:

$$C_\infty = C_0 = C_1 = 5.02 \times 10^{17} \text{ molecules cm}^{-3}$$

$$\beta_0 = \beta_1 = 2.44 \times 10^{-5} \text{ sec cm}^{-1}$$

$$n_0 = 2.50 \times 10^{19} \text{ molecules cm}^{-3}$$

$$\rho_a = 1.20 \times 10^{-3} \text{ gms cm}^{-3}$$

$$\rho_w = 0.998 \text{ gm cm}^{-3}$$

and taking $D = 1.9 \times 10^{-5}$ cm^2 sec^{-1} for air in water

$$K = \frac{n_0\delta}{2\sqrt{\pi}\beta_0 C_1 D} = \frac{2.50 \times 10^{19} \ \delta \text{ cm}^{-1}}{2 \times 1.77 \times 2.44 \times 10^{-5} \times 5.02 \times 10^{17} \times 1.9 \times 10^{-5}}$$

$$= 3.03 \times 10^{10} \ \delta \text{ cm}^{-1}$$

The average relative rate of growth of the bubble is

$$\overline{\frac{dN/N_1}{dt}} = \frac{1}{\beta_1 R_1} \overline{\frac{d\eta}{d\tau}} = \frac{3}{2\sqrt{\pi}\beta_1 R_1 K} \left[\frac{C_\infty}{C_1} \overline{\xi^2} - \overline{\xi^{-3/2}} \right]$$

$$= \frac{3\,[2.736 - 0.667]\ \mathrm{cm^2\ sec^{-1}}}{2 \times 1.77 \times 2.44 \times 10^{-5} \times 3.03 \times 10^{10}\ \delta R_1}$$

$$= \frac{2.37 \times 10^{-6}\ \mathrm{cm^2\ sec^{-1}}}{\delta R_1}$$

and

$$\overline{\frac{dN/N_1}{dt}} = \frac{3}{2\sqrt{\pi}\beta_1 R_1 K} \left[\frac{C_\infty}{C_1} \overline{\xi} - \overline{\xi^{-5/2}} \right]$$

$$= \frac{3\,[1.599 - 0.755]\ \mathrm{cm^2\ sec^{-1}}}{2 \times 1.77 \times 2.44 \times 10^{-5} \times 3.03 \times 10^{10}\ \delta_1 R_1}$$

$$= \frac{0.967 \times 10^{-6}\ \mathrm{cm^2\ sec^{-1}}}{\delta_1 R_1}$$

for the case of constant and linearly varying diffusion layer thickness respectively.

From numerical integration of the equation of motion it is found that the period of a pulse

$$\Delta t = \beta_1 R_1 \Delta \tau = \beta_1 R_1 \sqrt{\frac{3\rho_w}{\rho_a}}(3.20) = 3.2 \times 2.44 \times 10^{-5} \sqrt{\frac{3 \times 0.998}{1.20 \times 10^{-3}}}\ R_1\ \mathrm{cm^{-1}\ sec}$$

$$= 3.90 \times 10^{-3}\ R_1\ \mathrm{cm^{-1}\ sec}$$

so that the relative growth during the time of one pulse is given by

$$\Delta N/N = \Delta \eta = \frac{2.37 \times 10^{-6} \times 3.90 \times 10^{-3}\ \mathrm{cm}}{\delta} = \frac{9.24 \times 10^{-9}\ \mathrm{cm}}{\delta}$$

for constant diffusion layer thickness and

$$\Delta \eta = \frac{0.967 \times 10^{-6} \times 3.90 \times 10^{-3}\ \mathrm{cm}}{\delta} = \frac{3.77 \times 10^{-9}\ \mathrm{cm}}{\delta_1}$$

for linearly varying diffusion layer thickness.

It is observed that the relative change of the number of molecules in the bubble during a pulse depends upon the thickness of the diffusion layer and not directly upon the size of the bubble. However it may be expected that the thickness of the diffusion layer will be smaller for the smaller bubble, and, furthermore, since the period of the pulse of the smaller bubble is shorter it can execute many more pulses in a given time so that the size of the bubble enters

into the rate of change of the relative number of molecules when the rate is referred to an absolute time scale.

The numerical results indicate that diffusion would not have much effect upon larger bubbles subjected to a few pulses of the relative energy considered. However, very tiny bubbles, i.e., micronucleii, could be very considerably affected by the diffusion occurring during a single pulse. Moreover whenever a bubble is so situated that it is subjected to many pulses, such as is the case for a bubble in a container of water that is being continually and vigorously struck or for a bubble that lies in the track of an ultrsonic beam, the effect of diffusion may be quite pronounced even if the bubble is large.

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