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THE NEAR-SOUND FIELD OF TURBULENCE

by

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SUMMARY

STATEMENT OF PROBLEM

Determine some of the theoretical properties of the multipole sound fields generated by both free turbulence and turbulence in the presence of solid boundaries at distances that are less than a wavelength from the turbulence. Compare the frequency spectra and spatial properties of the induction near-sound fields with the frequency spectra and spatial properties of the far-sound fields of these multipole sound sources.

FINDINGS

There are significant differences in the frequency spectra and the spatial properties of the various multipole components of the sound field at distances greater than and less than a wavelength from a turbulent region. Approximate corrections to data taken in the near-sound field of multipole sound sources can be made if the geometric extent, type, and orientation of the multipole sources are known or can be estimated.

RECOMMENDATIONS

If possible, measurements of the sound from multipole sound sources should be taken at a distance that is many wavelengths and many "diameters" from the sound sources. If this is not possible or desirable, information about the geometric extent, type, and orientation of the sound sources should be obtained.

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NOTATION

c	Velocity of sound in medium outside turbulent region
c_f	Drag coefficient
$F_i(\overline{y}, t)$	Instantaneous applied force or acoustic dipole strength per unit volume at point $\overline{y}$ in the turbulent region
F_i	$F_i \equiv F_i(\overline{y}, t-r/c) \equiv \dot{f}_i$
f_i	$f_i \equiv f_i(\overline{y}, t-r/c)$
h	Perpendicular distance from point to plane
I_k	Acoustic intensity in the x_k coordinate direction at point $\overline{x}$
I_r, I_θ, I_ϕ	Acoustic intensity in the $\overline{r}$, $\overline{\theta}$, and $\overline{\phi}$ directions at point $\overline{x}$
K	Mass rate density of sources
L	Typical linear dimension of the flow
M	Mach number based on c and the convection velocity of the forces or the stresses
$\overline{M}$	Vector Mach number based on c and the convection velocity of the forces or the stresses
M_i	ith component of the vector Mach number
M_0	$M_0 \equiv U/c$, typical Mach number of the flow
P	Sound power radiated by the turbulent region
p	Instantaneous sound pressure at point $\overline{x}$
$\overline{p^2}$	Mean-square sound pressure at point $\overline{x}$
R	Reynolds number
r	$r \equiv \lvert\overline{x} - \overline{y}\rvert$, spherical polar coordinate centered at point $\overline{y}$
r_i	$r_i \equiv x_i - y_i$, ith component of $\overline{r}$
$\overline{r}$	$\overline{r} \equiv \overline{x} - \overline{y}$
S	Correlation area
$dS(\overline{y})$	Element of area at $\overline{y}$
$T_{ij}(\overline{y}, t)$	Instantaneous applied stress or acoustic quadrupole strength per unit volume at point $\overline{y}$ in the turbulent region
T_{ij}	$T_{ij} \equiv T_{ij}(\overline{y}, t-r/c) \equiv \dot{t}_{ij}$
t_{ij}	$t_{ij} \equiv t_{ij}(\overline{y}, t-r/c)$
t	Time
U	Typical velocity in the flow

u	$u \equiv \overline{u} \cdot \overline{r}/r$, the component of $\overline{u}$ in the $\overline{r}$-direction
$\overline{u}$	Vector velocity at $\overline{y}$ at retarded time $t-r/c$
u_i	$u_i \equiv u_i(\overline{y},\ t-r/c)$, fluid velocity at $\overline{y}$ at retarded time $t-r/c$
V	Correlation volume
v_k	kth component of the instantaneous particle velocity at $\overline{x}$
$\overline{x}$	Vector position of a point outside of the turbulent region
x_i	ith coordinate of a point outside of the turbulent region
$\overline{y}$	Vector position of a point in the turbulent region
y_i	ith coordinate of a point in the turbulent region
$d\overline{y}$	Element of volume at $\overline{y}$
δ_{ij}	Unit diagonal tensor
η_d	Efficiency of conversion of mechanical to acoustic energy for dipole
η_q	Efficiency of conversion of mechanical to acoustic energy for quadrupole
$\overline{\eta}$	$\overline{\eta} = \overline{y} + \overline{M}r$
θ	$\theta \equiv \lvert\overline{\theta}\rvert$, spherical polar coordinate
$\overline{\theta}$	Vector in the direction of increasing θ
θ_i	ith component of $\overline{\theta}$
ρ_0	Mean density of the medium
Φ	Instantaneous velocity potential at $\overline{x}$
ϕ	$\phi \equiv \lvert\overline{\phi}\rvert$, spherical polar coordinate
$\overline{\phi}$	Vector in the direction of increasing ϕ
ϕ_i	ith component $\overline{\phi}$
ω	Angular frequency or 2π times the frequency of the radiated sound
ω^*	Angular frequency of the velocity fluctuations
i, j, k, l, m	Subscripts which take on values 1, 2, or 3 for the three coordinate directions
$'$	Primes indicate condition at independent position $\overline{y}'$ in the turbulent region
$\cdot$	Dots indicate partial derivatives with respect to time
—	Overbar on individual quantities indicates a vector
——	Overbar on products of time-dependent quantities indicates a time average

ABSTRACT

General expressions for the velocity potential, particle velocity, instantaneous sound pressure, mean-square sound pressure, and sound intensity for the sound generated by turbulence are derived, starting from Lighthill's fundamental relations for the density fluctuations in the medium outside a turbulent region. The dipole radiation from the turbulent boundary layer of a rigid boundary, the quadrupole radiation from isotropic turbulence, and the lateral quadrupole radiation from turbulence in the presence of a large mean shear are discussed. The frequency spectra and directional patterns of the sound observed at points in the sound field that are much less than a wavelength from multipole sound sources are shown to differ significantly from the frequency spectra and directional patterns of the sound observed at points in the sound field that are many wavelengths from the multipole sound sources.

INTRODUCTION

Significant progress has been made toward an understanding of the subject of turbulence noise since the publication of Lighthill's fundamental relations[1] for the density fluctuations in the medium outside a turbulent region. The expressions defining the radiated or far-sound field of a turbulent region in terms of multipole fields have been extensively discussed by Lighthill,[2] Proudman,[3] Phillips,[4] and others.[5-8] The purpose of this report is to present expressions which also explicitly define the near-sound field at distances either not large compared with a wavelength of the radiated sound or not large compared with the geometric extent of the turbulent region. It is hoped that these expressions will be useful in interpreting noise data taken close to turbulent jets and boundary layers.

Acoustic multipoles are also associated with phenomena other than turbulence. The general expressions for the sound fields of multipoles, of course, are also applicable to these sound fields. For instance, the near-sound fields produced by rotating propellers[9] and by surface disturbances[10] have been investigated and analyzed using acoustic dipole theory.

Because the far-sound field is simpler and usually of more interest, measurements in the near-sound field of a turbulent region have usually been avoided. Some reports on the sound from turbulent jets, however, do contain data on the sound pressures in the near field.[11-18] In general, the data indicate an enhancement of the low-frequency portion of the sound spectra and a different sound pressure-distance dependence than has been observed for the far-sound field. In the future, careful measurements of the noise close to turbulent boundary layers may also show near-field effects.[19] Since observations in the near-sound field are often necessary or convenient, it is advantageous to have means of interpreting the data. This report is intended to facilitate the making of corrections to existing and future data and the planning of new experiments.

[1]References are listed on page 33.

GENERAL SOUND FIELDS OF DIPOLES AND QUADRUPOLES

DEFINITION OF NEAR FIELD

As used in this report, the unqualified expression "near-sound field" refers either to the region called the "induction near field" at distances less than a wavelength from the multipole source, or to the region called the "geometric near field" at distances from the sound source that are less than the order of the geometric extent of the source. In the induction near field the particle velocity and pressure fields are not in phase in time. These fields contain a relatively large amount of unradiated energy that alternates between the kinetic and potential forms in the immediate vicinity of the multipole source. The geometric near field exists because an average distance from the measuring point to the source is usually used in presenting data for the region close to an extended source, even though the actual distances to the various parts of the source should be used.

In many practical situations the regions defined by the two types of near field overlap and are comparable in extent, so that the effects of being in each region separately may be difficult to determine. The situation is further complicated by the fact that the effects of being in each region may tend to compensate each other. For example, if the distance from the nearest part of an extended multipole source is used as the characteristic distance from the source,[11] the rms sound pressure in the induction near field varies inversely as the square or some higher power of the distance from the source, and in the geometric near field the rms sound pressure may be nearly independent of the distance from the source.

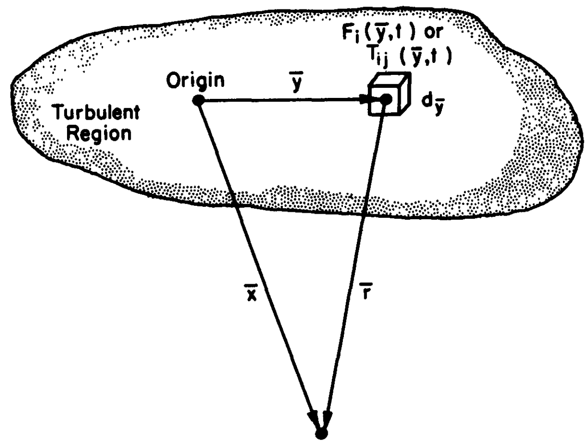
SOUND FIELD OUTSIDE A FIELD OF FLUCTUATING FORCES AND STRESSES

The general expressions derived by Lighthill for density fluctuations in the medium outside a turbulent region have been expanded and applied, up to now, only for the region many wavelengths from the turbulent region. The purpose of this report is to expand these general expressions also for the region defined by the induction near field. As has been done for the radiation field, a number of special cases will also be discussed because under some practical conditions one type of multipole may predominate sufficiently to permit an approximate description of the sound field by one set of simplified equations.

By using the relation between density fluctuations and pressure fluctuations, the instantaneous sound pressure p at time t and point $\bar{x}$ outside a region of fluctuating forces and a region of fluctuating stresses, as sketched in Figure 1, is given, respectively, by the volume integrals

$$p = -\frac{1}{4\pi}\int \frac{\partial}{\partial x_i}\left(\frac{F_i}{r}\right) d\bar{y} \qquad \text{(Equation 11, Reference 1)} \quad [1]$$

Figure 1 – Location of Origin and Volume Element in a Turbulent Region



and

$$p = \frac{1}{4\pi}\int \frac{\partial^2}{\partial x_i \partial x_j}\left(\frac{T_{ij}}{r}\right) d\overline{y} \qquad \text{(Equation 15, Reference 1)} \qquad [2]$$

where $F_i(\overline{y}, t)$ is the ith component of the instantaneous applied force or acoustic dipole strength per unit volume at time t and at point $\overline{y}$ in the turbulent region,

$T_{ij}(\overline{y}, t)$ is the ij component of the instantaneous applied stress or acoustic quadrupole strength per unit volume at time t and at point $\overline{y}$ in the turbulent region,

$F_i \equiv F_i(\overline{y}, t-r/c)$,

$T_{ij} \equiv T_{ij}(\overline{y}, t-r/c)$,*

$r \equiv |\overline{x}-\overline{y}|$, and

c is the velocity of sound in the medium.

If the applied forces are distributed over a surface,[4] F_i is considered to be a surface density and the integral is a surface integral instead of a volume integral.

If the differentiation in Equation [1] is performed, the general expression for the instantaneous sound pressure due to the distribution of dipoles becomes

$$p = \frac{1}{4\pi}\int \frac{r_i}{r}\left(\frac{\dot{F}_i}{rc} + \frac{F_i}{r^2}\right) d\overline{y} \qquad [3]$$

where $r_i \equiv x_i - y_i$ and dots indicate partial differentiation with respect to time. The term in $\dot{F}_i$ represents the radiated sound pressure, and the term in F_i represents induction near-field sound pressure. The effect of the geometric near field is manifested in the factor r_i/r and in the dependence on r. In other words, if r_i can be approximated by x_i, geometrical near-field effects are negligible.

*This is a slight variation of the conventional notation used by Lighthill, necessitated by the need for a short symbol.

Since the sound intensity, defined as the time average of the product of the particle velocity and the sound pressure at a point, cannot be approximated in the induction near field by $\overline{p^2}/\rho_0 c$, where ρ_0 is the mean density and the overbar on the squared quantity signified a time average, it is necessary to obtain an expression for the particle velocity. This is most easily obtained by taking the gradient of the velocity potential Φ which is first obtained by integrating the equation

$$p = -\rho_0 \dot{\Phi} \qquad [4]$$

After neglecting the constants of integration, the velocity potential for the distribution of dipoles is

$$\Phi = -\frac{1}{4\pi\rho_0}\int \frac{r_i}{r}\left(\frac{F_i}{rc} + \frac{f_i}{r^2}\right) d\overline{y} \qquad [5]$$

where $f_i \equiv f_i\,(\overline{y},\, t - r/c)$ and $\dot{f}_i = F_i$.* The kth component of the particle velocity $\overline{v}$ or the particle velocity in the x_k coordinate direction is then

$$v_k = \frac{1}{4\pi\rho_0}\int\left[\frac{r_i r_k}{r^2}\left(\frac{\dot{F}_i}{rc^2} + \frac{3F_i}{r^2 c} + \frac{3f_i}{r^3}\right) - \delta_{ik}\left(\frac{F_i}{r^2 c} + \frac{f_i}{r^3}\right)\right] d\overline{y} \qquad [6]$$

where δ_{ik} is the unit diagonal tensor and repeated subscripts are to be summed (i = 1, 2, 3). The kth component of the vector sound intensity is then given by

$$I_k = \frac{1}{16\pi^2\rho_0}\iint\left[\frac{r_i r_j' r_k'}{r r'^2}\overline{\left(\frac{\dot{F}_i}{rc} + \frac{F_i}{r^2}\right)\left(\frac{\dot{F}_j'}{r'c^2} + \frac{3F_j'}{r'^2 c} + \frac{3f_j'}{r'^3}\right)}\right.$$
$$\left. - \frac{r_i\delta_{jk}}{r}\overline{\left(\frac{\dot{F}_i}{rc} + \frac{F_i}{r^2}\right)\left(\frac{F_j'}{r'^2 c} + \frac{f_j'}{r'^3}\right)}\right] d\overline{y}\, d\overline{y}' \qquad [7]$$

where primes and different subscripts are used in the expression for the particle velocity to indicate functions of $\overline{y}'$. Similarly, the mean-square sound pressure is given by

$$\overline{p^2} = \frac{1}{16\pi^2}\iint\frac{r_i r_j'}{r r'}\overline{\left(\frac{\dot{F}_i}{rc} + \frac{F_i}{r^2}\right)\left(\frac{\dot{F}_j'}{r'c} + \frac{F_j'}{r'^2}\right)}\, d\overline{y}\, d\overline{y}' \qquad [8]$$

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* $-\dfrac{1}{4\pi\rho_0}\int f_i\,(\overline{y},\, t)\, d\overline{y} = \mu_i\,(t)$, the dipole strength as usually defined.

In like manner, if the double differentiation in Equation [2] is performed, the general expression for the instantaneous sound pressure due to the distribution of quadrupoles becomes

$$p = \frac{1}{4\pi}\int\left[\frac{r_i r_j}{r^2}\left(\frac{\ddot{T}_{ij}}{rc^2} + \frac{3\dot{T}_{ij}}{r^2 c} + \frac{3T_{ij}}{r^3}\right) - \delta_{ij}\left(\frac{\dot{T}_{ij}}{r^2 c} + \frac{T_{ij}}{r^3}\right)\right] d\bar{y} \qquad [9]$$

The term in $\ddot{T}_{ij}$ represents the radiated sound pressure, and the terms in T_{ij} represent the induction near-field sound pressure. The terms in $\dot{T}_{ij}$ are significant only in the transition region between the induction near field and the far field, and are conveniently lumped with the induction near-field terms. After again neglecting the constants of integration, the velocity potential is

$$\Phi = -\frac{1}{4\pi\rho_0}\int\left[\frac{r_i r_j}{r^2}\left(\frac{\dot{T}_{ij}}{rc^2} + \frac{3T_{ij}}{r^2 c} + \frac{3t_{ij}}{r^3}\right) - \delta_{ij}\left(\frac{T_{ij}}{r^2 c} + \frac{t_{ij}}{r^3}\right)\right] d\bar{y} \qquad [10]$$

where $t_{ij} \equiv t_{ij}(\bar{y}, t - r/c)$ and $\dot{t}_{ij} = T_{ij}$. This velocity potential, as well as the previous one, satisfies the wave equation

$$\nabla^2 \Phi = \frac{1}{c^2}\ddot{\Phi} \qquad [11]$$

The kth component of the particle velocity is

$$v_k = \frac{1}{4\pi\rho_0}\int\left[\frac{r_i r_j r_k}{r^3}\left(\frac{\ddot{T}_{ij}}{rc^3} + \frac{6\dot{T}_{ij}}{r^2 c^2} + \frac{15T_{ij}}{r^3 c} + \frac{15t_{ij}}{r^4}\right)\right.$$

$$\left. - \left(\delta_{ij}\frac{r_k}{r} + 2\delta_{ik}\frac{r_j}{r}\right)\left(\frac{\dot{T}_{ij}}{r^2 c^2} + \frac{3T_{ij}}{r^3 c} + \frac{3t_{ij}}{r^4}\right)\right] d\bar{y} \qquad [12]$$

By again using primes and different subscripts in the expression for the particle velocity and taking the time average of the product of p and v_k, the kth component of the sound intensity becomes

$$I_k = \frac{1}{16\pi^2\rho_0}\iint\left[\frac{r_i r_j r'_k r'_l r'_m}{r^2 r'^3}\overline{\left(\frac{\ddot{T}_{ij}}{rc^2} + \frac{3\dot{T}_{ij}}{r^2 c} + \frac{3T_{ij}}{r^3}\right)\left(\frac{\ddot{T}'_{lm}}{r'c^3} + \frac{6\dot{T}'_{lm}}{r'^2 c^2} + \frac{15T'_{lm}}{r'^3 c} + \frac{15t'_{lm}}{r'^4}\right)}\right.$$

$$- \frac{r_i r_j}{r^2}\left(\delta_{lm}\frac{r'_k}{r'} + 2\delta_{lk}\frac{r'_m}{r'}\right)\overline{\left(\frac{\ddot{T}_{ij}}{rc^2} + \frac{3\dot{T}_{ij}}{r^2 c} + \frac{3T_{ij}}{r^3}\right)\left(\frac{\dot{T}'_{lm}}{r'^2 c^2} + \frac{3T'_{lm}}{r'^3 c} + \frac{3t'_{lm}}{r'^4}\right)} \qquad [13]$$

$$+ \delta_{ij}\left(\delta_{lm}\frac{r_k'}{r'} + 2\,\delta_{lk}\frac{r_m'}{r'}\right)\overline{\left(\frac{\dot T_{ij}}{r^2 c} + \frac{T_{ij}}{r^3}\right)\left(\frac{\dot T_{lm}'}{r'^2 c^2} + \frac{3\,T_{lm}'}{r'^3 c} + \frac{3\,t_{lm}'}{r'^4}\right)}$$

$$- \delta_{ij}\frac{r_k' r_l' r_m'}{r'^3}\overline{\left(\frac{\dot T_{ij}}{r^2 c} + \frac{T_{ij}}{r^3}\right)\left(\frac{\ddot T_{lm}'}{r' c^3} + \frac{6\,\dot T_{lm}'}{r'^2 c^2} + \frac{15\,T_{lm}'}{r'^3 c} + \frac{15\,t_{lm}'}{r'^4}\right)}\Bigg]\,d\bar y\,d\bar y' \qquad [13]$$

Similarly, the mean-square sound pressure is given by

$$\overline{p^2} = \frac{1}{16\,\pi^2}\iint\Bigg[\frac{r_i r_j r_l' r_m'}{r^2 r'^2}\overline{\left(\frac{\ddot T_{ij}}{r c^2} + \frac{3\,\dot T_{ij}}{r^2 c} + \frac{3\,T_{ij}}{r^3}\right)\left(\frac{\ddot T_{lm}'}{r' c^2} + \frac{3\,\dot T_{lm}'}{r'^2 c} + \frac{3\,T_{lm}'}{r'^3}\right)}$$

$$- \delta_{ij}\frac{r_l' r_m'}{r'^2}\overline{\left(\frac{\dot T_{ij}}{r^2 c} + \frac{T_{ij}}{r^3}\right)\left(\frac{\ddot T_{lm}'}{r' c^2} + \frac{3\,\dot T_{lm}'}{r'^2 c} + \frac{3\,T_{lm}'}{r'^3}\right)}$$

$$- \delta_{lm}\frac{r_i r_j}{r^2}\overline{\left(\frac{\ddot T_{ij}}{r c^2} + \frac{3\,\dot T_{ij}}{r^2 c} + \frac{3\,T_{ij}}{r^3}\right)\left(\frac{\dot T_{lm}'}{r'^2 c} + \frac{T_{lm}'}{r'^3}\right)}$$

$$+ \delta_{ij}\delta_{lm}\overline{\left(\frac{\dot T_{ij}}{r^2 c} + \frac{T_{ij}}{r^3}\right)\left(\frac{\dot T_{lm}'}{r'^2 c} + \frac{T_{lm}'}{r'^3}\right)}\Bigg]\,d\bar y\,d\bar y' \qquad [14]$$

SIMPLIFYING ASSUMPTIONS

These are indeed formidable expressions. A few reasonable assumptions may be made, however, to reduce them to more tractable forms. First, it may be assumed that the turbulence is sufficiently steady over intervals of time required for sound to travel over distances for which the velocity fluctuations are appreciably correlated so that *simultaneous correlations* may be taken at $\bar y$ and $\bar y'$. This means that the wavelength of the sound in question must be long compared to the "eddy size." Observed average eddy sizes are approximately a wavelength for frequencies of the order of ten times the frequency of the peak in observed noise spectra of turbulent jets. Since it is reasonable to assume that smaller than average eddies contribute most to the high-frequency components, the assumption of simultaneous correlations seems to be justified. If, in addition, it is assumed that *r is much greater than the eddy diameter*, then $r' \cong r$.

Another reasonable assumption that permits a considerable reduction of the equations is that the *fluctuating forces and fluctuating stresses are approximately stationary random functions of time*. In other words, the turbulence is assumed to be essentially steady and not decaying appreciably.* This assumption still permits the existence of both gradual space

*Proudman[3] has shown that, for the case of isotropic turbulence, the "decay" terms are indeed negligible in comparison with the "instantaneous" terms.

variations and gradual time variations in the mean values of the turbulent fluctuations. Since the operations of differentiating and averaging permute, this assumption is expressed by the equation[20,21]

$$\overline{\frac{\partial}{\partial t}\,[g(t)]} = \frac{\partial}{\partial t}\,[\overline{g(t)}] = 0 \qquad [15]$$

Therefore, reductions of the following type may be made:

$$\overline{T_{ij}\,\ddot{T}_{lm}'} + \overline{\dot{T}_{ij}\,\dot{T}_{lm}'} = 0 \qquad [16]$$

and

$$\overline{\dot{T}_{ij}\,T_{lm}'} = 0 \qquad [17]$$

SIMPLIFIED GENERAL EXPRESSIONS FOR THE SOUND FIELD

By making the above assumptions, Equations [7], [8], [13], and [14] become

$$I_k = \frac{1}{16\pi^2 \rho_0 c^3} \iint \frac{1}{r^2}\, \frac{r_i r_j r_k}{r^3}\, \overline{\dot{F}_i\,\dot{F}_j'}\, d\bar{y}\, d\bar{y}' \qquad [18]$$

$$\overline{p^2} = \frac{1}{16\pi^2 c^2} \iint \frac{1}{r^2}\, \frac{r_i r_j}{r^2} \left(\overline{\dot{F}_i\,\dot{F}_j'} + \frac{c^2}{r^2}\, \overline{F_i\,F_j'} \right) d\bar{y}\, d\bar{y}' \qquad [19]$$

$$I_k = \frac{1}{16\pi^2 \rho_0 c^5} \iint \frac{1}{r^2} \left[\frac{r_i r_j r_k r_l r_m}{r^5}\, \overline{\ddot{T}_{ij}\,\ddot{T}_{lm}'} + \frac{c^2}{r^2}\, \delta_{ij} \left(\delta_{lm}\frac{r_k}{r} + 2\,\delta_{lk}\frac{r_m}{r} - 5\,\frac{r_k r_l r_m}{r^3} \right) \overline{\dot{T}_{ij}\,\dot{T}_{lm}'} \right] d\bar{y}\, d\bar{y}' \qquad [20]$$

and

$$\overline{p^2} = \frac{1}{16\pi^2 c^4} \iint \frac{1}{r^2} \left[\frac{r_i r_j r_l r_m}{r^4}\, \overline{\ddot{T}_{ij}\,\ddot{T}_{lm}'} + \frac{c^2}{r^2} \left(3\,\frac{r_i r_j r_l r_m}{r^4} - 4\,\delta_{ij}\frac{r_l r_m}{r^2} + \delta_{ij}\,\delta_{lm} \right) \overline{\dot{T}_{ij}\,\dot{T}_{lm}'} + \frac{c^4}{r^4} \left(9\,\frac{r_i r_j r_l r_m}{r^4} - 6\,\delta_{ij}\frac{r_l r_m}{r^2} + \delta_{ij}\,\delta_{lm} \right) \overline{T_{ij}\,T_{lm}'} \right] d\bar{y}\, d\bar{y}' \qquad [21]$$

Integrating the intensities in Equations [18] and [20] over the surface of a sphere gives the sound power radiated. The induction near-field terms in Equation [20] average out to zero over the sphere and thus do not represent radiated sound. The sound power radiated in the two cases is

$$P = \frac{1}{16\pi^2 \rho_0 c^3} \iint \frac{4\pi}{3} \overline{\dot{F}_i \dot{F}_j'} \, d\bar{y} \, d\bar{y}' \qquad [22]$$

and

$$P = \frac{1}{16\pi^2 \rho_0 c^5} \iint \frac{4\pi}{15} \left(\overline{\ddot{T}_{ii} \ddot{T}_{jj}'} + 2 \overline{\ddot{T}_{ij} \ddot{T}_{ij}'} \right) d\bar{y} \, d\bar{y}' \qquad [23]$$

(Equation 21, Reference 1)

To examine the nature of the sound field due to a small volume of turbulence, it is helpful to change to spherical coordinates centered in the small turbulent region. For convenience, θ will be measured from the x_3-axis and ϕ will be measured from the x_1-axis in the $x_1 x_2$-plane. If $r_i \cong x_i$, the following relations obtain for the components in the directions of increasing r, θ, and ϕ, respectively:

$$\begin{array}{lll} \qquad \bar{r} & \qquad \bar{\theta} & \qquad \bar{\phi} \\ \dfrac{r_1}{r} = \sin\theta \cos\phi & \dfrac{\theta_1}{\theta} = \cos\phi \cos\theta & \dfrac{\phi_1}{\phi} = -\sin\phi \\ \dfrac{r_2}{r} = \sin\theta \sin\phi & \dfrac{\theta_2}{\theta} = \sin\phi \cos\theta & \dfrac{\phi_2}{\phi} = \cos\phi \\ \dfrac{r_3}{r} = \cos\theta & \dfrac{\theta_3}{\theta} = -\sin\theta & \dfrac{\phi_3}{\phi} = 0 \end{array} \qquad [24]$$

where $\theta = |\bar{\theta}|$ and $\phi = |\bar{\phi}|$.

These expressions can be used to show that, for the dipole case, the intensity is purely radial; namely,

$$I_r = \frac{1}{16\pi^2 \rho_0 c^3} \iint \frac{1}{r^2} \frac{r_i r_j}{r^2} \overline{\dot{F}_i \dot{F}_j'} \, d\bar{y} \, d\bar{y}' \qquad [25]$$

For the quadrupole case, however, the intensity has components in the direction of increasing r, θ, and ϕ as follows:

$$I_r = \frac{1}{16\pi^2 \rho_0 c^5} \iint \frac{1}{r^2} \left[\frac{r_i r_j r_l r_m}{r^4} \overline{\ddot{T}_{ij} \ddot{T}_{lm}'} \right.$$

$$\left. + \frac{c^2}{r^2} \delta_{ij} \left(\delta_{lm} - 3 \frac{r_l r_m}{r^2} \right) \overline{\dot{T}_{ij} \dot{T}_{lm}'} \right] d\bar{y}\, d\bar{y}'$$

$$I_\theta = \frac{1}{16\pi^2 \rho_0 c^5} \iint \frac{1}{r^2} \frac{2c^2}{r^2} \delta_{ij} \frac{r_m \theta_l}{r\theta} \overline{\dot{T}_{ij} \dot{T}_{lm}'}\, d\bar{y}\, d\bar{y}' \qquad [26]$$

and

$$I_\phi = \frac{1}{16\pi^2 \rho_0 c^5} \iint \frac{1}{r^2} \frac{2c^2}{r^2} \delta_{ij} \frac{r_m \phi_l}{r\phi} \overline{\dot{T}_{ij} \dot{T}_{lm}'}\, d\bar{y}\, d\bar{y}'$$

In order to facilitate their use in actual cases, Equations [19], [21], [25], and [26] have been expanded and are given in Appendix A. Some physical insight is obtained if the equations are examined in their vector form. For this reason, the equations for the mean-square sound pressure and the intensity are given in vector form in Appendix B.

SPECIAL CASES

The general sound field is too complicated to visualize readily, so in order to gain insight into the general sound field a few special cases will now be examined. Four cases will be examined in some detail: (1) single dipole; (2) single lateral quadrupole; (3) single longitudinal quadrupole; and (4) quadrupole radiation from isotropic turbulence. Here the word "single" signifies a distributed multipole of one type and one orientation rather than a point multipole. A discussion of the changes in the spectra as determined by pressure measurements at various distances from these sources will be found on page 17 of this report.

The sound field of a single *dipole* oriented along the x_3-axis will be discussed first. Substituting $i = j = 3$ into Equations [19] and [25] gives

$$\overline{p^2} = \frac{1}{16\pi^2 c^2} \iint \frac{1}{r^2} \cos^2\theta \left(\overline{\dot{F}_3 \dot{F}_3'} + \frac{c^2}{r^2} \overline{F_3 F_3'} \right) d\bar{y}\, d\bar{y}' \qquad [27]$$

and

$$I_r = \frac{1}{16\pi^2 \rho_0 c^3} \iint \frac{1}{r^2} \cos^2\theta\, \overline{\dot{F}_3 \dot{F}_3'}\, d\bar{y}\, d\bar{y}' \qquad [28]$$

The intensity is purely radial and has no induction near-field component.

For a narrow band of frequencies around the angular frequency ω in the spectrum of F_3 or in the sound radiated by the dipole, Equation [27] may be written in the form

$$\overline{p^2} = \frac{1}{16\pi^2c^2}\iint \frac{1}{r^2} \cos^2\theta \, \overline{\dot{F}_3\dot{F}_3'} \left(1 + \frac{c^2}{r^2\omega^2}\right) d\bar{y}\, d\bar{y}' \qquad [29]$$

This indicates that there is an induction near-field term which changes the usual $1/r^2$ variation of the mean-square sound pressure when the square of the quantity, wavelength divided by $2\pi r$, is significant in comparison with one. The sound intensity shows no change in variation with r upon entering the induction near field. In any plane containing the axis of the dipole, the intensity pattern is the familiar "figure eight" of $\cos^2\theta$.

The sound field of a *lateral quadrupole* oriented along the x_1- and x_3-axes will now be examined. Equation [21] gives the mean-square sound pressure for this case to be

$$\overline{p^2} = \frac{4}{16\pi^2c^4}\iint \frac{1}{r^2} \sin^2\theta\cos^2\theta\cos^2\phi\left(\overline{\ddot{T}_{13}\ddot{T}_{13}'} + 3\frac{c^2}{r^2}\overline{\dot{T}_{13}\dot{T}_{13}'} + 9\frac{c^4}{r^4}\overline{T_{13}T_{13}'}\right) d\bar{y}\, d\bar{y}' \qquad [30]$$

Similarly, from Equation [26], the sound intensity for this case is

$$I_r = \frac{4}{16\pi^2\rho_0c^5}\iint\frac{1}{r^2} \sin^2\theta\cos^2\theta\cos^2\phi \, \overline{\ddot{T}_{13}\ddot{T}_{13}'} \, d\bar{y}\, d\bar{y}' \qquad [31]$$

The intensity is purely radial and has no induction near-field components.

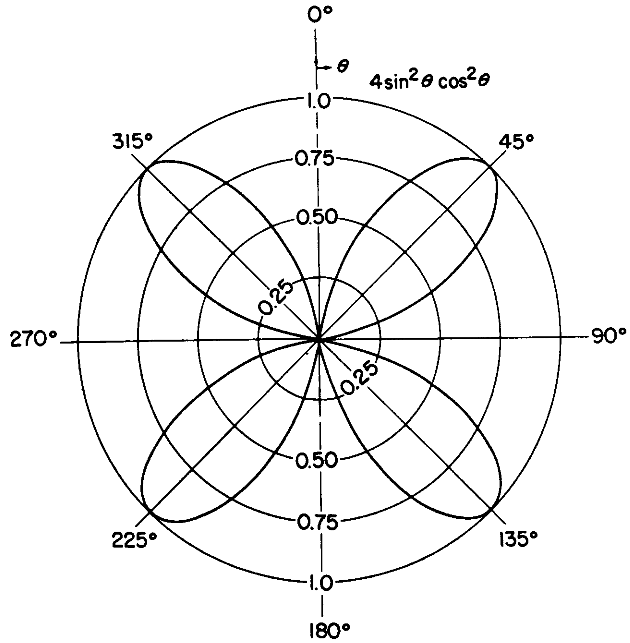
For a narrow band of frequencies around the angular frequency ω in the spectrum of T_{13} or the sound radiated by the lateral quadrupole, Equation [30] may be written as

$$\overline{p^2} = \frac{4}{16\pi^2c^4}\iint \frac{1}{r^2} \overline{\ddot{T}_{13}\ddot{T}_{13}'} \sin^2\theta\cos^2\theta\cos^2\phi \left(1 + 3\frac{c^2}{r^2\omega^2} + 9\frac{c^4}{r^4\omega^4}\right) d\bar{y}\, d\bar{y}' \qquad [32]$$

The polar patterns of all terms in the mean-square sound pressure and the intensity are the same for the lateral quadrupole. In the x_1x_3-plane the pattern has the "clover leaf" shape shown in Figure 2. On the surface of any cone of constant θ the pattern is a "figure-eight" shape. The three-dimensional pattern may be described as four "beaver tails."

The sound field of a *longitudinal quadrupole* oriented along the x_3-axis will be examined next. The mean-square sound pressure, by substituting $i = j = l = m = 3$ in Equation [21], is

Figure 2 – Directional Pattern of Intensity and All Components of Mean-Square Sound Pressure in Plane of the Two Axes of a Lateral Quadrupole



$$\overline{p^2} = \frac{1}{16\pi^2 c^4}\iint \frac{1}{r^2}\left[\cos^4\theta\,\overline{\ddot{T}_{33}\,\ddot{T}'_{33}} + \frac{c^2}{r^2}\,(3\cos^4\theta - 4\cos^2\theta + 1)\,\overline{\dot{T}_{33}\,\dot{T}'_{33}} + \frac{c^4}{r^4}(9\cos^4\theta - 6\cos^2\theta + 1)\,\overline{T_{33}\,T'_{33}}\right] d\bar{y}\,d\bar{y}' \qquad [33]$$

Similarly, from Equation [26], the sound intensity for this case is

$$I_r = \frac{1}{16\pi^2\rho_0 c^5}\iint \frac{1}{r^2}\left[\cos^4\theta\,\overline{\ddot{T}_{33}\,\ddot{T}'_{33}} + \frac{c^2}{r^2}\,(1 - 3\cos^2\theta)\,\overline{\dot{T}_{33}\,\dot{T}'_{33}}\right] d\bar{y}\,d\bar{y}'$$

$$I_\theta = \frac{1}{16\pi^2\rho_0 c^5}\iint \frac{1}{r^2}\left(-2\,\frac{c^2}{r^2}\,\sin\theta\cos\theta\,\overline{\dot{T}_{33}\,\dot{T}'_{33}}\right) d\bar{y}\,d\bar{y}' \qquad [34]$$

and

$$I_\phi = 0$$

For a narrow band of frequencies around the angular frequency ω in the spectrum of T_{33} or the sound radiated by this longitudinal quadrupole, these equations may be written:

$$\overline{p^2} = \frac{1}{16\pi^2 c^4} \iint \frac{1}{r^2} \; \overline{\ddot{T}_{33}\,\ddot{T}_{33}'} \left[\cos^4\theta + \frac{c^2}{r^2\omega^2}\,(3\cos^4\theta - 4\cos^2\theta + 1) + \frac{c^4}{r^4\omega^4}\,(9\cos^4\theta - 6\cos^2\theta + 1)\right] d\bar{y}\, d\bar{y}' \qquad [35]$$

$$I_r = \frac{1}{16\pi^2\rho_0 c^5} \iint \frac{1}{r^2} \; \overline{\ddot{T}_{33}\,\ddot{T}_{33}'} \left[\cos^4\theta + \frac{c^2}{r^2\omega^2}\,(1 - 3\cos^2\theta)\right] d\bar{y}\, d\bar{y}'$$

$$I_\theta = \frac{1}{16\pi^2\rho_0 c^5} \iint \frac{1}{r^2} \; \overline{\ddot{T}_{33}\,\ddot{T}_{33}'} \left[\frac{c^2}{r^2\omega^2}\,(-2\sin\theta\cos\theta)\right] d\bar{y}\, d\bar{y}' \qquad [36]$$

and

$$I_\phi = 0$$

If three equal, mutually orthogonal, longitudinal quadrupoles are combined, the mean-square sound pressure is

$$\overline{p^2} = \frac{1}{16\pi^2 c^4} \iint \frac{1}{r^2} \; \frac{1}{9} \; \overline{\ddot{T}_{ii}\,\ddot{T}_{ll}'} \; d\bar{y}\, d\bar{y}' \qquad [37]$$

As was pointed out by Lighthill,[1] these represent the equivalent applied fluctuating pressures in the turbulent region. The induction near-field terms in the intensity also disappear.

Figure 3 gives the polar patterns for the three terms in Equation [35] for all planes containing the x_3-axis. The two induction near-field terms have zeros when $\theta = \cos^{-1}\frac{1}{\sqrt{3}} \cong 55$ deg. The middle term has a negative value for $\cos^2\theta > 1/3$ but at no distance or angle from the quadrupole does this make $\overline{p^2}$ negative. It may be seen that the induction near-field terms always predominate at $\theta = 90$ deg and the far-field term always predominates at $\theta \cong 55$ deg.

The polar patterns of the intensity due to a single longitudinal quadrupole are given in Figure 4, where the arrows indicate magnitude and direction. The far-field intensity is purely radial. The term for the induction near-field intensity, however, indicates a flow of sound energy out perpendicular to the axis and in again along the axis of the quadrupole, giving no additional net outflow of energy or power radiated. The induction near-field intensity is perpendicular to the surface of the cone generated by the $\theta \cong 55$-deg line.

The sound field of a region of *isotropic turbulence* shows no angular variation, but the induction near-field effects are still present. The assumption of isotropy permits the elimination of many of the terms in such quantities as $\overline{\ddot{T}_{ij}\,\ddot{T}_{lm}'}$. To affect the simplification, it will

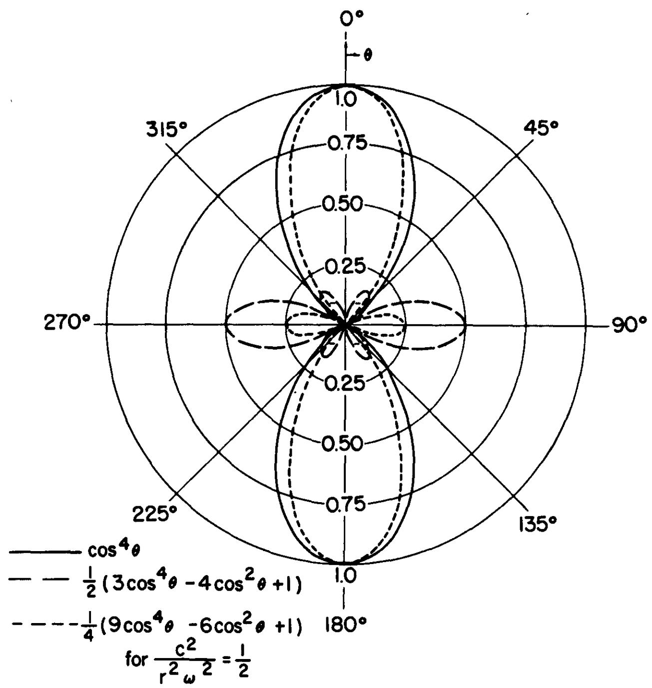


Figure 3 – Directional Patterns of Three Components of Mean-Square Sound Pressure in All Planes Containing the Axis of a Longitudinal Quadrupole

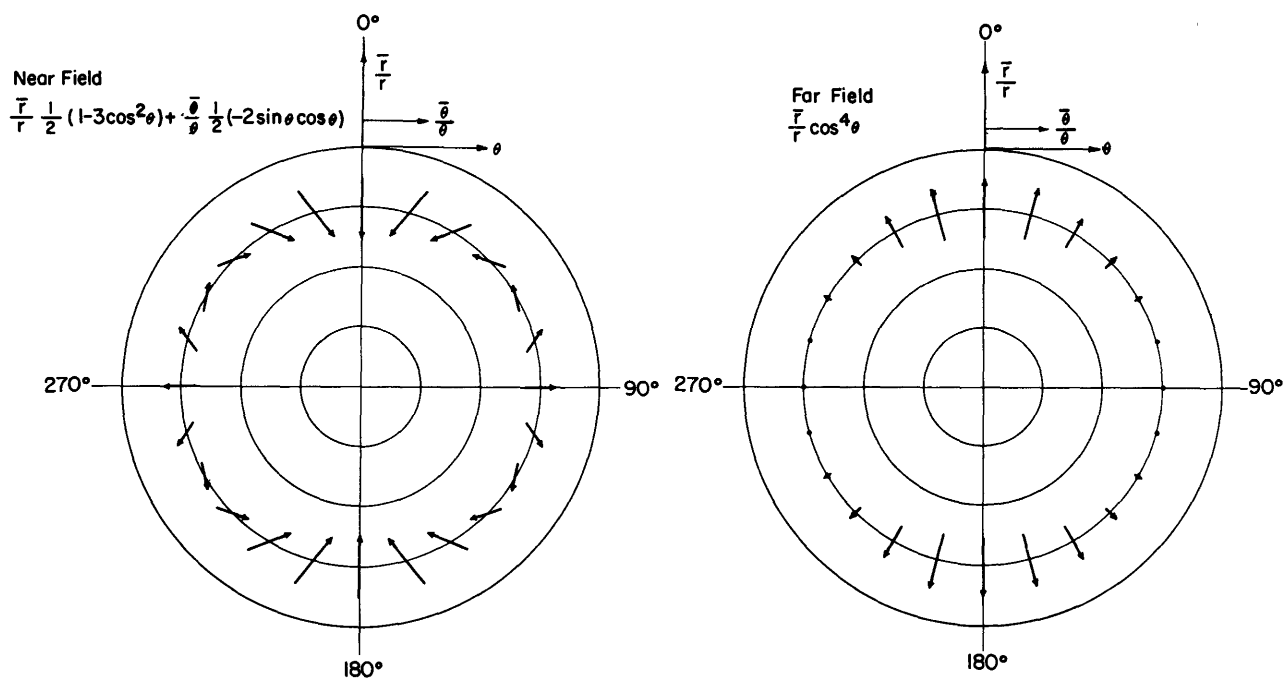


Figure 4a

Figure 4b

Figure 4 – The Induction Near- and Far-Field Components of the Intensity Field in All Planes Containing the Axis of a Longitudinal Quadrupole

Arrows indicate magnitude and direction.

be assumed that:

$$T_{ij} = \rho_0 u_i\,(\bar{y},\, t - r/c)\; u_j\,(\bar{y},\, t - r/c) \equiv \rho_0 u_i u_j \qquad [38]$$

$$\overline{u_i u_l'} = 0 \text{ if } i = l \quad \text{(isotropic turbulence)}^{22} \qquad [39]$$

$$\overline{u_i u_j u_l' u_m'} = \overline{u_i u_j}\;\overline{u_l' u_m'} + \overline{u_i u_l'}\;\overline{u_j u_m'} + \overline{u_i u_m'}\;\overline{u_j u_l'} \qquad [40]$$

(normal probability distribution)[23]

and

$$\overline{u_i \ddot{u}_i'} = -\overline{\dot{u}_i \dot{u}_i}, \quad \overline{u_i \dot{u}_i'} = 0 \text{ etc.} \qquad [41]$$

(stationary random functions of time)

Also, let $\bar{u} \cdot \bar{r}/r = u$ be the component of the turbulent velocity fluctuation $\bar{u}$ in the $\bar{r}$-direction at point $\bar{y}$ and at retarded time $(t - r/c)$. The velocity u is also, on the average, equal to the component of $\bar{u}$ in any other direction.

By using these equations, Equation [21] reduces to

$$\overline{p^2} = \frac{\rho_0^2}{16\,\pi^2 c^4} \iint \frac{4}{r^2} \left\{ \left[\overline{uu'}\;\overline{\ddot{u}\ddot{u}'} + 3\,(\overline{\dot{u}\dot{u}'})^2 \right] + 2\,\frac{c^2}{r^2}\;\overline{uu'}\;\overline{\dot{u}\dot{u}'} + 3\,\frac{c^4}{r^4}\,(\overline{uu'})^2 \right\} d\bar{y}\; d\bar{y}' \qquad [42]$$

Similarly, the intensity from Equation [26] reduces to

$$I_r = \frac{\rho_0^2}{16\,\pi^2 \rho_0 c^5} \iint \frac{4}{r^2} \left[\overline{uu'}\;\overline{\ddot{u}\ddot{u}'} + 3\,(\overline{\dot{u}\dot{u}'})^2 \right] d\bar{y}\; d\bar{y}' \qquad [43]$$

The θ and ϕ components of the intensity are zero. Equation [43] corresponds to the result of Proudman, Equations 4.5 and 5.1,[3] with the "decay" terms neglected.

As was pointed out by Lighthill,[1] the spectrum of the sound radiated is different from that of the velocity fluctuations because the radiated sound pressure depends on the square of the velocity fluctuations. Therefore, the sound pressure will include sum and difference tones of the velocity frequencies and, in general, will have a flatter spectrum. If the velocity fluctuations are confined to a narrow band of frequencies around the angular frequency ω^* one might approximate the effect of sum and difference frequencies by assuming that the angu- frequency ω of the radiated sound is approximately equal to $2\,\omega^*$. The mean-square sound pressure and intensity under this assumption then reduce to

$$\overline{p^2} \cong \frac{\rho_0^2}{16\,\pi^2 c^4}\iint \frac{1}{r^2}\ \omega^4\,(\overline{uu'})^2\left(1 + 2\,\frac{c^2}{r^2\omega^2} + 12\,\frac{c^4}{r^4\omega^4}\right)d\bar{y}\,d\bar{y}' \tag{44}$$

or

$$\overline{p^2} \cong \frac{\rho_0^2}{16\,\pi^2 c^4}\iint \frac{16}{r^2}\ (\overline{\ddot{u}\ddot{u}'})^2\left(1 + 2\,\frac{c^2}{r^2\omega^2} + 12\,\frac{c^4}{r^4\omega^4}\right)d\bar{y}\,d\bar{y}' \tag{45}$$

and

$$I_r \cong \frac{\rho_0^2}{16\,\pi^2 \rho_0 c^5}\iint \frac{16}{r^2}\ (\overline{\ddot{u}\ddot{u}'})^2\,d\bar{y}\,d\bar{y}' \tag{46}$$

As one would expect, there is no angular variation in the sound field, but the mean-square sound pressure has two induction near-field terms. This sound field turns out to be equivalent to the individual contributions of a longitudinal quadrupole in each of the coordinate directions and two lateral quadrupoles in each of the pairs of coordinate directions.

The modifications in the equations required when the flow is analyzed with respect to a moving frame have been discussed by Lighthill[2] and Phillips[4] for the far-field radiation. Similar modifications may be made for the general sound field. Because of the complexity of the equations, however, this has been done only for the instantaneous and mean-square sound pressures. These will be found in Appendix C.

The utility of the foregoing theoretical results depends on the ability of observers to estimate certain, up to now, unmeasured quantities. These estimates may be made more realistic by using a number of useful concepts and by keeping in mind the special properties of the near-sound field. These concepts will be discussed in the next section for a few practical applications.

DISCUSSION

CORRELATION VOLUME AND CORRELATION AREA

For purposes of estimating some of the integrals introduced previously, the concepts of correlation volume and correlation area are useful.[1,2,4,5] The correlation volume and correlation area as introduced by Lighthill refer to an average eddy volume and area inside of which the fluctuating quantities are considered correlated and outside of which the fluctuating quantities are considered uncorrelated. The correlation volume and correlation area may be given, respectively, as

$$V = \int \frac{\overline{T_{ij}\,T_{lm}'}}{\overline{T_{ij}\,T_{lm}}}\,d\bar{y}' \tag{47}$$

and

$$S = \int \frac{\overline{F_i F_j'}}{\overline{F_i F_j}} dS(\bar{y}') \qquad [48]$$

These expressions also hold for the time derivatives of the quantities under the integrals. Since eddies of different sizes have different frequency scales, V and S defined for a narrow band of frequencies would be expected to be functions of the frequency. One would expect V and S to be smaller for the higher frequencies; that is, that V would be approximately proportional to the inverse cube of the frequency and S approximately proportional to the inverse square of the frequency. This exact proportionality, of course, neglects the effects of sum-and-difference frequencies in the velocity fluctuations and ignores the fact that time fluctuations are not synonymous with the space variations that are usually measured. The frequency dependence of V and S partially offsets the higher efficiencies of quadrupole and dipole radiation at the higher frequencies. As a result, the frequency of the peak in the sound spectrum should be about the same as the frequency of the peak in the spectrum of the fluctuating velocities or fluctuating forces. The correlation volume and correlation area might also depend on the particular velocity components and force components used in the integrals. In spite of these uncertainties, the effect of integrating over $d\bar{y}'$ may be approximated by replacing $d\bar{y}'$ by V or S and removing the primes from the integrals that give the mean-square sound pressure and intensity.

MEAN-SQUARE SOUND PRESSURE IN TERMS OF SOUND POWER RADIATED

The mean-square sound pressure at any point in the sound field outside the geometric near field of like-oriented dipoles or quadrupoles may also be given in terms of the sound power P radiated by the multipoles. For the four special cases discussed previously (namely, dipoles, lateral quadrupoles, longitudinal quadrupoles, and a region of isotropic turbulence), the mean-square sound pressure is given, respectively, by

$$\overline{p^2} = \frac{\rho_0 c P}{4\pi r^2} (3\cos^2\theta) \left(1 + \frac{c^2}{r^2\omega^2}\right) \qquad [49]$$

$$\overline{p^2} = \frac{\rho_0 c P}{4\pi r^2} (15\sin^2\theta\cos^2\theta\cos^2\phi) \left(1 + 3\frac{c^2}{r^2\omega^2} + 9\frac{c^4}{r^4\omega^4}\right) \qquad [50]$$

$$\overline{p^2} = \frac{\rho_0 c P}{4\pi r^2} (5\cos^4\theta) \left[1 + \frac{c^2}{r^2\omega^2}\left(3 - \frac{4}{\cos^2\theta} + \frac{1}{\cos^4\theta}\right) + \frac{c^4}{r^4\omega^4}\left(9 - \frac{6}{\cos^2\theta} + \frac{1}{\cos^4\theta}\right)\right] \qquad [51]$$

$$\overline{p^2} = \frac{\rho_0 c P}{4\pi r^2}\left(1 + 2\frac{c^2}{r^2\omega^2} + 12\frac{c^4}{r^4\omega^4}\right) \qquad [52]$$

The quantity $\frac{\rho_0 c P}{4\pi r^2}$ in each case gives the mean-square sound pressure at all points in the sound field of an equivalent simple source. The terms that depend on θ and ϕ indicate the variations of $\overline{p^2}$ from the average over all directions. The last bracket in each equation gives the induction near-field correction to the radiated mean-square sound pressure.

SLOPE OF FREQUENCY SPECTRUM

It is apparent that the effect of the induction near-field sound pressure is to enhance the low-frequency components in the sound spectrum as it appears in the far field. Therefore, the sound pressure spectrum determined from data on the induction near field will differ from the spectrum determined from data on the far field. The effect of the induction near field is to decrease the slope of the spectrum by 6 db/octave in the case of dipoles and 12 db/octave in the case of quadrupoles. For a narrow band of frequencies the approximate far-field sound pressure level may be obtained from the induction near-field sound pressure level by subtracting the quantity

$$10 \log \left(1 + a_1 \frac{c^2}{r^2\omega^2} + a_2 \frac{c^4}{r^4\omega^4}\right) \qquad [53]$$

in each case using a mean frequency of the band and an average distance r. The quantities a_1 and a_2 are to be taken from the last bracket term in Equations [49], [50], [51], and [52] for the four cases discussed. The application of these equations to the study of the sound from turbulent jets and turbulent boundary layers will now be discussed.

SPECIAL CASES

The case of a *turbulent jet* is of special interest because of its widespread application. The sound produced by a jet is usually considered to consist of two parts: (1) from a cylindrical region of strong shear near the exit of the jet; and (2) from the core of the jet and a larger mixing region downstream. The sound from the shear layer is predominantly from lateral quadrupoles with axes along the jet and perpendicular to the shear layer of the jet. Thus, if θ is measured from the axis of a circular jet and if the observation point is at least several jet diameters from the jet, the term $\cos^2\phi$ may be replaced by its average, namely ½, in the equations pertaining to lateral quadrupoles. As a first approximation, it may be assumed that the sound from the core and the downstream part of the turbulent region of the jet is given by the equations for the quadrupole radiation from isotropic turbulence.

To obtain reasonable corrections for data taken in the near-sound field of a jet, an assumed distribution of quadrupoles along the jet is integrated with respect to varying distance and angle from the measuring point. This procedure automatically includes the effects of the geometric near field, the induction near field, and the angular dependence of the quadrupoles. To a first approximation an average distance and angle may be used. For high convection Mach numbers it may be necessary to use the equations in Appendix C.

For the typical sound spectrum of the far-field sound of an air jet, given by the solid curve in Figure 5, the approximate spectrum expected from data taken at a distance of 1 ft from the jet is indicated by the dashed curve in the figure. The frequency corresponding to a 1-ft wavelength in air is about 1100 cps. The eventual change in slope at the lower frequencies is 12 db/octave, but the change in slope in the transition region between the near field and the far field is somewhat indeterminate because of the assumptions made as to the average distance r and the types of quadrupoles. These estimates can be made sufficiently accurate, however, to enable one to make reasonable corrections to data taken in the near-sound field.

The case of sound from a *turbulent boundary layer* is also of interest. Aspects of this problem have been discussed by Lighthill,[24] Powell,[5] Phillips,[4] and Curle.[7] Curle has shown, in general, how dipole radiation results when a solid body or boundary is present in the turbulent field. For curved boundaries dipoles with axes perpendicular to the boundary arise from the fluctuating pressures on the boundary, and dipoles with axes parallel to the boundary arise from the fluctuating shear forces. The dipoles from the fluctuating forces exerted by a plane boundary on a fluid have been shown by Phillips to have their axes parallel to the boundary. If these dipoles are assumed to be of approximately equal strength in the two coordinate directions in the surface and if, for convenience, the origin of coordinates is

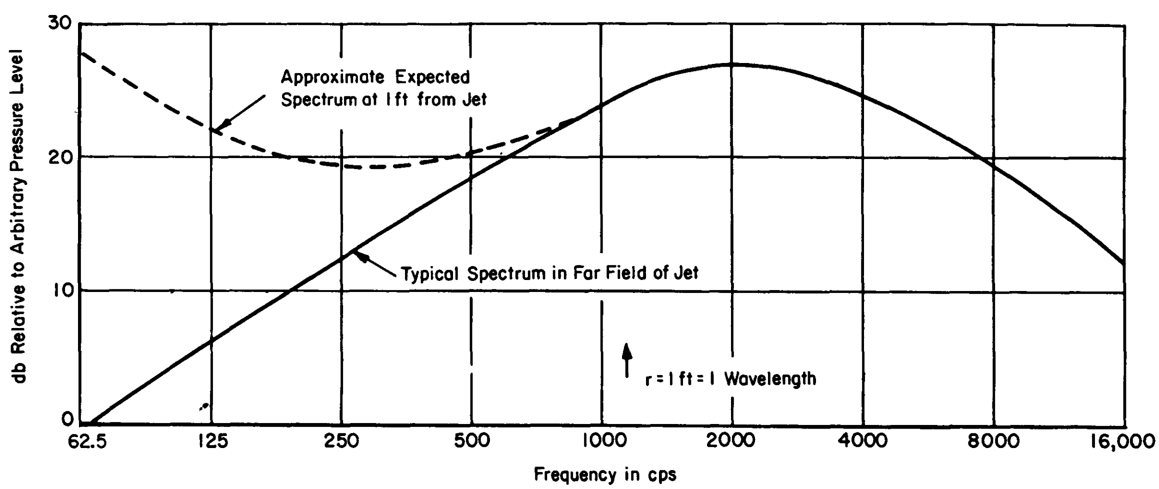


Figure 5 – Typical Air Jet Noise Spectrum and Approximate Changes that Occur if Data Are Taken at an Average Distance of 1 Foot from the Jet

located at the measuring point, the contributions from a given area of uniform boundary layer may be easily found by integration. For instance, by using Phillips' tentative estimate for an x-Reynolds number of approximately 10^8 (sound power radiated per unit area equals approximately $4(10^{-7})\,\rho_0 U^3 M_0^3$) and by using Equation [49] with $P(3\cos^2\theta)\left(1+\frac{c^2}{r^2\omega^2}\right)/r^2$ replaced by

$$\int_{\theta_1'}^{\theta_2'} \frac{8(10^{-7})\rho_0 U^3 M_0^3 \left(\frac{3}{2}\sin^2\theta'\right)\cos^2\theta'}{h^2}\left(1+\frac{c^2\cos^2\theta'}{h^2\omega^2}\right)2\pi h^2\,\frac{\sin\theta'}{\cos^3\theta'}\,d\theta'$$

the broad-band mean-square sound pressure a perpendicular distance h from the center of a circular ring area between $\theta' = \theta_1'$ and $\theta' = \theta_2'$ is

$$\overline{p^2} \cong 6(10^{-7})\rho_0^2 U^4 M_0^2 \left[-\ln\left(\frac{\cos\theta_2'}{\cos\theta_1'}\right)+\frac{1}{2}(\cos^2\theta_2' - \cos^2\theta_1') + \frac{c^2}{4\,\omega^2 h^2}(\sin^4\theta_2' - \sin^4\theta_1')\right] \quad [54]$$

where $M_0 = U/c$,

U is the free-stream velocity, and

θ' is the angle with the perpendicular to the plane boundary.

Phillips has shown further, however, that the dipole contribution is negligible if the boundary layer is homogeneous in layers parallel to the boundary. Thus, for this condition the predominant sound would be from quadrupoles in the surface and in the turbulent boundary layer. A procedure similar to the above might be used to estimate the contributions of these quadrupoles. The quadrupole radiation from the turbulence proper might be treated approximately as contributions from isotropic turbulence outside the shear layer and from lateral quadrupoles in the shear layer just outside the laminar sublayer. Further, the effects of a convection velocity may be included by using the equations in Appendix C.

The above discussion of the noise from a turbulent boundary layer holds only for points in the fluid outside rigid bodies. In the case of nonrigid bodies, motion of the boundary, induced by the fluctuating pressure field on it, will create a system of simple sound sources over the boundary and will permit propagation of dipole and quadrupole sound through the boundary. A thin pliant boundary with turbulent fluid motion along one side and no net fluid motion along the other side, for instance, might radiate simple source, dipole, and quadrupole sound both into the moving fluid and into the motionless fluid, with the multipole sound radiated into the motionless fluid modified by the transmission properties of the boundary. The simple sources do not have an induction near field but they can have a geometric near field. The transmission and reflection properties of a nonrigid boundary in the induction near field of dipoles and quadrupoles are, of course, not as simple as for a plane sound wave. Because of their greater efficiency, the simple sources should predominate at low Mach numbers for many actual cases of turbulence on nonrigid boundaries. Any induction near-field correction thus will depend significantly on the relative contributions of the sources, dipoles, and quadrupoles in the boundary layer. To facilitate the determination of these relative contributions, the effects of the Mach number and other factors should be considered.

DIMENSIONLESS RATIOS

A little dimensional analysis facilitates the understanding of some of the properties of the induction near-sound field. By using a typical velocity U and a typical linear dimension L in the flow, as in Reference 1, and by ignoring the numerical factors, the expressions for the *ratio of the mean-square sound pressure to the square of the typical Bernoulli pressure* for the source, dipole, and quadrupole, respectively, may be written

$$\frac{\overline{p^2}}{\left(\frac{1}{2}\rho_0 U^2\right)^2} \sim \frac{L^2}{r^2} \qquad [55]$$

$$\frac{\overline{p^2}}{\left(\frac{1}{2}\rho_0 U^2\right)^2} \sim \frac{L^2}{r^2} M_0^2 + \frac{L^4}{r^4} \qquad [56]$$

and

$$\frac{\overline{p^2}}{\left(\frac{1}{2}\rho_0 U^2\right)^2} \sim \frac{L^2}{r^2} M_0^4 + \frac{L^4}{r^4} M_0^2 + \frac{L^6}{r^6} \qquad [57]$$

These expressions indicate that very near the turbulent region, where L/r is of the order of one or more, the root-mean-square sound pressure is of the order of the typical Bernoulli pressure in the flow itself and independent of the Mach number at low Mach numbers. This also indicates that very near the turbulent region the density changes are of the order of the density changes in the flow itself.

It is of interest also to know the *ratio of the absolute value of the fluctuating Bernoulli pressure to the absolute value of the sound pressure* at any point in the field of sources, dipoles, and quadrupoles. In the far field one can show that this ratio is always equal to the ratio of the particle velocity to twice the sound velocity or

$$\left|\frac{\frac{1}{2}\rho_0 v^2}{p}\right| = \frac{|\int K \, d\bar{y}|}{8\pi\rho_0 r c^2} \qquad [58]$$

for a source,

$$\left|\frac{\frac{1}{2}\rho_0 v^2}{p}\right| = \frac{|\int \dot{F}_3\, d\bar{y}|\cos\theta}{8\pi\rho_0 r c^3} \qquad [59]$$

for a dipole,

$$\left|\frac{\frac{1}{2}\rho_0 v^2}{p}\right| = \frac{|\int \ddot{T}_{13}\, d\bar{y}|\sin\theta\cos\theta\cos\phi}{8\pi\rho_0 r c^4} \qquad [60]$$

for a lateral quadrupole, and

$$\left|\frac{\frac{1}{2}\rho_0 v^2}{p}\right| = \frac{|\int \ddot{T}_{33}\, d\bar{y}|\cos^2\theta}{8\pi\rho_0 r c^4} \qquad [61]$$

for a longitudinal quadrupole, where $K \equiv K(\bar{y}, t-r/c)$ is the "mass rate density" or acoustic source strength per unit volume of the sources.

In the induction near field the ratio $\left|\frac{1}{2}\rho_0 v^2/p\right|$ is equal to the ratio of the particle displacement to twice the radial distance from the multipole, with an additional angular dependente for the higher-order multipoles. In the induction near field of the four cases above, the ratios are, respectively:

$$\left|\frac{\frac{1}{2}\rho_0 v^2}{p}\right| = \frac{|\int K\, d\bar{y}|}{8\pi\rho_0 \omega^2 r^3} \qquad [62]$$

$$\left|\frac{\frac{1}{2}\rho_0 v^2}{p}\right| = \frac{|\int \dot{F}_3\, d\bar{y}|}{8\pi\rho_0 \omega^3 r^4}\ \frac{(1+3\cos^2\theta)}{|\cos\theta|} \qquad [63]$$

$$\left|\frac{\frac{1}{2}\rho_0 v^2}{p}\right| = \frac{|\int \ddot{T}_{13}\, d\bar{y}|}{8\pi\rho_0 \omega^4 r^5}\ \frac{3\left[9\sin^2\theta\cos^2\theta\cos^2\phi + \cos^2\phi(\cos^2\theta - \sin^2\theta)^2 + \cos^2\theta\sin^2\phi\right]}{|\sin\theta\cos\theta\cos\phi|}$$

[64]

$$\left| \frac{\frac{1}{2}\rho_0 v^2}{p} \right| = \frac{|\int \ddot{T}_{33}\, d\bar{y}|}{8\pi \rho_0 \omega^4 r^5} \frac{9[(1-3\cos^2\theta)^2 + 4\cos^2\theta \sin^2\theta]}{|3\cos^2\theta - 1|} \tag{65}$$

In general, the sound pressure from randomly oriented multipoles predominates over the fluctuating Bernoulli pressure in the induction near field as long as the magnitudes of the particle motions induced by the multipoles are small compared to the distance from the multipoles.

The dependence of the *ratio of the sound power radiated to the mechanical power dissipated* (the so-called "efficiency of conversion of mechanical power to sound power") on the Mach number and other factors may also be used to estimate the relative contributions of sources, dipoles, and quadrupoles. Except for secondary effects, this efficiency is proportional to M_0 for sources, M_0^3 for dipoles, and M_0^5 for quadrupoles.

For jets the efficiency of conversion of mechanical power to sound power has been defined in terms of the total mechanical power of the jet. The efficiency of conversion to sound for a boundary layer, however, is more conveniently expressed in terms of the time rate of energy dissipation by the boundary layer; namely, the drag times the free-stream velocity. In terms of the drag coefficient c_f, the efficiency of conversion to sound for the dipole and quadrupole radiation from a boundary layer may be written, respectively:[4]

$$\eta_d = \frac{\text{acoustic power per unit area}}{\frac{1}{2}\rho_0 U^3 c_f} = M_0^3 g_d(R, M_0) \tag{66}$$

$$\eta_q = \frac{\text{acoustic power per unit area}}{\frac{1}{2}\rho_0 U^3 c_f} = M_0^5 g_q(R, M_0) \tag{67}$$

Thus, in ranges of conditions in which each multipole source predominates, data may be consolidated and η_d and η_q evaluated. These efficiencies would be expected to show secondary Mach number effects and Reynolds number dependences through the functions $g_d(R, M_0)$ and $g_q(R, M_0)$, especially at low Reynolds numbers.

For a uniform boundary layer, the rate of energy dissipation by the layer should be a more realistic standard than the total power of the jet as used to compare jet noise data. Thus, the efficiency η_q would not be expected to be the same as the acoustic efficiency as defined for a jet. The presence of high viscous dissipation of energy very near the boundary would tend to make η_q lower than jet efficiencies. But the fact that much of the energy dissipated in a jet is in a region with lower Mach number than that used to define the jet sound conversion efficient, would tend to make η_q higher than jet efficiencies. The separation of the contributions of the fluctuating forces and the fluctuating stresses and the evaluation of

η_d and η_q may be facilitated by using the special properties of the induction near-sound field of dipoles and quadrupoles.

SUMMARY

In summary, the near-field properties of the general sound field of dipoles, lateral quadrupoles, and longitudinal quadrupoles, as well as isotropic turbulence, have been discussed for various assumed conditions. The general expressions for the rms sound pressure are expected to be the most useful in practical applications. These applications would include the analysis of internal noise and structural loads caused by jets, propellers, and the boundary layer of aircraft and ships; the analysis of surface and turbulence noise in the close vicinity of wind tunnels and water tunnels; and the analysis of surface and turbulence noise in the vicinity of the boundary layer of objects moving in air or water. To facilitate the use of the equations in these applications, a number of useful concepts are also given.

APPENDIX A

EXPANDED FORM OF EQUATIONS FOR MEAN-SQUARE SOUND PRESSURE AND INTENSITY

Equations [19], [25], [21], and [26], respectively, may be expanded by letting i, j, l, and m take on the integral values of 1, 2, and 3 in all combinations and summing the terms. These equations then become

$$\overline{p^2} = \frac{1}{16\,\pi^2 c^2} \iint \frac{1}{r^2} \left(\overline{\dot{A}\dot{A}'} + \frac{c^2}{r^2}\, \overline{AA'} \right) d\bar{y}\; d\bar{y}' \qquad [68]$$

$$I_r = \frac{1}{16\,\pi^2 \rho_0 c^3} \iint \frac{1}{r^2}\, \overline{\dot{A}\dot{A}'}\, d\bar{y}\; d\bar{y}'$$

$$I_\theta = I_\phi = 0 \qquad [69]$$

$$\overline{p^2} = \frac{1}{16\,\pi^2 c^4} \iint \frac{1}{r^2} \left\{ \overline{\ddot{B}\ddot{B}'} + \frac{c^2}{r^2} \left[3\,\overline{\dot{B}\dot{B}'} + \overline{\dot{D}\,(\dot{D}' - 4\,\dot{B}')} \right] + \frac{c^4}{r^4} \left[9\,\overline{BB'} + \overline{D\,(D' - 6\,B')} \right] \right\} d\bar{y}\; d\bar{y}'$$

[70]

$$I_r = \frac{1}{16\,\pi^2 \rho_0 c^5} \iint \frac{1}{r^2} \left[\overline{\ddot{B}\ddot{B}'} + \frac{c^2}{r^2}\, \overline{\dot{D}\,(\dot{D}' - 3\,\dot{B}')} \right] d\bar{y}\; d\bar{y}'$$

$$I_\theta = \frac{1}{16\,\pi^2 \rho_0 c^5} \iint \frac{1}{r^2}\; 2\,\frac{c^2}{r^2}\, \overline{\dot{D}\,(\dot{E}' + \dot{F}')}\, d\bar{y}\; d\bar{y}' \qquad [71]$$

and

$$I_\phi = \frac{1}{16\,\pi^2 \rho_0 c^5} \iint \frac{1}{r^2}\; 2\,\frac{c^2}{r^2}\, \overline{\dot{D}\,(\dot{G}' + \dot{H}')}\, d\bar{y}\; d\bar{y}'$$

where

$$A = \sin\theta \cos\phi\, F_1 + \sin\theta \sin\phi\, F_2 + \cos\theta\, F_3 \qquad [72]$$

$$B = \sin^2\theta \cos^2\phi\, T_{11} + \sin^2\theta \sin^2\phi\, T_{22} + \cos^2\theta\, T_{33}$$

$$+\, 2\,(\sin^2\theta \sin\phi \cos\phi\, T_{12} + \sin\theta \cos\theta \cos\phi\, T_{13} + \sin\theta \cos\theta \sin\phi\, T_{23} \qquad [73]$$

$$D = T_{11} + T_{22} + T_{33} \quad [74]$$

$$E = \sin\theta \cos\theta \cos^2\phi \; T_{11} + \sin\theta \cos\theta \sin^2\phi \; T_{22} - \sin\theta \cos\theta \; T_{33} \quad [75]$$

$$F = 2 \sin\theta \cos\theta \sin\phi \cos\phi \, T_{12} - (1 - 2\cos^2\theta) \cos\phi \, T_{13} - (1 - 2\cos^2\theta) \sin\phi \, T_{23} \quad [76]$$

$$G = -\sin\theta \cos\phi \sin\phi \; T_{11} + \sin\theta \cos\phi \sin\phi \; T_{22} \quad [77]$$

and

$$H = \sin\theta \, (1 - 2\sin^2\phi) \; T_{12} - \cos\theta \sin\phi \; T_{13} + \cos\theta \cos\phi \; T_{23} \quad [78]$$

It may be seen that, in general, terms involving interaction between lateral and longitudinal quadrupoles, longitudinal quadrupoles in different directions, and lateral quadrupoles in different directions, are present.

APPENDIX B

VECTOR FORM OF EQUATIONS FOR MEAN-SQUARE SOUND PRESSURE AND INTENSITY

By defining the fluctuating force vector $\overline{F}$ by the relation $F_i F_i = \overline{F} \cdot \overline{F}$, Equations [19] and [25] may be written in the vector form

$$\overline{p^2} = \frac{1}{16\,\pi^2 c^2} \iint \frac{1}{r^2} \left[\overline{\left(\frac{\partial}{\partial t}\,\frac{\overline{F}\cdot\overline{r}}{r}\right)\left(\frac{\partial}{\partial t}\,\frac{\overline{F}'\cdot\overline{r}}{r}\right) + \frac{c^2}{r^2}\,\frac{\overline{F}\cdot\overline{r}}{r}\,\frac{\overline{F}'\cdot\overline{r}}{r}} \right] d\overline{y}\, d\overline{y}' \qquad [79]$$

and

$$I_r = \frac{1}{16\,\pi^2 \rho_0 c^3} \iint \frac{1}{r^2}\, \overline{\left(\frac{\partial}{\partial t}\,\frac{\overline{F}\cdot\overline{r}}{r}\right)\left(\frac{\partial}{\partial t}\,\frac{\overline{F}'\cdot\overline{r}}{r}\right)}\; d\overline{y}\, d\overline{y}' \qquad [80]$$

By using Equation [38] and the relation $u_i u_i = \overline{u} \cdot \overline{u}$, Equations [21], [20], and [26] may be written in the form

$$\begin{aligned}
\overline{p^2} = \frac{\rho_0^2}{16\,\pi^2 c^4} \iint \frac{1}{r^2} \Bigg\{ & \overline{\left[\frac{\partial^2}{\partial t^2}\left(\frac{\overline{u}\cdot\overline{r}}{r}\right)^2\right]\left[\frac{\partial^2}{\partial t^2}\left(\frac{\overline{u}'\cdot\overline{r}}{r}\right)^2\right]} \\
& + \frac{c^2}{r^2}\left\{ 3\,\overline{\left[\frac{\partial}{\partial t}\left(\frac{\overline{u}\cdot\overline{r}}{r}\right)^2\right]\left[\frac{\partial}{\partial t}\left(\frac{\overline{u}'\cdot\overline{r}}{r}\right)^2\right]} - 4\,\overline{\left[\frac{\partial}{\partial t}\left(\frac{\overline{u}\cdot\overline{r}}{r}\right)^2\right]\left[\frac{\partial}{\partial t}\,(\overline{u}'\cdot\overline{u}')\right]} + \overline{\left[\frac{\partial}{\partial t}\,(\overline{u}\cdot\overline{u})\right]\left[\frac{\partial}{\partial t}\,(\overline{u}'\cdot\overline{u}')\right]} \right\} \\
& + \frac{c^4}{r^4}\left[9\,\overline{\left(\frac{\overline{u}\cdot\overline{r}}{r}\right)^2\left(\frac{\overline{u}'\cdot\overline{r}}{r}\right)^2} - 6\,\overline{\left(\frac{\overline{u}\cdot\overline{r}}{r}\right)^2 (\overline{u}'\cdot\overline{u}')} + \overline{(\overline{u}\cdot\overline{u})\,(\overline{u}'\cdot\overline{u}')} \right] \Bigg\}\, d\overline{y}\, d\overline{y}'
\end{aligned} \qquad [81]$$

$$\begin{aligned}
I_k = \frac{\rho_0^2}{16\,\pi^2 \rho_0 c^5} \iint \frac{1}{r^2} \Bigg\{ & \overline{\left[\frac{\partial^2}{\partial t^2}\left(\frac{\overline{u}\cdot\overline{r}}{r}\right)^2\right]\left[\frac{\partial^2}{\partial t^2}\left(\frac{\overline{u}'\cdot\overline{r}}{r}\right)^2\right]}\,\frac{r_k}{r} \\
& + \frac{c^2}{r^2}\left\{ 2\,\overline{\frac{\partial}{\partial t}\left[u_k\left(\frac{\overline{u}\cdot\overline{r}}{r}\right)\right]\left[\frac{\partial}{\partial t}\,(\overline{u}'\cdot\overline{u}')\right]} + \overline{\left[\frac{\partial}{\partial t}\,(\overline{u}\cdot\overline{u})\right]\left[\frac{\partial}{\partial t}\,(\overline{u}'\cdot\overline{u}')\right]}\,\frac{r_k}{r} \right. \\
& \left. - 5\,\overline{\left[\frac{\partial}{\partial t}\left(\frac{\overline{u}\cdot\overline{r}}{r}\right)^2\right]\left[\frac{\partial}{\partial t}\,(\overline{u}'\cdot\overline{u}')\right]}\,\frac{r_k}{r} \right\} \Bigg\}\, d\overline{y}\, d\overline{y}'
\end{aligned} \qquad [82]$$

$$I_r = \frac{\rho_0^2}{16\pi^2\rho_0 c^5}\iint \frac{1}{r^2}\left\{\overline{\left[\frac{\partial^2}{\partial t^2}\left(\frac{\bar{u}\cdot\bar{r}}{r}\right)^2\right]\left[\frac{\partial^2}{\partial t^2}\left(\frac{\bar{u}'\cdot\bar{r}}{r}\right)^2\right]}\right.$$

$$\left. + \frac{c^2}{r^2}\left\{\overline{\left[\frac{\partial}{\partial t}(\bar{u}\cdot\bar{u})\right]\left[\frac{\partial}{\partial t}(\bar{u}'\cdot\bar{u}')\right]} - 3\,\overline{\left[\frac{\partial}{\partial t}\left(\frac{\bar{u}\cdot\bar{r}}{r}\right)^2\right]\left[\frac{\partial}{\partial t}(\bar{u}'\cdot\bar{u}')\right]}\right\}\right\} d\bar{y}\, d\bar{y}'$$

$$I_\theta = \frac{\rho_0^2}{16\pi^2\rho_0 c^5}\iint \frac{2c^2}{r^4}\overline{\left\{\frac{\partial}{\partial t}(\bar{u}\cdot\bar{u})\right\}\left\{\frac{\partial}{\partial t}\left[\left(\frac{\bar{u}'\cdot\bar{r}}{r}\right)\left(\frac{\bar{u}'\cdot\bar{\theta}}{\theta}\right)\right]\right\}} d\bar{y}\, d\bar{y}' \qquad [83]$$

and

$$I_\phi = \frac{\rho_0^2}{16\pi^2\rho_0 c^5}\iint \frac{2c^2}{r^4}\overline{\left\{\frac{\partial}{\partial t}(\bar{u}\cdot\bar{u})\right\}\left\{\frac{\partial}{\partial t}\left[\left(\frac{\bar{u}'\cdot\bar{r}}{r}\right)\left(\frac{\bar{u}\cdot\bar{\phi}}{\phi}\right)\right]\right\}} d\bar{y}\, d\bar{y}'$$

APPENDIX C

EFFECT OF CONVECTION OF TURBULENCE

For the case of fluctuating forces being convected at vector Mach number $\overline{M}$ the transformations used by Lighthill[1] and Equation [1] lead to

$$p = \frac{1}{4\pi}\int\left[\frac{r_i \dot{F}_i}{c\,(r-\overline{M}\cdot\overline{r})^2} + \frac{(1-M^2)\,r_i F_i}{(r-\overline{M}\cdot\overline{r})^3} - \frac{M_i F_i}{(r-\overline{M}\cdot\overline{r})^2}\right] d\overline{\eta} \qquad [84]$$

where $\overline{\eta} = \overline{y} + \overline{M}r$ and $F_i \equiv F_i\,(\overline{\eta},\ t - r/c)$. By taking the time average of the square of the instantaneous sound pressure, the mean-square sound pressure becomes

$$\overline{p^2} = \frac{1}{16\,\pi^2 c^2}\iint \frac{1}{(r-\overline{M}\cdot\overline{r})^2}\left\{\frac{r_i r_j}{(r-\overline{M}\cdot\overline{r})^2}\,\overline{\dot{F}_i \dot{F}_j'} + \frac{c^2}{(r-\overline{M}\cdot\overline{r})^2}\left[\frac{(1-M^2)^2\, r_i r_j}{(r-\overline{M}\cdot\overline{r})^2} - \frac{2\,(1-M^2)\,r_i M_j}{(r-\overline{M}\cdot\overline{r})} + M_i M_j\right]\overline{F_i F_j'}\right\} d\overline{\eta}\; d\overline{\eta}' \qquad [85]$$

where terms involving odd numbers of time derivatives have been neglected as before.

If the fluctuating forces are being transported in the x_3-direction, $|\overline{M}| = M = M_3$ and $\overline{M}\cdot\overline{r}/r = M \cos\theta$. Therefore, the mean-square sound pressure is

$$\overline{p^2} = \frac{1}{16\,\pi^2 c^2}\iint \frac{1}{r^2\,(1-M\cos\theta)^2}\left\{\frac{r_i r_j}{r^2\,(1-M\cos\theta)^2}\,\overline{\dot{F}_i \dot{F}_j'} + \frac{c^2}{r^2\,(1-M\cos\theta)^2}\left[\frac{(1-M^2)^2\, r_i r_j}{r^2\,(1-M\cos\theta)^2}\,\overline{F_i F_j'} - \frac{2\,M\,(1-M^2)\,r_i}{r\,(1-M\cos\theta)}\,\overline{F_i F_3'} + M^2\,\overline{F_3 F_3'}\right]\right\} d\overline{\eta}\; d\overline{\eta}' \qquad [86]$$

For the special case of dipoles having axes in the x_3-direction and being transported at Mach number M in the x_3-direction, the mean-square sound pressure is

$$\overline{p^2} = \frac{1}{16\,\pi^2 c^2}\iint \frac{1}{r^2\,(1-M\cos\theta)^4}\left\{\cos^2\theta\,\overline{\dot{F}_3\,\dot{F}_3'} + \frac{c^2}{r^2}\left[\frac{\cos^2\theta\,(1-M^2)^2}{(1-M\cos\theta)^2} - \frac{2\cos\theta\, M\,(1-M^2)}{1-M\cos\theta} + M^2\right]\overline{F_3\,F_3'}\right\} d\overline{\eta}\; d\overline{\eta}' \qquad [87]$$

Similarly, for the case of turbulence being convected at vector Mach number $\overline{M}$, the instantaneous sound pressure, by using Equation [2], is

$$p = \frac{1}{4\pi}\int\left\{\frac{r_i r_j}{(r-\overline{M}\cdot\overline{r})^2}\left[\frac{\ddot{T}_{ij}}{c^2(r-\overline{M}\cdot\overline{r})} + \frac{3(1-M^2)\,\dot{T}_{ij}}{c(r-\overline{M}\cdot\overline{r})^2} + \frac{3(1-M^2)^2\,T_{ij}}{(r-\overline{M}\cdot\overline{r})^3}\right]\right.$$

$$-\frac{2r_iM_j}{r-\overline{M}\cdot\overline{r}}\left[\frac{2\dot{T}_{ij}}{c(r-\overline{M}\cdot\overline{r})^2} + \frac{3(1-M^2)\,T_{ij}}{(r-\overline{M}\cdot\overline{r})^3}\right]$$

$$\left.-\delta_{ij}\left[\frac{\dot{T}_{ij}}{c(r-\overline{M}\cdot\overline{r})^2} + \frac{(1-M^2)\,T_{ij}}{(r-\overline{M}\cdot\overline{r})^3}\right] + M_iM_j\left[\frac{2T_{ij}}{(r-\overline{M}\cdot\overline{r})^3}\right]\right\}d\overline{\eta} \qquad [88]$$

where now $T_{ij} \equiv T_{ij}\,(\overline{\eta},\ t-r/c)$. By again neglecting terms involving an odd number of time derivatives, the mean-square sound pressure becomes

$$\overline{p^2} = \frac{1}{16\pi^2c^4}\iint\frac{1}{(r-\overline{M}\cdot\overline{r})^2}\left\{\frac{r_ir_jr_lr_m}{(r-\overline{M}\cdot\overline{r})^4}\,\overline{\ddot{T}_{ij}\ddot{T}_{lm}'}\right.$$

$$+\frac{c^2}{(r-\overline{M}\cdot\overline{r})^2}\left[\frac{3(1-M^2)^2\,r_ir_jr_lr_m}{(r-\overline{M}\cdot\overline{r})^4} - \frac{4(1-M^2)\,\delta_{ij}r_lr_m}{(r-\overline{M}\cdot\overline{r})^2} + \delta_{ij}\delta_{lm} - \frac{12(1-M^2)\,r_ir_jr_lM_m}{(r-\overline{M}\cdot\overline{r})^3}\right.$$

$$\left.+\frac{16\,r_ir_lM_jM_m}{(r-\overline{M}\cdot\overline{r})^2} - \frac{4\,r_ir_jM_lM_m}{(r-\overline{M}\cdot\overline{r})^2} + \frac{8\,\delta_{lm}r_iM_j}{(r-\overline{M}\cdot\overline{r})}\right]\overline{\dot{T}_{ij}\dot{T}_{lm}'}$$

$$+\frac{c^4}{(r-\overline{M}\cdot\overline{r})^4}\left[\frac{9(1-M^2)^4\,r_ir_jr_lr_m}{(r-\overline{M}\cdot\overline{r})^4} - \frac{6(1-M^2)^3\,\delta_{ij}r_lr_m}{(r-\overline{M}\cdot\overline{r})^2} + (1-M^2)\,\delta_{ij}\delta_{lm}\right.$$

$$-\frac{36(1-M^2)^3\,r_ir_jr_lM_m}{(r-\overline{M}\cdot\overline{r})^3} + \frac{36(1-M^2)^2\,r_ir_lM_jM_m}{(r-\overline{M}\cdot\overline{r})^2} + \frac{12(1-M^2)^2\,r_ir_jM_lM_m}{(r-\overline{M}\cdot\overline{r})^2}$$

$$+\frac{12(1-M^2)^2\,\delta_{lm}r_iM_j}{r-\overline{M}\cdot\overline{r}} - \frac{24(1-M^2)\,r_iM_jM_lM_m}{r-\overline{M}\cdot\overline{r}} - 4(1-M^2)\,\delta_{ij}M_lM_m$$

$$\left.\left.+\,4M_iM_jM_lM_m\right]\overline{T_{ij}T_{lm}'}\right\}d\overline{\eta}\,d\overline{\eta}' \qquad [89]$$

If the turbulence is being transported in the x_3-direction, the mean-square sound pressure becomes

$$
\begin{aligned}
\overline{p^2} = \frac{1}{16\pi^2 c^4}\iint \frac{1}{r^2(1-M\cos\theta)^2}\Bigg\{ & \left[\frac{r_i r_j r_l r_m}{r^4(1-M\cos\theta)^4}\,\overline{\ddot{T}_{ij}\ddot{T}'_{lm}} + \frac{c^2}{r^2(1-M\cos\theta)^2}\Bigg\{\Bigg[\frac{3(1-M^2)^2 r_i r_j r_l r_m}{r^4(1-M\cos\theta)^4}\right. \\
& - \frac{4(1-M^2)\delta_{ij} r_l r_m}{r^2(1-M\cos\theta)^2} + \delta_{ij}\delta_{lm}\Bigg]\overline{\dot{T}_{ij}\dot{T}'_{lm}} - \frac{12M(1-M^2) r_i r_j r_l}{r^3(1-M\cos\theta)}\,\overline{\dot{T}_{ij}\dot{T}'_{l3}} + \frac{16M^2 r_i r_l}{r^2(1-M\cos\theta)^2}\,\overline{\dot{T}_{i3}\dot{T}'_{l3}} \\
& - \frac{4M^2 r_i r_j}{r^2(1-M\cos\theta)^2}\,\overline{\dot{T}_{ij}\dot{T}'_{33}} + \frac{8M r_i}{r(1-M\cos\theta)}\,\overline{\dot{T}_{i3}\dot{T}'_{ll}}\Bigg\} + \frac{c^4}{r^4(1-M\cos\theta)^4}\Bigg\{\Bigg[\frac{9(1-M^2)^4 r_i r_j r_l r_m}{r^4(1-M\cos\theta)^4} \\
& - \frac{6(1-M^2)^3\delta_{ij} r_l r_m}{r^2(1-M\cos\theta)^2} + (1-M^2)^2\delta_{ij}\delta_{lm}\Bigg]\overline{T_{ij}T'_{lm}} - \frac{36(1-M^2)^3 r_i r_j r_l}{r^3(1-M\cos\theta)^3}\,\overline{T_{ij}T'_{l3}} \\
& + \frac{36M^2(1-M^2)^2 r_i r_l}{r^2(1-M\cos\theta)^2}\,\overline{T_{i3}T'_{l3}} + \frac{12M^2(1-M^2)^2 r_i r_j}{r^2(1-M\cos\theta)^2}\,\overline{T_{ij}T'_{33}} + \frac{12M(1-M^2)^2 r_i}{r(1-M\cos\theta)}\,\overline{T_{i3}T'_{ll}} \\
& \left. - \frac{24M^3(1-M^2) r_i}{r(1-M\cos\theta)}\,\overline{T_{i3}T'_{33}} - 4M^2(1-M^2)\,\overline{T_{ii}T'_{33}} + 4M^4\,\overline{T_{33}T'_{33}}\Bigg\}\right]\Bigg\}\, d\bar{\eta}\, d\bar{\eta}'
\end{aligned}
\qquad [90]
$$

For the case of lateral quadrupoles having axes in the x_1- and x_3-directions and being transported in the x_3-direction at Mach number M, the mean-square sound pressure is given by

$$
\begin{aligned}
\overline{p^2} = \frac{1}{16\pi^2 c^4}\iint \frac{4\sin^2\theta\cos^2\phi}{r^2(1-M\cos\theta)^6}\Bigg\{ & \cos^2\theta\,\overline{\ddot{T}_{13}\ddot{T}'_{13}} \\
& + \frac{c^2}{r^2}\left[\frac{3(1-M^2)^2\cos^2\theta}{(1-M\cos\theta)^2} - \frac{6M(1-M^2)\cos\theta}{1-M\cos\theta} + 4M^2\right]\overline{\dot{T}_{13}\dot{T}'_{13}} \\
& + \frac{c^4}{r^4}\left[\frac{9(1-M^2)^4\cos^2\theta}{(1-M\cos\theta)^4} - \frac{18M(1-M^2)^3\cos\theta}{(1-M\cos\theta)^3} + \frac{9M^2(1-M^2)^2}{1-M\cos\theta)^2}\right]\overline{T_{13}T'_{13}}\Bigg\}\, d\bar{\eta}\, d\bar{\eta}'
\end{aligned}
\qquad [91]
$$

Beyond several diameters from a circular jet, $\cos^2\phi$ may be replaced by its average value; namely ½, as before.

For the case of longitudinal quadrupoles having their axes in the x_3-direction and being transported in the x_3-direction at Mach number M, the mean-square sound pressure is given by

$$\overline{p^2} = \frac{1}{16\pi^2 c^4} \iint \frac{1}{r^2(1-M\cos\theta)^4} \left\{ \cos^4\theta\, \overline{\ddot{T}_{33}\, \ddot{T}_{33}'} \right.$$

$$+ \frac{c^2}{r^2} \left[\frac{3(1-M^2)^2 \cos^4\theta}{(1-M\cos\theta)^2} - \frac{12M(1-M^2)\cos^3\theta}{1-M\cos\theta} - (4-9M^2)\cos^2\theta + 6M\cos\theta + 1 \right] \overline{\dot{T}_{33}\, \dot{T}_{33}'}$$

$$+ \frac{c^4}{r^4} \left[\frac{9(1-M^2)^4 \cos^4\theta}{(1-M\cos\theta)^4} - \frac{36M(1-M^2)^3\cos^3\theta}{(1-M\cos\theta)^3} - \frac{6(1-9M^2)(1-M^2)^2\cos^2\theta}{(1-M\cos\theta)^2} \right.$$

$$\left. \left. + \frac{12(1-3M^2)(1-M^2)M\cos\theta}{1-M\cos\theta} + (1-3M^2)^2 \right] \overline{T_{33}\, T_{33}'} \right\} d\bar{\eta}\, d\bar{\eta}' \qquad [92]$$

It may be seen that introducing a convection velocity somewhat complicates the expressions for the general sound field. The intensity could be obtained for the case of convection of dipoles and quadrupoles, but the complexity would be great indeed.

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