# NAVY DEPARTMENT

## THE DAVID W. TAYLOR MODEL BASIN

WASHINGTON 7, D.C.

# THE LOSS OF ENERGY OF A PROPELLER IN A LOCALLY VARYING WAKE FIELD

by

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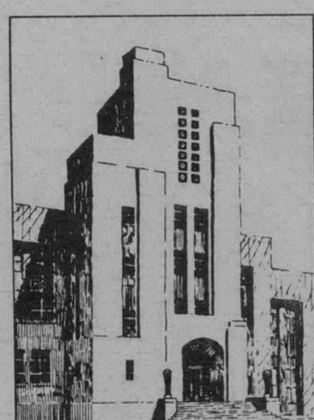

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## NOTATION

| | |
|---|---|
| $P$ | Power |
| $E$ | Energy |
| $D = 2\,R$ | Propeller diameter |
| $n$ | Number of revolutions |
| $p$ | Number of blades |
| $c$ | Chord length of a section |
| $\left.\begin{matrix} r \\ z \\ \varphi \end{matrix}\right\}$ | Cylindrical coordinates |
| $x$ | $r/R$ |
| $t$ | Time |
| $v_s,\ V_s$ | Speed of the ship |
| $\psi$ | Wake factor |
| $\overline{\psi}$ | Average of $\psi$ relative to $\varphi$ |
| $v_a = v_s\,(1 - \psi_a)$ | Axial component of the inflow |
| $v_t = v_s\,\psi_t$ | Tangential component of the inflow |
| $V$ | Resultant relative velocity at a propeller section |
| $\overline{V}$ | Time average of $V$ |
| $N$ | Component of the inflow perpendicular to $\overline{V}$ |
| $L$ | Component of the inflow parallel to $\overline{V}$ |
| $w$ | Induced velocity |
| $x',\ \vartheta$ | Coordinates on the chord of the propeller section |
| $s,\ \sigma$ | Coordinates on a free vortex sheet measured along a helical path |
| $H$ | Pitch of $V$ |
| $\beta_i$ | Pitch angle of $V$ |
| $\lambda_s = v_s/\pi\,n\,D$ | Rate of advance |
| $\Gamma$ | Total circulation |
| $\gamma$ | Local value of circulation per unit length |
| $G$ | Amplitude of $\Gamma$ |
| $g$ | Amplitude of $\gamma$ |
| $l$ | Wave length of velocity |

| | |
|---|---|
| $\delta$ | Phase angle |
| $\alpha$ | Angle of attack |
| $k = \pi\, c/l$ | Reduced frequency |
| $i$ | Imaginary unit |
| $J_0$, $J_1$ | Bessel functions of the first kind |
| $Y_0$, $Y_1$ | Bessel functions of the second kind |
| $K_0$, $K_1$ | Modified Bessel functions of the second kind |
| $\phi$ | Potential function |

## ABSTRACT

A wake field is assumed in the plane of the propeller which depends on both the radial and peripheral coordinate in an arbitrary manner. The energy which is associated with the free vorticity of the nonsteady flow at the propeller is expressed in terms of the wake field and of quantities related to the propeller. The analysis is based on the airfoil theory for nonsteady motion as developed by von Kármán and Sears. The result is that the energy depends on a certain function of the reduced frequency and is proportional to the number of blades, to the chord length, and to the square of the amplitudes of the harmonics of the wake. A numerical example shows that the power related to the energy is small, about 1.1 percent of the power input.

## INTRODUCTION

In the following considerations, a wake field is assumed which depends both on the radial and on the angular coordinate in an arbitrary manner. In such a field, the relative velocity at a certain section of the propeller is continuously varying; it follows from this that the bound circulation on the section depends on time. According to a fundamental law of hydromechanics, viz., the law of conservation of vorticity, free vortices, which are radially directed in the case considered, arise from the time-dependent variations of the bound circulation. Associated with these free vortices is a velocity field and, therefore, a certain amount of kinetic energy within the fluid. To this energy, there corresponds an equivalent amount of power input which is unavailable for the propulsion of the ship. The purpose of the following considerations is to express this loss of propelling energy in terms of both the wake field and the propeller parameters.

The significance of such consideration is twofold. In the first place, an order of magnitude is obtained for the possible improvement of the propulsion of a ship when axisymmetrical inflow to the propeller is realized. In the second place, if the distribution of the loss of energy over the radius of the propeller is known, it becomes possible to judge whether or not a theory of the wake-adapted propeller which is based on the assumption of axisymmetrical inflow is sufficiently realistic.

The result obtained in this paper is qualitative since certain quantities are determined by means of the concept of a propeller with infinitely many blades. Further, a negligibly small loading of the propeller is assumed. These simplifications are introduced to avoid complicated integrations. However, there are reasons to expect that the influence of these simplifications on the order of magnitude of the result is immaterial.

## FORMULATION OF THE PROBLEM

The motion which is associated with the free vortex sheets of the propeller is a potential motion. The kinetic energy of the field is then expressed by the well-known relation:

$$E = \frac{\rho}{2}\int \phi \frac{\partial \phi}{\partial n}\, dF*$$

The integral is to be taken over the surface of the volume under consideration. In our case, where the motion is related to vortex sheets, the integral may be transformed so that the integration is carried out over the elements $dF$ of the vortex sheets:

$$E = \frac{\rho}{2}\int \Delta\phi\, w_n\, dF \qquad [1]$$

In this integral, $w_n$ represents the component of the induced velocity perpendicular to the vortex sheets. $\Delta\phi$ represents the potential difference at a vortex sheet which is related to the circulation $\gamma$ of the sheet by the equation

$$\Delta\phi = \int \gamma\, ds \qquad [2]$$

where $s$ is a length coordinate on a free vortex sheet, the length being measured along a helical line in the case of a propeller. Since it is also possible to express $w_n$ in terms of $\gamma$, the problem of determining the energy of the induced velocity field is reduced to the determination of the circulation of the free vortex sheets.

To establish an expression for the free circulation, the resultant relative velocity at a propeller section $V$ is resolved, at any instant, into components perpendicular and parallel to the time average $\overline{V}$ of $V$ (Figure 1). The perpendicular component $N$ consists of several parts which arise from the axial and tangential wake fields in a direct way and from the parallel component $L$ in an indirect way. The reason for a part of $L$ being included in $N$ is that the effect of the parallel component may also be obtained by an equivalent normal component, as will be discussed later. Including in $N$ all perpendicular components:

$$N = N_a + N_t + N_L \qquad [3]$$

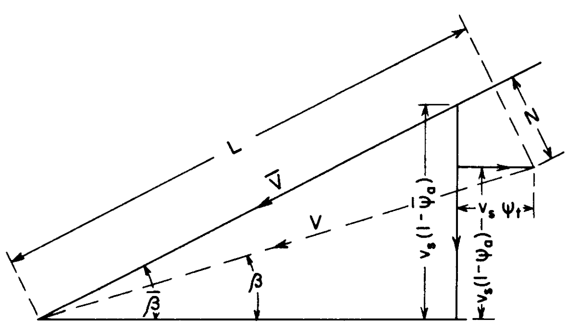


Figure 1 - Velocity Diagram

the unsteady relative flow at a propeller section consists of the flow $\overline{V}$ which is independent of time and the perpendicular flow $N$ which depends on time. That is, the problem concerned is considered as a gust problem. In order to determine the free circulation for such a flow, $N$ is resolved into a Fourier series at any radius:

$$N = a_0 + \sum_{m=1}^{\infty} a_m \sin(m\varphi) + \sum_{m=1}^{\infty} b_m \cos(m\varphi) \qquad [4a]$$

or, written differently:

$$\left.\begin{aligned} N &= a_0 + \sum_{m=1}^{\infty} A_m \sin(m\varphi + \delta_m) \\ A_m &= \sqrt{a_m^2 + b_m^2} \end{aligned}\right\} \qquad [4b]$$

The coefficients $a_m$ and $b_m$ depend on the radius. For a single harmonic $N_m$, the corresponding free circulation $\gamma_m$ can be established on a basis of unsteady wing theory. Summing $\gamma_m$ over all harmonics, the free circulation $\gamma$ which belongs to the perpendicular component $N$ of the inflow at radius $r$ is obtained provided that the relation between $\gamma_m$ and $N_m$ is linear.

## RESULTS OF NONSTEADY WING THEORY

For the purpose of this note, the representation of nonsteady wing theory as given by von Kármán and Sears is particularly suited.[1,2] We consider the flow relative to a section of the propeller and introduce coordinates $x'$ and $\vartheta$ on the section (Figure 2). They are related by

$$x' = \frac{c}{2} \cos \vartheta \qquad [5]$$

Relative to the bound circulation, the instantaneous value of the nonsteady circulation $\Gamma$ and the instantaneous value of the quasi-steady circulation $\Gamma_0$ are introduced. The latter is obtained when neglecting the influence of the free vortices on the flow at the section and, thus, on the bound circulation. In the case under consideration, both $\Gamma_0$ and $\Gamma$ are periodic functions of time. Correspondingly there is written:

$$\Gamma_0 = G_0 \exp(i\omega t) \exp(i\delta) * \qquad [6]$$

If $\Gamma$ is a periodic function of time, the free circulation $\gamma(s, t)$ is then a periodic function of position and time in the relative flow which is considered. It is one of the fundamental

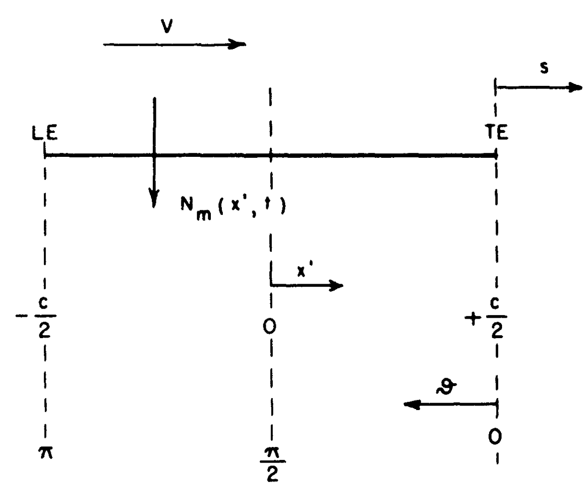


Figure 2 - Coordinates

[1]References are listed on page 14.

*exp $(i\omega t)$ means $e^{i\omega t}$.

assumptions in the theory by von Kármán and Sears that the frequencies of the bound and free circulation are equal. This is justifiable if the motion has taken place for a sufficient length of time so that transient phenomena have vanished. Accordingly, the free circulation is represented by the following expression:

$$\gamma(s,t) = g \exp\left[i\omega\left(t - \frac{s}{\overline{V}}\right)\right]\exp(i\delta) \qquad [7]$$

That is, the free circulation follows the law of a permanent wave in the relative flow.

On account of the concept that the bound circulation, at any instant, is the sum of the quasi-steady circulation and a part which is caused by the influence of the free vortex sheet on the velocity distribution on the section, the following relation between the amplitudes $G_0$ and $g$ is deduced:

$$g\left[K_0(ik) + K_1(ik)\right] = -G_0 \qquad [8]$$

This relation is independent of the phase $\delta$.

By means of this relation, the problem is reduced to the determination of the quasi-steady circulation which belongs to a given distribution of the perpendicular velocity component $N_m$ at the propeller profile. This relation is known from the theory of thin profiles and reads as follows:[3]

$$(\gamma_0)_m = \frac{2(\Gamma_0)_m}{\pi c \sin\vartheta} + \sum_{\mu=1}^{\infty} B_\mu \frac{\cos\mu\vartheta}{\sin\vartheta} \qquad [9a]$$

$$N_m = \sum_{\mu=1}^{\infty} B_\mu \frac{\sin\mu\vartheta}{2\sin\vartheta} \qquad [9b]$$

That is, the bound quasi-steady circulation which is associated with the $m^{th}$ harmonic of $N$ may be found from Equation [9a] after the coefficients $B_\mu$ of the series [9b] for $N_m$ have been determined.

Corresponding to the fact that the field of the $m^{th}$ harmonic of $N$ moves with velocity $\overline{V}$ over the propeller section, $N_m$ may be represented by the following expression for a permanent wave:

$$N_m(x', t) = A_m \exp\left[i\omega_m\left(t - \frac{x'}{\overline{V}}\right)\right]\exp(i\delta_m)$$

In this expression,

$$\omega_m = \frac{2\pi\overline{V}}{l_m}$$

is the number of velocity waves of length $l_m$ which travel past a fixed point of the profile in $2\pi$ seconds. Of this expression for $N_m$, only the imaginary part which corresponds to the series [4b] for $N$ is of interest.

Introducing the coordinate $\vartheta$ and the reduced frequency:

$$k_m = \frac{\omega_m \frac{c}{2}}{\overline{V}} = \frac{\pi c}{l_m} \qquad [10]$$

into the above expression for $N_m$, we obtain

$$N_m(\vartheta, t) = A_m \exp(i\omega_m t)\exp(-ik_m \cos\vartheta)\exp(i\delta_m)$$

The determination of the coefficients $B_\mu$ from Equation [9] is difficult with this representation of $N_m$. Kármán and Sears resolve $\exp(-ik_m \cos\vartheta)$ into the following series which is known from the theory of Bessel functions:

$$\exp(-ik_m \cos\vartheta) = J_0(k_m) + 2\sum_{\mu=1}^{\infty}(-i)^\mu J_\mu(k_\mu)\cos\mu\vartheta$$

With this series, the $m^{\text{th}}$ harmonic of $N$ becomes:

$$N_m(\vartheta, t) = \exp(i\delta_m)\Big\{A_m J_0(k_m)\exp(i\omega_m t)$$

$$-2A_m J_1(k_m)\cos\vartheta i\exp(i\omega_m t) - 2A_m J_2(k_m)\cos 2\vartheta\exp(i\omega_m t) + - \ldots\Big\}$$

For each term of this series, the quasi-steady circulation may be determined from the relations [9] by means of the Kutta condition of smooth flow at the trailing edge, i.e., $\gamma_0 = 0$ for $\vartheta = 0$. The result is obtained that a periodic quasi-steady circulation with phase $\delta_m$ corresponds to each member of the series. The amplitudes are as follows:

$$\left.\begin{array}{ll} \text{for the first term} & G_{0,1} = 2\pi A_m J_0(k_m) \\ \text{for the second term} & G_{0,2} = 2\pi A_m J_1(k_m) \\ \text{for the rest} & G_{0,3\ldots} = 0 \end{array}\right\} \qquad [11]$$

It should be mentioned that each term of the series for $N_m$ may be interpreted physically. The first term corresponds to perpendicular velocity components at the section such as occur, with negative sign, with harmonic oscillations of the sections in a direction perpendicular to $\overline{V}$. The second term corresponds to perpendicular components of a rotational oscillation. The rest of the terms represents standing oscillations which are antisymmetrical deformations about the midpoint of the section; thus, they do not generate lift in the quasi-steady case.

## LOSS OF ENERGY OF A PROPELLER IN LOCALLY VARYING WAKE

After these preparations, the amplitude of the free circulation may be determined for a section of the propeller at radius $r$ along which the $m^{\text{th}}$ harmonic of $N$ passes by. For these calculations, it is useful to express the left-hand side of Equation [8] by functions with a real argument. Applying certain integral representations of Bessel functions one obtains:

$$g(\alpha - i\beta)\,\pi = 2\,G_0, \text{ where}$$
$$\alpha = Y_0(k) + J_1(k), \quad \beta = Y_1(k) - J_0(k) \qquad [12]$$

Introducing Equation [12] into the expression [7] for the free circulation, the imaginary part of $\gamma_m$ becomes:

$$\gamma_m(s, t) = \frac{2}{\pi} \frac{(G_0)_m}{(\alpha_m^2 + \beta_m^2)} (C_m \cos\delta_m + D_m \sin\delta_m) \qquad [13a]$$

In this expression, the following abbreviations are used:

$$C_m = h_m \cos\sigma_m - f_m \sin\sigma_m \qquad [13b]$$

$$D_m = h_m \sin\sigma_m + f_m \cos\sigma_m \qquad [13c]$$

$$\sigma_m = 2\,k_m\ s/c \qquad [13d]$$

$$(G_0)_m = (G_{0,1})_m + (G_{0,2})_m \qquad [13e]$$

The functions $f_m$ and $h_m$ depend on time:

$$f_m = \alpha_m \cos(\omega_m t) - \beta_m \sin(\omega_m t)$$
$$h_m = \alpha_m \sin(\omega_m t) + \beta_m \cos(\omega_m t) \qquad [13f]$$

Summing $\gamma_m$ from Equations [13], over all $m$, the circulation $\gamma$ of the free vortex sheet which is caused by the perpendicular component of the inflow at radius $r$ is obtained:

$$\gamma(s, t) = \sum_{m=1}^{\infty} \gamma_m(s, t) \qquad [14]$$

This summation is justifiable since both the relations [9] between $N_m$ and $(G_0)_m$ and the relations [8] between $(G_0)_m$ and $g_m$ are linear.

With the expression for $\gamma$ so obtained, the energy of the free vortex sheet may be determined from Equation [1]. First, it follows for $\Delta\phi$ from Equation [2]:

$$\Delta \phi = \frac{c}{\pi} \sum_{m=1}^{\infty} \frac{(G_0)_m}{k_m(\alpha_m^2 + \beta_m^2)} \left(D_m \cos \delta_m - C_m \sin \delta_m\right)$$

An expression for the velocity $w_n$, i.e., the velocity induced by the free vortex sheets perpendicular to themselves; is still missing. To avoid extensive integrations, the number of blades is assumed to be very great. Then, the radially directed vortices which make up the vortex sheets are close together and form a vortex cylinder. A radial cut through this cylinder represents a plane vortex sheet, the circulation of which is a periodic function of the coordinate $z$ at any instant and depends, further, on the radial coordinate. The self-induction of such a vortex plane consists of a velocity component perpendicular to the plane, which is a tangential component in the propeller system. At points outside of the plane, velocity components are induced which are equal but of opposite sign at equidistant points above and below the plane. As a consequence, the outside components cancel within the vortex cylinder and only the self-induction remains.

For an estimate of the self-induced velocity, we start from the Biot-Savart integral and simplify the integrand, introducing average values of certain functions, the variation of which is small within the limits of integration. The error on the final result associated with these simplifications is small when the loading is small. The following approximation is obtained:

$$w_t = \frac{1}{2\pi} \int_0^{\infty} \frac{\gamma \, d z_1}{z - z_1}$$

Introducing the expressions [13] for $\gamma$, the principal value of this improper integral may be calculated; this gives:

$$w_t = \frac{1}{\pi} \sum_{m=1}^{\infty} \frac{(G_0)_m}{\alpha_m^2 + \beta_m^2} \left(D_m \cos \delta_m - C_m \sin \delta_m\right)$$

The component which is normal to the vortex sheets amounts to:

$$w_n = w_t \sin \bar{\beta}_i$$

For $\beta_i$, its time average is introduced, that is, the deformations of the free vortex sheets which arise from the time dependence of $\beta_i$, are considered small, in accordance with the assumptions of nonsteady airfoil theory. Corresponding to Equation [1], the energy is related to the product $(\Delta \phi \, w_n)$ which, from the just derived expressions for $\Delta \phi$ and $w_n$, is a function of time. Of interest is the time average of the energy which follows from the time average of $(\Delta \phi \, w_n)$. For this average, one obtains:

$$\overline{\Delta \phi \, w_n} = \frac{\sin \bar{\beta}_i}{2\pi^2} \, c \sum_{m=1}^{\infty} \frac{(G_0)_m^2}{k_m(\alpha_m^2 + \beta_m^2)}$$

This time average is independent both of the phase $\delta_m$ and of the coordinate $s$.

We consider the time average of the kinetic energy in an annular ring cylinder at radius $r$ the height of which equals the hydromechanic pitch $H$. If $p$ vortex sheets of equal shape are present, we obtain from Equation [1] for this amount of energy:

$$\overline{dE} = p\,\frac{\rho}{2}\int_0^{2\pi} \frac{\overline{\Delta\phi\, w_n}\, r\, dr}{\cos\overline{\beta}_i}\, d\varphi$$

Since the average $(\overline{\Delta\phi\, w_n})$ is independent of $s$, it is also independent of $\varphi$ if $r$ is kept constant. Therefore, it follows that:

$$\overline{dE} = p\,\frac{\rho}{2}\,\frac{\tan\overline{\beta}_i}{\pi}\, r\, c\, dr \sum_{m=1}^{\infty} \frac{(G_0)_m^2}{k_m(\alpha_m^2 + \beta_m^2)}$$

If this expression is multiplied by $v/H = v_s\,(1 - \overline{\psi}_a)/H$ there is obtained the time average of the energy within an annular cylinder of that length which a fluid element travels in one second in the axial direction. This energy equals the element of power. Considering that

$$H = 2r\pi\tan\overline{\beta}_i$$

and that

$$(G_0)_m^2 = [(G_{0,1})_m + (G_{0,2})_m]^2 = 4\pi^2\,(a_m^2 + b_m^2)\,(J_0 + J_1)^2$$

it follows that

$$dP = p\,\rho\, v_s\,(1 - \overline{\psi}_a)\, c\, dr \sum_{m=1}^{\infty} (a_m^2 + b_m^2)\, F(k_m)$$

Within the function

$$F(k_m) = \frac{(J_0 + J_1)^2}{k_m[(Y_0 + J_1)^2 + (Y_1 - J_0)^2]}$$

the argument of the Bessel functions is the reduced frequency $k_m$. This function is represented on Figure 3 from which it is seen that the effect of the harmonics of higher order decreases.

To obtain the loss of power input $\Delta P$, the element $dP$ must be integrated over the radius. The result in nondimensional form is as follows:

$$\Delta c_P = \frac{\Delta P}{\frac{\rho}{2} R^2 \pi v_s^3} = \frac{2p}{\pi}\int_{x_h}^{1} (1 - \overline{\psi}_a)\,\frac{c}{R}\, dx \sum_{m=1}^{\infty}\left[\left(\frac{a_m}{v_s}\right)^2 + \left(\frac{b_m}{v_s}\right)^2\right] F(k_m)$$

$$k_m = \frac{m}{2}\,\frac{c}{R}\,\frac{\cos\overline{\beta}_i}{x} \qquad [15]$$

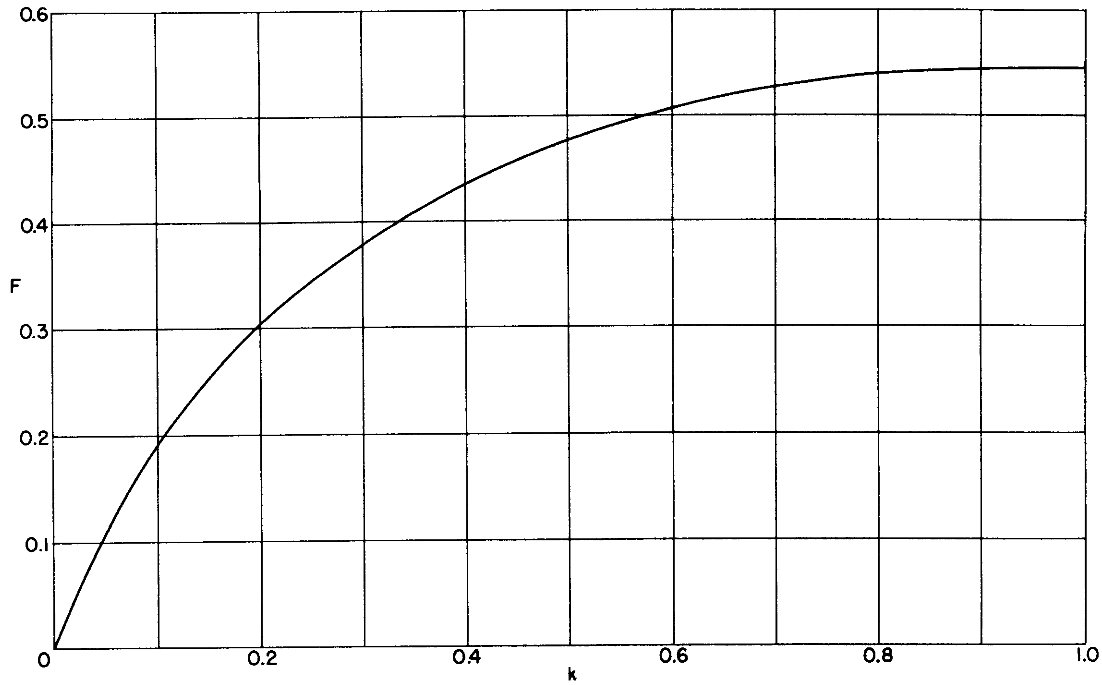


Figure 3a

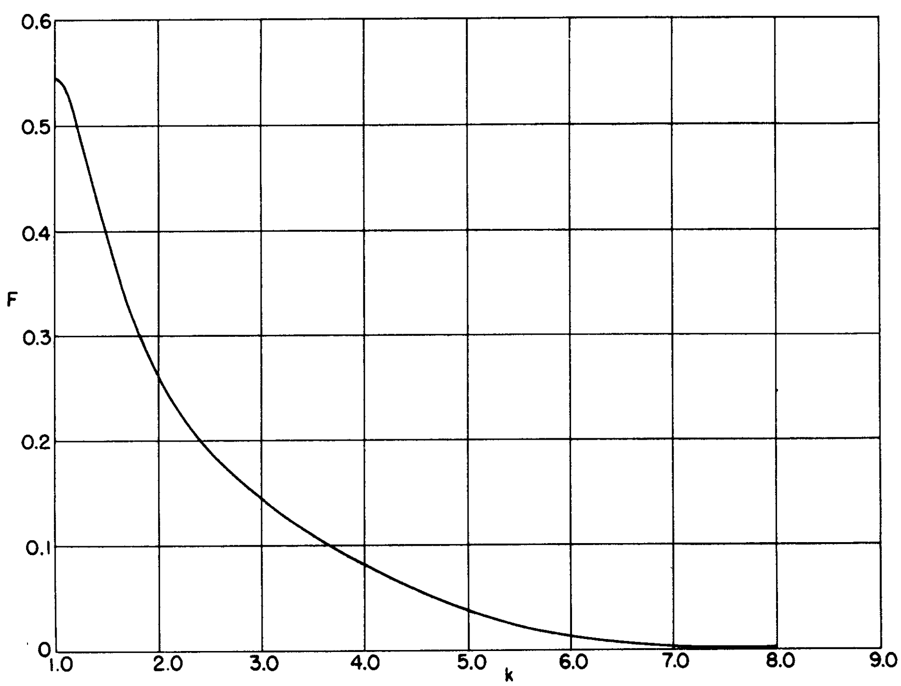


Figure 3b

Figure 3.- $F$ as a Function of the Reduced Frequency $k$

It remains to express the Fourier coefficients $a_m$ and $b_m$ of $N$ in terms of both the axial wake field $\psi_a$ and the tangential wake field $\psi_t$ which are given functions of $r$ and $\varphi$ Corresponding to Equation [3], $N$ includes parts from the longitudinal component $L$. These parts arise in the following way: If $L$ depends on time, both the bound and free circulation which are associated with $L$ depend on time. The same variations of these circulations may be produced by proper variations of the angle of attack, the longitudinal component being kept constant in time. This means that the longitudinal component is equivalent to a perpendicular component of equal frequency relative to time variations of the circulations. Since the circulation is proportional to the product of velocity and angle of attack, the equivalent perpendicular component follows from the relation

$$N_L = (L - \bar{L})\,\bar{\alpha}$$

Usually, $N_L$ is small since $\bar{\alpha}$, the time average of the angle of attack, is small. Thus, the associated loss is a small quantity.

In order to relate $N$ to the wake fields, small loading is assumed with which the approximation $\beta_i = \beta$ holds. It then follows that

$$\frac{N(r,\varphi)}{v_s} = (\psi_a - \bar{\psi}_a)\,(\cos\bar{\beta} - \bar{\alpha}\sin\bar{\beta}) \pm (\psi_t - \bar{\psi}_t)\,(\sin\bar{\beta} + \bar{\alpha}\cos\bar{\beta}) \qquad [16]$$

In this expression:

$$\cos\bar{\beta} = \frac{x \pm \lambda_s \bar{\psi}_t}{\sqrt{(x \pm \lambda_s \bar{\psi}_t)^2 + \lambda_s^2 (1 - \bar{\psi}_a)^2}}, \qquad \sin\bar{\beta} = \frac{\lambda_s (1 - \bar{\psi}_a)}{\sqrt{(x \pm \lambda_s \bar{\psi}_t)^2 + \lambda_s^2 (1 - \bar{\psi}_a)^2}}$$

The positive sign applies when the direction of the tangential wake is such that the tangential component of the relative velocity at the section is increased and vice versa.

From Equation [16], the Fourier coefficients $a_m/v_s$ and $b_m/v_s$ may be calculated as functions of the radius. Then, $\Delta c_P$ follows from Equation [15]. The result is that the loss depends both on the geometry of the propeller and on the shape of the hull. The loss is proportional to the number of blades and to the blade width and is, further, proportional to the square of the amplitudes of the harmonics of the perpendicular velocity $N$. It depends also on the average of the axial wake factor and on the advance coefficient.

For axisymmetrical inflow, the coefficients $a_m$ and $b_m$ equal zero and, further, the function $F(k_m)$ equals zero since $k_m = 0$ on account of Equation [10]. Hence, from Equation [15], the loss of energy equals zero, as is necessary.

## NUMERICAL RESULTS

For a single screw ship ($V_s$ = 15.5 knots, $D$ = 19.7 ft, $p$ = 4, $n$ = 1.58 rps) the axial and tangential wake fields, as represented on Figures 4 and 5, respectively, have been measured as functions of the radial and angular coordinates. Figure 5 shows that $\bar{\psi}_t = 0$ at

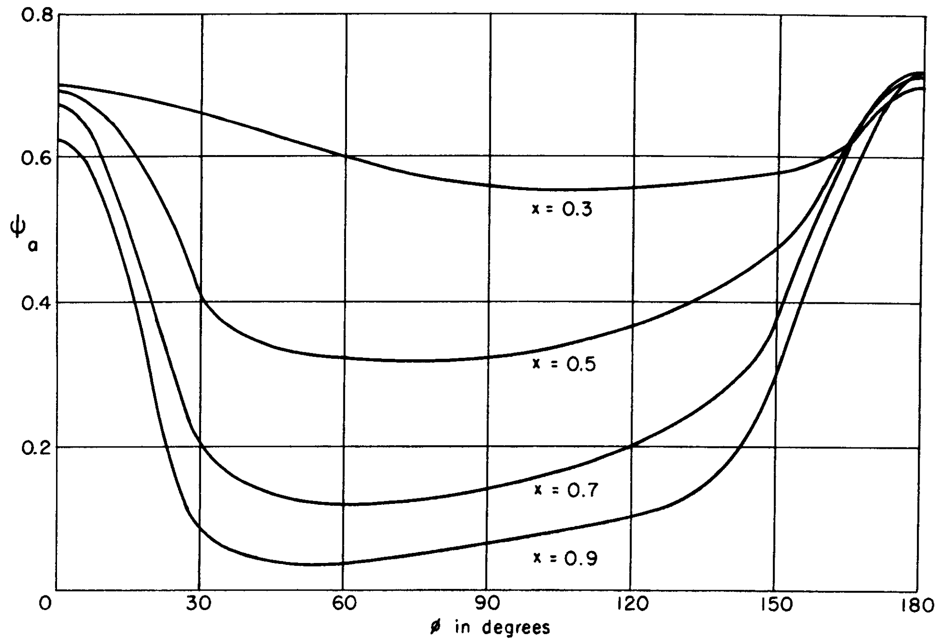


Figure 4 - Axial Wake Factor

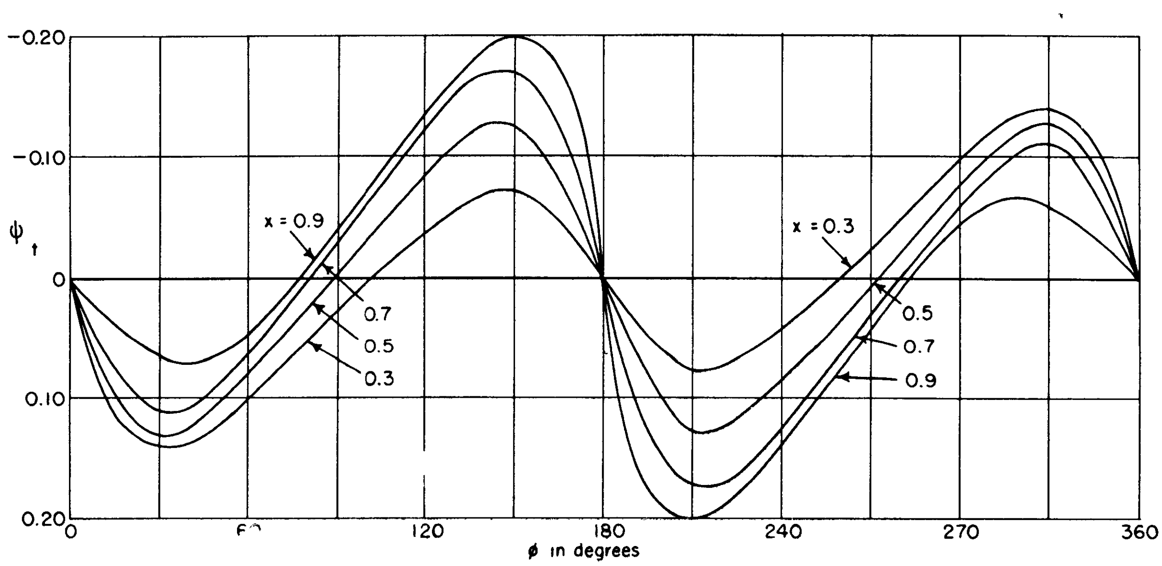


Figure 5 - Tangential Wake Factor

each radius. For the axial wake factor, the following averages are obtained:

| $x$ | 0.3 | 0.5 | 0.7 | 0.9 |
|---|---|---|---|---|
| $\overline{\psi}_a$ | 0.608 | 0.439 | 0.296 | 0.217 |

The expression [16] for $N$ is calculated at each radius, assuming $\overline{\alpha} = 0$, and this function is resolved into its Fourier series with respect to the angular coordinate. The result is as follows:

| $x$ | $\frac{a_1}{v_s}$ | $\frac{a_2}{v_s}$ | $\frac{a_3}{v_s}$ | $\frac{a_4}{v_s}$ | $\frac{a_5}{v_s}$ |
|---|---|---|---|---|---|
| 0.3 | 0.0117 | 0.0352 | 0.0047 | 0.0068 | 0.0020 |
| 0.5 | −0.0030 | 0.0353 | −0.0027 | 0.0080 | −0.0010 |
| 0.7 | −0.0138 | 0.0465 | −0.0163 | 0.0182 | −0.0118 |
| 0.9 | −0.0153 | 0.0270 | −0.0147 | 0.0003 | 0 |

| $x$ | $\frac{b_1}{v_s}$ | $\frac{b_2}{v_s}$ | $\frac{b_3}{v_s}$ | $\frac{b_4}{v_s}$ | $\frac{b_5}{v_s}$ | $\frac{b_6}{v_s}$ |
|---|---|---|---|---|---|---|
| 0.3 | 0.0277 | 0.0572 | −0.0127 | 0.0195 | −0.0157 | 0.0097 |
| 0.5 | −0.0310 | 0.1507 | 0.0113 | 0.0787 | 0.0063 | 0.0308 |
| 0.7 | −0.0468 | 0.2077 | 0.0175 | 0.1387 | 0.0065 | 0.0609 |
| 0.9 | −0.0883 | 0.2360 | 0.0057 | 0.1520 | 0.0337 | 0.0577 |

At each radius, the sum

$$\sum_{1}^{6} \left[ \left(\frac{a_m}{v_s}\right)^2 + \left(\frac{b_m}{v_s}\right)^2 \right] F(k_m)$$

is calculated. At $x = 0.7$, for instance, one obtains:

| $m$ | $k_m$ | $F$ | $\left(\frac{a_m}{v_s}\right)^2$ | $\left(\frac{b_m}{v_s}\right)^2$ | $\left[\left(\frac{a_m}{v_s}\right)^2 + \left(\frac{b_m}{v_s}\right)^2\right] F$ |
|---|---|---|---|---|---|
| 1 | 0.359 | 0.414 | 0.00018 | 0.00219 | 0.00098 |
| 2 | 0.717 | 0.529 | 0.00216 | 0.04314 | 0.02396 |
| 3 | 1.08 | 0.537 | 0.00027 | 0.00031 | 0.00031 |
| 4 | 1.43 | 0.417 | 0.00033 | 0.01924 | 0.00816 |
| 5 | 1.80 | 0.303 | 0.00014 | 0.00004 | 0.00006 |
| 6 | 2.15 | 0.234 | 0 | 0.00371 | 0.00087 |
| | | | | | $\Sigma = 0.03434$ |

Analogous calculations are carried out at each radius and the sum obtained is then multiplied by $(1 - \overline{\psi}_a)\, c/R$. The following tabulation gives the result of these calculations:

| $x$ | $\Sigma$ | $\frac{c}{R}$ | $(1-\overline{\psi}_a)$ | $\frac{d}{dx}(\Delta c_p)$ |
|---|---|---|---|---|
| 0.3 | 0.0028 | 0.403 | 0.392 | 0.0011 |
| 0.5 | 0.0155 | 0.495 | 0.561 | 0.0110 |
| 0.7 | 0.0343 | 0.521 | 0.704 | 0.0322 |
| 0.9 | 0.0409 | 0.387 | 0.783 | 0.0318 |

The distribution of the loss of propelling energy over the radius is shown on Figure 6. This curve complies in character with that of the distribution of the propeller forces. This means that the loss is great at stations where the forces are great and vice versa.

The result of the integration is $\Delta c_p = 0.015$. Since the power coefficient of the ship concerned amounts to $c_p = 1.380$, it follows that about 1.1 percent of the power input is lost for the propulsion of the ship on account of the variation of the wake. This order of magnitude of the loss is in compliance with results of experimental findings which, however, are very meager. In general, one would expect the experimentally determined loss to be somewhat greater than the analytical result since the latter is based on nonviscous flow. Thus, the effects of unsteady flow on the boundary layer of a section and on its drag coefficient are not taken into account.

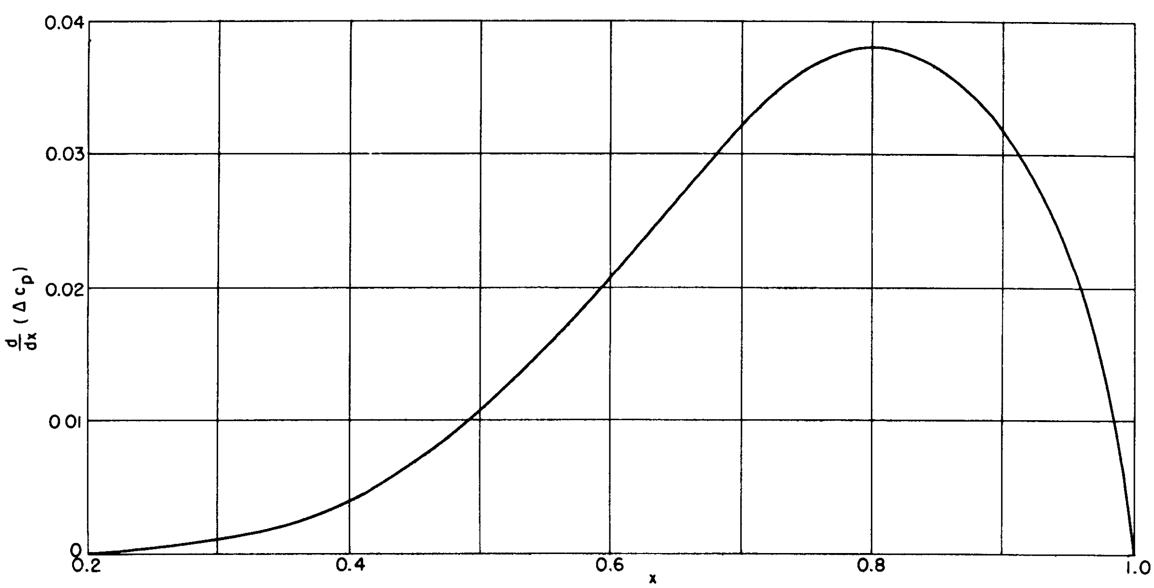


Figure 6 - Loss of Propelling Energy as Function of the Propeller Radius

## REFERENCES

1. von Kármán, Th. and Sears, W.R., "Airfoil Theory for Nonuniform Motion," Journal Aeronautical Sciences, 5, 1938.

2. Sears, W.R., "Some Aspects of Nonstationary Airfoil Theory and Its Practical Application," Journal Aeronautical Sciences, 8, 1941.

3. von Kármán, Th. and Burgers, J., "General Aerodynamic Theory of Perfect Fluids," Volume II, Aerodynamic Theory, Springer, Berlin, 1934.

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