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WASHINGTON 7, D.C.

FEASIBILITY STUDIES OF THE ROLL STABILIZATION
OF THE USS BOSTON (CAG-1)

by
Grant R. Hagen

September 1955
Report 950
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**NOTATION**

The notation in this report has been chosen to agree in general with the notation of References 3, 4 and 5. Wherever necessary, additional notation has been introduced and defined.

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<thead>
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<th>Symbol</th>
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<tr>
<td>ψ</td>
<td>Effective instantaneous waveslope amplitude, in radians</td>
</tr>
<tr>
<td>ψ_{max}</td>
<td>Actual capacity of stabilizer at any frequency (for tank stabilizer), or capacity at any speed U (for fin stabilizer)</td>
</tr>
<tr>
<td>ψ_{static}</td>
<td>Maximum (static) capacity of tank stabilizer</td>
</tr>
<tr>
<td>ψ_0</td>
<td>Required peak capacity of tank stabilizer at any time</td>
</tr>
<tr>
<td>ω</td>
<td>Apparent frequency of waves</td>
</tr>
<tr>
<td>ω_s</td>
<td>Natural frequency of ship in roll</td>
</tr>
<tr>
<td>ω_t</td>
<td>Natural frequency of tanks</td>
</tr>
<tr>
<td>ω_{st}</td>
<td>&quot;Decoupling&quot; frequency</td>
</tr>
<tr>
<td>Ω</td>
<td>Normalized apparent frequency of waves = ω/ω_s</td>
</tr>
<tr>
<td>Ω_t</td>
<td>Normalized frequency of tank fluid = ω_t/ω_s</td>
</tr>
<tr>
<td>Ω_{st}</td>
<td>Normalized &quot;decoupling frequency&quot; = ω_{st}/ω_s</td>
</tr>
<tr>
<td>Δ</td>
<td>Displacement of ship in long tons</td>
</tr>
<tr>
<td>VOL</td>
<td>Volume displacement of ship</td>
</tr>
<tr>
<td>B</td>
<td>Maximum beam of ship at load waterline</td>
</tr>
<tr>
<td>GM</td>
<td>Transverse metacentric height</td>
</tr>
<tr>
<td>T_s</td>
<td>Natural roll period of ship</td>
</tr>
<tr>
<td>b</td>
<td>Normalized beam of ship = B/(VOL)^{1/3}</td>
</tr>
<tr>
<td>R</td>
<td>Radius of roll = (T_s/2π)^2g</td>
</tr>
<tr>
<td>P_0</td>
<td>&quot;Standard Horsepower&quot; = Δ(VOL)^{1/6}(g)^{1/2} 2240/550</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of roll of ship</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Angle of tank-water level with respect to ship</td>
</tr>
<tr>
<td>$K_{SS}$</td>
<td>Ship-sea coefficient. Torque of waves = $K_{SS} \psi$</td>
</tr>
<tr>
<td>$K_s$</td>
<td>Static righting coefficient of ship. Righting moment = $K_s \theta$</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Static righting coefficient of tank water. Torque = $K_t \phi$</td>
</tr>
<tr>
<td>$A_0$</td>
<td>Average cross-sectional area of side tank</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Area of U-tube at any point</td>
</tr>
<tr>
<td>$D_s$</td>
<td>Perpendicular distance from center of rotation to any tank-water element</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>Average athwartships dimension of side tank</td>
</tr>
<tr>
<td>$l$</td>
<td>Tank lever arm</td>
</tr>
<tr>
<td>$H$</td>
<td>Maximum change of water level in side tanks</td>
</tr>
<tr>
<td>$S'$ and $S''$</td>
<td>&quot;Weighted&quot; lengths of U-tube</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of sets of U-tubes</td>
</tr>
<tr>
<td>$K_p$</td>
<td>Power coefficient</td>
</tr>
<tr>
<td>$Q_{t'}$</td>
<td>Magnification ratio of tanks (without pump)</td>
</tr>
<tr>
<td>$\rho_{fl}$</td>
<td>Density of stabilizing fluid</td>
</tr>
<tr>
<td>$\rho_{sw}$</td>
<td>Density of salt water</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Cavitation number</td>
</tr>
<tr>
<td>$P_0$</td>
<td>Static pressure in fluid</td>
</tr>
<tr>
<td>$e$</td>
<td>Vapor pressure of fluid</td>
</tr>
<tr>
<td>$V_k$</td>
<td>Ship speed in knots</td>
</tr>
<tr>
<td>$U$</td>
<td>Ship speed</td>
</tr>
<tr>
<td>$C_N$</td>
<td>Normal force coefficient of fins</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Lift coefficient of fins</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Total drag coefficient of fins (= C_{D_{\text{min}}} + C_{D_{\text{stab}}} C_L^2 )</td>
</tr>
<tr>
<td>$C_{D_{\text{min}}}$</td>
<td>Minimum drag coefficient of fins</td>
</tr>
<tr>
<td>$C_{D_{\text{stab}}}$</td>
<td>Stabilizing drag coefficient of fins</td>
</tr>
<tr>
<td>AR_{eff}</td>
<td>Effective aspect ratio of fins</td>
</tr>
<tr>
<td>$A$ or $A^*$</td>
<td>Projected area of fin(s)</td>
</tr>
<tr>
<td>$L$ or $L^*$</td>
<td>Lift of fin(s)</td>
</tr>
<tr>
<td>$D$ or $D^*$</td>
<td>Drag of fin(s)</td>
</tr>
<tr>
<td>$M_1$ and $M_2$</td>
<td>Moment components of fin</td>
</tr>
<tr>
<td>$c$</td>
<td>Chord length of fin</td>
</tr>
<tr>
<td>$s$</td>
<td>Span of fin</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance from fin axis to center of pressure</td>
</tr>
<tr>
<td>$k_1$ and $k_2$</td>
<td>Coefficients in moment equation for fins</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle of attack of fins</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
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ABSTRACT

The feasibility of stabilizing the roll of large naval vessels has been studied by actually making preliminary designs of several systems of stabilization for USS BOSTON. These include a system using activated U-tube tanks and four systems using activated fins. The weight, space, and power requirements of these systems have been estimated. Stabilization by passive systems, by moving solid weights, and by rotating fins are discussed briefly.

Weight and space requirements of activated fins appear to be less than one half the requirements for activated U-tube tanks. The auxiliary power required to activate the fins is far below that required to activate the tanks.

INTRODUCTION

The Bureau of Ships requested the Taylor Model Basin to make feasibility studies of the roll stabilization* of USS BOSTON (CAG-1). The purpose of these studies was to provide a basis for making recommendations to the Bureau of Ships concerning the method of stabilization which appears to have the greatest long-range interest to the Navy for application to ships comparable in size to USS BOSTON(1)**. It was requested specifically that the studies include: an activated-fin system, an activated-tank system, and a heavily-damped, passive tank system using tuned, moving weights. Interest in such studies arose because it had been estimated that the weight of an activated-fin system would be as high as 4 percent of the displacement of a vessel as large as USS BOSTON, and this was considered not acceptable.

This report offers independent estimates of weight and space requirements for systems using activated fins, activated tanks, and moving, solid weights. The estimates are based on design studies of an activated tank system, four activated-fin systems, and a moving-weight system for USS BOSTON. Rotating fins are

*In this report the expression "roll stabilization" is defined to mean stabilization of the ship in the roll degree of freedom.

**References are listed on pages 50 and 51 of this report.
discussed as a possible means of stabilization, but no weight estimate is made for a rotating-fin system.

The major portion of the report is concerned with the design studies of the tank system and various fin systems. These studies are based largely on the work done at Stanford University by Chadwick and Morris under contract with the Office of Naval Research (2, 3, 4). Only a superficial study was made of the heavily-damped, passive tank system using tuned, moving weights. In this system the stabilization is accomplished by shifting solid weights and the only purpose of the tank is to provide damping.

For activated systems the studies assume that adequate controls can be devised so that rolling can be virtually extinguished, provided the design capacity is not exceeded. A passive system cannot extinguish all roll no matter what the weight of the system may be, and its effectiveness is dependent on the apparent wave period.

The report offers comparisons of the several systems with respect to: effectiveness in stabilizing, weight and space requirements, effect on ship operations, and cost. On the basis of these comparisons, recommendations for a ship stabilization program are offered.

DESIGN OF ROLL STABILIZATION SYSTEMS FOR THE USS BOSTON (CAG-1)

Roll stabilization is accomplished by producing torques which counteract the torques imposed on the ship by the sea. Three ways of producing the stabilizing torques are considered: utilization of lift produced by fins which project outward from the vessel in way of the bilges, transfer of water between tanks on opposite sides of the vessel, and transfer of solid weights from side to side of the vessel. All of the design studies contained herein apply to USS BOSTON. The pertinent particulars for this vessel are given in Table 1.

A summary of the principal results of the studies for USS BOSTON are presented in Table 2, following which the report continues with the studies themselves. It should be borne in mind that the numbers in Table 2 can vary somewhat depending on the details of the systems.
**TABLE 1**

Particulars for USS BOSTON

<p>| | |</p>
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<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Displacement, full load, tons</td>
<td>17,550</td>
</tr>
<tr>
<td>LBP, ft.</td>
<td>664.0</td>
</tr>
<tr>
<td>Extreme breadth, ft.</td>
<td>69.7</td>
</tr>
<tr>
<td>Draft (from model displacement curves), ft.</td>
<td>24.6</td>
</tr>
<tr>
<td>Metacenter above baseline, ft.</td>
<td>32.0</td>
</tr>
<tr>
<td>Center of gravity above baseline, ft.</td>
<td>27.6</td>
</tr>
<tr>
<td>Metacentric height, ft.</td>
<td>4.4</td>
</tr>
<tr>
<td>Center of buoyancy above baseline (approx.), ft.</td>
<td>14.75</td>
</tr>
<tr>
<td>Natural period of roll, seconds</td>
<td>16.5</td>
</tr>
<tr>
<td>Shaft horsepower</td>
<td>120,000</td>
</tr>
</tbody>
</table>

**TABLE 2**

Summary of Results of Design Studies for USS BOSTON

<table>
<thead>
<tr>
<th></th>
<th>Retractable Fins</th>
<th>U-tube Tanks</th>
<th>Moving Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Capacity</strong></td>
<td>5° at 15 knots</td>
<td>7°</td>
<td>7°</td>
</tr>
<tr>
<td><strong>Number of Fins</strong></td>
<td>6</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td><strong>Number of Tanks</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Number of Weights</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Power of Positioning Motors, horsepower</strong></td>
<td>120</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Propulsive Power Absorbed, horsepower</strong></td>
<td>1900</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Pump Output $\Omega_t=1.0$</strong></td>
<td></td>
<td>16,200</td>
<td></td>
</tr>
<tr>
<td><strong>Rating, hp $\Omega_r=1.3$</strong></td>
<td></td>
<td>7,640</td>
<td>660</td>
</tr>
<tr>
<td><strong>Positioning $\Omega=1.0$</strong></td>
<td></td>
<td></td>
<td>5,30C</td>
</tr>
<tr>
<td><strong>Power, hp $\Omega=2.0$</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Source of Power</strong></td>
<td></td>
<td>Auxiliaries and Main Propulsion Plant</td>
<td>Auxil.</td>
</tr>
<tr>
<td><strong>Weight (% of $\Delta$)</strong></td>
<td>2.0</td>
<td>4.4</td>
<td>3.0</td>
</tr>
<tr>
<td><strong>Space (% of VOL)</strong></td>
<td>2.0</td>
<td>5.5</td>
<td>4.0</td>
</tr>
</tbody>
</table>
ACTIVATED FIN SYSTEM

The lift, and hence the stabilizing torque, produced by activated fins can be varied by changing the angles of attack of the fins. The angles are changed by motors which position the fins in response to automatic controls.

Capacity

A system which will effect adequate stabilization in a given sea may be unsatisfactory when the ship encounters heavier seas. Thus, a stabilization system is designed for a certain capacity, and the designers' first task is to select such capacity.

In line with the development presented in Reference 2, the capacity of a stabilization system is defined as the maximum effective waveslope that can be neutralized. This is assumed to be the maximum static list that can be eliminated by the system. Reference 3 offers the following empirical formula as a guide in selecting a desirable system capacity:

\[
\text{Desirable capacity} = 0.36 / \log_{10} A \text{ radians} \quad [1]
\]

where \( A \) is the ship's displacement in long tons. For USS BOSTON, Formula [1] yields a capacity of 4.8 degrees.

For a fin system, the stabilizing torque is a function of the lift of the fins. The lift in turn, and consequently the capacity of a fin system, varies with the speed of the ship. Therefore, it is necessary to relate capacity to a particular speed. On the basis of Formula [1] and consideration of past practice a system capacity of 5 degrees for a moderate speed appears to be reasonable for USS BOSTON. The lift developed by fins at a given angle of attack varies with the square of the speed up to the speed of incipient cavitation, beyond which it continues to increase with speed, but at a lesser rate. The fins considered in this study begin to cavitate at about 15 knots when they are operating at the maximum angle of 20 degrees. At lesser angles of attack the critical cavitation speed will be higher. Therefore, for the fin systems considered, the variation of lift with speed is such that if the capacity is 5 degrees at 15 knots, it will be about 14 degrees at 30 knots and 2.4 degrees at 10 knots.

The economics of the problem also should be considered, and it is clear that cost of stabilizing tends to increase with
capacity. Perhaps it would be more economical to select a capacity of 6 degrees at 20 knots corresponding to a capacity of about 1.8 degrees at 10 knots, 3.6 degrees at 15 knots, and 10.4 degrees at 30 knots. The capacity at the lower speeds is very small, and it is this characteristic of fin systems which is probably the most unattractive. It will be instructive to design systems both for a capacity of 5 degrees at 15 knots and 6 degrees at 20 knots.

Location of Fins

Figure 1 shows the outline of the maximum transverse section of USS BOSTON and gives three favorable locations for the fins. If the fins are made retractable the internal arrangements for housing and operating them should be most favorable when the fins are mounted horizontally as in Scheme A on Figure 1. However, the lift cannot be utilized fully if the ship has a large bilge radius (as CAG-1) because if they are located in way of the bilge (and it is not practical to have them above the turn of the bilge because of proximity to the surface) the turn of the bilge shortens the moment arms so that the arms for Scheme A are less than for Schemes B and C. It is likely too that there will be undesirable interference effects between the fins and the hull.

![Figure 1 - Maximum Transverse Section of USS BOSTON, Showing Three Possible Fin Locations](image-url)
Scheme B is an arrangement with the fins more nearly normal to the hull. The axes of the fins intersect the assumed axis of roll of the ship*. In this design study the BOSTON'S axis of roll is assumed to be midway between the ship's center of gravity and center of buoyancy. This assumption is the same as was made by Chadwick and Morris in Reference 4.

In the case of BOSTON the error in this approximation to the actual axis of roll is not serious. If the correct position is higher than assumed the fins will be more effective, for the moment arms will be greater. If, however, the axis of roll is as low as the center of buoyancy then the fin area need be increased by only 7 per cent to compensate for the smaller moment arm. The resulting increase in weight in the latter case would amount to about 0.2 per cent of the ship's displacement.

With the axis of roll in the assumed position the moment arms for the fins of Scheme B are somewhat longer than for Scheme A. In both of these schemes the fins would have a rather large outreach, and should therefore be made retractable.

Scheme C also has the axes of the fins passing through the assumed roll axis of the vessel. The fins extend only to the corner of the rectangle which bounds the midship section. Here non-retractable fins may be acceptable with consequent marked simplification of internal arrangements as well as large savings in weight and space. However, the smaller span will require larger chords or more fins, so the savings may not be as large as it first seems. Non-retractable fins have the disadvantage that they will always cause a drag force when the vessel is underway. However, as indicated on Figure 8, the power loss due to fins is small when they are not stabilizing and are thus operating at zero angle of attack. At 20 knots it is less than two tenths of one per cent of the design SHP for the assumed drag coefficient. Reference 8 concludes that the loss is entirely negligible, and Reference 2 states that the choice between retractable fins and non-retractable fins is essentially a matter of space and complexity versus the problem of avoiding damage.

*The axis of roll does not pass through the center of gravity of the ship, but rather through the "virtual" center of gravity of the ship combined with the water mass which it influences. Wendel offers an approximate method for determining the axis of rotation of a body such as a ship, and in an illustrative example the "virtual" center of gravity is found to be below the ship's center of gravity (7).
The location of the fins governs the number and size of fins, and consequently the weight of a system. These are quantities which must be known before one system can be selected in preference to another.

Lift Coefficients

Reference 1 states that interest in the present study sprang "from the fact that as the size of a ship is increased without change of geometry, activated fins have less and less attraction from the standpoint of required weight and space." The fewer fins required, the smaller is the encroachment on weight and space, and the more attractive the system becomes. Evidently the smallest number of fins will be required with fins having the largest lift per unit area.

Three types of fins appear to be practicable for roll stabilization. Doubly all movable fins yield the most lift; all movable fins rank next; and flapped (fixed front) fins are poorest. These designations are used in the absence of any universally accepted designations. Sections of each type are shown on Figure 2.

Figure 2 - Sections of Three Types of Fins
The relatively low lift of the flapped fin with a fixed front rules out this type. An idea of the relative effectiveness of the all movable fin and doubly all movable fin can be obtained from Figure 3. Two curves of lift coefficient $C_L$ versus fin angle $\alpha$ are shown for a fin which could be operated as either an all movable fin (Curve A) or as a doubly all movable fin (Curve B). The data for the curves were obtained from a report by William Denny and Brothers, Ltd (9). The doubly all movable fin is preferable in that it produces much more lift per unit area than the all movable fin. A more complex mechanism is required to control the deflection of the tail fin for the doubly all movable fin; but the increased complexity did not prevent the selection of this type of fin for the stabilization system designed for USS TIMMERMAN. (The system was not installed on TIMMERMAN, but a fin system is to be installed on another naval vessel).

![Figure 3 - Lift Coefficients for a Doubly All Movable Fin Operating Normally and Operating as an All Movable Fin](image-url)
This study will assume, therefore, that the fins are doubly all movable. Except as discussed later, the lift characteristics of the fins will be assumed the same as those given in Reference 9. These characteristics were obtained from open water tests and cavitation tests on two fins, one of constant section along the span, the other with varying thickness ratio giving an increased thickness at the root. Both fins are of rectangular planform and have the same geometric aspect ratio, viz., 2.22. When operating without cavitation, or when operating at the same cavitation number the lift coefficients for both fins are very close.

Figure 4 - Lift Coefficients for Doubly All Movable Fins, With Aspect Ratio = 2.22, when Operating Without Cavitation

Figure 4 shows lift coefficients as a function of main fin angle and tail fin angle for the doubly all movable fins without...
cavitation. Curve 1 gives data from tests conducted at a Reynolds number of $0.77 \times 10^6$. In these tests one end of the fin was close to a boundary plate which was taken to represent the hull adjacent to the fin. The other end was in three-dimensional flow. The lift coefficients should therefore be applicable to full-scale installations, except for the effects introduced by the wide difference in the Reynolds numbers. The maximum lift coefficient attained was 1.46. Curve 2 shows an assumed increase in stall angle up to 20 degrees that would be expected to result at large Reynolds numbers, and perhaps with some change in section profile.

The possibility of attaining a lift coefficient as large as 1.70 at a main fin angle of 20 degrees is evident from Figure 20 of Reference 8. Here for a doubly all movable fin having a constant ratio of 1.5 between main fin angle and tail fin angle relative to main fin a lift coefficient of 1.75 at a main fin angle of 20 degrees is attained. The geometric aspect ratio of this fin is 1.72, but it was tested with an end plate. The chord of the tail fin is 25 per cent of the total chord, as it is for the fin of Curve 1. Wind tunnel tests at the Taylor Model Basin have shown that doubly all movable fins having a geometric aspect ratio as low as 0.37 can also attain lift coefficients as large as 1.70, but the fin angle must be about 40 degrees, and the drag is large (10).

On the basis of the foregoing it will be assumed that for doubly all movable fins having geometric aspect ratios between 1.75 and 2.25 it is possible to attain a lift coefficient of 1.70 at a fin angle of 20 degrees when there is no cavitation. For estimating the effects of cavitation in the design studies which follow, it is assumed that the fins of all three Schemes, A, B, and C are at an immersion depth of 18 feet, the actual depth of the fins of Scheme A. The cavitation number $\sigma$ is given by

$$\sigma = \frac{P_o - e}{\frac{1}{2} \rho U^2}$$

[2]

where $\sigma$ is the cavitation number at a designated point in the fluid,

$P_o$ is the static pressure at the designated point, and is comprised of the atmospheric pressure and the head of water over the fin,

$e$ is the vapor pressure of the fluid, and is taken for a temperature of 50°F,

$\rho$ is the mass density of the fluid, and

$U$ is the relative speed of the fluid at the designated point, and is taken to be the forward speed of the ship.
For the USS BOSTON problem, the cavitation number is

\[ \sigma = \left( \frac{51.0 - 0.41}{0.5 \times 2.0 \times 2.85 V_k^2} \right) \frac{64}{V_k^2} = 1135 \]

where \( V_k \) is the ship speed in knots.

Figure 5 shows curves of lift coefficients of two doubly all movable fins for various cavitation numbers which have been taken from Reference 9. The lift characteristics for the fin of constant section as well as for the tapered fin are shown. Figure 6 was derived from Figure 5 except that the maximum lift coefficient of 1.70 has been assumed in lieu of 1.46, as discussed previously. Figure 6 gives a curve of maximum lift coefficient \( C_{L_{max}} \) versus the cavitation number \( \sigma \) and is an average curve for the two type of fins for \( \sigma \) up to 2.0. No test data were given for \( \sigma \) between 2.0 and 9.06 and the value of \( \sigma \) for inception of cavitation was not stated so that in this interval the character of the curve is not known. The photographs in Reference 9 show relatively little cavitation at a \( \sigma \) of 2.0 as compared with lower values of \( \sigma \). Therefore the curve between \( \sigma \) of 2.0 and 9.06 was drawn on the basis that the cavitation became progressively less over this range and was non-existent when \( \sigma \) reached 9.06. Inasmuch as the inception point could have been lower than 9.06, the curve of Figure 6 may be somewhat conservative. In all probability the actual curve will lie above the one shown. The values of \( C_{L_{max}} \) of Table 3 were obtained from the curve in Figure 6.

The lift \( L \) of the fin can be obtained from the formula:

\[ L = C_L \frac{1}{2} \rho A U^2 \]  

where \( C_L \) is the lift coefficient of the fin and \( A \) is the projected area of the fin. The maximum lift of a given fin can be expressed as a function of the cavitation number as follows:

\[ \text{Lift}_{\text{max}} = KV_k^2 C_{L_{\text{max}}} \]

where \( K \) is a constant and equals \( \frac{1}{2} \rho A x \frac{1}{2} (1.688)^2 \) and \( C_{L_{\text{max}}} \) is a function of the cavitation number. For the doubly all movable fin being considered, the values of \( C_{L_{\text{max}}} \) for various cavitation numbers are taken from Table 3. The dimensional values of maximum lift in terms of \( K \) are also listed in Table 3 and plotted against speed in knots in Figure 7.
TABLE 3

Effect of Cavitation on Lift Characteristics of Fins

<table>
<thead>
<tr>
<th>(V_k)</th>
<th>(\sigma)</th>
<th>(C_{L_{\text{max}}})</th>
<th>(\text{Lift}<em>{\text{max}} = KV_k^2C</em>{L_{\text{max}}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>45.45</td>
<td>1.70</td>
<td>43K</td>
</tr>
<tr>
<td>10</td>
<td>11.35</td>
<td>1.70</td>
<td>170K</td>
</tr>
<tr>
<td>13</td>
<td>6.72</td>
<td>1.67</td>
<td>282K</td>
</tr>
<tr>
<td>15</td>
<td>5.05</td>
<td>1.63</td>
<td>367K</td>
</tr>
<tr>
<td>20</td>
<td>2.84</td>
<td>1.48</td>
<td>592K</td>
</tr>
<tr>
<td>25</td>
<td>1.82</td>
<td>1.30</td>
<td>812K</td>
</tr>
<tr>
<td>30</td>
<td>1.26</td>
<td>1.10</td>
<td>990K</td>
</tr>
<tr>
<td>33</td>
<td>1.04</td>
<td>1.01</td>
<td>1100K</td>
</tr>
</tbody>
</table>

Figure 5 - Lift Coefficients for Doubly All Movable Tapered Fin and Non-Tapered Fin at Several Cavitation Numbers
Design of Fins

It was proposed earlier to investigate the three Schemes, A, B and C, shown on Figure 1. As two capacities have also been proposed, a total of six fin systems require investigation. However, after consideration of the several systems Scheme A can be eliminated without making a detailed investigation. The fins of Scheme A are horizontal and it may be easier to provide space for the installation of this system than it is for the others. However, Scheme A would require more fin area to compensate for the shorter moment arm as well as for interference effects at the hull (see page 5).

The procedure followed in designing the fins is presented in detail for Scheme B using a capacity of 5 degrees at 15 knots. For the other schemes, only a tabulation of the pertinent quantities which enter into the design process is given by Table 4. The capacity of 5 degrees at 15 knots is designated by subscript 1 and the capacity of 6 degrees at 20 knots by subscript 2.

![Figure 6](image_url)  
**Figure 6 - Maximum Lift Coefficients of Doubly All Movable Fins Expressed as a Function of the Cavitation Number**
Scheme B₁ - Capacity: 5° at 15 knots

Required stabilizing torque = $\Delta GM_{y_{\text{max}}}$

\[ = \frac{17,550 \times 4.4 \times 5}{57.3} \]

\[ = 6740 \text{ ton-feet} \]

Assume a fin span of 12.0 feet to give a relatively high aspect ratio and consequently a relatively high lift/drag ratio. The moment arm is then 38.5 feet and the required lift per side is

\[ \frac{(6740 \times 2240)}{(38.5 \times 2)} = 196,500 \text{ lbs} \]

Figure 7 - Maximum Lift of Doubly All Movable Fins, Expressed as a Function of Speed, Showing the Effect of Cavitation
If the type of fin applicable to Curve 2 of Figure 4 is used \( C_{l_{\text{max}}} \) will be 1.70 if separation limited, or 1.63 if cavitation limited (at 15 knots). As the fin is cavitation limited, then at 15 knots the lift per square foot will be 
\[
(15 \times 1.688)^2 \times 1.63 = 1045 \text{ lbs}
\]
and the required fin area per side = 196,500/1045 = 188.0 square feet. If 3 fins per side are used, the fins will have a chord of 5.22 feet and a geometric aspect ratio of 2.30. It appears that a stabilization system designed for a capacity of 6 degrees at 20 knots would be easier to provide for than one designed for a capacity of 5 degrees at 15 knots. Of course the system designed for 15 knots would provide better stabilization at all speeds than would the system designed for 20 knots.

The fins were designed on the assumption that if more than one fin were required on each side they would be arranged in tandem. With this arrangement the fins which operate in the downwash of preceding fins experience a serious loss of lift unless they are spaced a considerable distance apart. If the spacing is 10 chord lengths or more the loss of lift probably is of no serious concern. Assuming chords of 6 feet, even 3 fins per side would extend over a length of only 120 feet, and installation probably would not be particularly difficult. Furthermore the fins could be confined to a region near amidships where the moment arm of the fins is affected only slightly by change in form of the sections of the ship, and where the pitching action of the ship would have relatively little effect on the depth of immersion of the fins.

Although Scheme C requires one more fin per side over Scheme B, for both capacities, a maximum of 4 fins per side is not unreasonable. The relatively small span and loading of these fins makes the structural problem simpler than for larger fins. Of the three schemes considered, the fins of Scheme C have the deepest immersion and consequently they would be affected the least by cavitation. Furthermore, since the fins of Scheme C do not extend beyond the limits of the rectangle which bounds the maximum transverse section, the possibility of damage is minimized even if the fins are not retractable. The advantages of this scheme thus make it a very interesting possibility for a system of ship stabilization. It may be noted that the British cruiser CUMBERLAND has a stabilization system much like this.

**Structural Considerations**

The problem of designing a fin system has been considered so far without regard for any structural limitations which might
<table>
<thead>
<tr>
<th></th>
<th>Scheme B₁</th>
<th>Scheme B₂</th>
<th>Scheme C₁</th>
<th>Scheme C₂</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Capacity</strong></td>
<td>5° at 15 knots</td>
<td>6° at 20 knots</td>
<td>5° at 15 knots</td>
<td>6° at 20 knots</td>
</tr>
<tr>
<td>Required stabilizing torque, tons-feet</td>
<td>6740</td>
<td>8090</td>
<td>6740</td>
<td>8090</td>
</tr>
<tr>
<td>Assumed span of fin, feet</td>
<td>12.0</td>
<td>12.0</td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td>Lever arm, feet</td>
<td>38.5</td>
<td>38.5</td>
<td>36.0</td>
<td>36.0</td>
</tr>
<tr>
<td>Required lift per side, pounds</td>
<td>196,500</td>
<td>236,000</td>
<td>210,000</td>
<td>253,000</td>
</tr>
<tr>
<td>$C_{L_{max}}$ (Cavitation limited)</td>
<td>1.63</td>
<td>1.48</td>
<td>1.63</td>
<td>1.48</td>
</tr>
<tr>
<td>Required fin area per side, feet²</td>
<td>188</td>
<td>140</td>
<td>201</td>
<td>150</td>
</tr>
<tr>
<td>Number of fins per side</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Chord of fins, feet</td>
<td>5.22</td>
<td>5.84</td>
<td>5.3</td>
<td>5.3</td>
</tr>
<tr>
<td>Geometric aspect ratio of fins</td>
<td>2.30</td>
<td>2.06</td>
<td>1.79</td>
<td>1.79</td>
</tr>
</tbody>
</table>
exist. The largest loads and bending moments in the shaft of the fins would occur for Scheme B2 with the ship travelling at top speed, say 30 knots. At this speed the maximum lift coefficient is 1.10. The drag coefficient $C_D$ can be estimated as follows:

$$C_D = C_{D\text{min}} + \frac{C_L^2}{\pi \times \text{AR}_{\text{eff}}} \approx 0.01 + \frac{(1.10)^2}{\pi \times 2 \times 2.06} = 0.1034 \quad [5]$$

where $\text{AR}_{\text{eff}}$ is the effective aspect ratio. Then

- the maximum lift is $(30 \times 1.688)^2 \times 12 \times 5.84 \times 1.10 = 198,000$ lbs,
- the maximum drag is $198,000 \times \frac{0.1034}{1.10} = 18,600$ lbs, and
- the maximum resultant load is $201,000$ lbs.

The flexural stress in the shaft of the fin will be a maximum at the outer bearing which is assumed to be located where the shaft enters the hull. The resultant load on the fin is assumed to act at the middle of the span, although the usual span-wise distribution of lift is such as to cause the resultant to be inboard of midspan. The maximum allowable flexural stress is taken to be 40,000 psi, according to BuShips specifications for the design of rudderstocks. An allowable working stress of 20,000 psi is assumed for stress due to torsion.

Figures 9 and 10 show that the axis of the shaft can be located so that the distance from the axis to the center of action of the resultant force will always be far less than one half of the chord. Then the torque about the axis of the stock will always be less than a maximum torque of $201,000 \times 5.84/2 \times 12 = 7,040,000$ lb-in. The maximum bending moment is $201,000 \times 6 \times 12 = 14,460,000$ lb-in. The preceding loads and allowable stresses may be used in well-known formulas for combined stresses to find that a shaft diameter of 16 inches is required. The maximum thickness of the fin is considerably larger than 16 inches; thus it is feasible to build a fin which will accommodate the loads required to stabilize USS BOSTON even if only two fins per side are used.

**Weight of Installation**

It is difficult to estimate the weight of a fin installation on the basis of available data. In Reference 3 the view is express-
that, other things being equal, the per cent weight of fin installations should vary approximately as \((\text{normalized area per side})^{3/2}\). The normalized area/side is defined as: \((\text{fin area per side}) + (\text{displacement volume})^{2/3}\). To use this technique for estimating the weight of a proposed installation requires that the pertinent data for some other installation be known and that the "existing" and proposed installations be similar, i.e., both systems should have retractable fins or both should have non-retractable fins. Of the meager data which are available, the weights for USS TIMMERMAN are perhaps the most pertinent for projecting to other ships. The installation designed for this vessel employs doubly all movable fins having aspect ratios of about the same magnitude as proposed for USS BOSTON.

### DATA FOR USS TIMMERMAN

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Displacement, tons</strong></td>
<td>3409</td>
</tr>
<tr>
<td><strong>Stabilization weights, tons</strong></td>
<td></td>
</tr>
<tr>
<td>Machinery</td>
<td>60.7</td>
</tr>
<tr>
<td>Lost buoyancy</td>
<td>9.5</td>
</tr>
<tr>
<td>Total weight</td>
<td>70.2</td>
</tr>
<tr>
<td><strong>Weight of system as per cent</strong></td>
<td></td>
</tr>
<tr>
<td>of displacement</td>
<td>2.1</td>
</tr>
<tr>
<td><strong>Area of one 10' x 4½' fin</strong></td>
<td></td>
</tr>
<tr>
<td>(retractable), feet²</td>
<td>45</td>
</tr>
<tr>
<td>((\text{Normalized fin area/side})^{3/2})</td>
<td>2.52 x 10⁻³</td>
</tr>
</tbody>
</table>

*Stabilization designed for 5½° at 20 knots

Then for USS BOSTON the \((\text{normalized fin area/side})^{3/2}\) with Scheme B₁ is 4.40 x 10⁻³, and with Scheme B₂ is 2.72 x 10⁻³. Then the weights added for Schemes B₁ and B₂ are 3.7 and 2.3 per cent of the displacement, respectively. Scheme A would require more fin area and accordingly would weigh more by the above procedure. Scheme C would weigh substantially less than the preceding estimates if the fins were made non-retractable.

It should be appreciated that the foregoing method provides only a rough approximation to the actual weight. It may be possible to obtain a better approximation by another method of extrapolation. If the weight per fin can be estimated with reasonable accuracy, on the basis of existing installations, then the total weight can be estimated with comparable accuracy. Consider first Scheme B₁. The proposal was made that 3 fins per side be used, the fins having the dimensions of 5.22 feet by 12 feet, or an area of 62.7 square
feet. The fins proposed for USS BOSTON are of very nearly the same proportions as those for USS TIMMERMAN, for which the weights per fin are:

\[
\begin{align*}
\text{Machinery} & \quad 30.35 \text{ tons} \\
\text{Lost buoyancy} & \quad 4.75 \text{ tons}
\end{align*}
\]

Each fin of USS TIMMERMAN has an area of 45 square feet. If the relative magnitudes of the linear dimensions for the fins of the two ships are designated by the symbol \( \lambda \), then \( \lambda^2 = \frac{62.7}{45} = 1.39 \), and \( \lambda = 1.18 \).

The volume of the box required to house a fin may be assumed to vary as \( \lambda^3 \). Then for USS BOSTON the lost buoyancy is \( 4.75 \times (1.18)^3 = 7.8 \) tons per fin. Only the total machinery weight for USS TIMMERMAN is available, with no breakdown into housing and operating-machinery components. The weight of the structure for housing the fin may be expected to vary roughly as \( \lambda^3 \), but it does not seem likely that the weight of the operating machinery would vary by so large a factor. Indeed the power per fin should not differ appreciably since the ship speeds, period of oscillation of the fins, and fin dimensions for the two ships do not differ greatly. Furthermore the weights of the motors which drive the fins do not vary even linearly with power. It should be conservative, therefore, to assume that the total machinery weights vary as \( \lambda^3 \). The validity of this assumption is confirmed by Table VI of Reference 8, which gives weight data for the fin systems of two different ships.

On the basis of the preceding assumptions, the weight of Scheme B\(_1\) for USS BOSTON is estimated to be \( 35.1 \times (1.18)^3 = 57.7 \) tons per fin, and 346.2 tons, or 1.97 per cent of the displacement for the complete installation of 6 fins. Similarly, the weight of Scheme B\(_2\) is estimated to be 1.56 per cent of the displacement. These weight estimates are markedly less than those obtained before, and they are considered to be fairly accurate.

If the latter procedure is used to estimate the weights of Scheme C\(_1\) and Scheme C\(_2\), the lost buoyancy term is eliminated, for the fins are assumed to be non-retractable. For these schemes \( \lambda = 1.06 \), and the weight of Schemes C\(_1\) and C\(_2\) are 1.65 and 1.24 per cent of the displacement, respectively. These estimates are probably too high inasmuch as no allowance was made for the fact that Scheme C\(_1\) and C\(_2\) require no machinery for retracting the fins and no boxes for housing them.
Propulsive Power Absorbed by Fins

When the fins are extended they increase the resistance of the ship, and this additional drag must be overcome by the ship's propulsion plant. This additional power can be calculated by using the drag coefficient of the fin:

\[
CD = \frac{D}{\frac{1}{2}\rho U^2} = CD_{\text{min}} + CD_{\text{stab}} C_L^2 \quad [6]
\]

where

- \( D \) is the total drag of the fin,
- \( CD \) is the total drag coefficient,
- \( CD_{\text{min}} \) is the minimum drag coefficient (when the fin is producing no lift),

and

- \( CD_{\text{stab}} \) is the stabilizing drag coefficient (when the fin is stabilizing, or producing lift).

Equation [6] is an adequate representation of the drag coefficient unless separation or cavitation becomes serious. For the fins considered in this report, separation and cavitation are negligible for speeds below 15 knots at angles of attack less than 20 degrees. For speeds greater than 15 knots, the non-cavitation range is indicated on Figure 5.

Power is the product of drag and velocity, and from Formula [6] the

\[
\text{Instantaneous Power} = CD_{\text{min}} A^* U^3 + CD_{\text{stab}} C_L^* U \quad [7]
\]

where

- \( L^* \) is the total instantaneous lift of all the fins,
- \( A^* \) is the area of all the fins,

and

- \( U \) is the ship speed.

The first term of Equation [7] yields the minimum power loss with fins extended but not stabilizing, i.e., when the lift is zero. If the fins were retractable this term would be zero when not stabilizing. The second term gives the propulsive power absorbed due to the stabilizing action of the fins, i.e., due to the lift produced by the fins.
If the wave motion is assumed to be sinusoidal, then the motion of the fins should be sinusoidal, and should have the same period as the apparent period of the waves. Then $C_L$ and $L^*$ both should vary sinusoidally with the apparent period of the waves. If $C_{L_{\text{max}}}$ and $L_{\text{max}}^*$ are taken to express the maximum values of those quantities (for the combined fins) at a given speed $U$, the

\[
\text{Instantaneous Power} = C_{D_{\text{min}}} A^* U^3 + C_{D_{\text{stab}}} C_{L_{\text{max}}} L_{\text{max}}^* U \sin^2 \omega t \tag{9}
\]

and the

\[
\text{Peak Power} = C_{D_{\text{min}}} A^* U^3 + C_{D_{\text{stab}}} C_{L_{\text{max}}} L_{\text{max}}^* U \tag{10}
\]

where $L_{\text{max}}^*$ and $C_{L_{\text{max}}}$ are the design lift and corresponding lift coefficient for $U$ equal to or greater than the design speed of the system and the lift and lift coefficient, respectively, at a fin angle of 20 degrees for $U$ less than design speed.

Inasmuch as the average value of $\sin^2 \omega t$ over one period is one half, the

\[
\text{Average Power} = C_{D_{\text{min}}} A^* U^3 + 0.50 C_{D_{\text{stab}}} C_{L_{\text{max}}} L_{\text{max}}^* U \tag{11}
\]

Formula [11] has been used to calculate the power losses for Scheme B$_1$ and Scheme C$_2$. The fins are similar to the doubly all movable fins for which Reference 2 gives

\[
C_{L_{\text{max}}} = 1.75
\]

\[
C_{D_{\text{min}}} = 0.007
\]

\[
C_{D_{\text{stab}}} = 0.12
\]

These coefficients have been assumed to apply to the fins under consideration, except that the maximum lift coefficient has been assumed to be 1.70 instead of 1.75. Owing to cavitation under some conditions of operation, a lift coefficient
of 1.70 cannot be attained. Fortunately at speeds less than 15 knots cavitation is negligible, and at higher speeds the required lift can be attained with fin angles less than 20 degrees; so cavitation does not cause much of a problem for the schemes which were investigated. The results of the calculations of power loss are shown on Figure 8. For Scheme C₂ the effects of cavitation show up as a slight bending in the curve at speeds greater than 15 knots.

An interesting feature of the curves of Figure 8 is that the power loss due to stabilizing decreases as the speed exceeds the design speed. This is because the lift at higher speeds is held at a fixed value (the design lift) and thus a smaller lift coefficient is required. The drag coefficient varies approximately as the square of the lift coefficient, so that a small reduction in the lift coefficient results in a relatively large reduction in the drag coefficient, and the drag itself actually decreases with an increase in the speed.

![Figure 8 - Propulsive Power Absorbed by Fins](image)

At speeds greater than the design speeds of the systems the capacities are assumed to remain fixed at the design capacities.
Positioning Power

In addition to the power requirement for overcoming the drag of the fins, power is required also for positioning the fins; and this power must come from the ship's auxiliary power system. The quantities of particular interest in the calculation of this power are the maximum moment on the shaft of the fin, the maximum angular acceleration, and the maximum angular velocity. The requirement is made that full output of the fins is to be effected out to 0.35 cycles per second (2). For sinusoidal operation of the fins and a maximum angle of 20 degrees, the maximum acceleration is \(1.69\) radians/\(\text{second}^2\), and the maximum velocity is 0.77 radians/second.

The maximum moment can be determined only approximately. The moment includes the effects of steady flow as well as the effects of unsteady flow which results from the oscillation of the fin. On the basis of a review of the literature of aeronautics relating to the effects of unsteady flow, it is concluded in Reference 5 that the moment due to angular velocity of the control surface is no greater than the moment associated with steady flow at the same angle of attack.

However, there is an additional moment due to angular acceleration, and according to Reference 5 this can be expressed in the following form:

\[
|M_1| = \frac{\pi \rho c^2 s}{4} \left( \frac{k_1}{12} c^2 + k_2 a^2 \right) |\frac{d\alpha}{dt}| 
\]

where

- \(M_1\) is the moment due to angular acceleration,
- \(\rho\) is the mass density of the fluid,
- \(c\) is the chord length of the fin,
- \(s\) is the span of the fin,
- \(a\) is the distance between the half-chord and the location of the axis of rotation,
- \(\dot{\alpha}\) is the rotational velocity, and
- \(k_1, k_2\) are corrections primarily for finite aspect ratio, and are taken from curves given in Reference 5.
The power required for operating one fin will be calculated for Scheme B1 having 3 fins per side, the fins being 5.22 feet by 12 feet. Curves showing center-of-pressure location as a function of fin angle and cavitation number for the fins of Reference 9 are given on Figure 9. From these curves Figure 10 was derived to show the locations of the center of pressure as a function of cavitation number, with the angle of attack \( \alpha \) maintained at 20 degrees. It is evident that for the speeds of interest the steady-state moment will increase with \( \alpha \) since lift increases with \( \alpha \) and the center of pressure moves aft monotonically with \( \alpha \) as shown on Figure 9.

It also shows that for the speeds which are of interest the center of pressure can be taken to be at 0.3 of the chord aft of the leading edge at a fin angle of 5 degrees. It is assumed therefore that the axis of the fin is located at 0.3c even though the maximum fin thickness is at 0.25c. Then the distance \( d \) is found to be 1.04 feet. Thus the absolute value of the maximum moment due to angular acceleration is 1830 pound-feet.

To calculate the steady-state moment the lift coefficient is taken from Table 2, and the drag coefficient is calculated in the manner shown previously. Then the normal force coefficient at any angle of attack \( \alpha \) is

\[
C_N = C_L \cos \alpha + C_D \sin \alpha \tag{13}
\]

The steady-state moment is

\[
M_2 = \frac{1}{2} \rho AU^2 C_{N} d \tag{14}
\]

where \( d \) is the distance from the axis of rotation to the center of pressure, and can be found with the aid of Figure 9. For a speed of 15 knots \( M_2 \) is 20,000 pound-feet. The instantaneous power cannot exceed the product of maximum moment and maximum angular velocity. Then at 15 knots the peak power is

\[
(20,000 + 1830) (0.77)/550 = 31 \text{ horsepower.}
\]

For a speed of 30 knots at a capacity of 5 degrees the fins need develop a lift coefficient which is only one fourth that required for the speed of 15 knots. Therefore, a maximum angle of attack of only 4.5 degrees is required (see Figure 5). Figure 9 shows that at angles less than 20 degrees the center of pressure approaches the shaft of the fin at 0.3c. This causes the steady-state moment to decrease as the speed increases beyond 15 knots.
Figure 9 - Center-of-Pressure Location for Doubly All Movable Fins Expressed as a Function of Fin Angle and Cavitation Number

Figure 10 - Center-of-Pressure Location for Doubly All Movable Fins Expressed as a Function of Cavitation Number for a Main Fin Angle of 20 Degrees
At 30 knots the moment due to acceleration will also be much less than for 15 knots inasmuch as the maximum angular acceleration is smaller as a consequence of the smaller angle of attack. Indeed, even at a speed of 17 knots the peak power for positioning a fin is only 11 horsepower. Thus the marked reduction in positioning power which accompanies an increase in speed above 15 knots is the result of both the decrease in angular acceleration and the decrease in moment arm at which the resultant force acts. If the fins were operated at a maximum angle of 20 degrees, the peak power at 30 knots would be 113 horsepower and there would be a substantial increase in stabilizing capacity.

Assuming a continuous-duty rating of the fin-positioning motors equal to one half the peak power, and assuming a 25 per cent power increase to allow for mechanical losses, the continuous-duty rating for each of the motors should be 20 horsepower.

ACTIVATED U-TUBE TANKS

A stabilization system which uses anti-rolling tanks accomplishes its purpose by alternately filling and emptying tanks at the sides of the vessel with water or other liquid. If tanks on opposite sides of the vessel are connected at their lowest level by an athwartship duct the system is called a U-tube tank stabilizer.

If the transfer of water is accomplished simply by the rolling of the ship, the system is said to be passive. In this case the period of oscillation of the water is adjusted until it is equal to the natural period of roll of the ship, and it then oscillates with the same period as the ship, but with a phase lag of a quarter period. The ship does not always roll with its natural period, and the effectiveness of the anti-rolling tanks diminishes as the rolling of the ship departs from its natural period.

An activated U-tube tank system of stabilization is one in which a pump is used to force the water to oscillate at any desired amplitude and frequency within the limits for which the system is designed. The following study will deal only with an activated U-tube tank system.

As in the case of the activated fin studies, the following design of an activated tank system is based on the procedures developed in References 3 and 4. The degree of refinement of the design is dictated by the purpose of the study, namely, whether an activated tank system is feasible and how its general features
compare with other systems of stabilization. This study attempts
to determine with reasonable accuracy the number and size of tanks,
their locations, and the power required to operate them.

Capacity

The capacity for which a tank system is designed is rather
arbitrary, as for the case of activated fins. With activated
tanks, however, the capacity cannot be increased simply by in-
creasing the speed of the ship. It seems reasonable, therefore,
to design a tank system for a capacity somewhat larger than that
selected for a fin system. This would tend to place the two
systems on a more comparable basis.

Accordingly, a capacity of 7 degrees is assumed in the study
of a tank system. (A fin system with a capacity of 5 degrees at
15 knots would have a capacity of 7 degrees at about 18 knots).
This may seem to be excessive in the light of what has been done
in the past. However, insufficient capacity has been blamed for
the poor performance of many installations in the past, and it
does not seem advisable to base a feasibility study on an assumed
capacity which is not realistic (11).

Geometry

The particulars of the vessel which are needed for this study
have been given earlier in Table 1. Using the pertinent quantities,
the frequency of roll is
\[ \omega_s' = \frac{2\pi}{T_s'} = \frac{2\pi}{16.5} = 0.38 \text{ radians/second}, \]
the roll-righting coefficient is
\[ K_s' = \frac{WGM}{T_s'} = 17,550 \times 4.4 = 77,200 \text{ foot-tons/ radian}, \]
and the roll-inertia coefficient is
\[ J_s' = \frac{K_s'}{(\omega_s')^2} = \frac{77,200}{(0.38)^2} = 534,000 \text{ foot-tons sec}^2/\text{rad}. \]

*Primes refer to ship unmodified by tanks
It is assumed that the tank system can be designed on a static basis, so that

\[ K_t \phi_{\text{max}} = K_{ss} \psi_{\text{static}} \]  \hspace{1cm} [15]

where

- \( K_t \) is the static moment in roll produced by the tank water per unit of \( \phi \),
- \( K_{ss} \) is the static moment in roll produced by the sea per unit of \( \psi \),
- \( \phi_{\text{max}} \) is the maximum angle of tank-water level with respect to the ship, and
- \( \psi_{\text{static}} \) is the maximum effective waveslope (capacity).

Because the problem is one of dynamics rather than statics, the above assumption is permissible only if

\[ \omega_{st} > 2.5 \omega_s \] (see Reference 4)

where

- \( \omega_s \) is the resonant frequency of the ship system and
- \( \omega_{st} \) is the "decoupling" frequency; when the tank water oscillates sinusoidally at this frequency, the torque due to static head is exactly neutralized by the torque due to the acceleration forces acting on the water.

Whether the required condition is satisfied will appear later.

It is desirable that \( \phi_{\text{max}} \) be as large as possible for the particular ship under consideration to minimize tank area. As \( \phi_{\text{max}} \) increases, the tanks become deeper and eventually extend above the main deck, which does not seem desirable.
Consequently the system is designed for

\[ \phi_{\text{max}} = 20^\circ \]

Conversely, the required tank area (water surface) increases with a decrease in \( \phi_{\text{max}} \), so that it becomes increasingly difficult to fit the tank within the hull.

It is assumed (see Reference 4) that

\[ K_{sS} = K_S = K_S' \]

Then

\[ K_t = K_{sS} \times \frac{v_{\text{max}}}{\phi_{\text{max}}} = 77,200 \times \frac{7}{20} = 27,000 \text{ foot-tons/radian} \]

But also

\[ K_t = 2\rho g l^2 A_0 \times \frac{l}{2} = 2g \frac{l}{2} H A_0 \quad [16] \]

where

- \( \rho \) is mass density of liquid in tank,
- \( g \) is the acceleration due to gravity,
- \( l \) is the tank lever arm (to center of rotation), and
- \( A_0 \) is the tank area per side.

If the athwartship dimension \( y_0 \) of the tank is chosen to be 8 feet, then

\[ l = \frac{69.7}{2} - 4 = 30.8 \text{ feet} \]

\[ A_0 = \frac{K_t}{2\rho g l^2} = \frac{27,000 \times 2240}{2 \times 64 \times (30.8)^2} = 498 \text{ feet}^2 \]
With so large an area required per side it seems desirable to distribute it among four sets of tanks, giving an area per tank of \(498/4 = 124.5\) square feet. The length of each tank (fore and aft) is \(124.5/8 = 15.6\) feet.

The height of each tank is

\[
H = 2 \times 30.8 \times \tan 20^\circ = 22.4\text{ feet}
\]

It should be noted that if \(\phi_{\text{max}}\) had been chosen as \(30^\circ\), \(H\) would be \(35.6\) feet and the tanks would extend well above the main deck.

To prevent yawing moments on the ship, due to acceleration of water, the tanks should be equidistant fore and aft from the center of gravity \(CG\) of the ship. The \(CG\) is estimated to be \(17.4\) feet aft of amidship, or at about frame \(87.3\). The frame spacing is \(4\) feet, so each tank will fit conveniently into \(4\) frame spaces. Examination of the ship's plans indicates that the most likely locations for the tank pairs are:

<table>
<thead>
<tr>
<th>No.</th>
<th>Frames</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68(\frac{1}{2}) - 72(\frac{1}{2})</td>
</tr>
<tr>
<td>2</td>
<td>80 - 84</td>
</tr>
<tr>
<td>3</td>
<td>86 - 90</td>
</tr>
<tr>
<td>4</td>
<td>105 - 109</td>
</tr>
</tbody>
</table>

The \(CG\) of this tank system is at about frame \(86.9\). The tank arrangement is shown on Figure 11. In the following computations it is assumed that the tanks are identical.

The behavior of a set of tanks depends upon the frequency to which they are "tuned". After the tanks themselves are fixed, this depends on the length and cross-sectional area of the cross-connecting duct. It is suggested in Reference 4 that the most reasonable value for the resonant frequency of the tank system \(\omega_t\) is the same as the ship frequency, or for this case, \(\omega_t = \omega_s = 0.38\) rad/sec., as this satisfies the criterion for optimum passive damping. The resonant frequency of the tank system can be determined from the following formula:
Figure 11a - Profile of USS BOSTON Showing Proposed Location of Stabilizing Tanks
Figure 11b - Partial Inboard Profile of USS BOSTON
Showing Location of Tanks
where \( S' \) is the "weighted" length of the U-tube formed by a set of tanks and connecting duct and is defined as:

\[
S' = \int_0^S \left( \frac{A_o}{A_s} \right) ds
\]

From Equation [17] \( S' = \frac{2g}{(\omega_t)^2} = \frac{2 \times 32}{(0.38)^2} = 443 \) ft.

The total length of water in each tank pair is 22.4 feet. There must be a transition section connecting the tank to the duct which will be taken to be essentially vertical and 5 feet long on each side. The "weighted" length of the duct alone is 443 - 32.4 = 410.6 feet. The actual length of the cross duct is approximately 69.7 - 8 = 61.7 feet. Therefore the duct area equals (tank area)(actual length duct)/("weighted" length duct) = 124.5 x 61.7/410.6 = 18.7 square feet.

The centerline of each duct is 16.6 feet above the baseline. The validity of the earlier assumption which was based on the condition: \( \omega_{st} > 2.5 \omega_s \) can now be checked, since

\[
\omega_{st} = \sqrt{\frac{2g}{S'}}
\]

*The natural circular frequency of undamped oscillation of liquid in a U-tube of constant cross section is \( \omega_t = \sqrt{\frac{2g}{S}} \) where \( S \) is the length along the centerline of the tube between the free surfaces of the liquid. The U-tubes considered here are not of constant cross section, so the length to be used in calculating the natural frequency of oscillation is not the actual length \( S \), but a length \( S' \), denoted "weighted" length, which is found by integrating the element of length ds along the centerline of the tube using the factor \( A_o/A_s \) where \( A_o \) is the free surface area in the tanks and \( A_s \) is the cross-sectional area at any point in the U-tube.
where

\[ S_n^e = \int_0^S \left( \frac{D_S}{l} \right) ds \]  

and \( D_S \) is the perpendicular distance from the center of rotation to the velocity vector of the tank-water element under consideration. The center of rotation is assumed to be midway between the center of gravity and the center of buoyancy**, as was discussed earlier. In the present problem the center of rotation is 21.1 feet above the baseline and \( D_S \) for the cross duct is \( 21.1 - 16.6 = 4.5 \) feet.

The integration for finding \( S_n^e \) in Formula [20] will be somewhat approximate, but for present purposes it is not necessary to know the answer precisely but only whether \( \omega_{st} \) is sufficiently large. In view of the symmetry of the tank system

\[ S'' = 2 \int_0^{S/2} \left( \frac{D_S}{l} \right) ds = 2 \int_0^{16.2} \left( \frac{30.8}{30.8} \right) ds + 2 \int_0^{30.8} \left( \frac{4.5}{30.8} \right) ds \]

\[ = 32.4 + 9.0 = 41.4 \text{ feet} \]

**The ship-tank system is one having two degrees of freedom, and it is characterized by three natural frequencies: \( \omega_s \), \( \omega_t \), and \( \omega_{st} \), each of which can be written in terms of a moment coefficient \( K \) and an inertia coefficient \( J \). For example, \( \omega_{st} = \sqrt{K_t/J_{st}} \), where \( J_{st} \) is a "mutual" inertia coefficient. The derivation of the equations of motion shows that \( J_{st} \) is dependent on a "weighted" length of the fluid trajectory; that is, upon \( S'' = \int_0^S \left( \frac{D_S}{l} \right) ds \).

**It is pertinent to investigate the consequences of a mistake in estimating the position of the center of rotation \( CR \). If \( CR \) is lower than estimated, \( \omega_{st} \) will be larger than estimated. This is favorable. Conversely, if \( CR \) is higher than estimated, \( \omega_{st} \) will decrease but it will not become less than \( 2.5 \omega_s \) until \( CR \) is well above the ship's center of gravity.
so \[ \omega_{st} = \sqrt{\frac{2g}{S}} = \sqrt{\frac{64}{41.4}} = 1.24 \text{ radians/second} \]

then \[ \frac{\omega_{st}}{\omega_s} = \frac{1.24}{0.38} = 3.27 \]

It was only necessary that \( \omega_{st} > 2.5\omega_s \) and therefore the required condition is well satisfied. If an actual ship system was being designed it would now be necessary to determine whether the addition of the tanks changes the ship parameters to the extent that the tank dimensions and locations would have to be changed. The calculations made in Reference 4 for a tank installation indicate that such changes would be very small, and all that would be required is a change in the cross-sectional area of the cross duct. A small correction to the duct area is not of importance in the present study. Having determined rather closely the dimensions and location of a tank system which will stabilize USS BOSTON it is now of interest to determine the weight of the system.

Weight of Tank System

For each set of tanks with associated duct the weight of stabilizing water is (64) \( [(249 \times 16.2) + (61.7 \times 18.7)]/2240 = 14.6 \) tons per tank and the total weight of stabilizing water for the four sets is 592 tons. As a percentage of the displacement of the ship, the stabilizing water amounts to (100) (592)/17,550 = 3.4 per cent. Of this total weight, about 22 per cent is in the ducts.

The weight of the tank structure and the machinery cannot be estimated with accuracy owing to the lack of pertinent data. It is conceivable that in some ships a tank system could be so designed that not all of the weight would be charged to the tank system. One can hardly do better than adopt the suggestion of Reference 3 that, as an order of magnitude estimate, the weight of tank structure and machinery taken at about 1 per cent of the ship's displacement. The total weight of the system is then 4.4 per cent of the ship's displacement.

Required Power

The power required to oscillate the fluid in the tanks is the product of magnitude of flow and magnitude of head. The following calculation of power is based on the assumption that the wave motio
is sinusoidal so that the effective waveslope is given by $\Psi = \Psi_0 \sin \omega t$. At the tank's natural frequency the power required is small, but it increases rapidly at frequencies of oscillation above the frequency of tank resonance. However, the required capacity due to waveslope amplitude decreases rapidly for wave frequencies larger than twice the ship's natural frequency (3). Therefore, the calculation of peak power is based on the assumption that the tank water oscillates at twice the ship's natural frequency.

Inasmuch as the tank water is in motion, the head is comprised of dynamic head and head loss due to damping, as well as the static head. Reference 3 makes use of the equations of motion of the ship-tank system to develop an expression for complex power into the fluid, the general expression for which is

$$\text{Complex power} = \text{head} \times \text{flow}^*$$  \hspace{1cm} [21]

where * indicates the complex conjugate (3). The real and imaginary parts of the complex power are the real and reactive power, respectively (12). The average value of real power is one-half peak real power, and the average value of reactive power is zero. Peak total power is the product of peak magnitude of head and the peak magnitude of flow, no matter what the phase relation of these quantities may be. The expressions for peak powers are:

$$\text{Peak real power} = K_p \Psi_0 \frac{\Omega}{\Omega_t} \frac{\Omega}{\Omega_t}$$  \hspace{1cm} [22]

$$\text{Peak reactive power} = K_p \Psi_0 \Omega \left[1 - \left(\frac{\Omega}{\Omega_t}\right)^2\right]$$  \hspace{1cm} [23]

where

$$K_p = \frac{1}{2} \left(\frac{H}{B}\right) \left(\frac{GM}{B}\right) \left(\frac{B}{R}\right)^{1/2} \left(\frac{\Psi_0}{\Psi_{\text{static}}}\right)^2 (\Psi_{\text{static}}) b^{1/2}$$  \hspace{1cm} [24]

and

$$\frac{1}{Q_t} \approx 3.0 \times 10^{-2} \left[b^{3/2} \left(\frac{H}{B}\right)^2 \left(\frac{B}{h}\right)^{1/2} \left(\frac{R}{B}\right)^2 (\Psi_{\text{static}})^{-1/2} \frac{\sqrt{\Psi}}{\Omega_t}\right]$$

$$\left[\left(\frac{\rho_f}{\rho_{\text{SW}}}\right)^{1/2} \left(\frac{\Psi_0}{\Psi_{\text{static}}}\right) \left(\frac{\Omega}{\Omega_t}\right)\right]$$  \hspace{1cm} [25]
The various quantities used in Equations [22] through [25] are defined in the section on Notation. In this particular problem these quantities have the following values:

\[
\begin{align*}
H &= 22.4 \text{ ft} \\
\ell &= 30.8 \text{ ft} \\
GM &= 4.4 \text{ ft} \\
B &= 69.7 \text{ ft} \\
R &= 221.5 \text{ ft} \\
b &= 0.82 \\
\psi_{\text{static}} &= 7^\circ = 0.122 \text{ radians} \\
\end{align*}
\]

\[
K_p = 1.41 \times 10^{-8} \left( \frac{\psi_0}{\psi_{\text{static}}} \right)^2
\]

\[
P_o = 3.73 \times 10^6
\]

If \( \omega_t = \omega_3 \), then \( \Omega_t = \omega_t/\omega_3 = 1.0 \). The maximum power is required for \( \psi_0/\psi_{\text{static}} = 1 \). As stated previously, the power requirements are based on a frequency of tank-water oscillation equal to twice the ship's natural frequency. Then \( \omega/\omega_3 = \Omega; \Omega_{\text{max}} = 2.0 \). Using the quantities given, the combined required rating for all four sets of tanks is found:

- Maximum peak reactive power = 31,600 hp
- Maximum peak real power = 6,750 hp
- Pump output rating = \( \frac{1}{2} \left[ (31,600)^2 + (6,750)^2 \right]^{\frac{1}{2}} = 16,200 \text{ hp} \)

The average power input to the fluid will be much less than the peak power. Nevertheless, the capacity of the pumps must be great enough to handle the maximum possible load demands within the design range.

Evidently most of the capacity of the pumps is needed to handle the peak reactive power requirement when the frequency is appreciably greater than the natural frequency of the tanks. This means the average power is relatively small, but it has superimposed on it a large oscillating power flow, the reactive power. During part
of the cycle of operation the tank water will drive the pumps and return power to the line, so that for a system that is 100 per cent efficient the net reactive power would be zero. The efficiency of the pump while driving or being driven will often be very low, however, owing to the wide range of heads over which it must operate to meet the oscillating power demand. Consequently the power input to the pump will be much greater than the average power output.

It would seem that the large power requirement would be prohibitive. The power could be reduced by decreasing the magnitude of $H$ or by increasing $\Omega_t$, or both. It is not desirable to make $H$ smaller, for that requires an increase in the tank area, and an increase in the weight of tank water.

While Reference 4 indicates that $\omega_t = \omega_s$ is perhaps the most reasonable value for $\omega_t$, an acceptable range for $\omega_t$ was found to be $0.7 \omega_s \leq \omega_t \leq 1.3 \omega_s$. It would be interesting to assume that $\omega_t = 1.3 \omega_s$ and find what differences are made in the system. Then $\Omega_t = \omega_t / \omega_s = 1.3$; and the

Maximum peak reactive power $= 14,350$ hp

Maximum peak real power $= 5,200$ hp

Pump output rating $= \frac{1}{2} [(14,350)^2 + (5,200)^2]^{1/2}$

$= 7,640$ hp

Changing the magnitude of $\Omega_t$ affects the size of the cross ducts, so that

$$S' = \int_0^S \left( \frac{A_0}{A_s} \right) ds = \frac{2g}{(\omega_t)^2} = \frac{64}{(1.3 \times 0.38)^2} = 262 \text{ feet.}$$

The "weighted" length of duct is 262-32.4 = 229.6 feet, the actual length of duct = 61.7 feet, and the duct area = 124.5 x 61.7/229.6 = 33.5 square feet.

The increase in $\Omega_t$ has effected a reduction in pump output rating to less than one half its previous value, but the power is still very high. The size of the cross ducts has nearly doubled, so that they would need to be nearly 6 feet square, a size that is
probably prohibitive. The weight of water is now \((4)(64) \times \left(\frac{(249)(16.2) + (61.7)(33.5)}{2240}\right) = 697 \text{ tons, or } 4\text{ per cent of the ship's displacement. Assuming as before that the weight of the tank structure and machinery is } 1\text{ per cent of the displacement, the total weight of this tank system is } 5.0\text{ per cent of the displacement.}

**MOVING WEIGHTS**

The stabilization of vessels in roll by transferring solid weights from one side to the other has been investigated and has actually been attempted by early workers in this field. In his historical notes concerning automatic stabilization of ships, Chalmers mentions the active system employed by Sir John Thornycroft (apparently fairly successful) and a passive system tried by Monsieur Victor Crémieu (13). The latter system consisted of a four-wheeled truck which could oscillate on curved rails within a tank extending across the width of the vessel. The tank was filled with lime water, and the damping of the truck was varied by changing the clearance between it and the walls of the chamber. According to observers, including Crémieu, himself, the system was not successful, and the stabilizer was even said to aggravate the roll.

Dr. N. Minorsky in 1931 submitted to the Bureau of Construction and Repair a detailed analysis of a ship-stabilization system which bore some resemblance to Crémieu's system. The essential difference between the two systems was that Minorsky controlled the motion of the weight by a motor, and consequently his was an activated weight system. Indeed the whole purpose of his analysis was to show the importance of controlling properly the motion of the weight. In commenting on the importance of imparting to the weight the proper motion to get the best stabilizing effect he stated, "As regards the discussion ... that a good stabilization is not necessary but that only elimination of occasional excessive angles of roll is desirable I wish to state that it is just as easy to solve the problem correctly as to solve it partially on condition that the fundamental dynamical facts are properly applied."

It would appear that the system of Crémieu was essentially the same as that proposed by the Bureau of Ships, viz., a highly-damped, passive tank system using tuned, moving weights. The proposal suggests a modification of Crémieu's system. The weights are to move transversely in tubes which contain helical springs on either side of the weights, the springs extending from the weights to the sides of the vessel. Damping is to be accomplished by
movement of a fluid in the tube through orifices in the ends of the tube.

The theoretical analysis of such a system is complex, and a considerable amount of time would be necessary to make a full analysis. A similar, but simpler, problem is that of the dynamic vibration absorber discussed by Timoshenko (14). It is possible, however to arrive at some idea of the practicality of solid weights without an elaborate analysis. It can be assumed that an activated system will be more effective than a passive system, so that moving weights can be considered in the best possible light by examining an activated system.

In Minorsky's words, "The ....... control . . . . has for its primary purpose to produce such timing of motion of the weight whereby the moment produced by the latter is substantially equal and opposite to the disturbing moment of buoyancy produced by the waveslope - which prevents the cumulative effect of successive waves on the ship." A very considerable degree of stabilization can be achieved with less weight than that required to extinguish roll. However, it will be assumed that it is required that the system be able to virtually extinguish the roll, as was the case with the systems discussed earlier in this report. The controls are assumed to be adequate for this purpose, as was done before.

Assuming a capacity (maximum effective waveslope) of 7 degrees the required stabilizing torque is \((77,200)(7/57.3) = 9,440\) ton-feet. Now assume that lead, at a weight of 710 pounds per cubic foot, is to be used; and further that the weights are 10 feet in length and can move to within 5 feet of the sides of the vessel. The amount of movable weight required to produce the required torque is \((9,440)/(69.7/2 -10) = 380\) tons, or 2.2 per cent of the ship displacement. If a capacity of 6 degrees had been selected the weight would have been about 1.9 per cent of the displacement. The addition of the supporting structure and the activation machinery would increase the system weight to perhaps 3 per cent of the displacement. If it is assumed that 10 weights are used, each one being 10 feet long, then a total weight of 380 tons will require that the weights have a diameter of

\[
2 \sqrt{\frac{380 \times 2240}{\pi \times 710 \times 100}} = 3.91 \text{ feet}.
\]

At a cost of 10 cents per pound (present cost is greater than 11 cents) the cost of 380 tons of lead would be $85,000. This is in contrast with water in a tank or fin system where the mass used for stabilizing costs nothing. All of the systems require structural changes to a conventional vessel, and all require activation
mechanisms. The prospect of supporting and moving 10 weights of the size and weight contemplated is not an appealing one, but the scheme is within the realm of possibility. The cost of the weights could be reduced by using, say, cast iron, but that would require that 16 instead of 10 weights be used, and other costs would probably nullify any savings made in cost of materials.

The power required to activate the weights increases rapidly as the apparent frequency of the waves increases. If it is assumed that: (1) the wave motion is sinusoidal, (2) the ship remains level, (3) friction is negligible, (4) the weights move rectilinearly, and (5) the apparent frequency of the waves is twice the ship's natural frequency (as was done for the tanks), then the peak power of the system is about 6,500 horsepower and the average power is about 5,300 horsepower.

The preceding discussion of a moving weight system has dealt with only an activated system in order to make this system appear as attractive as possible. Nevertheless, it may be of interest to consider physical features of a passive system using solid weights which slide in transverse tubes having helical springs on either side of the weights, the springs extending to the sides of the ship. Inasmuch as the system is passive it is assumed that large angles of roll could occur at times. A rough estimate of spring size and stress was made assuming that: (1) the spring is 3 feet in diameter, (2) the weight is 20 tons and can move 20 feet to either side of the centerline, (3) the spring can be compressed into a length of about 5 feet, and (4) the maximum heel angle is 20 degrees. These assumptions lead to an estimate of about 2 inches for the wire size of the springs, and a maximum stress exceeding 400,000 pounds per square inch. The estimate is admittedly crude, but it gives an idea of the kind of problem which must be faced in designing such a system.

The spring stress is far beyond the working range. A reduction in stress could be effected by nesting springs, one inside another, but this would not be enough. The weights would have to be made smaller, and the number of them increased. Evidently an acceptable arrangement would require that the springs work at a high stress, and the possibility of spring failure becomes a matter of concern. Should the vessel's motion exceed that contemplated, the weights might "bottom" with an attendant increase in spring stress and a shock as the weights came to rest. All of these problems could be avoided by designing the system for a smaller movement of the weights. But to do this would decrease seriously the effectiveness of the system.

The passive, moving weight system has the quality of being a relatively simple installation, but its limited effectiveness combined with the problems of providing a suitable spring in the
proposed system do not make it an attractive possibility for roll stabilization. If a passive system should be considered acceptable for certain vessels, it may be that a suitable spring arrangement could be worked out in specific cases. For instance, the springs might run fore and aft, connecting to the weights by cables which pass around sheaves. This would permit the use of longer springs and thereby simplify the spring problem.

The discussion of moving weights has assumed that the stabilizing effect of weights is due only to the moment of the weight of the mass with respect to the center of rotation. This is strictly true only if the weights are at the same height as the center of rotation. If not, acceleration forces enter the problem. However, for long wave periods the acceleration forces become relatively ineffective because the maximum allowable velocity of the weights limits the maximum allowable integral of acceleration (and so of torque) with respect to time.

ROLL STABILIZATION OF SHIPS BY ROTATING FINS

It is not proposed here to present a design for stabilization by rotating fins, but to call attention to a possible means of roll stabilization which may have been overlooked.

The least attractive feature of stabilization by activated fins is the very marked decrease in effectiveness as the speed decreases. For instance, an activated fin system at 10 knots is less than one half as effective as it is at 15 knots. If the lift coefficients of fins could be increased to much larger values than are ordinarily attained, the greatest objection to the use of fins could be overcome. Furthermore, the fins could be made smaller so that they may not have to be retractable.

The fact that a rotating circular cylinder can produce very high lift coefficients when placed in a stream of fluid is well known. It is perhaps not so well known that a cylinder having hydrofoil sections behaves about the same as a circular cylinder when rotated in a fluid stream. The significant difference between the two "rotors" is that the circular cylinder has a very high drag coefficient when not rotating whereas the drag coefficient of the non-rotating cylinder with streamline sections is very low if the sections are oriented for zero angle of attack. In this fact lies the possibility of approaching the more ideal requirements for a stabilizing fin - a fin whose lift coefficients can be substantially increased without incurring large drag losses when not stabilizing.
Marbury has discussed the use of a rotating surface as a marine rudder and a paper by v. Holst contains a comprehensive survey of experimental work done on rotors up to 1941\(^1\). These investigators report lift coefficients as high as 15.0. If the lift coefficients of the fins discussed earlier in the report could be only doubled at the slower speeds, the effectiveness at 10 knots would be about the same as at 15 knots. It appears to be a simple matter to attain such an increase in lift coefficients simply by rotating the fins at the proper speed.

The idea of using rotating fins for roll stabilization has very attractive possibilities and it would seem to deserve consideration. Before a system can be designed, the literature needs to be reviewed carefully, and it may be necessary to make further experimental studies, particularly cavitation studies. At present there is no apparent reason for questioning the potentialities of rotating fins for roll stabilization.

The design of a rotating-fin system would pose certain problems not existing in "normal" fin installations. Whether the fins should rotate at all ship speeds or only the lower speeds would have to be settled. If serious cavitation occurred at the higher ship speeds when rotating, it might be necessary to operate the fins in normal fashion at those speeds. Tied in with this consideration is the question of whether the fins should be retractable. If cavitation is not a serious problem it would probably be well to rotate the fins at all ship speeds and take advantage of the high lift coefficients, permitting the fins to be small enough so that they would not have to be retractable. This would greatly simplify the internal arrangements. Otherwise, if the fins were to rotate at slow ship speeds, operate normally at high ship speeds, and were so large as to have to be retractable, the design problem would be very difficult. In any case the controls would have to be such that they could automatically position the fins at zero angle of attack. The motors for rotating the fins would have to be reversible. None of these requirements presents an insurmountable engineering problem. As with all activated systems, the adequacy of the controls will determine the effectiveness of the system.

**COMPARISON OF METHODS OF ROLL STABILIZATION**

This report has considered five methods of roll stabilization, although only two have been examined in detail. The five methods are:
1. Activated fins
2. Activated tanks
3. Activated solid moving weights
4. Passive solid moving weights
5. Rotating fins

Of these five methods, only the first two are in vogue today, and these are the methods which have been examined here. Before the systems can be evaluated and compared, there must exist a basis on which the evaluation and comparison can be made. Evidently the comparisons can be made with respect to the following considerations which are listed approximately in the order of their importance.

1. Effectiveness in stabilizing
2. Weight and space requirements
3. Effect on ship operations
4. Cost

There is no general agreement as to the relative importance of these items, particularly the last one. For naval vessels however there appears to be some justification for placing cost last. These four items will be used as a basis for comparing the several stabilizing systems.

EFFECTIVENESS IN STABILIZING

Consideration of this item again raises the issue of active stabilization versus passive stabilization. Active systems are much more effective than passive systems, and inasmuch as we are "buying" stabilization, consideration of effectiveness alone points to the selection of active systems. Historically, ship stabilization systems have tended to begin as passive systems and evolve toward active systems. This indicates that while no criterion for effectiveness has been established, the desire and the aim is to effect stabilization which is as complete as can be attained. Certainly this is true for naval vessels when they
are in combat. Consideration of passive systems must then be regarded as a backward step if one's purpose is to settle upon the best means of stabilizing large vessels. It may be that for a few ships having very specialized missions the results achieved by a passive system are adequate, but as a method on which to base a stabilization program passive systems must be ruled out.

In general, an active system of stabilization can be as effective as is desired provided the capacity is large enough and the controls are adequate. However, the effectiveness of activated fin systems varies with ship speed, and at zero speed fins afford only passive stabilization. Objections have been raised against the diminished effectiveness of fins at low speeds. The importance of this argument in any specific case depends on the mission of the vessel concerned. Unless a ship accomplishes its mission at low speeds it is of no great moment that the effectiveness is diminished at these speeds. Activated tanks have the advantage of being able to stabilize even at zero speed.

The foregoing discussion does not imply a yielding to the argument that fins cannot give a high degree of stabilization at low speeds. This cannot be said to be axiomatic until the possibilities of rotating fins have been thoroughly investigated and discarded. At present the system appears to have very real potentialities.

WEIGHT AND SPACE REQUIREMENTS

The activated fin system of stabilization for USS BOSTON weighs about 2 per cent of the ship's displacement as contrasted with about 5 per cent for activated U-tube tanks. Reference 3 estimates the space requirements at approximately 5.5 per cent for tanks, and 2 per cent for retractable fins. (It is assumed that the fins retract axially). These estimates appear to be reasonable if based on displacement volume. The weight and power requirements of the tank system evidently could be decreased by using diversified tanks, and marked saving in weight and space could be effected by using sea-ducted tanks. Even so, tanks cannot compete with fins on the bases of weight and space. If the fins were not retractable the space requirements would be only a fraction of one per cent of the displacement volume.

The argument has been advanced that fuel oil can be carried in the tanks of a stabilization system so that the weight and space need not be charged to stabilization. This would raise
other problems since fuel oil becomes very viscous at low temperatures and would have to be heated before pumping (17). According to Figure 55-1 of the Bureau of Ships Manual the optimum pumping temperature for Bunker "C" oil is 170 degrees F. Carrying oil in the tanks would create some of the same problems relative to fire that exist aboard oil tankers. Indeed, the danger may be greater, for the tanks cannot be isolated from the normal activities aboard a combatant ship as they are on tankers. In the event of battle damage which caused leakage of oil into the ship the danger of fire would become especially acute.

Because of the problems associated with carrying oil in the tanks it appears that water alone will be used for stabilizing, and there is no escape from charging the weight and space to the stabilizing system.

The weight of a system using solid, moving weights would be significantly larger than the weight of an activated fin system, and the space requirements would be considerably greater because of the number of weights required and the allocation of space to permit them to travel from side to side of the vessel. For USS BOSTON the space requirements would be at least 4 per cent of the displacement volume.

EFFECT ON OPERATION OF SHIP

The matter for concern here is whether the addition of a stabilizer to a ship has any adverse effects on the normal operations of the ship. All active systems of stabilization require power which would not be expended if the ship had no stabilizer, and the cruising radius is therefore decreased slightly. On the other hand the incremental resistance due to rolling is decreased by decreasing rolling, and the saving in power in this manner tends to offset the power consumption of the stabilizer (8). Activated tanks require far more power from the auxiliary power plant than do activated fins, but consume no power from the main propelling engines as fins do when stabilizing. The difference in fuel consumption for the various systems would be only a fraction of the total consumption for a given system, so the difference in cruising radii for two different systems would be very small. In any event the modern technique of refueling at sea tends to make "cruising radius" less significant on naval vessels than it was in times past.

Maximum speed also is affected slightly in the case of non-retractable fins which contribute an increment of drag even when they are not stabilizing. However the fins would normally replace
at least a part of the bilge keels, so the net change in drag due to fins should be very small. As shown on Figure 8 and in Reference 8, the power loss due to fins operating at zero angle of attack has a negligible effect on maximum speed.

Inasmuch as space inside a ship is always at a premium, the stabilizing system requiring the least internal space would be the most desirable. Fins require less space than tanks or moving weights, and if the fins are non-retractable they will take up much less internal space than if they are retractable. Tanks and moving weights require the exclusive use of spaces extending across the entire width of the vessel. In many ships these regions would block the free movement of personnel in a fore-and-aft direction. Retractable fins also interfere with arrangement of space within the ship, particularly if the fins are normal to the hull. However, the fin installation would not extend across the entire width of the ship, and it would be relatively low in the ship where there is not so much fore-and-aft movement of personnel.

A tank which was seriously damaged in battle would be put out of action, and in all likelihood some of its contents would discharge into the ship. Damage to a moving weight system could also have adverse effects on the operation of a ship. At best, one or more weights would be out of action. Should a weight become free to travel without restraint the results could be calamitous. A weight stranded off the centerline would produce a permanent heeling moment. Another weight could be used to counteract this if it were available; however this would diminish the effectiveness of the stabilizer. If a weight should be dislodged and fall, a hazardous situation would be created. Damage to a fin itself would at worst necessitate the use of a fin on the other side of the ship to counteract the lift of the damaged fin if it were locked at a fixed angle. Damage which included the hull probably would be no worse than if the fin were not there, and the damage would be low in the ship where damage to the stabilizing system would have relatively little effect on the activities of personnel.

If fins are non-retractable, conceivably they could interfere with certain operations such as docking, lying alongside, and negotiation of restricted channels, such as locks. These objections to fins do not seem to be insurmountable on USS BOSTON, and they would be minimized if the fins did not extend beyond the rectangle which bounds the maximum transverse section.
COST

The original cost and the cost of operation are involved here. A quantitative estimate of comparative costs for different systems is beyond the scope of this report, but a few general comments relative to costs can be made.

Activated tanks require much larger motors and structural work of greater magnitude than do activated fins. For retractable fins the structural work probably would be more detailed than for tanks, and therefore more costly per unit of volume or weight. Structural costs for a non-retractable fin system should be less than for all other types of systems.

At high frequencies the cost of operating tanks is much larger than the cost of operating the positioning motors of fins but near the ship's resonant frequency fins may require more power. Tanks do not increase the ship's resistance as do fins, but they require power from the ship's auxiliary system. On the whole, fins appear to have the advantage over U-tube tanks so far as cost is concerned. The cost of operating a moving weight system compares roughly with the cost of operating tanks.

The original costs and operating costs of stabilizing systems are only a small percentage of the original costs and operating costs of the vessel as a whole, and the differences in costs among the several systems are even a smaller percentage. Thus, it would seem that considerations other than cost are of much more importance in selecting a method of stabilization.

CONCLUSIONS AND RECOMMENDATIONS

Present interest in the stabilization of large naval vessels is a result of continuing effort to make fighting ships more effective in carrying out their missions. The evolution of the science of stabilization has tended toward a greater degree of stabilization, i.e., from passive to active systems. Passive systems can never give complete stabilization, and the effectiveness of a passive system varies markedly with the apparent wave frequency. The technical requirements for active stabilization can be met with today's knowledge and modern instrumentation. In the light of these considerations it is recommended that a program for the stabilization of large naval vessels be restricted to activated, rather than passive, stabilizers.
The weight and space requirements of an activated, retractable fin system for USS BOSTON are less than one half the requirements for an activated U-tube tank system using several identical tanks. Non-retractable fin systems require even less space and weight. For the same capacity, at moderate ship speeds, fin systems are less costly in weight and space than systems using solid moving weights. However, at very low speeds a "normal" fin system would have to be very large if it were to be effective.

Tanks and solid weights require space extending entirely across the vessel, and they interfere with the movement of personnel to a much greater extent than do fin installations. The consequences of battle damage to either a tank or moving weight installation would be more serious than they would be for a fin system. Damage to a fin installation would not be likely to affect the fighting efficiency any more than would damage in the same area of an unstabilized ship.

The principal objections to fins are their drag and their decrease in effectiveness when the ship speed is decreased. With respect to the first objection, however, the drag when not stabilizing is a very small part of the total ship resistance, and if the fins are retractable the drag disappears when not stabilizing. Power requirements due to drag of fins when stabilizing compare very favorably with power requirements of activated tanks, and most of the power required by fins is supplied by the main propelling engines rather than by auxiliaries. Whether decreased effectiveness of fins at low speeds is a serious objection depends on the mission of a ship.

All things considered, the activated fin system of stabilizing appears to be more attractive than other systems, and it is recommended that a program of roll stabilization be based on this system. More specifically, a non-retractable, multiple fin system such as Scheme C of Figure 1 appears to be optimum.

There is a good possibility that fin systems can be made even more attractive by resorting to rotating fins. The potentialities of rotating fins for a stabilization system have not been explored, and it is recommended that a thorough examination of a system employing rotating fins be made. Should the examination indicate the need for model studies of rotating fins, it is recommended that such studies be made.
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The feasibility of stabilizing the roll of large naval vessels has been studied by actually making preliminary designs of several systems of stabilization for USS BOSTON. These include a system using activated U-tube tanks and four systems using activated fins. The weight, space, and power requirements of these systems have been estimated. Stabilization by passive systems, by moving solid weights, and by rotating fins are discussed briefly.

Weight and space requirements of activated fins appear to be less than one half the requirements for activated U-tube tanks. The auxiliary power required to activate the fins is far below that required to activate the tanks.

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