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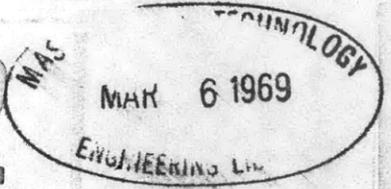


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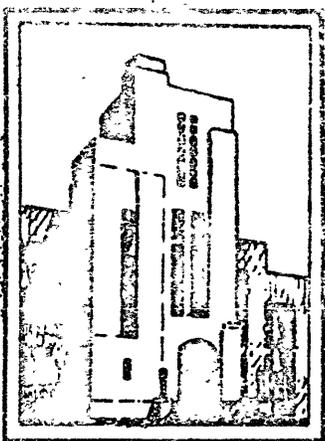
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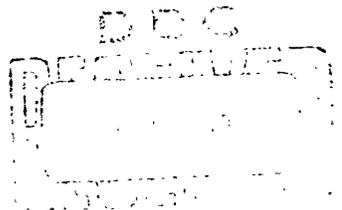
APPLIED  
MATHEMATICS

A GUIDE FOR THE SELECTION AND APPLICATION OF  
RESILIENT MOUNTINGS TO SHIPBOARD EQUIPMENT

REVISED

by

Francis F. Vane



STRUCTURAL MECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

February 1958

Report 880

**A GUIDE FOR THE SELECTION AND APPLICATION OF  
RESILIENT MOUNTINGS TO SHIPBOARD EQUIPMENT**

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**Francis F. Vane**

**February 1958**

**Report 800**

**NS 713-017**

## PREFACE

This manual was prepared at the request of Code 375 of the Applied Science Branch, Bureau of Ships, in order to provide guidance to design agencies for the selection and application of resilient mountings to shipboard equipment. All the mounting arrangements expected to be encountered in practical installations have been treated. For the more commonly encountered arrangements, the computations were simplified and systematized as much as possible, and charts were used to reduce further the computational work.

Natural frequencies and normal modes of resiliently mounted equipment can now be calculated by means of high-speed computers. The David Taylor Model Basin has coded one general form of the problem for the UNIVAC. Guidance on the data needed for UNIVAC calculation is given in BUSHIPS Notice 10482 dated 6 July 1956. The Electric Boat Division of the General Dynamics Corporation also has coded a general form of the problem, in this instance for IBM machines. This code has been made available to the Portsmouth and Philadelphia Naval Shipyards.

Numerous individuals of many groups contributed substantially to the compilation of this manual by many concrete suggestions on the scope of the manual, on information required for calculating natural frequencies, and on considerations and precautions in the selection and application of mountings. Further suggestions were made in comments on the several preliminary drafts of this manual. Those who should be specially mentioned include Messrs. W.W. Jackson and C.M. Banfield of the Portsmouth Naval Shipyard, CAPT P.G. Schultz, USN, Mr. A.C. McClure of the General Dynamics Corporation, Electric Boat Division, and Messrs. P.J. Shovestul and F. Schloss of the Engineering Experiment Station. Comments on drafts of the manual were also made by personnel of the Naval Research Laboratory, the Mare Island Naval Shipyard, the San Francisco Naval Shipyard, and Codes 371, 436, 508B, and 525 of the Bureau of Ships. Mr. Roy Henderson of Code 375 of the Bureau of Ships contributed greatly by suggesting sources of information, correlating the contents to Bureau of Ships developments, specifications, and instructions, and by suggesting changes in the arrangement of the manual.

Contributions of Taylor Model Basin personnel were as follows: Section A5.1 of Appendix 5 was prepared by Mr. R.T. McGoldrick and the remaining sections of Appendix 5 and Appendix 6 were prepared by Dr. E.H. Kennard. Dr. E.H. Bareiss developed a chart for the solution of cubic equations, and Mr. R.D. Ruggles suggested a combined procedure for determining centers of gravity and moments of inertia. Dr. Mark Harrison and Mr. Harry Rich were consultants on noise and shock. Messrs. W.D. Schutt, E.R. Wagner, and R.R. Milam helped to prepare and checked the illustrative problems, charts, and figures.

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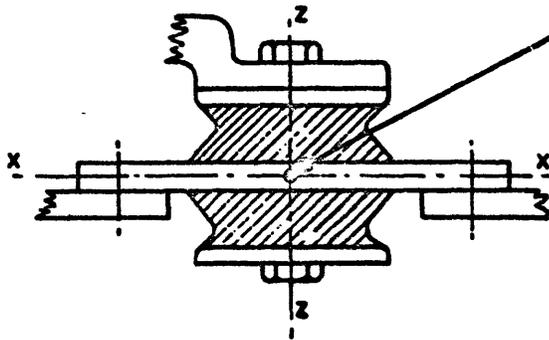
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## NOTATION

$B_1, C_1, D_1$ $B_2, C_2, D_2$ }	Coefficients in the cubic equations for solution of natural frequencies of mounted assemblies with one plane of symmetry
$C$	Clearance around resiliently mounted equipment, inches
$c$	Numerical factor or ratio; also used as viscous damping constant
$D$	Maximum expected deflection of the mounting in its axial direction in a base mounting arrangement, inches; also used as a constant in derivation of equations for frequencies of assemblies with symmetry
$D_X, D_Y, D_Z$	For four mountings with mirror symmetry of equipment with two planes of vibrational symmetry, $D_X, D_Y, D_Z$ are equal, respectively, to the absolute values of the coordinates of the effective points of attachment or $X, Y, Z$ , inches. For other cases $D_X, D_Y, D_Z$ are equal to the absolute values of $X, Y, Z$ for four equivalent mountings, and expressions are given for $D_X, D_Y, D_Z$ of these equivalent mountings under the different cases of mounting arrangements, inches
$d$	Perpendicular distance between parallel axes, inches
$E$	Maximum expected deflection of a mounting in the radial direction, inches
$F$	A function; or a force; or amplitude of a sinusoidal force
$F_x, F_y, F_z$	Forces in the $x, y$ , and $z$ directions, respectively, due to displacements of the effective point of attachment of a mounting
$f, f_n$	Natural frequency of vibration of a resiliently mounted assembly, cycles per second
$f_{max}$	Natural frequency of vibration of a resiliently mounted assembly in the higher rocking mode in a plane of symmetry, cycles per second
$f_{min}$	Natural frequency of vibration of a resiliently mounted assembly in the lower rocking mode in the same plane of symmetry, cycles per second
$f_{rot}$	Natural frequency of vibration of a resiliently mounted assembly in a rotational mode, cycles per second
$f_{tr}$	Natural frequency of vibration of a resiliently mounted assembly in a translational mode, cycles per second
$G$	Static load on each base mounting in a braced mounting arrangement, pounds

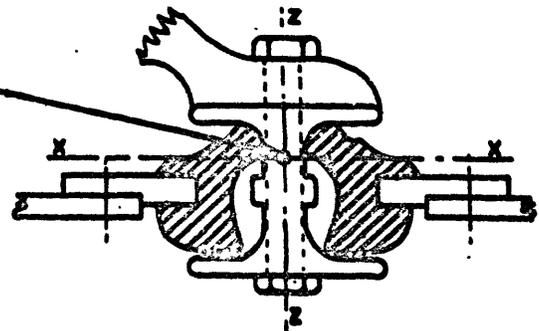
$g$	Acceleration of gravity, 386 inches per second squared
$g_y$	$(\sum k_y Y)^2$
$g_z$	$(\sum k_y Z)^2$
$H$	Perpendicular distance from the plane of the mountings* to a point on the equipment in a base mounting arrangement, inches
$h$	Height of equipment, inches

\*In more precise terms, this would be the effective point of attachment and may differ among various types of mountings from the geometrical center of a mounting, especially in the Z direction. To determine this point requires judgment on the part of the design engineer. Illustrative examples are given as follows for various designs of mountings:



Approximate effective point and XY plane of attachment for BST (15 cps) mounting. Point is about mid-height of mounting on Z axis neglecting small deflection under load.

Approximate effective point and XY plane of attachment for EES type A6L (6 cps) mounting. Due to design of mounting, point on Z axis is about midheight of the rubber on the compressed side of the mounting under load. Here the large static deflection under load must be considered.



$I$	Mass moment of inertia of equipment, pound-inch-seconds squared
$I_X, I_Y, I_Z$	Mass moment or inertia of a mounted assembly about the $X$ , $Y$ , and $Z$ axes, respectively, pound-inch-seconds squared
$I_x, I_y, I_z$	Mass moment of inertia of a mounted assembly about the $x$ , $y$ , and $z$ axes, respectively, pound-inch-seconds squared
$I_{xy}, I_{yz}, I_{xz}$	Products of inertia of a mounted assembly with respect to the $xy$ , $yz$ , and $xz$ axes, respectively
$K$	Constant used in determining the mass moment of inertia of an assembly by trifilar suspension
$K_{uv}$	Spring constant of an entire set of mountings relating a displacement in the $y$ direction with the restoring force in the $x$ direction and, conversely. A displacement $v$ in the positive $y$ direction evokes a force $-K_{uv}v$ in the $x$ direction; if $v$ and $K_{uv}$ are both positive, the force is directed toward $-x$ . Similarly, a displacement $u$ toward $+x$ evokes a force $-K_{uv}u$ in the $y$ direction
$K_{u\beta}$	Spring constant of an entire set of mountings giving either the restoring force in the $x$ direction due to unit rotation about the $y$ axis or the restoring torque about the $y$ axis due to unit displacement in the $x$ direction. The sign convention corresponds to that for $K_{uv}$
$K_{uv}, K_{u\alpha}$ , etc.	Spring constants of an entire set of mountings defined by obvious extension of above definitions. For $K_{uv}$ , $K_{u\alpha}$ , etc., the same axis is used twice
$k$	Dynamic spring constant or effective stiffness during vibration of a mounting with the same stiffness (restoring force divided by displacement) in all directions, pounds per inch
$k_X, k_Y, k_Z$	Spring constants of individual mountings in the direction of the $X$ , $Y$ , and $Z$ axes, respectively, (always considered positive)
$k_a$	Axial spring constant of an individual mounting (always considered positive)
$k_r$	Radial spring constant of an individual mount (always considered positive and independent of direction in a plane normal to the axis of elastic symmetry)
$k_{ij}$	"Cross stiffness" of an individual mounting, that is, a quantity determined by the restoring action with respect to the $i$ th coordinate due to a displacement with respect to the $j$ th coordinate. Either $i$ or $j$ may be a rectilinear or an angular coordinate

$k_{xx}, k_{yy}, \text{etc.}$	Spring constants of individual mountings giving the restoring force in the direction of one of the axes indicated in the subscript excited by a unit displacement of the effective point of attachment of the mounting in the positive direction of the other axis indicated in the subscript. The sign convention corresponds to that adopted for the $K_{uv}$ 's, etc.
$L$	Length of supporting wires of trifilar suspension, inches
$L_1, L_2$	Distance to planes of mountings parallel to the $XY$ plane in multiplane arrangements with three planes of symmetry
$l$	Length of equipment, inches; distance, inches
$M_x, M_y, M_z$	Restoring moments about $x$ , $y$ , and $z$ axes, respectively, due to the displacement of the effective point of attachment of a mounting
$m$	Mass of the unit, subbase, or assembly, $\frac{W}{g}$ , pound-seconds squared per inch
$N$	Number of mountings
$n$	$\frac{c}{2m}$
$p_i, q_i$	Terms derived from the $k$ 's and $m$ 's of a mounted assembly and used in the derivation of the frequencies for symmetrical cases
$R$	Reaction force, pounds
$r$	Distance from the supporting wires to the center of gravity, i.e., center of platform, of the trifilar suspension, inches
$r_x, r_y, r_z$	Radius of gyration of a mounted assembly about the $X$ , $Y$ , and $Z$ axes, respectively, inches
$S$	Distance between centers of the most widely spaced mountings in the direction being considered, inches; also used as constant derived from parameters of mounted assembly in frequency equations for symmetrical systems
$S_{11}, S_{12}, S_{13}$	Constants derived from spring constants of mountings used in analysis of symmetrical cases
$T$	Period of oscillation (time in seconds from one extremity to the other and back to the first, i.e., time of one cycle), seconds; also used for transmissibility

$u, v, w$	Small displacements of the center of mass of the mounted assembly in the $x$ , $y$ , and $z$ directions, respectively
$W$	Weight of mounted equipment and subbase, pounds
$w$	Width of mounted equipment or subbase, inches
$X, Y, Z$	Rectangular coordinates with origin at the center of mass of the mounted assembly when the axes are principal axes of inertia of this assembly; the coordinates of the effective point of attachment of an individual mounting with respect to these axes
$\bar{X}, \bar{Y}, \bar{Z}$	Coordinates of the center of gravity of an assembly of equipment and subbase with respect to an arbitrarily chosen set of axes, inches
$x$	Length, inches; distance, inches
$x, y, z$	Rectangular coordinates with respect to a set of axes of arbitrary orientation. In the dynamical equations this origin is taken at the center of mass of the mounted assembly
$x_1, y_1, z_1$	Rectangular coordinate axes parallel to the $x$ , $y$ , and $z$ axes but with origin $O_1$ at an arbitrary point on the axis of symmetry of an individual mounting
$\bar{x}, \bar{y}, \bar{z}$	Coordinates of center of gravity of mounted assembly or of an individual unit with respect to $x$ , $y$ , and $z$ axes
$\alpha, \beta, \gamma$	Small rotations of the mounted assembly about the $x$ , $y$ , and $z$ axes, respectively
$\Sigma$	Sign of summation over all mountings in an installation. When any factor is the same in all terms of a sum, this factor can be put in front of $\Sigma$ , for example, if $k_x$ is the same for all mountings, then $\Sigma k_x Y^2 = k_x \Sigma Y^2$
$\phi$	Phase angle by which the driving force leads the displacement in the steady-state vibration of a system with a single degree of freedom
$\phi_x, \phi_y, \phi_z$	Direction angles between the axis of symmetry of an individual mounting and the $X$ , $Y$ , and $Z$ axes, respectively
$\phi_x, \phi_y, \phi_z$	Direction angles between the axis of symmetry of an individual mounting and $x$ , $y$ , and $z$ axes, respectively
$\omega$	Circular frequency
$\omega_n$	Undamped natural circular frequency of a system of one degree of freedom

## INTRODUCTION

A resilient mounting is defined as an item designed to incorporate rigid members for attachment and resilient elements for the purpose of isolating shock, noise, and vibrations of a continuous or intermittent origin and to serve as a foundation support for an item of equipment to be so isolated. The general nomenclature has been chosen because of the difficulty in defining specifically a shock, noise, shock-noise, or vibration mounting. Mountings employed in one installation primarily to isolate noise or vibration may also provide adequate shock protection in the same or another installation.

In the application of resilient mountings to items of shipboard equipment, it should first be determined why mountings are needed and for what purpose. The chief aim is to have shipboard equipment designed to be inherently noiseproof, shockproof, and capable of withstanding normal shipboard vibration when the equipment is bolted down rigidly. Equipment that has been proved inherently shockproof by shock machine tests requires no resilient mountings, except when the equipment generates noise which must be attenuated because of a shipboard requirement. Equipment that has passed shock and vibration tests can be expected to withstand normal vibrations experienced aboard ship when bolted down rigidly. Laboratory tests<sup>1</sup> indicate that even electronic equipment will often satisfactorily pass shock and vibration tests without resilient mountings.

When it is necessary to employ mountings, care must be exercised to prevent excessive amplification of vibration resulting from excitations in the equipment itself or from propeller, hull, or adjacent machinery.

On submarines, it is desirable that certain natural frequencies of equipment-mounting installations be lower than the frequencies of exciting forces or motions in order to reduce noise transmission. Fortunately, low-frequency mountings can be employed on submarines because of the small vibration amplitudes that usually result from the propeller and hull excitations. On the other hand, for surface ships, it is often desirable to have the natural frequencies of equipment-mounting installations higher than the vibration frequencies excited by propeller forces in order to avoid resonances. In such installations, amplification of the exciting forces or motions will exist, but if such amplification is no greater than three, it is generally considered acceptable. The higher shaft speeds and the greater number of propeller blades on more recent ships make it more difficult to avoid resonances with vibration frequencies excited by propeller forces. In these instances, mountings with high damping characteristics may have to be used.

The calculations involved in the selection and application of mountings are, at best, time-consuming and tedious. Information must be available on the characteristics of the equipment and mountings and the exciting frequencies that may be encountered. To meet the requirements of specific conditions for a particular installation, a tentative selection and

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<sup>1</sup>References are listed on page 137.

positioning of resilient mountings must first be made. Natural frequencies of the resiliently mounted systems must then be calculated. If the design requirements are not met, the mountings will need to be repositioned or a new selection made, and the frequencies must be calculated again until the design requirements are satisfied.

Accordingly, this "Mounting Guide" has been prepared to aid design engineers and engineering draftsmen in solving problems dealing with the selection and application of resilient mountings to items of shipboard equipment. The main objective has been to present quick and practical methods of solving mounting-installation problems by utilizing charts, tables, and simple formulas wherever possible.

This guide contains three chapters and seven appendixes, the purposes of which are as follows:

- (a) To describe briefly the nature and principles of noise, shock, and vibration isolation—Chapter 1 and Appendix 1.
- (b) To discuss the physical constants needed for the selection and application of resilient mountings and to present methods for their determination—Chapter 2 and Appendix 3.
- (c) To emphasize the general considerations and list the precautions that should be observed in the selection and application of mountings—Chapter 1.
- (d) To present methods for calculating the natural frequencies of systems with various types of mounting arrangements—Chapter 3 and Appendixes 2 and 6.
- (e) To present additional information to engineers desiring to delve more deeply into the theory of isolation and the equations of motion—Appendixes 1, 5, and 6, References, and Bibliography.

## CHAPTER 1

### NATURE OF THE TRANSMISSION OF NOISE, SHOCK, AND VIBRATION

#### 1.1. NOISE, SHOCK, AND VIBRATION

In the selection and application of resilient mountings to shipboard equipment, the reduction of the transmission of noise, shock, and vibration is of concern only for certain paths of transmission. Applied shock and vibration are transmitted to shipboard machinery and equipment through the ship structure. In the opposite direction, noise and vibration are transmitted from machinery through the intervening ship structure to the hull and water. Such noise may then be transmitted to an enemy ship or to listening devices in the ship; see Figure 1.

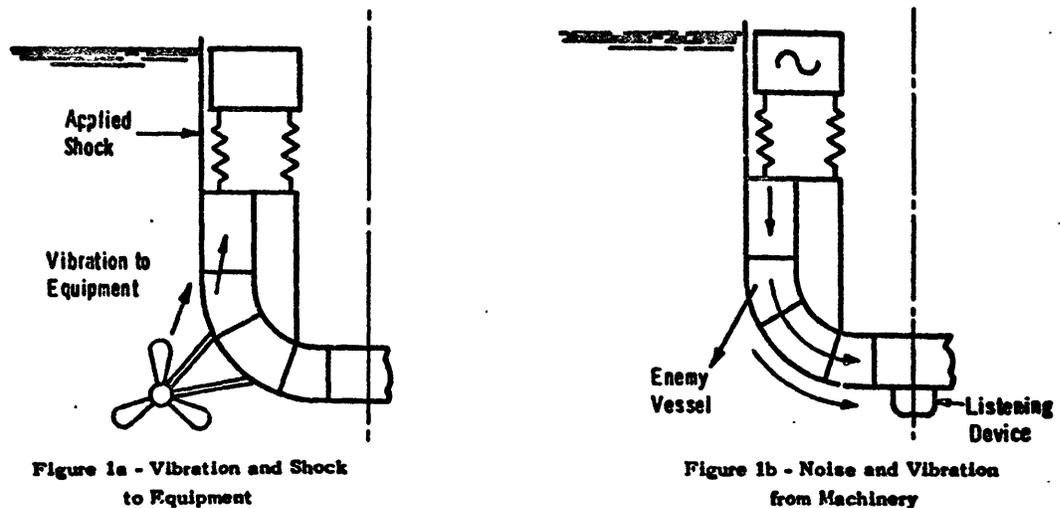


Figure 1 - Directions of Transmission of Noise, Shock, and Vibration on Shipboard

The reduction of noise, shock, and vibration is necessary for various reasons. Noise may either reduce the listening range of the ship itself or increase the chance of detection of the ship by listeners on other ships. Applied shock, such as that of noncontact underwater explosions, may damage or render inoperable certain types of equipment. Naval equipment, with the exception of some electronic units, is expected, however, to withstand the normal vibration existing on shipboard; therefore it is only necessary to prevent excessive amplification of these vibrations by proper design of mountings.

There are four approaches to the problem of reducing the severity of noise, shock, and vibration on shipboard:

1. Improvement or modification of the equipment design.
2. Relocation of equipment aboard ship.
3. Installation of resilient mountings.
4. Proper maintenance and balancing of equipment.

Design improvement embodies design of ship structures, such as machinery foundations, as well as design of the individual machinery items, particularly design that avoids resonances in structures and appurtenances. It is assumed that conditions resulting in abnormal operation, such as excessive unbalance or misalignment of propellers, shafting, or machinery, already have been remedied. Properly aligned equipment can easily become misaligned in service; therefore it is essential that equipment be checked frequently and realigned when necessary. As design of equipment for shipboard installations improves, fewer attenuating devices, such as resilient mountings, will be needed. There are certain locations on board ship where the noise, shock, and vibration excitations have smaller amplitudes than in other parts of the ship. Both design improvement and relocation can reduce the exciting forces and motions and consequently modify the response.

Meanwhile, the need for installation of machinery and equipment on resilient mountings continues. Resilient mountings can reduce the effects (transmissibility) of shock motion and vibration on equipment and can reduce the transmissibility of noise and vibration from machinery to the ship structure and the water. *Improperly selected resilient mountings, however, may increase rather than decrease the transmissibility.*

## 1.2. RESPONSE OF RESILIENTLY MOUNTED BODIES

A body that has extremely high stiffnesses between its components may be treated approximately as a rigid mass. If such a body is supported by a massless spring and constrained by frictionless, rigid guides to move only in a vertical direction in the plane of the paper, see Figure 2, it is a one-degree-of-freedom system, that is, the position of the mass at any instant can be described by one coordinate. The system has one natural frequency corresponding to a vertical mode.

If the vertical guides are now removed, with the motions still confined to the plane of the paper, and the spring has vertical, horizontal, and rotational stiffnesses, Figure 3, the system has three degrees of freedom and therefore has three natural frequencies. Two of the natural frequencies due to horizontal vibrations cause modes of rotation about two separate axes perpendicular to the plane of the paper. These are designated rocking modes. The rocking mode  $f_{\min}$  is caused by the mass attempting to move in a horizontal direction, but because of the restraint offered by the mountings, the mass rocks about an axis below the center of gravity. The rocking mode  $f_{\max}$  is initiated by the tendency of the mass to rotate

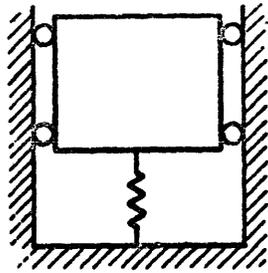


Figure 2 - A One-Degree-of-Freedom System

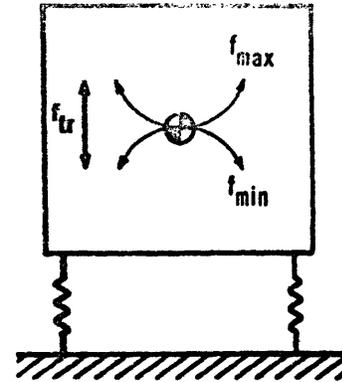


Figure 3 - A Three-Degree-of-Freedom System

about the center of gravity, but since the mountings constrain this rotation, the mass rocks about an axis above the center of gravity. In general, in a system of three degrees of freedom, each mode will involve all three coordinates, but because of the symmetry in this example, the third natural frequency  $f_{tr}$  is for a pure vertical translational mode.

Let it now be assumed that the center of gravity of the mass is at its geometrical center and that the principal axes of inertia are the  $X$ ,  $Y$ , and  $Z$  axes and let the system be supported on eight resilient mountings symmetrically positioned about the mass, as in Figure 4. Then the system has natural translational frequencies in the  $X$ ,  $Y$ , and  $Z$  directions and also natural rotational frequencies about the  $X$ ,  $Y$ , and  $Z$  axes. Thus the system has six natural frequencies, or six degrees of freedom and six natural frequencies in all.

If there is partial or complete lack of symmetry, certain of the translational and rotational modes may be replaced by rocking modes, each of which has translational and rotational components of motion. In general, a mass supported by resilient mountings has six normal modes of vibration and therefore six natural or resonance frequencies. It is possible, however, for two or more modes to have the same frequency. Each mode may have translational and rotational components. Such a system has the property that a free vibration may exist in any one of these modes without exciting any of the others.

Rocking modes, however, are not the most complicated types of motion that can occur in a normal mode of vibration of a rigid mass. The body may move in rotation about a particular axis and in translation along an

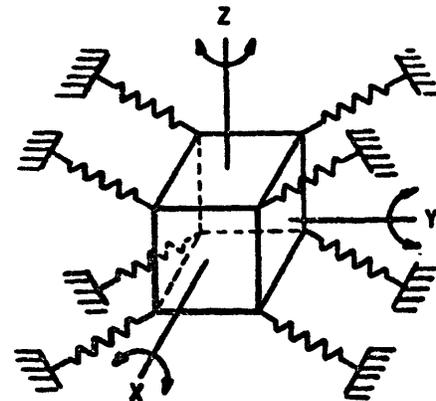


Figure 4 - A Six-Degree-of-Freedom System

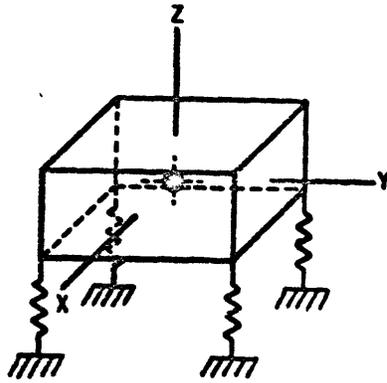


Figure 5 - Typical Base-Mounting Arrangement

axis inclined to the first axis. In general, a free vibration may exist in any mode without exciting any of the other modes.

For a frequently used arrangement, sometimes called "bottom mounting" or "base mounting," with four identical mountings symmetrically attached to the base of a unit which is also symmetrical, Figure 5, the horizontal translational mode of vibration along the Y axis and the rotational mode about the X axis are supplanted by two rocking modes about axes parallel to the X axis. Likewise, there are two rocking modes about axes parallel to the Y axis. There are two other modes, one rotational about the Z axis

and the other translational along the Z axis, each having its own natural frequency. There are six modes of vibration in all: four rocking, one translational, and one rotational.

### 1.3. TRANSMISSIBILITY

Consider the one-degree-of-freedom system shown in Figure 2. If the mass is displaced and then released, it will oscillate vertically at a substantially constant period or frequency, but the amplitude of vibration will gradually decrease because of damping forces.

If, now, instead of an initial displacement, a sinusoidal force of constant amplitude is applied to the mass, see Figure 6a, or if a sinusoidal motion of constant displacement is applied to the spring support, see Figure 6b, after a short transitional interval, the mass will move in a forced vibration of constant peak amplitude at the frequency of the exciting force

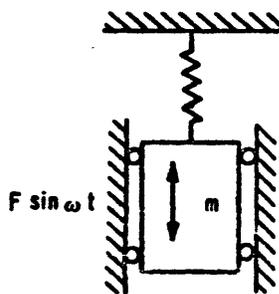


Figure 6a - Sinusoidal Force Applied to the Mass

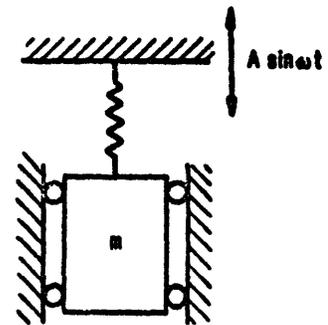


Figure 6b - Sinusoidal Motion of the Mounting Support

Figure 6 - Motions and Forces Applied to an Undamped One-Degree-of-Freedom System

or displacement. For a given amplitude of excitation, the amplitude of the forced vibration will depend upon the magnitude of the damping and upon the ratio of the exciting frequency to the natural frequency of vibration of the mass.

The variation of amplitude with frequency is shown for a typical one-degree-of-freedom system in Figure 7, in which the ordinate is proportional to the amplitude of vibration of the mass. As shown by the peak in the curve, the amplitude of vibration is a maximum when the frequency of vibration nearly equals the natural frequency of vibration of the mass. With less damping, the peak would be higher and the frequency closer to the natural frequency; with greater damping, it would be lower.

The ordinate in Figure 7 actually represents what is called the transmissibility for the mass on the spring. When the motion is excited by a sinusoidal force applied to the mass, sinusoidal force is also transmitted through the spring to the supporting base. The ratio of the amplitude of the force transmitted to the base to the exciting force is the transmissibility. The same number also represents the ratio of the amplitude of vibration of the mass to the amplitude of vibration of the base when the motion is excited by motions of the base. It will be seen that the transmissibility is unity for a static force or displacement, is greater than unity at frequencies between zero and resonance, and eventually becomes less than unity at frequencies well above resonance.

The phenomenon of forced vibration is described analytically in Appendix 1.

#### 1.4. SUITABLE NATURAL FREQUENCIES

Frequency requirements related to ship vibration, to shock motion, and to noise excitation will be described successively. First, however, the importance of these requirements should be discussed. On submarines, the primary purpose of resilient mountings is noise reduction. Equipment resiliently mounted to reduce noise must also be able to withstand shock motions of relatively large magnitude and vibration, with some allowable magnification of motion without failure or maloperation. On surface ships, at least for the present, noise reduction is less important, but the equipment must withstand shock motions and also vibrations moderately magnified in amplitude. Much shipboard equipment will perform satisfactorily under shock excitation, and even more equipment under vibration excitation if it is rigidly

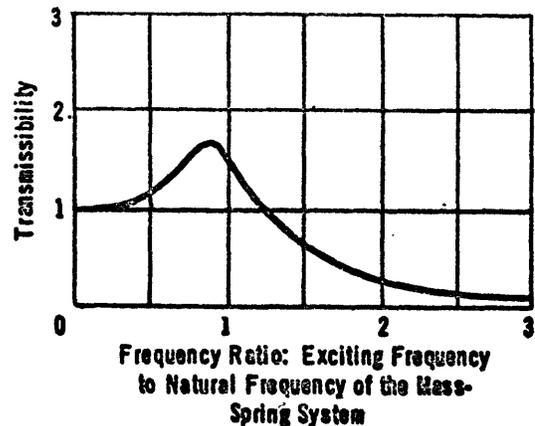


Figure 7 - Response of a One-Degree-of-Freedom System to Excitations at Various Frequencies

mounted. Therefore, for this sort of equipment, no resilient mountings need be provided except where noise reduction is required.

#### 1.4.1. VIBRATION

Two types of vibration excitation of resiliently mounted equipment occur on shipboard, excitation from ship structure and self-excitation of equipment by unbalanced moving parts.

The hull of a ship can be excited by vibration in the propulsion machinery system as well as by thrust variations due to the variation of pressure on each propeller blade as the propeller rotates. The lateral component of thrust variation causes vibration at frequencies equal to the shaft rpm multiplied by the number of blades of the propeller. If the excitation caused by thrust variation has a frequency equal or close to one of the natural frequencies of the hull in flexural vibration, then the hull may respond to this excitation at amplitudes large enough to be troublesome. For most naval ships, the frequency of this excitation has not exceeded 25 cps (1500 cpm), but, with the trend toward higher shaft speeds and 5- and 6-bladed propellers, it may become as high as  $33\frac{1}{3}$  cps (2000 cpm).

One way of avoiding resonances is to keep the resonance frequencies of the equipment above the excitation frequencies. As shown by the transmissibility curve, Figure 7, the steady-state vibration cannot then be kept from exceeding the vibration of the foundations, but it can be kept within reasonable limits. In the absence of damping, a satisfactory limit of magnification is obtained if all important natural or resonance frequencies of the equipment are made at least 1.4 times as great as the maximum excitation frequency.

If a surface ship, for which structure-borne noise is of minor importance, has a highest propeller-blade excitation frequency of 20 cps (1200 cpm), the natural frequencies of resiliently mounted equipment should be at least 25 cps (1500 cpm) and preferably 28 cps (1680 cpm). On this basis, many units of radio and radar equipment have been installed with mountings having a natural frequency of 25 cps under rated load. For many classes of surface ships, the typical vibratory displacement of the hull is about 15 mils single amplitude vertically, about two thirds of that horizontally, and less than one-third fore and aft in the few instances measured. On infrequent occasions, particularly during hard turns, magnitudes several times as great occur. Certain maximum environmental values are given in MIL-STD-167 (Ships), "Mechanical Vibrations of Shipboard Equipment," 20 Dec 1954.

Submarines have propeller-blade excitation frequencies up to 31 cps (1850 cpm). In order to reduce as much as possible the transmissibility of noise from machinery through the hull into the water, the trend is to install mountings with natural frequencies ranging from 8 to 15 cps at rated loads.

Steady-state vibration is noticeable throughout the entire hull only near the critical speeds, that is, when either the shaft frequency or blade frequency approximately coincides with one of the natural frequencies of the hull, but at intermediate excitation frequencies, there may be little or no vibration on the ship anywhere except at the stern. The forced vibration is usually most severe at high speeds and is aggravated on ships with large overhangs

at the stern. The vibrations may have larger amplitudes on superstructure members such as masts or directors. If these members are located near a nodal point of the hull for the particular mode of vibration that is present, they may be set in rotational motion because the nodal point is a point of maximum change of slope. If their own natural frequencies lie near one of the natural frequencies of the hull, the amplitude at the top will be much greater than that at the base.

The installation of equipment in structures such as masts, directors, high superstructures, and fantails should be avoided wherever feasible. Sometimes equipment that needs to be resiliently mounted can be located at a lower level in the ship away from the fantail. Locations, such as light bulkhead plating which may itself resonate at excitation frequencies within the operating speed range of the ship, should also be avoided whether or not the equipment is resiliently mounted.<sup>2</sup> This does not preclude supporting the equipment on bulkhead stiffeners.

The other aspect of vibration, excitation of equipment by its own moving parts, also requires some comment. Unbalances of internal rotating parts can cause excessive vibration if the resulting excitation frequency is close to one of the natural frequencies of vibration of the equipment on its mountings. If suitable natural frequencies of the equipment on its mountings cannot be obtained by rearrangement or substitution of mountings, then the vibration displacements can be reduced by substitution of different types of equipment when available, such as those with different operating speeds.

Ways to avoid excessive vibration have been described previously, but no quantitative statement has been offered as to what constitutes excessive vibration. Without attempting an exact definition, it may be said that vibration is excessive when it causes damage or there is danger of damage to structural components, when it interferes with the operation or causes malfunction of equipment, or when it is an unnecessary and avoidable nuisance.

Some resonance frequencies of resiliently mounted equipment prove to be relatively unimportant because of the absence of excitation for these corresponding modes of vibration. Orientation of equipment may permit disregard of these modes. Equipment and machinery on shipboard are normally oriented in a fore-and-aft direction. Since there is little excitation of equipment in this direction, either internally or externally, modes of vibration in a fore-and-aft direction are not as important as other modes with the exception of those of longitudinal vibration of the propulsion machinery.

#### **1.4.2. SHOCK MOTION**

Shock excitation of resiliently mounted equipment is produced by motions of the supports of the resilient mountings. These supports or foundations have shock motions involving sudden velocity changes which may be as great as 75 fps and may occur in about 2 msec. The mechanism of shock excitation is treated in several references and will not be repeated except to mention that its effect varies considerably depending on many factors including size and

weight of the equipment, its location in the ship, the class of ship, the time history of the shock excitation, and the response characteristics of the equipment and its components. Only the following topics will be discussed: the significance of the Military and Bureau of Ships specifications, acceptance testing for shock motion, the influence of equipment location on severity of shock, and design changes that may improve resistance of equipment to shock motion.

The following specifications have been issued for evaluating the performance of equipment under shock and vibration:

- (a) Military Specification MIL-S-901<sup>3</sup> describes shock tests only for Class HI (High-Impact) shockproof equipment.
- (b) Military Specification MIL-T-17113 (SHIPS)<sup>4</sup> prescribes shock tests and is used primarily for evaluating electronic equipment. In this specification, tests on the mediumweight shock machines are divided into Class A and Class B shock tests. Tests on lightweight shock machines are conducted with reduced heights of hammer blows as compared with tests specified in Specification MIL-S-901. In MIL-T-17113 (SHIPS), provisions are also made for testing equipment with or without resilient mountings.

These specifications and testing machines are used by the Navy to give some assurance that equipment passing such tests will give satisfactory performance in service.

The location of the equipment aboard ship has considerable influence on the severity of shock motion that must be withstood. For excitations caused by underwater explosions, the most severe condition occurs when the equipment is mounted on the hull itself, especially the hull plating, which is subject to direct impact of the explosion pressure wave. There is a trend toward decrease in shock severity with increased distance of the equipment from the hull. Air blast caused by gun fire or exploding bombs can also cause shock excitation of exposed structures such as decks, side plating of superstructures, and directors. Wherever possible, equipment should not be exposed to such excitations by attachment to the inner sides of directly exposed structures.

Shock tests and "striking" tests show that the vibration of structures, such as hull, side plating, and decks, is excited at many natural frequencies simultaneously. In a number of tests, the dominant vibrations (the components of vibrations having the largest amplitudes) for surface ships were in the range from 35 to 100 cps, with many at about 50 cps. For submarines, the dominant frequencies were higher, principally in the range of 100 to 400 cps.<sup>2</sup> These are frequencies at which the structures respond, and they would be the principal frequencies of shock motion at the points of attachment of mountings if equipment were installed here.

Modifications in the design of equipment have often made it capable of withstanding shock excitation without the use of resilient mountings.<sup>5</sup> Items such as switches and levers have been designed with counterweights to prevent maloperation. Methods of support and

fastening have been altered, and materials have been substituted so that now many items can be solidly mounted. Equipment with rotating or reciprocating parts has been strengthened by increasing bearing diameters and lengths and shaft diameters, so that shock damage is eliminated or reduced and the machinery operates satisfactorily despite the excitations. However, equipment of this sort for submarine service and, eventually, for certain types of surface vessels may still require resilient mountings to reduce the transmission of noise to the water and to sonar gear.

### **1.4.3. NOISE**

Structure-borne and air-borne noise is generated on shipboard by rotating and reciprocating parts of mechanical equipment, by electrical equipment, and by moving fluids. The structure-borne noise is of primary importance at present in the study of methods for reducing noise. In resiliently mounted equipment, structure-borne noise is transmitted through the mountings and ship structure to other parts of a ship and radiated into the water, generating water-borne noise. Structure-borne noise may be loosely defined to be vibration with small amplitudes and with frequencies above about 20 cps; structure-borne noise is the vibration of structures that will generate sound waves in the surrounding fluid mediums. In a broader sense, it may be defined as vibration at any frequency and amplitude that interferes with a ship's own listening devices or that may be detected by another ship. Other sources of noise on shipboard, such as propeller noise, hydrodynamic noise, and propeller-shaft squeals,<sup>6</sup> are not discussed since they are not of concern in the selection of mountings.

At present, resilient mountings are used for machinery on submarines primarily to reduce noise. It is expected, however, that certain types of surface vessels with specialized service requirements may also require this protection. More extensive discussion of the influence of location, design, and mountings on noise transmission has been given elsewhere.<sup>6</sup>

Equipment with rotating and reciprocating parts can generate noise of three types:<sup>6</sup>

**1. Noise of definite frequencies, for example:**

- (a) Running frequency caused by unbalanced rotating parts;**
- (b) Gear noise at a frequency equal to shaft speed times the number of teeth;**
- (c) Noise at armature slot frequency, which is a frequency equal to the shaft speed times the number of slots; and**
- (d) Noise at the commutator frequency.**

**2. Noises of definite frequencies which do not change with the speed of the machines, such as free vibrations of machine components, the subbase, or the foundation, caused by impact of machine parts such as cams or valves.**

**3. Random noises caused by impacts occurring at irregular intervals.**

Resilient mountings can reduce the structure-borne noise transmitted to the ship structure and to the water. The lower the rated natural frequency of the mounting, the greater will

be the overall attenuation of noise. Reducing the rated natural frequency may, on the other hand, create problems in the stability of equipment, particularly under such motions as roll and pitch.

The loading on the mountings cannot always be exactly the design loading. For reduced noise transmission, it is better for a mounting to be moderately overloaded than considerably underloaded. Subject to experimental verification, it is believed that overloads in the order of 10 percent will not materially affect the strength of a mounting under shock excitation.

A characteristic of mountings that must be considered in attempting to reduce noise transmission is the phenomenon of standing waves or resonance vibration within the mountings themselves. For this reason, a reduction of the rated natural frequency of the mounting does not necessarily guarantee a reduction in transmission throughout the noise spectrum. It may happen that one of the excitation frequencies coincides with one of the standing wave resonances, causing greater noise transmission at this frequency.

It is not necessary to reduce the noise of all equipment; it would be futile to reduce noise of certain units while units with much greater noise levels are not improved. The priority of items in a noise-reduction program depends also on the operating condition of the ship. In submarines, the first step has been to improve noisy machinery items that must be operated during listening condition. Further study is needed on the effects of transmission of noise into the water by intervening structure between a machine and the hull and the sub-problem of the design of the mounting supports, that is, the foundation of the machine. But if the more important noisemakers on submarines, especially those that need to be operated during listening condition, can be segregated and located remotely from listening devices, the listening ability of a submarine can be improved considerably.

In selecting mountings and in planning the installation of machinery, precautions should be taken lest the beneficial effects be spoiled by the incidental introduction of acoustical shorts. For example, flexible couplings for shafting and piping and flexible conduit for electrical wiring need to be provided. Piping may need to be further isolated by suspending it in rubber-lined hangers.

Redesign of machinery can reduce its noise generation.<sup>6</sup> The use of helical and worm gears instead of spur gears, nonmetallic gears or belts instead of metallic gears, and sleeve bearings instead of ball bearings often helps to reduce noise at its source. Dynamic as well as static balancing of rotating parts reduces the exciting forces, as does proper design of slots and poles of motors and generators.

The final test of the effectiveness of a noise-reduction program is a noise survey.

## 1.5. ELASTIC FOUNDATIONS AND EQUIPMENT

In presenting methods for the calculation of natural frequencies of vibration in Chapter 3, it will be assumed that the mountings are attached to infinitely stiff foundations and that

the equipment is also infinitely stiff internally. To make these calculations applicable, the subbase, legs, frame, and principal components must be rigid enough so that the assumed infinite stiffness is validly approximated. These assumptions are not valid for all shipboard installations.<sup>7</sup>

If the foundation to which an isolation mounting is attached is flexible and relatively massless, then the natural frequencies of the assembly will be lower than those calculated, without correcting for flexibility of foundations. If the foundation has appreciable mass as well as flexibility, the assembly will have additional degrees of freedom and the normal analysis will not be entirely valid. In this case, because of resonances in higher modes, magnification may appear at frequencies where the analysis, without correction, would predict attenuation. This effect may result in greater noise transmission than anticipated in certain frequency ranges.

The possibility of increased transmissibility due to elasticity of foundations can be minimized, particularly for equipment such as machinery with internal vibration excitation, by avoiding the installation of equipment on light bulkhead or deck plating. Where resilient mountings are used, equipment foundations should be integrally stiff and securely attached to those stiffeners of the ship structure which carry the equipment load. These precautions reduce the response of equipment to propulsion-system excitation and may reduce noise transmission.

Flexibility of equipment has effects on the transmissibility curve similar to the effects of flexibility of foundations.<sup>8</sup> Resonances will appear superimposed on the attenuation portion of the transmissibility curve depending on the number of components in the equipment with supports of low relative stiffness and on the weights of these components. Normally the magnification factor of the equipment will not greatly exceed unity; therefore vibration transmission need not be severe. Damaging effects may occur, however, if internal components are in resonance with vibration excitations. Testing the equipment, when not too large, in shock machines or even vibration machines will indicate whether damage may occur because of relatively low stiffness within the equipment itself. If there is internal excitation, reduction of noise transmission from the equipment may be difficult unless some of its components can be redesigned.

Experience has indicated that for installations where equipment was relatively limber, such as radio units with light frames and heavy transformer components, the actual fundamental natural frequency of the unit of equipment on four mountings was 15 to 20 percent lower than the computed natural frequency. If the stiffness of the equipment, as determined by vibration tests on its mountings, is compared with that of an equal solid or stiff mass on the same mountings, then a stiffness ratio less than unity is obtained. The reciprocal of this stiffness ratio, the flexibility factor, when multiplied by the actual mounting stiffness results in a required stiffness on the basis of which mountings may be selected. The actual mounting stiffness would still be used in calculations, but the selection of a somewhat stiffer mounting,

Factor should be multiplying the actual stiffness of the mounting by the flexibility factor, would compensate for the flexibility of the equipment. In a similar manner, if the foundation itself is flexible, the flexibility factor may be determined from the stiffness ratio by a vibration test of the equipment on flexible foundations in comparison with an infinitely rigid base. The flexibility factor is never less than unity.

For lightly constructed equipment on four mountings, flexibility factors as great as 1.4 have been observed. For sturdy radio and radar units, the ratio may be 1.1 or 1.2. The use of more than four mountings tends to reduce this factor because of support provided by the mounting fittings. For motors and machinery of small to medium size, the factor can be assumed to be unity, particularly if components, such as rotors and shafting, have comparatively large diameters. Whether this will hold true for larger units, such as diesel engines, remains to be determined by design computations and shipboard vibration tests.

It must be admitted that there are very few experimental data concerning the flexibility of equipment and of foundations. However, the approach discussed above is believed to result in better approximations to desired natural frequencies of installed resiliently mounted equipment. In any case, the amount of cut-and-try should be reduced.

## 1.6. CLEARANCE

Adequate clearances around mountings and equipment are necessary to permit the mountings to function properly and to prevent damage to the mountings and equipment because of excessive shock or vibratory displacements. On the other hand, since equipment with resilient mountings occupies more space than rigidly mounted equipment, no more space must be requisitioned than is actually necessary.

The value of the maximum expected deflection of mountings, including allowable deformation of metal parts, must be known in order to determine the necessary clearance for a unit of equipment. According to data now available, the greatest deflection occurs during shock machine tests and the least when the mounting is subjected to the prescribed static test load. Deflections obtained during full-scale shock tests are between these extremes. As a general rule, mountings should not be installed aboard naval ships until samples have been subjected to at least shock, static load-deflection, and vibration tests to obtain information for calculating clearances. Specific information on deflections for mountings should be obtained by reference to reported characteristics of resilient mountings or to the latest Bureau of Ships Instruction 9110.4.

Another consideration in the selection of resilient mountings for equipment, particularly for shock isolation, is the amount of permissible travel or deformation of the mounting. This depends upon design and, in the case of rubber mountings, on the thickness of rubber in the direction of travel. A mounting of good design will have minimum overall size for a particular load rating and spring constant, thus conserving space in the installation of equipment. A necessary precaution is the avoidance of hard bottoming of the equipment either

because of inadequate mounting design or because of inadequate clearances of adjacent structures.

The selected value of deflection gives directly the clearances needed for translational modes of motion. From this value and the dimensions of the equipment, clearances for rotational modes may be computed, and those for rocking modes may be estimated.

The required clearance  $C$  (in inches) around equipment may be determined approximately for various mounting arrangements as follows:

1. For three planes of symmetry as illustrated by a center-of-gravity mounting arrangement, Figure 8a, the clearance  $C$  around the equipment should be equal to the maximum deflection that occurs across the mounting in either the axial or radial direction. In cases where the center of gravity is asymmetrically located within the equipment, greater clearance may have to be provided at points remote from the center of gravity.

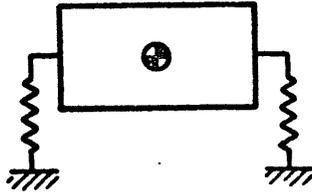


Figure 8a - Center of Gravity

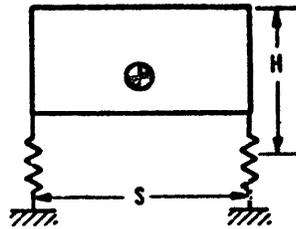


Figure 8b - Base

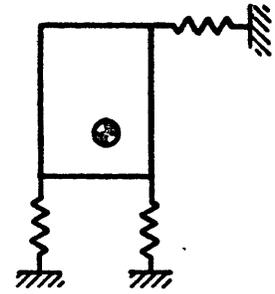


Figure 8c - Braced

Figure 8 - Mounting Arrangements

2. For two planes of symmetry as illustrated by a base or bottom mounting arrangement, Figure 8b, the clearance  $C$  can be calculated from the approximate formula<sup>9</sup>

$$C = \frac{2DH}{S} + E$$

where  $D$  is the maximum expected deflection of the mounting in the axial direction, inches,  
 $E$  is the maximum expected deflection of the mounting in the radial direction, inches,  
 $H$  is the perpendicular distance from the plane of the mountings to the point on the equipment (see footnote on page iv of Reference 9) in inches, and  
 $S$  is the distance between centers of the most widely spaced mountings in the direction being considered, inches.

3. For one plane of symmetry as illustrated by a braced mounting arrangement, Figure 8c, the clearance  $C$  around the equipment should be equal to the maximum deflection that occurs

across the mounting in either the axial or radial directions. If the mountings are attached away from the corners of the equipment, that is, if the top mountings are lowered or the bottom mountings are spaced closer together, re-evaluation of the mounting geometry must be considered and allowance must be made for greater clearance.

If two resiliently mounted units are placed next to each other, the clearance between them must be the sum of the clearances required for each of the units since the motions of the units may be out of phase. Protuberances such as knobs, sockets, nuts, dimples, and flanges must be accounted for in providing clearances. If there is a rigid conduit or pipe attached to the equipment, then adequate clearance must be provided around the piping up to points of flexibility, such as rubber or bellows joints or flexible U-bends, to prevent striking. In some instances, clearance may have to be provided beyond these points of flexibility in the form of rubber-lined hangers for the pipe or conduit.

In detailing the attachment of the resilient mounting to the equipment or subbase and to the foundation or supports, provision must be made for free motion of the resilient elements of the mounting. This is illustrated in Figure 9 for a Portsmouth BST mounting. If the hole in the subbase plate were small so that the plating extended to points  $A_1$ ,  $A_2$ , the rubber of the mounting would strike the subbase plate and eventually fail because of the cutting and abrading action of the plate. The hole in the subbase plate should be made as large as possible, consistent with strength, leaving sufficient material at the bolt holes. Chamfering the plate edge is also beneficial.

Finally, all the care, effort, money, and time expended in selecting and applying resilient mountings goes for naught if indiscriminate permission is given to use the clearance spaces for other purposes. It is true that certain parts of the clearance spaces can be more fully utilized, but the mounting engineer should always have authority as well as responsibility to control this space in order to prevent not only the nullification of isolation but also damage to the equipment.

It is difficult to establish criteria of acceptable maximum motion for resiliently mounted equipment because each installation has to be judged separately. Previously, a procedure for calculating the clearances required for motion of the equipment due to shock excitation was presented. If sufficient space is not available for the clearance required for a bottom mounting arrangement, or if the violent motion is such that the equipment intrudes in space where personnel normally are working, then the equipment may be damaged or personnel injured. In these instances, it is often better to use braced mounting arrangements or stabilizing mountings to reduce the excessive rocking motion of the equipment.

There are other circumstances under which motions may be even larger, usually for bottom mounting arrangements. The equipment may sway from one extreme position to another because of a ship's rigid-body motions such as rolling and pitching. It is not likely that installations will be encountered in which gravity will have an important effect on any of the natural frequencies of the mounted equipment. Moreover, since the lowest frequency of the

assembly will usually be at least several times as high as the frequency of the ship's rigid-body motions, no resonance magnification of these slow motions into a large movement of the assembly relative to the ship structure is likely to occur. However, in mounting equipment with a high center of gravity relative to the width of the base, the static deflection due to inclination of the ship in a seaway should be estimated for an angle of, say, 30 deg. If it is found that the upper corners of the assembly would take up a large part of the clearance, additional clearance should be allowed. If the required clearance becomes excessive, the effect of static inclination can be decreased by setting mountings farther apart, introducing stabilizing mountings, or changing to a braced-mounting arrangement. When the mountings are set farther apart, the natural frequencies of the system, particularly for rocking and rotational modes, should be checked to insure that the relocation of the mountings does not result in unsatisfactorily high natural frequencies.

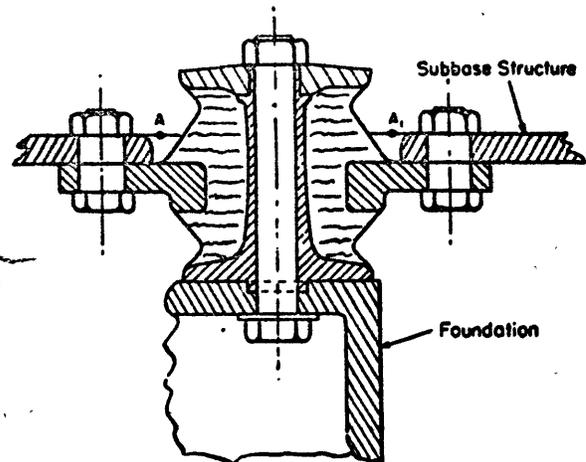


Figure 9 - Detail of Portsmouth BST Mounting Attached to Subbase and Foundation

## 1.7. PRECAUTIONS

The following check list of precautions that should be taken in the use of resilient mountings is appended for convenience. The list includes the principal points already mentioned and additional minor items.

### 1.7.1. INSTALLATION

1. The rubber resilient elements of mountings should not be painted.
2. The rubber elements of mountings should be protected from the effects of oil. Where this is inconvenient, the rubber should be of oil-resistant shock.
3. All welding or flame cutting of structures in way of mounting locations shall be performed prior to the installation of resilient mountings.
4. The installation of mountings and the alignment of mounting surfaces of equipment and foundations shall be such as to insure that all load-carrying mountings with the same rated load capacity and stiffness have equal deflection under load. Holes for bolts for securing mountings to foundations and holes in the feet or subbases of equipment shall be aligned to prevent any distortion of the mountings.

5. Some mountings are designed to be loaded in a specific direction. Care should be taken to insure that all such mountings are installed so as to be loaded in the correct direction.

6. Mountings used as stabilizers are not to share the dead load of the equipment and should not be statically deflected after the equipment is installed.\* Mountings in a stabilizer arrangement are not to be confused with braced-mounting arrangements. In the latter arrangement, all mountings share their design portion of the dead load.

7. Bolts designed to be stressed in shear shall be installed in holes with minimum clearance.

8. Bolt material should be as specified in the Interim Military Specification MIL-M-17185 (SHIPS) dated 12 May 1952.

9. Sufficient clearances should be provided around equipment installed on mountings to prevent the equipment from striking ship structure or other fixed or resiliently mounted equipment.

10. No conduit, pipe, or other item should be located in the clearance spaces around resiliently mounted equipment without approval of the mounting engineer.

11. Piping connected to resiliently mounted equipment should have long runs, preferably two 90-deg bends, and rubber-lined hangers close to the equipment, where necessary, in order to minimize restraint of the equipment by the piping. Flexible couplings should be installed in each line close to the mounted machinery, and their effect on natural frequency should be estimated.

12. Electric ground straps shall be provided for all resiliently mounted equipment for the safety of personnel. All cables, flexible connectors, and ground straps shall be so attached that no pull is exerted on the equipment, and sufficient slack shall be allowed for movement of the equipment on mountings under shock conditions.

### **1.7.2. SERVICING OF MOUNTINGS**

13. In regard to overhaul, inspection, drift, and replacement of mountings, shipbuilding and design activities should refer to Bureau of Ships Instructions 9110.4 and 9110.5 for latest information.

### **1.7.3. EQUIPMENT**

14. The manufacturer should determine and furnish the weight, location of the center of gravity, moments of inertia about the principal axes, and operating speeds of his equipment.

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\*Even though they carry no static load, snubbers, stabilizers, braces, and all pipe connections affect the natural frequencies of a system to some degree. All that can be said about their effects is that they tend to raise the natural frequencies of the system; not enough work has been done to evaluate quantitatively the spring constants of these flexible connections. By imposing no static load on these devices, their effects are minimized.

15. The weight of liquid, if any, should be considered in determining the data in 14.

16. Components of an item of equipment as well as the assembly of a number of units on a common subbase should be arranged so that the moments of inertia do not differ by a ratio of more than 5 : 1; preferably they should be as close to 1 : 1 as is possible.

17. Manufacturers and technical sections responsible for procurement of shipboard equipment have joint responsibility in determining in the early design stages whether it is likely that resilient mountings will be used: If it is decided that there is a possibility that mountings will be used, the equipment designer should give consideration to and provide for several alternate methods for attaching mountings to equipment.

#### **1.7.4. SELECTION AND APPLICATION OF MOUNTINGS**

18. The effect of liquid in the equipment, if any, and the effect of piping and conduit associated with the equipment should be considered in calculating natural frequencies.

19. For individual units and for subbase assemblies, 16 applies.

20. Resilient mountings for shipboard use must be designed with "captive features." This is required to prevent the equipment from coming adrift in the event of failure of the resilient element due to normal service conditions or to shock.

21. Acoustical shorts across mountings, such as rigid conduit and straps, must be avoided.

22. Resonance of mountings with exciting frequencies of the equipment should be avoided.

23. Mountings should be fastened to foundations or stiffeners, not to bulkheads, decks, or tank plating unless they are sufficiently stiffened.

24. Subbases and foundations should be designed to be rigid and yet light in weight.

25. In calculating natural frequencies, allowance should be made for nonrigid foundations and equipment.

26. Clearances around mountings and around equipment should be determined as described previously.

## CHAPTER 2

### INFORMATION REQUIRED FOR CALCULATING NATURAL FREQUENCIES

Before the natural frequencies of a resiliently mounted assembly can be calculated, certain properties of the equipment and resilient mountings must be known or determined. This information is also of use in selecting from various suitable mountings the one that best satisfies the design requirements for the particular installation.

#### 2.1. PERTINENT PROPERTIES OF EQUIPMENT

Information needed and methods for obtaining dimensions, weight, center of gravity, moments and products of inertia, radii of gyration, principal axes of equipment, and speed of machinery will be presented.

##### 2.1.1. DIMENSIONS

The profiles and dimensions of equipment in three views are needed to determine space and clearance requirements. If a subbase is to be used, its length and width are needed to determine possible positionings of resilient mountings; however, the subbase dimensions may have to be changed later to obtain an assembly with appropriate natural frequencies. The positions of bolt holes in the supporting legs of the equipment also must be known, either for locating the mountings or for designing a subbase.

##### 2.1.2. WEIGHT

The weight must be determined for each item of machinery and equipment. The simplest method of weighing is with a platform scale. A crane scale or hook dynamometer, if available, is often convenient for larger units. Equipment already installed can be weighed in place with a calibrated hydraulic jack or with a weighing capsule. It must be remembered that equipment does not always have equal weight distribution among its supporting legs. If equipment is fastened to a resiliently mounted subbase, then the weight of the subbase must also be determined and added to the weight of the equipment.

The weight of externally unsupported piping ducts and electrical cables connected to machinery must be estimated, as it contributes to the total weight supported by resilient mountings. On machinery items such as pumps, the normal weight of fluid must be included. If there are partially filled tanks or spaces, a correction<sup>10</sup> may have to be made for the free-surface effect. This would increase the effective height of the center of gravity of the liquid in the machinery. If the free-surface area is small and the volume and weight of the liquid are small compared with the weight of the machinery, the free-surface effect is generally negligible.

Often the mountings must be selected and positioned and a subbase must be designed before information on the weight can be obtained. In these instances, if drawings are available,

the weight can be calculated. Usually, calculation of the weight of the principal parts plus an estimate for the minutiae will give a good approximation. When insufficient design details are furnished, the weight must be estimated. Comparisons may be made with known weights of similar equipment.

### 2.1.3 CENTER OF GRAVITY

For each unit of equipment or for an assembly installed on a subbase that is to be resiliently mounted, the location of the center of gravity must be known in order to determine moments of inertia and to position resilient mountings.

The center of gravity of equipment may be determined by balancing the unit on a knife edge or bar three times. When the unit is balanced, its center of gravity lies in a vertical plane through the knife edge. If the unit is rotated about a vertical axis approximately 90 deg with respect to the knife edge and balanced again, another vertical plane passing through the center of gravity and the knife edge is established. The two planes intersect in a line through the center of gravity. The procedure is repeated with the unit turned on its side, giving a third plane intersecting the line at the center of gravity of the unit.

It may be difficult or impossible to balance a unit on a knife edge. For instance, if there is a thin casing in way of the center of gravity, then a heavy unit may be damaged by being supported in this way. In this case, the unit may be supported near one end on a stationary knife edge and near the other end on a knife edge supported by a platform scale as in Figure 10. From the weight of the unit  $W$ , the weight on the scale  $R$ , and the known distance between the knife edges  $l$ , the distance between the center of gravity and one knife edge may be determined.

The moments about  $A$  are

$$Wx = Rl$$

so that, with the length measured in inches, the vertical plane through the center of gravity is  $Rl/W$  inches to the right of  $A$  and perpendicular to the paper. This weighing process must be done three times with different orientations of the unit in order to locate the center of gravity.

If the unit has a comparatively simple shape and uniform density, the center of gravity if not obvious by inspection, may be found by formulas given in any engineering handbook.

When several units are installed on a resiliently supported subbase, the same methods

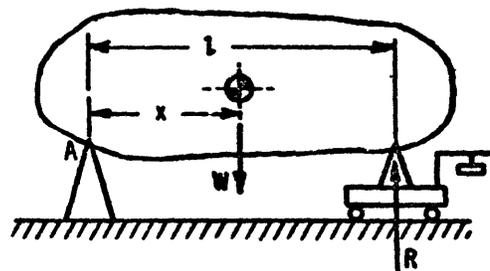


Figure 10 - Determination of Center of Gravity by Means of a Platform Scale

may be used to find the center of gravity of the assembly. If the assembly is too large, then the center of gravity of each unit and of the subbase may be determined separately, and, if the locations of the units upon the subbase are known, the center of gravity of the assembly may then be calculated. In the design stage, the use of a subbase permits the arrangement of the units so that the center of gravity is over the center of the subbase or at least located with some degree of symmetry. This simplifies the problem of selecting and positioning resilient mountings. A typical numerical example of the determination of the center of gravity is worked in Problem 1 of Appendix 2.

#### 2.1.4. MOMENTS AND PRODUCTS OF INERTIA

When the weight and the location of the center of gravity of a unit are known, the moments and products of inertia may be determined either experimentally or by computation.

For symmetrical bodies of uniform density, the desired quantities can usually be computed from formulas available in engineering handbooks. For irregularly shaped bodies, these quantities can be estimated by dividing the body up into a large number of small rectangular parallelepipeds of dimensions  $\Delta x$ ,  $\Delta y$ , and  $\Delta s$ , treating each element as of uniform density, and applying the basic relations

$$I_x = \Sigma (y^2 + z^2) \Delta m$$

$$I_y = \Sigma (x^2 + z^2) \Delta m$$

$$I_z = \Sigma (y^2 + x^2) \Delta m$$

$$I_{xy} = \Sigma xy \Delta m$$

$$I_{yz} = \Sigma yz \Delta m$$

$$I_{zx} = \Sigma zx \Delta m$$

When test apparatus is available, it is easier to determine moments of inertia experimentally. The trifilar suspension system<sup>11, 12, 13</sup> for the determination of moments of inertia is described in Appendix 3.

If an assembly is so heavy and large that its moments of inertia cannot easily be determined experimentally, then the moments of inertia of the assembly can be calculated after those for the individual units are experimentally determined. The moment of inertia of a rigid body about any axis may be obtained by determining the moment of inertia of the body about a parallel axis through the center of gravity and adding to it the product of the mass of the body and the square of the perpendicular distance between the axes; see Figure 11.

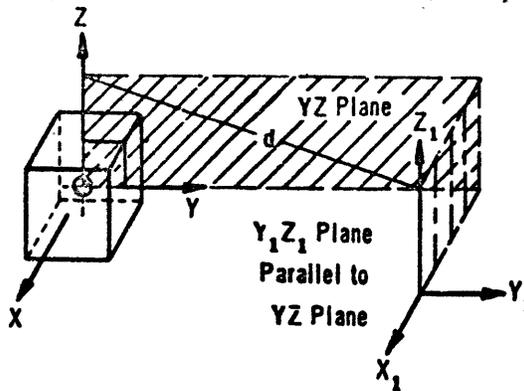


Figure 11 - Parallel Axes for Determination of Moment of Inertia

The expression is

$$I_{Z_1} = I_Z + md^2$$

where  $I_Z$  is the mass moment of inertia about the  $Z$  axis through the center of gravity of the unit or subbase, pound-inch-seconds squared,

$I_{Z_1}$  is the mass moment of inertia about the  $Z_1$  axis in pound-inch-seconds squared,

$m = \frac{W}{g}$  is the mass of the unit or subbase in pound-seconds squared per inch,

$d$  is the perpendicular distance between the  $Z$  and  $Z_1$  axes in inches, and

$g$  is 386 inches per second squared.

A numerical problem illustrating the calculation of the moments of inertia of an assembly and one combining center-of-gravity and moment-of-inertia computations are presented in Appendix 2, Problems 2 and 3.

It is recommended that equipment manufacturers note the moments of inertia on their mounting installation drawings.

### 2.1.5. RADII OF GYRATION

Once the moments of inertia about the desired axes are determined, the radii of gyration about these axes may be calculated from

$$I = r^2 m \text{ or } r = \sqrt{\frac{I}{m}} \text{ or } r = 19.65 \sqrt{\frac{I}{W}}$$

where  $I$  is the mass moment of inertia in pound-inch-seconds squared,  
 $m$  is the mass in pound-seconds squared per inch,  
 $r$  is the radius of gyration in inches, and  
 $W$  is the weight of the equipment in pounds.

### 2.1.6. PRINCIPAL AXES

So far in this chapter the moments and products of inertia of units and assemblies were determined with respect to three mutually perpendicular axes passing through the center of gravity. In solid geometrical configurations with uniform density, such as cubes and rectangular parallelepipeds, the axes would be parallel to the sides or edges of the object. In the solution of practical problems dealing with equipment and machinery of varying configurations it is recognized, for example, that the centerline axis of a motor armature is not necessarily congruent with a principal axis of the motor, but it is so close that for all practical purposes, they can be assumed congruent. The fact that the principal moments of inertia and the principal axes are not exactly determined for equipment usually results in little error in calculating natural frequencies.

A relation between moments of inertia about principal axes and about any other set of rectangular axes through the center of gravity is

$$I_X + I_Y + I_Z = I_x + I_y + I_z = \text{a constant}$$

If the moments of inertia about any set of rectangular axes are represented as vectors from the center of gravity, they terminate at the surface of an ellipsoid of inertia, see Figure 12.

If  $xyz$  is a set of rectangular axes,  $A$ ,  $B$ , and  $C$  the moments of inertia with respect to these axes, and  $D$ ,  $E$ , and  $F$  the corresponding products of inertia, the moment of inertia with respect to any other axis through  $O$  will be  $1/\rho^2$  where  $\rho$  is the distance along this axis from  $O$  to the surface of the ellipsoid whose equation is

$$Ax^2 + By^2 + Cz^2 - 2Dyz - 2Ezx - 2Fxy = 1$$

If the axes are principal axes, the semimajor axes of the ellipsoid have lengths equal to  $\sqrt{1/A}$ ,  $\sqrt{1/B}$ , and  $\sqrt{1/C}$ , respectively. In Figure 12 the  $X$ ,  $Y$ , and  $Z$  axes are the principal axes of inertia and also the major axes of the ellipsoid. The axes  $x$ ,  $y$ , and  $z$  are axes with arbitrary inclination to the principal axes.

If there is angular displacement of the order of 5 deg between the two sets of axes, the difference in the moments of inertia is negligible. The less the difference among the three moments of inertia, the greater is the angle between the sets of axes that can be tolerated for reasonably accurate calculation of natural frequencies.

In practical applications, one can usually proceed by selecting axes parallel to the sides of installations and passing through the center of gravity. This will help to reduce cost and complexity of the mounting installation. In units such as electric motors, one principal

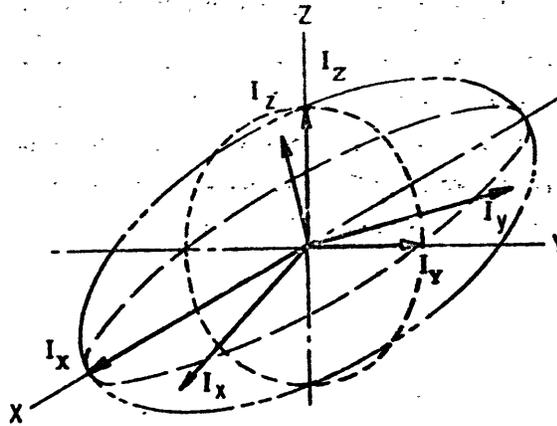


Figure 12 - Ellipsoid of Inertia

axis is very nearly parallel and congruent to the centerline of the rotor, and the moments of inertia about the other two axes are very nearly equal. In the assembly of various items on a common subbase, principal axes parallel to the sides of the subbase can be approximated by positioning the units to satisfy the conditions for symmetry; see Chapter 3. When the axes chosen are not principal axes, the calculation will also require the evaluation of the products of inertia  $I_{xy}$ ,  $I_{yz}$ , and  $I_{zx}$ .

### 2.1.7. EXCITING FREQUENCIES (SPEED)

The normal operating speeds of rotating or reciprocating machinery must be known since machinery can be the generator of excessive noise or vibration. If the speed ranges are known, remedial steps can be taken to reduce the transmission of noise and vibration by suitable selection and positioning of resilient mountings. The machinery manufacturer can best furnish this information, and it should be provided on identification plates, in instruction books, and on machinery and mounting installation drawings.

Not only must the operating speeds of machinery be known but also certain design features that will contribute to excitations at frequencies which are multiples of the machinery speed (Reference 6). These include the number of poles and slots in motors and generators, the number of teeth in gears, and the number of balls and rollers in bearings. On the basis of noise-transmissibility data, it may be possible to select from among otherwise acceptable mountings the ones more suitable for minimizing the noise transmission from the machinery at these machinery speeds.

## 2.2. PROPERTIES OF RESILIENT MOUNTINGS

The performance characteristics of resilient mountings can be determined by tests prescribed in the Interim Military Specification MIL-M-17185 (SHIPS). As performance data are obtained from tests on various mountings, reports showing these data will be distributed to naval shipbuilding activities. Ready reference sheets which summarize experimental data considered essential in the selection and application of resilient mountings are reproduced in Appendix 4.

## CHAPTER 3

### CALCULATION OF NATURAL FREQUENCIES OF RESILIENTLY MOUNTED EQUIPMENT

In this chapter, formulas will be given for calculating the natural frequencies of vibration of resiliently mounted equipment. Derivation of the formulas may be found in Appendix 5. Formulas for some additional cases are presented in Appendixes 5 and 6.

The magnitudes of the natural frequencies depend upon the weight of the equipment, its radii of gyration about its principal axes of inertia, and the directions of these axes. These quantities and directions are usually already established, and the engineer selecting and applying mountings can do little about them. The directions of the principal axes are likely to be the most uncertain quantity, but some error may usually be made in the assumed directions without affecting materially the calculated frequencies, as has been shown in Chapter 2. The frequencies also depend upon the number, the elastic stiffnesses, and the arrangement of the mountings. For the most part, these variables are under the control of the engineer.

In considering different mounting arrangements, the following general principles may be of use:

1. The number of independent modes of free vibration of a rigid body upon its mountings is always six. In special cases, two or more of the natural frequencies may be equal.
2. Increasing the stiffness of the mounting arrangement at any point, as by adding another mounting, generally raises all the frequencies and decreasing the stiffness has the opposite effect. The only exceptions are those modes in which the mountings with increased stiffness are not subjected to additional strain due to vibration; in such cases, no change in frequency occurs.
3. An increase of mass, or an increase of a radius of gyration, generally lowers all frequencies. Decrease of mass or of a radius of gyration has the opposite effect. The only exceptions are those modes in which an added mass lies at a nodal point and therefore is at rest during vibration, or for the modes in which there is no rotation about the axis to which the altered radius of gyration is referred; in such cases, no change in frequency occurs.

The calculation of the six natural frequencies is always possible, but it is laborious unless the mountings are arranged in a relatively simple manner or unless computing machines such as the IBM or UNIVAC are available. First, the necessary simplicity in terms of vibrational symmetry will be explained. Then the most useful types of arrangements and the methods of calculating the natural frequencies will be described, and numerical illustrations will be presented. Explicit formulas will be given here for certain simple classes of arrangements.

The  $XYZ$  axes will always be assumed to be drawn from the center of gravity of the equipment as the origin and in the direction of the principal axes of inertia and the axes of the mountings will be assumed parallel to the coordinate axes unless  $k_a = k_r$ . Damping in the

mountings will be ignored since its effect upon the frequencies is slight.

### 3.1. VIBRATIONAL SYMMETRY

As presented in this guide, the calculation of natural frequencies will be based on mounting arrangements having at least one plane of symmetry. The greater the number of planes of symmetry, the easier are the natural frequency calculations.

A plane through the center of gravity of a mounted assembly is a plane of vibrational symmetry when:

1. Vibrational motions parallel to the plane do not evoke reactions tending to generate displacements perpendicular to the plane. Such motions may involve translations parallel to the plane and rotations about an axis through the center of gravity and perpendicular to the plane. These motions may appear singly or may be combined in each of the vibrational modes.
2. Vibrational motions perpendicular to the plane may involve perpendicular translations and rotation about an axis lying in the plane. These motions may appear singly or may be combined in each of the vibrational modes.

A vibration of Type 1 has no tendency to excite a vibration of Type 2 and vice versa.

Vibrational symmetry is usually accomplished as follows:

1. Each plane of vibrational symmetry must contain two of the principal axes of inertia of the equipment or assembly.
2. The mountings must be arranged to provide suitable symmetry in their elastic reaction to displacement of the equipment or assembly. A simple way to achieve elastic symmetry is to arrange the mountings in geometrical symmetry, that is, so that each mounting is matched by another identical mounting located at the mirror image of the first with respect to the plane of vibrational symmetry. If a mounting has unequal axial and radial stiffnesses, its axis and the axis of its mate may be either both parallel or both perpendicular to the plane of symmetry. Geometrical symmetry of this sort satisfies the requirement for elastic symmetry since forces and moments associated with vibrational displacements are equal on both sides of a plane of symmetry. Any mounting that has equal stiffnesses in all directions may have its axis oriented in any direction. A few relatively simple cases of inclined mountings will be described in Appendixes 5 and 6.

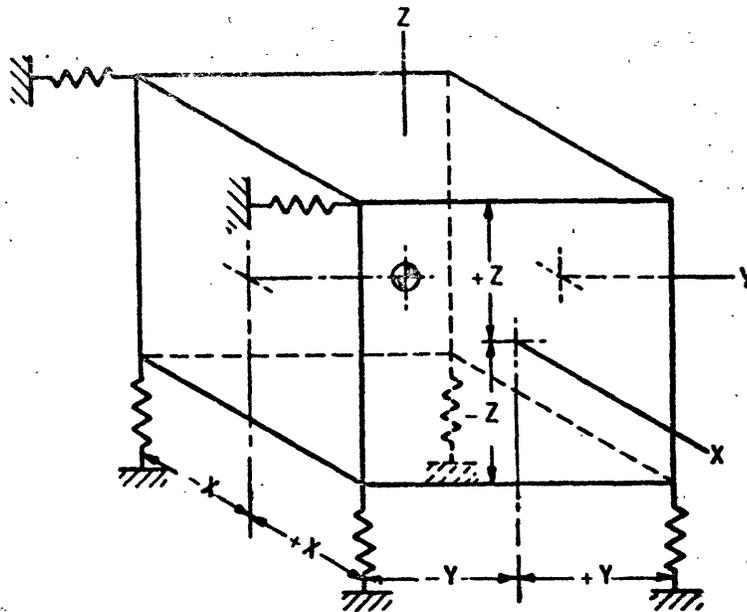


Figure 13 - One Plane of Vibrational Symmetry

For at least one plane of vibrational symmetry to exist, taken as the  $YZ$ -plane as in Figure 13, the conditions of elastic symmetry are as follows:

1. The summation for all mountings of the algebraic product of the stiffness of the mounting in the  $Y$  direction and its  $X$  coordinate must equal zero and similarly for the stiffnesses in the  $Z$  direction.

$$\sum k_y X = 0; \quad \sum k_z X = 0$$

2. The summation for all mountings of the stiffness in the  $Z$  direction multiplied by the algebraic product of the  $X$  and  $Y$  coordinates must equal zero, and similarly with  $Y$  and  $Z$  interchanged:

$$\sum k_z XY = 0; \quad \sum k_y XZ = 0$$

If each mounting has equal spring constants  $k$  in all directions, then three equations suffice:

$$\sum k X = 0, \quad \sum k XY = 0, \quad \text{and} \quad \sum k XZ = 0$$

If  $k$  is the same for all mountings, the above equations can be simplified further:

$$\sum X = 0, \quad \sum XY = 0, \quad \text{and} \quad \sum XZ = 0$$

### 3.2. THREE PLANES OF VIBRATIONAL SYMMETRY

Equipment that has the simplest vibrational motion and therefore whose natural frequencies are the easiest to calculate, has its mountings arranged so that there are three mutually perpendicular planes of vibrational symmetry containing the center of gravity of the mounted assembly. Then three translational modes of vibration occur, each with motion in the direction of one of the lines of intersection of the planes of symmetry, and three rotational modes occur with the lines of intersection of the planes of symmetry as the axes of rotation. If each mounting has the same stiffness in all directions, then the three translational frequencies are equal and a translational mode at this frequency may occur in any direction.

The  $XYZ$  axes will be drawn as usual through the center of gravity and along the principal axes of inertia of the mounted equipment. The three planes of vibrational symmetry will then be the  $XY$ ,  $YZ$ , and  $XZ$  planes. Any mounting not having equal axial and radial stiffnesses is assumed to have its axis oriented parallel to a coordinate axis.

If  $k_X$ ,  $k_Y$ , and  $k_Z$  are not the same for all mountings, the conditions for elastic symmetry relative to all three planes are

$$\Sigma k_Y X = 0, \quad \Sigma k_X Y = 0, \quad \Sigma k_X Z = 0$$

$$\Sigma k_Z X = 0, \quad \Sigma k_Z Y = 0, \quad \Sigma k_Y Z = 0$$

$$\Sigma k_Z XY = 0, \quad \Sigma k_Y XZ = 0, \quad \Sigma k_X YZ = 0$$

If  $k_X$  has the same value for all mountings and if the same is true of  $k_Y$  and  $k_Z$ , then the conditions for elastic symmetry relative to all three planes are

$$\Sigma X = 0, \quad \Sigma Y = 0, \quad \Sigma Z = 0$$

$$\Sigma XY = 0, \quad \Sigma XZ = 0, \quad \Sigma YZ = 0$$

For inclined mountings, conditions for three planes of elastic symmetry are stated in Section A5.4.2. of Appendix 5.

#### 3.2.1. COMMON CENTER-OF-GRAVITY ARRANGEMENTS OF FOUR MOUNTINGS

As a special case, the effective points of attachment of the mountings may lie in a plane containing the center of gravity of the equipment, forming a "center-of-gravity arrangement." If the mounting plane contains two of the axes of inertia of the equipment, it is a plane of vibrational symmetry; the mountings may also be so arranged in the plane that two other planes of symmetry exist, all three planes being mutually perpendicular. See Figure 14 for typical arrangements. Such arrangements have been used in mounting many electronic

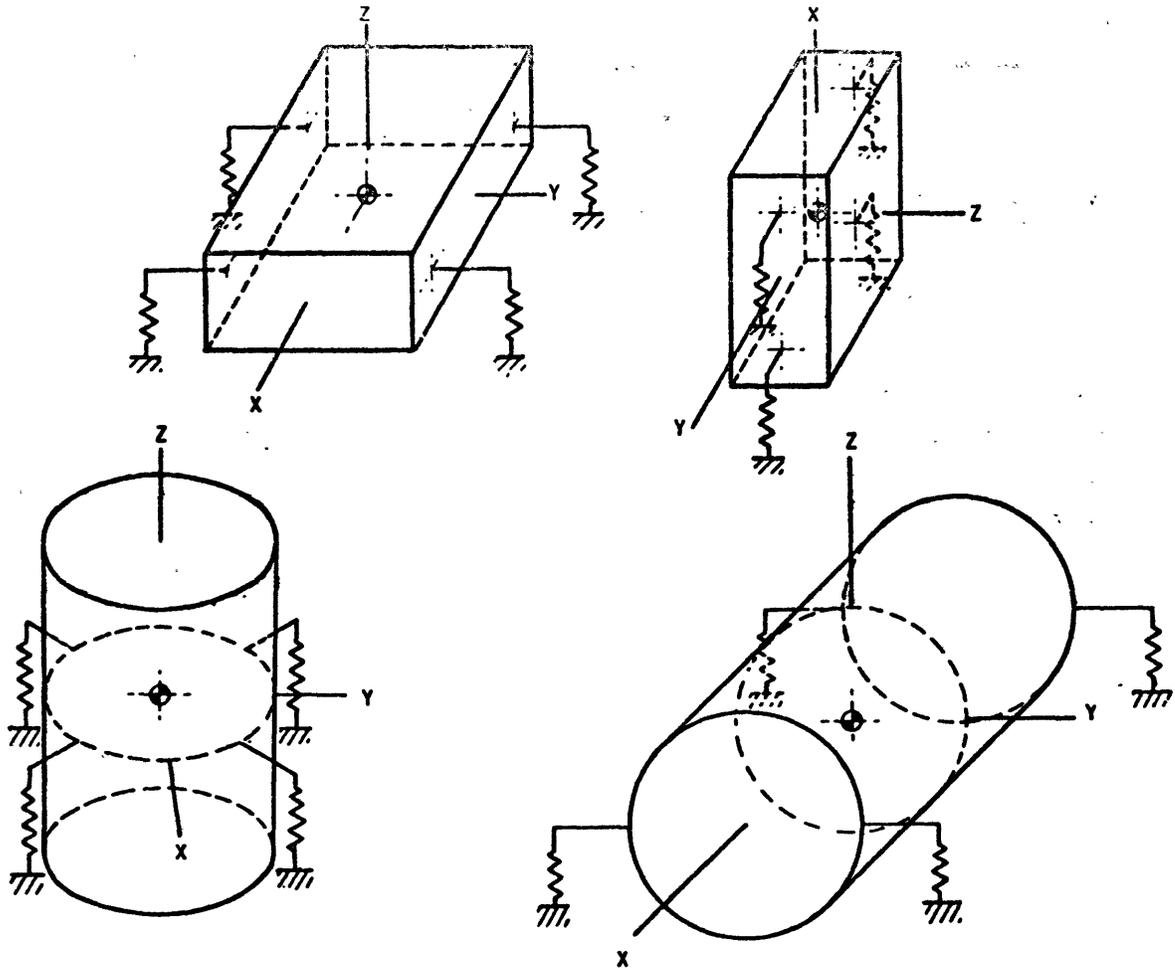


Figure 14 - Common Arrangement of Four Mountings with Three Planes of Vibrational Symmetry

devices and even a few items of heavier equipment. A more general type of center-of-gravity arrangement in which the mounting plane is inclined to two of the principal axes is described in Section A5.5.

Case (a):  $k_a = k_r$

The simplest arrangement with three planes of vibrational symmetry is a center-of-gravity arrangement consisting of four identical mountings having equal axial and radial stiffnesses  $k$  with the effective points of attachment falling at the corners of a rectangle whose sides have lengths  $2D_x$  and  $2D_y$  and whose center is at the center of gravity of the equipment. The sides of the rectangle are parallel to two of the principal axes of inertia, taken as the  $X$  and  $Y$  axes. The formulas for the frequency  $f_{tr}$  of translational vibration in any direction and the frequencies  $f_{rot,X}$ ,  $f_{rot,Y}$ , and  $f_{rot,Z}$  of rotational vibrations about the axes indicated by the subscripts are

$$f_{tr} = \frac{1}{2\pi} \sqrt{\frac{N g k}{W}} \text{ or } 6.26 \sqrt{\frac{k}{W}}$$

where  $N$  equals the number of mountings, here four, and

$$f_{rot, X} = \frac{6.26 D_Y}{r_X} \sqrt{\frac{k}{W}}, \quad f_{rot, Y} = \frac{6.26 D_X}{r_Y} \sqrt{\frac{k}{W}}, \quad f_{rot, Z} = \frac{6.26}{r_Z} \sqrt{\frac{k}{W} (D_X^2 + D_Y^2)}$$

Here, for four identical mountings,  $6.26 = \sqrt{4g}/2\pi$ ,  $W$  is the weight of the equipment in pounds,  $D_X$ ,  $D_Y$ , and  $D_Z$  are the absolute values of the coordinates of the point of attachment of a mounting in inches, and  $r_X$ ,  $r_Y$ , and  $r_Z$  are the radii of gyration of the mounted equipment about the  $X$ ,  $Y$ , and  $Z$  axes, respectively. If  $D_X = r_Y$  and  $D_Y = r_X$ ,

$$f_{tr} = f_{rot, X} = f_{rot, Y}$$

Case (b):  $k_a \neq k_r$ ,

If the four mountings have unequal radial and axial stiffnesses but are identical and have their axes parallel, the formulas for the three translational frequencies of vibration parallel to the  $X$ ,  $Y$ , and  $Z$  axes and for the three rotational frequencies about these axes are

$$f_{tr, X} = 6.26 \sqrt{\frac{k_X}{W}}, \quad f_{tr, Y} = 6.26 \sqrt{\frac{k_Y}{W}}, \quad f_{tr, Z} = 6.26 \sqrt{\frac{k_Z}{W}}$$

$$f_{rot, X} = \frac{6.26 D_Y}{r_X} \sqrt{\frac{k_Z}{W}}, \quad f_{rot, Y} = \frac{6.26 D_X}{r_Y} \sqrt{\frac{k_Z}{W}}, \quad f_{rot, Z} = \frac{6.26}{r_Z} \sqrt{\frac{1}{W} (k_Y D_X^2 + k_X D_Y^2)}$$

For most mountings, one of the three stiffnesses,  $k_X$ ,  $k_Y$ , or  $k_Z$  is equal to the axial stiffness of the mounting while the other two are equal to its radial stiffness. In the plane of the directions of the two equal stiffnesses, translational vibration can occur in any direction or the motions may be elliptical depending on the direction or directions of the initial exciting forces as well as their relative magnitudes.

Computations in the use of such formulas as those for  $f_{tr}$  or  $f_{rot, X}$ , and  $f_{rot, Y}$  may be reduced by the use of a chart, Figure 15, when four mountings are employed.

The application of the frequency equations is illustrated in Problem 4, Section A2.4, where four identical mountings are attached in a plane containing the center of gravity of the equipment. In this problem, procedures are indicated for modifying the frequencies if the initially assumed positions of the mountings result in natural frequencies unsuitable to the

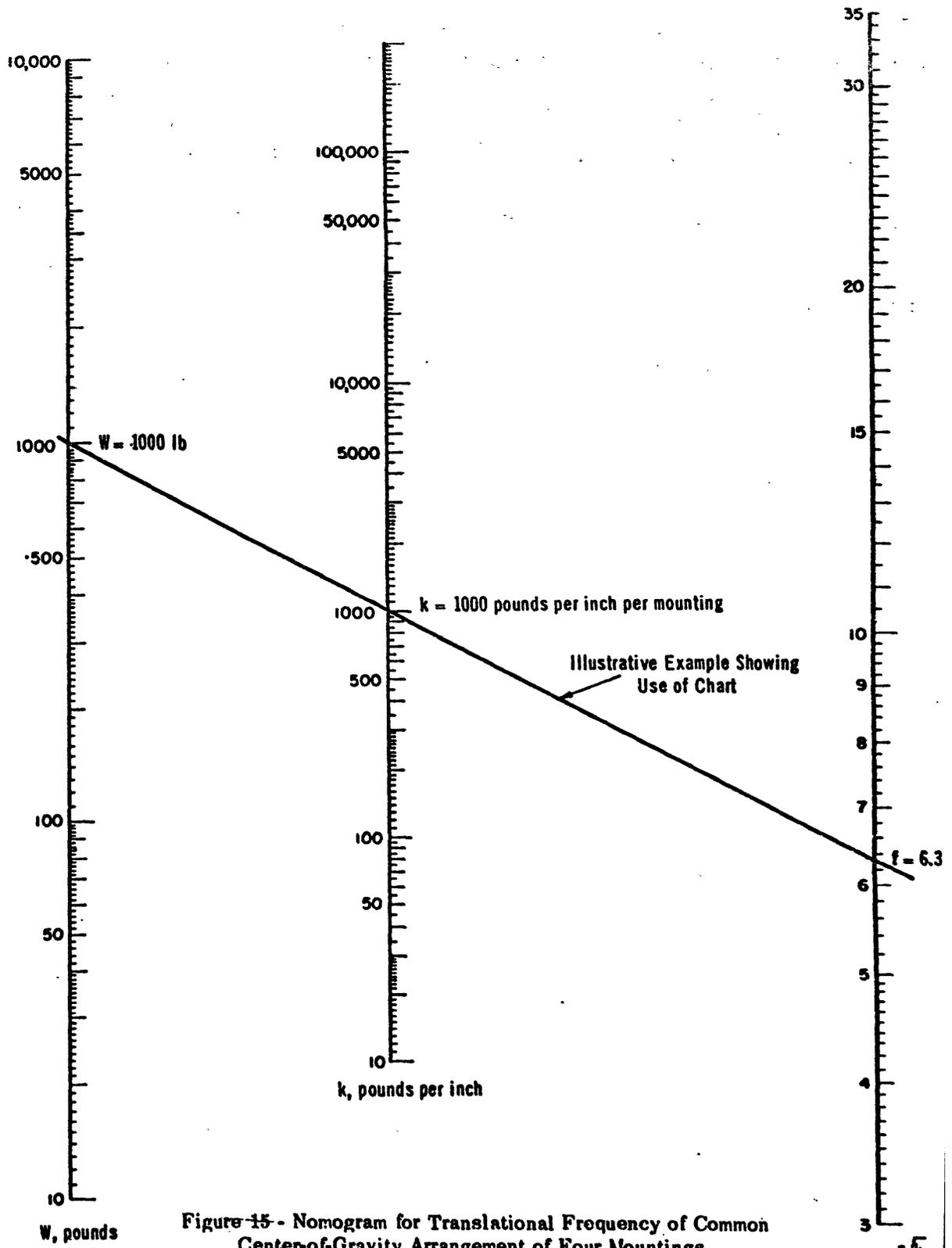


Figure 15 - Nomogram for Translational Frequency of Common Center-of-Gravity Arrangement of Four Mountings

requirements imposed by environmental conditions for the equipment.

Another illustration of the application of the formulas for equipment having three planes of vibrational symmetry is Problem 5, Section A2.5. In this problem,  $k_X = k_Y$  and  $XZ$  and  $YZ$  planes of symmetry may pass either through the axes of the mountings or halfway between them; the same frequencies are obtained with either choice of the planes of symmetry.

### 3.2.2. OTHER ARRANGEMENTS WITH THREE PLANES OF SYMMETRY

Frequency equations will now be presented for more general cases, to be used where these may provide the most satisfactory solution to the problem of arranging mountings.

Ways of generalizing center-of-gravity arrangements of mountings in the  $XY$  plane starting initially with four parallel mountings at the corners,  $D_X, D_Y$ , of the equipment and still maintaining the three planes of vibrational symmetry, see Figure 16, are:

1. Any number of additional sets of four mountings, with parallel axes if  $k_a \neq k_r$ , may be arranged at  $\pm a_1 D_X, \pm b_1 D_Y; \pm a_2 D_X, \pm b_2 D_Y$ ; etc; see Figure 16a.
2. Two identical mountings, with parallel axes if  $k_a \neq k_r$ , may be added at  $X = \pm a D_X$  on the  $X$  axis, or at  $Y = \pm b D_Y$  on the  $Y$  axis. Any number of such pairs may be added; see Figure 16b.
3. Any two identical mountings with the same  $Y$ , with parallel axes if  $k_a \neq k_r$ , may be moved equal and opposite distances parallel to the  $X$  axis without changing  $Y$ ; or if they have the same  $X$ , they may be moved equal and opposite distances parallel to the  $Y$  axis; see Figure 16c.
4. Any two identical mountings, with parallel axes if  $k_a \neq k_r$ , having the same  $X$  but equal and opposite  $Y$  may be moved so as to change  $X$  to  $(1/c)X$  for both provided the elastic constants are changed by a factor of  $c$ ; and the converse statement holds with  $X$  and  $Y$  interchanged. For example, if the two mountings are at  $X = A, Y = B$  and  $X = -A, Y = B$ , respectively, with stiffness  $k$ , they may be moved to  $X = A, Y = B/c$  and  $X = -A, Y = B/c$  with stiffness  $ck$ ; see Figure 16d.
5. Any mounting may be replaced by two mountings having equal total stiffness and placed at suitable points whose location may be determined from the conditions for symmetry. For example, if the two have stiffnesses  $c$  and  $(1-c)$  times the stiffness of the original mounting, then they may be placed with the same  $Y$  and the same orientations of axes as the original but with coordinates  $X'$  for the first and  $X''$  for the second mounting where

$$X' = bX, \quad X'' = \frac{1-bc}{1-c} X$$

and  $X$  is the original coordinate,  $b$  is any number, and  $1 > c > 0$ ; see Figure 16e. By such changes as 4 and 5, it may be possible to adjust the stiffness of each mounting to the load

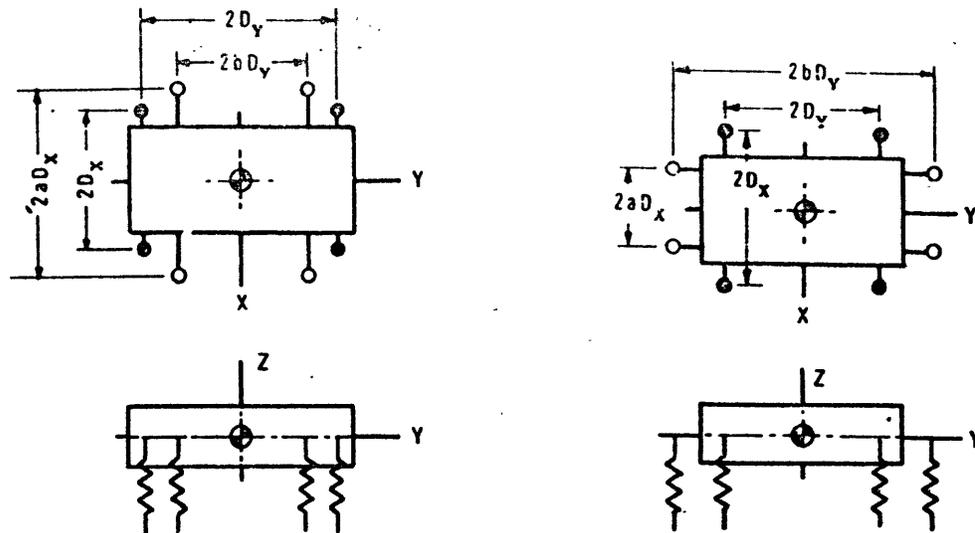


Figure 16a - Several Sets of Four Mountings

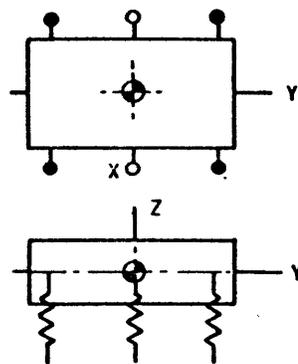


Figure 16b - Pair of Additional Mountings on the  $\lambda$  Axis

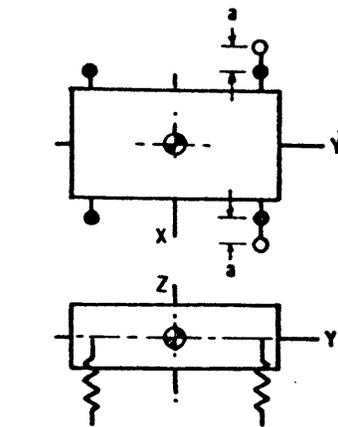


Figure 16c - Shift of a Pair of Mountings Equally and in Opposite Directions Parallel to an Inertial Axis

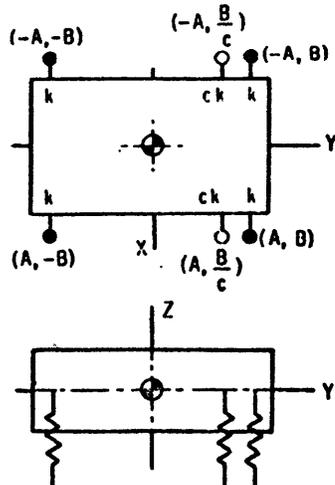


Figure 16d - Shift of a Pair of Mountings without Changing Frequencies

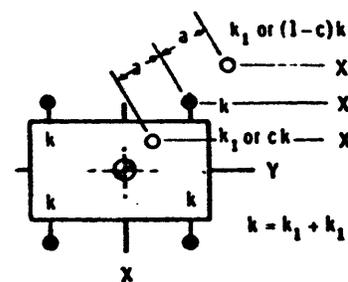


Figure 16e - Replacement of One Mounting by Two

Figure 16 - Other Arrangements - Three Planes of Vibrational Symmetry

it supports. Many other arrangements are possible, but the process of design to secure the requisite symmetry may be tedious.

Frequency formulas for any number of mountings, identical or different and with equal or unequal axial and radial stiffnesses arranged in the  $XY$  plane drawn through the center of gravity of the mounted equipment with vibrational symmetry relative to the  $XZ$  and  $YZ$  planes, are

$$f_{tr,X} = 3.13 \sqrt{\frac{1}{W} \Sigma k_X}, \quad f_{tr,Y} = 3.13 \sqrt{\frac{1}{W} \Sigma k_Y}, \quad f_{tr,Z} = 3.13 \sqrt{\frac{1}{W} \Sigma k_Z}$$

$$f_{rot,X} = \frac{3.13}{r_X} \sqrt{\frac{1}{W} \Sigma k_Z D_Y^2}, \quad f_{rot,Y} = \frac{3.13}{r_Y} \sqrt{\frac{1}{W} \Sigma k_Z D_X^2}, \quad f_{rot,Z} = \frac{3.13}{r_Z} \sqrt{\frac{1}{W} (\Sigma k_Y D_X^2 + \Sigma k_X D_Y^2)}$$

Here  $3.13 = \sqrt{g/2\pi}$ , and the position of a mounting is at  $X = \pm D_X$ ,  $Y = \pm D_Y$ . An illustration is Problem 6, in Section A2.6., in which the mountings are not identical.

### 3.2.2.1. Multiplane and Other Arrangements with Three Planes of Symmetry

Multiplane arrangements with three planes of symmetry can be formed by taking two or more center-of-gravity arrays in the  $XY$  plane and translating them parallel to the  $Z$  axis so that they lie in parallel planes at suitable distances from  $XY$ ; see Figure 17. Then, of the conditions for three planes of symmetry, those not containing  $Z$  are already satisfied; so are those containing  $XZ$  or  $YZ$  since  $Z$  has a constant value for each array. There remain to be satisfied the equations containing  $Z$  alone.

If there are just two arrays consisting of  $N_1$  mountings in a plane at a distance  $L_1$  from the  $XY$  plane and  $N_2$  mountings in a plane at a distance  $L_2$  from the  $XY$  plane and on the opposite side of it, then, provided  $k_X$  and  $k_Y$  are the same for all mountings, the only restriction on  $L_1$  and  $L_2$  is that  $N_1 L_1 = N_2 L_2$ . If, however, there is variation in  $k_X$  or in  $k_Y$ , then it is necessary that the ratio  $\Sigma k_Y / \Sigma k_X$  have the same value for both arrays and that  $L_1 (\Sigma k_X)_1 = L_2 (\Sigma k_X)_2$ , the subscripts indicating the plane to which each sum refers.

More generally, if several center-of-gravity arrays in the  $XY$  plane are moved into parallel planes at  $Z = Z_1, Z_2, \dots$ , respectively, then  $\Sigma k_Y / \Sigma k_X$  must have the same value for all of them and they must be placed so that

$$Z_1 (\Sigma k_X)_1 + Z_2 (\Sigma k_X)_2 + Z_3 (\Sigma k_X)_3 + \dots = 0$$

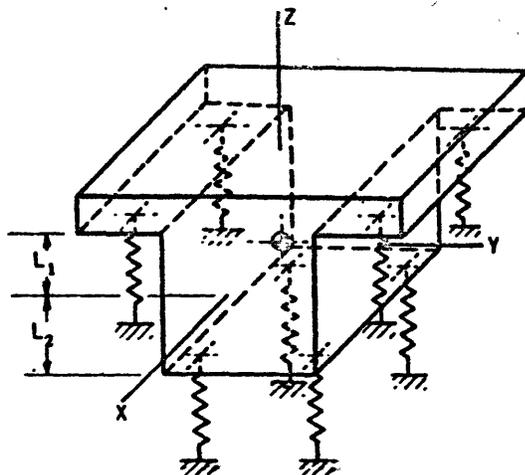


Figure 17 - Multiplane Arrangement with Three Planes of Symmetry

The formulas for the frequencies are those that hold for any arrangement having three planes of vibrational symmetry and not including inclined mountings. These formulas are the same as the last set given above, except for these two:

$$f_{rot, X} = \frac{3.13}{r_X} \sqrt{\frac{1}{W} (\Sigma k_Z D_Y^2 + \Sigma k_Y D_Z^2)}$$

$$f_{rot, Y} = \frac{3.13}{r_Y} \sqrt{\frac{1}{W} (\Sigma k_Z D_X^2 + \Sigma k_X D_Z^2)}$$

### 3.2.3. SUMMARY FOR THREE PLANES OF SYMMETRY

The conditions for three planes of vibrational symmetry and the frequency equations for equipment resiliently mounted are summarized by types of arrangements in Table 1. Separate equations are given for assemblies having mountings with equal and unequal spring constants in axial and radial directions. Sketches showing typical arrangements for each set of equations are given also.

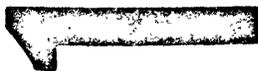
### 3.3. TWO PLANES OF VIBRATIONAL SYMMETRY

Many arrangements of mountings have only two planes of vibrational symmetry. Examples are the base arrangements described in Section 3.3.1. When two planes of symmetry exist, there occurs a single translational mode of vibration parallel to the line of intersection of the two perpendicular planes of symmetry and a single rotational mode about this intersection as an axis. The other four modes of motion are, in general, rocking modes, two in each plane of symmetry. Each rocking mode, however, is actually a mode of rotation about an axis which

TABLE 1 - Frequency Equations for Three Planes of Symmetry, Including Common Four-Mounting Arrangements

Each mounting either has  $k_a = k$ , or has its axis parallel to X, Y, or Z.

Mountings				Conditions of Elastic Stiffness	Frequency Equations	
Identical or Different	Arrangement	Direction of Mounting Axes	Mounting Stiffnesses		Four Mountings with Geometrical Symmetry	Any Number N of Mountings
Identical	Coplanar in XY Plane	Any	$k_a = k_r = k = \text{const.}$ $k_x = k_y = k_z = k$ $\Sigma k_x = \Sigma k_y = \Sigma k_z = \Sigma k$	$\Sigma X = \Sigma Y = \Sigma Z = 0$ $\Sigma XY = \Sigma XZ = \Sigma YZ = 0$	$f_{tr, X} = f_{tr, Y} = f_{tr, Z} = 6.26 \sqrt{\frac{k}{W}}$ $f_{rot, X} = \frac{6.26 D_Y}{r_X} \sqrt{\frac{k}{W}}$ $f_{rot, Y} = \frac{6.26 D_X}{r_Y} \sqrt{\frac{k}{W}}$ $f_{rot, Z} = \frac{6.26}{r_Z} \sqrt{\frac{k}{W} (D_X^2 + D_Y^2)}$	$f_{tr, X} = f_{tr, Y} = f_{tr, Z} = 3.13 \sqrt{\frac{k}{W}}$ $f_{rot, X} = \frac{3.13}{r_X} \sqrt{\frac{k}{W} \Sigma D_Y^2}$ $f_{rot, Y} = \frac{3.13}{r_Y} \sqrt{\frac{k}{W} \Sigma D_X^2}$ $f_{rot, Z} = \frac{3.13}{r_Z} \sqrt{\frac{k}{W} \Sigma (D_X^2 + D_Y^2)}$
		All Mountings Parallel to Same Axis X, Y, or Z	$k_a = k_r$ $k_a = \text{const.}$ $k_r = \text{const.}$	$\Sigma X = \Sigma Y = \Sigma Z = 0$ $\Sigma XY = \Sigma XZ = \Sigma YZ = 0$	$f_{tr, X} = 6.26 \sqrt{\frac{k_X}{W}}, f_{tr, Y} = 6.26 \sqrt{\frac{k_Y}{W}}$ $f_{tr, Z} = 6.26 \sqrt{\frac{k_Z}{W}}$ $f_{rot, X} = \frac{6.26 D_Y}{r_X} \sqrt{\frac{k_Z}{W}}$ $f_{rot, Y} = \frac{6.26 D_X}{r_Y} \sqrt{\frac{k_Z}{W}}$ $f_{rot, Z} = \frac{6.26}{r_Z} \sqrt{\frac{1}{W} (k_Y D_X^2 + k_X D_Y^2)}$	$f_{tr, X} = 3.13 \sqrt{\frac{N k_X}{W}}, f_{tr, Y} = 3.13 \sqrt{\frac{N k_Y}{W}}$ $f_{tr, Z} = 3.13 \sqrt{\frac{N k_Z}{W}}$ $f_{rot, X} = \frac{3.13}{r_X} \sqrt{\frac{k_Z \Sigma D_Y^2}{W}}$ $f_{rot, Y} = \frac{3.13}{r_Y} \sqrt{\frac{k_Z \Sigma D_X^2}{W}}$ $f_{rot, Z} = \frac{3.13}{r_Z} \sqrt{\frac{1}{W} (k_Y \Sigma D_X^2 + k_X \Sigma D_Y^2)}$



Frequency Equations		Examples
	Any Number N of Mountings	
	$f_{cr, X} = f_{cr, Y} = f_{cr, Z} = 3.13 \sqrt{\frac{Nk}{W}}$ $f_{rot, X} = \frac{3.13}{r_X} \sqrt{\frac{k \Sigma D_Y^2}{W}}$ $f_{rot, Y} = \frac{3.13}{r_Y} \sqrt{\frac{k \Sigma D_X^2}{W}}$ $f_{rot, Z} = \frac{3.13}{r_Z} \sqrt{\frac{k \Sigma (D_X^2 + D_Y^2)}{W}}$	<p>(a)</p>
$\sqrt{\frac{k_Y}{W}}$	$f_{cr, X} = 3.13 \sqrt{\frac{Nk_X}{W}}, f_{cr, Y} = 3.13 \sqrt{\frac{Nk_Y}{W}}$ $f_{cr, Z} = 3.13 \sqrt{\frac{Nk_Z}{W}}$ $f_{rot, X} = \frac{3.13}{r_X} \sqrt{\frac{k_Z \Sigma D_Y^2}{W}}$ $f_{rot, Y} = \frac{3.13}{r_Y} \sqrt{\frac{k_Z \Sigma D_X^2}{W}}$	<p>(b) Examples (a) and the following:</p>
$\overline{D_Y^2}$	$f_{rot, Z} = \frac{3.13}{r_Z} \sqrt{\frac{1}{W} (k_Y \Sigma D_X^2 + k_X \Sigma D_Y^2)}$	

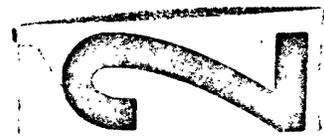


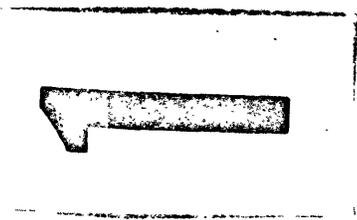
TABLE 1 (continued)

Mountings				Conditions of Elastic Stiffness	Frequency Equations
Identical or Different	Arrangement	Direction of Mounting Axes	Mounting Stiffnesses		Any Number of Mountings
Either Identical or Different	Coplanar in XY Plane	Each Axis Parallel to X, Y, or Z if $k_x \neq k_y$	Any	$\Sigma k_y X - \Sigma k_x Y - \Sigma k_x Z = 0$ $\Sigma k_z X - \Sigma k_z Y - \Sigma k_y Z = 0$ $\Sigma k_z XY - \Sigma k_y XZ - \Sigma k_x YZ = 0$	$f_{tr, X} = 3.13 \sqrt{\frac{1}{W} \Sigma k_x} \cdot f_{tr, Y} = 3.13 \sqrt{\frac{1}{W} \Sigma k_y} \cdot$ $f_{rot, X} = \frac{3.13}{r_x} \sqrt{\frac{1}{W} \Sigma k_z D_Y^2}$ $f_{rot, Y} = \frac{3.13}{r_y} \sqrt{\frac{1}{W} \Sigma k_z D_X^2}$ $f_{rot, Z} = \frac{3.13}{r_z} \sqrt{\frac{1}{W} (\Sigma k_y D_X^2 + \Sigma k_x D_Y^2)}$
	Any Arrangement	Each Axis Parallel to X, Y, or Z if $k_x \neq k_y$	If mountings are attached in two planes parallel to XY plane, (a) if also $k_x = \text{const.}$ and $k_y = \text{const.}$ , then $N_1 L_1 = N_2 L_2$ . (b) Otherwise, $\Sigma k_y / \Sigma k_x$ must have the same value for both planes and also $L_1 (\Sigma k_x)_1 = L_2 (\Sigma k_x)_2$	$\Sigma k_y X - \Sigma k_x Y - \Sigma k_x Z = 0$ $\Sigma k_z X - \Sigma k_z Y - \Sigma k_y Z = 0$ $\Sigma k_z XY - \Sigma k_y XZ - \Sigma k_x YZ = 0$	$f_{tr, X} = 3.13 \sqrt{\frac{1}{W} \Sigma k_x} \cdot f_{tr, Y} = 3.13 \sqrt{\frac{1}{W} \Sigma k_y} \cdot f_{tr, Z} = 3.13 \sqrt{\frac{1}{W} \Sigma k_z}$ $f_{rot, X} = \frac{3.13}{r_x} \sqrt{\frac{1}{W} (\Sigma k_z D_Y^2 + \Sigma k_y D_X^2)}$ $f_{rot, Y} = \frac{3.13}{r_y} \sqrt{\frac{1}{W} (\Sigma k_z D_X^2 + \Sigma k_x D_Y^2)}$ $f_{rot, Z} = \frac{3.13}{r_z} \sqrt{\frac{1}{W} (\Sigma k_y D_X^2 + \Sigma k_x D_Y^2)}$

See Sections A5.6 and A6.4 for additional cases.

Principal Axes X, Y, Z

Planes of Vibrational Symmetry YZ, XZ, XY



Frequency Equations

Any Number of Mountings

$$f_{tr, X} = 3.13 \sqrt{\frac{1}{W} \Sigma k_X}, \quad f_{tr, Y} = 3.13 \sqrt{\frac{1}{W} \Sigma k_Y}, \quad f_{tr, Z} = 3.13 \sqrt{\frac{1}{W} \Sigma k_Z}$$

$$f_{rot, X} = \frac{3.13}{r_X} \sqrt{\frac{1}{W} \Sigma k_Z D_Y^2}$$

$$f_{rot, Y} = \frac{3.13}{r_Y} \sqrt{\frac{1}{W} \Sigma k_Z D_X^2}$$

$$f_{rot, Z} = \frac{3.13}{r_Z} \sqrt{\frac{1}{W} (\Sigma k_Y D_X^2 + \Sigma k_X D_Y^2)}$$

$$f_{tr, X} = 3.13 \sqrt{\frac{1}{W} \Sigma k_X}, \quad f_{tr, Y} = 3.13 \sqrt{\frac{1}{W} \Sigma k_Y}, \quad f_{tr, Z} = 3.13 \sqrt{\frac{1}{W} \Sigma k_Z}$$

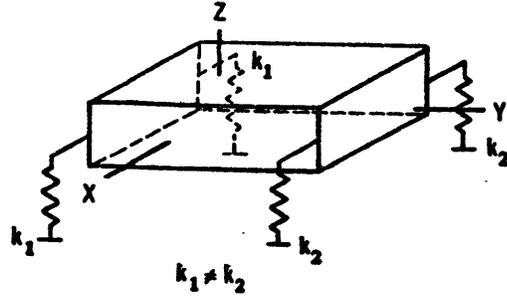
$$f_{rot, X} = \frac{3.13}{r_X} \sqrt{\frac{1}{W} (\Sigma k_Z D_Y^2 + \Sigma k_Y D_Z^2)}$$

$$f_{rot, Y} = \frac{3.13}{r_Y} \sqrt{\frac{1}{W} (\Sigma k_Z D_X^2 + \Sigma k_X D_Z^2)}$$

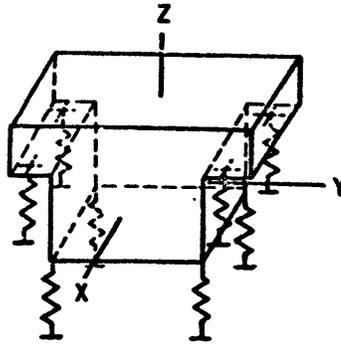
$$f_{rot, Z} = \frac{3.13}{r_Z} \sqrt{\frac{1}{W} (\Sigma k_Y D_X^2 + \Sigma k_X D_Y^2)}$$

Examples

(c) Examples (a), (b), and the following:



(d) Examples (a), (b), and (c), and the following:



2

does not pass through the center of gravity of the equipment. Calculation of the natural frequencies requires the solution of quadratic equations.

The XYZ axes will be drawn as usual from the center of gravity of the equipment along the principal axes of inertia of the equipment, with the Z axis perpendicular to the plane of the mountings; and the planes of symmetry will be taken to be the XZ and YZ planes. The single translational mode is thus in the direction of Z. Any mounting not having equal axial and radial stiffnesses will be assumed to have its axis parallel to a coordinate axis except when the contrary is stated.

For two planes of vibrational symmetry to exist, the following equations must be satisfied:

$$\Sigma k_Y X = 0, \quad \Sigma k_Z X = 0, \quad \Sigma k_X Y = 0, \quad \Sigma k_Z Y = 0$$

$$\Sigma k_Z XY = 0, \quad \Sigma k_Y XZ = 0, \quad \Sigma k_X YZ = 0$$

If each of the constants  $k_X$ ,  $k_Y$ , and  $k_Z$  has the same values for all the mountings as, for example, for identical mountings with parallel axes, these conditions may be simplified to

$$\Sigma X = 0, \quad \Sigma Y = 0, \quad \Sigma XY = 0, \quad \Sigma XZ = 0, \quad \Sigma YZ = 0$$

One way to satisfy the requirements of these equations is to start with mountings in the XY plane as in the case of center-of-gravity mountings described in Sections 3.2.1 or 3.2.2. If such an array is translated parallel to Z, then Z has the same value for all mountings in the array, so that Z may be taken outside the summations in the equations containing Z and these equations are also satisfied. Two or more such arrays in parallel planes can then be combined if necessary. If the distances between the planes and the spring constants are properly adjusted, the arrangement becomes that of the multiplane center-of-gravity type; see Section 3.2.2.1.

Another variation in arrangement permissible without upsetting the vibrational symmetry is to give equal and opposite displacements, parallel to Z, to any two identical mountings provided they have the same X and Y but different values of Z, and provided also that they have parallel axes if  $k_a \neq k_r$ .

### 3.3.1. BASE OR BOTTOM ARRANGEMENTS IN A PLANE

Base or bottom arrangements of mountings are used in many of the resilient-mounting arrangements on shipboard. In such arrangements, the points of attachment of the mountings lie in a plane parallel to the base of the equipment. Usually the mountings are placed under the equipment, but the equipment may also be suspended from the overhead or fastened to bulkhead stiffeners; see Figure 18. A modification sometimes used is a stepped-base arrangement in which the mountings are in two or more parallel planes, Figure 19, but in this case there

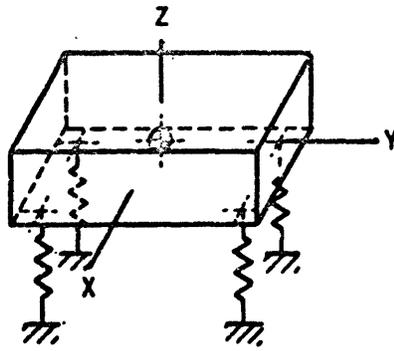


Figure 18a - Four Identical Mountings  
beneath Equipment

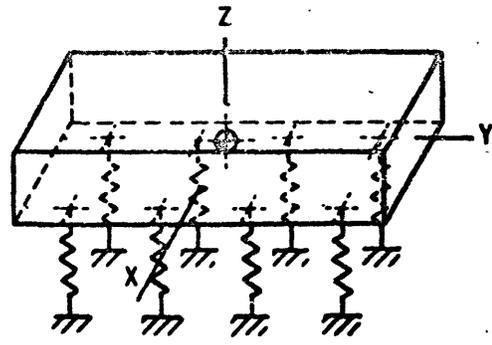


Figure 18b - Typical Arrangement of More  
Than Four Mountings

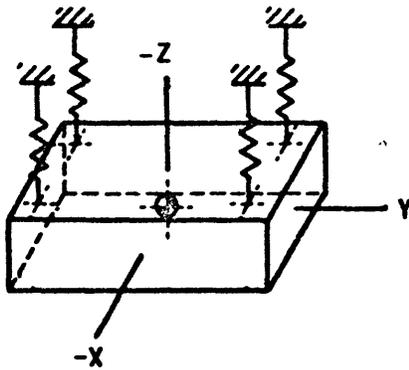


Figure 18c - Equipment Suspended from the  
Overhead by Identical Mountings

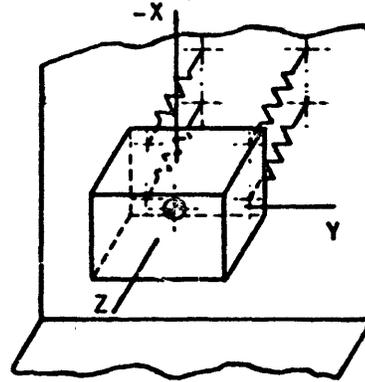


Figure 18d - Equipment Supported by a Bulkhead  
with Identical Mountings

Figure 18 - Normal Arrangements of Mountings for Equipment  
with Two Planes of Vibrational Symmetry

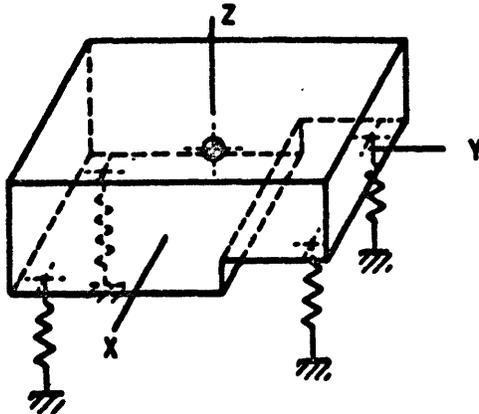


Figure 19a - Identical Mountings

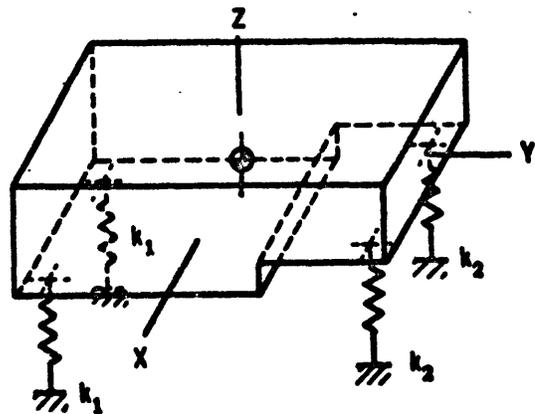


Figure 19b - Different Mountings

Figure 19 - Stepped-Base Arrangements

is only one plane of symmetry and an exact calculation requires the formulas of Section 3.4.

Let the mountings have all their effective points of attachment in a plane parallel to the  $XY$  plane, and let them be arranged to have vibrational symmetry relative to the  $XZ$  and  $YZ$  planes. Let the mounting plane be at a distance  $D_z$  from the center of gravity of the mounted equipment.

(a) Four Identical Mountings Symmetrically Placed, With  $k_a = k_r$

The simplest example of such an arrangement consists of four identical mountings having equal stiffness in all directions, placed at symmetrically disposed points  $X = \pm D_x$ ,  $Y = \pm D_y$ . The frequencies  $f_{tr}$  of the translational mode in the  $Z$  direction and of the rotational mode about the  $Z$  axis are

$$f_{tr} = 6.26 \sqrt{\frac{k}{W}}, \quad f_{rot} = \frac{6.26}{r_z} \sqrt{\frac{k}{W} (D_x^2 + D_y^2)}$$

Here  $6.26 = 2\sqrt{g}/2\pi$ .

The frequencies of the four rocking modes may be found with less computational labor by use of a chart which will be called the Base Mounting Chart;<sup>\*14</sup> see Figure 20. For rocking modes in the  $XZ$  plane the abscissa represents  $D_x/r_y$ , and for rocking modes in the  $YZ$  plane, it represents  $D_y/r_x$ . The ordinate is  $D_z/r$ , where  $r$  stands for the radius of gyration,  $r_x$  or  $r_y$ , whichever is being used for the abscissa. The origin represents the center of gravity, while any point on the chart represents the position of one of the four mountings with respect to the  $X$  or  $Y$  and  $Z$  axes in either the  $XZ$  or  $YZ$  plane. Since the coordinate distances have been divided by the respective radii of gyration, the chart coordinates are in nondimensional form. Circular arcs are drawn on the chart representing constant values of the ratio of the maximum frequency  $f_{max}$  to the minimum frequency  $f_{min}$  for the two rocking modes in the same plane; see Section 1.2. Hyperbolas are also drawn representing constant values of the ratio  $f_{min}/f_{tr}$ , where  $f_{min}$  is always less than  $f_{tr}$ .

In calculating frequencies, the value of  $f_{tr}$  is first obtained from the formula. Then values of the two ratios  $f_{min}/f_{tr}$  and  $f_{max}/f_{min}$  are read off the chart, usually by interpolation between curves. The two rocking frequencies are then calculated as  $f_{max}$  and  $f_{min}$ . This procedure is followed for each plane of symmetry in turn.

The chart may also be used by the designer in planning the arrangement. It is helpful to lay off as a horizontal line on the chart the half-length ( $l/2$ ) or half-width ( $w/2$ ) of the base of the equipment divided by the appropriate radius of gyration, thus reducing it to nondimensional form. This line then indicates the possible positions where a mounting can be attached to the equipment without extending the base. If the line is drawn at the proper ordinate  $D_z/r$ , then the circular arc that is tangent to it represents the minimum value of

<sup>\*</sup>This chart is similar to the upper right quarter of the Isomode Chart devised by Lewis and Unholtz and issued by the MB Company.

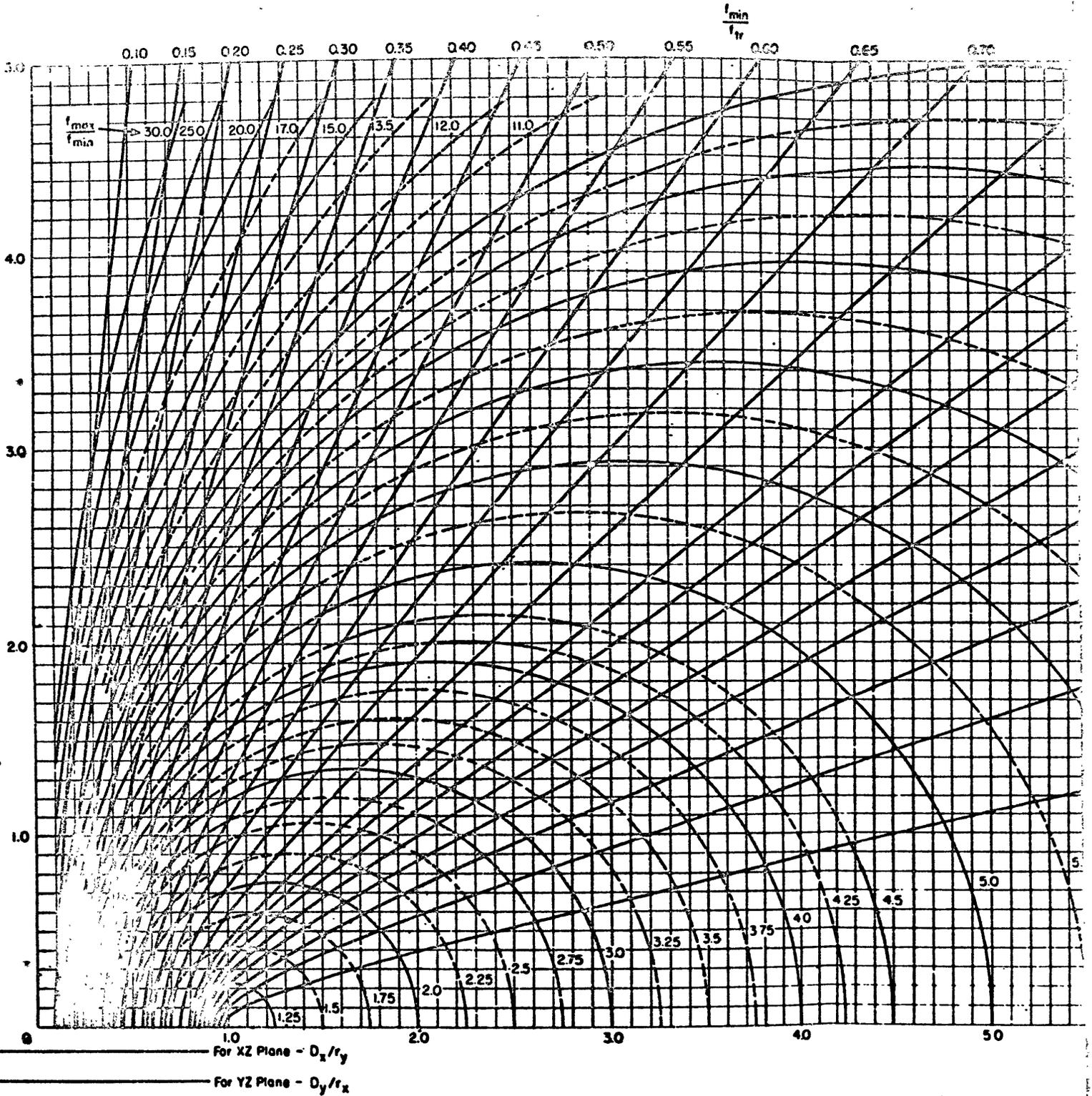
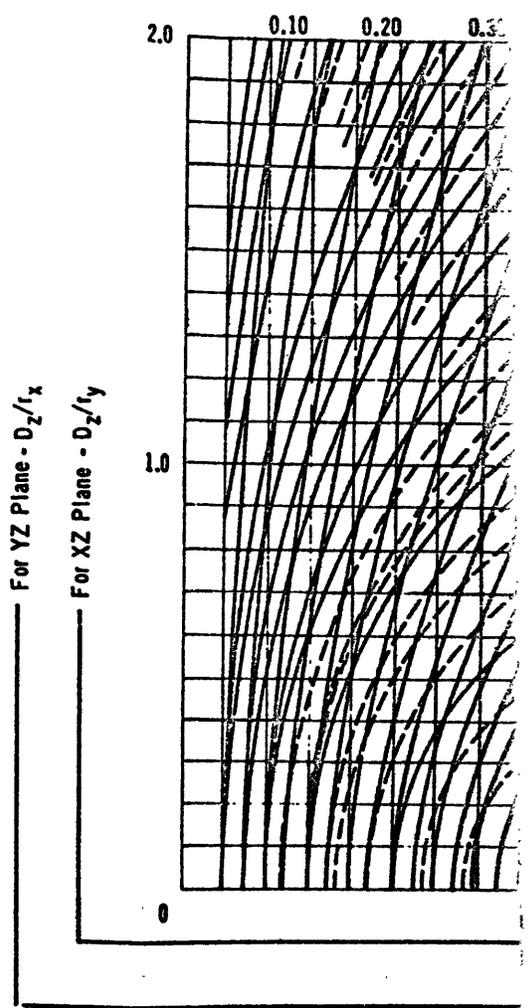
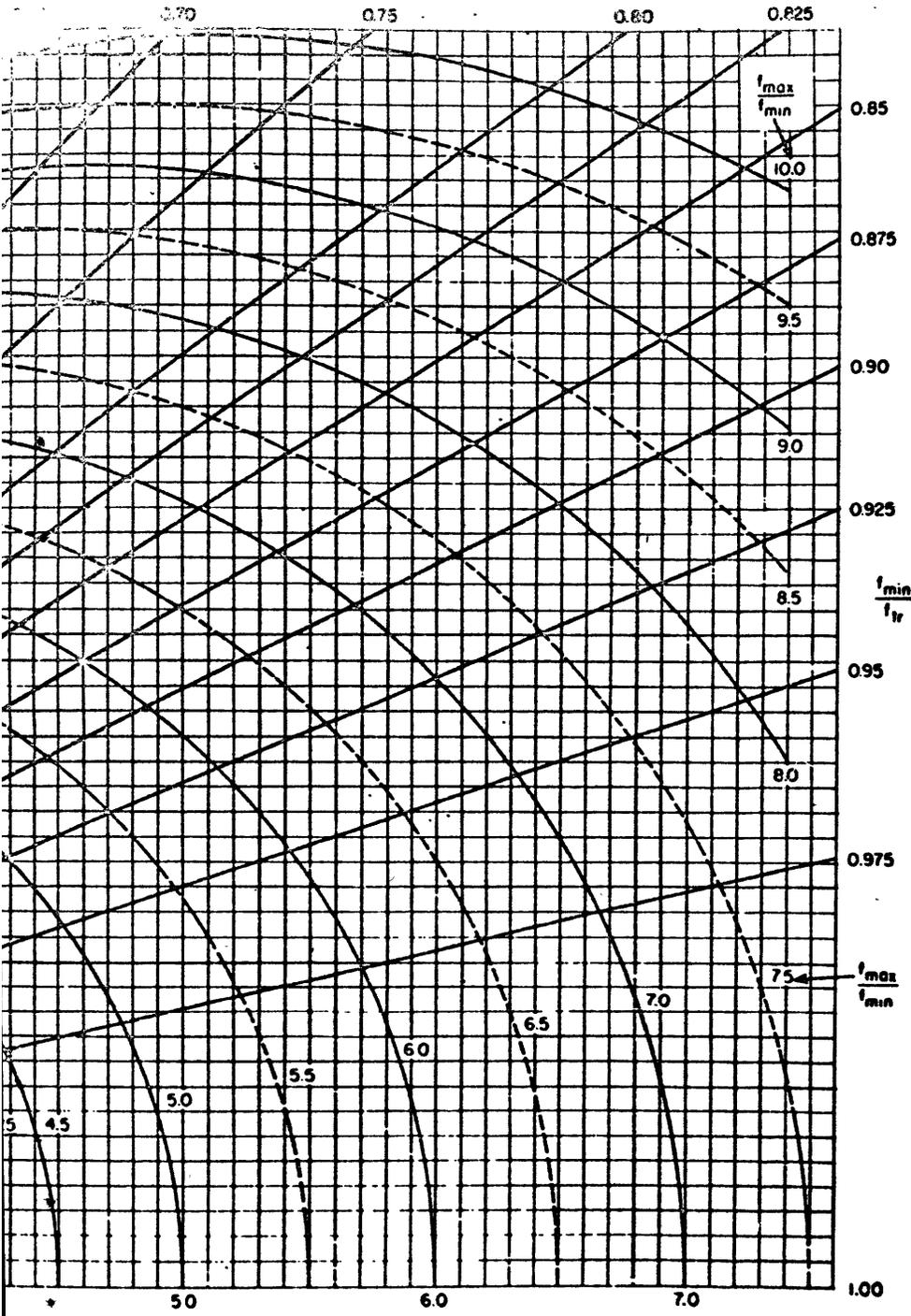
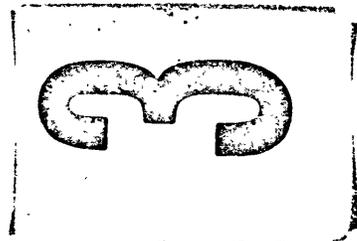
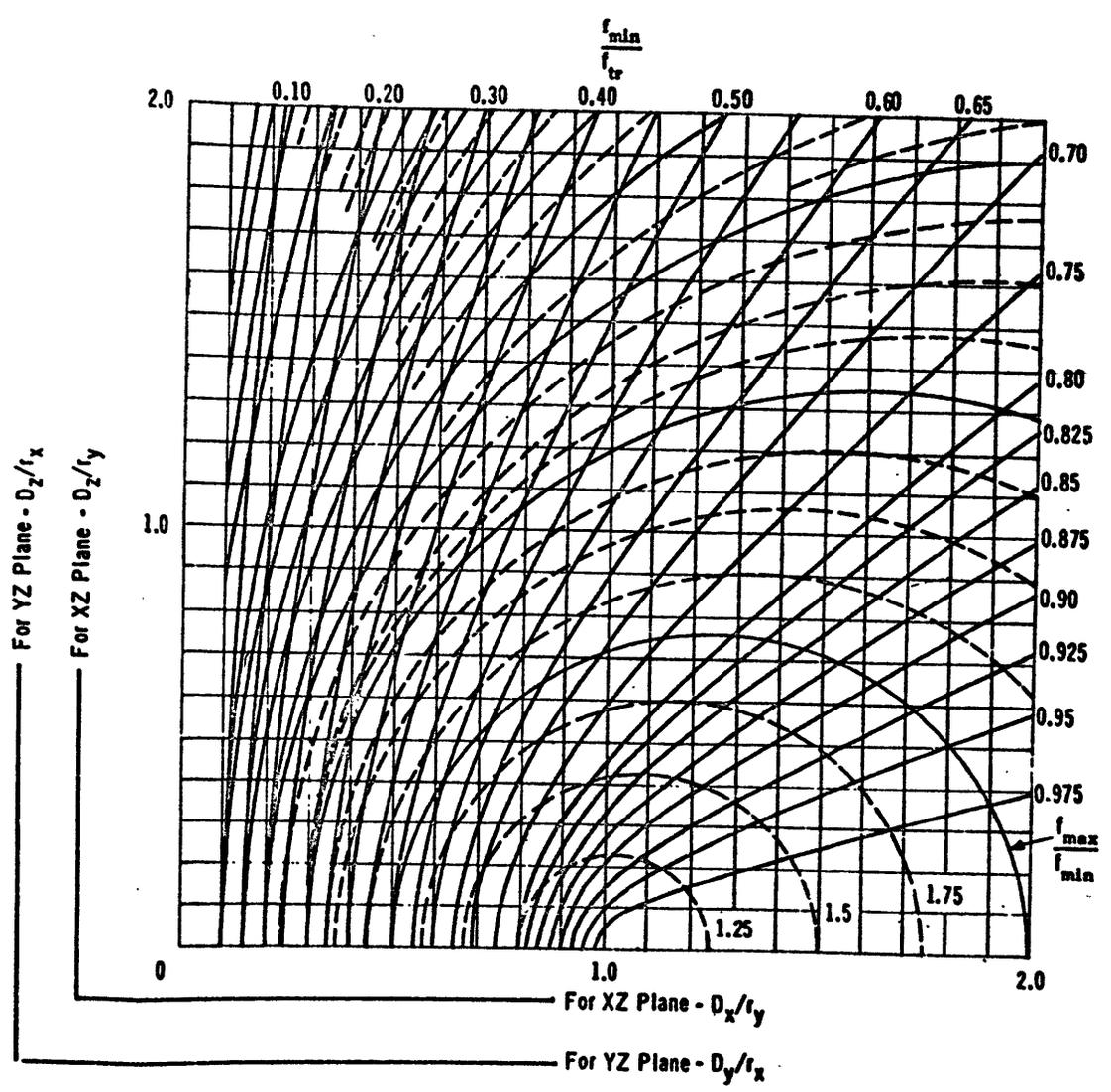
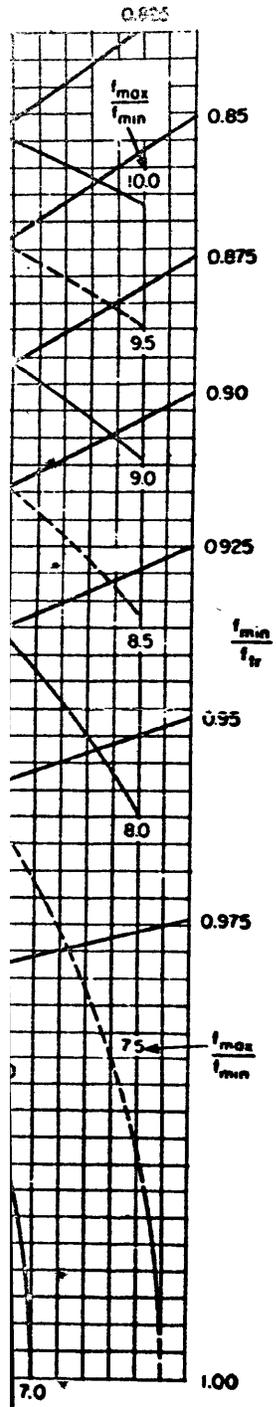


Figure 20 - Base Mounting Chart Showing Frequency Ratios for Rocking Modes of Equipment with Base Mounting Arrangements Having Two Planes of Vibrational Symmetry





$f_{\max}/f_{\min}$  that can be attained under the given conditions. The abscissa of the point of tangency indicates the proper value of  $D_X$  or  $D_Y$  to secure this minimum.

As an alternative to the chart, the formulas for the two rocking modes in the  $XZ$  plane are

$$\frac{f_{\max}^2}{f_{tr}^2} = \frac{1}{2} \left[ 1 + \frac{D_X^2 + D_Z^2}{r_Y^2} + \sqrt{\left(1 - \frac{D_X^2 + D_Z^2}{r_Y^2}\right)^2 + \frac{4D_Z^2}{r_Y^2}} \right]$$

$$\frac{f_{\min}^2}{f_{tr}^2} = \frac{1}{2} \left[ 1 + \frac{D_X^2 + D_Z^2}{r_Y^2} - \sqrt{\left(1 - \frac{D_X^2 + D_Z^2}{r_Y^2}\right)^2 + \frac{4D_Z^2}{r_Y^2}} \right]$$

Here  $D_X = |X|$ ,  $D_Y = |Y|$  while  $D_Z = |Z|$  is the distance from the center of gravity to the plane of the mountings. The formulas for the  $YZ$  plane are obtained by changing  $D_X$  to  $D_Y$  and  $r_Y$  to  $r_X$ .

A numerical illustration for four mountings using the Base Mounting Chart is presented in Problem 7, Section A2.7. This problem, typifying many shipboard installations, is carried out in considerable detail including calculation of the clearances required around the equipment.

(b) More General Base Arrangements

The Base Mounting Chart or the formulas just stated for  $f_{\max}$  and  $f_{\min}$  also can be used for determining the natural frequencies of the rocking modes of vibration for other types of base arrangements merely by reinterpreting certain symbols. In particular,  $D_X$  and  $D_Y$  will now be the distances, not to the actual mountings, but to a set of four identical mountings in rectangular array that would be equivalent to the actual mountings. The general formulas for  $D_X$  and  $D_Y$  are

$$D_X = \sqrt{\frac{\Sigma k_Z X^2}{\Sigma k_X}}, \quad D_Y = \sqrt{\frac{\Sigma k_Z Y^2}{\Sigma k_Y}}$$

Assuming that the mountings are arranged in a plane parallel to the  $XY$  plane with vibrational symmetry relative to the  $XZ$  and  $YZ$  planes, the principal cases are as follows:

1. For any number  $N$  of identical mountings each having stiffness  $k$  in any direction, take, on the chart or in any formula,

$$D_X = \sqrt{\frac{1}{N} \Sigma X^2}, \quad D_Y = \sqrt{\frac{1}{N} \Sigma Y^2}$$

also

$$f_{tr} = 3.13 \sqrt{\frac{Nk}{W}}, \quad f_{rot} = \frac{3.13}{r_Z} \sqrt{\frac{k}{W} (\Sigma X^2 + \Sigma Y^2)}$$

Here  $3.13 = \sqrt{g/2\pi}$ .

2. The base arrangement of four mountings each having its radial and axial stiffnesses equal may be arranged in two pairs, one pair at  $X = \pm D_X$  and  $Y = Y_1$ , the other at  $X = \pm D_X$  and  $Y = Y_2$ . To preserve the symmetry in the  $XZ$  plane, the stiffnesses  $k_1$  and  $k_2$  of the two pairs must be such that

$$k_1 Y_1 + k_2 Y_2 = 0 \text{ or } \frac{k_1}{k_2} = -\frac{|Y_2|}{|Y_1|}$$

Hence

$$\frac{k_1 Y_1^2 + k_2 Y_2^2}{k_1 + k_2} = \frac{k_2 |Y_2| |Y_1| + k_1 |Y_1| |Y_2|}{k_1 + k_2} = |Y_1| |Y_2|$$

and

$$D_X = |X|, \quad D_Y = \sqrt{|Y_1| |Y_2|}$$

Also

$$f_{tr} = 3.13 \sqrt{\frac{2(k_1 + k_2)}{W}}, \quad f_{rot} = \frac{3.13}{r_Z} \sqrt{\frac{2(k_1 + k_2)}{W} (X^2 + |Y_1| |Y_2|)}$$

Here  $3.13 = \sqrt{g/2\pi}$ . An example is treated in Section A2.8, Problem 8.

3. For any number of nonidentical mountings, each having equal stiffnesses in all directions, the corresponding equations are

$$D_X = \sqrt{\frac{\sum k X^2}{\sum k}}, \quad D_Y = \sqrt{\frac{\sum k Y^2}{\sum k}}$$

$$f_{tr} = 3.13 \sqrt{\frac{1}{W} \sum k}, \quad f_{rot} = \frac{3.13}{r_Z} \sqrt{\frac{1}{W} (\sum k X^2 + \sum k Y^2)}$$

4. For any number of mountings with unequal axial and radial stiffnesses and each mounting having its axis parallel to a coordinate axis, on the chart or in any formula,  $D_X$  and  $D_Y$  must have the values

$$D_X = \sqrt{\frac{\sum k_z X^2}{\sum k_x}}, \quad D_Y = \sqrt{\frac{\sum k_z Y^2}{\sum k_y}}$$

The equations for the translational and rotational frequencies are

$$f_{tr} = 3.18 \sqrt{\frac{1}{W} \Sigma k_z}, \quad f_{rot} = \frac{3.18}{r_z} \sqrt{\frac{1}{W} (\Sigma k_y X^2 + \Sigma k_x Y^2)}$$

and, for the rocking modes,  $f_{tr}$  is to be replaced by  $f_x$  in calculating frequencies in the XZ plane, or by  $f_y$  for modes in the YZ plane, where  $f_x$  and  $f_y$  have the values

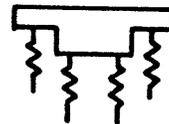
$$f_x = 3.18 \sqrt{\frac{\Sigma k_x}{W}}, \quad f_y = 3.18 \sqrt{\frac{\Sigma k_y}{W}}$$

### 3.3.2. NEARLY COPLANAR ARRANGEMENTS

When the mountings do not all lie in a single plane parallel to a principal plane of inertia, here taken as the XY plane, there may be only one plane of vibrational symmetry, and the more complicated formulas for this condition of symmetry presented in Section 3.4 should be used for an accurate calculation. If, however, the departure from a single mounting plane is rather small and if the conditions for two planes of symmetry are satisfied when the variation in the value of Z is ignored, then a sufficiently accurate estimate of the frequencies may often be obtained by using the chart or the formulas for the base mounting arrangement with an average value of  $D_z$ . This is illustrated for four noncoplanar identical mountings in Problem 9 of Section A2.9, and for four noncoplanar different mountings in Problem 10 of Section A2.10.

### 3.3.3. NONCOPLANAR ARRANGEMENTS WITH TWO PLANES OF SYMMETRY

The Base Mounting Chart is not directly applicable to noncoplanar arrangements with two planes of vibrational symmetry where the mountings lie in more than one plane perpendicular to these two planes. Exact formulas for the frequencies are as follows, the first two being the same as for the plane base arrangement:



$$f_{tr, z} = 3.18 \sqrt{\frac{1}{W} \Sigma k_z}, \quad f_{rot, z} = \frac{3.18}{r_z} \sqrt{\frac{1}{W} (\Sigma k_y X^2 + \Sigma k_x Y^2)}$$

and for the rocking modes in the XZ plane,

$$f_{max} = 3.18 \sqrt{\frac{P_1 + Q_2}{2} + L_x}, \quad L_x = \sqrt{\left(\frac{P_1 - Q_2}{2}\right)^2 + \left(\frac{\Sigma k_x Z}{W r_y}\right)^2}$$

$$f_{min} = 3.18 \sqrt{\frac{P_1 + Q_2}{2} - L_x} = \frac{9.78}{f_{max}} \sqrt{P_1 Q_2 - \left(\frac{\Sigma k_x Z}{W r_y}\right)^2}$$

where

$$P_1 = \frac{\Sigma k_X}{W}, \quad Q_2 = \frac{1}{W r_Y^2} (\Sigma k_Z X^2 + \Sigma k_X Z^2)$$

For the rocking modes in the YZ plane,

$$f_{\max} = 3.13 \sqrt{\frac{P_1 + Q_1}{2}} + L_Y, \quad L_Y = \sqrt{\left(\frac{P_2 - Q_1}{2}\right)^2 + \left(\frac{\Sigma k_Y Z}{W r_X}\right)^2}$$

$$f_{\min} = 3.13 \sqrt{\frac{P_2 + Q_1}{2}} - L_Y = \frac{9.78}{f_{\max}} \sqrt{P_2 Q_1 - \left(\frac{\Sigma k_Y Z}{W r_X}\right)^2}$$

where

$$P_2 = \frac{\Sigma k_Y}{W}, \quad Q_1 = \frac{1}{W r_X^2} (\Sigma k_Z Y^2 + \Sigma k_Y Z^2)$$

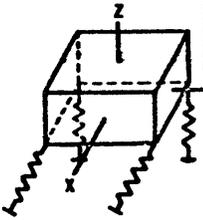
Here  $3.13 = \sqrt{g}/2\pi$ ,  $9.78 = g/(2\pi)^2$ .

Equations for certain arrangements of *inclined* mountings with unequal axial and radial stiffnesses and with two planes of vibrational symmetry are given in Sections A5.3.2 and A6.3.

### 3.3.4. SUMMARY FOR TWO PLANES OF SYMMETRY

The conditions for two planes of vibrational symmetry and the frequency equations for the resiliently mounted equipment are summarized in Table 2 by types of arrangements. Sketches showing typical mounting arrangements for each set of equations are given also.

TABLE 2 - Frequency Equations for Two Planes of Vibrational Symmetry

Mountings				Conditions of Elastic Symmetry	Frequency Equations	
Midental or Different	Arrangement	Direction of Mounting Axes	Mounting Stiffnesses			
Midental	Coplanar in Plane Parallel to XY	Any Direction	$k_x = k, -k$ $k_y = \text{const.}$	$\Sigma X - \Sigma Y = 0$ $\Sigma XY = 0$	<p><u>Four Mountings with Geometrical Symmetry</u></p> <p><u>Any Number of Mountings</u></p> $f_w = 6.26 \sqrt{\frac{k}{W}} \qquad f_w = 3.13 \sqrt{\frac{Nk}{W}}$ $f_{\text{res}} = \frac{6.26}{r_2} \sqrt{\frac{k}{W} (X^2 + Y^2)} \qquad f_{\text{res}} = \frac{3.13}{r_2} \sqrt{\frac{k}{W} (\Sigma X^2 + \Sigma Y^2)}$ <p><u>Any Number N of Mountings</u></p> <p>Rocking Frequencies <math>f_{\text{max}}, f_{\text{min}}</math> On Base Mounting Chart or in Formulas</p> <p>for XZ plane take <math>D_x = \sqrt{\frac{1}{N} \Sigma X^2}</math> for YZ plane take <math>D_y = \sqrt{\frac{1}{N} \Sigma Y^2}</math></p> $D_z =  Z $ <p>By Formula for XZ plane</p> $\frac{f_{\text{max}}^2}{f_w^2} = \frac{1}{2} \left[ 1 + \frac{D_x^2 + D_z^2}{r^2} + \sqrt{\left(1 - \frac{D_x^2 + D_z^2}{r^2}\right)^2 + \frac{4D_z^2}{r^2}} \right]$ $\frac{f_{\text{min}}^2}{f_w^2} = \frac{1}{2} \left[ 1 + \frac{D_x^2 + D_z^2}{r^2} - \sqrt{\left(1 - \frac{D_x^2 + D_z^2}{r^2}\right)^2 + \frac{4D_z^2}{r^2}} \right]$ <p>for YZ plane</p> $\frac{f_{\text{max}}^2}{f_w^2} = \frac{1}{2} \left[ 1 + \frac{D_y^2 + D_z^2}{r^2} + \sqrt{\left(1 - \frac{D_y^2 + D_z^2}{r^2}\right)^2 + \frac{4D_z^2}{r^2}} \right]$ $\frac{f_{\text{min}}^2}{f_w^2} = \frac{1}{2} \left[ 1 + \frac{D_y^2 + D_z^2}{r^2} - \sqrt{\left(1 - \frac{D_y^2 + D_z^2}{r^2}\right)^2 + \frac{4D_z^2}{r^2}} \right]$	
Different	Coplanar in Plane Parallel to XY	Any Direction	$k_x = k_1$	$\Sigma kX - \Sigma kY = 0$ $\Sigma kXY = 0$	<p><u>Four Mountings</u> with <math>k_1</math> at <math>X = \pm D_x, Y = Y_1 &lt; 0</math>, but <math>k_2</math> at <math>X = \pm D_x, Y = Y_2 &gt; 0</math>; and <math>\frac{k_1}{k_2} = \frac{ Y_2 }{ Y_1 }</math></p> $f_w = 3.13 \sqrt{\frac{2}{W} (k_1 + k_2)} \qquad f_{\text{res}} = \frac{3.13}{r_2} \sqrt{\frac{2(k_1 + k_2)}{W} (X^2 +  Y_1   Y_2 )}$ <p>and on Base Mounting Chart or in Above Formulas for <math>f_{\text{max}}</math> and <math>f_{\text{min}}</math></p> $D_x =  X , \quad D_y = \sqrt{ Y_1   Y_2 }$ <p><u>Any Number of Mountings:</u></p> $f_w = 3.13 \sqrt{\frac{1}{W} \Sigma k}, \quad f_{\text{res}} = \frac{3.13}{r_2} \sqrt{\frac{1}{W} (\Sigma kX^2 + \Sigma kY^2)}$ <p>and on Base Mounting Chart or in Above Formulas for <math>f_{\text{max}}</math> and <math>f_{\text{min}}</math></p> $D_x = \sqrt{\frac{\Sigma kX^2}{\Sigma k}}, \quad D_y = \sqrt{\frac{\Sigma kY^2}{\Sigma k}}, \quad D_z =  Z $	(b) Examples (a) and

Frequency Equations for Two Planes of Vibrational Symmetry

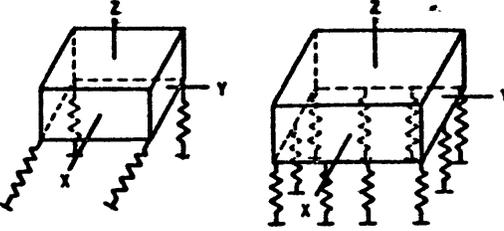
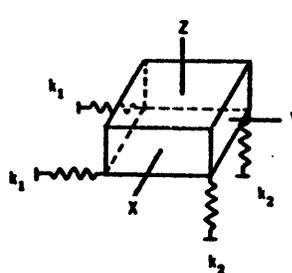
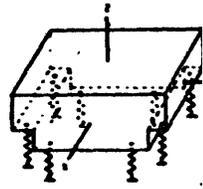
Mounting Conditions	Conditions of Elastic Symmetry	Frequency Equations	Examples
$k_x, -k_y$ const.	$\Sigma X - \Sigma Y = 0$ $\Sigma XY = 0$	<p><u>Four Mountings with Geometrical Symmetry</u></p> <p style="text-align: center;"><u>Any Number of Mountings</u></p> $f_w = 6.26 \sqrt{\frac{k}{W}} \qquad f_w = 3.13 \sqrt{\frac{Nk}{W}}$ $f_{rot} = \frac{6.26}{r_z} \sqrt{\frac{k}{W} (X^2 + Y^2)} \qquad f_{rot} = \frac{3.13}{r_z} \sqrt{\frac{k}{W} (\Sigma X^2 + \Sigma Y^2)}$ <p style="text-align: center;"><u>Any Number N of Mountings</u></p> <p>Rocking Frequencies <math>f_{max}</math>, <math>f_{min}</math> On Base Mounting Chart or in Formulas</p> <p>for XZ plane take <math>D_x = \sqrt{\frac{1}{N} \Sigma X^2}</math> for YZ plane take <math>D_y = \sqrt{\frac{1}{N} \Sigma Y^2}</math></p> <p style="text-align: center;"><math>D_z =  Z </math></p> <p>By Formula for XZ plane</p> $\frac{f_{max}^2}{f_w^2} = \frac{1}{2} \left[ 1 + \frac{D_x^2 + D_z^2}{r_x^2} + \sqrt{\left(1 - \frac{D_x^2 + D_z^2}{r_x^2}\right)^2 + \frac{4D_z^2}{r_x^2}} \right]$ $\frac{f_{min}^2}{f_w^2} = \frac{1}{2} \left[ 1 + \frac{D_x^2 + D_z^2}{r_x^2} - \sqrt{\left(1 - \frac{D_x^2 + D_z^2}{r_x^2}\right)^2 + \frac{4D_z^2}{r_x^2}} \right]$ <p>for YZ plane</p> $\frac{f_{max}^2}{f_w^2} = \frac{1}{2} \left[ 1 + \frac{D_y^2 + D_z^2}{r_y^2} + \sqrt{\left(1 - \frac{D_y^2 + D_z^2}{r_y^2}\right)^2 + \frac{4D_z^2}{r_y^2}} \right]$ $\frac{f_{min}^2}{f_w^2} = \frac{1}{2} \left[ 1 + \frac{D_y^2 + D_z^2}{r_y^2} - \sqrt{\left(1 - \frac{D_y^2 + D_z^2}{r_y^2}\right)^2 + \frac{4D_z^2}{r_y^2}} \right]$	<p>(a)</p> 
$k_x$	$\Sigma kX - \Sigma kY = 0$ $\Sigma kXY = 0$	<p><u>Four Mountings</u> with <math>k_1</math> at <math>X = \pm D_x</math>, <math>Y = Y_1 &lt; 0</math>, but <math>k_2</math> at <math>X = \pm D_x</math>, <math>Y = Y_2 &gt; 0</math>; and <math>\frac{k_1}{k_2} = \frac{ Y_2 }{ Y_1 }</math></p> $f_w = 3.13 \sqrt{\frac{2}{W} (k_1 + k_2)} \qquad f_{rot} = \frac{3.13}{r_z} \sqrt{\frac{2(k_1 + k_2)}{W} (X^2 +  Y_1   Y_2 )}$ <p>and on Base Mounting Chart or in Above Formulas for <math>f_{max}</math> and <math>f_{min}</math></p> $D_x =  X , \quad D_y = \sqrt{ Y_1   Y_2 }$ <p><u>Any Number of Mountings:</u></p> $f_w = 3.13 \sqrt{\frac{1}{W} \Sigma k}, \quad f_{rot} = \frac{3.13}{r_z} \sqrt{\frac{1}{W} (\Sigma k X^2 + \Sigma k Y^2)}$ <p>and on Base Mounting Chart or in Above Formulas for <math>f_{max}</math> and <math>f_{min}</math></p> $D_x = \sqrt{\frac{\Sigma k X^2}{\Sigma k}}, \quad D_y = \sqrt{\frac{\Sigma k Y^2}{\Sigma k}}, \quad D_z =  Z $	<p>(b) Examples (a) and</p> 



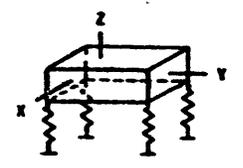
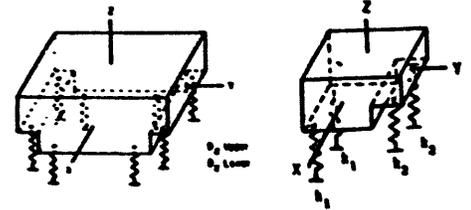
TABLE 2 (continued)

Mountings				Conditions of Elastic Symmetry	Frequency Equations	
Mechanical of Different	Arrangement	Direction of Mounting Axes	Mounting Stiffnesses			
Different	Coplanar in Plane Parallel to XY	Each Mounting Axis Parallel to One of XYZ if $k_x \neq k_y$	$k_x \neq k_y$ , or $k_x = k_y$	$\Sigma k_y X - \Sigma k_x X - \Sigma k_x Y - \Sigma k_y Y = 0$ $\Sigma k_x X Y - \Sigma k_y X Z - \Sigma k_x Y Z = 0$	<p>Any Number of Mountings</p> $f_x = 3.13 \sqrt{\frac{1}{W} \Sigma k_x}; \quad f_y = 3.13 \sqrt{\frac{1}{W} (\Sigma k_y X^2 + \Sigma k_x Y^2)}$ <p>For Rocking Frequencies <math>f_{max}, f_{min}</math> take</p> <p>for XZ plane <math>D_x = \sqrt{\frac{\Sigma k_x X^2}{\Sigma k_x}}</math>, <math>D_z =  X </math> for YZ plane <math>D_y = \sqrt{\frac{\Sigma k_y Y^2}{\Sigma k_y}}</math>, <math>D_z =  Y </math></p> <p>On Base Mounting Chart Replace <math>f_x</math> for XZ Plane by <math>f_x</math>, for YZ plane by <math>f_y</math> where</p> $f_x = 3.13 \sqrt{\frac{\Sigma k_x}{W}}, \quad f_y = 3.13 \sqrt{\frac{\Sigma k_y}{W}}$ <p>By Formula for XZ Plane</p> $\frac{f_{min}^2}{f_x^2} = \frac{1}{2} \left[ 1 - \frac{D_z^2 + D_x^2}{r_x^2} - \sqrt{\left( 1 - \frac{D_z^2 + D_x^2}{r_x^2} \right)^2 + \frac{4D_z^2}{r_x^2}} \right]$ $\frac{f_{max}^2}{f_x^2} = \frac{1}{2} \left[ 1 + \frac{D_z^2 + D_x^2}{r_x^2} + \sqrt{\left( 1 - \frac{D_z^2 + D_x^2}{r_x^2} \right)^2 + \frac{4D_z^2}{r_x^2}} \right]$ <p>for YZ Plane</p> $\frac{f_{min}^2}{f_y^2} = \frac{1}{2} \left[ 1 - \frac{D_z^2 + D_y^2}{r_y^2} - \sqrt{\left( 1 - \frac{D_z^2 + D_y^2}{r_y^2} \right)^2 + \frac{4D_z^2}{r_y^2}} \right]$ $\frac{f_{max}^2}{f_y^2} = \frac{1}{2} \left[ 1 + \frac{D_z^2 + D_y^2}{r_y^2} + \sqrt{\left( 1 - \frac{D_z^2 + D_y^2}{r_y^2} \right)^2 + \frac{4D_z^2}{r_y^2}} \right]$	(a) E
Mechanical or Different	Any Arrangement	Each Axis Parallel to One of X, Y or Z if $k_x \neq k_y$	$k_x \neq k_y$ , or $k_x = k_y$	$\Sigma k_y X - \Sigma k_x X - \Sigma k_x Y - \Sigma k_y Y = 0$ $\Sigma k_x X Y - \Sigma k_y X Z - \Sigma k_x Y Z = 0$	<p>Any Number of Mountings</p> $f_x = 3.13 \sqrt{\frac{1}{W} \Sigma k_x}; \quad f_y = 3.13 \sqrt{\frac{1}{W} (\Sigma k_y X^2 + \Sigma k_x Y^2)}$ <p>Rocking Frequencies</p> <p>By Formula, for XZ plane</p> $f_{min} = 3.13 \sqrt{\frac{P_1 + Q_1}{2} - L_x}; \quad L_x = \sqrt{\left( \frac{P_1 - Q_1}{2} \right)^2 + \left( \frac{\Sigma k_x Z^2}{W r_x} \right)^2}$ $f_{max} = 3.13 \sqrt{\frac{P_1 + Q_1}{2} - L_x} + \frac{3.13}{f_{min}} \sqrt{P_1 Q_1 - \left( \frac{\Sigma k_x Z^2}{W r_x} \right)^2}$ <p>for YZ plane</p> $f_{min} = 3.13 \sqrt{\frac{P_2 + Q_2}{2} - L_y}; \quad L_y = \sqrt{\left( \frac{P_2 - Q_2}{2} \right)^2 + \left( \frac{\Sigma k_y Z^2}{W r_y} \right)^2}$ $f_{max} = 3.13 \sqrt{\frac{P_2 + Q_2}{2} - L_y} + \frac{3.13}{f_{min}} \sqrt{P_2 Q_2 - \left( \frac{\Sigma k_y Z^2}{W r_y} \right)^2}$ <p>where <math>P_1 = \frac{\Sigma k_x}{W}</math> <math>Q_1 = \frac{1}{W r_x^2} (\Sigma k_x X^2 + \Sigma k_y Y^2)</math> <math>P_2 = \frac{\Sigma k_y}{W}</math> <math>Q_2 = \frac{1}{W r_y^2} (\Sigma k_y Y^2 + \Sigma k_x X^2)</math></p> <p>If Nearly Coplanar, Use Appropriate Coplanar Method</p> <p>with Approximation <math>D_x = \frac{D_z \text{ Mass} + D_x L_{max}}{2}</math></p>	(d) Example



See Sections A5.5. and A6.3. for additional cases.  
Principal Axes X, Y, Z Planes of Vibrational Symmetry YZ, XZ

(8d)

Frequency Equations	Examples
<p style="text-align: center;"><u>Any Number of Mountings</u></p> $f_x = 3.13 \sqrt{\frac{1}{W} \Sigma k_x}; \quad f_{rot} = \frac{3.13}{r_z} \sqrt{\frac{1}{W} (k_y X^2 + \Sigma k_x Y^2)}$ <p>Rocking Frequencies <math>f_{max}</math>, <math>f_{min}</math> (also)</p> <p>XZ plane <math>D_x = \sqrt{\frac{\Sigma k_x X^2}{\Sigma k_x}}</math>, <math>D_z =  Z </math> for YZ plane <math>D_y = \sqrt{\frac{\Sigma k_y Y^2}{\Sigma k_y}}</math>, <math>D_z =  Z </math></p> <p>Mounting Chart Replace <math>f_x</math> for XZ Plane by <math>f_x</math>, for YZ plane by <math>f_y</math></p> $f_x = 3.13 \sqrt{\frac{\Sigma k_x}{W}}, \quad f_y = 3.13 \sqrt{\frac{\Sigma k_y}{W}}$ <p>charts for XZ Plane</p> $\frac{f_{max}^2}{f_x^2} = \frac{1}{2} \left[ 1 + \frac{D_x^2 + D_z^2}{r_x^2} + \sqrt{\left(1 - \frac{D_x^2 + D_z^2}{r_x^2}\right)^2 + \frac{4D_z^2}{r_x^2}} \right]$ $\frac{f_{min}^2}{f_x^2} = \frac{1}{2} \left[ 1 + \frac{D_x^2 + D_z^2}{r_x^2} - \sqrt{\left(1 - \frac{D_x^2 + D_z^2}{r_x^2}\right)^2 + \frac{4D_z^2}{r_x^2}} \right]$ <p>for YZ Plane</p> $\frac{f_{max}^2}{f_y^2} = \frac{1}{2} \left[ 1 + \frac{D_y^2 + D_z^2}{r_y^2} + \sqrt{\left(1 - \frac{D_y^2 + D_z^2}{r_y^2}\right)^2 + \frac{4D_z^2}{r_y^2}} \right]$ $\frac{f_{min}^2}{f_y^2} = \frac{1}{2} \left[ 1 + \frac{D_y^2 + D_z^2}{r_y^2} - \sqrt{\left(1 - \frac{D_y^2 + D_z^2}{r_y^2}\right)^2 + \frac{4D_z^2}{r_y^2}} \right]$	<p style="text-align: center;">(a) Examples (a) and</p> 
<p style="text-align: center;"><u>Any Number of Mountings</u></p> $f_w = 3.13 \sqrt{\frac{1}{W} \Sigma k_w}; \quad f_{rot} = \frac{3.13}{r_z} \sqrt{\frac{1}{W} (\Sigma k_y X^2 + \Sigma k_x Y^2)}$ <p>Rocking Frequencies By Formula, for XZ plane</p> $-3.13 \sqrt{\frac{P_1 + Q_1}{2} + L_x}; \quad L_x = \sqrt{\left(\frac{P_1 - Q_1}{2}\right)^2 + \left(\frac{\Sigma k_x Z^2}{W r_x}\right)^2} \quad \left. \begin{array}{l} \text{where} \\ P_1 = \frac{\Sigma k_x}{W} \end{array} \right\}$ $-3.13 \sqrt{\frac{P_1 + Q_1}{2} - L_x} - \frac{3.13}{f_{min}} \sqrt{P_1 Q_1 - \left(\frac{\Sigma k_x Z^2}{W r_x}\right)^2} \quad \left. \begin{array}{l} Q_1 = \frac{1}{W r_x^2} (\Sigma k_x X^2 + \Sigma k_y Z^2) \end{array} \right\}$ <p>for YZ plane</p> $-3.13 \sqrt{\frac{P_2 + Q_1}{2} + L_y}; \quad L_y = \sqrt{\left(\frac{P_2 - Q_1}{2}\right)^2 + \left(\frac{\Sigma k_y Z^2}{W r_y}\right)^2} \quad \left. \begin{array}{l} \text{where} \\ P_2 = \frac{\Sigma k_y}{W} \end{array} \right\}$ $-3.13 \sqrt{\frac{P_2 + Q_1}{2} - L_y} - \frac{3.13}{f_{min}} \sqrt{P_2 Q_1 - \left(\frac{\Sigma k_y Z^2}{W r_y}\right)^2} \quad \left. \begin{array}{l} Q_1 = \frac{1}{W r_y^2} (\Sigma k_y Y^2 + \Sigma k_x Z^2) \end{array} \right\}$ <p>Identify Columns, Use Appropriate Column Method</p> <p>th Approximation <math>D_z = \frac{D_x Upper + D_z Lower}{2}</math></p>	<p style="text-align: center;">(d) Examples (a), (b), (c), and</p> 



## 3.4. ONE PLANE OF VIBRATIONAL SYMMETRY

Mounting arrangements with one plane of vibrational symmetry are usually braced arrangements where the base mountings are supplemented by upper lateral mountings. Such arrangements are particularly suited to equipment whose height is large compared with its base dimensions and which is placed so that lateral mountings may be attached to an adjacent bulkhead; see Figure 21. These arrangements have been used for installations of many electronic and electrical devices and even for a few items of heavy equipment.

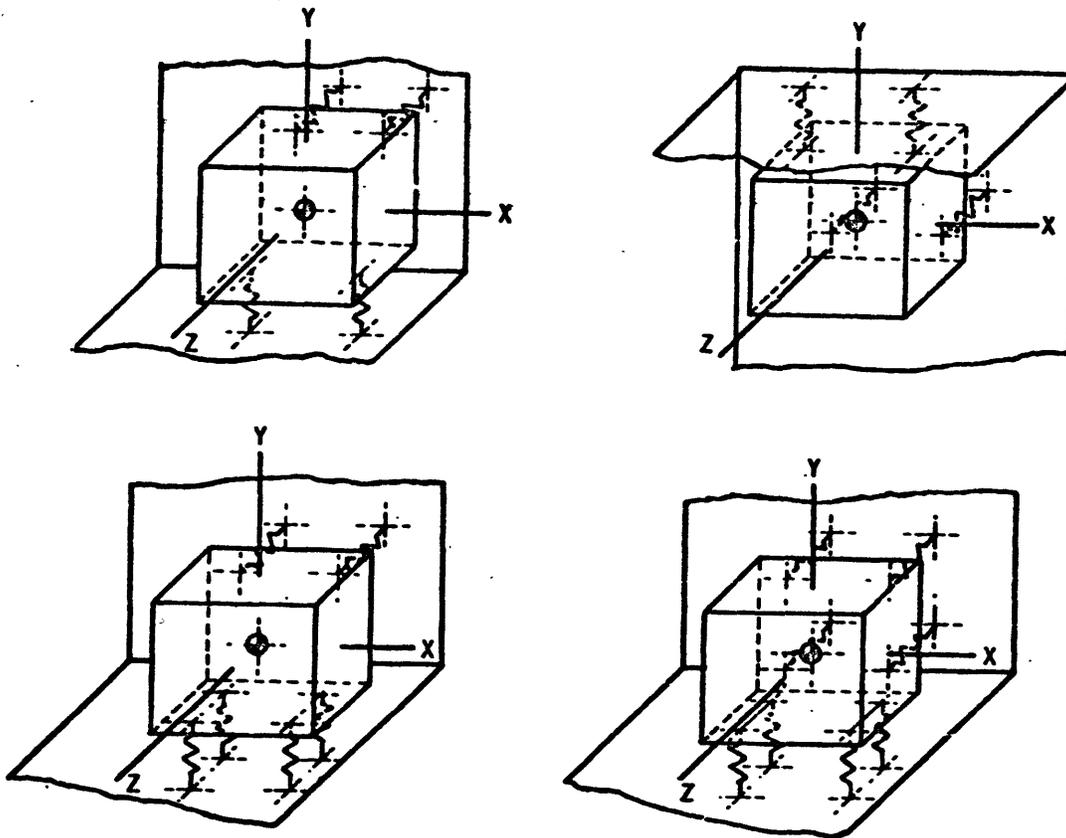


Figure 21 - Common Arrangements of Mountings for Equipment with One Plane of Vibrational Symmetry (the YZ Plane)

As usual, the XYZ axes pass through the center of gravity of the equipment along the principal axes of inertia, and any mounting with unequal axial and radial stiffnesses has its axis parallel to one of the coordinate axes. The single plane of vibrational symmetry is taken as the YZ plane. The general conditions for elastic symmetry are

$$\Sigma k_Y X = 0, \quad \Sigma k_Z X = 0, \quad \Sigma k_Z XY = 0, \quad \Sigma k_Y XZ = 0$$

If every mounting has the same value of  $k_Y$  and  $k_Z$ , then

$$\Sigma X = 0, \quad \Sigma XY = 0, \quad \Sigma XZ = 0$$

Special arrangements satisfying these equations are easily designed; see Figure 22. The simplest methods are the following:

1. Single mountings can be added anywhere in the  $YZ$  plane; see Figure 22a.
2. A pair of identical mountings, with parallel axes when  $k_a \neq k_r$ , can be placed at any pair of points having the same  $Y$  and  $Z$  but having equal and opposite  $X$ . Any number of such pairs can be added; see Figure 22b.
3. Equal and opposite translational shifting of any magnitude parallel to the  $Y$  axis can be given to any two mountings having the same value of  $k_Z X$ , or equal and opposite shifting parallel to  $Z$  if they have the same value of  $k_Y X$ ; see Figure 22c.
4. The elastic constants of any mounting can be changed by a factor of  $c$  provided its coordinate  $X$  is simultaneously changed by the factor  $1/c$ , without change of  $Y$  or  $Z$  or of the orientation of its axis; see Figure 22d.

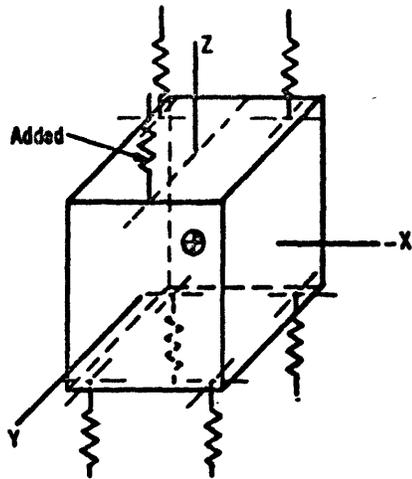


Figure 22a - Addition of Single Mountings

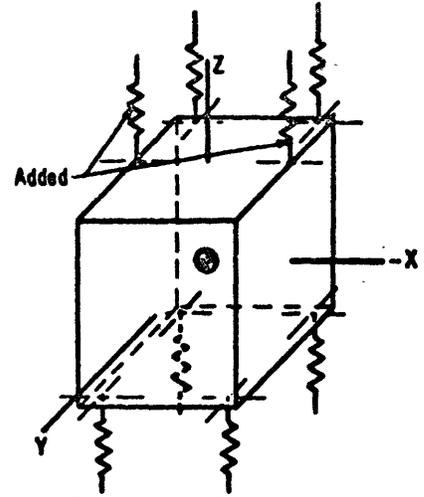


Figure 22b - Addition of a Pair of Mountings

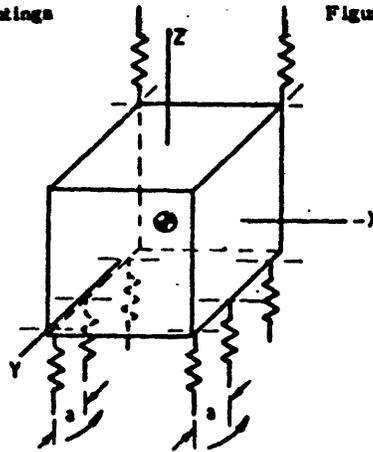


Figure 22c - Translation of a Pair of Mountings

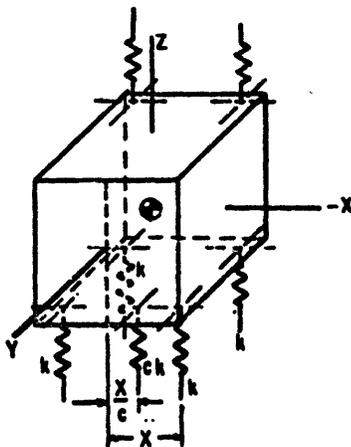


Figure 22d - Change of Elastic Constant of a Mounting

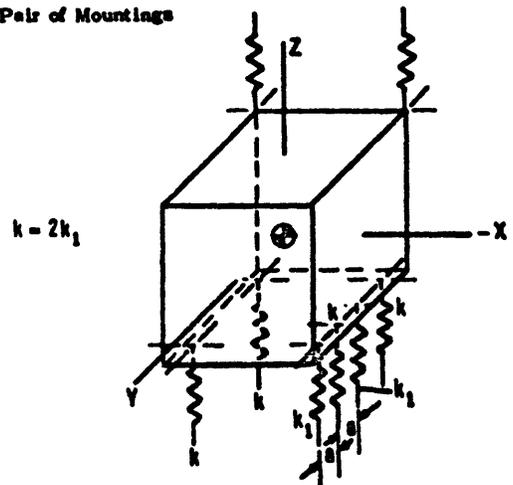


Figure 22e - Replacement of One by a Pair of Mountings

Figure 22 - Other Arrangements of Mountings for Equipment with One Plane of Vibrational Symmetry (the YZ Plane)

5. Finally, any mounting can be replaced by two or more mountings of equal total stiffness suitably located. For example, if the two have stiffnesses  $c$  and  $(1-c)$  times those of the original, then they may be replaced at the same  $X$  but at  $(Y', Z')$ ,  $(Y'', Z'')$ , respectively, where

$$Y' = cY, \quad Z' = bZ, \quad Y'' = \frac{1-ac}{1-c} Y, \quad Z'' = \frac{1-bc}{1-c}$$

Here  $X, Y, Z$  is the original position,  $a$  and  $b$  are any numbers, and  $1 > c > 0$ .

Many further modifications of the arrangement are possible, but it is more tedious to insure that the conditions for symmetry are satisfied.

When vibrational symmetry exists relative to the  $YZ$  plane, vibration in this plane is independent of vibration perpendicular to the plane. The motion in the plane can be resolved into translational vibration parallel to the plane and rotational vibration about a perpendicular axis containing the center of gravity of the equipment. Vibrational motion perpendicular to the plane may be resolved into translation and rotation about an axis lying in the plane of symmetry.

Calculation of the six frequencies  $f$  requires the solution of the two following cubic equations in  $f^2$ :

For motion in the  $YZ$  plane,

$$f^6 - B_1 f^4 + C_1 f^2 - D_1 = 0$$

where

$$B_1 = p_2 + p_3 + q_1$$

$$C_1 = p_2 p_3 + p_2 q_1 + p_3 q_1 - \frac{95.6}{W^2 r_X^2} \left[ (\Sigma k_Z Y)^2 + (\Sigma k_Y Z)^2 \right]$$

$$D_1 = p_2 p_3 q_1 - \frac{95.6}{W^2 r_X^2} \left[ p_2 (\Sigma k_Z Y)^2 + p_3 (\Sigma k_Y Z)^2 \right]$$

and where  $p_2, p_3,$  and  $q_1$  are three of the six quantities

$$p_1 = \frac{9.78}{W} \Sigma k_X, \quad p_2 = \frac{9.78}{W} \Sigma k_Y, \quad p_3 = \frac{9.78}{W} \Sigma k_Z$$

$$q_1 = \frac{9.78}{W r_X^2} (\Sigma k_Z Y^2 + \Sigma k_Y Z^2), \quad q_2 = \frac{9.78}{W r_Y^2} (\Sigma k_Z X^2 + \Sigma k_X Z^2)$$

$$q_3 = \frac{9.78}{W r_Z^2} (\Sigma k_Y X^2 + \Sigma k_X Y^2)$$

For motion perpendicular to YZ, including rotation about an axis lying in YZ,

$$f^6 - B_2 f^4 + C_2 f^2 - D_2 = 0$$

where

$$B_2 = p_1 + q_2 + q_3$$

$$C_2 = p_1 q_2 + p_1 q_3 + q_2 q_3 - \frac{95.6}{W^2} \left[ \frac{1}{r_Z^2} (\Sigma k_X Y)^2 + \frac{1}{r_Y^2} (\Sigma k_X Z)^2 + \frac{1}{r_Y^2 r_Z^2} (\Sigma k_X YZ)^2 \right]$$

$$D_2 = p_1 q_2 q_3 - \frac{95.6}{W^2} \left[ \frac{q_2}{r_Z^2} (\Sigma k_X Z)^2 + \frac{q_3}{r_Y^2} (\Sigma k_X Y)^2 + \frac{p_1}{r_Y^2 r_Z^2} (\Sigma k_X YZ)^2 \right] \\ + \frac{1870}{W^3 r_Y^2 r_Z^2} (\Sigma k_X Y) (\Sigma k_X Z) (\Sigma k_X YZ)$$

Here  $9.78 = g/(2\pi)^2$ ,  $95.6 = g^2/(2\pi)^4$ ,  $1870 = 2g^3/(2\pi)^6$ .

If the mountings have equal radial and axial stiffnesses, then  $k_X = k_Y = k_Z = k$ , a single constant for each mounting; if all the mountings are identical,  $(\Sigma k_X Y)^2 = k_X^2 (\Sigma Y)^2$ , etc.

The  $p$ 's and  $q$ 's serve as a basis for the quantitative consideration of the frequencies.

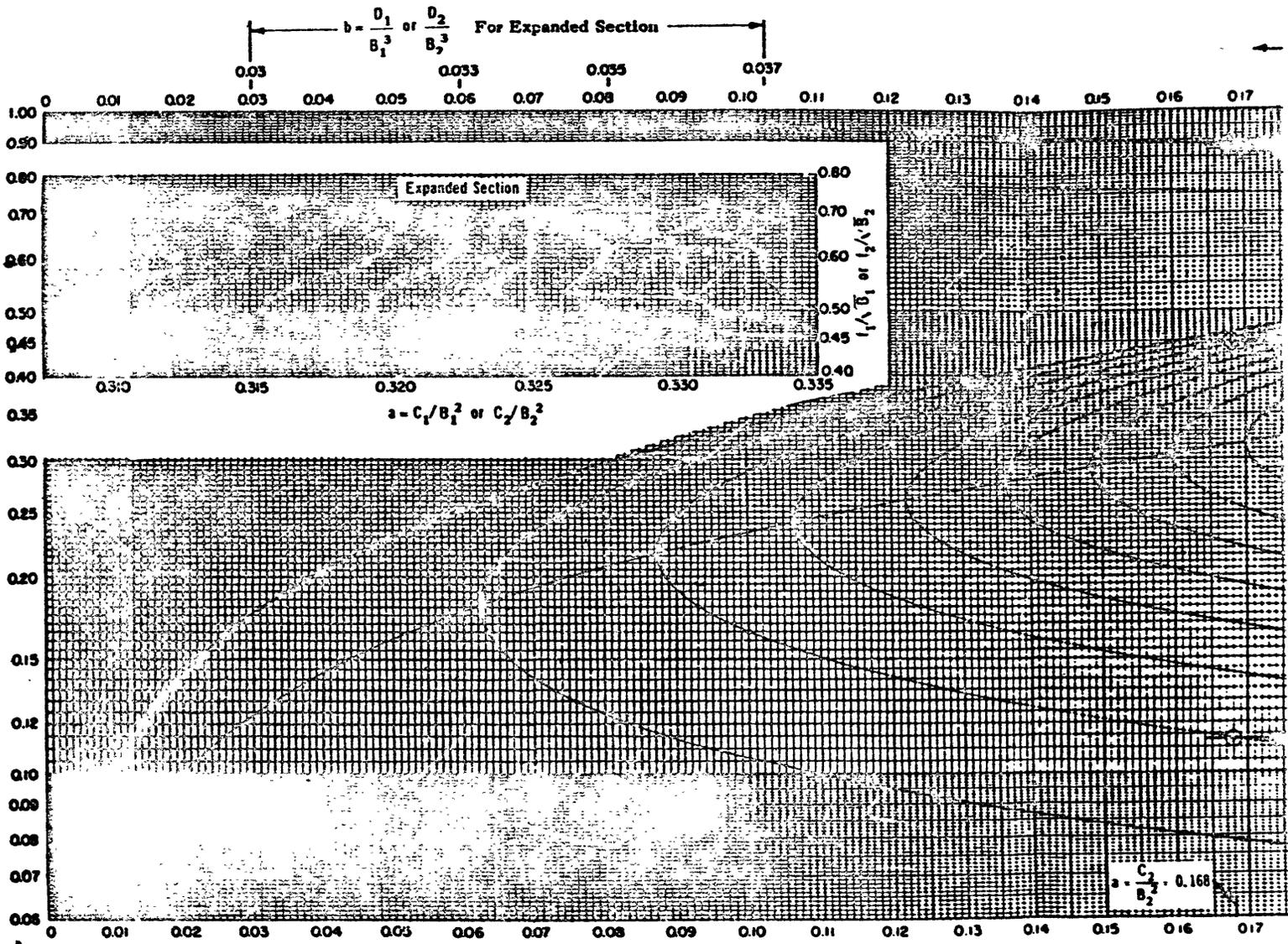
Let  $f_1, f_2, f_3$  denote the frequencies for motions parallel to YZ. Then

$$f_1^2 + f_2^2 + f_3^2 = p_2 + p_3 + q_1$$

$$(f_1^2 - f_2^2)^2 + (f_1^2 - f_3^2)^2 + (f_2^2 - f_3^2)^2 = (p_2 - p_3)^2 + (p_2 - q_1)^2 + (p_3 - q_1)^2 \\ + \frac{6 \times 95.6}{W r_X^2} \left[ (\Sigma k_Z Y)^2 + (\Sigma k_Y Z)^2 \right]$$

From the last equation,  $f_1 = f_2 = f_3$  only if  $p_2 = p_3 = q_1$  and also  $\Sigma k_Z Y = \Sigma k_Y Z = 0$ .

If the latter sums do not vanish, their effect is to spread the roots further apart. Thus, if it is desired to bring the frequencies closer together, the differences between  $p_2, p_3$ , and  $q_1$  and the sums just mentioned should be decreased. More precisely, if  $\Sigma k_Z Y = \Sigma k_Y Z = 0$ , then  $f_1^2 = p_2, f_2^2 = p_3, f_3^2 = q_1$ . Otherwise the square of the lowest of the three frequencies  $f_1, f_2, f_3$  cannot exceed the least of  $p_2, p_3$ , and  $q_1$  and will in general be less, whereas the square of the highest frequency usually exceeds the greatest of  $p_2, p_3$ , and  $q_1$ , and at least cannot be less.



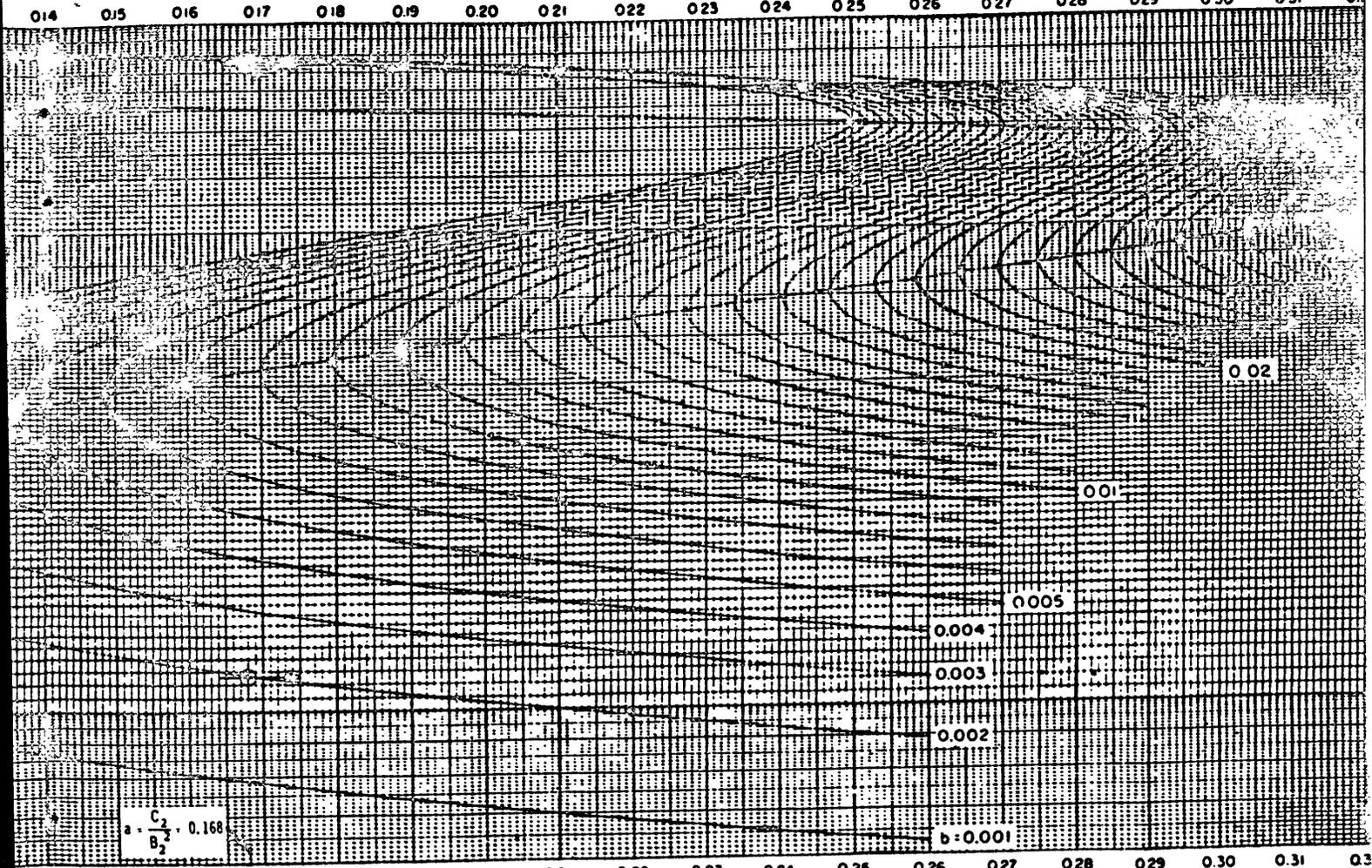
**Figure 23 - Chart for the solution of the Cubic Equations Applicable to Cases Involving One Plane of Vibrational Symmetry**

The equations are given on pages 53 and 54. The values of  $b$  shown at the top are the values pertaining to the curve whose upper or right-hand turning point lies directly below the number.

1

$$a = \frac{C_1}{B_1^2} \text{ or } \frac{C_2}{B_2^2}$$

$$b = \frac{D_1}{B_1^3} \text{ or } \frac{D_2}{B_2^3}$$

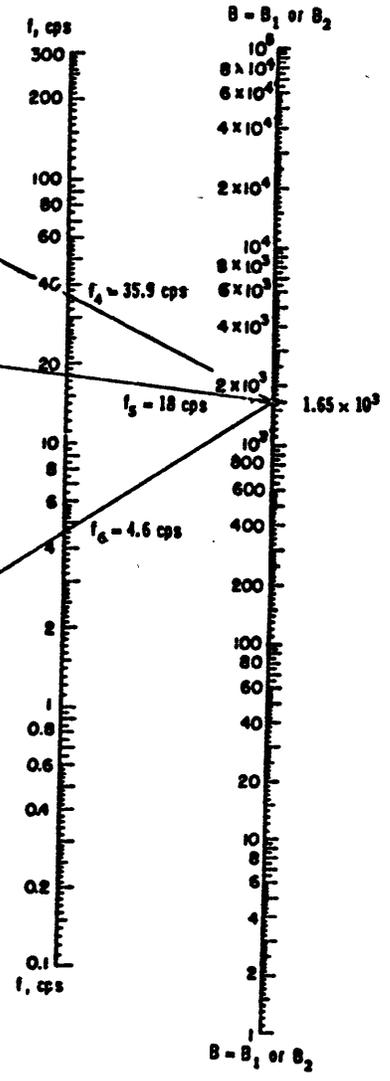
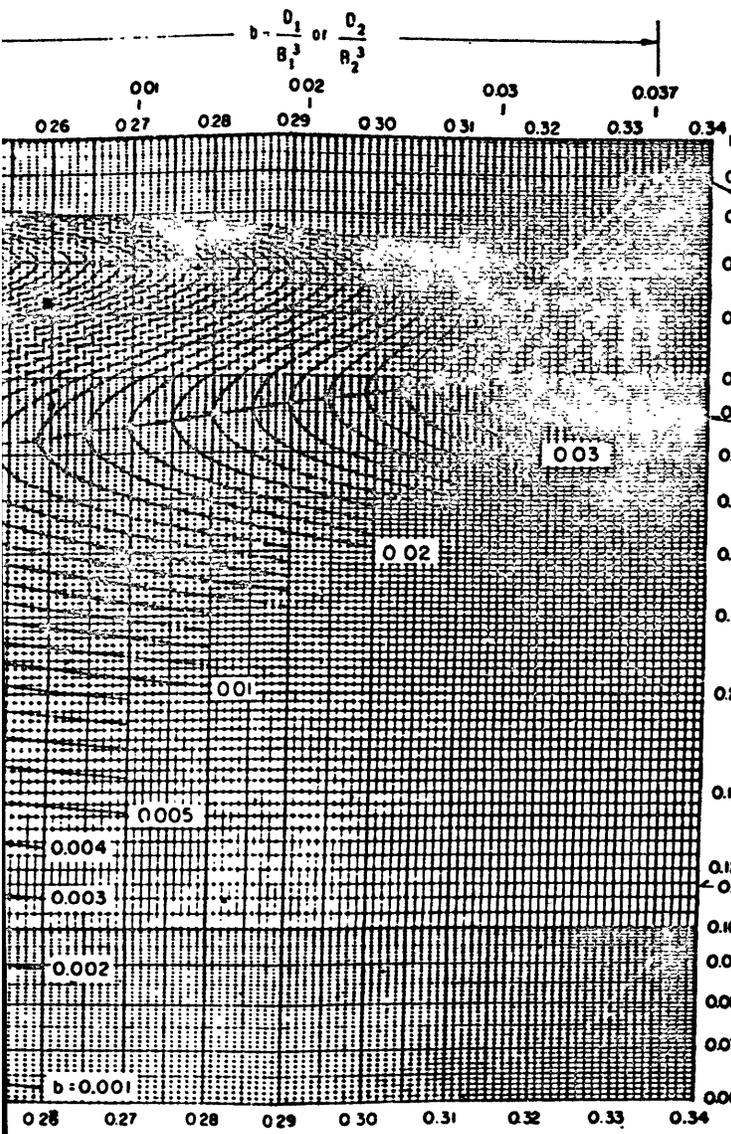


$a = \frac{C_1}{B_1^2} = 0.168$

$b = 0.001$

$$a = \frac{C_1}{B_1^2} \text{ or } \frac{C_2}{B_2^2}$$





3

In the case of vibration involving at least some motion perpendicular to  $YZ$ , three sums are involved:

$$\Sigma k_X Y, \quad \Sigma k_X Z, \quad \Sigma k_X YZ$$

To make the three frequencies equal, it is necessary that all three of these sums vanish and also that  $p_1 = q_2 = q_3$ . If all three sums vanish, then the three frequencies  $f_4, f_5, f_6$  have the values

$$f_4^2 = p_1, \quad f_5^2 = q_2, \quad f_6^2 = q_3$$

In any case

$$f_4^2 + f_5^2 + f_6^2 = p_1 + q_2 + q_3$$

$$(f_4^2 - f_5^2)^2 + (f_4^2 - f_6^2)^2 + (f_5^2 - f_6^2)^2 = (p_1 - q_2)^2 + (q_2 - q_3)^2 + (p_1 - q_3)^2 \\ + \frac{6 \times 95.6}{W^2} \left[ \frac{1}{r_Z^2} (\Sigma k_X Y)^2 + \frac{1}{r_Y^2} (\Sigma k_X Z)^2 + \frac{1}{r_Y^2 r_Z^2} (\Sigma k_X YZ)^2 \right]$$

A chart, Figure 23, has been prepared to reduce the numerical work in calculating natural frequencies of equipment. First, the coefficients  $B_1, C_1, D_1$  and  $B_2, C_2, D_2$  of the two cubic equations in  $f^2$  are evaluated. Then the ratios  $a = C_1/B_1^2$  or  $C_2/B_2^2$ , and  $b = D_1/B_1^3$  or  $D_2/B_2^3$  are calculated. The abscissa of the families of curves in Figure 23 is  $a = C_1/B_1^2$  or  $C_2/B_2^2$  while the ordinate is  $f/\sqrt{B_1}$  or  $f/\sqrt{B_2}$  depending upon whether the roots of the cubic equation are being determined for motion in plane  $YZ$  or perpendicular to plane  $YZ$ , respectively. Curves representing discrete values of  $b = D_1/B_1^3$  or  $D_2/B_2^3$  are plotted; to assist in identifying these curves, values of  $b$  at the upper or right-hand turning points of the curves are shown on scales above the plot. If the ordinates  $f/\sqrt{B}$  are read for the three intersections of the calculated value of  $a = C/B^2$  with the appropriate curve  $b = D/B^3$ , these ordinates multiplied by  $\sqrt{B}$  give three of the frequencies. The multiplication may be avoided by use of the alignment chart shown at the right, in Figure 23.

A typical numerical example for calculating natural frequencies of resiliently mounted equipment with one plane of vibrational symmetry is Problem 11 in Section A2.11.

### 3.4.1. SUMMARY FOR ONE PLANE OF SYMMETRY

The conditions for one plane of vibrational symmetry and the frequency equations for resiliently mounted equipment are summarized in Table 3. Sketches showing typical applicable arrangements for each set of equations are given also.

TABLE 3 - Frequency Equations for One Plane of Vibrational Symmetry

Mountings			Conditions of Elastic Symmetry	Frequency Equations
Identical or Different	Direction of Mounting Axes	Mounting Stiffnesses		
Identical	Any	$k_a = k_r = k$ $k_r = \text{const.}$	$\Sigma X = \Sigma XY = \Sigma XZ = 0$	<p>By Formula Motion Parallel to YZ Plane</p> <p style="text-align: right;"><u>Any Number of Mountings</u></p> $B_1 = p_2 + p_3 + q_1$ $C_1 = p_2 p_3 + p_2 q_1 + p_3 q_1 - \frac{95.6}{W^2 r_x^2} [(\Sigma k_z Y)^2 + (\Sigma k_y Z)^2]$ $D_1 = p_2 p_3 q_1 - \frac{95.6}{W^2 r_x^2} [p_2 (\Sigma k_z Y)^2 + p_3 (\Sigma k_y Z)^2]$
	Each Axis Parallel to One of X, Y, Z	$k_a \neq k_r$	$\Sigma k_y X - \Sigma k_z X - \Sigma k_z XY - \Sigma k_y XZ = 0$	
Different	Any	$k_a = k_r = k$	$\Sigma k X = \Sigma k XY = \Sigma k XZ = 0$	<p>Translation <math>\perp</math> to YZ Plane or Rotation About an Axis in YZ Plane</p> $B_2 = p_1 + q_2 + q_3$ $C_2 = p_1 q_2 + p_1 q_3 + q_2 q_3 - \frac{95.6}{W^2} \left[ \frac{1}{r_z^2} (\Sigma k_x Y)^2 + \frac{1}{r_y^2} (\Sigma k_x Z)^2 + \frac{1}{r_y^2 r_z^2} (\Sigma k_x Y Z)^2 \right]$ $D_2 = p_1 q_2 q_3 - \frac{95.6}{W^2} \left[ \frac{q_2}{r_z^2} (\Sigma k_x Y)^2 + \frac{q_3}{r_y^2} (\Sigma k_x Z)^2 + \frac{p_1}{r_y^2 r_z^2} (\Sigma k_x Y Z)^2 \right] + \frac{95.6}{W^2} (\Sigma k_x X)^2$ <p>where</p> $p_1 = \frac{9.78}{W} \Sigma k_x, \quad p_2 = \frac{9.78}{W} \Sigma k_y, \quad p_3 = \frac{9.78}{W} \Sigma k_z$ $q_1 = \frac{9.78}{W r_x^2} (\Sigma k_z Y^2 + \Sigma k_y Z^2), \quad q_2 = \frac{9.78}{W r_y^2} (\Sigma k_z X^2 + \Sigma k_x Z^2), \quad q_3 = \frac{9.78}{W r_z^2} (\Sigma k_y X^2 + \Sigma k_x Y^2)$ <p>Then for obtaining frequencies from the chart, Figure 23.</p> <p>Three frequencies - YZ Plane <span style="float: right;">Three frequencies</span></p> $a = \frac{C_1}{B_1^2}, \quad b = \frac{D_1}{B_1^3}, \quad B = B_1$ $a = \frac{C_2}{B_2^2}, \quad b = \frac{D_2}{B_2^3}, \quad B = B_2$
	Each Axis Parallel to One of X, Y, Z	$k_a \neq k_r$	$\Sigma k_y X - \Sigma k_z X - \Sigma k_z XY - \Sigma k_y XZ = 0$	

See Sections A5.4, A5.7, and A6.2 for additional cases.

Principal Axes X, Y, Z

Plane of Vibrational Symmetry YZ

Frequency Equations

Examples

Any Number of Mountings

Plane

$$-\frac{95.6}{W^2 r_x^2} [(\Sigma k_z Y)^2 + (\Sigma k_y Z)^2]$$

$$\Sigma k_z Y^2 + p_3 (\Sigma k_y Z)^2$$

or Rotation About an Axis in YZ Plane

$$\frac{95.6}{W^2} \left[ \frac{1}{r_z^2} (\Sigma k_y Y)^2 + \frac{1}{r_y^2} (\Sigma k_x Z)^2 + \frac{1}{r_y^2 r_z^2} (\Sigma k_x Y Z)^2 \right]$$

$$(\Sigma k_x Y)^2 + \frac{q_3}{r_y^2} (\Sigma k_x Z)^2 + \frac{p_1}{r_y^2 r_z^2} (\Sigma k_x Y Z)^2 \left] + \frac{1870}{W^3 r_y^2 r_z^2} (\Sigma k_x Y) (\Sigma k_x Z) (\Sigma k_x Y Z) \right.$$

$$\Sigma k_y, p_3 = \frac{9.78}{W} \Sigma k_z$$

$$q_2, q_3 = \frac{9.78}{W r_y^2} (\Sigma k_z X^2 + \Sigma k_x Z^2), q_3 = \frac{9.78}{W r_z^2} (\Sigma k_y X^2 + \Sigma k_x Y^2)$$

from the chart, Figure 23.

Plane

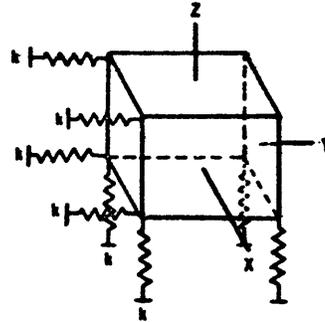
Three frequencies  $\perp$  to YZ Plane

$$B = B_1$$

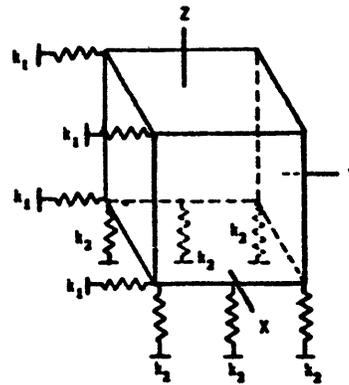
$$a = \frac{C_2}{B_2^2}, b = \frac{D_2}{B_2^3}, B = B_2$$

k's identical and

$$k_o = k_r$$



k's different but  $k_o = k_r$



## APPENDIX I

### TRANSMISSIBILITY AND ISOLATION THEORY

In the case of a mass attached to a spring, whether a sinusoidal force acts directly on the mass or a sinusoidal motion is impressed on the support the amplitude of the mass will vary with the frequency.<sup>15,16</sup> This physical principle is utilized in isolating machinery or equipment from motions of a support or in isolating the support from motions or forces generated by the machinery. Isolation is attained by making the natural frequency of the mass-spring system such that the response of either the equipment or the support is sufficiently small to be satisfactory for the particular installation.

If the mass is considered concentrated at a point, and the supporting spring is almost weightless, and if the mass is constrained to move in one direction only, then the system has one degree of freedom. Its behavior can be approximated by analysis based on simple vibration theory, even for nonlinear springs such as resilient mountings if their displacements are small.

#### A1.1. EXCITATION BY AN IMPRESSED FORCE

From Newton's second law, the acceleration of a given particle is proportional to the force applied to it and acts in the direction of the force. This may be expressed as  $F = ma$ , or for more than one force,  $\Sigma F = ma$  or  $\Sigma F - ma = 0$ . First let a sinusoidal force be impressed on the mass of a one-degree-of-freedom system. Then the forces on the mass are: the sinusoidal force  $P_0 \sin \omega t$ , a damping force proportional to the velocity of the mass  $c\dot{x}$ , and the restoring force of the spring proportional to the displacement of the mass relative to the support  $kx$ ; see Figure 24.

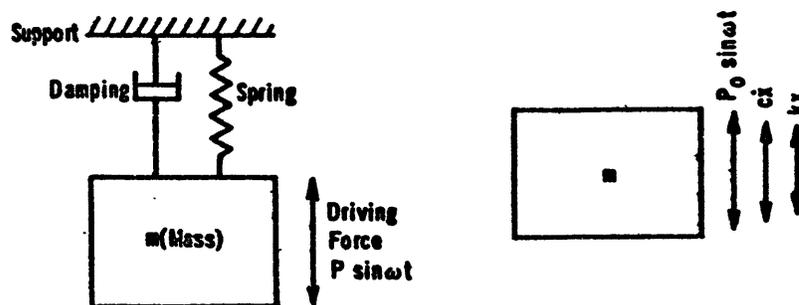


Figure 24 - Free-Body Diagram, One-Degree-of-Freedom System

Therefore, by Newton's second law, the equation of motion is

$$m\ddot{x} = -c\dot{x} - kx + P_0 \sin \omega t$$

or

$$m\ddot{x} + c\dot{x} + kx - P_0 \sin \omega t = 0$$

The solution of this equation for steady vibrations with transient terms omitted is

$$x_1 = \frac{P_0 \sin (\omega t - \phi)}{m \sqrt{(\omega_n^2 - \omega^2)^2 + (2n\omega)^2}}$$

where  $\phi = \tan^{-1} \frac{2n\omega}{\omega_n^2 - \omega^2}$ ,

$$n = \frac{c}{2m},$$

$\omega_n = \sqrt{k/m}$  is the natural frequency in radians per second, and  $\omega$  is the exciting frequency in radians per second.

The equation can be written

$$x_1 = A \sin (\omega t - \phi)$$

where

$$A = \frac{P_0}{m \omega_n^2 \sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \frac{4n^2 \omega^2}{\omega_n^4}}}$$

The amplitude can also be expressed in terms of the static deflection  $x_{st}$  that would be produced by a steady force of magnitude  $P_0$  applied to the mass

$$x_{st} = \frac{P_0}{k} = \frac{P_0}{m \omega_n^2}$$

The amplitude becomes

$$A = x_{st} \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \frac{4n^2 \omega^2}{\omega_n^4}}}$$

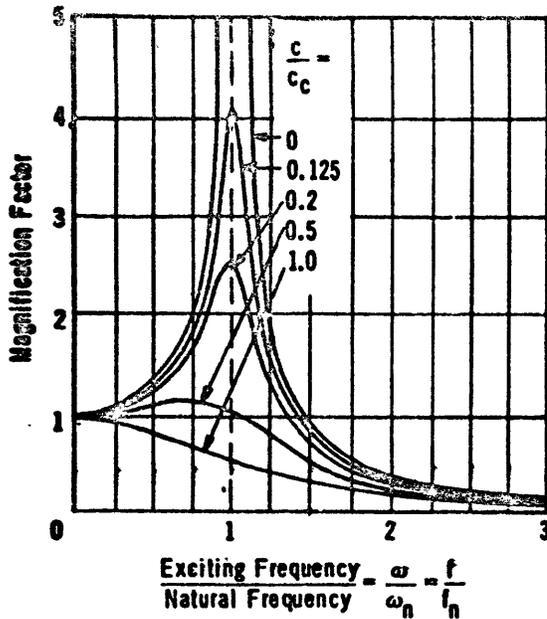


Figure 25 - Curves of Magnification Factors versus Frequency Ratios in Steady-State Vibration of a System of One Degree of Freedom

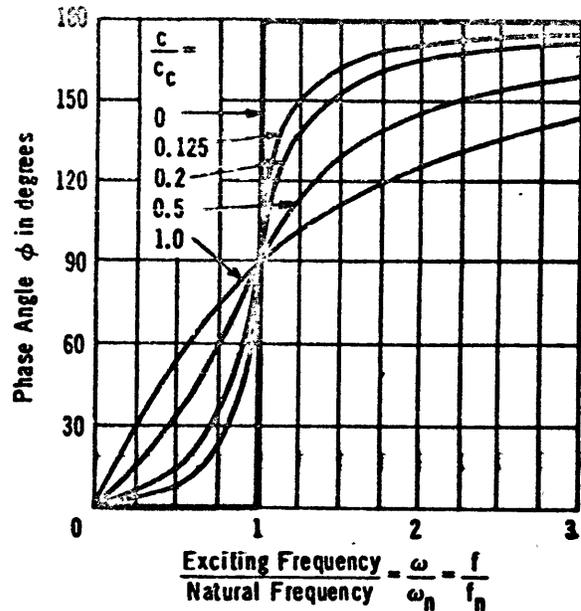


Figure 26 - Curves of Phase Angle versus Frequency Ratios in Steady-State Vibration of a System of One Degree of Freedom

The quantity

$$\frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \frac{4n^2\omega^2}{\omega_n^4}}}$$

is called the magnification factor. This factor times the static deflection gives the maximum displacement of the mass. Normally the term  $4n^2\omega^2/\omega_n^4$  is much less than unity. As the ratio  $\omega/\omega_n$  approaches 1, the magnification factor for zero damping becomes infinite. In practical cases, damping usually reduces the factor to 10 or less at resonance. If the ratio  $\omega/\omega_n$  is much less than 1, the factor approaches 1; if the ratio  $\omega/\omega_n$  is much greater than 1, the factor approaches zero; see Figure 25.

Since the force on the mass is  $P_0 \sin \omega t$  and the displacement of the mass is  $A \sin(\omega t - \phi)$ ,  $\phi$  is the phase angle between the force and the displacement. In another form,

$$\tan \phi = \frac{2 \frac{\sigma}{\sigma_c} \frac{\omega}{\omega_n}}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)}$$

where  $\sigma_c$  is the value of  $\sigma$  that produces critical damping and has the value  $\sigma_c = 2m\omega_n = 2k/\omega_n$ . Critical damping is the damping just sufficient to cause the system to return to its rest position without oscillation after an initial displacement. The angle  $\phi$  is small for small values of  $\omega/\omega_n$ , and the force  $P \sin \omega t$  leads the displacement  $X$  by a small amount. For values of

$\omega/\omega_n$  greater than 1,  $\phi$  approaches 180 deg, and the displacement is almost completely out of phase with the exciting force; see Figure 26.

The transmissibility  $T$  is commonly defined as the ratio of the force transmitted to the support to that imposed on the mass. The transmitted force is

$$c \dot{x}_1 + k x_1 = \frac{P_0}{m \sqrt{(\omega_n^2 - \omega^2)^2 + (2n\omega)^2}} [k \sin(\omega t - \phi) + c \omega \cos(\omega t - \phi)]$$

or

$$\frac{P_0 \sqrt{k^2 + c^2 \omega^2}}{m \sqrt{(\omega_n^2 - \omega^2)^2 + (2n\omega)^2}} \sin(\omega t + \phi')$$

where  $\phi'$  is a new phase angle. Hence by definition

$$T = \frac{\sqrt{k^2 + c^2 \omega^2}}{m \sqrt{(\omega_n^2 - \omega^2)^2 + (2n\omega)^2}} = \frac{\sqrt{1 + \left(2 \frac{c}{c_c} \frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2 \frac{c}{c_c} \frac{\omega}{\omega_n}\right)^2}}$$

Since  $k/m = \omega_n^2$  when the damping factor  $c/c_c$  is small, the equation reduces, except for very large  $\omega$ , to  $T = 1/(1 - \omega^2/\omega_n^2)$ . When  $\omega_n$  becomes small, the transmissibility approaches zero and as  $\omega_n$  becomes large, the transmissibility approaches 1.

## A1.2. EXCITATION BY MOTION OF SUPPORT

For application to shock excitation, it is useful to consider also the case in which there is no impressed force on the vibrating mass, but the support is given a sinusoidal motion. Let the displacement  $x_b$  of the support be

$$x_b = b \sin \omega t$$

The equation of motion for the mass is then

$$m \ddot{x} + c(\dot{x} - \dot{x}_b) + k(x - x_b) = 0$$

The solution for the steady-state vibration is

$$x_1 = \frac{b \sqrt{k^2 + c^2 \omega^2}}{m \sqrt{(\omega_n^2 - \omega^2)^2 + (2n\omega)^2}} \sin \left[ \omega t - \tan^{-1} \frac{c \omega}{k - m \omega^2} - \tan^{-1} \frac{c \omega}{k} \right]$$

This equation is the same as for the previous case except that  $P_0$  is replaced by  $b \sqrt{k^2 + c^2 \omega^2}$  and a phase angle is added to  $\omega t - \phi$ .

The ratio of the amplitude of vibration of the mass to the amplitude of vibration  $b$  of the support is thus

$$\frac{\sqrt{k^2 + c^2 \omega^2}}{m \sqrt{(\omega_n^2 - \omega^2)^2 + (2n\omega)^2}} = T$$

Hence the transmissibility  $T$  can also be defined as the ratio of the amplitude of vibration of the mass to the amplitude of vibration of the support when the motion of the mass is excited by motion of the support and not by an impressed force. The ratio of the velocities and the ratio of the accelerations are also equal to  $T$ .

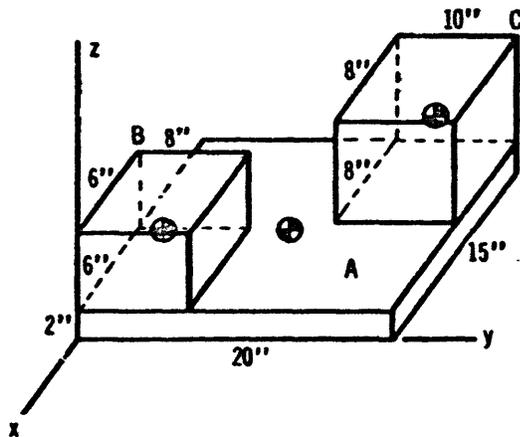
## APPENDIX 2

### NUMERICAL EXAMPLES

Typical numerical examples illustrating the determination of centers of gravity, moments of inertia, and natural frequencies of resiliently mounted equipment with various symmetrical arrangements of mountings are presented in this Appendix.

#### A2.1. PROBLEM 1 - CENTER OF GRAVITY

Assume that the centers of gravity of the subbase A and of the individual units B and C, Figure 27, are at their respective centers. Determine the center of gravity of the assembly.

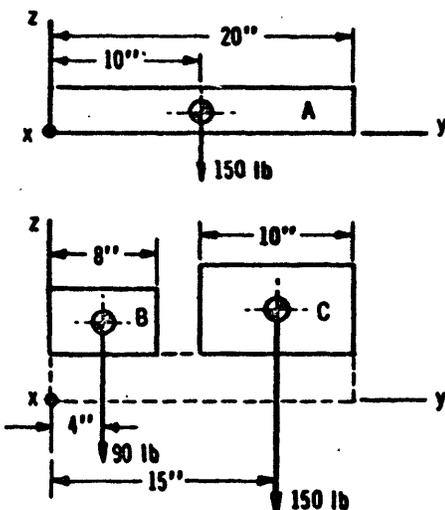


The weights are:

Subbase A	150 lb
Unit B	90 lb
Unit C	150 lb
Total Weight = $\Sigma W$	390 lb

Figure 27 - Problem 1 - Determination of Center of Gravity of Assembly of Two Units and a Subbase

Take as the moment about the z axis the weight of the individual unit in pounds times the y coordinate of its center of gravity in inches.

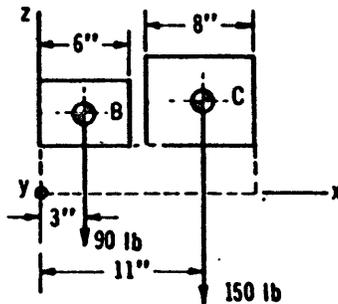
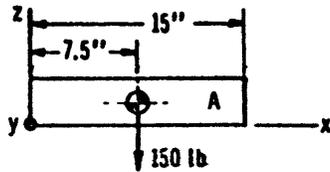


	<u><math>W \times y</math></u>
Subbase A	$150 \times 10 = 1500$
Unit B	$90 \times 4 = 360$
Unit C	$150 \times 15 = 2250$
	$\Sigma W y = 4110 \text{ in.-lb}$

Dividing the total moment  $\Sigma Wy$  by the weight  $\Sigma W$ , the distance  $\bar{y}$  from the  $x$  axis to the center of gravity is determined.

$$\frac{\Sigma Wy}{\Sigma W} = \frac{4110}{390} = 10.5 \text{ in.} = \bar{y}$$

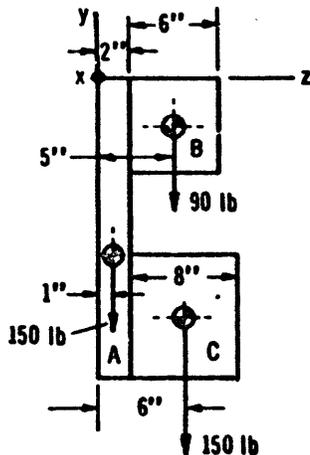
Take as the moment about the  $y$  axis the weight of the individual unit in pounds times the  $x$  coordinate of its center of gravity in inches.



	<u>W x x</u>
Subbase A	150 x 7.5 = 1125
Unit B	90 x 3 = 270
Unit C	150 x 11 = 1650
	<u>Σ Wx = 3045 in. lb</u>

$$\frac{\Sigma Wx}{\Sigma W} = \frac{3045}{390} = 7.8 \text{ in.} = \bar{x}$$

Take as the moment about the  $x$  axis the weight of the individual unit in pounds times the  $z$  coordinate of its center of gravity in inches.



	<u>W x z</u>
Subbase A	150 x 1 = 150
Unit B	90 x 5 = 450
Unit C	150 x 6 = 900
	<u>Σ Wz = 1500 in. lb</u>

$$\frac{\Sigma Wz}{\Sigma W} = \frac{1500}{390} = 3.8 \text{ in.} = \bar{z}$$

The center of gravity of the assembly is

- 7.8 in. back of the front edge of the subbase,
- 10.5 in. to the right of the left edge of the subbase, and
- 3.8 in. up from the bottom edge of the subbase,

that is, the coordinates of the center of gravity with respect to the reference axes are  $\bar{x} = 7.8$  in.,  $\bar{y} = 10.5$  in.,  $\bar{z} = 3.8$  in.

## A2.2. PROBLEM 2 - MOMENT OF INERTIA

Assume the same units of equipment, B and C, and subbase as in Problem 1, see Figure 27. Determine the moments of inertia about axes in the  $x$ ,  $y$ , and  $z$  directions through the center of gravity of the assembly.

In Problem 1, the location of the center of gravity of the assembly was calculated as

- 7.8 in. back of the front edge of the subbase,
- 10.5 in. to the right of the left edge of the subbase, and
- 3.8 in. up from the bottom of the subbase.

Assume the moments of inertia of the individual units about axes through their centers of gravity in the  $x$ ,  $y$ , and  $z$  directions to be

$I$ , lb in. sec <sup>2</sup>	Unit A	Unit B	Unit C
$I_x$	13.08	1.94	5.31
$I_y$	7.42	1.40	4.15
$I_z$	20.24	1.94	5.31

Now that the moments of inertia of the individual units about axes through their centers of gravity are known, the transformation formula, for example,  $I_x = I_{x, c.g.} + md^2$  for axes parallel to the  $X$  axis, may be used to determine moments of inertia about parallel axes through the center of gravity of the assembly. The formula shows that the moment of inertia about any axis is equal to the moment of inertia about a parallel axis through the center of gravity of the unit plus the mass of the unit multiplied by the square of the distance between the axes.

Before the transformation formula can be used, the squares of these distances must be calculated. But first the distances of the center of gravity of each unit in the  $x$ ,  $y$ , and  $z$  directions from the center of gravity of the assembly must be determined; they are

Direction	Unit A	Unit B	Unit C
$x$	0.3 in.	4.8 in.	3.2 in.
$y$	0.5 in.	6.5 in.	4.5 in.
$z$	2.8 in.	1.2 in.	2.2 in.

The squares of the distances between the axes of the units and those of the assembly and the moments of inertia of the assembly may now be calculated.\*

Axes Parallel to	Unit A	Unit B	Unit C
<i>x</i>	$(\sqrt{(0.5)^2 + (2.8)^2})^2$ = 8.09 in. <sup>2</sup>	$(\sqrt{(1.2)^2 + (6.5)^2})^2$ = 43.69 in. <sup>2</sup>	$(\sqrt{(4.5)^2 + (2.2)^2})^2$ = 25.09 in. <sup>2</sup>
<i>y</i>	$(\sqrt{(0.3)^2 + (2.8)^2})^2$ = 7.93	$(\sqrt{(1.2)^2 + (4.8)^2})^2$ = 24.48	$(\sqrt{(3.2)^2 + (2.2)^2})^2$ = 15.08
<i>z</i>	$(\sqrt{(0.3)^2 + (0.5)^2})^2$ = 0.34	$(\sqrt{(6.5)^2 + (4.8)^2})^2$ = 65.29	$(\sqrt{(4.5)^2 + (3.2)^2})^2$ = 30.49

About	Unit A	Unit B	Unit C	Total
	$I_x = I_{x, c.g.} + md^2$			
<i>x</i> Axis	$= 13.08 + \frac{150}{386} \times 8.09$ $= 13.08 + 3.14$ $= 16.22 \text{ lb in. sec}^2$	$= 1.94 + \frac{90}{386} \times 43.69$ $= 1.943 + 10.19$ $= 12.13 \text{ lb in. sec}^2$	$= 5.31 + \frac{150}{386} \times 25.09$ $= 5.31 + 9.75$ $= 15.06 \text{ lb in. sec}^2$	= 43.41 call 43.4 lb in. sec <sup>2</sup>
	$I_y = I_{y, c.g.} + md^2$			
<i>y</i> Axis	$= 7.42 + \frac{150}{386} \times 7.93$ $= 7.42 + 3.08$ $= 10.50 \text{ lb in. sec}^2$	$= 1.40 + \frac{90}{386} \times 24.48$ $= 1.40 + 5.71$ $= 7.11 \text{ lb in. sec}^2$	$= 4.15 + \frac{150}{386} \times 15.08$ $= 4.15 + 5.86$ $= 10.01 \text{ lb in. sec}^2$	= 27.62 call 27.6 lb in. sec <sup>2</sup>
	$I_z = I_{z, c.g.} + md^2$			
<i>z</i> Axis	$= 20.24 + \frac{150}{386} \times 0.34$ $= 20.24 + 0.13$ $= 20.37 \text{ lb in. sec}^2$	$= 1.94 + \frac{90}{386} \times 65.29$ $= 1.94 + 15.23$ $= 17.17 \text{ lb in. sec}^2$	$= 5.31 + \frac{150}{386} \times 30.49$ $= 5.31 + 11.85$ $= 17.16 \text{ lb in. sec}^2$	= 54.70 call 54.7 lb in. sec <sup>2</sup>

\*No attempt was made to arrange units on the subbase so that the *x*, *y*, *z* axes would be principal axes. In practice, the units should be arranged so that the center of gravity is close to the geometrical center of the assembly and the principal axes are parallel to the sides of the assembly.

**A2.3. PROBLEM 3 - COMBINATION OF PROBLEMS 1 AND 2**  
**By Rearrangement of Operations, the Centers of Gravity and Moments of Inertia**  
**are Worked Together, Reducing the Computations**

Unit	$\psi$	$m$	$x$	$mx$	$mx^2$	$y$	$my$	$my^2$	$z$	$mz$	$mz^2$	$I_x$	$I_y$	$I_z$
A	150	0.389	7.5	2.92	21.90	10.0	3.89	38.90	1.0	0.39	0.39	13.08	7.42	20.24
B	90	0.233	3.0	0.70	2.10	4.0	0.93	3.73	5.0	1.17	5.83	1.94	1.40	1.94
C	150	0.389	11.0	4.28	47.07	15.0	5.84	87.53	6.0	2.33	14.00	5.31	4.15	5.31
$\Sigma$	390	1.011		7.90	71.07		10.66	130.2		3.89	20.22	20.33	12.97	27.49

$$\bar{x} = \frac{\Sigma mx}{\Sigma m} = \frac{7.90}{1.011} = 7.80 \text{ in.}; \quad \bar{x}^2 = 60.8$$

$$\bar{y} = \frac{\Sigma my}{\Sigma m} = \frac{10.66}{1.011} = 10.54 \text{ in.}; \quad \bar{y}^2 = 111.1$$

$$\bar{z} = \frac{\Sigma mz}{\Sigma m} = \frac{3.89}{1.011} = 3.85 \text{ in.}; \quad \bar{z}^2 = 14.8$$

$$I_x = \Sigma I_{x, c.g.} + \Sigma m y^2 + \Sigma m z^2 - \Sigma m (\bar{y}^2 + \bar{z}^2)$$

$$= 20.33 + 130.2 + 20.22 - 1.011 (111.1 + 14.8) = 43.6 \text{ lb-in.-sec}^2$$

$$I_y = \Sigma I_{y, c.g.} + \Sigma m x^2 + \Sigma m z^2 - \Sigma m (\bar{x}^2 + \bar{z}^2)$$

$$= 12.97 + 71.07 + 20.22 - 1.011 (60.8 + 14.8) = 27.8 \text{ lb-in.-sec}^2$$

$$I_z = \Sigma I_{z, c.g.} + \Sigma m x^2 + \Sigma m y^2 - \Sigma m (\bar{x}^2 + \bar{y}^2)$$

$$= 27.49 + 71.07 + 130.2 - 1.011 (60.8 + 111.1) = 55.2 \text{ lb-in.-sec}^2$$

**A2.4. PROBLEM 4 - CENTER-OF-GRAVITY MOUNTING ARRANGEMENT**  
**THREE PLANES OF VIBRATIONAL SYMMETRY**  
**FOUR COPLANAR IDENTICAL MOUNTINGS**

The unit is a solid homogeneous rectangular body and weighs 4000 lb; see Figure 28. The center of gravity is at the geometrical center of the unit. The X, Y, and Z axes coincide with the principal axes of inertia. The dimensions, moments of inertia, radii of gyration, and

Initial conditions of the resilient mounts are

$$\begin{aligned}
 l &= 50 \text{ in.} & I_X &= 1280 \text{ lb-in.-sec}^2 & r_X &= 10.9 \text{ in.} & D_X &= 28 \text{ in.} \\
 w &= 32 \text{ in.} & I_Y &= 2500 \text{ lb-in.-sec}^2 & r_Y &= 15.6 \text{ in.} & D_Y &= 19 \text{ in.} \\
 h &= 20 \text{ in.} & I_Z &= 8040 \text{ lb-in.-sec}^2 & r_Z &= 17.1 \text{ in.} & D_Z &= 0 \text{ in.}
 \end{aligned}$$

Four 1000-lb mountings are attached to brackets of the unit in a horizontal plane containing the center of gravity of the unit.

Using the value of the rated frequency of the mounting obtained from the standard report form, here taken to be 15 cps for the supported load of 1000 lb, the dynamic stiffness of a single 1000-lb mounting may be determined from the formula

$$k = (2\pi f)^2 \frac{W}{g} = \frac{W f^2}{9.8} = \frac{1000 (15)^2}{9.8} = 23,000 \text{ lb/in.}$$

The translational and rotational natural frequencies of the equipment supported by these four mountings are

$$f_{tr} = 6.26 \sqrt{\frac{k}{W}} = 6.26 \sqrt{\frac{23,000}{4,000}} = 15 \text{ cps}$$

$$f_{rot, X} = \frac{6.26 D_Y}{r_X} \sqrt{\frac{k}{W}} = \frac{6.26 \times 19}{10.9} \sqrt{\frac{23,000}{4,000}} = 26.1 \text{ cps}$$

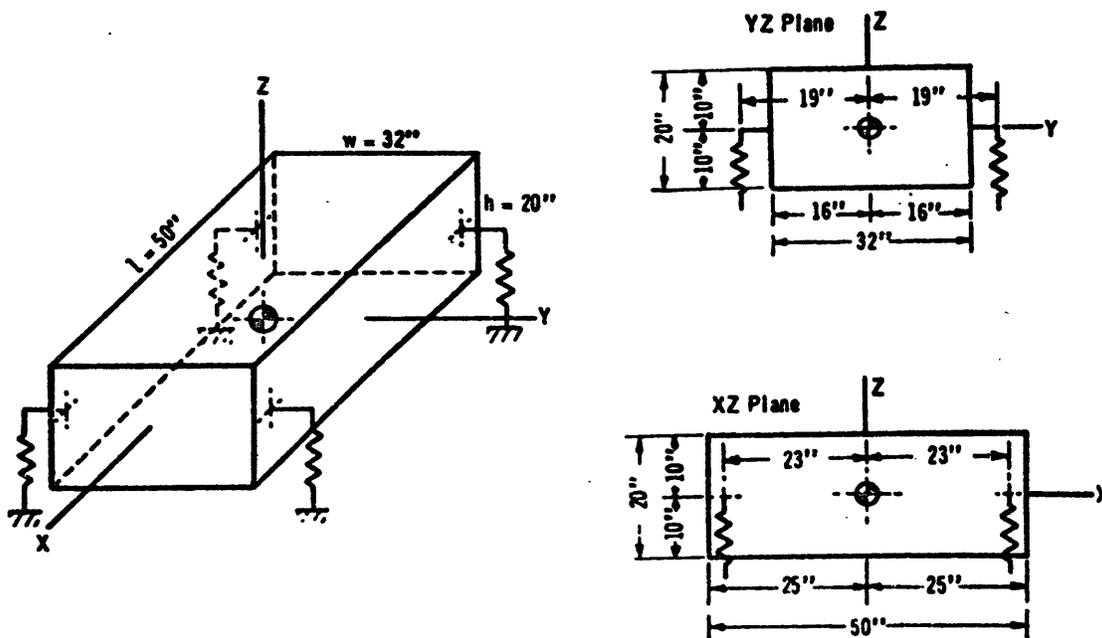


Figure 28 - Problem 4 - Three Planes of Vibrational Symmetry

$$f_{rot,Y} = \frac{6.26 D_X}{r_Y} \sqrt{\frac{k}{W}} = \frac{6.26 \times 23}{15.5} \sqrt{\frac{23,000}{4,000}} = 22.3 \text{ cps}$$

$$f_{rot,Z} = \frac{6.26}{r_Z} \sqrt{\frac{k}{W} (D_X^2 + D_Y^2)} = \frac{6.26}{17.1} \sqrt{\frac{23,000}{4,000} [(23)^2 + (19)^2]} = 26.1 \text{ cps}$$

The  $f_{rot,X}$  and  $f_{rot,Z}$  natural frequencies are the same solely by coincidence.

If the maximum deflection across the mounting under shock is assumed to be 1 in. either side of an equilibrium position, the clearance around the equipment should be 1 in.; see Section 1.6.

If the mountings can be positioned closer in the  $X$  and  $Y$  directions, reducing  $D_X$  and  $D_Y$ , the rotational frequencies  $f_{rot,X}$  and  $f_{rot,Y}$  are reduced proportionally, while  $f_{tr}$  remains the same. If the translational frequency is not satisfactory with respect to noise transmission and a lower natural frequency is desired in this mode, other mountings with rated frequencies lower than 15 cps must be selected. In this problem the  $D_Y$  distance, and therefore  $f_{rot,X}$ , cannot be reduced with a solid body. However  $D_X$ , and therefore  $f_{rot,Y}$  and to a lesser extent  $f_{rot,Z}$ , can be reduced. If the equipment were not solid and the components could be redistributed, an increase in  $r_X$ , a smaller increase in  $r_Z$ , and a decrease in  $D_X$  would reduce all the rotational frequencies.

Another illustration of the application of the formulas for equipment having three planes of vibrational symmetry, Problem 5, is also presented in this appendix. In this problem moments of inertia about any axis in the  $XY$  plane through the center of gravity are the same. The question here is the location of the  $XZ$  and  $YZ$  planes of symmetry. If the planes are taken at 45 deg to the mountings, the formulas for four mountings previously presented may be used. If these planes of symmetry are taken through the mountings, then the coefficient of the equations becomes 4.43 rather than 6.26 because two mountings instead of four are acting with respect to either the  $XZ$  or  $YZ$  plane of vibrational symmetry. The natural frequencies obtained by calculation are the same for either procedure. Still another illustration of a problem with mountings arranged with three planes of vibrational symmetry is Problem 6, also in this appendix.

#### A2.5. PROBLEM 5 - CENTER-OF-GRAVITY MOUNTING ARRANGEMENT THREE PLANES OF VIBRATIONAL SYMMETRY FOUR COPLANAR IDENTICAL MOUNTINGS

The unit is a cylindrical body and weighs 5000 lb; see Figure 29. The center of gravity is on the axis of the cylinder, 26 in. from its bottom. The  $X$ ,  $Y$ , and  $Z$  axes coincide with the principal axes of inertia of the body. In this instance the moment of inertia about any axis

through the center of gravity in the XY plane is the same. The dimensions, moments of inertia, and radii of gyration of the body and initial locations of the resilient mountings are

$$\begin{array}{llll}
 l = 48 \text{ in.} & I_X = 8.76 \times 10^3 \text{ lb-in.-sec}^2 & r_X = 26.0 \text{ in.} & D_X = 19.8 \text{ in.} \\
 w = 48 \text{ in.} & I_Y = 8.76 \times 10^3 \text{ lb-in.-sec}^2 & r_Y = 26.0 \text{ in.} & D_Y = 19.8 \text{ in.} \\
 h = 72 \text{ in.} & I_Z = 3.72 \times 10^3 \text{ lb-in.-sec}^2 & r_Z = 17.0 \text{ in.} & D_Z = 0 \text{ in.}
 \end{array}$$

Four 1200-lb mountings are attached to brackets of the unit in a horizontal plane containing the center of gravity of the unit. The dynamic stiffness of one 1200-lb mounting may be determined by the formula

$$k = (2\pi f)^2 \frac{W}{g} = \frac{W f^2}{9.8}$$

in which the value of the natural frequency obtained from the standard report form, and here assumed to be 15 cps for the 1250-lb load to be supported, is used.

$$k = \frac{W f^2}{9.8} = \frac{1250 (15)^2}{9.8} = 28,700 \text{ lb/in.}$$

The translational and rotational natural frequencies of the equipment supported by the four mountings are

$$f_{tr} = 6.26 \sqrt{\frac{k}{W}} = 6.26 \sqrt{\frac{28,700}{5000}} = 15 \text{ cps}$$

$$f_{rot, X} = \frac{6.26 D_Y}{r_X} \sqrt{\frac{k}{W}} = \frac{6.26 \times 19.8}{26.0} \sqrt{\frac{28,700}{5000}} = 11.4 \text{ cps}$$

$$f_{rot, Y} = \frac{6.26 D_Y}{r_Y} \sqrt{\frac{k}{W}} = \frac{6.26 \times 19.8}{26.0} \sqrt{\frac{28,700}{5000}} = 11.4 \text{ cps}$$

$$f_{rot, Z} = \frac{6.26}{r_Z} \sqrt{\frac{k}{W} (D_X^2 + D_Y^2)} = \frac{6.26}{17.0} \sqrt{\frac{28,700}{5000} [(19.8)^2 + (19.8)^2]} = 24.7 \text{ cps}$$

In this problem, when the moments of inertia or the radii of gyration are the same about the X and Y axes, the rotational frequencies are the same.

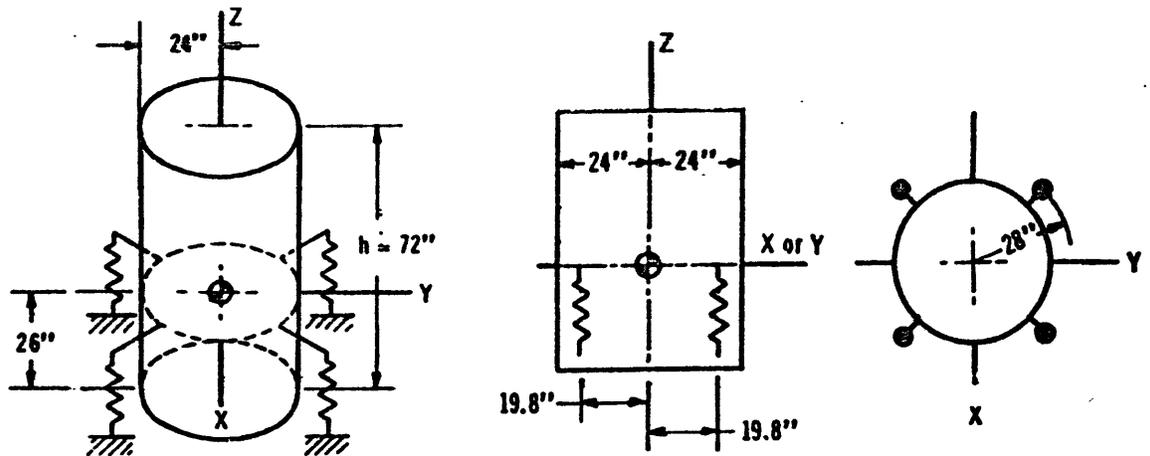


Figure 29 - Problem 5 - Three Planes of Vibrational Symmetry

The translational frequency  $f_{tr}$  is a function of the mounting frequency. To reduce this frequency, mountings with lower natural frequencies under normal load must be selected. The rotational frequency  $f_{rot, Z}$  can be reduced by redistribution of the components of the unit so that  $r_z$  is larger.

For any of this group of problems, it should be noted that  $f_{tr}$  need not be calculated if the load per mounting is the same as the load associated with the natural frequency of the mounting, which is obtained from the report form to calculate the spring constant. It should also be noted that  $f_{tr}$  may not be exactly the same in the X, Y, and Z directions and depends again on the mounting characteristics in the axial and radial directions. Nevertheless, for mountings having nominally equal radial and axial stiffnesses, it can be assumed that the natural frequencies in the two directions are close enough to be represented by one number for most practical problems.

**42.6. PROBLEM 6 - CENTER-OF-GRAVITY MOUNTING ARRANGEMENT  
THREE PLANES OF VIBRATIONAL SYMMETRY  
FOUR COPLANAR DIFFERENT MOUNTINGS**

The unit is a nonhomogeneous rectangular body and weighs 3600 lb; see Figure 30. The center of gravity is 10 in. back of the front, 10 in. to the left of the right side, and 10 in. up from the bottom of the unit. The X, Y, and Z axes coincide with the principal axes of inertia. The dimensions, moments of inertia, and radii of gyration of the body are

$$l = 50 \text{ in.} \quad I_X = 1500 \text{ lb-in. sec}^2 \quad r_X = 12.0 \text{ in.}$$

$$w = 30 \text{ in.} \quad I_Y = 2800 \text{ lb-in. sec}^2 \quad r_Y = 16.4 \text{ in.}$$

$$h = 20 \text{ in.} \quad I_Z = 3600 \text{ lb-in. sec}^2 \quad r_Z = 18.6 \text{ in.}$$

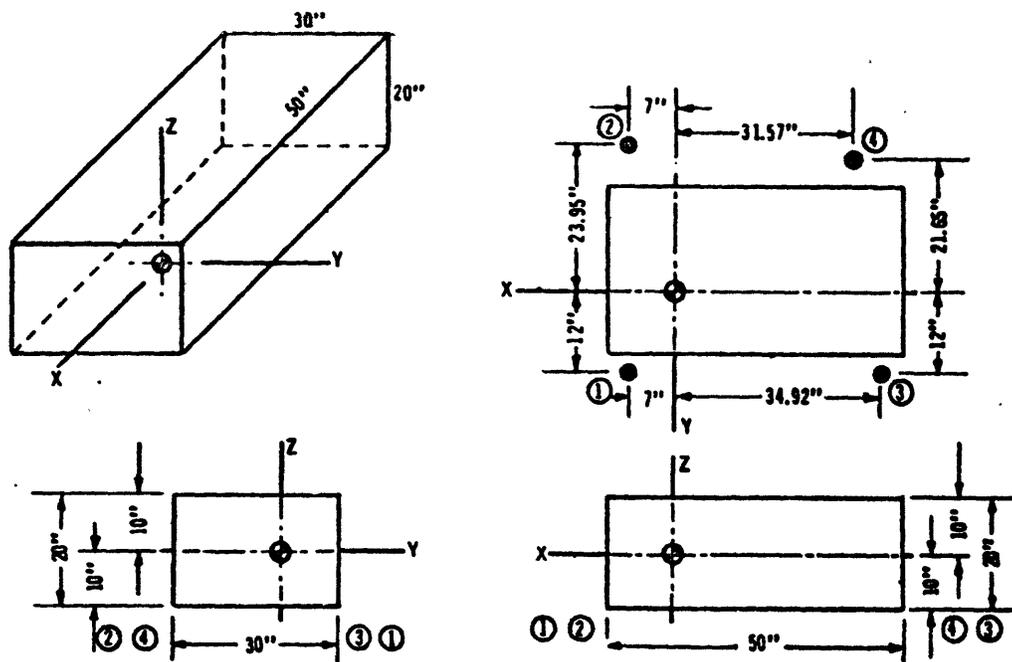


Figure 30 - Problem 6 - Three Planes of Vibrational Symmetry

Four different mountings, each having equal axial and radial stiffnesses, are attached to brackets at distances from the center of gravity of the equipment inversely proportional to their stiffnesses in order that elastic symmetry be obtained. The mountings selected were 2000-, 1000-, 400-, and 220-lb mountings.

Their stiffnesses are:

For the 2000-lb mounting,

$$k_1 = \frac{W f^2}{9.8} = \frac{2000 (15)^2}{9.8} = 45,900 \text{ lb/in.}$$

For the 1000-lb mounting,

$$k_2 = \frac{W f^2}{9.8} = \frac{1000 (15)^2}{9.8} = 23,000 \text{ lb/in.}$$

For the 400-lb mounting,

$$k_3 = \frac{W f^2}{9.8} = \frac{400 (15)^2}{9.8} = 9200 \text{ lb/in.}$$

For the 220-lb mounting,

$$k_4 = \frac{W f^2}{9.8} = \frac{220 (15)^2}{9.8} = 5100 \text{ lb/in.}$$

The mounting locations and their spring constants are

Mounting No.	$X = \pm D_X$	$Y = \pm D_Y$	$Z = \pm D_Z$	$k(k_X = k_Y = k_Z)$
1	7	12	0	45,900
2	7	-23.95	0	23,000
3	-34.92	12	0	9,200
4	-31.57	-21.65	0	5,100

$$\Sigma k = 8.32 \times 10^4, \quad \Sigma k D_X^2 = 1.967 \times 10^7, \quad \Sigma k D_Y^2 = 2.357 \times 10^7, \quad \Sigma k D_Z^2 = 0$$

The natural frequencies are

$$f_{tr} = 3.13 \sqrt{\frac{1}{W} \Sigma k} = 3.13 \sqrt{\frac{1}{3600} \times 8.32 \times 10^4} = 15.0 \text{ cps}$$

since  $\Sigma k_X = \Sigma k_Y = \Sigma k_Z$ ,  $f_{tr,X} = f_{tr,Y} = f_{tr,Z}$

$$f_{rot,X} = \frac{3.13}{r_X} \sqrt{\frac{1}{W} \Sigma k D_Y^2} = \frac{3.13}{12.0} \sqrt{\frac{2.357 \times 10^7}{3600}} = 21.1 \text{ cps}$$

$$f_{rot,Y} = \frac{3.13}{r_Y} \sqrt{\frac{1}{W} \Sigma k D_X^2} = \frac{3.13}{16.4} \sqrt{\frac{1.967 \times 10^7}{3600}} = 14.1 \text{ cps}$$

$$f_{rot,Z} = \frac{3.13}{r_Z} \sqrt{\frac{1}{W} (\Sigma k D_X^2 + \Sigma k D_Y^2)} = \frac{3.13}{18.6} \sqrt{\frac{1.967 \times 10^7 + 2.357 \times 10^7}{3600}} = 18.4 \text{ cps.}$$

#### A2.7. PROBLEM 7 - BOTTOM MOUNTING ARRANGEMENT TWO PLANES OF VIBRATIONAL SYMMETRY FOUR COPLANAR IDENTICAL MOUNTINGS

The unit is a rectangular nonhomogeneous body and weighs 7500 lb; see Figure 31. The center of gravity is 6 in. above the bottom of the unit and at its midlength and midwidth. The X, Y, and Z axes coincide with the principal axes of inertia. The dimensions, moments of inertia, and radii of gyration are

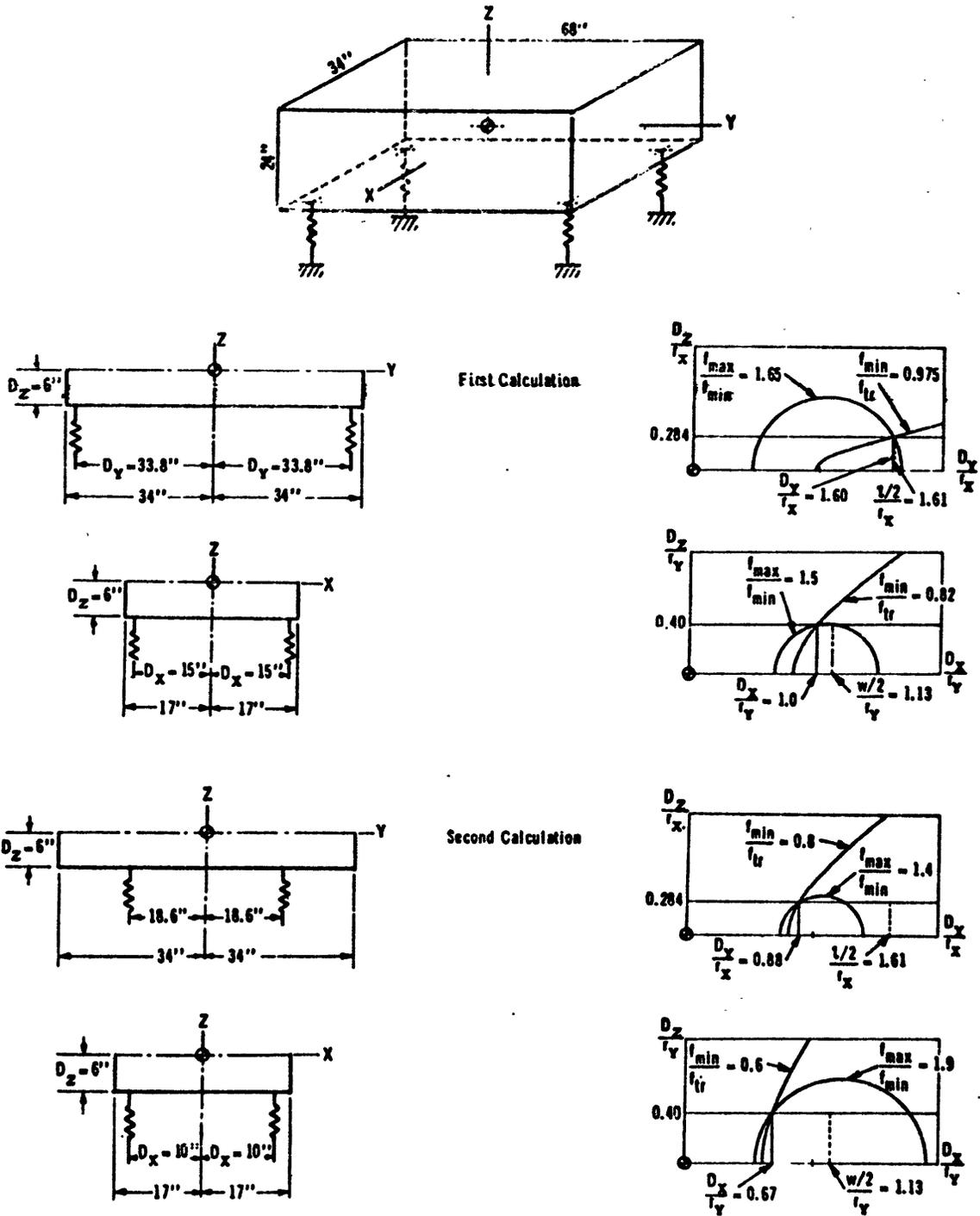


Figure 31 - Problem 7 - Two Planes of Vibrational Symmetry

$$\begin{aligned}
 l &= 68 \text{ in.} & I_x &= 8.66 \times 10^3 \text{ lb-in.-sec}^2 & r_x &= 21.1 \text{ in.} \\
 w &= 34 \text{ in.} & I_y &= 4.37 \times 10^3 \text{ lb-in.-sec}^2 & r_y &= 15.0 \text{ in.} \\
 h &= 24 \text{ in.} & I_z &= 9.36 \times 10^3 \text{ lb-in.-sec}^2 & r_z &= 21.9 \text{ in.}
 \end{aligned}$$

Let it be assumed that this unit was subjected to vibration tests in accordance with Military Specification MIL-T-17118 (SHIPS);<sup>4</sup> the most severe excitation of the components of the equipment occurred at an exciting frequency of 24 cps. Therefore it is desired to avoid resonance at this frequency aboard ship. In addition the equipment has an exciting frequency of 40 cps (2400 rpm) due to a rotating part. It is desired to avoid this frequency in order to reduce the transmission of structure-borne noise. To avoid any magnification of vibration, the rated frequency of the mounting selected, with its share of the dead load of the equipment applied, should be less than the exciting frequency divided by  $\sqrt{2}$ , or less than  $f/\sqrt{2} = 24/\sqrt{2} = 17$  cps. This mounting, for the moment, may be assumed to be satisfactory with respect to the 40-cps excitation in the equipment.

By referring to performance data in report form on various mountings, as illustrated in Appendix 4, a 2000-lb mounting with a rated frequency of 15 cps obtained for a dead load of 7500/4 or 1875 lb can be found. Assume that the performance data for this mounting shows equal stiffnesses in the axial and radial directions, therefore permitting easier use of the Base Mounting Chart in determining the natural frequencies of the rocking modes of the equipment. Assume also that under tests on a shock machine the mountings have adequate shock strength, and the maximum deflection across the mounting is 0.75 in. from the equilibrium position in all directions.

Solving the frequency equation for the dynamic spring constant  $k$  gives

$$k = (2\pi f)^2 \frac{W}{g} = \frac{W f^2}{9.8} = 0.1022 (15)^2 1875 = 43,000 \text{ lb/in. for one mounting}$$

where  $W = 7500/4 = 1875$ , and the translational frequency,  $f_{tr}$ , of the equipment with four mountings is

$$f_{tr} = 6.26 \sqrt{\frac{k}{W}} = 6.26 \sqrt{\frac{43,000}{7500}} = 15.0 \text{ cps}$$

where  $W = 7500$ .

Now, for the YZ plane, locate on the Base Mounting Chart, Figure 20, the half-length of the base divided by the radius of gyration,  $(l/2)/r_x = 34/21.1 = 1.61$ , and the distance from the center of gravity to the plane of attachment of the mountings,  $D_z/r_x = 6/21.1 = 0.284$  in nondimensional units, as shown in Figure 29. One position of the mounting along this line will give a value for  $f_{max}/f_{min} = 1.65$  and  $D_y/r_x = 1.60$ , or  $D_y = 1.60 r_x = 1.60 \times 21.1 = 33.8$  in. Therefore the four mountings should be a distance  $D_y = 33.8$  in. in the Y direction from the center of gravity of the equipment. For this mounting location the chart gives the value  $(f_{min}/f_{tr}) = 0.975$ . With the  $f_{tr}$  already computed, the values of  $f_{max}$  and  $f_{min}$ , the two

rocking modes in the YZ plane, may now be computed

$$\frac{f_{\min}}{f_{\max}} = \frac{f_{\min}}{15.0} = 0.975, \quad f_{\min} = 14.6 \text{ cps}$$

$$\frac{f_{\max}}{f_{\min}} = \frac{f_{\max}}{14.6} = 1.65, \quad f_{\max} = 24.1 \text{ cps}$$

In a similar way, the natural frequencies of the rocking modes in the XZ plane may now be computed, again using the Base Mounting Chart, Figure 20. The half-width of the base is laid off  $(w/2)/r_Y = 17/15.0 = 1.13$  at a height  $D_Z/r_Y = 6/15 = 0.40$ . The value at the mounting position is taken as  $f_{\max}/f_{\min} = 1.5$ . This point is  $D_X/r_Y = 1.0$ , or  $D_X = 1.0 \times r_Y = 1.0 \times 15.0 = 15$  in. Therefore the four mountings are a distance  $D_X = 15$  in. in the X direction from the center of gravity of the equipment. For this mounting location the chart gives the value  $f_{\min}/f_{tr} = 0.82$ . With  $f_{tr}$  already computed, the values of  $f_{\max}$  and  $f_{\min}$ , the two rocking modes in the XZ plane, may be computed as follows:

$$\frac{f_{\min}}{f_{tr}} = \frac{f_{\min}}{15.0} = 0.82, \quad f_{\min} = 12.3 \text{ cps}$$

$$\frac{f_{\max}}{f_{\min}} = \frac{f_{\max}}{12.3} = 1.5, \quad f_{\max} = 18.4 \text{ cps}$$

The rotational frequency with respect to the Z axis is

$$f_{\text{rot}} = \frac{6.26}{r_Z} \sqrt{\frac{k}{W} (D_X^2 + D_Y^2)} = \frac{6.26}{21.9} \sqrt{\frac{43,000}{7500} [(15)^2 + (33.8)^2]} = 25.2 \text{ cps}$$

Some of these frequencies are close to the 24 cps excited by ship hull vibration. By moving the points of attachment for the same mountings closer together, the natural frequencies of the mounted equipment may be reduced to more satisfactory values with respect to the exciting frequencies. In the YZ plane,  $D_Z/r_X = 0.284$  and if a point on the Base Mounting Chart  $D_Y/r_X = 0.88$  be chosen for the location of the mounting, then  $f_{\max}/f_{\min} = 1.4$  and  $f_{\min}/f_{tr} = 0.8$ . For this chart point, the mounting would be a distance in the Y direction  $D_Y = 0.88 r_X = 18.6$  in. from the center of gravity. The two rocking frequencies are  $f_{\min} = 12.0$  cps and  $f_{\max} = 16.8$  cps. In the XZ plane  $D_Z/r_Y = 0.4$ . Choose a point on the chart such that  $D_X/r_Y = 0.67$ , then  $f_{\max}/f_{\min} = 1.9$  and  $f_{\min}/f_{tr} = 0.6$ . For this point, the mounting would be a distance in the X direction,  $D_X = 0.67 r_Y = 10$  in., from the center of gravity. The two rocking frequencies are 9.0 cps and 17.1 cps and the rotational frequency

with respect to the Z axis is

$$f_{rot} = \frac{6.26}{r_Z} \sqrt{\frac{k}{W} (D_X^2 + D_Y^2)} = \frac{6.26}{21.9} \sqrt{\frac{43,000}{7500} [(10)^2 + (18.6)^2]} = 14.5 \text{ cps}$$

Problem 7 is written in tabular form, including calculations for the two sets of mounting locations, in Appendix 7. There, too, clearances around the equipment are calculated.

### A2.8. PROBLEM 8 - BOTTOM MOUNTING ARRANGEMENT TWO PLANES OF VIBRATIONAL SYMMETRY FOUR COPLANAR DIFFERENT MOUNTINGS

The unit is a rectangular nonhomogeneous body and weighs 800 lb; see Figure 32. The center of gravity is 7.5 in. from the bottom of the unit, 14 in. from one end, and midway between the sides. The X, Y, and Z axes coincide with the principal axes of inertia. The dimensions, moments of inertia, and radii of gyration of the body are

$$l = 50 \text{ in.} \quad I_X = 174 \text{ lb-in.-sec}^2 \quad r_X = 9.17 \text{ in.}$$

$$w = 20 \text{ in.} \quad I_Y = 108 \text{ lb-in.-sec}^2 \quad r_Y = 7.22 \text{ in.}$$

$$h = 15 \text{ in.} \quad I_Z = 213 \text{ lb-in.-sec}^2 \quad r_Z = 10.1 \text{ in.}$$

Since the mountings are not geometrically symmetrical with respect to the XZ plane, an equivalent  $D_Y$  is needed in order to use the Base Mounting Chart, Figure 20. The equivalent  $D_Y = \sqrt{\sum k Y^2 / \sum k} = \sqrt{(Y_{Left})(Y_{Right})}$  for four mountings is  $D_Y = \sqrt{(|Y_{Left}|)(|Y_{Right}|)} = \sqrt{(|Y_1|)(|Y_2|)}$ . Because of the unequal distances of the mountings from the center of gravity in the Y direction, the two pairs of mountings must differ in stiffness such that  $k_1/k_2 = |Y_2|/|Y_1|$  so that a translational mode parallel to the Z axis and a rotational mode about the Z axis exist.

$$\frac{k_1}{k_2} = \frac{|Y_2|}{|Y_1|} = \frac{3}{1}, \quad k_1 = 3k_2$$

resulting in two 300-lb and two 100-lb mountings, with  $k_a = k_r$ , located as shown in Figure 34. The dynamic stiffnesses of the mountings are, with natural frequencies of 15 cps for all mountings,

$$k_1 = \frac{W f^2}{9.8} = \frac{300(15)^2}{9.8} = 6900 \text{ lb/in.}$$

$$k_2 = \frac{W f^2}{9.8} = \frac{100(15)^2}{9.8} = 2300 \text{ lb/in.}$$

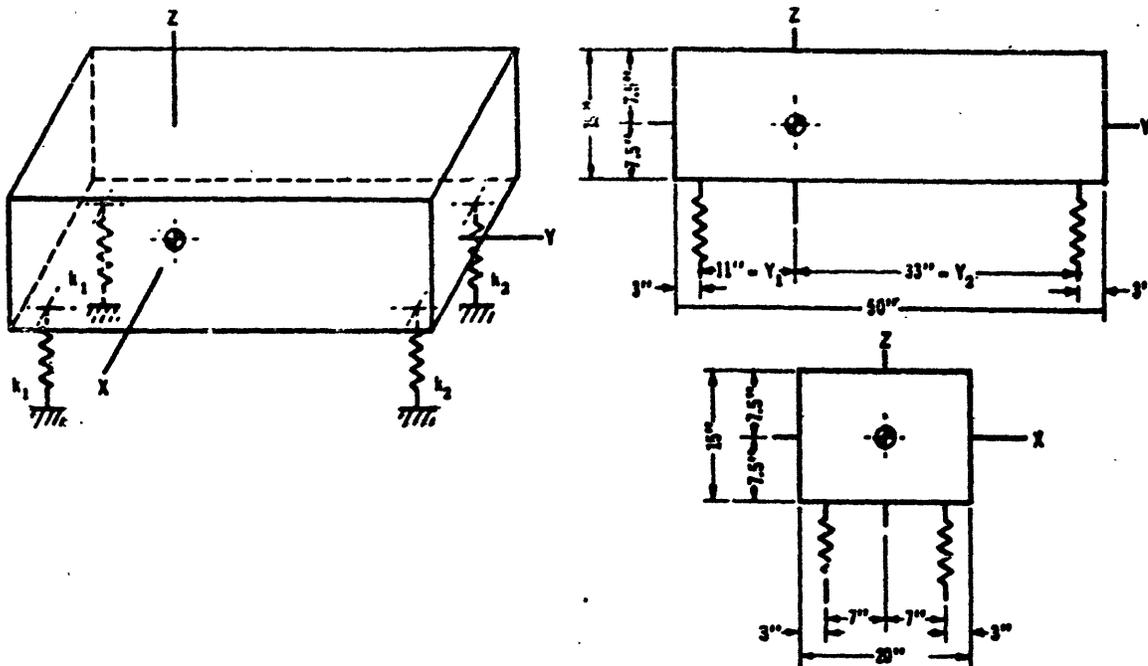


Figure 32 - Problem 8 - Two Planes of Vibrational Symmetry

Then from Figure 34,

$$D_Y = \sqrt{(|Y_1|)(|Y_2|)} = \sqrt{(11)(33)} = 19.1 \text{ in.}$$

Now the natural frequencies may be calculated. The vertical natural frequency  $f_{tr}$  is dependent on the stiffnesses

$$f_{tr} = 3.13 \sqrt{\frac{1}{W} \sum k} = 3.13 \sqrt{\frac{6900 \times 2 + 2300 \times 2}{800}} = 15 \text{ cps}$$

The rocking frequencies in the YZ plane are then obtained. First, converting to nondimensional units,

$$\frac{l}{r_X} = \frac{22.5}{9.17} = 2.45, \quad \frac{D_Y}{r_X} = \frac{19.1}{9.17} = 2.08, \quad \frac{D_Z}{r_X} = \frac{7.5}{9.17} = 0.82$$

Draw the elevation in the YZ plane on the Base Mounting Chart, Figure 20, in these nondimensional units and for the mounting location read off the frequency ratios:

$$\frac{f_{\min}}{f_{tr}} = \frac{f_{\min}}{15} = 0.92, \quad f_{\min} = 13.8 \text{ cps}$$

$$\frac{f_{\max}}{f_{\min}} = \frac{f_{\max}}{13.8} = 2.48, \quad f_{\max} = 34.2 \text{ cps}$$

Similarly for the XZ plane

$$\frac{W}{r_Y} = \frac{10}{7.22} = 1.39, \quad \frac{D_X}{r_Y} = \frac{7}{7.22} = 0.97, \quad \frac{D_Z}{r_Y} = \frac{7.5}{7.22} = 1.04$$

$$\frac{f_{\min}}{f_{tr}} = \frac{f_{\min}}{15.0} = 0.59, \quad f_{\min} = 8.9 \text{ cps}$$

$$\frac{f_{\max}}{f_{\min}} = \frac{f_{\max}}{8.9} = 2.79, \quad f_{\max} = 24.7 \text{ cps}$$

The rotational frequency about the Z axis is then obtained

$$f_{rot,Z} = \frac{3.13}{r_Z} \sqrt{\frac{1}{W} \sum (k X^2 + k Y^2)} = \frac{3.13}{10.1} \sqrt{\frac{1}{800} [6900(49 + 121) + 2300(49 + 1089)]} = 30.2 \text{ cps}$$

The natural frequencies are:

Translation, Z direction	$f_{tr} = 15.0 \text{ cps}$
Rocking, in the YZ plane	$f_{\min} = 13.8 \text{ cps}, f_{\max} = 34.2 \text{ cps}$
Rocking, in the XZ plane	$f_{\min} = 8.9 \text{ cps}, f_{\max} = 24.7 \text{ cps}$
Rotation, about the Z axis	$f_{rot} = 30.0 \text{ cps}$

#### A2.9. PROBLEM 9 - BOTTOM MOUNTING ARRANGEMENT TWO PLANES OF VIBRATIONAL SYMMETRY FOUR NONCOPLANAR IDENTICAL MOUNTINGS

The unit is a rectangular body with a stepped bottom and weighs 3000 lb; see Figure 83. The center of gravity is 9.in. from the bottom of the body and midway between its sides.

The  $X$ ,  $Y$ , and  $Z$  axes coincide with the principal axes of inertia. The dimensions, moments of inertia, and radii of gyration are

$$\begin{aligned}
 l &= 48 \text{ in.} & I_X &= 1700 \text{ lb-in.-sec}^2 & r_X &= 14.7 \text{ in.} \\
 w &= 30 \text{ in.} & I_Y &= 792 \text{ lb-in.-sec}^2 & r_Y &= 10.1 \text{ in.} \\
 h &= 18 \text{ in.} & I_Z &= 2070 \text{ lb-in.-sec}^2 & r_Z &= 16.3 \text{ in.}
 \end{aligned}$$

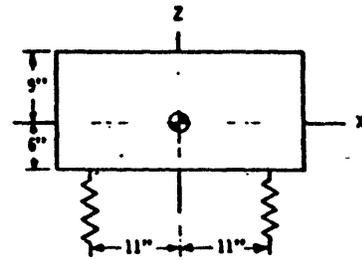
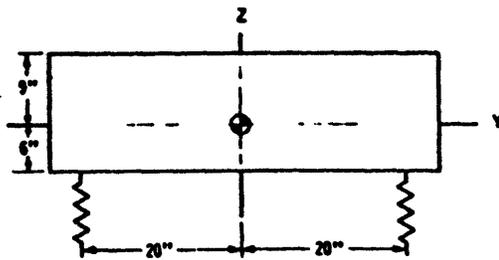
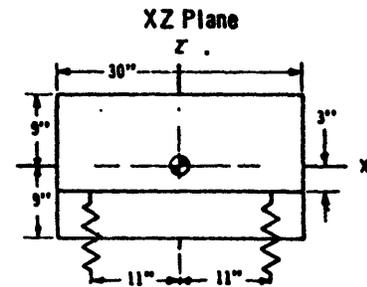
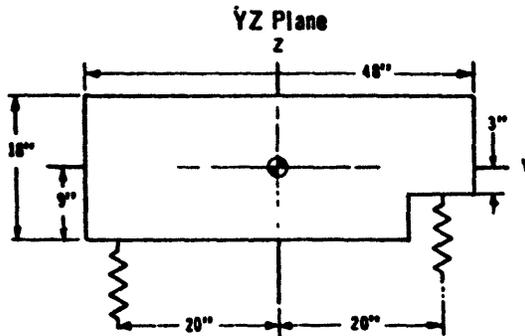
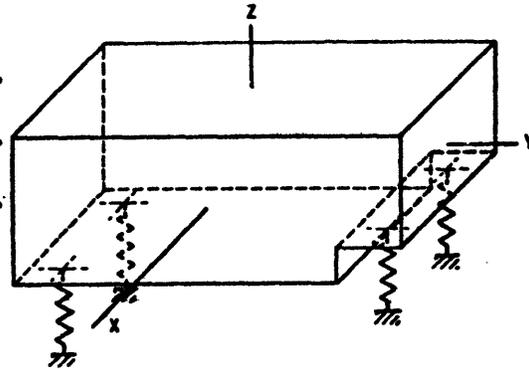


Figure 33 - Problem 9 - Two Planes of Vibrational Symmetry

Since the mountings are not in the same plane, an equivalent  $D_Z$  is needed to use the Base Mounting Chart, Figure 20. The equivalent  $D_Z = (Z_{\text{Upper}} + Z_{\text{Lower}})/2$  approximately. With the use of identical mountings in the arrangement, the remainder of the solution is similar to that for previous problems.

The dynamic stiffness of each of the four 800-lb mountings selected, assuming that 15 cps is the natural frequency of the mounting with a dead load of 750 lb, is

$$k = \frac{Wf^2}{9.8} = \frac{750(15)^2}{9.8} = 17,220 \text{ lb/in.}$$

Now the natural frequencies may be calculated. The vertical natural frequency  $f_{tr}$  is

$$f_{tr} = 3.13 \sqrt{\frac{1}{W} \sum k_z} = 3.13 \sqrt{\frac{4 \times 17,220}{3000}} = 15.0 \text{ cps}$$

The rocking frequencies in the YZ plane are then obtained. First, converting to nondimensional units,

$$\frac{l}{r_x} = \frac{24}{11.7} = 1.63, \quad \frac{D_y}{r_x} = \frac{20}{14.7} = 1.36, \quad \frac{D_z}{r_x} = \frac{9.0+3.0}{14.7} = 0.41$$

From the Base Mounting Chart, Figure 20,

$$\frac{f_{min}}{f_{tr}} = \frac{f_{min}}{15} = 0.92, \quad f_{min} = 13.8 \text{ cps}$$

$$\frac{f_{max}}{f_{min}} = \frac{f_{max}}{13.8} = 1.58, \quad f_{max} = 21.8 \text{ cps}$$

The rocking frequencies in the XZ plane are obtained next. Converting to nondimensional units,

$$\frac{w}{r_y} = 1.49, \quad \frac{D_x}{r_y} = 1.09, \quad \frac{D_z}{r_y} = 0.59$$

$$\frac{f_{min}}{f_{tr}} = \frac{f_{min}}{15.0} = 0.79, \quad f_{min} = 11.9 \text{ cps}$$

$$\frac{f_{max}}{f_{min}} = \frac{f_{max}}{11.9} = 1.8, \quad f_{max} = 21.4 \text{ cps}$$

Finally, the rotational frequency with respect to the Z axis is obtained:

$$f_{rot} = \frac{3.13}{r_z} \sqrt{\frac{1}{W} (\sum k_y X^2 + \sum k_x Y^2)} = \frac{3.13}{16.3} \sqrt{\frac{1}{3000} [17,220 (4 \times 121 + 4 \times 400)]} = 21.0 \text{ cps}$$

The natural frequencies are:

Translation, Z direction	$f_{tr} = 15.0 \text{ cps}$
Rocking, in the YZ plane	$f_{min} = 13.8 \text{ cps}, f_{max} = 21.8 \text{ cps}$
Rocking, in the XZ plane	$f_{min} = 11.9 \quad f_{max} = 21.4 \text{ cps}$
Rotation, about the Z axis	$f_{rot} = 21.0 \text{ cps}$

**A2.10. PROBLEM 10 - BOTTOM MOUNTING ARRANGEMENT  
TWO PLANES OF VIBRATIONAL SYMMETRY  
FOUR NONCOPLANAR DIFFERENT MOUNTINGS**

The unit is a rectangular body with a stepped bottom and weighs 2500 lb; see Figure 84. The center of gravity is 16 in. from the left end and 9.2 in. above the bottom of the body. The X, Y, and Z axes coincide with the principal axes of inertia. The dimensions, moments of inertia, and radii of gyration are

$$l = 48 \text{ in.} \quad I_X = 842 \text{ lb-in.-sec}^2 \quad r_X = 11.4 \text{ in.}$$

$$w = 32 \text{ in.} \quad I_Y = 673 \text{ lb-in.-sec}^2 \quad r_Y = 10.4 \text{ in.}$$

$$h = 20 \text{ in.} \quad I_Z = 1100 \text{ lb-in.-sec}^2 \quad r_Z = 13.0 \text{ in.}$$

This problem is similar to Problem 10, both have a stepped bottom. In Problem 10, the four identical mountings were located symmetrically to the YZ and XZ planes and were equally loaded; in this problem the nonidentical mountings are symmetrical to the YZ plane but not to the XZ plane. They are arranged so that  $|Y_2| = 2|Y_1|$  along the Y axis, or

$$\frac{k_2}{k_1} = \left| \frac{Y_1}{Y_2} \right|$$

using two 800-lb and two 400-lb mountings with equal axial and radial stiffnesses. Effective  $D_Y$  and  $D_Z$  can be computed, and the rocking frequencies can be obtained using the Base Mounting Chart, Figure 20.

$$D_Y = \sqrt{|Y_{\text{Left}}| |Y_{\text{Right}}|} = \sqrt{|Y_1| |Y_2|} = \sqrt{11.4 \cdot 22.8} = 16.1 \text{ in.}$$

where the 800-lb mountings are 11.4 in. and the 400-lb mountings are 22.8 in. from the center of gravity in the Y direction. The individual mountings have a natural frequency of 15 cps with dead loads of 833 and 417 lb for the 800-lb and 400-lb mountings, respectively

$$k_{800} = \frac{Wf^2}{9.8} = \frac{833(15)^2}{9.8} = 19,125 \text{ lb/in.}$$

$$k_{400} = \frac{Wf^2}{9.8} = \frac{417(15)^2}{9.8} = 9514 \text{ lb/in.}$$

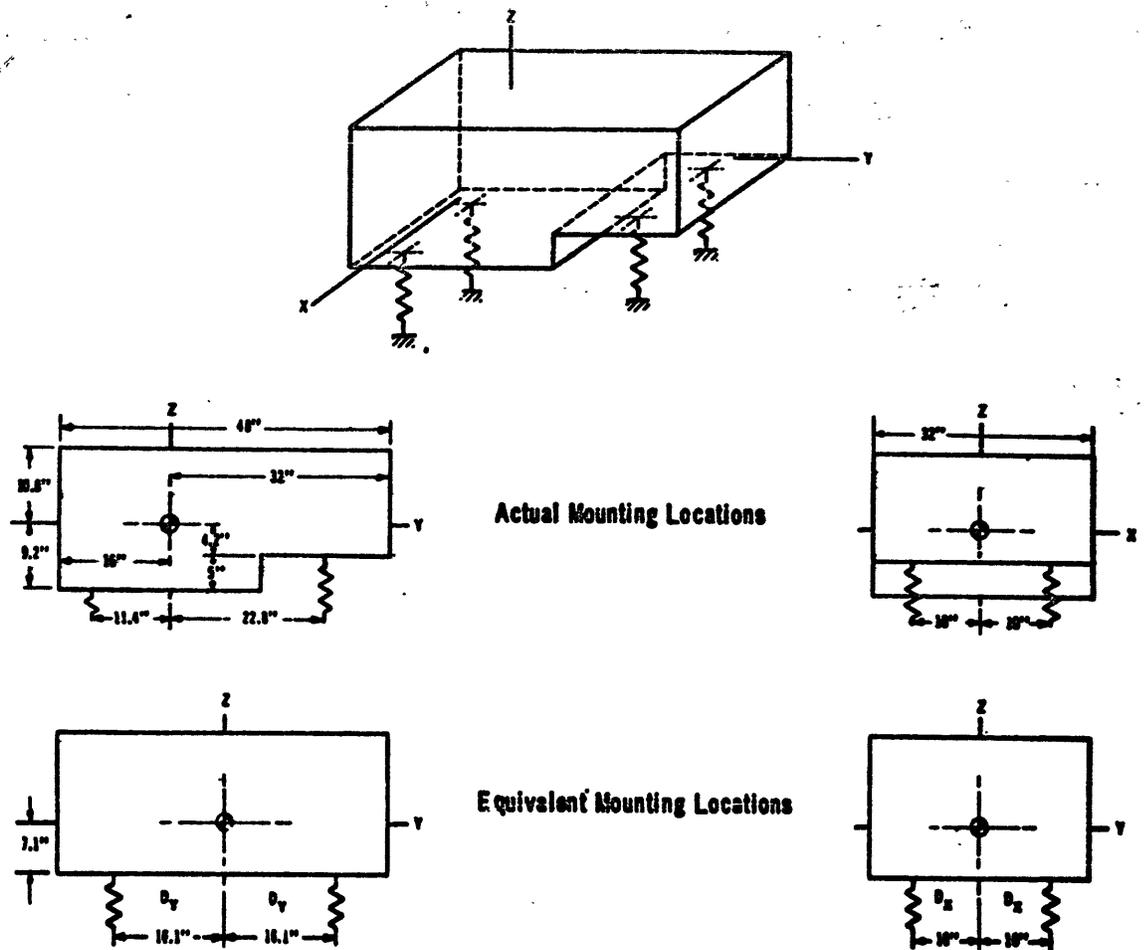


Figure 84 - Problem 10 - Two Planes of Vibrational Symmetry

$$D_Z = \frac{Z_{\text{Upper}} + Z_{\text{Lower}}}{2} = \frac{9.2 + 5}{2} = 7.1 \text{ in.}$$

Translational frequency:

$$f_{tr} = 3.13 \sqrt{\frac{1}{W} \sum k_z} = 3.13 \sqrt{\frac{1}{2500} (2 \times 19,125 + 2 \times 9574)} = 15.0 \text{ cps}$$

The rocking frequencies in the YZ plane, converting dimensions to nondimensional units by

$$\frac{l}{r_X} = \frac{22.6}{11.4} = 1.98, \quad \frac{D_Y}{r_X} = \frac{16.1}{11.4} = 1.41, \quad \frac{D_Z}{r_X} = \frac{7.1}{11.4} = 0.62$$

are, from the chart,

$$\frac{f_{\min}}{f_{tr}} = \frac{f_{\min}}{14.7} = 0.85, \quad f_{\min} = 12.5 \text{ cps}$$

$$\frac{f_{\max}}{f_{\min}} = \frac{f_{\max}}{12.8} = 1.85, \quad f_{\max} = 23.7 \text{ cps}$$

The rocking frequencies in the XZ plane, converting dimensions to nondimensional units by

$$\frac{r}{r_y} = \frac{16}{10.2} = 1.57, \quad \frac{D_X}{r_y} = \frac{10}{10.2} = 0.98, \quad \frac{D_Z}{r_y} = \frac{7.1}{10.2} = 0.70,$$

are, from the chart,

$$\frac{f_{\min}}{f_{tr}} = \frac{f_{\min}}{14.7} = 0.71, \quad f_{\min} = 10.4 \text{ cps}$$

$$\frac{f_{\max}}{f_{\min}} = \frac{f_{\max}}{10.4} = 2.00, \quad f_{\max} = 20.8 \text{ cps}$$

Finally the rotational frequency with respect to the Z axis is obtained,

$$f_{\text{rot}} = \frac{3.13}{r_z} \sqrt{\frac{1}{W} (\sum k_y X^2 + \sum k_x Y^2)} = \frac{3.13}{13.0} \sqrt{\frac{1}{2500} [2 \times 19,125 [(10)^2 + (11.4)^2] + 2 \times 9574 [(10)^2 + (22.8)^2]}]}$$

$$= 21.3 \text{ cps}$$

The natural frequencies are

Translation, Z direction	$f_{tr} = 14.7 \text{ cps}$
Rocking, in the YZ plane	$f_{\min} = 12.5 \text{ cps}, f_{\max} = 23.7 \text{ cps}$
Rocking, in the XZ plane	$f_{\min} = 10.4 \text{ cps}, f_{\max} = 20.8 \text{ cps}$
Rotation, about the Z axis	$f_{\text{rot}} = 21.3 \text{ cps}$

#### A211. PROBLEM 11 - BRACED MOUNTING ARRANGEMENT - ONE PLANE OF VIBRATIONAL SYMMETRY - FOUR IDENTICAL BOTTOM MOUNTINGS TWO IDENTICAL BACK MOUNTINGS

The unit is a trim pump and motor with vertical in-line shafts and weighs 2035 lb; see Figure 35. The center of gravity is 40 in. above the bottom and is on the vertical centerline of the unit. The X, Y, and Z axes coincide with the principal axes of inertia. The dimensions and moments of inertia are

$$l = 22 \text{ in.} \quad I_X = 2150 \text{ lb-in.-sec}^2$$

$$w = 22 \text{ in.} \quad I_Y = 2150 \text{ lb-in.-sec}^2$$

$$h = 84\frac{1}{2} \text{ in.} \quad I_Z = 321 \text{ lb-in.-sec}^2$$

The four 550-lb bottom mountings and the two 50-lb back mountings are initially located as shown in Figure 35.

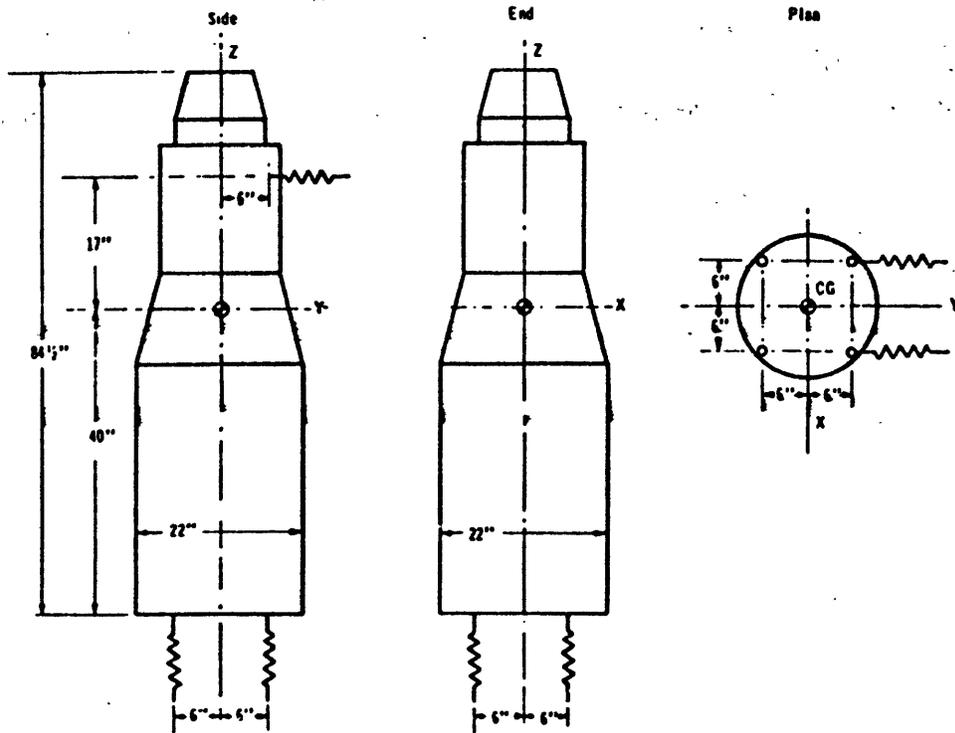


Figure 35 - Problem 11 - One Plane of Vibrational Symmetry

The dynamic stiffness of one 550-lb mounting may be determined, using an assumed value of the average rated frequency in radial and axial directions for the load to be supported, by the formula

$$k = \frac{W f^2}{9.8} = \frac{484 (16.7)^2}{9.8} = 13,860 \text{ lb/in.}$$

and similarly for the 50-lb mountings

$$k = \frac{44 (16.0)^2}{9.8} = 1150 \text{ lb/in.}$$

The values of the loads are found as follows:

Let  $F$  be the load on each back mounting and  $G$  the load on each base mounting. Then  $4G + 2F = 2035$  lb, but each base mounting is rated at 550 lb and each back mounting at 50 lb. Therefore

$$\frac{G}{550} = \frac{F}{50}, \text{ or } G = 11F$$

and

$$44F + 2F = 46F = 2035, \quad F = 44 \text{ lb, and } G = 484 \text{ lb.}$$

The problem may be solved in two general steps; the first, solving for  $a$ ,  $b$ , and  $B$  in terms of the coefficients of the two cubic equations  $B_1, C_1, D_1$  and  $B_2, C_2, D_2$ , and the second, taking off the frequencies from the plot of the cubic equations in terms of parameters  $a, b, B$ ; Figure 23.

Solve for the three natural frequencies in the  $YZ$  plane of vibrational symmetry by determining

$$a = \frac{C_1}{B_1^2}, \quad b = \frac{D_1}{B_1^3}, \quad B = B_1$$

where

$$B_1 = p_2 + p_3 + q_1$$

$$C_1 = p_2 p_3 + p_2 q_1 + p_3 q_1 - \frac{95.6}{W^2 r_X^2} \left[ (\Sigma k_Z Y)^2 + (\Sigma k_Y Z)^2 \right]$$

$$D_1 = p_2 p_3 q_1 - \frac{95.6}{W^2 r_X^2} \left[ p_2 (\Sigma k_Z Y)^2 + p_3 (\Sigma k_Y Z)^2 \right]$$

and

$$p_1 = \frac{9.78}{W} \Sigma k_X, \quad p_2 = \frac{9.78}{W} \Sigma k_Y, \quad p_3 = \frac{9.78}{W} \Sigma k_Z$$

$$q_1 = \frac{9.78}{W r_X^2} (\Sigma k_Z Y^2 + \Sigma k_Y Z^2), \quad q_2 = \frac{9.78}{W r_Y^2} (\Sigma k_Z X^2 + \Sigma k_X Z^2),$$

$$q_3 = \frac{9.78}{W r_Z^2} (\Sigma k_Z X^2 + \Sigma k_X Y^2)$$

It may be seen that in these equations only the constant  $W$ , the spring constants  $k_X, k_Y, k_Z$ , and the positions of the mountings are known. The expressions  $r_X^2, r_Y^2, r_Z^2, (\Sigma k_Z Y)^2, (\Sigma k_Y Z)^2, \Sigma k_X, \Sigma k_Y, \Sigma k_Z, \Sigma k_Z Y^2, \Sigma k_Y Z^2, \Sigma k_X X^2, \Sigma k_X Z^2$  and  $\Sigma k_X Y^2$  must be evaluated.

Since each mounting has equal stiffness in all directions,

$$\Sigma k_X = \Sigma k_Y = \Sigma k_Z, \quad \Sigma k_Z Y^2 = \Sigma k_X Y^2, \quad \text{and} \quad \Sigma k_Y Z^2 = \Sigma k_X Z^2$$

To determine  $r_X, r_Y$ , and  $r_Z$ :

$$l = m r^2$$

$$r_X^2 = \frac{l_X}{m} = \frac{2150}{5.272} = 407.8 \text{ in.}^2$$

$$r_Y^2 = \frac{l_Y}{m} = \frac{2150}{5.272} = 407.8 \text{ in.}^2$$

$$r_Z^2 = \frac{l_Z}{m} = \frac{321}{5.272} = 60.89 \text{ in.}^2$$

and to determine the summations:

$$\sum k_X = \sum k_Y = \sum k_Z = (4 \times 13,600 + 2 \times 1150) = 5.670 \times 10^4$$

$$k_1 Y_1 = -8.160 \times 10^4$$

$$k_1 Y_1^2 = k_1 X_1^2 = 0.4896 \times 10^6$$

$$k_2 Y_2 = 8.160 \times 10^4$$

$$k_2 Y_2^2 = k_2 X_2^2 = 0.4896 \times 10^6$$

$$k_3 Y_3 = 8.160 \times 10^4$$

$$k_3 Y_3^2 = k_3 X_3^2 = 0.4896 \times 10^6$$

$$k_4 Y_4 = -8.160 \times 10^4$$

$$k_4 Y_4^2 = k_4 X_4^2 = 0.4896 \times 10^6$$

$$k_5 Y_5 = 0.690 \times 10^4$$

$$k_5 Y_5^2 = k_5 X_5^2 = 0.0414 \times 10^6$$

$$k_6 Y_6 = 0.690 \times 10^4$$

$$k_6 Y_6^2 = k_6 X_6^2 = 0.0414 \times 10^6$$

$\sum k Y = 1.380 \times 10^4$
--------------------------------

$\sum k Y^2 = \sum k X^2 = 2.041 \times 10^6$
---

$(\sum k Y)^2 = 1.904 \times 10^8$
------------------------------------

$$k_1 Z_1 = -5.440 \times 10^5$$

$$k_1 Z_1^2 = 2.176 \times 10^7$$

$$k_2 Z_2 = -5.440 \times 10^5$$

$$k_2 Z_2^2 = 2.176 \times 10^7$$

$$k_3 Z_3 = -5.440 \times 10^5$$

$$k_3 Z_3^2 = 2.176 \times 10^7$$

$$k_4 Z_4 = -5.440 \times 10^5$$

$$k_4 Z_4^2 = 2.176 \times 10^7$$

$$k_5 Z_5 = 0.196 \times 10^5$$

$$k_5 Z_5^2 = 0.033 \times 10^7$$

$$k_6 Z_6 = 0.196 \times 10^5$$

$$k_6 Z_6^2 = 0.033 \times 10^7$$

$\sum k Z = -2.137 \times 10^6$
---------------------------------

$\sum k Z^2 = 8.770 \times 10^7$
----------------------------------

$(\sum k Z)^2 = 4.567 \times 10^{12}$
---------------------------------------

Then

$$p_1 = p_2 = p_3 = \frac{9.78}{W} \sum k_X = \frac{9.78}{2035} \times 5.670 \times 10^4 = 0.2722 \times 10^3$$

and

$$q_1 = \frac{9.78}{W r_X^2} (\sum k_Z Y^2 + \sum k_Y Z^2) = \frac{9.78}{2035 \times 407.8} (2.041 \times 10^6 + 8.770 \times 10^7) = 1.056 \times 10^3$$

$$q_2 = \frac{9.78}{W r_y^2} (\sum k_z X^2 + \sum k_x Z^2) \quad \text{and since} \quad \sum k_z X^2 = \sum k_z Y^2, \quad \sum k_x Z^2 = \sum k_y Z^2,$$

and

$$r_y^2 = r_x^2$$

then

$$q_2 = q_1$$

$$q_3 = \frac{9.78}{W r_z^2} (\sum k_y X^2 + \sum k_x Y^2) = \frac{9.78}{2035 \times 60.89} (2.041 \times 10^6 + 2.041 \times 10^6) = 0.3219 \times 10^3$$

The coefficients  $B_1$ ,  $C_1$ ,  $D_1$  may now be determined.

$$B_1 = p_2 + p_3 + q_1 = 0.2722 \times 10^3 + 0.2722 \times 10^3 + 1.056 \times 10^3 = 1.600 \times 10^3$$

$$C_1 = p_2 p_3 + p_2 q_1 + p_3 q_1 - \frac{95.6}{W^2 r_x^2} [(\sum k_z Y)^2 + (\sum k_y Z)^2]$$

$$= 0.2722 \times 10^3 (0.2722 \times 10^3) + 0.2722 \times 10^3 (1.056 \times 10^3) + 0.2722 \times 10^3 (1.056 \times 10^3)$$

$$- \frac{95.6}{(2035)^2 (407.8)} [1.904 \times 10^8 + 4.567 \times 10^{12}] = 3.904 \times 10^5$$

$$D_1 = p_2 p_3 q_1 - \frac{95.6}{W^2 r_x^2} [p_2 (\sum k_z Y)^2 + p_3 (\sum k_y Z)^2]$$

$$= (0.2722 \times 10^3) (0.2722 \times 10^3) (1.056 \times 10^3) - \frac{95.6}{(2035)^2 (407.8)} [(0.2722 \times 10^3) (1.904 \times 10^8)$$

$$+ (0.2722 \times 10^3) (4.567 \times 10^{12})] = 7.89 \times 10^6$$

Now that  $B_1$ ,  $C_1$ , and  $D_1$  are known, the ratios  $a$ ,  $b$ , and  $B$  may be calculated, and the three natural frequencies for the modes of vibration in the  $YZ$  plane may be taken off the chart, Figure 23.

$$a = \frac{C_1}{B_1^2} = \frac{3.904 \times 10^5}{(1.600 \times 10^3)^2} = 0.153$$

$$b = \frac{D_1}{B_1^3} = \frac{7.89 \times 10^6}{(1.600 \times 10^3)^3} = 0.00193$$

$$B = B_1 = 1.600 \times 10^3$$

From the chart, Figure 23, the natural frequencies are

$$f_1 = 36.0 \text{ cps}, \quad f_2 = 16.5 \text{ cps}, \quad f_3 = 4.7 \text{ cps}.$$

To calculate the natural frequencies of the modes of vibration not in the  $YZ$  plane of vibrational symmetry, a similar procedure is followed:

$$k_1 Y_1 Z_1 = 32.64 \times 10^5$$

$$k_2 Y_2 Z_2 = -32.64 \times 10^5$$

$$k_3 Y_3 Z_3 = -32.64 \times 10^5$$

$$k_4 Y_4 Z_4 = 32.64 \times 10^5$$

$$k_5 Y_5 Z_5 = 1.173 \times 10^5$$

$$k_6 Y_6 Z_6 = 1.173 \times 10^5$$

$$\Sigma k Y Z = 2.346 \times 10^5$$

$$(\Sigma k Y Z)^2 = 5.504 \times 10^{10}$$

$$B_2 = p_1 + q_2 + q_3 = 0.2722 \times 10^3 + 1.056 \times 10^3 + 0.3219 \times 10^3 = 1.650 \times 10^3$$

$$C_2 = p_1 q_2 + p_1 q_3 + q_2 q_3 - \frac{95.6}{W^2} \left[ \frac{1}{r_Z^2} (\Sigma k_X Y)^2 + \frac{1}{r_Y^2} (\Sigma k_X Z)^2 + \frac{1}{r_Y^2 r_Z^2} (\Sigma k_X Y Z)^2 \right]$$

$$= (0.2722 \times 10^3)(1.056 \times 10^3) + (0.2722 \times 10^3)(0.3219 \times 10^3) + (1.056 \times 10^3)(0.3219 \times 10^3)$$

$$- \frac{95.6}{(2035)^2} \left[ \frac{1}{60.89} (1.904 \times 10^8) + \frac{1}{407.8} (4.567 \times 10^{12}) + \frac{1}{487.8 \times 60.89} (5.504 \times 10^{10}) \right]$$

$$= 4.561 \times 10^5$$

$$D_2 = p_1 q_2 q_3 - \frac{95.6}{W^2} \left[ \frac{q_2}{r_Z^2} (\Sigma k_X Y)^2 + \frac{q_3}{r_Z^2} (\Sigma k_X Z)^2 + \frac{p_1}{r_Y^2 r_Z^2} (\Sigma k_X Y Z)^2 \right]$$

$$+ \frac{1870}{W^3 r_Y^2 r_Z^2} (\Sigma k_X Y)(\Sigma k_X Z)(\Sigma k_X Y Z)$$

$$= (0.2722 \times 10^3)(1.056 \times 10^3)(0.3219 \times 10^3) - \frac{95.6}{(2035)^2} \left[ \frac{1.056 \times 10^3}{60.89} (1.904 \times 10^8) \right.$$

$$\left. + \frac{0.3219 \times 10^3}{407.8} (4.567 \times 10^{12}) + \frac{0.2722 \times 10^3}{60.89 \times 407.8} (5.504 \times 10^{10}) \right]$$

$$+ \frac{1870}{(2035)^3 (60.89)(407.8)} (1.380 \times 10^4)(-2.137 \times 10^6)(2.346 \times 10^5) = 9.12 \times 10^6$$

$$a = \frac{C_2}{B_2^2} = \frac{4.561 \times 10^5}{(1.650 \times 10^3)^2} = 0.168$$

$$b = \frac{D_2}{B_2^3} = \frac{9.12 \times 10^6}{(1.650 \times 10^3)^2} = 0.00203$$

$$B = B_2 = 1.65 \times 10^3$$

From the chart, Figure 23, the natural frequencies are

$$f_4 = 35.9 \text{ cps}, \quad f_5 = 18.0 \text{ cps}, \quad f_6 = 4.6 \text{ cps}.$$

Problem 11 is also presented in tabular form, Appendix 7, where calculations of the clearances needed around the equipment are also made.

### APPENDIX 3

## EXPERIMENTAL DETERMINATION OF MOMENT OF INERTIA WITH A TRIFILAR SUSPENSION

The most practical experimental arrangement for determining the moment of inertia is the trifilar suspension<sup>11</sup> where three wires or three cables support a table or platform upon which different units can be placed in various orientations; see Figure 36. In the simplest case the center of gravity of the unit is placed over the center of the platform which is equidistant from the three supporting wires. If the platform suspended from the three supporting wires is symmetrical, its center will also be its center of gravity.

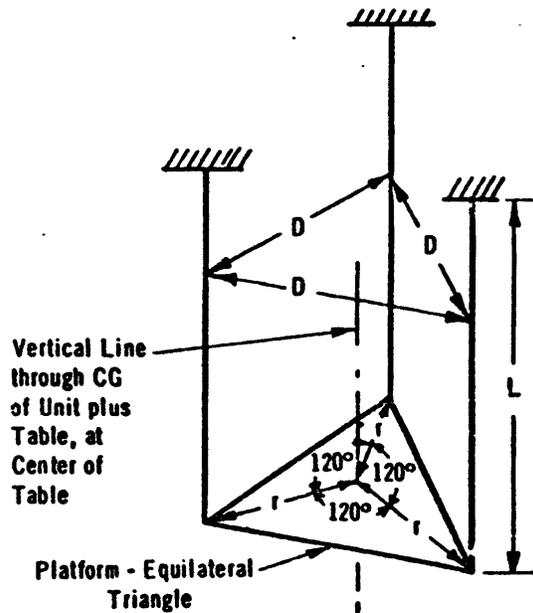


Figure 36 - Trifilar Suspension for Experimentally Determining Moments of Inertia

The equation for the moment of inertia about the vertical axis through the center of gravity of both the unit and the table is

$$I = \frac{Wr^2T^2}{4\pi^2L}$$

where  $W$  is the weight of the body plus the platform, pounds,

$T$  is the period of oscillation, time in seconds from one extremity of rotation to the other and back to the first, i.e., time of one cycle,

$L$  is the length of supporting wires, inches,

$r$  is the distance from center of gravity, i.e., center of platform, to supporting wires, inches, and

$I$  is the mass moment of inertia of the unit plus platform, pound-inch seconds squared.

If the platform is rotated 5 deg or less ( $L$  must be at least three times as large as  $r$ ) and released, the period,  $T$ , may be measured. The accuracy may be increased by measuring the time of several oscillations and dividing by the number of cycles. Once the trifilar suspension platform is built,  $L$  and  $r$  are fixed and the moments of inertia,  $I$ , now becomes .

$$I = KWT^2$$

where

$$K = \frac{r^2}{4\pi^2 L}$$

The moment of inertia of the table alone may be determined in a similar manner without any equipment installed on it. This moment of inertia is then subtracted from that for the unit plus the table to give the moment of inertia of the unit alone. The determination for the table alone need only be made once since it will remain constant. If blocks are needed to support the unit in an appropriate orientation to the table, then the period,  $T$ , should be determined with the table and blocks and this moment of inertia should be subtracted from that of the table, blocks, and unit together.

The principal inertial axes of the equipment may be determined by repeated tests with changes in the angular position of the equipment without a shift of the center of gravity: The position of the equipment resulting in the maximum period establishes the maximum principal axis. The position with minimum period establishes the minimum principal axis. The third axis is at right angles to the other two.

### A3.1. GENERAL CASE

In general when the unit is placed on the platform, vertical lines through the center of gravity of the equipment and through the center of gravity of the platform do not coincide. The table should be so designed that it is symmetrical and its center of gravity is equidistant from the suspension wires; Figure 37.

When the equipment is installed on the platform, the combined center of gravity of the equipment and the platform is determined; Figure 38. Then the distances,  $r_1$ ,  $r_2$ , and  $r_3$ , from the combined center of gravity of the equipment and the table to each of the suspension wires

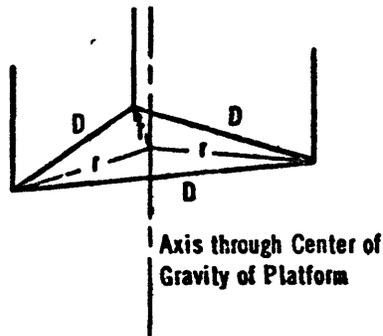


Figure 37 - Location of Center of Gravity of Platform

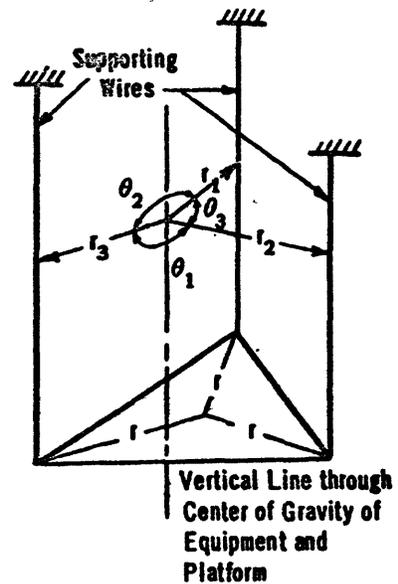


Figure 38 - Location of Center of Gravity of Equipment Plus Platform

may be determined. The equation for the general case of moment of inertia is

$$I = \frac{W r_1 r_2 r_3 T^2}{4\pi^2 L} \left[ \frac{r_1 \sin \theta_1 + r_2 \sin \theta_2 + r_3 \sin \theta_3}{r_2 r_3 \sin \theta_1 + r_1 r_3 \sin \theta_2 + r_1 r_2 \sin \theta_3} \right]$$

where  $W$  is the weight of the body plus platform, pounds,

$T$  is the period of oscillation, seconds,

$L$  is the length of the supporting wires, inches,

$r$  is the distance from the center of gravity to the supporting wire, inches,

$\theta$  is the angle between radial lines from the center of gravity to the supporting wires, degrees, and

$I$  is the mass moment of inertia of the body plus platform, pound-inch-seconds squared.

### A3.2. DESIGN NOTES

In the design of the cable-suspended platform, a number of factors must be considered. The dimensions  $L$  and  $r$  should be such that reasonable differences in period of oscillation can be obtained for the apparatus alone and for the apparatus plus the equipment so that  $I$  may be determined with suitable accuracy. The distance  $r$  from the center of the platform to each of the cables is determined largely by the size of the equipment being tested. This distance should be kept as small as possible, since the  $I$  of the platform alone increases as the square of  $r$ . This increase in  $I$  can be counteracted by relatively larger increases in  $L$ . For the

linear approximation of the analysis to hold when the oscillation is started by an angular displacement of 5 deg or less,  $L$  must be at least three times as large as  $r$ . The  $I$  of the platform for a given  $r$  can be minimized by fabricating it from light members, but the size of the platform components is limited not only by the size of the equipment but also by its density. A pointer attached to the platform is handy for measuring the time of oscillations. The weight of the platform should not exceed the weight of the equipment. The trifilar suspension should be calibrated using objects whose moments of inertia are known to determine accuracy and capacity of the apparatus.<sup>12,13</sup>

For units of approximately the same size and weight, only one trifilar suspension is required. If, however, the sizes and weights vary considerably, two suspensions should suffice, one for the larger units and the other for the smaller.

## APPENDIX 4

### REPORT FORM FOR RESILIENT MOUNTING CHARACTERISTICS

This report form indicates the type of information available on all types and sizes of mountings as a result of tests made at a naval activity. The mountings must pass these tests in order to be acceptable for shipboard use. The reports may be obtained through the Bureau of Ships. In reports published by the U.S. Naval Engineering Experiment Station these sheets are inserted in the back and are intended to be removed and retained by design groups engaged in the solution of mounting problems.

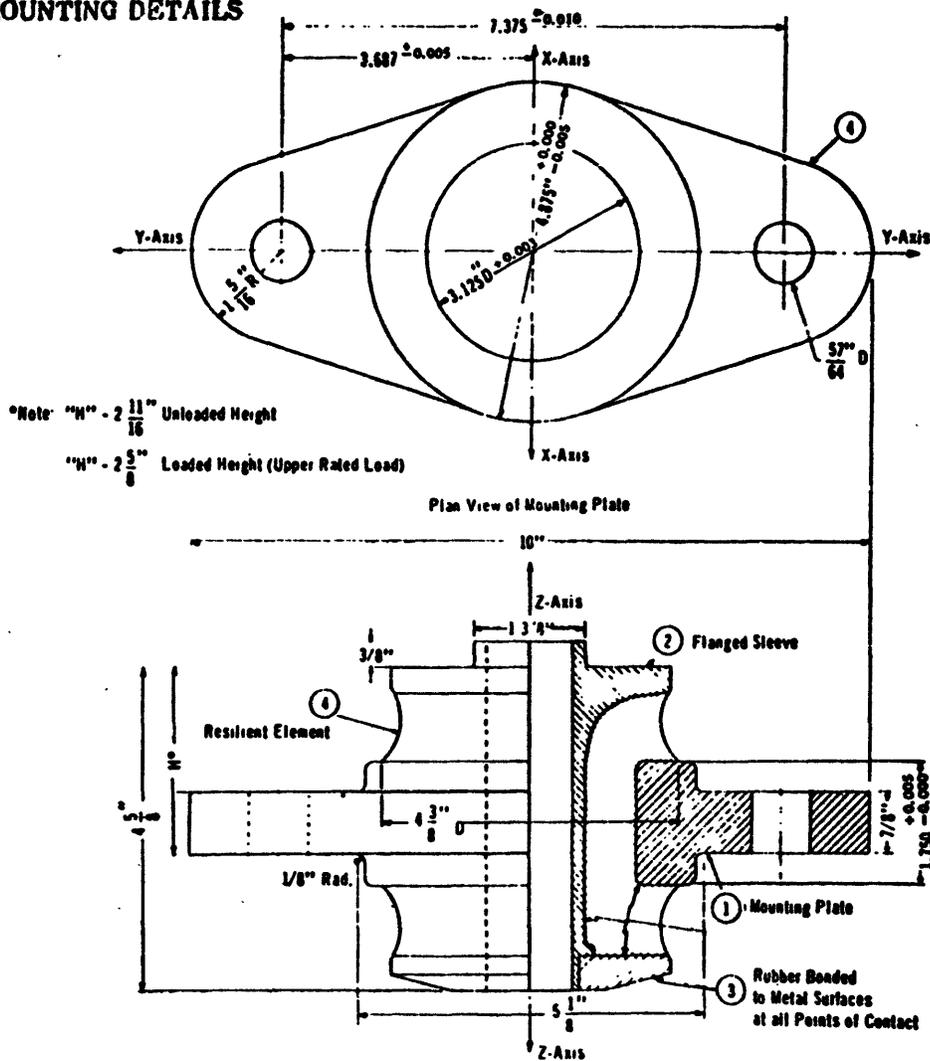
The  $X$ ,  $Y$ , and  $Z$  axes here are local axes applicable to individual mounts, and the use of capital letters does not imply the significance attached to the notation that is used for mounted assemblies in other sections of this manual.

**SUMMARY OF PERFORMANCE DATA ON TESTS OF  
RESILIENT MOUNTINGS**  
(Ready Reference Sheets - Abstract from EES Report 050095E)

**MOUNTING IDENTIFICATION**

1. Type of Mounting: Portsmouth BST No. 1000, Design A
2. Mounting Mfr: Hood Rubber Company, Watertown, Mass.
3. Mounting Dwg No.: (BUSHIPS) 5000-S1112-F-1385777-C
4. Applicable Specification: (Procurement) MIL-M-17191A
5. Standard Navy Stock No.: P17-M-75887-2126
6. Rated Load: 1000 lb
7. Natural Frequency: Z Direction = 13.8 cps  
X and Y Directions = 16.0 cps

**MOUNTING DETAILS**



**SUMMARY OF TEST DATA**  
(Ready Reference Sheets - Abstract from EES Report 050095E)

**1. Vibration:**

Characteristics	Load on Mountings 1,000 lb		Load on Mountings 900 lb		Load on Mountings 800 lb		Load on Mountings 700 lb	
	Z	X and Y	Z	X and Y	Z	X and Y	Z	X and Y
Natural Frequency <i>f<sub>n</sub></i> (cps)	13.6	16.0	14.4	16.8	15.3	17.8	16.1	19.0
Critical Damping, percent	5.2	5.3					4.7	4.6
Maximum Transmissibility	9.7	9.5					10.7	10.9
Spring Constant <i>K<sub>d</sub></i> (lb/in.)	18,900	22,200	19,165	26,000	19,200	25,900	18,600	25,800

**2. Drift:**

Direction	Load lb	Temperature deg F	Deflection After 1 Hour in.	Drift 1 Hour to 200 Hours in.	Natural Frequency - CPS		
					Before Test	3 Minutes After Test	Room Temperature After Test
Z	1,000	160	0.079	0.014	14.2	-	14.2
	1,000	30	0.070	0.014	14.2	-	14.2
X and Y	1,000	160	0.050	0.029	-	-	17.7
	1,000	30	0.050	0.008	17.7	-	17.3

**3. Static Load Deflection:**

Load lb	Deflection - Inch	
	Z	X and Y
700	0.05	0.04
1,000	0.08	0.06
5,000	0.41	0.31
10,000	0.62	0.54
15,000	0.67	0.61
20,000	0.70	0.65

**4. Shock Deflection:**

Direction	Load lb	Deflection - Inch	
		Up	Down
Z	4,000	0.80	0.80
	2,800	-	-
X and Y	4,000	1.00	0.80
	2,800	-	-

**5. Brief Remarks:**

- (a) Inspection: Mountings conformed to drawing details. Neoprene rubber stock Type C used. Mountings manufactured in June 1950 and June 1952. Tests completed May 1953.
- (b) Test Specification: MIL-M-17185 (SHR 5).
- (c) Transmissibility: No remarks.
- (d) Uniformity: *f<sub>n</sub>* varied less than 0.5 cps from average of 4 mountings tested.
- (e) Static Load Deflection: No remarks.
- (f) Noise Isolation: Satisfactory. See EES Report 050095F.
- (g) Shock Damage: There was no damage observed to the rubber or metal parts or to the load bolts (1-1/4 in. 12NF-3 of Alloy No. 2) or foundation bolts (7/8 in. 9NC-2 of mild steel).
- (h) Salt Spray: No damage or significant change.
- (i) Oil Immersion: No damage or significant change.
- (j) Drift: No significant change in *f<sub>n</sub>* as a result of these tests.
- (k) Cold Storage: No damage or significant change.

## APPENDIX 5

### DERIVATION OF EQUATIONS OF MOTION AND FREQUENCY FORMULAS FOR A RESILIENTLY MOUNTED RIGID ASSEMBLY

#### A5.1. BASIC THEORY

For generality there is given here the analysis with respect to an arbitrarily oriented set of axes, which presumably would be chosen with reference to some compartment on the ship but which need not line up either with the axes of the mountings or with the principal axes of inertia of the mounted assembly. This procedure may require the evaluation of both moments and products of inertia. It will be assumed that all mountings have at least one axis of elastic symmetry, by which is meant that the spring constant of the mount is the same in any direction normal to this axis.

The analysis is based on linear theory and hence can be considered valid only for small motions. Under large shock motions the mountings may be displaced into their nonlinear ranges. Since one of the main objects in design is to avoid steady-state resonance under excitations of known frequency, the linear theory is useful in predicting whether large vibrations are likely to build up.

##### A5.1.1. THE EQUATIONS OF MOTION

Let a right-hand system of fixed axes be taken with the origin at the center of mass of the mounted assembly when the system is in its rest position. Let  $u$ ,  $v$ , and  $w$  be displacements in the  $x$ ,  $y$ , and  $z$  directions, respectively, of the center of mass, and  $\alpha$ ,  $\beta$ , and  $\gamma$  the components of angular displacement about the  $x$ ,  $y$ , and  $z$  axes, respectively; see Figure 39.

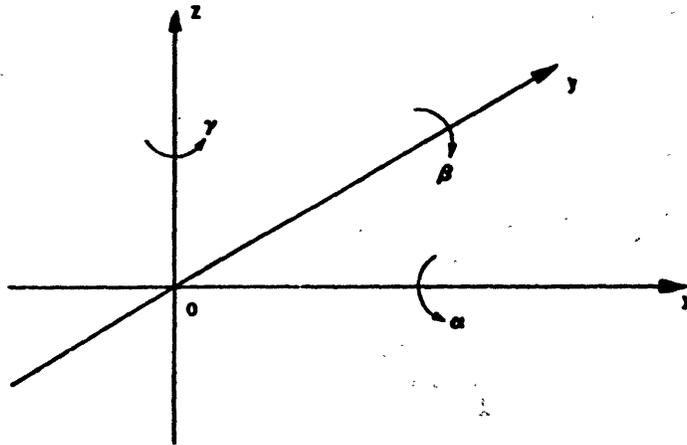


Figure 39 - Right-Hand Coordinate System Used in the Analysis  
with Arbitrarily Oriented Axes

With the restriction that the motions remain small, the dynamical equations are

$$\Sigma F_x = m\ddot{u}$$

$$\Sigma F_y = m\ddot{v}$$

$$\Sigma F_z = m\ddot{w}$$

$$\Sigma M_x = I_x \ddot{\alpha} - I_{xy} \ddot{\beta} - I_{xz} \ddot{\gamma}$$

$$\Sigma M_y = I_y \ddot{\beta} - I_{yz} \ddot{\gamma} - I_{xy} \ddot{\alpha}$$

$$\Sigma M_z = I_z \ddot{\gamma} - I_{xz} \ddot{\alpha} - I_{yz} \ddot{\beta}$$

where  $I_x$ ,  $I_y$ , and  $I_z$  are moments of inertia and  $I_{xy}$ ,  $I_{xz}$ , and  $I_{yz}$  are products of inertia with respect to the fixed axes when the body is in the rest position. The restriction to small motions permits treating the  $I$ 's as constants. The  $F$ 's and  $M$ 's are sums of the forces and moments acting on the body due to the elastic distortions within the individual mountings and can be expressed by equations of the type  $F = -Ku$  where  $K$  is an elastic constant of the entire set of mounts.

#### A5.1.2. CALCULATION OF THE ELASTIC CONSTANTS

The elastic constants of the individual mountings must first be determined. The "effective point of attachment" of the body to any mount will be assumed to be a point about which a rotation of the axis of the movable element of the mounting evokes only a torque, and this torque will be assumed to be negligible in comparison with the moments on the mounted assembly resulting from the rectilinear displacement of the effective points of attachment. Thus only the axial and radial stiffnesses of the individual mountings have to be taken into account in the analysis. It is to be noted that a mounting can have radial elastic symmetry with respect to a certain axis without necessarily having polar symmetry in its geometrical construction.

The elastic constants that must be calculated for the entire set of mountings are designated here by a  $K$  labeled with two subscripts, e.g.,  $K_{x\alpha}$  and each constant represents a restoring action in the direction of or about one coordinate axis due to a displacement in the direction of or about the same or another axis. The sign convention used conforms with the usual convention applied to the simple system of one degree of freedom, according to which, if a displacement in the positive direction results in a force in the negative direction, the spring constant is taken as positive. Exact definitions are given in the Notation.

It can be shown that in such an elastic system conservation of energy requires that  $K_{ij} = K_{ji}$  so that the total number of  $K$ 's required is only the number of possible combinations of the six coordinates defining the displacement of the mounted body taken two at a time, plus the six of the form  $K_{ii}$ . This gives 15 + 6 or 21  $K$ 's to be evaluated.

Let  $x$ ,  $y$ , and  $z$  be the coordinates of the effective point of attachment of an individual mounting,  $k_a$  and  $k_r$  its axial and radial spring constants, respectively, and  $\phi_x$ ,  $\phi_y$ , and  $\phi_z$  the direction angles which its axis makes with the  $x$ ,  $y$ , and  $z$  axes, respectively. For the individual mountings there are only six elastic constants to be evaluated since local torques generated by the displacement of a mounting and forces due to its rotation are considered negligible. These constants are denoted by  $k_{xx}$ ,  $k_{yy}$ ,  $k_{zz}$ ,  $k_{xy}$ ,  $k_{yz}$ , and  $k_{xz}$ . Exact definitions are given in the Notation.

In Figure 40 the axis of the movable element of an inclined mounting is shown by the line  $O_1P$  making direction angles  $\phi_x$ ,  $\phi_y$ , and  $\phi_z$  with axes  $x_1$ ,  $y_1$ , and  $z_1$  parallel to the  $x$ ,  $y$ , and  $z$  axes, respectively. The effective point of attachment is assumed to be  $P$  and through this point is passed a plane normal to  $O_1P$  intersecting the  $x_1$  axis at  $Q$  and the  $y_1$  axis at  $R$ .

If  $P$  is displaced a small distance  $u_1$  in the positive  $x_1$  direction, there will be developed a restoring force whose axial and radial components are indicated by  $F_a$  and  $F_r$  in Figure 40. In this case  $F_a$  is (algebraically) equal to  $u_1 k_a \cos \phi_x$  and  $F_r$  to  $u_1 k_r \sin \phi_x$ . The sum of the  $x$  components of the restoring force is thus

$$-u_1 k_a \cos^2 \phi_x - u_1 k_r \sin^2 \phi_x$$

By the definition of  $k_{xx}$  this sum equals  $-u_1 k_{xx}$ .

Hence

$$k_{xx} = k_a \cos^2 \phi_x + k_r \sin^2 \phi_x$$

Similarly

$$k_{yy} = k_a \cos^2 \phi_y + k_r \sin^2 \phi_y$$

$$k_{zz} = k_a \cos^2 \phi_z + k_r \sin^2 \phi_z$$

To obtain the constant  $k_{xy}$  the components of  $F_a$  and  $F_r$  in the  $y$  direction must be found. The former is  $F_a \cos \phi_y = u_1 k_a \cos \phi_x \cos \phi_y$ . To obtain the  $y$  component of  $F_r$ , account must be taken of the fact that  $F_r$  lies in the plane determined by  $O_1P$  and  $O_1x_1$  so that its line of action coincides with  $QP$ .  $F_r$  may then be considered as the resultant of two vectors  $F_{OQ}$  and  $F_{OP}$  (shown dotted in Figure 40) in the  $PO_1Q$  plane, one parallel to  $O_1x_1$  and the other parallel to  $O_1P$ ; the angle between these two vectors is  $\phi_x$ . Of these two vectors only  $F_{OP}$  has a component in the  $y_1$  direction. Since the force triangle is similar to the triangle  $PO_1Q$ , numerically

$$F_{OP} = \frac{F_r}{\tan \phi_x} = u_1 k_r \sin \phi_x \times \frac{\cos \phi_x}{\sin \phi_x} = u_1 k_r \cos \phi_x$$

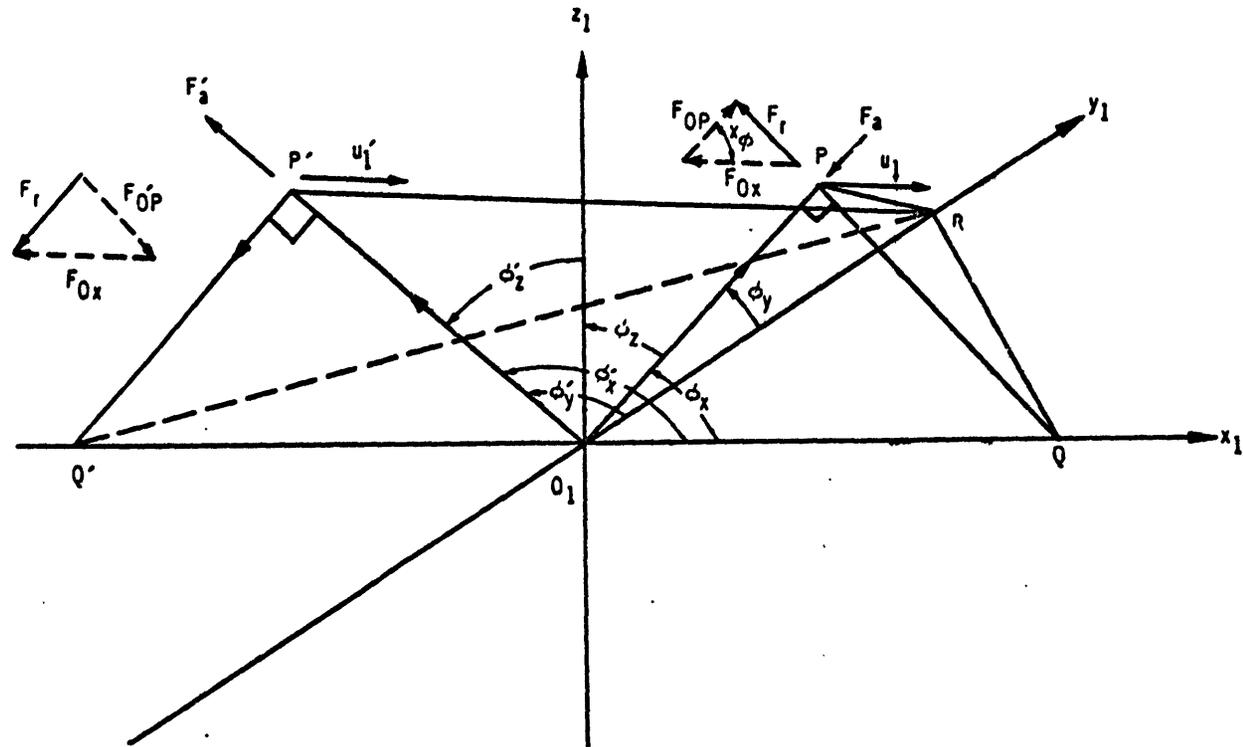


Figure 40 - Diagram for Deriving Elastic Constants of Individual Mountings When Inclined to the  $x_1, y_1, z_1$  Axes

Since the coordinates of the point of attachment are not involved, any point on the axis of a mounting may be chosen as the origin of local coordinates  $x_1, y_1, z_1$  parallel to  $xyz$ .  $O_1P'$  represents the axis of a mounting which is the mirror image of  $O_1P$  in the  $y_1z_1$  plane.

and the component in the  $y_1$  direction, which is also the component of  $F_r$  in this direction, is  $u_1 k_r \cos \phi_x \cos \phi_y$ . If account is taken of the directions of the force components and the convention for the  $k$ 's, it follows that

$$k_{xy} = (k_a - k_r) \cos \phi_x \cos \phi_y$$

similarly

$$k_{yz} = (k_a - k_r) \cos \phi_y \cos \phi_z$$

and

$$k_{xz} = (k_a - k_r) \cos \phi_x \cos \phi_z$$

It may be noted that the same values of the  $k$ 's are obtained if the line  $O_1P$  is drawn in the opposite direction along the axis of the mounting. Furthermore, they depend only on the direction of the axis and are not changed if the mounting is reversed end for end, or if the attachments to the assembly and to the foundation are interchanged. In Figure 40,  $O_1P'$  represents the axis of a mounting which is the mirror image of  $O_1P$  in the  $y_1s_1$  plane. The same formulas are obtained with  $\phi$  replaced by  $\phi'$ . The mirror mounting has the same value of  $k_{yz}$  as the original mounting but equal and opposite values of  $k_{xy}$  and  $k_{xz}$ .

For the entire assembly, then,  $K_{uu} = \sum k_{xx}$  summed for all the mountings, and similarly for five other  $K$ 's.

For  $K$ 's involving rotation, however, the effects of small rotations of the mounted assembly must also be taken into account; this involves the position coordinates of the effective points of attachment. For example, consider the evaluation of  $K_{u\alpha}$ . This requires the determination of the force in the  $x$  direction due to a unit positive rotation of the mounted assembly about  $Ox$ . A small rotation  $d\theta$  about  $Ox$  causes a displacement of the effective point of attachment of each mounting in a plane through this point drawn parallel to the  $yz$  plane; the  $s$  component of this displacement is  $yd\theta$  and its  $y$  component is  $-zd\theta$ . These displacements evoke forces in the  $x$  direction of magnitude  $-k_{xz}yd\theta$  and  $+k_{xy}zd\theta$ . The total force in the  $x$  direction is the sum of these forces or  $-k_{xz}yd\theta + k_{xy}zd\theta$ . The resulting force due to the entire system of mountings is denoted by  $-K_{u\alpha}d\theta$ . Hence, for the summation of all mounts

$$K_{u\alpha} = \sum (k_{xz}y - k_{xy}z)$$

Again, consider the evaluation of  $K_{\alpha\beta}$ . This requires the determination of the moment about  $Ox$  due to a unit positive rotation of the assembly about  $Oy$ . A small rotation  $d\theta$  about  $Oy$  causes displacements of the effective point of attachment  $(x, y, s)$  of any mount in a plane parallel to the  $xs$  plane of magnitude  $sd\theta$  in the  $x$  direction and  $-zd\theta$  in the  $s$  direction. Each of these displacements, in turn, may evoke forces in both the  $y$  and  $s$  directions. Thus the displacement in the  $x$  direction gives forces  $-k_{xz}sd\theta$  in the  $s$  direction and  $-k_{xy}sd\theta$  in the  $y$  direction, whereas the displacement in the  $s$  direction gives forces  $+k_{xz}xd\theta$  in the  $s$  direction and  $+k_{yz}xd\theta$  in the  $y$  direction. These four forces yield the following moments about  $Ox$  per unit angular displacement:

$$-k_{zz}zy; +k_{zy}z^2; +k_{zz}xy; \text{ and } -k_{yz}zs$$

The resultant moment equals  $-K_{\alpha\beta}d\theta$ . Hence for the entire set of mountings

$$K_{\alpha\beta} = \Sigma (-k_{zz}xy - k_{xy}z^2 + k_{zz}yz + k_{yz}zs)$$

It turns out that, as is required by the conservation of energy, the order of the subscripts on the  $K$ 's is immaterial; thus  $K_{\alpha\alpha} = K_{\alpha\alpha}$ ;  $K_{\beta\alpha} = K_{\alpha\beta}$ ; etc.

By such processes the following set of  $K$ 's was derived:

$$K_{uu} = \Sigma k_{xx}$$

$$K_{vv} = \Sigma k_{yy}$$

$$K_{ww} = \Sigma k_{zz}$$

$$K_{uv} = \Sigma k_{xy}$$

$$K_{uw} = \Sigma k_{xz}$$

$$K_{vw} = \Sigma k_{yz}$$

$$K_{\alpha\alpha} = \Sigma (k_{zz}y^2 + k_{yy}z^2 - 2k_{yz}yz)$$

$$K_{\beta\beta} = \Sigma (k_{xx}z^2 + k_{zz}x^2 - 2k_{xz}xz)$$

$$K_{\gamma\gamma} = \Sigma (k_{yy}x^2 + k_{xx}y^2 - 2k_{xy}xy)$$

$$K_{\alpha\beta} = \Sigma (-k_{zz}xy - k_{xy}z^2 + k_{zz}yz + k_{yz}zs)$$

$$K_{\alpha\gamma} = \Sigma (-k_{yy}xz - k_{xz}y^2 + k_{yz}xy + k_{xy}yz)$$

$$K_{\beta\gamma} = \Sigma (-k_{xx}yz - k_{yz}x^2 + k_{xy}zs + k_{xz}xy)$$

$$K_{u\alpha} = \Sigma (k_{zz}y - k_{xy}z)$$

$$K_{u\beta} = \Sigma (k_{xx}z - k_{xz}x)$$

$$K_{u\gamma} = \Sigma (k_{xy}x - k_{xz}y)$$

$$K_{v\alpha} = \Sigma (k_{yz}y - k_{yy}z)$$

$$K_{v\beta} = \Sigma (k_{xy}z - k_{yz}x)$$

$$K_{v\gamma} = \Sigma (k_{yy}x - k_{xy}y)$$

$$K_{w\alpha} = \Sigma (k_{zz}y - k_{yz}z)$$

$$K_{w\beta} = \Sigma (k_{xx}z - k_{xz}x)$$

$$K_{w\gamma} = \Sigma (k_{yz}x - k_{xz}y)$$

### A5.1.3. THE FREQUENCY EQUATIONS

On substituting terms of the type  $-u\omega^2$  for  $u$  in the equations of motion, since the motions in question are simple harmonic, the equations of motion become in algebraic form:

$$-mu\omega^2 + K_{uu}u + K_{uv}v + K_{uw}w + K_{u\alpha}\alpha + K_{u\beta}\beta + K_{u\gamma}\gamma = 0$$

$$-mv\omega^2 + K_{uv}u + K_{vv}v + K_{vw}w + K_{v\alpha}\alpha + K_{v\beta}\beta + K_{v\gamma}\gamma = 0$$

$$-mw\omega^2 + K_{uw}u + K_{vw}v + K_{ww}w + K_{w\alpha}\alpha + K_{w\beta}\beta + K_{w\gamma}\gamma = 0$$

$$-I_x\alpha\omega^2 + I_{xy}\beta\omega^2 + I_{xz}\gamma\omega^2 + K_{u\alpha}u + K_{v\alpha}v + K_{w\alpha}w + K_{\alpha\alpha}\alpha + K_{\alpha\beta}\beta + K_{\alpha\gamma}\gamma = 0$$

$$-I_y\beta\omega^2 + I_{xy}\alpha\omega^2 + I_{yz}\gamma\omega^2 + K_{u\beta}u + K_{v\beta}v + K_{w\beta}w + K_{\alpha\beta}\alpha + K_{\beta\beta}\beta + K_{\beta\gamma}\gamma = 0$$

$$-I_z\gamma\omega^2 + I_{xz}\alpha\omega^2 + I_{yz}\beta\omega^2 + K_{u\gamma}u + K_{v\gamma}v + K_{w\gamma}w + K_{\alpha\gamma}\alpha + K_{\beta\gamma}\beta + K_{\gamma\gamma}\gamma = 0$$

The following determinant of the coefficients of the displacement coordinates, when set equal to zero, gives the frequency equation.

$K_{uu} - m\omega^2$	$K_{uv}$	$K_{uw}$	$K_{u\alpha}$	$K_{u\beta}$	$K_{u\gamma}$
$K_{uv}$	$K_{vv} - m\omega^2$	$K_{vw}$	$K_{v\alpha}$	$K_{v\beta}$	$K_{v\gamma}$
$K_{uw}$	$K_{vw}$	$K_{ww} - m\omega^2$	$K_{w\alpha}$	$K_{w\beta}$	$K_{w\gamma}$
$K_{u\alpha}$	$K_{v\alpha}$	$K_{w\alpha}$	$K_{\alpha\alpha} - I_x\omega^2$	$K_{\alpha\beta} + I_{xy}\omega^2$	$K_{\alpha\gamma} + I_{xz}\omega^2$
$K_{u\beta}$	$K_{v\beta}$	$K_{w\beta}$	$K_{\alpha\beta} + I_{xy}\omega^2$	$K_{\beta\beta} - I_y\omega^2$	$K_{\beta\gamma} + I_{yz}\omega^2$
$K_{u\gamma}$	$K_{v\gamma}$	$K_{w\gamma}$	$K_{\alpha\gamma} + I_{xz}\omega^2$	$K_{\beta\gamma} + I_{yz}\omega^2$	$K_{\gamma\gamma} - I_z\omega^2$

One method of solving the problem is to find by trial the values of  $\omega$  (the circular frequency) for which the determinant of the coefficients of the displacement coordinates vanishes. If the values of the determinant calculated for various values of  $\omega$  are plotted against  $\omega$ , a curve is obtained which crosses the axis at each of the natural circular frequencies, provided the six frequencies are distinct. When, however, certain frequencies coincide, the plotted curve may only touch the axis at the corresponding points without crossing. If, when the six frequencies are distinct, the set of simultaneous equations in  $u, v, w, \alpha, \beta$ , and  $\gamma$  above is solved, in each case with the appropriate value of  $\omega$ , the values of the ratios of the coordinates found will represent the normal mode pattern corresponding to that particular circular frequency. If two or more frequencies coincide, the mode pattern for that frequency is not unique. If a number  $n$  of frequencies are equal,  $n$  different basic mode patterns can be found,

and any other mode pattern that is possible at that frequency can be regarded as a linear combination of the  $n$  basic modes.

Under special conditions it may be found that various  $K$ 's in the frequency determinant vanish. If all the  $K$ 's of the type  $K_{ij}$  (where  $i \neq j$ ) vanish and the axes are principal axes of inertia, there will remain only terms falling on the main diagonal. In this case the three motions of translation in the  $x$ ,  $y$ , and  $z$  directions and the three motions of rotation about the  $x$ ,  $y$ , and  $z$  axes are all independent, and each frequency may be found from an equation of the type  $K_{uu} - m\omega^2 = 0$ . Although the motions are independent because of the symmetry of the mountings, the frequencies of different modes may be the same.

Where only certain terms not on the main diagonal are zero, it may be found that the set of six simultaneous equations in the six unknowns breaks down into smaller sets in fewer variables which are independent of each other. These sets may then be treated separately, and in such cases it may also be found that the frequencies of different modes coincide.

The evaluation of the elastic parameters requires the tabulation for each mounting of the following quantities:

$$k_a, k_r, x, y, z, \phi_x, \phi_y, \text{ and } \phi_z$$

These are, respectively, the axial stiffness, the radial stiffness, the three position coordinates of the effective point of attachment, and the three direction angles which the axis of the mount makes with the  $x$ ,  $y$ , and  $z$  axes. It is obvious that, regardless of the physical shape of the mounting, if any two of its principal stiffnesses are equal, the analysis given here is applicable by taking the mounting axis as normal to the directions of equal stiffness.

#### A5.1.4. VIBRATIONAL SYMMETRY

The solution of the general frequency equation is so laborious that it is seldom undertaken without the use of a high-speed computer. Usually, however, the mountings themselves are so arranged that one or more planes of vibrational symmetry exist, and then the sixth-order determinant can be broken down into independent determinants of lower order. Such cases will be treated in the following sections.

Vibrational symmetry exists with respect to a given plane passing through the center of gravity of the mounted assembly when motion parallel to that plane has no tendency to excite motion perpendicular to the plane; then, also, translation in a perpendicular direction or rotation about an axis lying in the plane does not excite motion parallel to the plane. Such symmetry usually requires that two of the principal axes of inertia of the mounted body lie in the plane of symmetry and also that there be elastic symmetry with respect to this plane. It is convenient in such cases to take the axes of coordinates along the principal axes of inertia. Coordinates so defined will be denoted by  $X$ ,  $Y$ ,  $Z$  to distinguish them from the more general coordinates  $x$ ,  $y$ ,  $z$ . The subscripts on the  $k$ 's and  $I$ 's will therefore be capital letters.

For computational purposes in these simpler cases, it is more convenient to write

explicit symbols in place of  $K_{uu}$ , etc., and for simplicity  $k_X$ ,  $k_Y$ , and  $k_Z$  will be written for the individual mountings in place of  $k_{XX}$  (or  $k_{zz}$ ), etc. The appropriate sums are easily read off from the definitions of the  $K$ 's, for example

$$K_{uu} = \Sigma k_X$$

$$K_{uv} = \Sigma k_{XY}$$

$$K_{u\beta} = \Sigma (k_X Z - k_{XZ} X)$$

$$K_{\alpha\alpha} = \Sigma (k_Z Y^2 + k_Y Z^2 - 2k_{YZ} YZ)$$

Summation over all mountings is understood in each case.

Explicit frequency formulas will now be given for the principal cases that may arise in practice,  $X, Y, Z$  axes being assumed in all cases.

## A5.2. ONE PLANE OF VIBRATIONAL SYMMETRY ( $YZ$ )

### A5.2.1. SIMPLY ORIENTED MOUNTINGS

Let every mounting have either  $k_a = k$ , or its axis parallel to  $X, Y$ , or  $Z$ . Let the plane of symmetry be the  $YZ$  plane. Then the conditions for elastic symmetry are

$$\Sigma k_Y X = 0, \Sigma k_Z X = 0, \Sigma k_Y XZ = 0, \Sigma k_Z XY = 0$$

When these four conditions are satisfied, it is apparent that, of the six equations of motion in Section A5.1.1., the second, third, and fourth contain only the variables  $v, w$ , and  $\alpha$ , so that these equations can be solved independently of the other three. Similarly, the first one and the last two contain only  $u, \beta$ , and  $\gamma$ .

Equate the determinant of each set of three equations to zero.

$$\begin{vmatrix} \Sigma k_Y - m\omega^2 & 0 & -\Sigma k_Y Z \\ 0 & \Sigma k_Z - m\omega^2 & \Sigma k_Z Y \\ -\Sigma k_Y Z & \Sigma k_Z Y & \Sigma k_Z Y^2 + \Sigma k_Y Z^2 - I_X \omega^2 \end{vmatrix} = 0; \quad \begin{vmatrix} \Sigma k_Z - m\omega^2 & \Sigma k_X Z & -\Sigma k_X Y \\ \Sigma k_X Z & \Sigma k_Z X^2 + \Sigma k_X Z^2 - I_Y \omega^2 & -\Sigma k_X YZ \\ -\Sigma k_X Y & -\Sigma k_X YZ & \Sigma k_Y X^2 + \Sigma k_X Y^2 - I_Z \omega^2 \end{vmatrix} = 0$$

Expansion of the first determinant gives the equation

$$\begin{aligned} & (\Sigma k_Y - m\omega^2)(\Sigma k_Z - m\omega^2)(\Sigma k_Z Y^2 + \Sigma k_Y Z^2 - I_X \omega^2) \\ & - (-\Sigma k_Y Z)(\Sigma k_Z - m\omega^2)(-\Sigma k_Y Z) - (\Sigma k_Y - m\omega^2)(\Sigma k_Z Y)^2 = 0 \end{aligned}$$

After expanding further, the equation has the form of a cubic in  $\omega^2$ .

Introduce the notation

$$p_1 = \frac{1}{4\pi^2 m} \Sigma k_X, \quad p_2 = \frac{1}{4\pi^2 m} \Sigma k_Y, \quad p_3 = \frac{1}{4\pi^2 m} \Sigma k_Z$$

$$q_1 = \frac{1}{4\pi^2 m r_X^2} (\Sigma k_Z Y^2 + \Sigma k_Y Z^2), \quad q_2 = \frac{1}{4\pi^2 m r_Y^2} (\Sigma k_Z X^2 + \Sigma k_X Z^2),$$

$$q_3 = \frac{1}{4\pi^2 m r_Z^2} (\Sigma k_Y X^2 + \Sigma k_X Y^2)$$

in which  $r_X$ ,  $r_Y$ , and  $r_Z$  are the radii of gyration of the body about the  $X$ ,  $Y$ , and  $Z$  axes, respectively, so that  $I_X = m r_X^2$ , etc. Then the cubic equation can be written, after dividing through by  $-64\pi^6 m^2 I_X$  and replacing  $\omega$  by  $2\pi f$ ,

$$f^6 - B_1 f^4 + C_1 f^2 - D_1 = 0$$

where

$$B_1 = p_2 + p_3 + q_1$$

$$C_1 = p_2 p_3 + p_2 q_1 + p_3 q_1 - \frac{1}{(2\pi)^4 m^2 r_X^2} [(\Sigma k_Z Y)^2 + (\Sigma k_Y Z)^2]$$

$$D_1 = p_2 p_3 q_1 - \frac{1}{(2\pi)^4 m^2 r_X^2} [p_2 (\Sigma k_Z Y)^2 + p_3 (\Sigma k_Y Z)^2]$$

The roots of this cubic equation, which are necessarily real and positive, give three of the natural frequencies of the body on its mountings. In each of these three modes of vibration only  $v$ ,  $w$ , and  $\alpha$  differ from zero.

If the roots of the cubic equation are  $f_1^2$ ,  $f_2^2$ ,  $f_3^2$ , then  $f_1^2 + f_2^2 + f_3^2 = p_2 + p_3 + q_1$ . Further facts can be inferred from the following: Write  $g_Y = (\Sigma k_Z Y)^2$ ,  $g_Z = (\Sigma k_Y Z)^2$ . Then, if  $g_Y = g_Z = 0$ , the roots for  $f^2$  are  $p_2$ ,  $p_3$ , and  $q_1$ . If  $F(f^2)$  represents the function of  $f^2$  including  $D_1$  that appears on the left in this cubic equation, then, when  $f^2$  is held constant,

$$\frac{\partial F}{\partial g_Y} = \frac{1}{16\pi^4 m^2 r_X^2} (p_2 - f^2), \quad \frac{\partial F}{\partial g_Z} = \frac{1}{16\pi^4 m^2 r_X^2} (p_3 - f^2)$$

Now, when  $g_Y = g_Z = 0$ , the maximum root is the greatest of the three quantities  $p_2$ ,  $p_3$ , and  $q_1$ . Then, as  $g_Y$  and  $g_Z$  increase from zero, at the maximum root  $f^2 \geq p_2$  and  $\partial F / \partial g_Y \leq 0$ , and similarly  $\partial F / \partial g_Z \leq 0$ ; thus the curve for  $F(f^2)$  sinks, or does not move if  $p_2 = p_3$  and  $q_1 \leq p_2$ . It follows since the slope of the curve is positive at the maximum root, see Figure 41, that the maximum root increases progressively or, perhaps, does not change. Similarly, the minimum root decreases or does not change. One root or the other, however, must change,

unless  $p_2 = p_3 = q_1$ , in which case all three roots for  $f^2$  are equal to  $p_2$ . Therefore, unless  $p_2 = p_3 = q_1$ , the effect of  $g_Y$  and  $g_Z$  is to increase the difference between the maximum and minimum frequencies.

At the middle root, on the other hand, the slope of the curve for  $F(f^2)$  is negative; therefore, increasing  $g_Y$  moves the middle root toward  $p_2$ , while increasing  $g_Z$  moves it toward  $p_3$ .

Finally, the three roots can be equal only if  $p_2 = p_3 = q_1$ , and also  $g_Y = g_Z = 0$ . For, from the general relation between the roots of a cubic and its coefficients,

$$f_1^2 + f_2^2 + f_3^2 = p_2 + p_3 + q_1$$

$$f_1^2 f_2^2 + f_1^2 f_3^2 + f_2^2 f_3^2 = C_1 - p_2 p_3 + p_2 q_1 + q_3 q_1 - W$$

where  $W$  stands for the last term in the expression for  $C_1$ . If the first equation is squared and multiplied by 2, and if 6 times the second equation is then subtracted from it, the result can be written

$$(f_1^2 - f_2^2)^2 + (f_1^2 - f_3^2)^2 + (f_2^2 - f_3^2)^2 = (p_2 - p_3)^2 + (p_2 - q_1)^2 + (p_3 - q_1)^2 + 6W$$

Here  $W \geq 0$ . Therefore,  $f_1^2 - f_2^2 = f_3^2$  only if  $p_1 = p_3 = q_1$  and  $W = 0$ .

The cubic equation can be solved by standard methods or with the help of a chart.\* In special cases the standard methods are easier. If  $p_2 = p_3$  or if  $\sum k_Y Z = 0$ , one root for  $f^2$  is  $p_2$ ; if  $\sum k_Z Y = 0$ , one root is  $p_3$ . In either case, a quadratic equation for the other two roots can be formed from the cubic equations by dropping  $D_1$  and all terms that contain as a factor the root already found,  $p_2$  or  $p_3$ , and dividing through by  $f^2$ .

Similar treatment of the second determinant gives

$$f^6 - B_2 f^4 + C_2 f^2 - D_2 = 0$$

where

$$B_2 = p_1 + q_2 + q_3$$

$$C_2 = p_1 q_2 + p_1 q_3 + q_2 q_3 - \frac{1}{(2\pi)^4 m^2} \left[ \frac{1}{r_2^2} (\sum k_X Y)^2 + \frac{1}{r_Y^2} (\sum k_X Z)^2 + \frac{1}{r_Y^2 r_Z^2} (\sum k_X YZ)^2 \right]$$

\*Figure 23.

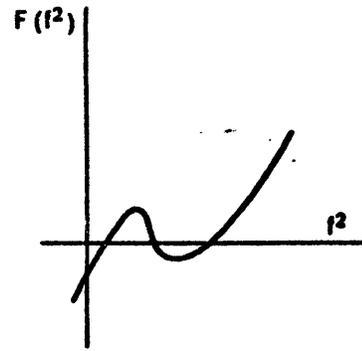


Figure 41 - Plot of a Cubic Equation

$$D_2 = p_1 q_2 q_3 - \frac{1}{(2\pi)^4 m^2} \left[ \frac{q_2}{r_Z^2} (\Sigma k_X Y)^2 + \frac{q_3}{r_Y^2} (\Sigma k_X Z)^2 + \frac{p_1}{r_Y^2 r_Z^2} (\Sigma k_X YZ)^2 \right] + \frac{2}{(2\pi)^6 m^3 r_Y^2 r_Z^2} (\Sigma k_X Y) (\Sigma k_X Z) (\Sigma k_X YZ)$$

where  $p_1$ ,  $q_2$ , and  $q_3$  are quantities previously defined. The roots of this equation give the frequencies of the three modes in which only the quantities  $u$ ,  $\beta$ , and  $\gamma$  occur.

When all terms, except the first, are zero in both  $C_2$  and  $D_2$ , the roots for  $f^2$  are  $p_1$ ,  $q_2$ , and  $q_3$ . In any case, the sum of the roots is  $f_1^2 + f_2^2 + f_3^2 = p_1 + q_2 + q_3$ . Further reasoning is complicated by the presence of the last term in  $D_2$ . The other terms in  $C_2$  and  $D_2$  containing  $\Sigma$ , as can be seen by reasoning as before, raise the upper root and reduce the lower root, except that in special cases one root may not be affected. This effect becomes greater as those terms are increased. The last term in  $D_2$ , if the product  $(\Sigma k_X Y) (\Sigma k_X Z) (\Sigma k_X YZ)$  is positive, lowers the curve for  $F(f^2)$  and thereby increases both the upper and lower roots, with the opposite effect if the product is negative. Therefore, if this product does not vanish, increasing the numerical value of  $\Sigma k_X Y$  or  $\Sigma k_X Z$  or  $\Sigma k_X YZ$  will lower the lowest root if the product is negative, or will raise the upper root if the product is positive. The general effect of these sums is to spread the roots farther apart.

The same general methods for solving this cubic equation apply as for the previous one. As special cases: if all three of the sums occurring in  $C_2$  and  $D_2$  vanish, then the roots of the equation regarded as a cubic in  $f^2$  are  $p_1$ ,  $q_2$ ,  $q_3$ ; if two sums vanish, whichever of the quantities  $p_1$ ,  $q_2$ , or  $q_3$  multiplies the third sum in  $D_2$  is a root; if only one sum vanishes, a root lies between the two of the quantities  $p_1$ ,  $q_2$ ,  $q_3$  that multiply the other two sums in  $D_2$ . These statements can be verified by writing the cubic expression as the sum of the product  $(f^2 - p_1)(f^2 - q_2)(f^2 - q_3)$  and a remainder.

### A5.2.2. INCLINED MOUNTINGS WITH ONE PLANE OF SYMMETRY (YZ)

When one or more mountings have unequal axial and radial stiffnesses and are not parallel to a coordinate axis, the conditions for vibrational symmetry relative to the YZ plane become

$$\Sigma k_{XY} = 0, \quad \Sigma k_{XZ} = 0$$

$$\Sigma k_Y X = \Sigma k_{XY} Y, \quad \Sigma k_Z X = \Sigma k_{XZ} Z$$

$$\Sigma k_{YZ} X = \Sigma k_{XZ} Y - \Sigma k_{XY} Z$$

$$\Sigma k_Z XY + \Sigma k_{XY} Z^2 - \Sigma k_{YZ} XZ + \Sigma k_{XZ} YZ$$

$$\Sigma k_Y XZ + \Sigma k_{XZ} Y^2 - \Sigma k_{YZ} XY + \Sigma k_{XY} YZ$$

The zeros in the first determinant are now replaced by  $\Sigma k_{YZ}$ , and skew force additions occur in many other elements. The expressions for  $p_1, p_2, p_3, B_1$ , and  $B_2$  are unaltered. The remainder of the quantities in the two cubic equations become

$$q_1 = \frac{1}{(2\pi)^2 m r_X^2} (\Sigma k_Z Y^2 + \Sigma k_Y Z^2 - 2\Sigma k_{YZ} YZ)$$

$$q_2 = \frac{1}{(2\pi)^2 m r_Y^2} (\Sigma k_Z X^2 + \Sigma k_X Z^2 - 2\Sigma k_{XZ} XZ)$$

$$q_3 = \frac{1}{(2\pi)^2 m r_Z^2} (\Sigma k_Y X^2 + \Sigma k_X Y^2 - 2\Sigma k_{XY} XY)$$

$$C_1 = p_2 p_3 + p_2 q_1 + p_3 q_1 - \frac{1}{(2\pi)^4 m^2} \left( \frac{S_{11}^2}{r_X^2} + \frac{S_{12}^2}{r_X^2} + S_{13}^2 \right)$$

$$D_1 = p_2 p_3 q_1 - \frac{1}{(2\pi)^4 m^2} \left( p_2 \frac{S_{11}^2}{r_X^2} + p_3 \frac{S_{12}^2}{r_X^2} + q_1 S_{13}^2 \right) - \frac{2 S_{11} S_{12} S_{13}}{(2\pi)^6 m^3 r_X^2}$$

$$C_2 = p_1 q_2 + p_1 q_3 + q_1 q_2 - \frac{1}{(2\pi)^4 m^2} \left( \frac{S_{21}^2}{r_Y^2} + \frac{S_{22}^2}{r_Y^2} + \frac{S_{23}^2}{r_Y^2 r_Z^2} \right)$$

$$D_2 = p_1 q_2 q_3 - \frac{1}{(2\pi)^4 m^2} \left( q_2 \frac{S_{21}^2}{r_Y^2} + q_3 \frac{S_{22}^2}{r_Y^2} + p_1 \frac{S_{23}^2}{r_Y^2 r_Z^2} + \frac{2 S_{21} S_{22} S_{23}}{(2\pi)^6 m^3 r_Y^2 r_Z^2} \right)$$

where

$$S_{11} = \Sigma k_Z Y - \Sigma k_{YZ} Z, \quad S_{12} = \Sigma k_Y Z - \Sigma k_{YZ} Y, \quad S_{13} = \Sigma k_{YZ}$$

$$S_{21} = \Sigma k_X Y - \Sigma k_{XY} X, \quad S_{22} = \Sigma k_X Z - \Sigma k_{XZ} X,$$

$$S_{23} = \Sigma k_X YZ + \Sigma k_{YZ} X^2 - \Sigma k_{XZ} XY - \Sigma k_{XY} XZ$$

The discussion of the roots of the equation regarded as a cubic in  $f^2$  follows the same general lines as in the absence of the skew forces.

### A5.3. TWO PLANES OF VIBRATIONAL SYMMETRY (XZ AND YZ)

#### A5.3.1. SIMPLY ORIENTED MOUNTINGS

Let each mounting have either  $k_o = k_r$ , or its axis parallel to  $X, Y$ , or  $Z$ , and let both the  $XZ$  and  $YZ$  planes be planes of vibrational symmetry. Then, by extension of the conditions previously written for one plane, it is necessary that

$$\Sigma k_Y X = \Sigma k_Z X = \Sigma k_X Y = \Sigma k_Z Y = 0$$

$$\Sigma k_Z XY = \Sigma k_Y XZ = \Sigma k_X YZ = 0$$

When these conditions are satisfied, the six equations of motion, Section A5.1.1., become three independent pairs; one pair contains only  $u$  and  $\beta$ , another only  $v$  and  $\alpha$ , and the third only  $w$  and  $\gamma$ . The three pairs are

$$(\Sigma k_X - m\omega^2)u + (\Sigma k_X Z)\beta = 0$$

$$(\Sigma k_X Z)u + (\Sigma k_Z X^2 + \Sigma k_X Z^2 - \omega^2 I_Y)\beta = 0$$

$$(\Sigma k_Y - m\omega^2)v - (\Sigma k_Y Z)\alpha = 0$$

$$-(\Sigma k_Y Z)v + (\Sigma k_Z Y^2 + \Sigma k_Y Z^2 - \omega^2 I_X)\alpha = 0$$

$$(\Sigma k_Z - m\omega^2)w = 0$$

$$(\Sigma k_Y X^2 + \Sigma k_X Y^2 - \omega^2 I_Z)\gamma = 0$$

Equating the determinant for the first two equations to zero gives the equation

$$\begin{vmatrix} \Sigma k_X - m\omega^2 & \Sigma k_X Z \\ \Sigma k_X Z & \Sigma k_Z X^2 + \Sigma k_X Z^2 - \omega^2 I_Y \end{vmatrix} = 0$$

or in terms of the notation defined in Section A5.2.1.,

$$f^4 - (p_1 + q_2)f^2 + p_1 q_2 - \frac{1}{(2\pi)^4 m^2 r_Y^2} (\Sigma k_X Z)^2 = 0$$

and, by the usual quadratic formula, the frequencies for the two rocking modes in the  $XZ$  plane are given by the equation

$$f^2 = \frac{1}{2} \left[ p_1 + q_2 \pm \sqrt{(p_1 - q_2)^2 + \frac{4}{(2\pi)^4 m^2 r_Y^2} (\Sigma k_X Z)^2} \right]$$

The second pair of equations gives

$$\begin{vmatrix} \Sigma k_Y - m\omega^2 & -\Sigma k_Y Z \\ -\Sigma k_Y Z & \Sigma k_Z Y^2 + \Sigma k_Y Z^2 - \omega^2 I_X \end{vmatrix} = 0$$

or

$$f^4 - (p_2 + q_1)f^2 + p_2 q_1 - \frac{1}{(2\pi)^4 m^2 r_X^2} (\Sigma k_Y Z)^2 = 0$$

and for the frequencies of the two rocking modes in the YZ plane,

$$f^2 = \frac{1}{2} \left[ p_2 + q_1 \pm \sqrt{(p_2 - q_1)^2 + \frac{4}{(2\pi)^4 m^2 r_X^2} (\Sigma k_Y Z)^2} \right]$$

For the fifth equation, it may be assumed that  $\omega \neq 0$  but all other five variables are zero. Then,

$$\Sigma k_Z - m \omega^2 = 0$$

and

$$f^2 = \frac{\omega^2}{4\pi^2} = p_3 = \frac{1}{4\pi^2 m} \Sigma k_Z$$

where  $f$  is the frequency of a translational mode of vibration in the direction of the Z axis at the intersection of the two planes of symmetry. Similarly, the last equation gives

$$f^2 = q_3 = \frac{1}{4\pi^2 m r_Z^2} (\Sigma k_Y X^2 + \Sigma k_X Y^2)$$

for a rotational mode about the Z axis.

In the most important practical case, that of arrangements with base mountings, all the mountings lie in a single plane perpendicular to both planes of symmetry. For this case, the base-mounting chart, Figure 20, is useful. The relevant theory is as follows:

Let the distance from the center of gravity of the mounted assembly to the plane of the mountings be  $D_Z$ . Then  $Z = -D_Z$  for all mountings, the Z axis being assumed drawn from the center of gravity away from the mountings. Write

$$D_X^2 = \frac{\Sigma k_Z X^2}{\Sigma k_X}$$

Then, using the definitions of  $p_1$  and  $q_2$ ,

$$\Sigma k_X Z = -D_Z \Sigma k_X = -4\pi^2 m D_Z p_1$$

$$\frac{q_2}{p_1} = \frac{1}{r_Y^2 \Sigma k_X} (\Sigma k_Z X^2 + \Sigma k_X Z^2) = \frac{D_X^2}{r_Y^2} + \frac{D_Z^2}{r_Y^2}$$

and, dividing through by  $p_1^2$ , the first quadratic equation becomes

$$\left(\frac{f^2}{p_1}\right)^2 - \left(1 + \frac{D_X^2}{r_Y^2} + \frac{D_Z^2}{r_Y^2}\right) \frac{f^2}{p_1} + \frac{D_X^2}{r_Y^2} = 0$$

Therefore, if  $f_{\max}^2$  is the greater and  $f_{\min}^2$  the smaller of the two roots of this equation,

$$\frac{f_{\max}^2}{p_1} + \frac{f_{\min}^2}{p_1} = 1 + \frac{D_X^2}{r_Y^2} + \frac{D_Z^2}{r_Y^2}, \quad \frac{f_{\max}^2 f_{\min}^2}{p_1^2} = \frac{D_X^2}{r_Y^2}$$

and, as is seen after multiplying out,

$$\left(1 - \frac{f_{\max}^2}{p_1}\right) \left(1 - \frac{f_{\min}^2}{p_1}\right) = -\frac{D_Z^2}{r_Y^2}$$

On the left in this equation, the second factor cannot be negative since then the first factor would have to be a negative quantity of greater magnitude and the product would be positive. Therefore

$$f_{\min}^2 \leq p_1$$

It follows that, if  $f_{\min}^2$  is substituted for  $f^2$  in the quadratic equation and if  $D_X/r_Y$  and  $D_Z/r_Y$  are taken as coordinates on a plot, curves drawn for particular values of  $f_{\min}^2/p_1$  will be hyperbolas.

Furthermore, if

$$R = \frac{f_{\max}}{f_{\min}}$$

by substitution for  $f_{\max}$  in terms of  $R$  in the preceding two equations

$$(1 + R^2) \frac{f_{\min}^2}{p_1} = 1 + \frac{D_X^2}{r_Y^2} + \frac{D_Z^2}{r_Y^2}, \quad R^2 \left(\frac{f_{\min}^2}{p_1}\right)^2 = \frac{D_X^2}{r_Y^2}$$

and, by elimination of  $f_{\min}^2$ ,

$$\frac{D_X^2}{r_Y^2} + \frac{D_Z^2}{r_Y^2} - \left(R + \frac{1}{R}\right) \frac{D_X}{r_Y} + 1 = 0$$

Curves for a given value of  $R$  are, therefore, semicircles centered on the  $D_X/r_Y$  axis.

Finally, dividing the first of the equations of motion by  $\Sigma k_X$  or  $4\pi^2 m p_1$ , and substituting  $Z = -D_Z$

$$\left(1 - \frac{f^2}{p_1}\right) u - \beta D_Z = 0, \quad \text{hence } \frac{f^2}{p_1} = 1 - \frac{\beta D_Z}{u}$$

Substitute this expression for  $f^2/p_1$  in the quadratic equation and multiply through by  $u/\beta$

$$\frac{D_X^2}{r_Y^2} + \frac{D_Z^2}{r_Y^2} - \left(\frac{u}{\beta r_Y} - \frac{\beta r_Y}{u}\right) \frac{D_Z}{r_Y} - 1 = 0$$

Therefore, if curves were drawn on the chart for a fixed value of  $u/\beta r_Y$ , they would be circular arcs centered in the  $D_Z/r_Y$  axis; see Figure 30. The same arc would represent  $u/\beta r_Y = a_1 > 0$  and  $u/\beta r_Y = -1/a_1$ . Since  $f_{\min}^2/p_1 \leq 1$ , it is evident that for vibrations at frequency  $f_{\min}$ ,  $u/\beta r_Y > 0$ , the vibrations with  $u/\beta r_Y < 0$  must therefore be those at frequency  $f_{\max}$ .

A similar analysis holds for rocking motions in the  $YZ$  plane;  $r_Y$  is replaced by  $r_X$ ,  $p_1$  is replaced by  $p_2$ ,  $u/\beta r_Y$  by  $(-v/\alpha r_X)$  and  $D_X$  by  $D_Y$  where

$$D_Y^2 = \frac{\sum k_Z Y^2}{\sum k_Y}$$

### A5.3.2. INCLINED MOUNTINGS WITH TWO PLANES OF SYMMETRY ( $XZ$ AND $YZ$ )

When one or more mountings have  $k_a \neq k$ , or are not parallel to a coordinate axis, in addition to the requirements stated in Section A5.2.2. for symmetry relative to the  $YZ$  plane, the following conditions must be met to secure elastic symmetry relative to the  $XZ$  plane as well:

$$\sum k_{YZ} = 0$$

$$\sum k_{XY} = \sum k_{YX}$$

$$\sum k_{ZY} = \sum k_{YZ}$$

$$\sum k_{XYZ} + \sum k_{YZX} = \sum k_{XZY} + \sum k_{ZYX}$$

The formulas for the frequencies are:

For the rocking modes in the  $XZ$  plane,

$$f^2 = \frac{1}{2} \left[ p_1 + q_2 \pm \sqrt{(p_1 - q_2)^2 + \frac{4}{(2\pi)^4 m^2 r_Y^2} (\sum k_{XZ} - \sum k_{ZX})^2} \right]$$

where

$$p_1 = \frac{1}{(2\pi)^2 m} \sum k_X, \quad q_2 = \frac{1}{(2\pi)^2 m r_Y^2} (\sum k_Z X^2 + \sum k_X Z^2 - 2 \sum k_{XZ} XZ)$$

For the rocking modes in the  $YZ$  plane,

$$f^2 = \frac{1}{2} \left[ p_2 + q_1 \pm \sqrt{(p_2 - q_1)^2 + \frac{4}{(2\pi)^4 m^2 r_X^2} (\sum k_{YZ} - \sum k_{ZY})^2} \right]$$

where

$$p_2 = \frac{1}{(2\pi)^2 m} \sum k_Y, \quad q_1 = \frac{1}{(2\pi)^2 m r_X^2} (\sum k_Z Y^2 + \sum k_Y Z^2 - 2 \sum k_{YZ} YZ)$$

For translational vibration parallel to Z, as before,  $f^2 = p_3$ , but for the rotational mode about Z

$$f^2 = q_3 = \frac{1}{(2\pi)^2 m r_z^2} (\Sigma k_y X^2 + \Sigma k_x Y^2 - 2\Sigma k_{xy} XY)$$

The Base Mounting Chart is not usually applicable to arrangements including inclined mountings.

#### A5.4. THREE PLANES OF VIBRATIONAL SYMMETRY

##### A5.4.1. SIMPLY ORIENTED MOUNTINGS

Let every mounting have either  $k_a = k_r$  or its axis parallel to X, Y, or Z, and let the XY, XZ, and YZ planes all be planes of vibrational symmetry. Then the conditions for elastic symmetry are

$$\Sigma k_y X = \Sigma k_z X = \Sigma k_x Y = \Sigma k_z Y = \Sigma k_x Z = \Sigma k_y Z = 0$$

$$\Sigma k_z XY = \Sigma k_y XZ = \Sigma k_x YZ = 0$$

When these sums are all made zero in the frequency equations, Section A5.1.3., only the first two terms remain in each equation. Therefore, each of the six modes of motion is either translational or rotational. By permitting in turn only  $u$ ,  $v$ ,  $w$ ,  $\alpha$ ,  $\beta$ , or  $\gamma$  to be different from zero and solving for  $f^2$ , the following values are found for the frequencies of the three translational and the three rotational modes of vibration.

$$f_1^2 = \frac{1}{(2\pi)^2 m} \Sigma k_x, \quad f_2^2 = \frac{1}{(2\pi)^2 m} \Sigma k_y, \quad f_3^2 = \frac{1}{(2\pi)^2 m} \Sigma k_z$$

$$f_4^2 = \frac{1}{(2\pi)^2 I_x} (\Sigma k_z Y^2 + \Sigma k_y Z^2), \quad f_5^2 = \frac{1}{(2\pi)^2 I_y} (\Sigma k_z X^2 + \Sigma k_x Z^2),$$

$$f_6^2 = \frac{1}{(2\pi)^2 I_z} (\Sigma k_y X^2 + \Sigma k_x Y^2)$$

##### A5.4.2. INCLINED MOUNTINGS WITH THREE PLANES OF SYMMETRY

When at least one mounting has either  $k_a \neq k_r$  or its axis not parallel to a coordinate axis, in addition to the conditions for the XZ and YZ planes as stated for inclined mountings with two planes of symmetry in Sections A5.2.2 and A5.3.2, the following conditions must be met to secure symmetry relative also to the XY plane:

$$\Sigma k_x Z = \Sigma k_{xz} X, \quad \Sigma k_y Z = \Sigma k_{yz} Y$$

The formulas for the translational frequencies  $f_1, f_2, f_3$  as just presented for simply oriented mountings are unaffected by the inclination of the mountings, but the formulas for the rotational frequencies become

$$f_4^2 = \frac{1}{(2\pi)^2 I_X} (\Sigma k_Z Y^2 + \Sigma k_Y Z^2 - 2 \Sigma k_{YZ} YZ)$$

$$f_5^2 = \frac{1}{(2\pi)^2 I_Y} (\Sigma k_Z X^2 + \Sigma k_X Z^2 - 2 \Sigma k_{XZ} XZ)$$

$$f_6^2 = \frac{1}{(2\pi)^2 I_Z} (\Sigma k_Y X^2 + \Sigma k_X Y^2 - 2 \Sigma k_{XY} XY)$$

### A5.5. OBLIQUE CENTER-OF-GRAVITY ARRANGEMENTS

It can happen that special features of an arrangement, although not giving rise to an additional plane of symmetry, nevertheless greatly simplify the formulas for the frequencies. One important practical case, which includes two distinct subcases, will be treated.

Let the points of attachment of the mountings all be in a plane containing the center of gravity of the mounted body; let this plane, although containing the  $X$  axis, be inclined at an angle  $\theta$  to the  $Y$  axis. Thus two of the principal planes of inertia are inclined to the plane of the mountings. If mountings having unequal axial and radial stiffnesses are present, let their axes at least be perpendicular to the  $X$  axis, so that  $k_{XY} = k_{XZ} = 0$  for all mountings.

Let the arrangement be such that vibrational symmetry actually exists relative to the  $YZ$  plane and would also exist relative to the  $XZ$  plane if all terms containing  $Z$  or  $k_{YZ}$  were replaced by zero in the conditions for symmetry relative to  $XZ$ , that is,

$$\Sigma k_X Y = \Sigma k_Z Y = 0, \quad \Sigma k_Z XY = 0$$

in addition to the conditions for  $YZ$  symmetry with inclined mountings as stated in Section A5.2.2. Note that here  $k_{XY} = k_{XZ} = 0$  for all mountings. Since also  $Z = Y \tan \theta$  for each mounting, it follows also that

$$\Sigma k_X Z = \Sigma k_Z Z = 0, \quad \Sigma k_Z XZ = 0$$

Case 1: Assume also either that all the mountings have equal stiffness in all directions, so that  $k_Y = k_Z$  and  $k_{YZ} = 0$ , or that the mountings,  $N$  in number, are identical and have parallel axes. Then the following additional equations hold:

$$\Sigma k_Y Y = \Sigma k_{YZ} X = \Sigma k_{YZ} Y = 0, \quad \Sigma k_Y XY = \Sigma k_{YZ} XY = 0$$

$$\Sigma k_Y Z = \Sigma k_{YZ} Z = 0, \quad \Sigma k_Y XZ = \Sigma k_{YZ} XZ = 0$$

On the second assumption these equations hold because all  $k$ 's cancel and the equations state nothing new.

The equations of motion, Section A5.1.1, now give, for solutions containing  $\sin \omega t$ ,

$$\begin{aligned}(\Sigma k_X - m\omega^2) u &= 0 \\(\Sigma k_Y - m\omega^2) v + (\Sigma k_{YZ}) w &= 0 \\(\Sigma k_Z - m\omega^2) w + (\Sigma k_{YZ}) v &= 0 \\[\Sigma k_Z Y^2 + (\Sigma k_Y Y^2) \tan^2 \theta - 2(\Sigma k_{YZ} Y^2) \tan \theta - I_X \omega^2] \alpha &= 0 \\[\Sigma k_Z X^2 + (\Sigma k_X Y^2) \tan^2 \theta - I_Y \omega^2] \beta - [(\Sigma k_X Y^2) \tan \theta + \Sigma k_{YZ} X^2] \gamma &= 0 \\[\Sigma(k_Y X^2 + k_X Y^2) - I_Z \omega^2] \gamma - [(\Sigma k_X Y^2) \tan \theta + \Sigma k_{YZ} X^2] \beta &= 0\end{aligned}$$

The equations are solved for the six frequencies; the last four give two quadratic equations for  $\omega^2$ .

Case 2: Instead of the additional assumptions of Case 1, let it be required, in addition to the previously stated conditions, that

$$\Sigma k_{YZ} X = 0, \quad \Sigma k_Y XY = 0, \quad \Sigma k_{YZ} XY = 0$$

whence it follows also that

$$\Sigma k_Y XZ = 0, \quad \Sigma k_{YZ} XZ = 0$$

Then the equations containing  $u$ ,  $\beta$ , or  $\gamma$  are the same as in Case 1 but the other three are as follows:

$$\begin{aligned}(\Sigma k_Y - m\omega^2) v + (\Sigma k_{YZ}) w - \Sigma[(k_Y \tan \theta - k_{YZ}) Y] \alpha &= 0 \\(\Sigma k_Z - m\omega^2) w + (\Sigma k_{YZ}) v - (\Sigma k_{YZ} Y) (\tan \theta) \alpha &= 0 \\[\Sigma k_Z Y^2 + (\Sigma k_Y Y^2) \tan^2 \theta - 2(\Sigma k_{YZ} Y^2) \tan \theta - I_X \omega^2] \alpha \\- \Sigma[(k_Y \tan \theta - k_{YZ}) Y] v - (\Sigma k_{YZ} Y) (\tan \theta) w &= 0\end{aligned}$$

These three equations lead to a cubic equation in  $\omega^2$  like those encountered with only  $YZ$  symmetry and can be treated in the same way.

## APPENDIX 6

### PRACTICAL ARRANGEMENTS OF INCLINED MOUNTINGS

The equations of motion and the general conditions of elastic symmetry of arrangements of inclined mountings were given in Sections A5.2.2, A5.3.2, and A5.4.2. Possible practical arrangements will be discussed here. Some further remarks will be made on vibrational symmetry and on specific cases of arrangements for three, two, and one planes of vibrational symmetry. Arrangements of inclined mountings with equal axial and radial stiffnesses are included in the cases treated in Chapter 3.

#### A6.1. VIBRATIONAL SYMMETRY WITH INCLINED MOUNTINGS

Let a mounting have axial and radial stiffnesses  $k_a$  and  $k_r$ , respectively, and let its axis make angles  $\phi_X, \phi_Y, \phi_Z$  with the  $X, Y, Z$  axes, respectively, where  $\cos^2 \phi_X + \cos^2 \phi_Y + \cos^2 \phi_Z = 1$ . Then, for this mounting,

$$k_X = k_a \cos^2 \phi_X + k_r \sin^2 \phi_X, \quad k_{XY} = (k_a - k_r) \cos \phi_X \cos \phi_Y$$

Similar formulas for  $k_Y, k_Z, k_{XZ}, k_{YZ}$  can be found by changing either  $Y$  to  $Z$  or  $X$  to  $Y$  or  $Z$ . Note that  $k_{YX} = k_{XY}, k_{ZX} = k_{XZ}, k_{ZY} = k_{YZ}$ . A small displacement  $u$  of the equipment toward positive  $X$  causes a restoring force  $k_X u$  on the equipment directed toward negative  $X$ , and also skew forces  $k_{XY} u, k_{XZ} u$  directed toward negative  $Y$  and  $Z$ , respectively. If the axis of the mounting lies in the  $XY$  plane, then  $\cos \phi_Y = \sin \phi_X, \cos \phi_Z = 0$ , and  $k_{XZ} = k_{YZ} = 0$ , and similarly for other planes.

To secure geometrical or mirror symmetry relative to a given plane when  $k_a \neq k_r$ , any mounting and its image mounting must have their axes on lines which are mirror images of each other in the plane, e.g., either 2a or 2b for the image of 1 in Figure 42. If the  $X$  axis is drawn perpendicular to the plane of symmetry, and if  $\phi_X, \phi_Y, \phi_Z$  refer to a mounting and  $\phi'_X, \phi'_Y, \phi'_Z$  to its image, then  $\phi'_X = \pi - \phi_X, \phi'_Y = \phi_Y, \phi'_Z = \phi_Z$ , so that  $\cos \phi'_X = -\cos \phi_X, \cos \phi'_Y = \cos \phi_Y, \cos \phi'_Z = \cos \phi_Z, k'_{XY} = -k_{XY}, k'_{XZ} = -k_{XZ}$ .

Vibrational symmetry can exist, however, without geometrical symmetry; a few cases will be mentioned. The general conditions for elastic symmetry for inclined mountings are too complicated for practical use. The difficulties of allowing for the effects of inclination of mounting axes to the principal axis of inertia of the mounted equipment is the principal advantage of using mountings parallel to the inertial axes, or, if they must be inclined, using mountings with equal axial and radial stiffnesses.

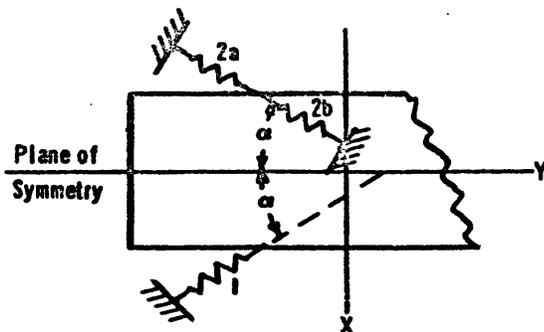


Figure 42 - Mirror Image of an Inclined Mounting

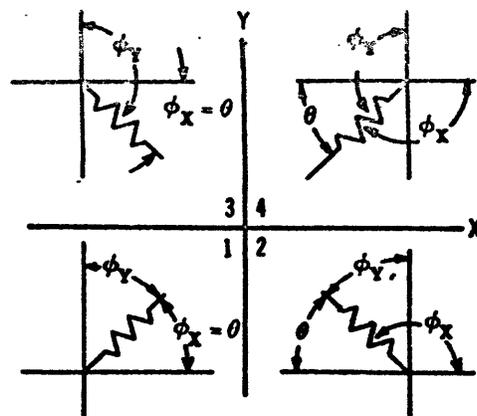


Figure 43 - Inclined Mountings with Axes Lying in the XY Plane

### A6.2. ARRANGEMENTS OF INCLINED MOUNTINGS WITH THREE PLANES OF VIBRATIONAL SYMMETRY

Inclined mountings with  $k_a \neq k_r$  in center-of-gravity arrangements with three planes of vibrational symmetry lead to great complications unless their axes all lie in the plane of attachment of the mountings. Even then, the only practical design seems to be the insertion of groups of four mountings, all in a group being identical and so located and oriented that each has a mirror image of itself in both of the two planes of symmetry that are perpendicular to the mounting plane. The locations and orientations of the four, if they are in the XY plane, see Figure 43, can be written

$$\begin{aligned}
 X_1 &= -A, & Y_1 &= -B, & \phi_X &= \theta, & \phi_Y &= \frac{\pi}{2} - \theta \\
 X_2 &= A, & Y_2 &= -B, & \phi_X &= \pi - \theta, & \phi_Y &= \frac{\pi}{2} - \theta \\
 X_3 &= -A, & Y_3 &= B, & \phi_X &= \theta, & \phi_Y &= \frac{\pi}{2} + \theta \\
 X_4 &= A, & Y_4 &= B, & \phi_X &= \pi - \theta, & \phi_Y &= \frac{\pi}{2} + \theta
 \end{aligned}$$

In such an arrangement, for each mounting

$$k_X = k_a \cos^2 \theta + k_r \sin^2 \theta, \quad k_Y = k_a \sin^2 \theta + k_r \cos^2 \theta$$

$$k_Z = k_r,$$

while  $k_{XY}$  has the same sign as  $XY(k_a - k_r)$ , and  $|k_{XY}| = |(k_a - k_r)| \sin \theta \cos \theta$ . Also

$$\phi_Z = \frac{\pi}{2} \text{ and } k_{XZ} = k_{YZ} = 0.$$

The frequency formulas for such arrangements are the same as those given in terms of  $k_X, k_Y, k_Z$  in the normal arrangements, Section 3.2.1.

### A6.3. ARRANGEMENTS OF INCLINED MOUNTINGS WITH TWO PLANES OF VIBRATIONAL SYMMETRY (XZ AND YZ)

In arrangements with two planes of vibrational symmetry, inclined mountings with unequal axial and radial stiffnesses can be introduced readily only in groups of four, the members of each group being identical and arranged so that each has a mirror image in each of the two planes of symmetry. The image must correspond both in the position of the mounting and in the orientation of its axis. The positions and orientations for a group, see Figure 44, can be specified in terms of arbitrary numbers  $A$ ,  $B$ , and arbitrary angles of  $\alpha$ ,  $\beta$  radians as follows, the planes of symmetry being the  $XZ$  and  $YZ$  planes:

$$X_1 = -A, Y_1 = -B, \phi_X = \alpha, \phi_Y = \beta; \quad X_2 = A, Y_2 = -B, \phi_X = \pi - \alpha, \phi_Y = \beta$$

$$X_3 = -A, Y_3 = B, \phi_X = \alpha, \phi_Y = \pi - \beta; \quad X_4 = A, Y_4 = B, \phi_X = \pi - \alpha, \phi_Y = \pi - \beta$$

The values of  $Z$ ,  $\phi_Z$ ,  $k_X$ ,  $k_Y$ , and  $k_Z$  are the same for all members of a group, whereas those members having equal and opposite  $X$  have also equal and opposite values of  $k_{XZ}$ , and  $k_{YZ}$  changes sign similarly with  $Y$  and  $k_{XY}$  with  $XY$ .

If at least some of the mountings have their axes parallel to a coordinate plane, further generalization may perhaps be accomplished easily without destroying the symmetry. If any two identical mountings have axes parallel to the  $YZ$  plane, so that  $\phi_X = \pi/2$  and  $k_{XY} = k_{XZ} = 0$  for both, and if these mountings have the same values of  $X^2$ ,  $Z$ , and  $\phi_Z$  but have equal and opposite  $Y$  and  $\cos \phi_Y$ , and

hence also equal and opposite  $k_{YZ}$ , then the elastic constants of these two mountings may be changed in any common arbitrary ratio  $n$  provided  $X$  is simultaneously changed for each in the ratio  $1/n$ . Note that for the two mountings  $X$  may have either the same value or equal and opposite values. Similarly, if the axes are parallel to  $XZ$ , so that  $k_{XY} = k_{YZ} = 0$ , and if  $Y^2$ ,  $Z$ , and  $\phi_Z$  are the same but  $X$ ,  $\cos \phi_X$ , and, hence,  $k_{XZ}$  are equal and opposite, then the elastic constants may be changed in the ratio  $n$  provided each  $Y$  is changed in the ratio  $1/n$ .

The general conditions for two planes of symmetry with inclined mountings were stated in Section A5.3.2.

For any of the arrangements just described with two planes of symmetry, the working formula for  $f_{tr,Z}$  stated in Section 3.3.3 for noncoplanar arrangements still holds, but here

$$f_{rot,Z} = \frac{3.13}{r_Z} \sqrt{\frac{1}{W} (\sum k_Y X^2 + \sum k_X Y^2 - 2 \sum k_{XY} XY)}$$

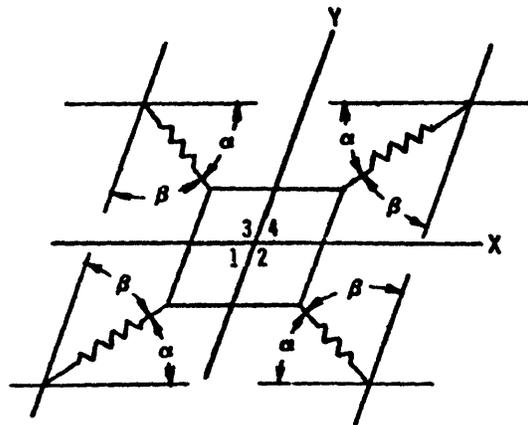


Figure 44 - Orientation of Inclined Mountings - Two Planes of Vibrational Symmetry

The formulas for  $f_{\max}$  and  $f_{\min}$  as there stated in terms of  $L_X$  and  $L_Y$  also still hold as well as those for  $P_1$  and  $P_2$ , but here  $L_X$ ,  $L_Y$ ,  $Q_1$ , and  $Q_2$  have the values:

For the  $XZ$  rocking modes,

$$L_X = \sqrt{\left(\frac{P_1 - Q_2}{2}\right)^2 + \frac{1}{W^2 r_Y^2} (\Sigma k_X Z - \Sigma k_{XZ} X)^2}$$

$$Q_2 = \frac{1}{W r_Y^2} (\Sigma k_Z X^2 + \Sigma k_X Z^2 - 2 \Sigma k_{XZ} XZ)$$

For the  $YZ$  rocking modes

$$L_Y = \sqrt{\left(\frac{P_2 - Q_1}{2}\right)^2 + \frac{1}{W^2 r_X^2} (\Sigma k_Y Z - \Sigma k_{YZ} Y)^2}$$

$$Q_1 = \frac{1}{W r_X^2} (\Sigma k_Z Y^2 + \Sigma k_Y Z^2 - 2 \Sigma k_{YZ} YZ)$$

#### A6.4. ARRANGEMENTS OF INCLINED MOUNTINGS WITH ONE PLANE OF VIBRATIONAL SYMMETRY ( $YZ$ )

The general conditions for vibrational symmetry relative to a single plane  $YZ$  are too involved for practical use; see Section A5.2.2. Mention will be made only of certain methods for designing such arrangements including inclined mountings with  $k_a \neq k_r$ .

Of the procedures listed in Section 3.4, Nos. 1, 2, and 4 can be used for any inclined mountings whose axes are parallel to the  $YZ$  plane; No. 3 can be used provided also  $k_{YZ} X$  has the same value for both members of a pair.

Complete geometrical symmetry with respect to the  $YZ$  plane is always sufficient; the mountings are then located in pairs. The members of a pair have equal and opposite values of  $X$ , supplementary values of  $\phi_Y$ , and the same values of  $Y$ ,  $Z$ ,  $\phi_Y$ ,  $\phi_Z$ ; therefore they have equal and opposite values of  $k_{XY}$  and  $k_{XZ}$  but the same  $k_{YZ}$ .

The frequencies  $f_1$  to  $f_6$  are obtained by solving the two cubic equations given in Section 3.4 or A5.2.2 with the values of  $B_1$ ,  $C_1$ ,  $D_1$ ,  $B_2$ ,  $C_2$ ,  $D_2$  as given in Section A5.2.2. The formulas may be converted into a numerical form like those in Section 3.4 by substituting

$$\frac{1}{4 \pi^2 m} = \frac{0.78}{W}, \quad \frac{1}{(2 \pi)^4 m^2} = \frac{95.6}{W^2}$$

## AA.4.1. OBLIQUE CENTER-OF-GRAVITY ARRANGEMENTS

Sometimes it is not convenient to have the mounting plane coincide with a principal plane of inertia of the equipment. It may happen that only one of the principal axes is horizontal, or it may be more convenient to incline the mounting plane. Then only one plane of vibrational symmetry will exist, and the methods of Section 3.4 can be employed in design and in the calculation of the frequencies. In special cases, however, the problem is simpler because of certain other features; several such cases will be given separate treatment here.

With the axes taken as usual along principal axes of inertia of the equipment, let the plane containing the points of attachment contain also the  $X$  axis but be at an angle  $\theta$  with the  $Y$  axis. Thus the mounting plane contains only one of the principal axes of inertia.

Since it may be desirable to have the mounting axes perpendicular to the mounting plane, any mounting with unequal axial and radial stiffnesses will only be required to have its axis parallel to the  $YZ$  plane. Then for any mounting  $k_{XY} = k_{XZ} = 0$ . Furthermore,  $Z = Y \tan \theta$  for all mountings; the origin is at the center of gravity.

Vibrational symmetry will be assumed to exist relative to the  $YZ$  plane, and it will also be assumed that both the  $YZ$  and the  $XZ$  planes would be planes of vibrational symmetry, if  $Z$  and  $k_{YZ}$  were made zero for each mounting without other changes. This necessitates, in addition to the requirements for  $YZ$  symmetry stated in Section 3.4, that  $\sum k_X Y = 0$ ,  $\sum k_Z Y = 0$ ,  $\sum k_Z XY = 0$ .

Two cases will now be treated.

Case 1: Assume in addition either

- (a) that every mounting has equal stiffness in all directions, so that  $k_{YZ} = 0$ , or
- (b) that the mountings are identical and have parallel axes so that  $k_{YZ}$  has the same value for all.

Under these conditions two translational modes occur in the  $YZ$  plane but probably not in the  $Y$  and  $Z$  directions, and two rotational modes occur about perpendicular axes in this plane. The frequency formulas for this case are:

For translation in the  $X$  direction,

$$f_{tr} = 3.13 \sqrt{\frac{1}{W} \sum k_X}$$

For rotation about  $X$ ,

$$f_{rot} = 3.13 \sqrt{\frac{1}{W} \{ \sum k_Z Y^2 + (\sum k_Y Y^2) \tan^2 \theta - 2k_{YZ} (\sum Y^2) \tan \theta \}}$$

For translations in certain directions in the  $YZ$  plane,

$$f = \sqrt{p' + q'^2} L, \quad L = \sqrt{(p' - q')^2 + s^2}$$

where

$$p' = \frac{4.89}{W} \Sigma k_Y, \quad q' = \frac{4.89}{W} \Sigma k_Z, \quad s = \frac{9.78}{W} \Sigma k_{YZ}$$

For rotations with axis in YZ plane,

$$f = \sqrt{p' + q'} L, \quad L = \sqrt{(p' - q')^2 + s^2}$$

where

$$p' = \frac{4.89}{W r_Y^2} [\Sigma k_Z X^2 + (\Sigma k_X Y^2) \tan^2 \theta]$$

$$q' = \frac{4.89}{W r_Z^2} (\Sigma k_Y X^2 + \Sigma k_X Y^2).$$

$$s = \frac{9.78}{W r_Y r_Z} [(\Sigma k_X Y^2) \tan \theta + \Sigma k_{YZ} X^2]$$

$$\text{Here } 3.13 = \sqrt{g}/2\pi, \quad 4.89 = \frac{1}{2} g/(2\pi)^2, \quad 9.78 = g/(2\pi)^2.$$

Case 2: As an alternative to Case 1, it may be assumed, in addition to the requirements previously stated, that the mountings are arranged with mirror symmetry relative to the YZ plane, so that each one is matched by another with equal and opposite  $x$  but the same values of  $Y$  and  $Z$  and of the elastic constants including  $k_{YZ}$ . Or, more generally, make

$$\Sigma k_Y XY = \Sigma k_{YZ} X = \Sigma k_{YZ} XY = 0$$

The formulas for  $f_{tr}$  and for rotations with the axis in the YZ plane are the same as in Case 1. The other three motions are rocking modes, the frequencies being roots of the cubic

$$f^6 - Bf^4 + Cf^2 - D = 0$$

where

$$B = p_2 + p_3 + q_1$$

$$C = p_2 p_3 + p_2 q_1 + p_3 q_1 - \frac{95.6}{W^2} \left[ \frac{1}{r_X^2} (\Sigma k_Y Y \tan \theta - \Sigma k_{YZ} Y)^2 + (\Sigma k_{YZ})^2 + \frac{1}{r_X^2} (\Sigma k_{YZ} Y)^2 \tan^2 \theta \right]$$

$$D = p_2 p_3 q_1 - \frac{95.6}{W^2} \left[ \frac{p_3}{r_X^2} (\Sigma k_Y Y \tan \theta - \Sigma k_{YZ} Y)^2 + q_1 (\Sigma k_{YZ})^2 + \frac{p_2}{r_X^2} (\Sigma k_{YZ} Y)^2 \tan^2 \theta \right]$$

$$+ \frac{1870}{W^3 r_X^2} (\Sigma k_{YZ}) (\Sigma k_{YZ} Y) [\Sigma (k_Y Y) \tan \theta - \Sigma k_{YZ} Y]$$

$$p_2 = \frac{9.78}{W} \Sigma k_Y, \quad p_3 = \frac{9.78}{W} \Sigma k_Z$$

$$q_1 = \frac{9.78}{W r_X^2} [\Sigma k_Z Y^2 + (\Sigma k_Y Y^2) \tan^2 \theta - 2(\Sigma k_{YZ} Y^2) \tan \theta]$$

Here  $9.78 = g/(2\pi)^2$ ,  $95.6 = g^2/(2\pi)^4$ ,  $1870 = 2g^3/(2\pi)^6$ .

For discussion and method of solution of the cubic equation, see Section 3.4.

## APPENDIX 7

### DESIGN WORK SHEETS FOR COMMON CASES OF MOUNTINGS

Sample work sheets are presented for calculating the natural frequencies and the required clearances for resiliently mounted equipment. These work sheets offer arrangements that minimize the repeated writing of numerical values and the amount of desk calculator computation. The sheets are offered as a convenience for those who are just starting work in this field and who have available only desk calculators. It is recognized that other forms may be more convenient under certain circumstances and that those who use electrical computers must code the problems to suit the particular machines.

A proposed design work sheet is presented for base mounting arrangements with two planes of vibrational symmetry,  $YZ$  and  $XZ$ , using the calculations of Problem 2, Appendix 2.2, as an illustration. Another proposed design work sheet is presented for braced mounting arrangements with one plane of vibrational symmetry,  $YZ$ , using the calculations of Problem 3, Appendix 2.3, as an illustration.

## BASE MOUNTING ARRANGEMENT - TWO PLANES OF VIBRATIONAL SYMMETRY, YZ AND XZ

### DESIGN WORK SHEET FOR NATURAL FREQUENCY AND CLEARANCE CALCULATIONS

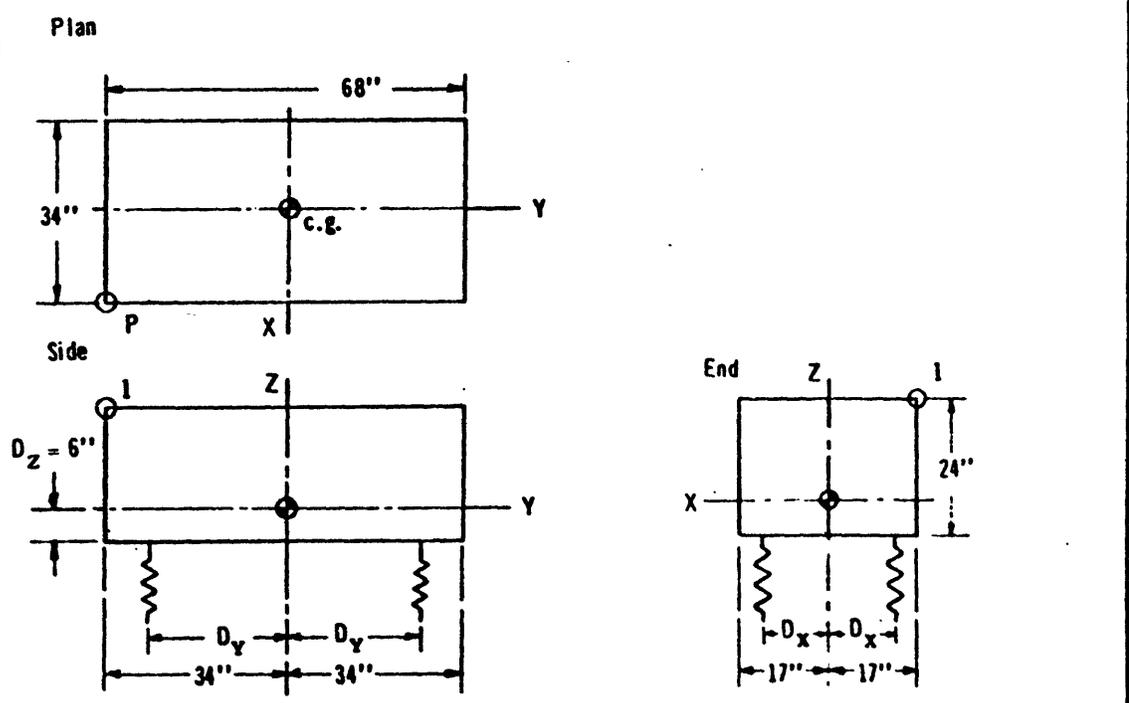
1. SHIP: \_\_\_\_\_ 2. COMPARTMENT \_\_\_\_\_

3. EQUIPMENT LOCATION: Frame No. \_\_\_\_\_ Port Side \_\_\_\_\_ Stbd. Side \_\_\_\_\_

4. EQUIPMENT: Name \_\_\_\_\_ Mod. No. \_\_\_\_\_ Ser. No. \_\_\_\_\_  
Mfr. \_\_\_\_\_ Dwg. No. \_\_\_\_\_  
WEIGHT: Equipment 7500 lb Subbase \_\_\_\_\_ Total 7500 lb  
Speeds or Exciting Frequencies of Equipment \_\_\_\_\_

5. FOUNDATION DWG. NO: BuShips \_\_\_\_\_ Shipbuilder \_\_\_\_\_

6. SKETCH OF MOUNTING ARRANGEMENT:



**RECOMMENDATIONS:**  
Use four 2000-lb mountings located at  $D_x = 10$  in.,  $D_y = 18.6$  in.,  $D_z = 6.0$  in.

Clearances	Calculated from Shock Data	Calculated from Static Load-Deflection Data
$C_{yz}$	2 5/16 in.	1 3/8 in.
$C_{xz}$	3 7/16 in.	2 in.
Vertical	1.0 in.	0.6 in.
Horizontal	1.0 in.	0.6 in.

CALCULATIONS BY: \_\_\_\_\_ ACTIVITY OR SHIPBUILDER: \_\_\_\_\_  
APPROVED BY: \_\_\_\_\_ DATE: \_\_\_\_\_  
DESIGN NUMBER: \_\_\_\_\_

## NATURAL FREQUENCY CALCULATIONS

<b>Frequencies to be avoided:</b> 24-cps excitation by ship 40-cps excitation by equipment			
<b>KNOWN DATA</b>			
Weight	7500 lb	$I_X$	8.66 × 10 <sup>3</sup> lb-in.-sec <sup>2</sup> $r_X$ 21.1 in.
Length	68 in.	$I_Y$	4.37 × 10 <sup>3</sup> lb-in.-sec <sup>2</sup> $r_Y$ 15.0 in.
Width	34 in.	$I_Z$	9.36 × 10 <sup>3</sup> lb-in.-sec <sup>2</sup> $r_Z$ 21.9 in.
Height	24 in.		
<b>SELECTION AND CHARACTERISTICS OF MOUNTINGS</b>			
<b>First Trial</b>		<b>Second Trial</b>	
2000-lb mountings for dead load of 1875 lb per mounting, natural frequency = 15 cps  $k_a = k_r = k_r + \pi/2$  <b>MOUNTING DEFLECTION:</b> from shock tests - $D = E = 1.0$ in. from static-load deflection tests - $D = E = 0.6$ in.		2000-lb mountings for dead load of 1875 lb per mounting, natural frequency = 15 cps  $k_a = k_r = k_r + \pi/2$  <b>MOUNTING DEFLECTION:</b> from shock tests - $D = E = 1.0$ in. from static-load deflection tests - $D = E = 0.6$ in.	
<b>DYNAMIC STIFFNESSES OF MOUNTINGS</b>			
<b>First Trial</b>		<b>Second Trial</b>	
$k = \frac{W f^2}{9.8} = \frac{1875 \times (15)^2}{9.8} = 43,000 \text{ lb/in.}$		$k = \frac{W f^2}{9.8} = \frac{1875 \times (15)^2}{9.8} = 43,000 \text{ lb/in.}$	
<b>CALCULATION OF TRANSLATIONAL FREQUENCY</b>			
<b>First Trial</b>		<b>Second Trial</b>	
$f_{tr} = 3.13 \sqrt{\frac{\sum k}{W}} = 6.26 \sqrt{\frac{k}{W}} =$  $= 6.26 \sqrt{\frac{43000}{7500}} = 15 \text{ cps}$		$f_{tr} = 3.13 \sqrt{\frac{\sum k}{W}} = 6.26 \sqrt{\frac{k}{W}} =$  $= 6.26 \sqrt{\frac{43000}{7500}} = 15 \text{ cps}$	

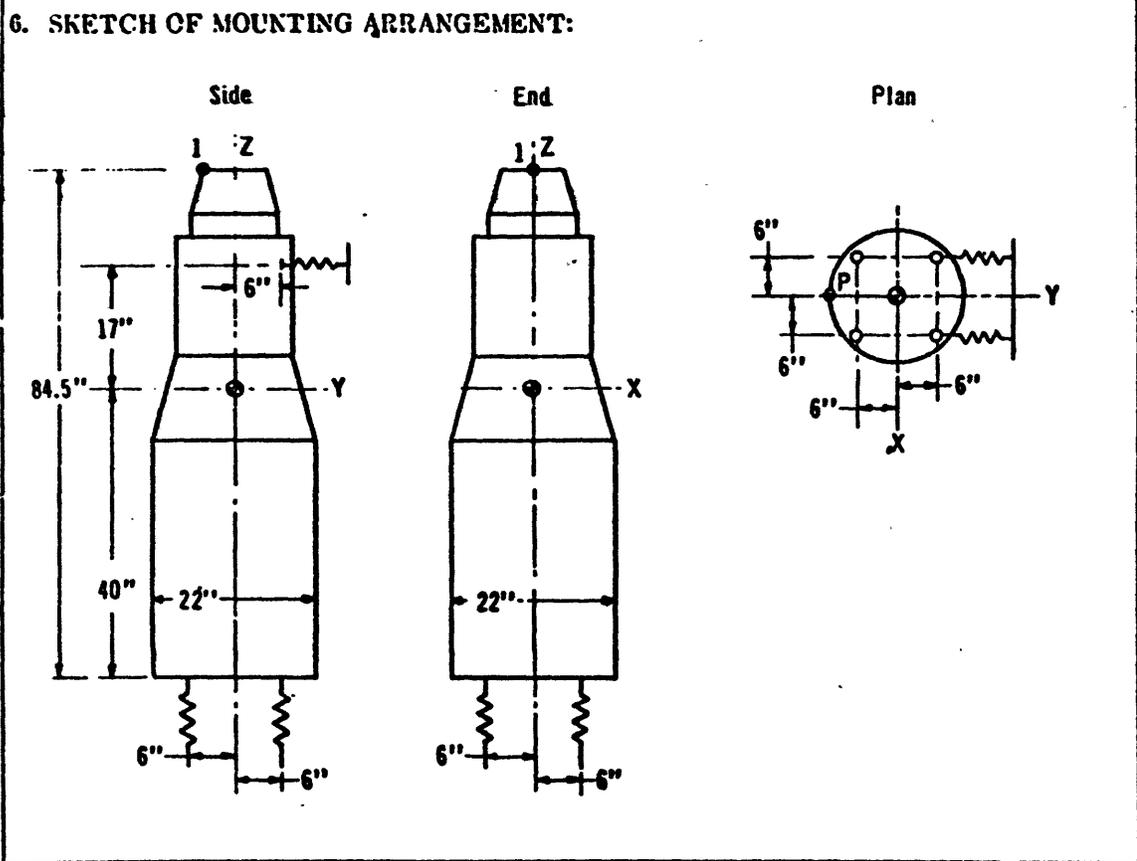
CALCULATION OF ROCKING FREQUENCIES IN YZ PLANE	
Half-Length of Base	
First Trial	Second Trial
$\frac{l/2}{r_x} = \frac{34}{21.1} = 1.61$	$\frac{l/2}{r_x} = \frac{34}{21.1} = 1.61$
Z Distance from Center of Gravity to Plane of Mountings	
First Trial	Second Trial
$\frac{D_z}{r_x} = \frac{6}{21.1} = 0.284$	$\frac{D_z}{r_x} = \frac{6}{21.1} = 0.284$
Selection of Mounting Position	
First Trial	Second Trial
$\frac{D_y}{r_x} = \frac{D_y}{21.1} = 1.60, D_y = 33.8 \text{ in.}$	$\frac{D_y}{r_x} = \frac{D_y}{21.1} = 0.88, D_y = 18.6 \text{ in.}$
Rocking Frequencies in YZ Plane	
First Trial	Second Trial
$\frac{f_{\min}(\text{Chart})}{f_{tr}} = \frac{f_{\min}}{15.0} = 0.975, f_{\min} = 14.6 \text{ cps}$	$\frac{f_{\min}}{f_{tr}} = \frac{f_{\min}}{15.0} = 0.8, f_{\min} = 12.0 \text{ cps}$
$\frac{f_{\max}(\text{Chart})}{f_{\min}(\text{Fig 22})} = \frac{f_{\max}}{14.6} = 1.65, f_{\max} = 24.1 \text{ cps}$	$\frac{f_{\max}}{f_{\min}} = \frac{f_{\max}}{12.0} = 1.4, f_{\max} = 16.8 \text{ cps}$
CALCULATION OF ROCKING FREQUENCIES IN XZ PLANE	
Half-Width of Base	
First Trial	Second Trial
$\frac{w/2}{r_y} = \frac{17}{15.0} = 1.13$	$\frac{w/2}{r_y} = \frac{17}{15.0} = 1.13$
Z Distance from Center of Gravity to Plane of Mountings - See Above Selection of Mounting Position	
First Trial	Second Trial
$\frac{D_x}{r_y} = \frac{D_x}{15} = 1.0, D_x = 15 \text{ in.}$	$\frac{D_x}{r_y} = \frac{D_x}{15} = 0.67, D_x = 10 \text{ in.}$
Rocking Frequencies in XZ Plane	
First Trial	Second Trial
$\frac{f_{\min}(\text{Chart})}{f_{tr}} = \frac{f_{\min}}{15.0} = 0.82, f_{\min} = 12.3 \text{ cps}$	$\frac{f_{\min}}{f_{tr}} = \frac{f_{\min}}{15.0} = 0.6, f_{\min} = 9.0 \text{ cps}$
$\frac{f_{\max}(\text{Chart})}{f_{\min}(\text{Fig 22})} = \frac{f_{\max}}{12.3} = 1.50, f_{\max} = 18.4 \text{ cps}$	$\frac{f_{\max}}{f_{\min}} = \frac{f_{\max}}{9.0} = 1.9, f_{\max} = 17.1 \text{ cps}$

CALCULATION OF ROTATIONAL FREQUENCY			
First Trial		Second Trial	
$f_{rot} = \frac{3.13}{r_z} \sqrt{\frac{1}{W} (\sum k_y X^2 + \sum k_x Y^2)}$ $= \frac{6.26}{r_z} \sqrt{\frac{k}{W} (X^2 + Y^2)} =$ $= \frac{6.26}{21.9} \sqrt{\frac{43000}{7500} [(15)^2 + (33.8)^2]} = 25.2 \text{ cps}$		$f_{rot} = \frac{3.13}{r_z} \sqrt{\frac{1}{W} (\sum k_y X^2 + \sum k_x Y^2)}$ $= \frac{6.26}{r_z} \sqrt{\frac{k}{W} (X^2 + Y^2)} =$ $= \frac{6.26}{21.9} \sqrt{\frac{43000}{7500} [(10)^2 + (18.6)^2]} = 14.5 \text{ cps}$	
SUMMARY OF RESULTS			
First Trial		Second Trial	
$D_X = 15.0 \text{ in.}, D_Y = 33.8 \text{ in.}, D_Z = 6.0 \text{ in.}$ $f_{tr} = 15.0 \text{ cps}$ $f_{min, X} = 12.3 \text{ cps}, f_{max, X} = 18.4 \text{ cps}$ $f_{min, Y} = 14.6 \text{ cps}, f_{max, Y} = 24.1 \text{ cps}$ $f_{rot} = 25.2 \text{ cps}$		$D_X = 10 \text{ in.}, D_Y = 18.6 \text{ in.}, D_Z = 6.0 \text{ in.}$ $f_{tr} = 15.0 \text{ cps}$ $f_{min, X} = 9.0 \text{ cps}, f_{max, X} = 17.1 \text{ cps}$ $f_{min, Y} = 12.0 \text{ cps}, f_{max, Y} = 16.8 \text{ cps}$ $f_{rot} = 14.5 \text{ cps}$	
CLEARANCE CALCULATIONS			
Approximate Formula: $C = \frac{2DH}{S} + E$ - See Section 1.6.			
$C$ = Clearance for Movement of Point 1 Shown on Sketch			
First Trial		Second Trial	
Shock Test Data	Static Load Deflection Data	Shock Test Data	Static Load Deflection Data
		$D = 1.0 \text{ in.}, E = 1.0 \text{ in.}$ $H = 24 \text{ in.}$ $W_{YZ} = 37.2 \text{ in.}$ $W_{XZ} = 20 \text{ in.}$ $C_{YZ} = \frac{2 \times 1.0 \times 24}{37.2} + 1.0$ $= 1.29 + 1 = 2.29$ Call $2 \frac{5}{16} \text{ in.}$ $C_{XZ} = \frac{2 \times 1.0 \times 24}{20} + 1.0$ $= 2.4 + 1 = 3.4$ Call $3 \frac{7}{16} \text{ in.}$	$D = 0.6 \text{ in.}, E = 0.6 \text{ in.}$ $H = 24 \text{ in.}$ $W_{YZ} = 37.2 \text{ in.}$ $W_{XZ} = 20 \text{ in.}$ $C_{YZ} = \frac{2 \times 0.6 \times 24}{37.2} + 0.6$ $= 0.78 + 0.6 = 1.38$ Call $1 \frac{3}{8} \text{ in.}$ $C_{XZ} = \frac{2 \times 0.6 \times 24}{20} + 0.6$ $= 1.44 + 0.6 = 2.04$ Call $2 \text{ in.}$
Clearances: 			

**BRACED MOUNTING ARRANGEMENT - ONE PLANE OF  
VIBRATIONAL SYMMETRY, YZ**

**DESIGN WORK SHEET FOR NATURAL FREQUENCY AND CLEARANCE CALCULATIONS**

1. SHIP: \_\_\_\_\_ 2. COMPARTMENT \_\_\_\_\_  
 3. EQUIPMENT LOCATION: Frame No. \_\_\_\_\_ Port Side \_\_\_\_\_ Stbd Side \_\_\_\_\_  
 4. EQUIPMENT: Name Trim Pump and Motor Mod. No. \_\_\_\_\_ Ser. No. \_\_\_\_\_  
 Mfr. \_\_\_\_\_ Dwg. No. \_\_\_\_\_  
 WEIGHT: Equipment 2035 lb Subbase \_\_\_\_\_ Total 2035 lb  
 Speeds or Exciting Frequencies of Equipment \_\_\_\_\_  
 5. FOUNDATION DWG. NO: BuShips \_\_\_\_\_ Shipbuilder \_\_\_\_\_



**RECOMMENDATIONS:** Use four 550-lb base mountings and two 50-lb back mountings whose locations are shown on page 131 under Selection of Mounting Positions.

Clearances	Calculated from Shock Data	Calculated from Static Load
		Deflection Data
$C_{yz}$	1.1 in.	0.7 in.
$C_{xz}$	1.1 in.	0.7 in.
Vertical	1.0 in.	0.6 in.
Horizontal	1.0 in.	0.6 in.

CALCULATIONS BY: \_\_\_\_\_ ACTIVITY OR SHIPBUILDER: \_\_\_\_\_  
 APPROVED BY: \_\_\_\_\_ DATE: \_\_\_\_\_  
 DESIGN NUMBER: \_\_\_\_\_

## NATURAL FREQUENCY CALCULATIONS

Frequencies to be avoided:						
<b>KNOWN DATA</b>						
Weight	2035 lb	$I_x$	2150 lb-in.-sec <sup>2</sup>			
Length	22.0 in.	$I_y$	2150 lb-in.-sec <sup>2</sup>			
Width	22.0 in.	$I_z$	321 lb-in.-sec <sup>2</sup>			
Height	84.5 in.					
<b>SELECTION AND CHARACTERISTICS OF MOUNTINGS</b>						
First Trial				Second Trial		
550-lb bottom mountings 50-lb back mountings for 550-lb mountings, $f_a = f_r = 16.7$ cps for 50-lb mountings, $f_a = f_r = 16.0$ cps $k_a = k_r = k_r + \pi/2$ <b>MOUNTING DEFLECTION:</b> from shock tests 1.0 in. from static load deflection tests 0.6 in.						
<b>DYNAMIC STIFFNESSES OF MOUNTINGS</b>						
First Trial				Second Trial		
$k = \frac{W f^2}{9.8} =$  for 550-lb mtgs $k_a = k_r = \frac{484 (16.7)^2}{9.8} = 13,860$ lb/in.  for 50-lb mtgs $k_a = k_r = \frac{44 (16.0)^2}{9.8} = 1150$ lb/in.				$k = \frac{W f^2}{9.8} =$		
<b>SELECTION OF MOUNTING POSITIONS</b>						
First Trial				Second Trial		
Mtg	X	Y	Z	$k_x$	$k_y$	$k_z$
1	+6	-6	-40	13860	13860	13860
2	+6	+6	-40	13860	13860	13860
3	-6	+6	-40	13860	13860	13860
4	-6	-6	-40	13860	13860	13860
5	+6	+6	+17	1150	1150	1150
6	-6	+6	+17	1150	1150	1150
7						
8						
9						
10						
Mtg	X	Y	Z	$k_x$	$k_y$	$k_z$
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

CALCULATIONS FOR  $a_1, b_1, c_1$  and  $a_2, b_2, c_2$

Radius of Gyration					
First Trial			Second Trial		
$r_x^2 = \frac{I_x}{m} = \frac{2150}{5.272} = 407.8 \text{ in.}^2$			$r_x^2 = \frac{I_x}{m} =$		
$r_y^2 = \frac{I_y}{m} = \frac{2150}{5.272} = 407.8 \text{ in.}^2$			$r_y^2 = \frac{I_y}{m} =$		
$r_z^2 = \frac{I_z}{m} = \frac{321}{5.272} = 60.89 \text{ in.}^2$			$r_z^2 = \frac{I_z}{m} =$		
Summations					
	First Trial	Second Trial		First Trial	Second Trial
$k_1 Y_1$	$-8.160 \times 10^4$		$k_1 Y_1^2$	$0.4896 \times 10^6$	
$k_2 Y_2$	$+8.160 \times 10^4$		$k_2 Y_2^2$	$0.4896 \times 10^6$	
$k_3 Y_3$	$+8.160 \times 10^4$		$k_3 Y_3^2$	$0.4896 \times 10^6$	
$k_4 Y_4$	$-8.160 \times 10^4$		$k_4 Y_4^2$	$0.4896 \times 10^6$	
$k_5 Y_5$	$+0.690 \times 10^4$		$k_5 Y_5^2$	$0.0414 \times 10^6$	
$k_6 Y_6$	$+0.690 \times 10^4$		$k_6 Y_6^2$	$0.0414 \times 10^6$	
$k_7 Y_7$			$k_7 Y_7^2$		
$k_8 Y_8$			$k_8 Y_8^2$		
$k_9 Y_9$			$k_9 Y_9^2$		
$k_{10} Y_{10}$			$k_{10} Y_{10}^2$		
$\Sigma k Y$	$1.380 \times 10^4$		$\Sigma k Y^2$	$2.041 \times 10^6$	
$(\Sigma k Y)^2$	$1.904 \times 10^8$				
$k_1 X_1^2$	$0.4896 \times 10^6$		$k_1 Y_1 Z_1$	$32.64 \times 10^5$	
$k_2 X_2^2$	$0.4896 \times 10^6$		$k_2 Y_2 Z_2$	$32.64 \times 10^5$	
$k_3 X_3^2$	$0.4896 \times 10^6$		$k_3 Y_3 Z_3$	$32.64 \times 10^5$	
$k_4 X_4^2$	$0.4896 \times 10^6$		$k_4 Y_4 Z_4$	$32.64 \times 10^5$	
$k_5 X_5^2$	$0.0414 \times 10^6$		$k_5 Y_5 Z_5$	$1.173 \times 10^5$	
$k_6 X_6^2$	$0.0414 \times 10^6$		$k_6 Y_6 Z_6$	$1.173 \times 10^5$	
$k_7 X_7^2$			$k_7 Y_7 Z_7$		
$k_8 X_8^2$			$k_8 Y_8 Z_8$		
$k_9 X_9^2$			$k_9 Y_9 Z_9$		
$k_{10} X_{10}^2$			$k_{10} Y_{10} Z_{10}$		
$\Sigma k X^2$	$2.041 \times 10^6$		$\Sigma k Y Z$	$2.346 \times 10^5$	
			$(\Sigma k Y Z)^2$	$5.504 \times 10^{10}$	
$k_1 Z_1$	$-5.440 \times 10^5$		$k_1 Z_1^2$	$2.176 \times 10^7$	
$k_2 Z_2$	$-5.440 \times 10^5$		$k_2 Z_2^2$	$2.176 \times 10^7$	
$k_3 Z_3$	$-5.440 \times 10^5$		$k_3 Z_3^2$	$2.176 \times 10^7$	
$k_4 Z_4$	$-5.440 \times 10^5$		$k_4 Z_4^2$	$2.176 \times 10^7$	
$k_5 Z_5$	$+0.196 \times 10^5$		$k_5 Z_5^2$	$0.033 \times 10^7$	
$k_6 Z_6$	$+0.196 \times 10^5$		$k_6 Z_6^2$	$0.033 \times 10^7$	
$k_7 Z_7$			$k_7 Z_7^2$		
$k_8 Z_8$			$k_8 Z_8^2$		
$k_9 Z_9$			$k_9 Z_9^2$		
$k_{10} Z_{10}$			$k_{10} Z_{10}^2$		
$\Sigma k Z$	$-2.137 \times 10^6$		$\Sigma k Z^2$	$8.770 \times 10^7$	
$(\Sigma k Z)^2$	$4.567 \times 10^{12}$				

CALCULATIONS FOR $p_1, p_2, p_3$ and $q_1, q_2, q_3$		
	First Trial	Second Trial
$p_1 = 9.78 \frac{\sum k_X}{W}$	$\frac{9.78 \times 5.670 \times 10^4}{2035} = 2.722 \times 10^2$	
$p_2 = 9.78 \frac{\sum k_Y}{W}$	$= 2.722 \times 10^2$	
$p_3 = 9.78 \frac{\sum k_Z}{W}$	$= 2.722 \times 10^2$	
$q_1 = \frac{9.78}{W r_X^2} [\sum k_Z Y^2 + \sum k_X Z^2]$	$\frac{9.78}{2035 \times 407.8} [2.04 \times 10^6 + 8.770 \times 10^7]$ $= 1.056 \times 10^3$	
$q_2 = \frac{9.78}{W r_Y^2} [\sum k_Z X^2 + \sum k_X Z^2] = v_1$	$= 1.056 \times 10^3$	
$q_3 = \frac{9.78}{W r_Z^2} [\sum k_Y X^2 + \sum k_X Y^2]$	$\frac{9.78}{2035 \times 60.89} [2.041 \times 10^6 + 2.041 \times 10^6]$ $= 3.219 \times 10^2$	
CALCULATIONS FOR $B_1, C_1, D_1$ and $B_2, C_2, D_2$		
$B_1 = p_2 + p_3 + q_1$		
First Trial		
$B_1 = 2.722 \times 10^2 + 2.722 \times 10^2 + 1.056 \times 10^3 = 1.600 \times 10^3$		
Second Trial		
$C_1 = p_2 p_3 + p_2 q_1 + p_3 q_1 - \frac{95.6}{W^2 r_X^2} [(\sum k_Z Y)^2 + (\sum k_Y Z)^2]$		
First Trial.		
$C_1 = 2.722 \times 10^2 \times 2.722 \times 10^2 + 2.722 \times 10^2 \times 1.056 \times 10^3 + 2.72 \times 10^2 \times 1.056 \times 10^3 - \frac{95.6}{(2035)^2 \times 407.8}$		
Second Trial		
$\times [1.904 \times 10^8 + 4.567 \times 10^{12}] = 3.904 \times 10^5$		

$$D_1 = p_2 p_3 q_1 - \frac{95.6}{W^2 r_X^2} [p_2 (\sum k_Z Y)^2 + p_3 (\sum k_Y Z)^2]$$

First Trial

$$D_1 = 2.722 \times 10^2 \times 2.722 \times 10^2 \times 1.056 \times 10^3 - \frac{95.6}{(2035)^2 \times 607.8} [2.722 \times 10^2 \times 1.904 \times 10^8 + 2.722 \times 10^2 \times 4.567 \times 10^{12}]$$

$$= 7.89 \times 10^6$$

Second Trial

$$B_2 = p_1 + q_2 + q_3$$

First Trial

$$B_2 = 2.722 \times 10^2 + 1.056 \times 10^3 + 3.219 \times 10^2 = 1.650 \times 10^3$$

Second Trial

$$C_2 = p_1 q_2 + p_1 q_3 + q_2 q_3 - \frac{95.6}{W^2} \left[ \frac{1}{r_Z^2} (\sum k_X Y)^2 + \frac{1}{r_Y^2} (\sum k_X Z)^2 + \frac{1}{r_Y^2 r_Z^2} (\sum k_X Y Z)^2 \right]$$

First Trial  $C_2 = 2.722 \times 10^2 \times 1.056 \times 10^3 + 2.722 \times 10^2 \times 3.219 \times 10^2 + 1.056 \times 10^3 \times 3.219 \times 10^2$

$$- \frac{95.6}{(2035)^2} \left[ \frac{1.904 \times 10^8}{60.89} + \frac{4.567 \times 10^{12}}{407.8} + \frac{5.504 \times 10^{10}}{407.8 \times 60.89} \right] = 4.561 \times 10^5$$

Second Trial

$$D_2 = p_1 q_2 q_3 - \frac{95.6}{W^2} \left[ \frac{q_2}{r_Z^2} (\sum k_X Y)^2 + \frac{q_3}{r_Z^2} (\sum k_X Z)^2 + \frac{p_1}{r_Y^2 r_Z^2} (\sum k_X Y Z)^2 \right] + \frac{1870}{W^3 r_Y^2 r_Z^2} (\sum k_X Y)(\sum k_X Z)$$

$\times (\sum k_X Y Z)$

First Trial  $D_2 = 2.722 \times 10^2 \times 1.056 \times 10^3 \times 3.219 \times 10^2 - \frac{95.6}{(2035)^2} \left[ \frac{1.056 \times 10^3}{60.89} (1.904 \times 10^8) \right.$

$$\left. + \frac{3.219 \times 10^2}{407.8} (4.567 \times 10^{12}) + \frac{2.722 \times 10^2}{60.89 \times 407.8} (5.504 \times 10^{10}) \right] + \frac{1870}{(2035)^3 60.89 \times 407.8} (1.380 \times 10^4) (-2.137 \times 10^6)$$

Second Trial

$$\times (2.346 \times 10^5) = 9.12 \times 10^6$$

Constants $a_1, b_1, c_1$ and $a_2, b_2, c_2$		
YZ Plane	First Trial	Second Trial
$a_1 = \frac{C_1}{B_1^2}$	$\frac{3.904 \times 10^5}{(1.600 \times 10^3)^2} = 0.153$	
$b_2 = \frac{D_1}{B_1^3}$	$\frac{7.89 \times 10^6}{(1.600 \times 10^3)^3} = 0.00193$	
$c_1 = B_1$	$= 1.600 \times 10^3$	
Other		
$a_2 = \frac{C_2}{B_2^2}$	$\frac{4.561 \times 10^5}{(1.650 \times 10^3)^2} = 0.168$	
$b_2 = \frac{D_2}{B_2^3}$	$\frac{9.12 \times 10^6}{(1.650 \times 10^3)^3} = 0.00203$	
$c_2 = B_2$	$= 1.65 \times 10^3$	

**NATURAL FREQUENCIES OBTAINED FROM CHART, FIGURE 23**

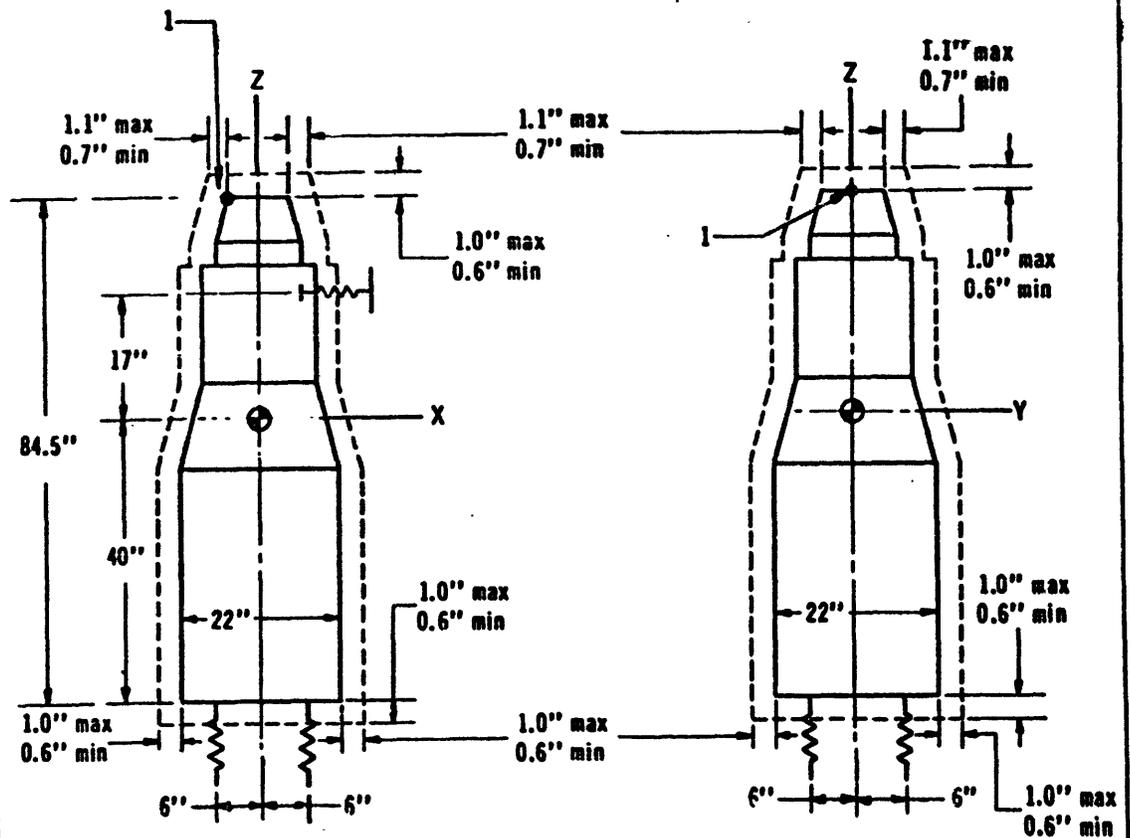
First Trial		Second Trial	
YZ Plane	Other	YZ Plane	Other
$f_1 = 36.0$ cps	$f_4 = 35.9$ cps	$f_1 =$ cps	$f_4 =$ cps
$f_2 = 16.5$ cps	$f_5 = 18.0$ cps	$f_2 =$ cps	$f_5 =$ cps
$f_3 = 4.7$ cps	$f_6 = 4.6$ cps	$f_3 =$ cps	$f_6 =$ cps

### CLEARANCE CALCULATIONS

Approximate Formula:  $C = \text{Deflection of Mounting} + \text{a Factor}$   
 $C = \text{Clearance for Movement of Point 1 Shown on Sketch}$

	First Trial		Second Trial	
	Shock Test Data	Static Load Deflection Data	Shock Test Data	Static Load Deflection Data
$C_{YZ} =$	$1 + \frac{1}{8} = 1.1 \text{ in.}$	$0.6 + \frac{1}{16} = 0.7 \text{ in.}$		
$C_{XZ} =$	$1 + \frac{1}{8} = 1.1 \text{ in.}$	$0.6 + \frac{1}{16} = 0.7 \text{ in.}$		
$C_{\text{Vertical}} =$	1 in.	0.6 in.		
$C_{\text{Horiz}} =$	1 in.	0.6 in.		

#### Clearances



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**David Taylor Model Basin. Report 880.**

**A GUIDE FOR THE SELECTION AND APPLICATION OF RESILIENT MOUNTINGS TO SHIPBOARD EQUIPMENT, REVISED,** by Francis F. Vane. February 1958. xi, 143p. Tables, graphs, diagrams, refs. (Structural Mechanics Laboratory research and development report) UNCLASSIFIED

This manual has been prepared to provide guidance to design agencies for the selection and application of resilient mountings to shipboard equipment. All the mounting arrangements expected to be encountered in practical installations have been treated. For the more commonly encountered arrangements, the computations have been simplified and systematized as much as possible, and charts have been used to reduce further the computational work.

1. Vibration isolators - Applications
2. Ships - Equipment - Vibration
3. Compression shock - Reduction
4. Noise - Reduction
- I. Resilient mountings.
- II. Vane, Francis F.
- III. NS713-017

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