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# NAVY DEPARTMENT

## THE DAVID W. TAYLOR MODEL BASIN

### WASHINGTON 7, D.C.

# A PROCEDURE TO IMPART SPECIFIED DYNAMICAL PROPERTIES TO SHIP MODELS



by

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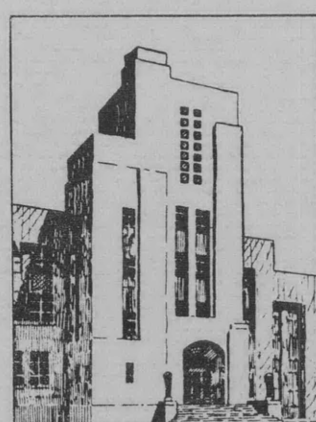

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# NOTATION

| | |
|---|---|
| $b$ | Length |
| $d_1, d_2$ | Lengths; see Figure 1 |
| $F$ | Elliptic integral of the first kind |
| $g$ | Gravitational acceleration |
| $H$ | $I - I_0$ |
| $I$ | Moment of inertia of model in pitch about its center of gravity |
| $I_0$ | Required value of $I$ |
| $I_r$ | See Equation [17] |
| $I_1, I_2, \overline{I}_1, \overline{I}_2$ | Moments of inertia of weights about their own centers of gravity |
| $K$ | Complete elliptic integral of the first kind |
| $k$ | Radius of gyration associated with $I$ |
| $L$ | $l - l_0$ |
| $l, l'$ | Distances from center of gravity to center of rotation |
| $l_0$ | Required value of $l$ |
| $P_1, P_2$ | $\rho_1 - \rho_1'$, $\rho_2 - \rho_2'$ |
| $r_1, r_2, s_1, s_2$ | Lengths; see Figure 1 |
| $t$ | Time |
| $W$ | Weight of model including only that portion of the towing bracket which rotates with the model |
| $W_b$ | Weight of that portion of the towing bracket which does not rotate with the model |
| $W_1, W_2$ | Weights of the ballast |
| $w_b$ | Weight of that portion of the towing bracket which rotates with the model |
| $w_1, w_2$ | Test weights |
| $\alpha$ | Maximum angle of rotation |
| $\delta$ | Logarithmic decrement |
| $\delta_1, \delta_2$ | Lengths; see Figure 1 |
| $\zeta$ | Damping factor |
| $\theta$ | Angle of rotation |
| $\lambda$ | See Equation [12] |

| | |
|---|---|
| $\rho_1, \rho_2, \sigma_1, \sigma_2$ | Lengths; see Figure 1 |
| $\Sigma_1, \Sigma_2$ | $\sigma_1 - \sigma_1', \sigma_2 - \sigma_2'$ |
| $\tau_f$ | Period |
| $\phi$ | Defined by $\sin\frac{\theta}{2} = \sin\frac{\alpha}{2} \sin\phi$ |
| $\omega, \omega', \omega_d, \omega_f$ | Frequencies |

## ABSTRACT

A procedure is presented to systematize the ballasting of ship models intended for studies of ship motions in waves. The method permits determining a model's center of gravity and its moment of inertia in pitch for any particular ballasting condition; it also yields a specification for the shift in ballast necessary to realize a required center of gravity and pitching moment of inertia.

## INTRODUCTION

Up to the present time, almost all testing of ship models in waves has been done with the model towed in head or following seas. If the resulting two-dimensional motion of the model is to be considered meaningful for the full-scale ship, the center of gravity of ship and model must correspond and the radius of gyration in pitch must be appropriately scaled. The ballasting of the model must then be such as to yield a specified displacement, trim, center of gravity, and pitching moment of inertia.

The first two of these four requirements may be met in a straightforward manner. The other two requirements can also be satisfied concurrently with the first two by a direct method such as the one described in this report. It has, however, been common practice to ignore the vertical location of the center of gravity and to arrive at the appropriate moment of inertia by a trial and error procedure. Further, one of the methods[1] frequently used for finding the moment of inertia rests on the assumption that moments of inertia in yaw and pitch are almost identical. Although this assumption seems to be a good one in most instances, it serves to introduce an indeterminate systematic error into the results. This unsatisfactory situation with respect to widely accepted ballasting procedures motivated the present work.

It is possible to prescribe a routine procedure for ballasting a ship model which will provide simultaneously the required displacement, trim, center of gravity, and moment of inertia in pitch by a direct, noniterative method such as the method described below. It is designed to take advantage of the widespread practice of using a towing bracket pivoted at a fixed point in the model. Although it depends on the possibility of suspending a model from a freely pivoting towing bracket, the analysis can be readily modified to correspond to other means of model suspension which may be more convenient for models equipped with other towing arrangements.

In the following sections, the required analysis is performed, some remarks are given about details of the procedure which might be helpful in practice, and finally, an outline for a suggested ballasting procedure is presented.

## BALLASTING PROCEDURE

It will be presumed that the model has been supplied with sufficient ballast to obtain the required total displacement and that this ballast is divided into two portions, one forward and

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[1] References are listed on page 14.

one aft of the center of gravity. If either the forward or after ballast is composed of more than one element, any shift in location specified for the ballast applies to each such element. It will further be presumed that the ballast is initially so positioned in the model as to place the center of gravity in its correct longitudinal position and that the towing bracket is installed so that its pivot point is directly above the center of gravity. The manner in which these stipulations may be satisfied and the possible requirements for locating the towing bracket pivot vertically are discussed in the section on practical details.

## ANALYSIS

### Equation of Motion

If the model is suspended in air by its towing bracket and allowed to rotate freely, it will come to rest in a horizontal orientation because of the location of the towing bracket pivot directly above the model's center of gravity. The model will then respond to a small initial angular displacement by oscillating about its position of equilibrium with a frequency $\omega$. The governing equation of motion can be written as

$$\frac{d^2 \theta}{dt^2} + \omega^2 \theta = 0 \qquad [1]$$

where $\theta$ represents the angular displacement from equilibrium and $\omega$ is given by

$$\omega^2 = -\frac{Wl}{I + \frac{W}{g} l^2} \qquad [2]$$

Here $W$ is the weight of the model which rotates about the towing bracket pivot (i.e., $W$ does not include the portion of the towing bracket which is attached to the member supporting the model and thus does not rotate with the model),

$l$ is the vertical coordinate measuring the distance (in the equilibrium position) between the towing bracket pivot and the model's center of gravity,and

$I$ is the moment of inertia in pitch about the center of gravity of the model.

Note that $l$ is always negative (see Figure 1). $W$ can be measured in a straightforward manner. The other two quantities are initially unknown.

Equation [1] can be written as shown only after making two assumptions. The first is that damping effects are negligible, and the second is that amplitudes of motion are small.

If a damping term were included in Equation [1] to represent friction in the bearings at the towing bracket pivot, then the damped frequency of oscillation $\omega_d$ would be related to the $\omega$ of Equation [2] by

$$\omega_d = \sqrt{1-\zeta^2}\, \omega$$

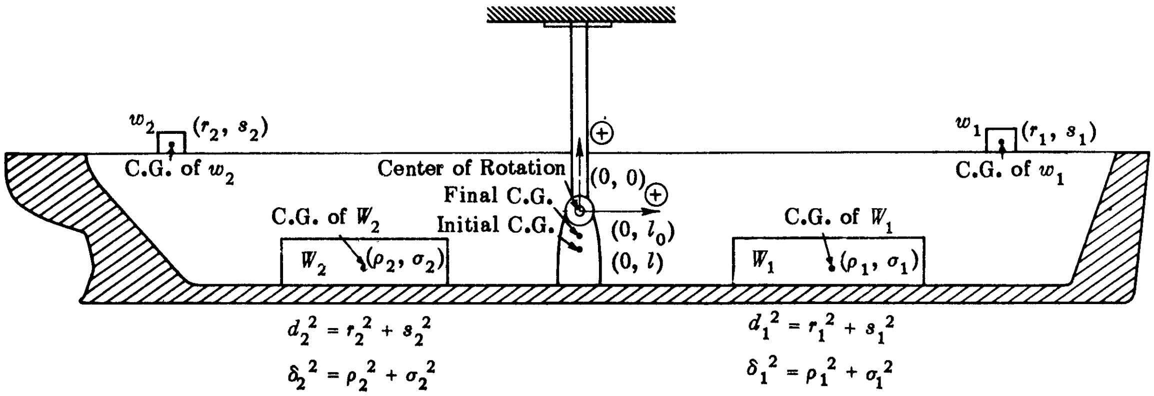


Figure 1 - Schematic Diagram of Model Suspended by Towing Bracket, Showing Notation

where $\zeta$ is the ratio of the coefficient of damping to the critical value of this coefficient and is called the damping factor. The damping factor will, of course, depend upon the particular bearings used, but experience at the Taylor Model Basin indicates that the amplitude of oscillation will decay to half its initial value in no less (usually much more) than about 5 to 10 cycles. Thus if 8 cycles is arbitrarily assumed to be a representative value, the logarithmic decrement $\delta$ is 1/8 log 2 = 0.087. Using the approximate relation $\delta = 2\pi\,\zeta$, $\zeta$ is 0.014 and $\omega_d = \sqrt{1-\zeta^2}\;\omega = 0.9999\omega$. Thus using $\omega$ in place of $\omega_d$ results in an error of only about one part in ten thousand, which is negligible as compared to errors of measurement. Sources of damping error other than in the towing bracket bearings, such as air drag, can safely be assumed to be negligible because of the slowness of the oscillations.

If damping is neglected, Equation [1] should still be written as

$$\frac{d^2\,\theta}{dt^2} + \omega^2 \sin\theta = 0 \qquad [3]$$

With the initial conditions that for $t = 0$, $\theta$ has its maximum value $\alpha$ and $d\theta/dt = 0$, the solution of Equation [3] is[2]

$$\omega t = F\left(\frac{\alpha}{2}, \phi\right) - K\left(\frac{\alpha}{2}\right) \qquad [4]$$

where $\phi$ is related to $\theta$ by

$$\sin\frac{\theta}{2} = \sin\frac{\alpha}{2}\sin\phi$$

$F\,(\alpha/2,\,\phi)$ is the elliptic integral of the first kind and $K\,(\alpha/2)$ is the complete elliptic integral of the first kind. From Equation [4],

$$\begin{aligned}\omega\,(t + T_f) &= F\left(\frac{\alpha}{2}, \phi + 2\pi\right) - K\left(\frac{\alpha}{2}\right)\\ &= 4\,K\left(\frac{\alpha}{2}\right) + F\left(\frac{\alpha}{2}, \phi\right) - K\left(\frac{\alpha}{2}\right)\end{aligned}$$

where $\tau_f$ is the period for finite amplitude motions and the second line above follows from the recurrence relation[3] for $F(\alpha/2, \phi)$. Therefore

$$\tau_f = \frac{4}{\omega} K\left(\frac{\alpha}{2}\right)$$

or

$$\omega_f = \frac{\pi \omega}{2K\left(\frac{\alpha}{2}\right)}$$

where $\omega_f$ is the frequency associated with the period $\tau_f$. If a limit of error of five parts in ten thousand is imposed upon replacing $\omega_f$ by $\omega$ in order to make such an error small as compared to errors of measurement, then by consulting tables[3] of $K(\alpha/2)$ it is found that $\alpha$ may be as large as about 5 degrees. Even if the initial displacement given to the model is somewhat greater, the motion will decay and most of the cycles timed will have amplitudes much smaller than 5 degrees.

Since it has been shown that the separate effects of damping and finite amplitudes of oscillation can be neglected, it seems safe to assume that any coupling between these two effects also has a negligible influence. Thus the use of Equation [1] is justified, and the angular frequency of the model motion is given by Equation [2].

### Moment of Inertia and Center of Gravity

Suppose that two weights $w_1$ and $w_2$ are placed on the deck of the model at distances $d_1$ and $d_2$ from the center of rotation (see Figure 1). Let these weights be small enough that the new configuration is still stable with respect to oscillations about the towing bracket pivot and let the weights be located such that $w_1 r_1 = -w_2 r_2$, where $r_1$ and $r_2$ are the horizontal components of $d_1$ and $d_2$ ($s_1$ and $s_2$ are the vertical components of $d_1$ and $d_2$). Note that the signs of the components of $d_1$ and $d_2$ are fixed by the choice of coordinates as shown in Figure 1, where the origin is at the center of rotation. The model will still possess the same equilibrium position but now has an angular frequency $\omega'$ where, from Equation [2],

$$\omega'^2 = -\frac{(W + w_1 + w_2)\, l'}{I + (I_1 + \frac{w_1}{g} d_1^2) + (I_2 + \frac{w_2}{g} d_2^2) + \frac{W}{g} l^2} \qquad [5]$$

Here, $l'$ is the new vertical coordinate measuring the distance between the centers of rotation and gravity, and $I_1$ and $I_2$ represent the moments of inertia of $w_1$ and $w_2$ about axes parallel to the towing bracket pivot but passing through the centers of gravity of the weights $w_1$ and $w_2$. From the definition for center of gravity, it follows that

$$l' = \frac{Wl + w_1 s_1 + w_2 s_2}{W + w_1 + w_2} \qquad [6]$$

Equations [2], [5], and [6] constitute a set of three simultaneous equations for $I$, $l$, and $l'$ in terms of the measured quantities $W$, $w_1$, $w_2$, $I_1$, $I_2$, $d_1$, $d_2$, $s_1$, $s_2$, $\omega$ and $\omega'$. Although the solutions for $I$ and $l$ from this set of equations can actually be used, the expressions are rather awkward, and it will usually be found more satisfactory to use some approximate relations which are considerably simpler. Thus, $l$ is ordinarily sufficiently small to expect that $(W/g)\,l^2$ can be neglected as compared to $I$. In fact, if $I$ and $l$ are assumed known, the error involved in calculating $\omega^2$ from

$$\omega^2 = -\frac{Wl}{I} \qquad [7]$$

instead of from Equation [2] is just

$$\frac{Wl^2/g}{I}$$

From an actual model which was ballasted by this method, and which may be considered typical: $W = 170$ lb, $l = -0.05$ ft, and $I = 13$ lb ft sec$^2$. Thus the error is approximately one part in a thousand. If this error is acceptable, and in general it should be, then $(W/g)\,l^2$ may also be dropped from Equation [5]. Equation [5] also contains the terms $I_1$ and $I_2$ which might be expected to be very small. The error in $\omega'^2$ caused by omitting $I_1$ and $I_2$ is (with the $(W/g)\,l^2$ term deleted)

$$\frac{I_1 + I_2}{I + \frac{w_1}{g} d_1^2 + \frac{w_2}{g} d_2^2}$$

If $w_1$ and $w_2$ are, say, square bars of cross section $b \times b$ laid on the deck with their length in the beamwise direction, then $I_1 = I_2 = \frac{1}{6}\frac{w_1}{g} b^2 = \frac{1}{6}\frac{w_2}{g} b^2$. The error is thus

$$\frac{\frac{1}{3}\frac{w_1}{g} b^2}{I + \frac{2w_1}{g} d_1^2}$$

for $w_1 = w_2$ and $d_1 = d_2$.

Again resorting to example: $w_1 = 5$ lb, $b = 0.15$ ft, $d_1 = 3$ ft, and $I = 13$ lb ft sec$^2$. Then the error in $\omega'^2$ is less than one part in ten thousand, and thus $I_1$ and $I_2$ could readily be neglected. In this connection, it should be noted that unlike the $(W/g)\,l^2$ term, $I_1$ and $I_2$ can be retained in the equations with little algebraic complication. They are dropped here because their effect is so small and because it would hardly pay to require that they be measured when actually ballasting a model. It should be noted that the errors quoted above were in $\omega^2$ (and $\omega'^2$) rather than in $\omega$ (and $\omega'$). However,

$$\Delta\omega = \frac{d\omega}{d(\omega^2)}\Delta(\omega^2) = \frac{1}{2\omega}\Delta(\omega^2)$$

or

$$\frac{\Delta \omega}{\omega} = \frac{1}{2} \frac{\Delta (\omega^2)}{\omega^2}$$

Thus the error in $\omega$ (and $\omega'$) is just one-half that in $\omega^2$ (and $\omega'^2$).

From Equations [6], [7] and

$$\omega'^2 = - \frac{(W + w_1 + w_2)\, l'}{I + \frac{w_1}{g} d_1^2 + \frac{w_2}{g} d_2^2} \qquad [8]$$

it follows that

$$I = \frac{(w_1 d_1^2 + w_2 d_2^2) \frac{\omega'^2}{g} + w_1 s_1 + w_2 s_2}{\omega^2 - \omega'^2} \qquad [9]$$

and

$$l = - I \frac{\omega^2}{W} \qquad [10]$$

Equations [9] and [10] represent the required results. They could be replaced by the solutions of Equations [2], [5], and [6] if the need for accuracy is exceptional or if the particular ballasting problem is unusual in that the error in using Equations [9] and [10] is much greater than was indicated earlier.

If the results are desired in terms of the radius of gyration $k$, then

$$k = \sqrt{\frac{I g}{W + W_b}} \qquad [11]$$

where $W_b$ is the weight of that portion of the towing bracket which does not oscillate with the model, i.e., $W + W_b$ is the total model weight.

## Required Ballast Shifts

The program to be followed in ballasting a model would be to obtain the required displacement and trim, then suspend the model by the towing bracket, and find $I$ and $l$ as described above. If, then, a moment of inertia $I_0$ and a distance of $l_0$ from towing bracket pivot to center of gravity are prescribed for the model in its final ballasted state, it is possible to specify the manner in which the forward and aft ballast should be shifted so that $I_0$ and $l_0$ will result from Equations [9] and [10] while the displacement and trim remain unchanged. To calculate the required ballast shift requires a knowledge of the center of gravity of the forward and after groups of ballast individually. To this end, it is convenient to have the individual weights forming the ballast of such geometrical shape that the center of gravity of each group of ballast can be determined readily.

If the distances from the center of rotation to the centers of gravity of the forward and after ballast groups are $\delta_1$ and $\delta_2$ respectively, with components $\rho_1$, $\sigma_1$, and $\rho_2$, $\sigma_2$ (Figure 1), then the equation for the original position of the model's center of gravity is, in vector notation

$$W\vec{l} = W_1 \vec{\delta}_1 + W_2 \vec{\delta}_2 + (W - W_1 - W_2)\vec{\lambda} \qquad [12]$$

Here $W_1$ and $W_2$ are the weights of the forward and after ballast and $\lambda$ is the distance from the center of rotation to the center of gravity of the model alone lacking $W_1$ and $W_2$.

After shifting the ballast, Equation [12] transforms to

$$W\vec{l}_0 = W_1 \vec{\delta}_1{}' + W_2 \vec{\delta}_2{}' + (W - W_1 - W_2)\vec{\lambda} \qquad [13]$$

where the primes indicate final values of $\delta_1$ and $\delta_2$. If Equation [13] is subtracted from Equation [12], the two component equations are

$$0 = W_1 (\rho_1 - \rho_1') + W_2 (\rho_2 - \rho_2') \qquad [14]$$

$$W(l - l_0) = W_1 (\sigma_1 - \sigma_1') + W_2 (\sigma_2 - \sigma_2') \qquad [15]$$

where again primes are used to indicate final values. If the vertical shifts for $W_1$ and $W_2$ are required to be the same, then

$$\sigma_1 - \sigma_1' = \sigma_2 - \sigma_2' \qquad [16]$$

Let $\bar{I}_1$ and $\bar{I}_2$ be the moments of inertia of $W_1$ and $W_2$ about axes parallel to the axis of the towing bracket pivot and passing through the centers of gravity of $W_1$ and $W_2$ respectively. Then, initially

$$I + \frac{W}{g} l^2 = \bar{I}_1 + \frac{W_1}{g}\delta_1^2 + \bar{I}_2 + \frac{W_2}{g}\delta_2^2 + I_r \qquad [17]$$

where $I_r$ is a "residual" moment of inertia about the center of rotation which is defined by Equation [17]. After the ballast shift,

$$I_0 + \frac{W}{g} l_0^2 = \bar{I}_1 + \frac{W_1}{g}\delta_1'^2 + \bar{I}_2 + \frac{W_2}{g}\delta_2'^2 + I_r \qquad [18]$$

The difference of Equations [17] and [18] is

$$I - I_0 = \frac{W_1}{g}(\delta_1^2 - \delta_1'^2) + \frac{W_2}{g}(\delta_2^2 - \delta_2'^2) - \frac{W}{g}(l^2 - l_0^2) \qquad [19]$$

Equations [14], [15], [16], and [19] comprise four equations in $\rho_1'$, $\rho_2'$, $\sigma_1'$, and $\sigma_2'$. Equivalently, they represent four equations in the four components of the ballast shifts $\rho_1 - \rho_1'$, $\rho_2 - \rho_2'$, $\sigma_1 - \sigma_1'$, and $\sigma_2 - \sigma_2'$.

Introduce the notation

$$
\begin{aligned}
P_1 &= \rho_1 - \rho_1' \\
P_2 &= \rho_2 - \rho_2' \\
\Sigma_1 &= \sigma_1 - \sigma_1' \\
\Sigma_2 &= \sigma_2 - \sigma_2' \\
H &= I - I_0 \\
L &= l - l_0
\end{aligned}
\qquad [20]
$$

Then since

$$
\begin{aligned}
\delta_1^2 - \delta_1'^2 &= -P_1^2 - \Sigma_1^2 + 2\rho_1 P_1 + 2\sigma_1 \Sigma_1 \\
\delta_2^2 - \delta_2'^2 &= -P_2^2 - \Sigma_2^2 + 2\rho_2 P_2 + 2\sigma_2 \Sigma_2 \\
l^2 - l_0^2 &\doteq L\,(L + 2\,l_0)
\end{aligned}
$$

Equations [14], [15], [16], and [19] can be rewritten as

$$W_1 P_1 + W_2 P_2 = 0 \qquad [21]$$

$$W_1 \Sigma_1 + W_2 \Sigma_2 = WL \qquad [22]$$

$$\Sigma_1 - \Sigma_2 = 0 \qquad [23]$$

$$
\begin{aligned}
W_1(P_1^2 + \Sigma_1^2 - 2\rho_1 P_1 - 2\sigma_1 \Sigma_1) + W_2(P_2^2 + \Sigma_2^2 - 2\rho_2 P_2 - 2\sigma_2 \Sigma_2)& \\
+ WL\,(L + 2\,l_0) = -\,g\,H&
\end{aligned}
\qquad [24]
$$

From Equations [22] and [23],

$$\Sigma_1 = \Sigma_2 = \frac{WL}{W_1 + W_2} \doteq \Sigma \qquad [25]$$

and from Equations [21], [24], and [25],

$$P_1 = \frac{1}{1 + \frac{W_1}{W_2}} \left\{ (\rho_1 - \rho_2) \pm \sqrt{(\rho_1 - \rho_2)^2 - gH\left(\frac{1}{W_1} + \frac{1}{W_2}\right) - \frac{W^2 L}{W_1 W_2}\left[L + \frac{W_1}{W}(L + 2\,l_0 - 2\sigma_1) + \frac{W_2}{W}(L + 2\,l_0 - 2\sigma_2)\right]} \right\}$$

To resolve the ambiguity in sign, consider the special case $W_1 = W_2$, $H = 0$, $L = 0$. Then

$$P_1 = \frac{1}{2}\left[(\rho_1 - \rho_2) \pm (\rho_1 - \rho_2)\right]$$

Using the upper sign leads to $\rho_1' = \rho_2$, and using the lower sign leads to $\rho_1' = \rho_1$. Clearly the lower sign is appropriate since the other possibility would require interchange of the forward and after ballast. Finally

$$P_1 = \frac{1}{1+\frac{W_1}{W_2}}\left\{(\rho_1-\rho_2) - \sqrt{(\rho_1-\rho_2)^2 - gH\left(\frac{1}{W_1}+\frac{1}{W_2}\right) - \frac{W^2L}{W_1W_2}\left[L+\frac{W_1}{W}\,(L+2l_0-2\sigma_1)+\frac{W_2}{W}\,(L+2l_0-2\sigma_2)\right]}\right\} \qquad [26]$$

$$P_2 = -\frac{1+\frac{W_1}{W_2}}{1+\frac{W_2}{W_1}}\,P_1 = -\frac{W_1}{W_2}\,P_1 \qquad [27]$$

Equations [25], [26], and [27] completely define the ballast shifts and are thus the required results.

Simplification of Equation [26] is possible though neither necessary nor advisable. Usually, the term containing $L$ in the square root is small compared to the second term, which is, in turn, small compared to the first term. If the square root is expanded in series and only the first two terms retained,

$$P_1 = \left(\frac{\rho_1-\rho_2}{1+\frac{W_1}{W_2}}\right)\frac{gH}{2}\left(\frac{1}{W_1}+\frac{1}{W_2}\right)\frac{1}{(\rho_1-\rho_2)^2}$$

or

$$P_1 = \frac{gH}{2W_1\,(\rho_1-\rho_2)} \qquad [28]$$

As indicated, Equations [25], [26], and [27] are to be preferred to Equations [25], [28], and [27] since under some circumstances, Equation [28] may introduce errors quite as large, or larger, than the errors of measurement.

## Propagation of Error

To estimate the error caused in the derived quantities of interest as a result of errors in the directly measured quantities, it is sufficient to work with the simplified forms of the equations obtained by setting $w_1 = w_2$, $d_1 = d_2$, and $s_1 = s_2$. Thus Equation [9] may be written

$$I = 2w_1\left(\frac{d_1^{\,2}\,\frac{\omega'^2}{g} + s_1}{\omega^2-\omega'^2}\right)$$

From the first-order terms of a Taylor expansion,

$$\Delta I = \frac{\partial I}{\partial w_1}\,\Delta w_1 + \frac{\partial I}{\partial d_1}\,\Delta d_1 + \frac{\partial I}{\partial \omega'}\,\Delta \omega' + \frac{\partial I}{\partial s_1}\,\Delta s_1 + \frac{\partial I}{\partial \omega}\,\Delta \omega$$

The partial derivatives are given by

$$\frac{\partial I}{\partial w_1} = \frac{I}{w_1}$$

$$\frac{\partial I}{\partial d_1} = \frac{2I'}{d_1}$$

$$\frac{\partial I}{\partial \omega'} = \frac{2I'}{\omega'} + \frac{2\omega' I}{\omega^2 - \omega'^2}$$

$$\frac{\partial I}{\partial s_1} = \frac{I''}{s_1}$$

$$\frac{\partial I}{\partial \omega} = -\frac{2\omega I}{\omega^2 - \omega'^2}$$

where $I'$ represents the first term in the expression for $I$ and $I''$ represents the second term. Then to ascertain separately the effect of errors in the measured quantities on errors in $I$,

$$\frac{(\Delta I/I)_{w_1}}{\Delta w_1/w_1} = 1$$

$$\frac{(\Delta I/I)_{d_1}}{\Delta d_1/d_1} = \frac{2I'}{I}$$

$$\frac{(\Delta I/I)_{\omega'}}{\Delta \omega'/\omega'} = \frac{2I'}{I} + \frac{2\omega'^2}{\omega^2 - \omega'^2}$$

$$\frac{(\Delta I/I)_{s_1}}{\Delta s_1/s_1} = \frac{I''}{I}$$

$$\frac{(\Delta I/I)_{\omega}}{\Delta \omega/\omega} = -\frac{2\omega^2}{\omega^2 - \omega'^2}$$

Since $I' << I''$ usually, then $w_1$ and $s_1$ propagate errors linearly into $I$, while $I$ is relatively insensitive to errors in $d_1$. Notice the importance of choosing $w_1$ sufficiently large to make $\omega^2 - \omega'^2$ at least of the magnitude of $\omega'^2$ and preferably greater. In fact, making $\omega^2$ three or four times as large as $\omega'^2$ is a considerable help in reducing the error. In any event, the errors in frequency measurement are the most important ones to minimize.

It can be seen from Equation [10] that errors in $l$ result from errors in $w_1$, $d_1$, $\omega'$, and $s_1$ in the same fashion as in the case of $I$, and the effect of errors in $W$ and $\omega$ on errors in $l$ can be ascertained from

$$\frac{\left(\frac{\Delta l}{l}\right)_W}{\frac{\Delta W}{W}} = -1$$

$$\frac{\left(\frac{\Delta l}{l}\right)_\omega}{\frac{\Delta \omega}{\omega}} = 2 - \frac{2\,\omega^2}{\omega^2 - \omega'^2}$$

Errors in $\Sigma$ can be examined readily from Equation [25]. Here it is sufficient to note that since $W$, $W_1$, and $W_2$ can usually be measured with very good precision, errors in $\Sigma$ result almost entirely from errors in $l$, upon which $\Sigma$ is linearly dependent.

Because the terms containing $L$ in Equation [26] are usually so small, any reasonably precise measurements of $\sigma_1$, $\sigma_2$, and $L$ are adequate to cause only negligibly small errors in $P_1$ (and $P_2$). It is thus sufficient to examine Equation [28], which shows that errors in $H$, $W_1$ and $\rho_1 - \rho_2$ propagate linearly into $P_1$, with analogous results obtaining for $P_2$ from Equations [28] and [27]. It should be noted that an error in $P_1$ (or $P_2$) is much greater than the resulting error in $\rho_1'$ (or $\rho_2'$), the factor being on the order of $\rho_1'/P_1$ (or $\rho_2'/P_2$).

## PRACTICAL DETAILS

### Ballasting Requirements

In the foregoing analysis it has been assumed (1) that the model could be ballasted initially to the correct displacement and trim condition and (2) that the towing bracket pivot could be fixed directly above the longitudinal location of the center of gravity. Achievement of the correct total model weight is entirely straightforward. If the actual location of the center of gravity is specified in advance, then it is a simple matter to locate the towing bracket at the appropriate position. It is more usual, however, to have a trim condition specified, with no a priori information about the center of gravity. In such a circumstance, the towing bracket could first be fastened in a position judged to be a good guess for the final center of gravity. The model is then floated, and the ballast is shifted to yield the required trim. Then the model is supported by its towing bracket and, since in general it is not located directly above the center of gravity, one end of the model will tend to drop.

Suppose, for example, it is the bow which drops. Then a weight is placed on deck astern of the towing bracket and moved to a position at which the model assumes a horizontal equilibrium position. If $W$ is the model weight excluding the movable portion of the towing bracket, $\Delta w$ the balancing weight placed on deck, and $x$ the horizontal distance between the center of gravity of $\Delta w$ and the towing bracket pivot, then the center of gravity of the model is a distance $y$ forward of the towing bracket pivot, where

$$y = \frac{\Delta w}{W}\, x \qquad [29]$$

The towing bracket could then be moved forward a distance $y$ and usually be acceptably close to the longitudinal position of the center of gravity. However, when the towing bracket is moved, the center of gravity of the model also moves slightly. To achieve a true balance, the towing bracket should be moved to the position which corresponds to the center of gravity of the model less the entire weight of the towing bracket. If $w_b$ is the weight of that portion of the towing bracket which is fixed to and rotates with the model, then the center of gravity of $W - w_b$ is a horizontal distance $z$ forward of the center of gravity of $W$, where $z$ is

$$z = \frac{w_b}{W - w_b} y$$

Hence, the towing bracket should be moved forward a distance $z + y$,

$$z + y = \left(1 + \frac{w_b}{W - w_b}\right) y = \left(\frac{W}{W - w_b}\right) \frac{\Delta w}{W} x = \left(\frac{\Delta w}{W - w_b}\right) x \qquad [30]$$

Usually, the difference between the results of Equations [29] and [30] is significant only if the initial guess for the towing bracket position is so poor that $(\Delta w)\, x$ is large.

The vertical location of the towing bracket pivot is at the discretion of the experimenter. He may choose to locate it at the vertical location of the center of gravity of the model, or he may favor some other location such as one lying along the thrust line of the ship. His choice determines the value of $l_0$, which is zero if the towing force is to be applied to the center of gravity of the model.

## Possible Difficulties

It may occur that the vertical location chosen for the towing bracket pivot is so low in the model that the model's initial center of gravity may lie above it. In this case the model is unstable when supported by the towing bracket, or at best only neutrally stable with no possibility of oscillations about an equilibrium position. Or, it may happen that the towing bracket pivot is so close to the center of gravity that the addition of weights on deck to determine a value for $\omega'$ makes the model unstable. In either of these cases, the recommended procedure would be to raise the towing bracket pivot by some known distance from its desired location, sufficient to insure stable oscillations. Then the entire program for finding $I$, $l$, $\Sigma$, $P_1$, and $P_2$ can be followed with the appropriately modified value of $l_0$, and finally, the towing bracket can be restored to its desired location as the last step in the ballasting procedure.

Although no specific information about the nature of the ballast is needed to determine $I$, $l$, and $\Sigma$, it is necessary to know the center of gravity of both forward and after groups of ballast to evaluate $P_1$ and $P_2$. Some attention should be devoted to arranging the ballast to facilitate finding the center of gravity. For example, if calibrated weights are used, it is simple to find the center of gravity if the weights are grouped in a symmetrical arrangement. If lead sheets are used for ballast, the sheets can be cut to a uniform geometrical planform and arranged in straight stacks. Similar convenient dispositions may be practical for other types of ballast.

A response to information supplied by values of $P_1$ and $P_2$ simply involves movement of the ballast along the longitudinal centerline of the model – a motion which generally involves no difficulties. However, the value of $\Sigma$ dictates how far the ballast should be moved upwards, and such a displacement of the ballast requires a means of support in the new position. It should be noted that generally the ballast will initially be placed as low in the model as possible to ensure a stable condition when the model is suspended by the towing bracket. Thus the required vertical displacement of the ballast is almost invariably upwards. In anticipation of this necessity, it is desirable to make allowance for the probable weight of blocks to raise the ballast to some new location and to include the weight of these blocks when adjusting the total model weight. Initially, when the ballast is low in the model, the weight and moment of inertia of the support blocks can be represented by small pieces of lead, for example, located as close as possible to the expected position of the support blocks.

A model used for testing in waves will usually be equipped with a deck which is fastened in place with screws and made watertight by the use of a filling material to plug small gaps around the deck. During the process of ballasting, it is impractical to remove and then replace the deck fasteners and filling material whenever the deck must be removed for access to the ballast. Hence it is convenient to use small weights placed on deck to represent the weight and moment of inertia of the deck fasteners and filling material during the ballasting operation.

## Precision

As was mentioned in an earlier section, the errors in the determination of $I$ and $l$ can be reduced by proper choice of $\omega'$ as compared to $\omega$. This choice in turn depends upon the selection of the added weights $w_1$ and $w_2$. These weights should be chosen to make $\omega^2$ at least two or three times as large as $\omega'^2$, i.e., $\omega$ should be at least one and a half to one and three quarter times greater than $\omega'$. When timing the period of oscillation, it is very important to time as long an interval as possible. Timing devices such as a stopwatch are capable of timing a given interval with zero error for present purposes, but errors are introduced by the experimenter in starting and stopping the timing device at the end points of the timed interval. These errors become relatively smaller the longer the interval timed.

The calculations required to solve for $I$, $l$, $\Sigma$, $P_1$, and $P_2$ are quite straightforward. A word of caution about units might be in order. It is usually found convenient to make linear measurements in inches, but Equations [9], [11], [26], and [28] contain $g$, the gravitational acceleration, which is usually quoted in units of ft/sec$^2$. A choice of feet or inches should be made at the beginning of the calculations and adhered to throughout.

Full exploitation of the capabilities of the measuring instruments available proves profitable in a procedure such as the present one. When ballasting changes are made in accordance with the values of $\Sigma$, $P_1$, and $P_2$, and then the final values of $I$ and $l$ checked against the required values, any discrepancies are very evident. This situation is in contrast to a trial and error procedure where no basis for consistency exists, i.e., where there is no predicted value to compare with the measured one. The basis for consistency which does exist in the present procedure serves to enforce the need for precision. Measured values of $I$ in agreement with the predicted values to within less than 1 percent are both possible and practical, though the figure would be expected to vary with the particular ballasting problem at hand.

## OUTLINE FOR SUGGESTED PROCEDURE

a. Fix axis of towing bracket at desired vertical location and at estimated horizontal location of model's center of gravity.

b. Ballast to correct total displacement, including ballast support blocks and fasteners for ballast.

(Items c and d can be omitted if the horizontal location of the center of gravity is known in advance.)

c. Float model and shift ballast to trim.

d. Relocate towing bracket to correct horizontal location of center of gravity; see Equation [30] and related text.

e. Suspend model by its towing bracket and measure frequency of oscillation.

f. Add weights $w_1$ and $w_2$ on deck so that equilibrium position is maintained, and find new frequency of oscillation.

g. Calculate $I$ and $l$; see Equations [9] and [10].

(The remaining steps should be omitted if a knowledge of $I$ and $l$ is all that is required.)

h. Calculate new positions for the forward and after groups of ballast to obtain the required $I_0$ and $l_0$; see Equations [25], [26], and [27].

i. Move ballast to calculated positions.

j. Check $I$ and $l$.

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