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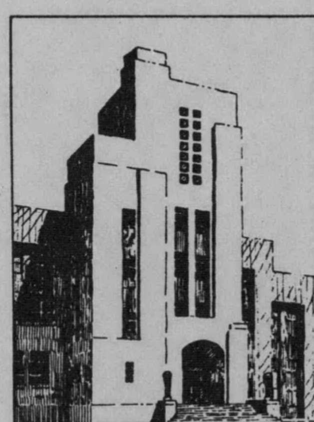
THE DETERMINATION OF EFFECTIVE STRESS BY MEANS OF SMALL CUBES TAKEN FROM PHOTOELASTIC MODELS

by

Joseph S. Brock



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INTRODUCTION

Three-dimensional photoelasticity has been developed along two general lines in the past decade. These two lines have been the so-called "stress-freezing" technique and the scattered-light technique. By far the most emphasis has been placed on the former method, and a number of papers have been written on the analysis of slices cut from "frozen" models.[1-4] These articles develop the equations necessary to calculate the five photoelastic unknowns from data obtained by viewing the slice normally and obliquely. However, no simple procedure has been developed, the computations required are somewhat tedious, and the formulas are often very sensitive to experimental errors. The method presented in this report minimizes these difficulties and, in addition, gives the effective (or uniaxial equivalent) stress in terms of photoelastically measurable quantities. The technique developed here is to freeze the stress into a three-dimensional photoelastic model by a standard stress-freezing technique and then, in addition to slicing the model, to cut the slices into small cubes. The cubes are analyzed individually, and thus the method is a point-by-point one. This report gives the essentials for the analysis of a general cube. The cube is taken as a matter of convenience but the analysis would apply equally well to a parallelepiped.

EFFECTIVE STRESS IN TERMS OF COORDINATE STRESS COMPONENTS

In order to derive the relations necessary to obtain a knowledge of the effective stress from photoelastic measurements made on an elementary cube in which a general stress condition has been frozen, we begin with Hooke's law

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[1] References are listed on page 13.

$$e_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$e_y = \frac{1}{E}[\sigma_y - \nu(\sigma_z + \sigma_x)]$$

$$e_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] \qquad [1]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}, \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}, \quad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

The total strain energy per unit volume of a stressed material may be written as

$$W = \frac{1}{2}(e_x\sigma_x + e_y\sigma_y + e_z\sigma_z + \gamma_{xy}\tau_{xy} + \gamma_{yz}\tau_{yz} + \gamma_{zx}\tau_{zx})$$

Using Equation [1] and the fact that $E = 2(1+\nu)G$ we find

$$W = \frac{1}{4G}\left[\frac{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\nu(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x)}{1+\nu} + 2(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right] \qquad [2]$$

This is the total strain energy per unit volume of a material subject to a general stress condition.

The strain energy of dilatation (hydrostatic strain energy) per unit volume is defined by

$$W_1 = \frac{1}{2} \times \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)(e_1 + e_2 + e_3)$$

where σ_1, σ_2, σ_3, e_1, e_2, e_3 are principal stresses and strains, respectively. Now

$$\sigma_1 + \sigma_2 + \sigma_3 = \sigma_x + \sigma_y + \sigma_z$$

and

$$e_1 + e_2 + e_3 = e_x + e_y + e_z$$

Therefore

$$W_1 = \frac{1}{6}(\sigma_x + \sigma_y + \sigma_z)(e_x + e_y + e_z)$$

From Equation [1]

$$e_x + e_y + e_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

so that

$$W_1 = \frac{1-2\nu}{6E}(\sigma_x + \sigma_y + \sigma_z)^2 = \frac{1-2\nu}{12(1+\nu)G}(\sigma_x + \sigma_y + \sigma_z)^2 \qquad [3]$$

This is the strain energy of dilatation per unit volume.

The strain energy of distortion (shear strain energy) is obtained simply by subtracting the strain energy of dilatation from the total strain energy, thus

$$W_2 = W - W_1$$

$$= \frac{1}{4G}\left[\frac{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\nu(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x)}{1+\nu} + 2(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]$$

$$- \frac{(1-2\nu)}{12(1+\nu)G}(\sigma_x + \sigma_y + \sigma_z)^2$$

After some simple algebraic reductions

$$W_2 = \frac{1}{12G}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right] \quad [4]$$

This is the shear strain energy per unit volume of a material subject to a general stress system.

Now consider a uniaxial stress state $\sigma_x = \sigma_0$, $\sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$

Then

$$(W_2)_0 = \frac{1}{12G}\left[2\sigma_0^2\right]$$

If we equate $(W_2)_0$ to W_2, we get

$$2\sigma_0^2 = (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \quad [5]$$

which is the well-known von Mises yield condition. The usual form of the above yield condition is

$$2\sigma_0^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

where σ_1, σ_2, σ_3 are the principal stresses. One interpretation of the yield condition is that if $\sigma_0 = \sigma_{Y.P.}$ then a material will yield when the combination of the stress components on the right of Equation [5] is just equal to the magnitude of the left member. Another way to think of Equation [5] is to let it define σ_0 as the effective (uniaxial equivalent) stress for any values of the coordinate stress components, thus

$$\sigma_0 = \sqrt{\frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]} \quad [6]$$

represents the effective stress for a general stress condition. This derivation is of course not new, but it serves to define the uniaxial equivalent of a three-dimensional stress system.

EFFECTIVE STRESS IN TERMS OF PHOTOELASTICALLY MEASURABLE QUANTITIES

Consider the question of the determination of the effective stress by means of photoelastic measurements. As mentioned before, the technique developed here is to freeze the stress in a three-dimensional photoelastic model by a standard stress-freezing technique and then, in addition to slicing the model, to cut the slices into small cubes. Each cube then gives the mean effective stress at the center of the cube over a gage length of the dimension of the edge of the cube. The measurements required are retardation (double refraction) measurements in the three coordinate directions and orientation (isoclinic parameter) measurements in these three directions. Thus there are six measurable quantities; however, only five are independent, as will be seen later.

It is evident from both theory and experiment that the stress in the direction of the propagation of the light has no effect on the retardation and orientation measurements.[5] Thus, if the direction of the light is in the z -direction, the components of stress having z as a subscript have no stress-optical effect, that is, σ_z , τ_{yz} , τ_{zx} , have no effect. One sees then the effect of the other stress components, σ_x , σ_y , τ_{xy}. These components give rise to the so-called secondary principal stresses for the z -direction, and the optical retardation is proportional to the difference in the secondary principal stresses just as in two-dimensional photoelasticity the optical retardation is proportional to the difference in the true principal stresses. The relations which define the differences in the secondary principal stresses and the orientation of these stresses are:

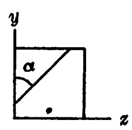
$$D_x = \sqrt{(\sigma_y - \sigma_z)^2 + 4\tau_{yz}^2}$$
$$D_y = \sqrt{(\sigma_z - \sigma_x)^2 + 4\tau_{zx}^2} \qquad [7]$$
$$D_z = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tan 2\alpha = \frac{2\tau_{yz}}{\sigma_y - \sigma_z}$$
$$\tan 2\beta = \frac{2\tau_{zx}}{\sigma_z - \sigma_x} \qquad [8]$$
$$\tan 2\gamma = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

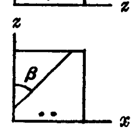
where D_x is the difference in the secondary principal stresses with the direction of the light in the x-direction, etc., and α is the angle between the y-axis and one of the secondary principal stresses, also with the direction of the light in the x-direction, etc.

According to the stress-optic law in three dimensions, D_x, D_y, D_z are proportional to the retardations as measured in the coordinate directions. Here we are neglecting the usually small effect due to the rotation of the plane of polarization. The isoclinic parameters α, β and γ may be determined directly, and thus we may consider D_x, D_y, D_z, α, β, γ as measurable quantities from a small cube of material which has been subjected to stress freezing.

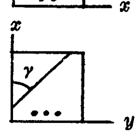
In practice a cube of convenient dimensions is chosen - small compared with the dimensions of the model, but large enough to have a retardation which can be measured with reasonable accuracy. A right-handed coordinate system is formed on the cube and identified in the following manner. Look in the x-direction with the y-axis vertical and place a dot just above the z-axis. Look in the y-direction with the z-axis vertical and place two dots just above the x-axis. Finally, look in the z-direction with the x-axis vertical and place three dots just above the y-axis. The dots may be placed on a blank cube by looking in the direction of a long diagonal drawn through the origin and placing dots on successive faces in one-two-three order in a clockwise sense. The dots have the following meaning:



looking in the x-direction, y-axis vertical



looking in the y-direction, z-axis vertical



looking in the z-direction, x-axis vertical

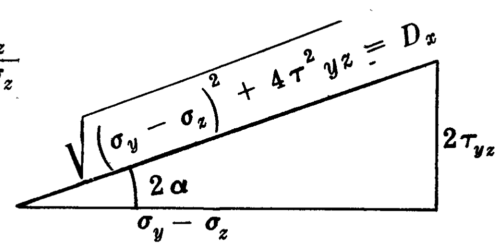
The quantities in Equation [6] are now required in terms of the measurable quantities. From the first equations of [7] and [8]

$$D_x = \sqrt{(\sigma_y - \sigma_z)^2 + 4\tau_{yz}^{\ 2}}$$

and

$$\tan 2\alpha = \frac{2\tau_{yz}}{\sigma_y - \sigma_z}$$

A figure will make the solution obvious.



From the figure

$$\sin 2\alpha = \frac{2\tau_{yz}}{D_x}$$

or

$$\tau_{yz} = \frac{1}{2} D_x \sin 2\alpha$$

Likewise [9]

$$\tau_{zx} = \frac{1}{2} D_y \sin 2\beta$$

and

$$\tau_{xy} = \frac{1}{2} D_z \sin 2\gamma$$

Also from the figure

$$\cos 2\alpha = \frac{\sigma_y - \sigma_z}{D_x}$$

Thus

$$(\sigma_y - \sigma_z) = D_x \cos 2\alpha$$

$$(\sigma_z - \sigma_x) = D_y \cos 2\beta \qquad [10]$$

$$(\sigma_x - \sigma_y) = D_z \cos 2\gamma$$

Placing [9] and [10] into [6] gives

$$\sigma_0 = \frac{1}{2} \sqrt{(2+\sin^2 2\alpha)\, D_x^{\,2} + (2+\sin^2 2\beta)\, D_y^{\,2} + (2+\sin^2 2\gamma)\, D_z^{\,2}} \qquad [11]$$

The effective stress is thus determined in terms of the photoelastically measurable quantities D_x, D_y, D_z, α, β, γ. Only five of these six quantities are independent, as may be seen by adding Equation [10], which gives

$$D_x \cos 2\alpha + D_y \cos 2\beta + D_z \cos 2\gamma = 0 \qquad [12]$$

This will be referred to as the check relation. Thus it is only necessary to make five measurements (any five of the six) to determine the effective stress. The usual practice is to make all six measurements and to use Equation [12] as a check on the consistency of the data. However, a least-squares solution will give rise to a higher probable accuracy with slightly more computations. This solution will be given at the end of this report.

Equation [11] requires some comment. It may be noted that the measured quantities or functions of them are squared, and thus all contribute to the magnitude of the effective stress σ_0 regardless of sign. The way in which the orientation angles enter is also worthy of note. If α, β, and γ are measured in the first quadrant it is obvious that 2α, 2β, $2\gamma \leqq 180$ deg. Also it can be shown that $\sin^2 2(\alpha \pm 90\,deg) = \sin^2 2\alpha$; therefore the loose definition of the orientation angles after Equation [8] is permissible for the present purpose. In other words, it is immaterial whether we measure the orientation angle from the major or the minor secondary principal stress. The question of the sign of the effective stress may be of some interest. From a mathematical standpoint there should be a plus or minus sign before the right side of Equations [6] and [11]. A definition of which sign to use is lacking, except for the simple case of uniaxial stress. Most authors apparently evade the question by leaving Equation [6] in its squared form. For the present purpose it is sufficient to consider σ_0 as a magnitude. No difficulty is encountered unless the material in question has a decidedly different yield strength in tension and compression. In this case we may consider the smaller yield strength as a safety measure.

DIRECTIONS OF PRINCIPAL STRESSES

In addition to the uniaxial equivalent or effective stress, it is also possible to find the directions of the principal stresses. In order to develop the required relations, consider the equilibrium of a rigid tetrahedral element (Figure 1), three faces of which are chosen as the coordinate planes The following familiar equations result:

$$
\begin{aligned}
(\sigma_x - \sigma)\, l + \tau_{xy}\, m + \tau_{zx}\, n &= 0 \\
\tau_{xy}\, l + (\sigma_y - \sigma)\, m + \tau_{yz}\, n &= 0 \\
\tau_{zx}\, l + \tau_{yz}\, m + (\sigma_z - \sigma)\, n &= 0
\end{aligned}
\qquad [13]
$$

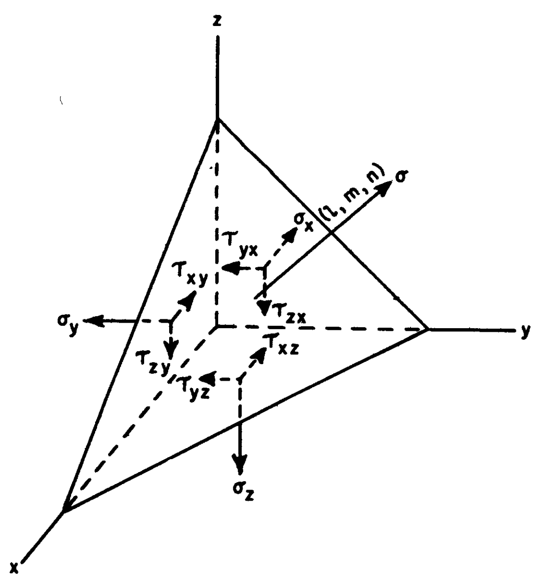


Figure 1 - Right Tetrahedral Element

The oblique face is a principal plane.

where l, m, and n are the direction cosines for the stress σ. Here the direction of σ is taken so that there is no shearing stress on planes perpendicular to it. Thus σ is a principal stress which, in general, has three distinct values determined by eliminating l, m, and n from Equation [13]. The cubic equation which relates the coordinate stresses to the principal stress results:

$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\,\sigma^2 + (\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}{}^2 - \tau_{yz}{}^2 - \tau_{zx}{}^2)\,\sigma$$
$$- (\sigma_x\,\sigma_y\,\sigma_z + 2\,\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}{}^2 - \sigma_y\tau_{zx}{}^2 - \sigma_z\tau_{xy}{}^2) = 0 \qquad [14]$$

The roots of this equation are real. A general stress state, σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{zx}, has principal stresses σ_1, σ_2, σ_3, which are roots of Equation [14].

It is perhaps obvious that an increase in each of the normal stresses by the same amount increases each of the principal stresses by the same amount. (This may be demonstrated by adding a hydrostatic stress state to a general stress state, finding the cubic equation similar to Equation [14], and comparing this equation with the one obtained by increasing the roots of Equation [14] by an amount equal to the hydrostatic stress state.) Due to this fact, it is possible to assign an arbitrary value to one of the normal stresses for the purpose of determining the directions of the principal stress. For convenience take $\sigma_z = 0$. Then Equation [10] becomes

$$\sigma_y = D_x \cos 2\alpha$$
$$\sigma_x = -D_y \cos 2\beta \qquad [15]$$
$$\sigma_z = 0$$

The measured valued of D_x, D_y, D_z, α, β, γ give the values of τ_{xy}, τ_{yz}, τ_{zx} from Equation [9], and the <u>relative</u> values of σ_x, σ_y, σ_z, are given by Equation [15]. These may be used in Equation [14] to determine the <u>relative</u> values of the principal stresses. Since the roots of the cubic Equation [14] are real, they may be obtained by a trigonometric solution.

Let the following scheme denote the direction cosines of the principal stresses with respect to the coordinate axes:

	σ_1-axis	σ_2-axis	σ_3-axis
x	l_1	l_2	l_3
y	m_1	m_2	m_3
z	n_1	n_2	n_3

where l_1 is the cosine of the angle between the x-axis and the σ_1-axis, etc. These direction cosines may now be obtained from Equation [13] subject to the condition

$$l^2+m^2+n^2=1 \qquad [16]$$

The solution of Equation [13] with [16] gives

$$l_{1,2,3}=\frac{\pm 1}{\sqrt{1+A^2_{1,2,3}+B^2_{1,2,3}}}$$

$$m_{1,2,3}=A_{1,2,3}\ l_{1,2,3} \qquad [17]$$

$$n_{1,2,3}=B_{1,2,3}\ l_{1,2,3}$$

where

$$A_{1,2,3}=\frac{(\sigma_x-\sigma_{1,2,3})\tau_{yz}-\tau_{zx}\tau_{xy}}{(\sigma_y-\sigma_{1,2,3})\tau_{zx}-\tau_{xy}\tau_{yz}}$$

and

$$B_{1,2,3}=\frac{(\sigma_x-\sigma_{1,2,3})\tau_{yz}-\tau_{zx}\tau_{xy}}{(\sigma_z-\sigma_{1,2,3})\tau_{xy}-\tau_{yz}\tau_{zx}}$$

Equation [17] uniquely determines the direction cosines for each of the principal stresses. The plus-minus sign denotes the "double" direction of the stress lines. The values obtained are finally checked in the following stress transformation equations:

$$\sigma_x=\sigma_1 l_1^2+\sigma_2 l_2^2+\sigma_3 l_3^2$$

$$\sigma_y=\sigma_1 m_1^2+\sigma_2 m_2^2+\sigma_3 m_3^2$$

$$\sigma_z=\sigma_1 n_1^2+\sigma_2 n_2^2+\sigma_3 m_3^2 \qquad [18]$$

$$\tau_{xy}=\sigma_1 l_1 m_1+\sigma_2 m_2 l_2+\sigma_3 l_3 m_3$$

$$\tau_{yz}=\sigma_1 m_1 n_1+\sigma_2 m_2 n_2+\sigma_3 m_3 n_3$$

$$\tau_{zx}=\sigma_1 n_1 l_1+\sigma_2 n_2 l_2+\sigma_3 n_3 l_3$$

LEAST-SQUARES SOLUTION OF BASIC EQUATIONS

The basic Equations [7] and [8], which relate the secondary principal stresses and their directions to the coordinate shear stresses and normal stress differences, are once redundant. That is, we have six equations and only five unknowns, $(\sigma_y-\sigma_z)$, $(\sigma_z-\sigma_x)$, τ_{yz}, τ_{zx}, τ_{xy}. In order to use the measurements to the best advantage, we employ the principle of least squares. Let the measured values of D_x, D_y, D_z, α, β, γ be denoted by primes. To satisfy least squares

$$\sum\left[(D_x'-D_x)^2+(D_y'-D_y)^2+(D_z'-D_z)^2+(\alpha'-\alpha)^2+(\beta'-\beta)^2+(\gamma'-\gamma)^2\right]=a\ minimum$$

Then

$$\frac{\partial\Sigma}{\partial(\sigma_y-\sigma_z)}=0,\quad \frac{\partial\Sigma}{\partial(\sigma_z-\sigma_x)}=0,\quad \frac{\partial\Sigma}{\partial\alpha}=0,\quad \frac{\partial\Sigma}{\partial\beta}=0,\quad \frac{\partial\Sigma}{\partial\gamma}=0 \qquad [19]$$

produce the following equations:

$$\left.\begin{aligned}
&\frac{-\left(D_x'-\sqrt{(\sigma_y-\sigma_z)^2+4\tau_{yz}^2}\right)\left(\sigma_y-\sigma_z\right)}{\sqrt{(\sigma_y-\sigma_z)^2+4\tau_{yz}^2}}+\frac{\left(D_z'-\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}\right)\left(\sigma_x-\sigma_y\right)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}=0\\
&\frac{-\left(D_y'-\sqrt{(\sigma_z-\sigma_x)^2+4\tau_{zx}^2}\right)\left(\sigma_z-\sigma_x\right)}{\sqrt{(\sigma_z-\sigma_x)^2+4\tau_{zx}^2}}+\frac{\left(D_z'-\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}\right)\left(\sigma_x-\sigma_y\right)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}=0\\
&\alpha'-\frac{1}{2}\arctan\frac{2\tau_{yz}}{\sigma_y-\sigma_z}=0\\
&\beta'-\frac{1}{2}\arctan\frac{2\tau_{zx}}{\sigma_z-\sigma_x}=0\\
&\gamma'-\frac{1}{2}\arctan\frac{2\tau_{xy}}{\sigma_x-\sigma_y}=0
\end{aligned}\right\}[20]$$

The solution of these equations gives

$$\sigma_y-\sigma_z=D_x'\cos 2\alpha'-\frac{1}{3}(D_x'\cos 2\alpha'+D_y'\cos 2\beta'+D_z'\cos 2\gamma')$$

$$\sigma_z-\sigma_x=D_y'\cos 2\beta'-\frac{1}{3}(D_x'\cos 2\alpha'+D_y'\cos 2\beta'+D_z'\cos 2\gamma')$$

$$\tau_{yz}=\frac{1}{6}(2D_x'\cos 2\alpha'-D_y'\cos 2\beta'-D_z'\cos 2\gamma')\tan 2\alpha'$$

$$\tau_{zx}=\frac{1}{6}(-D_x'\cos 2\alpha'+2D_y'\cos 2\beta'-D_z'\cos 2\gamma')\tan 2\beta'$$

$$\tau_{xy}=\frac{1}{6}(-D_x'\cos 2\alpha'-D_y'\cos 2\beta'+2D_z'\cos 2\gamma')\tan 2\gamma'$$

These equations may be written in the following form:

$$\sigma_y - \sigma_z = D_x' \cos 2\alpha' - \frac{1}{3}(D_x' \cos 2\alpha' + D_y' \cos 2\beta' + D_z' \cos 2\gamma')$$

$$\sigma_z - \sigma_x = D_y' \cos 2\beta' - \frac{1}{3}(D_x' \cos 2\alpha' + D_y' \cos 2\beta' + D_z' \cos 2\gamma')$$

$$\sigma_x - \sigma_y = D_z' \cos 2\gamma' - \frac{1}{3}(D_x' \cos 2\alpha' + D_y' \cos 2\beta' + D_z' \cos 2\gamma')$$

$$\tau_{yz} = \frac{1}{2}(\sigma_y - \sigma_z) \tan 2\alpha' = \frac{1}{2} D_x' \sin 2\alpha' - \frac{1}{6}(D_x' \cos 2\alpha' + D_y' \cos 2\beta' + D_z' \cos 2\gamma') \tan 2\alpha'$$

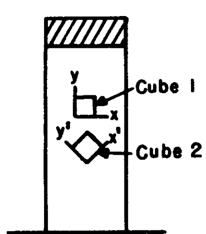
$$\tau_{zx} = \frac{1}{2}(\sigma_z - \sigma_x) \tan 2\beta' = \frac{1}{2} D_y' \sin 2\beta' - \frac{1}{6}(D_x' \cos 2\alpha' + D_y' \cos 2\beta' + D_z' \cos 2\gamma') \tan 2\beta'$$

$$\tau_{xy} = \frac{1}{2}(\sigma_x - \sigma_y) \tan 2\gamma' = \frac{1}{2} D_z' \sin 2\gamma' - \frac{1}{6}(D_x' \cos 2\alpha' + D_y' \cos 2\beta' + D_z' \cos 2\gamma') \tan 2\gamma'$$

It may be seen that, if the check relation (Equation [12]) holds, the least-squares solution reduces to the algebraic solution (Equations [9] and [10]). It is also apparent that the least-squares solution merely distributes the excess from the check relation equally to the coordinate normal stress differences.

EXPERIMENTAL VERIFICATION

In order to test the validity of Equation [11], the following simple experiment was performed. A test specimen (1/2 inch × 1/2 inch × 2 inches) was cut from a block of Fosterite and subjected to a uniaxial compression stress of 39.7 psi. The stress was frozen in by means of the usual technique, and two sample cubes were cut from the interior of the column in the following manner:



(1) One cube was taken with the edges parallel to the edges of the parent test specimen.

(2) One cube was taken with the edges at 45 deg to the edge of the test specimen (simple rotation about an axis perpendicular to the paper).

The cubes were sawed out slightly oversize and milled to 1/4 inch on a side. The following measurements were made:

Cube 1				
Observer	Observation Direction	Compensator Reading inches	Angle Reading degrees	Effective Compensator Reading, R_0 inches
1	x y z	+ 0.527 - 0.005 + 0.520	90 85 1	0.524*
2	x y z	+ 0.522 0 + 0.500	90 74 1	0.521*
Cube 2				
1	x y' z'	+ 0.530 - 0.271 - 0.262	45 6 87	0.532*
2	x y' z'	+ 0.520 - 0.286 - 0.282	45 5 88	0.533*
*Calculated from Equation [11].				

The retardation measurements were made with a quartz wedge compensator whose least count was 0.01 inch; however, the measurements were estimated to 0.005 inch. The values shown in the table were corrected for an exact 1/4-inch cube. Equation [11] written in terms of the compensator readings was used to calculate R_0, the average of which is 0.527 inch. The relation $\sigma_0 = k\,R_0$ serves to evaluate the compensator constant k. Thus

$$k = \frac{39.7}{0.527} = 75.4 \text{ psi/inch for 1/4-inch cubes}$$

The least count on the compensator is thus 0.754 psi for 1/4-inch cubes of Fosterite. Since the values of R_0 and thus σ_0 are essentially the same for cubes 1 and 2, the present theory is verified.

CONCLUSIONS

1. A method of analyzing small cubes to determine the internal stresses in a photoelastic model has been developed.

2. The present method gives the effective stress in terms of photoelastically measurable quantities. It also gives the directions of the principal stresses.

3. The method has been verified by experiment.

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