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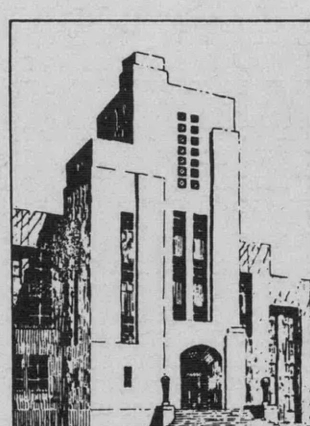
THEORY OF THE FORCED VIBRATIONS OF A RING WITH RADIAL ELASTIC SUPPORT SUBJECTED TO UNIFORM EXTERNAL PRESSURE

[redacted]

by

Thomas E. Reynolds



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NOTATION

A	Area of cross section of ring
a_i	Generalized coordinate
B, b	Constants
C, C_i, D_i	Constants
c	Damping coefficient
E	Young's modulus
F	Dissipation function
g	Acceleration of gravity
I	Moment of inertia of cross section of ring
i, j	Integers
k	Modulus of elastic support
P	External load
P_0	Amplitude of external load
$\bar{p}_{iq}$, p_{iq}	Damped and undamped circular frequencies for ith mode at pressure q
p_{i0}	Undamped frequency at pressure = 0
Q_i	Generalized force
q	External pressure
q_i	Critical buckling pressure for ith mode
R	Radius to median surface of ring
R_1, R_1', R_2, R_2'	Radial coordinates of a deflected element
s	Distance along centerline of ring
T	Kinetic energy
t	Time
U	Total potential energy
U_{int}, U_{ext}	Internal potential energy of ring, work done by external pressure
u	Radial displacement, positive inward
V	Volume enclosed in plane of ring
w	Tangential displacement, positive clockwise
γ	Density of ring material
θ	Angular coordinate
θ_1, θ_1', θ_2, θ_2'	Angular coordinates of a deflected element
ω	Circular frequency of exciting force

ABSTRACT

The theory for inextensional vibrations of a circular ring with radial elastic support is extended to include the effects of damping and external pressure. Equations are developed for free vibrations under external pressure and for the steady-state response of the ring to the simultaneous action of external pressure and a radial vibratory exciting force concentrated at one point. Results obtained include the reduction in natural frequency produced by the pressure with the buckling in different modes inferred from the zero frequency condition. Also given are the magnifications of response at resonance resulting from external pressure. Curves of these results show a nonuniform increase in displacements at resonance with rising pressure for a constant driving force such that peaks appear in the curves because of the coincidence of two natural frequencies at certain pressures.

INTRODUCTION

The equations developed by Wenk[1] for vibrations of a circular ring with radial elastic support have been used with some success in studying the lobar vibrations of stiffened cylinders.* In the hope of throwing further light upon such vibrations, an analytical study was made of the response of a ring with radial elastic support when subjected simultaneously to steady-state lobar vibration at resonance and uniform external pressure.**

Damping is assumed to be viscous, and the value of the damping constant used in the calculations was obtained from measurements of decay of transients observed during tests of stiffened cylinders. Equations are obtained for the natural frequencies and displacements at resonance and the variation of these quantities with pressure. It is found that at certain pressures the frequencies for two different modes become equal, with the result that for a constant driving force peaks appear in the displacement curves. An expression for the length of a deflected element is derived in Appendix 1. The work done by the external pressure during deflection of the ring appears in Appendix 2.

ANALYSIS

FREE VIBRATION

A plane circular ring having radial elastic support is subjected to uniform, static, external pressure. The elastic support is assumed to act along radii and only in proportion

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[1]References are listed on page 8.

*Cf. TMB CONFIDENTIAL Reports C-354 (1950) and C-445 (1953), also papers presented by R.B. Allnutt et al. and by E. Wenk, Jr. at the Sixth Symposium on Underwater Explosion Research, CONFIDENTIAL.

**Lobar vibrations of rings and stiffened cylinders are being investigated as one phase of an analysis of the response of submarines to enemy attack.[2]

to radial displacements, while the pressure is exerted always normal to the median surface of the ring and has no associated mass. The cross section of the ring is of unit width and constant depth, both small compared with the radius of the ring. Further, it is assumed that, during vibration, damping forces retard the motion in the direction of and in proportion to its velocity. Only inextensional vibrations which take place in the plane of the ring are considered, and the effects of shear and rotary inertia are neglected.

For a nonconservative system Lagrange's equation for free oscillation can be written

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{a}_i}\right) - \frac{\partial T}{\partial a_i} + \frac{\partial U}{\partial a_i} + \frac{\partial F}{\partial \dot{a}_i} = 0 \qquad [1]$$

where a_i is a generalized coordinate,

$\dot{a}_i$ is a generalized velocity, and

T, U, and F are the kinetic, potential and dissipation energies.

During deformation the motion of a ring element is described by a radial displacement u measured positively inward and a tangential displacement w measured positively clockwise. The length ds of a deflected element is given by

$$(ds)^2 = (d\theta)^2 \left[(R - u + w_\theta)^2 + (u_\theta + w)^2\right] \qquad [2]$$

This expression is derived in Appendix 1. If displacements are inextensional, the length of an element remains constant, or

$$ds = R d\theta \qquad [3]$$

During deformation, the external pressure q does work U_{ext}, proportional to the change in the volume bounded by the ring and the two planes normal to its surface, which is

$$U_{\text{ext}} = \frac{q}{2}\left\{\int_0^{2\pi} \left[(R - u)^2 + w_\theta(R - u) + w(w + u_\theta)\right] d\theta - \int_0^{2\pi} R^2 d\theta\right\} \qquad [4]$$

A derivation of this expression appears in Appendix 2. Solving Equations [2] and [3] for $(R - u)^2$ and substituting this value in Equation [4] yields

$$U_{\text{ext}} = \frac{q}{2}\int_0^{2\pi} \left[w_\theta\,(u - R - w_\theta) - u_\theta(w + u_\theta)\right] d\theta \qquad [5]$$

which is correct to the second order in u and w. Observing that w_θ is an exact differential, this reduces to

$$U_{\text{ext}} = \frac{q}{2}\int_0^{2\pi} \left[w_\theta(u - w_\theta) - u_\theta(w + u_\theta)\right] d\theta \qquad [6]$$

The internal energy of the ring U_{int} consists of the energy of bending and the energy of the elastic support, which are given in Reference 1 as

$$U_{\text{int}} = \frac{EI}{2R^3} \int_0^{2\pi} (u_{\theta\theta} + u)^2 \, d\theta + \frac{kR}{2} \int_0^{2\pi} u^2 \, d\theta \qquad [7]$$

where I is the moment of inertia of the cross section of the ring and k is the modulus of elastic support measured in pounds per inch per inch of circumference. The total potential energy is

$$U = U_{\text{ext}} + U_{\text{int}} \qquad [8]$$

As defined in Reference 1, the kinetic energy T is

$$T = \frac{A\gamma R}{2g} \int_0^{2\pi} (\dot{u}^2 + \dot{w}^2) \, d\theta \qquad [9]$$

where A is the area of cross section of the ring and γ is the density of the material. The dissipation energy is defined as

$$F = \frac{cR}{2} \int_0^{2\pi} (\dot{u}^2 + \dot{w}^2) \, d\theta \qquad [10]$$

where c is the damping coefficient measured in pound-seconds per inch.[3] In the evaluation of the foregoing integrals, terms higher than the second order in u and w will be neglected. A tangential displacement

$$w = \frac{a_i}{i} \sin i\,\theta \qquad [11a]$$

may be assumed, where a_i is a generalized time coordinate and i is a positive integer. The condition of inextensionality, Equation [3], will be satisfied to the extent that U, T, and F will be correct to the second order in a_i, if

$$u = w_\theta = a_i \cos i\,\theta \qquad [11b]$$

Evaluation of the integrals then yields the results

$$U = \frac{\pi EI}{2R^3} (i^2 - 1)^2 \, a_i^2 + \frac{\pi kRa_i^2}{2} - \frac{q\pi}{2} (i^2 - 1) \, a_i^2 \qquad [12a]$$

$$T = \frac{A\gamma\pi R}{2g} \frac{(i^2 + 1)}{i^2} \dot{a}_i^2 \qquad [12b]$$

$$F = \frac{c\pi R}{2} \frac{(i^2 + 1)}{i^2} \dot{a}_i^2 \qquad [12c]$$

Their derivatives become

$$\frac{\partial U}{\partial a_i} = \frac{\pi EI}{R^3} (i^2 - 1)^2 \, a_i + \pi kRa_i - q\pi (i^2 - 1) \, a_i \qquad [13a]$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{a}_i}\right) = \frac{A\gamma\pi R}{g} \frac{(i^2 + 1)}{i^2} \ddot{a}_i \qquad [13b]$$

$$\frac{\partial F}{\partial \dot{a}_i} = c\pi R \frac{(i^2+1)}{i^2} \dot{a}_i \qquad [13c]$$

Substitution of these quantities in Equation [1] results in the equation of motion

$$\frac{A\gamma}{g}\frac{(i^2+1)}{i^2}\ddot{a}_i + \frac{EI}{R^4}(i^2-1)^2 a_i + ka_i - q(i^2-1)a_i + c\frac{(i^2+1)}{i^2}\dot{a}_i = 0 \qquad [14]$$

A solution of the form

$$a_i = e^{bt}[B\cos \bar{p}_{iq}t + C\sin \bar{p}_{iq}t] \qquad [15]$$

will satisfy Equation [14] if

$$\bar{p}_{iq}^{\,2} = \frac{gi^2}{A\gamma(i^2+1)}\left[\frac{EI}{R^4}(i^2-1)^2 + k - \frac{q}{R}(i^2-1) - \frac{gc(i^2+1)}{4A\gamma i^2}\right] \qquad [16a]$$

and

$$b = \frac{-gc}{2A\gamma} \qquad [16b]$$

Here $\bar{p}_{iq}$ is the damped natural frequency at a pressure equal to q and exists for values of c less than critical. For most practical cases, c will be so small that the approximation

$$\bar{p}_{iq}^{\,2} = p_{iq}^{\,2} = \frac{gi^2}{A\gamma(i^2+1)}\left[\frac{EI}{R^4}(i^2-1)^2 + k - \frac{q}{R}(i^2-1)\right] \qquad [17]$$

can be made, where p_{iq} is the undamped natural frequency. The condition for buckling of an elastically supported ring may be obtained by setting $p_{iq} = 0$ which gives

$$q_i = \frac{EI}{R^3}(i^2-1) + \frac{kR}{i^2-1} \qquad [18]$$

where q_i is the buckling pressure at which static displacements would assume the form of the ith mode of vibration. Equation [17] can then be written

$$p_{iq}^{\,2} = p_{i0}^{\,2}\left(1 - \frac{q_i}{q}\right) \qquad [19]$$

where p_{i0} is the frequency at zero pressure.

FORCED VIBRATION

Lagrange's equation for forced oscillations can be written

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{a}_i}\right) - \frac{\partial T}{\partial a_i} + \frac{\partial U}{\partial a_i} + \frac{\partial F}{\partial \dot{a}_i} = Q_i \qquad [20]$$

where Q_i is a generalized force. If a simple harmonic force, $P_0 \sin \omega t$, acts in the radial direction at $\theta = 0$, then at that point, $u = a_i$, and the generalized force is

$$Q_i = P_0 \sin \omega t \qquad [21]$$

Lagrange's equation then becomes

$$\frac{A\gamma}{g}\frac{(i^2+1)}{i^2}\ddot{a}_i + \frac{EI}{R^4}(i^2-1)^2\, a_i + ka_i - q\,(i^2-1)\, a_i + c\,\frac{(i^2+1)}{i^2}\,\dot{a}_i = P_0 \sin \omega t \qquad [22]$$

In order to determine the steady-state response under the action of the external force, a particular solution of the form

$$a_i = C_i \sin \omega t + D_i \cos \omega t \qquad [23]$$

is chosen. This will satisfy Equation [22] if

$$C_i = \frac{P_0\left(\frac{p_{iq}^2}{\omega^2} - 1\right)\frac{A\gamma}{g}\,\frac{i^2}{i^2+1}}{c^2\pi R\left\{\left[\frac{p_{iq}^2}{\omega^2} - 1\right]^2\left(\frac{A\gamma\omega}{cg}\right)^2 + 1\right\}} \qquad [24a]$$

and

$$D_i = \frac{-P_0\,\frac{i^2}{i^2+1}}{\omega\pi Rc\left\{\left[\frac{p_{iq}^2}{\omega^2} - 1\right]^2\left(\frac{A\gamma\omega}{cg}\right)^2 + 1\right\}} \qquad [24b]$$

Since the assumed solution will satisfy Equation [22] for all positive integral values of i*, the complete solution, or total displacement, for the steady-state condition is

$$u = \sum_{i=1}^{\infty} a_i \cos i\theta = \sum_{i=1}^{\infty} [C_i \sin \omega t + D_i \cos \omega t] \cos i\theta \qquad [25]$$

Resonance will occur where $p_{iq} = \omega$, the frequency of the exciting force. Equation [23] will then reduce to

$$a_i = \frac{-P_0 i^2}{\pi Rcp_{iq}\,(i^2+1)} \cos p_{iq} t \qquad [26]$$

and the total displacement amplitude will be

$$u_{\max} = \frac{\frac{-P_0}{\pi Rcp_{iq}}\left[\frac{i^2 \cos i\theta}{i^2+1} + \sum_{j=1}^{\infty}\frac{j^2 \cos j\theta}{j^2+1}\right]}{\left\{\left[\left(\frac{p_{jq}}{p_{iq}}\right)^2 - 1\right]^2\left[\frac{A\gamma p_{iq}}{cg}\right]^2 + 1\right\}} \qquad [27]$$

where j is a positive integer not equal to i.

---

*The value $i = 1$ corresponds to rigid-body translation.

COINCIDENCE OF TWO RESONANT FREQUENCIES

The linear relationship between p_{iq}^2 and q expressed in Equations [17] and [19] can best be illustrated in graphical form. The following example was chosen, using dimensions and results identical with those appearing in Figure 16 of Reference 1.

$$r = 12 \text{ in.}$$

$$A = 0.125 \text{ in.}^2$$

$$I = 162.8 \times 10^{-6} \text{ in.}^4$$

$$E = 29.2 \times 10^6 \text{ psi}$$

$$\gamma = 0.322 \text{ lb/in.}^3$$

$$k = 31.2 \text{ psi}$$

The reductions in frequency with pressure, given by Equation [17], are plotted in Figure 1. Intercepts on the x-axis give the buckling pressures in different modes, $q = 66.2$ corresponding to $i = 4$ being the lowest. Also it is seen that several intersections of the curves occur so that, at certain pressures, frequencies for two different modes attain the same value. This has a curious effect on displacements at resonance. From Equation [27] it can be seen that, at zero pressure, the first term within the brackets is large compared with the other terms of the series, so that the resonance displacement amplitude is approximately given by

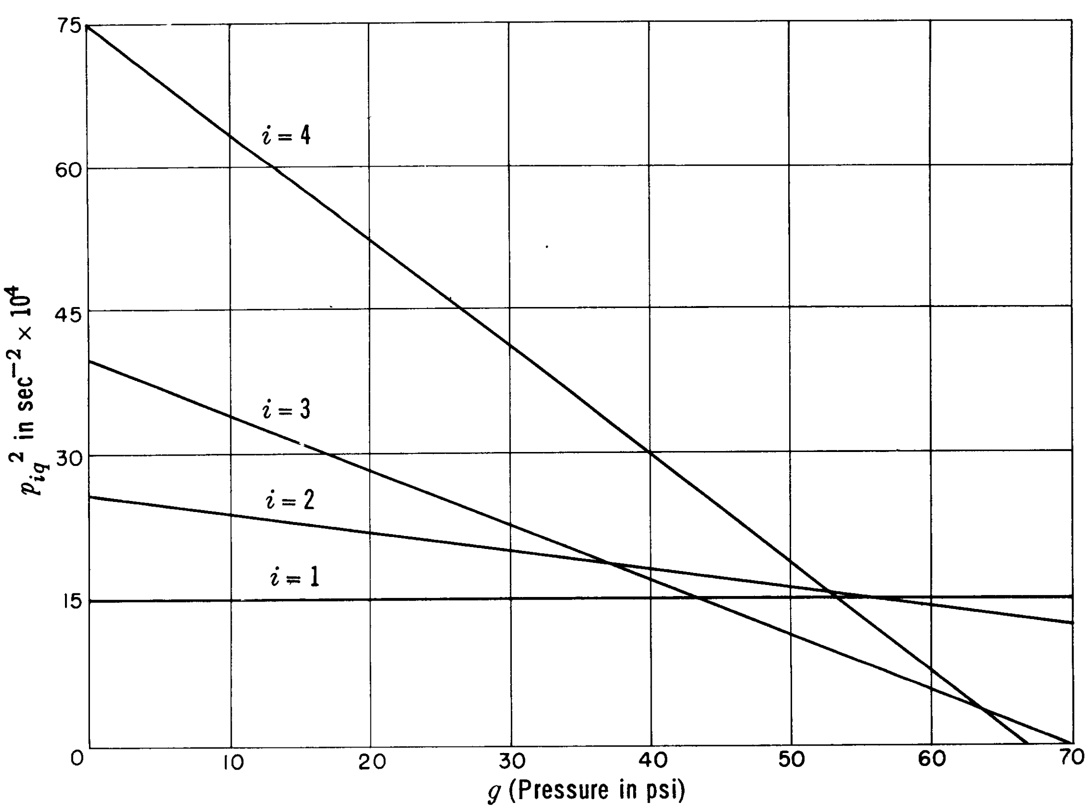

Figure 1 - Variation of Frequency with Pressure

$$u_{max} = \frac{-P_0\, i^2 \cos i\theta}{\pi R c p_{i0} \sqrt{1 - \frac{q}{q_i}}\, (i^2 + 1)} \qquad [28]$$

However, with increasing pressure, there are values of q for which $p_{iq} = p_{jq}$, as indicated in Figure 1, with the result that one of the terms of the series is greatly magnified and must be added to the result of Equation [28]. Under these conditions the displacement amplitude becomes approximately

$$u_{max} = \frac{-P_0}{\pi R c p_{i0} \sqrt{1 - \frac{q}{q_i}}} \left[\frac{i^2}{i^2 + 1} \cos i\theta + \frac{j^2}{j^2 + 1} \cos j\theta \right] \qquad [29]$$

This effect is illustrated in Figure 2 where the radial displacement amplitudes at resonance for the modes $i = 3$ and $i = 4$ have been plotted against pressure. For simplicity the calculations have been made for $\theta = 0$. The value for $cg/A\gamma$ was 15.8 sec/in.^{2}.* It is seen that peaks in the amplitude curve appear wherever two frequencies coincide. As q approaches q_4,

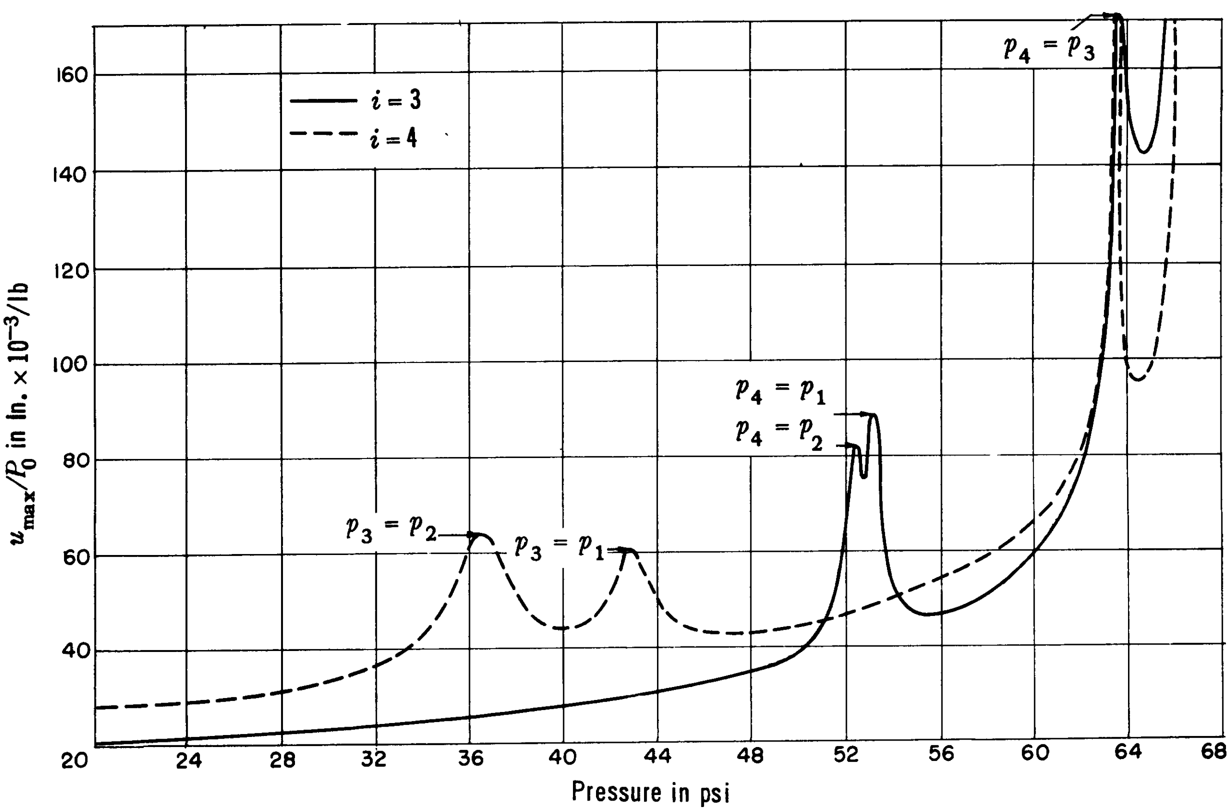

Figure 2 - Variation of Displacements at Resonance with Pressure at $\theta = 0$

*In order to calculate these displacements it was necessary to use a specific value for the damping coefficient per unit mass. Although any value consistent with the approximation $\bar{p}_i = p_i$ could have been used, it was preferable to choose a value obtained experimentally. Logarithmic decrements determined from vibration tests of a stiffened steel cylinder yielded the value 15.8 sec/in.^{2}

the curve for $i = 4$ becomes increasingly steep, corresponding to infinite displacements or buckling of the ring when q_4 is reached. Calculations were made for the first four modes only since the frequencies for the higher modes were of such a magnitude that their effect on the calculations would be negligible. The shape of the curves in Figure 2 depends, of course, on the particular orientation, or value of θ, being considered. The peaks are most pronounced at $\theta = 0$ since at that point the contributions to the displacement of all modes are in phase. At some orientations, valleys rather than peaks would be.found.

Qualitatively, the sort of behavior represented in Figure 2 has been observed during vibration tests of stiffened cylinders at resonance under pressure. While the foregoing presents only an analogous situation, it may suggest reasons for the phenomenon.

CONCLUSIONS

1. Under combined vibratory loading and uniform external pressure without an associated mass, a ring having radial elastic support exhibits an inverse linear relationship between the squares of its frequencies (corresponding to inextensional vibration) and the pressure.

2. The inclusion of damping permits the calculation of displacements at resonance.

3. Displacements per unit driving force at resonance are generally magnified with increasing pressure. However, due to the coincidence of two natural frequencies at certain pressures, the displacement amplitudes vary with pressure in a nonuniform manner, attaining several relative maximum and minimum values.

ACKNOWLEDGMENTS

This investigation was initiated at the suggestion of Mr. Ralph B. Allnutt of the David Taylor Model Basin staff. The author is indebted to Dr. Robert Bart, also of the Model Basin staff, who reviewed the theoretical work and offered constructive criticism.

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2. Bureau of Ships CONFIDENTIAL ltr C-S81/3(423) Ser 423-041 of 7 Jul 1952 to David Taylor Model Basin.

APPENDIX 1

EXPRESSION FOR THE LENGTH OF A DEFLECTED ELEMENT

In Figure 3, an undeflected element of the ring included within a central angle $d\theta$ is bounded by the coordinates (R, θ_1) and (R, θ_2). During radial deflection, these coordinates become (R_1, θ_1) and (R_2, θ_2). At the same time the element undergoes a tangential displacement so that its final position is given by (R_1', θ_1') and (R_2', θ_2'). The length of the deflected element is given by

$$(ds)^2 = R_1'^2 + R_2'^2 - 2R_1' R_2' \cos(\theta_1' - \theta_2') \qquad [30]$$

The coordinates can be expressed as follows:

$$R_1 = R - u \qquad [31a]$$

$$R_2 = R - u - u_\theta d\theta \qquad [31b]$$

$$R_1' = [R_1^2 + w^2]^{1/2} \qquad [31c]$$

$$R_2' = [R_2^2 + (w + w_\theta d\theta)^2]^{1/2} \qquad [31d]$$

$$\theta_1' = \theta_1 + \frac{w}{R_1'} \qquad [31e]$$

$$\theta_2 = \theta_1 + d\theta \qquad [31f]$$

$$\theta_2' = \theta_2 + \frac{w + w_\theta d\theta}{R_2'} \qquad [31g]$$

Employing series expansions for R_1', R_2', and $\cos(\theta_1' - \theta_2')$, these quantities become

$$R_1' = R_1 + \frac{w^2}{2R_1} \qquad [32a]$$

$$R_2' = R_1 - u_\theta d\theta + \frac{w^2}{2R_1} + \frac{ww_\theta d\theta}{R_1} \qquad [32b]$$

$$\cos(\theta_1' - \theta_2') = 1 - \frac{1}{2}\left[d\theta + \frac{w + w_\theta d\theta}{R_2'} - \frac{w}{R_1'}\right]^2 \qquad [32c]$$

Equation [30] then can be written

$$(ds)^2 = (d\theta)^2 \, [R_1^2 + u_\theta^2 + w^2 + w_\theta^2 + 2wu_\theta + 2R_1 w_\theta] \qquad [33]$$

or

$$(ds)^2 = (d\theta)^2 \, [(R-u)^2 + u_\theta^2 + w^2 + w_\theta^2 + 2wu_\theta + 2w_\theta (R-u)] \qquad [34a]$$

$$(ds)^2 = (d\theta)^2 \, [(R - u + w_\theta)^2 + (u_\theta + w)^2] \qquad [34b]$$

APPENDIX 2

EXPRESSION FOR THE WORK DONE BY THE EXTERNAL PRESSURE

The work done by the external pressure when the ring undergoes displacements is the product of the pressure and the change in the volume bounded by the median surface of the ring. In Figure 3, the volume dV enclosed by the ring element and the two radii, R_1' and R_2', is

$$dV = \frac{R_1' R_2'}{2} (\theta_2' - \theta_1') \quad [35]$$

From Equations [31e] and [31g],

$$\theta_2' - \theta_1' = \frac{R_1' w - R_2' w + d\theta \, (R_1' w_\theta + R_1' R_2')}{R_1' R_2'} \quad [36]$$

and

$$dV = \frac{1}{2} [R_1' d\theta \, (R_2' + w_\theta) + w(R_1' - R_2')] \quad [37]$$

Using Equations [31a] through [31d], this becomes

$$dV = \frac{d\theta}{2} [(R - u)^2 + w_\theta (R - u) + w(w + u_\theta)] \quad [38]$$

The change in volume is, therefore,

$$\Delta V = \frac{1}{2} \int_0^{2\pi} [(R - u)^2 + w_\theta (R - u) + w(w + u_\theta) - R^2] d\theta \quad [39]$$

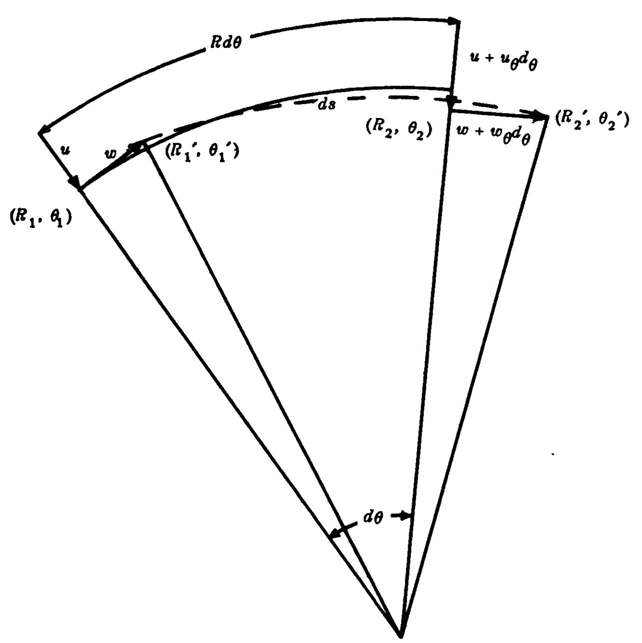

Figure 3 - Deflected Element of a Ring

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1 Mr. E.C. Korten, Hartford Steam Boiler Inspection & Insurance Co, Hartford, Conn.

1 Mr. H.L. O'Brien, Graver Tank & Manufacturing Co, East Chicago, Ind.

1 Dr. D.B. Wesstrom, Des Div, Engin Dept, E.I. duPont de Nemours & Co, Wilmington, Del.

1 The Babcock and Wilcox Co, Res & Dev Dept, Alliance, Ohio

3 Pressure Vessel Res Committee, New York, N.Y.

1 Dr. James R. Adams, Development Engineer, Midvale Co, Philadelphia, Pa.

1 Mr. Perry R. Cassidy, The Babcock & Wilcox Co, Boston, Mass.

1 Mr. Harold C. Hill, Asst Chief Engineer, Fabricated Steel Construction, Bethlehem Steel Co, Bethlehem, Pa.

1 Mr. Henry Liessenberg, Stress Res Engin, Combustion Engineering-Superheater, Inc., New York, N.Y.

1 Mr. Frank L. Maker, Engin Specialist, Standard Oil Co of Calif., San Francisco, Calif.

1 Mr. James J. Murphy, Section Engin, Dev Div, M.W. Kellogg Co, New York, N.Y.

1 Mr. Cyril O. Rhys, Sr., Independent Consultant, Westfield, N.J.

1 Mr. G.J. Schoessow, The Babcock & Wilcox Co, Barberton, Ohio

1 Mr. W. Spraragen, Dir, Welding Res Council, N.Y., N.Y.

1 Mr. George W. Watts, Dir of Engin, Standard Oil Co (Indiana), Chicago, Illinois

1 Armour Res Fdtn, Chicago, Ill.

1 General Dynamics Corp, Elec Boat Div, Groton, Conn.

1 BSRA, London, England

3 CJS

9 BJSM (NS)