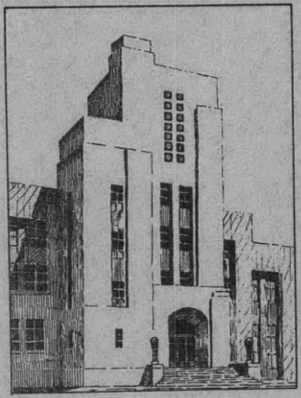
THE DAVID W. TAYLOR MODEL BASIN

UNITED STATES NAVY

A SUMMARY OF STATIC DEFLECTION DATA ON THREE SERIES OF PLASTIC DIAPHRAGMS

BY F. MINTZ



OCTOBER 1943

REPORT 506

RESTRICTED

NAVY DEPARTMENT
DAVID TAYLOR MODEL BASIN
WASHINGTON, D.C.

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PERSONNEL

The data presented in this report were collected from a number of sources. The Modugno gage calibration curves were prepared by the Norfolk Navy Yard. The tests on the 5-inch diaphragms were performed by W.S. Preston, Jr., and the 20-inch diaphragm data were obtained by E.P. Donoghue and W.S. Preston, Jr., of the David Taylor Model Basin staff. The theory on which this report is based was derived by Dr. A.N. Gleyzal. The calculations were made under the direction of F. Mintz, who prepared this report. Captain W.P. Roop, USN, who is in charge of the entire underwater explosion project at the David Taylor Model Basin, advanced many valuable suggestions in connection with the preparation of the text.

A SUMMARY OF STATIC DEFLECTION DATA ON THREE SERIES OF PLASTIC DIAPHRAGMS

ABSTRACT

Curves of deflection plotted against pressure are presented for three series of diaphragms classified by diameter; the nominal diameters are 1 3/16 inch,* 5 inches, and 20 inches. Stress values based on the simple theory of TMB Report 490 (1)** are plotted. According to this theory,

$$\sigma = \frac{p\,(a^2 + z^2)}{4hz}$$

where, in a diaphragm of radius a and initial thickness h, σ is the stress, assumed uniform throughout the diaphragm, and z is the deflection caused by a normal applied pressure p. Generalized curves of nominal plastic deflection are derived from the deflection-pressure data. It is concluded that the simple theory of TMB Report 490 is verified, approximately, within certain limits.

INTRODUCTION

In the underwater explosion research being carried on by the Bureau of Ships, the thin circular diaphragm has been adopted as an ideal target and damage gage, partly because it can be duplicated readily and partly because it lends itself to theoretical treatment. In an earlier David Taylor Model Basin Report (1), Dr. A.N. Gleyzal developed a nominal theory relating the normal loading and the energy of elastic and plastic deformation to the deflection of thin circular plates with clamped edges. The field of special interest in this study lies in the plastic range, where the deflections are so large that they are of the same order as the contour radius of the diaphragm.

Considerable experimental work has been performed on diaphragms of various types in connection with this problem. The results of some of this work, treated in conformity with the nominal theory, are presented here.

The pattern of all tests performed was of the same general nature; see Figure 1. A diaphragm of known dimensions was submitted to a series of known static pressures, and the resultant deflections on the central axis were measured. Data thus obtained were plotted as deflection on a basis of pressure.

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* These are Modugno gage disks.

** Numbers in parentheses indicate references on page 23 of this report.

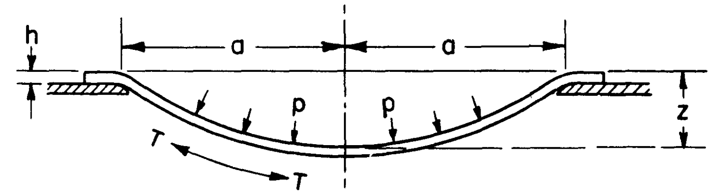

Figure 1 - Deformed Circular Plate Subjected to Pressure

GENERAL CONSIDERATIONS

One of the results derived in Reference (1) is the relationship among pressure, deflection, and stress

$$p = \frac{4\sigma h z}{a^2 + z^2} \qquad [1]$$

where p is the pressure necessary to cause a deflection z in a diaphragm of radius a and initial thickness h in which the stress σ is assumed uniform. The symbol a in Figure 1 denotes the radius of the rim boundary, or contour of the diaphragm; this is independent of the deflection in all cases.

From the data at hand, p and z are known concurrently, so that the uniform stress may then be calculated as

$$\sigma = p\left(\frac{a^2 + z^2}{4hz}\right) \qquad [1a]$$

It is a basic assumption of the nominal theory previously referred to, that the reduction in thickness as the area of the diaphragm increases plastically, is offset in this expression by the effect of strain-hardening in increasing the value of stress which the metal will carry. Thus, if the reduced value of h and the increased value of σ at pressure p are distinguished as h_p and σ_p, the nominal theory assumes that $\sigma_p h_p$ has a constant value. The nominal stress σ is then $\sigma_p h_p/h$. Since h is the initial thickness and is not subject to variation, the observed variations of σ thus indicate actual variations of $\sigma_p h_p$.

The calculation of σ in accordance with Equation [1a] has been performed for every measured point on the deflection-pressure curves of three series of diaphragms, and the results have been plotted as σ on a basis of pressure. The energy per unit volume of metal u, absorbed by the diaphragm at deflection z, is then, in accordance with Equation [6a] of Reference (1)

$$u = \sigma\left(\frac{z}{a}\right)^2 \text{ if } \sigma \text{ is constant}$$

Data of the sort presented here have two different uses:

(a) as reference calibration for the conversion of results obtained under explosive load;

(b) as an indication of the plastic behavior of a model structure of simple form.

From both points of view, a summarized presentation of the data is desirable.

For purposes of field measurements, the practice of utilizing the calibration curve exactly as obtained in the static test, for converting deflections into pressures, has the merit of simplicity and of reducing to a minimum the uncertainty as to the properties of the material, provided suitable precautions are taken to assure that the diaphragms calibrated have a close similarity to those used in the field measurement. The precaution commonly observed is to cut all diaphragms for a given series of tests from a given supply of metal, assumed to be homogeneous. When that supply is exhausted and a new supply must be drawn upon, a new calibration is necessary.

Precautions of this kind were taken in the present series, except that the starred and unstarred items in Tables 2 to 4 on pages 16, 17, and 18 belong in separate series.

In order to estimate dispersion in these series, however, it is necessary to reduce the elements of each curve to a restricted number of parameters, preferably a single one. A procedure of this kind is even more necessary if the data are to be made to afford a fair picture from the experimental point of view of the plastic behavior of diaphragms considered as models of large structures. The major relations between the variables must be separated from any random and incidental effects which may obscure them.

TYPES OF DIAPHRAGMS TESTED

Three series of diaphragms of various materials and thicknesses have been tested by various agencies; for convenience these may be classified by their diameters. The first series, known as Modugno gage disks, were 1 3/16 inch in diameter, the second series were thin steel and copper diaphragms 5 inches in diameter and the third series were special machined diaphragms, 20 inches in diameter, with heavy integral rims.

1 3/16 INCH DIAPHRAGMS OF MODUGNO GAGES

The plastic member of a Modugno gage consists of a disk, 1 3/16 inch in free diameter, of copper or steel, ranging in thickness from about 0.01 inch to about 0.17 inch, clamped in a holder as shown in Figure 2, reproduced

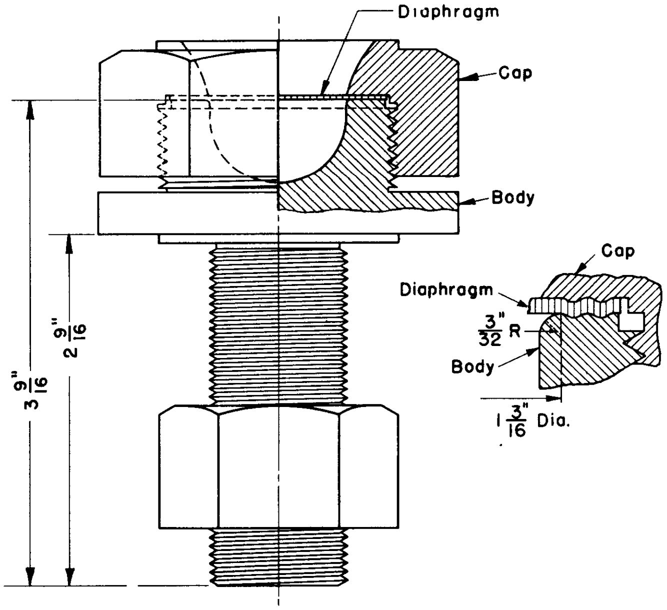

Figure 2 - Arrangement of Modugno Gage

The enlarged section shows the corrugations in the seats to hold the disk firmly around the edge.

from a Bureau of Ships report (2). When subjected to pressure, the initially flat disk deforms. After release of the pressure it assumes a permanent set which is used as a measure of the applied pressure. In present practice,[*] the disk is used for only one measurement and then discarded.

In the calibration of these gages, several disks of nearly equal thickness, are subjected to a known pressure. The only quantity measured is the permanent set *after* removal of the pressure. Data of this sort, obtained over a range of pressures up to the rupture point, are plotted; each point is labeled with the exact original thickness of the particular disk used. The curve which is faired through these points is the calibration curve for the range of thicknesses covered.

Actual tests of Modugno gages were not performed at the Taylor Model Basin; all the data presented in this report were obtained from calibration curves made at the Norfolk Navy Yard (3). The method of handling the data is described in a subsequent section.

5-INCH DIAPHRAGMS

Circular plates of copper and steel were tested at the Taylor Model Basin in a special loading device developed by the National Bureau of Standards (4) to allow an exposed surface 5 inches in diameter. The arrangement is shown in Figure 3. To each plate a series of increasing and known pressures was applied until rupture occurred, and the total deformation corresponding to each pressure was recorded. These data were then plotted as deflection on a basis of pressure.

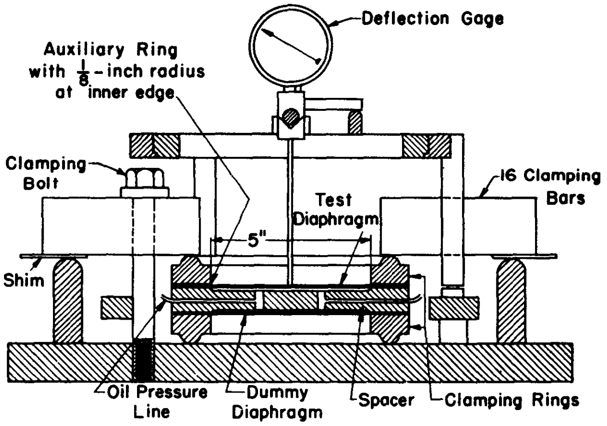

Figure 3 - Sketch showing Fixture for Testing 5-inch Diaphragms

Two identical thin diaphragms were assembled over a circular spacer block and clamped to it by the action of 16 radial clamping bars held to a base plate by a ring of bolts. The auxiliary ring prevents shearing of the diaphragm at its boundary. The deflection measuring apparatus is mounted on the base plate entirely independently of the diaphragms and clamping gear.

20-INCH DIAPHRAGMS

A loading device designed at the Bureau of Ships was used in the tests of 20-inch diaphragms machined from the solid with heavy integral rims (5) (6); see Figure 4. In these tests, each diaphragm was subjected to increasing pressures in more or less equal steps. At

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[*] At the time of writing, diaphragms were used only once whether in calibration or in field testing; later technique in calibration, however, is to use one disk throughout the entire range of pressures (2).

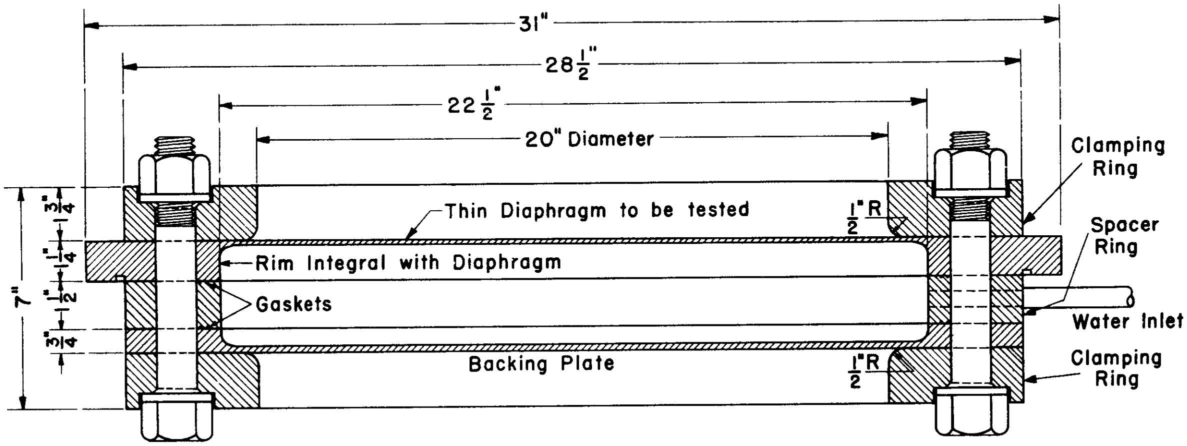

Figure 4 - Cross Section of 20-inch Diaphragm Test Assembly

each new pressure, deflections at many points distributed over the face of the diaphragm were measured, of which only the deflection taken at the center is presented here. In addition, the permanent set in Diaphragms 3, 4, and 5 was measured in the following manner. After the deflection for a given pressure had been read, the pressure was reduced to 10 pounds per square inch, and the residual deformation was observed. Readings were taken for pressures from 50 pounds per square inch upward at sufficiently frequent intervals to define the curve. The values thus obtained have been plotted, together with the deflection-pressure curve for the diaphragm. The characteristics of the various diaphragms for which data are given are set down in Table 1.

TEST DATA

The deflection-pressure data are shown graphically in Figures 5 to 13 inclusive. To these plots there have been added stress-pressure data derived by the following method:

Given the contour radius a and the initial thickness h of a diaphragm and the observed deflection z due to an applied pressure, the stress is calculated simply and directly by Equation [1a]. In the case of diaphragms such as the 5-inch and 20-inch, for which data were given directly in terms of pressure and deflection for a plate of given thickness, little numerical manipulation is required.

The Modugno gages, however, present a different problem, arising from the particular test procedure used. In handling these data, it was necessary to assume a certain standard thickness of disk; this was generally the average of the range of thicknesses used. Each deflection point for a given pressure was then converted to a deflection for the standard thickness by

TABLE 1

Characteristics of Various Diaphragms

Nominal Diameter inches	Material	Thickness, h inches			Results Plotted in Figure
1 3/16 Modugno Gage	Copper	0.0110	0.0164*	0.0214*	7a, 5a, 5b
		0.0225	0.0235	0.0333	7b, 7c, 7d
		0.0336*	0.0462*	0.0505	5c, 5d, 7e
		0.0514*	0.0637*	0.0650	5e, 6a, 7f
		0.0712*	0.0735	0.0818*	6b, 7g, 6c
		0.0845	0.1002*	0.1020*	7h, 6d, 6e
		0.1055	0.1665	0.1685	
	Medium Steel	0.022	0.0350	0.0485	8a, 8b, 8c
		0.0633	0.0785		8d, 8e
5	Copper	1/64†			9a, 9b, 9c
	Furniture Steel	0.023†			10a,10b,11a
		0.026†			11b
		0.030†			11c, 11e
		0.030†			11d, 11h
		0.030†			11f, 11g
20	Medium Steel	0.125			12c, 12b
		0.063			12a
	Special Treatment Steel	0.125			13b
		0.063			13a

* Disks identified by an asterisk were fabricated from a different lot of metal than the unmarked disks. The two series are referred to hereafter as the starred and unstarred series, respectively.

† These thicknesses are nominal.

multiplication by the factor h_0/h_i,* where h_0 is the standard thickness chosen and h_i the thickness of the disk for which the point is plotted. The average of the "converted" deflections was then used as the deflection at the given pressure for the standard thickness. The curve of average converted deflections against pressures is the one presented in this report, and the points on it are used in the calculation of the points on the stress-pressure curve.

COMMENTS ON TEST DATA

The deflection-pressure curves represent the observed data directly and relate the plastic deflection of a diaphragm to the pressure imposed on

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* The conversion factor h_0/h_i is based on the assumption of an inverse proportionality between deflection and thickness. From Equation [1] it is apparent that for values of z which are small compared to a, this assumption is fairly accurate. In view of the additional fact that the differences in thickness are small, the conversion is sufficiently valid.

it. The pressure-stress curves, representing the reduced data, relate the pressure with the nominal stress, where the stress is calculated by Equation [1a]. The figures for stress obtained here are exactly valid only when the assumptions on which the equation is based are correct. Constancy of the value would indicate accurate cancellation of strain-hardening by loss of thickness, and variation would indicate that this cancellation is not exact.

The deflection-pressure curves exhibit what might be termed "original" data, that is to say, they are based upon actual observations which are modified, if at all, only to a slight degree. They may therefore be used, if desired, either wholly independently of the theoretical considerations on which this report is based, or for possible further investigations involving energy analyses. It is for this latter reason that data on permanent set have been included where available.

With respect to the various curves it should be noted that the Modugno gage deflection-pressure curves are all actually curves of permanent set, taking account of spring-back, rather than of deflection under load. Although, strictly speaking, Equation [1] relates to total deflection under load, the error introduced by using permanent set rather than total deflection for Modugno gages, except at low pressures, is relatively insignificant.

In many of the stress-pressure curves, the points at low pressures are not plotted since the considerable variations in the stresses calculated for low pressures have no significance except as observational errors. Systematic deviations also exist because the bending effects and the deviation of the disk from initial flatness, both of which are ignored in the formula, have consequences which are most pronounced at low pressures and deflections.

Separate curves of permanent set are plotted for some of the 20-inch diaphragms. In the use of these curves for energy calculations, it should be kept in mind that the values of set were found by release only to a pressure of 10 pounds per square inch on the diaphragm. It might have been desirable from a theoretical standpoint to have removed all the load before measuring the set. However, the use of a small "retainer" load is a practical experimental necessity as consistent readings at zero pressure are not possible. The error introduced in this manner becomes relatively small for high pressures.

DISCUSSION OF GENERAL RESULTS

According to the nominal theory (1), the pressure-deflection curves should all be straight lines at small deflection and should pass through the origin. It is in fact possible to draw in such lines if the intersection

(Text continued on page 15)

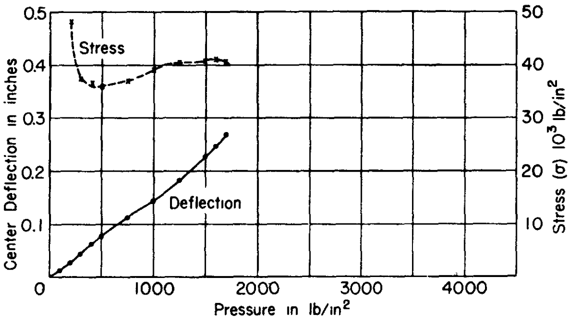

Figure 5a - Copper Diaphragm 0.0164 inch thick

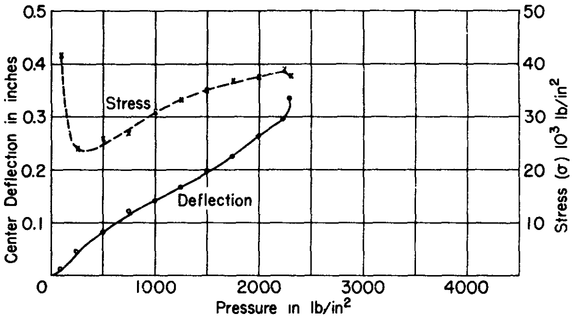

Figure 5b - Copper Diaphragm 0.0214 inch thick

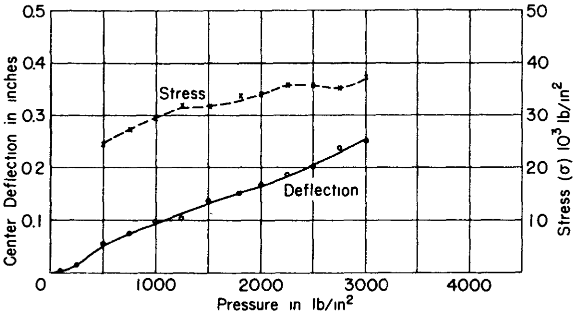

Figure 5c - Copper Diaphragm 0.0336 inch thick

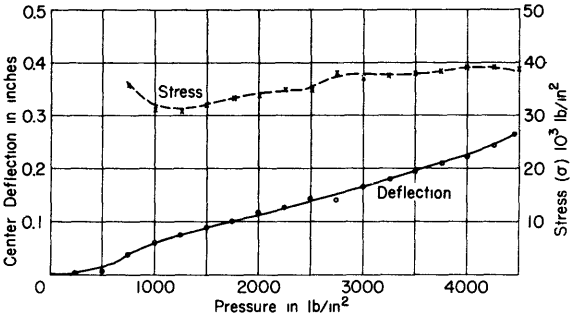

Figure 5d - Copper Diaphragm 0.0462 inch thick

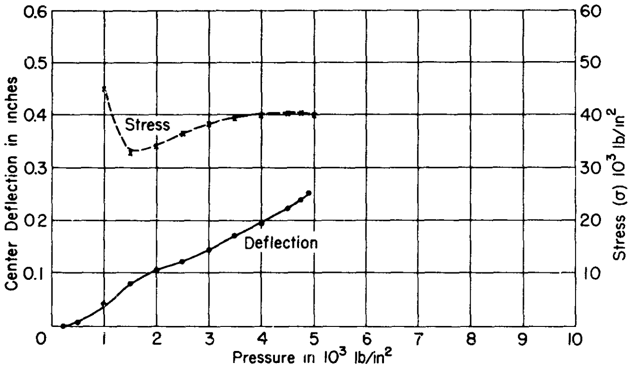

Figure 5e - Copper Diaphragm 0.0514 inch thick

Figure 5 - Curves of Deflection and Plastic Stress for Modugno Gage Disks

Starred Series

(Continued in Figure 6)

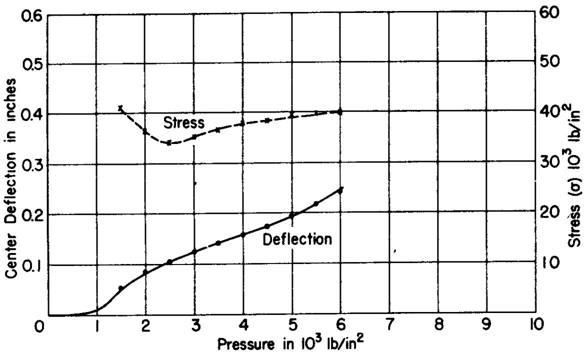

Figure 6a - Copper Diaphragm 0.0637 inch thick

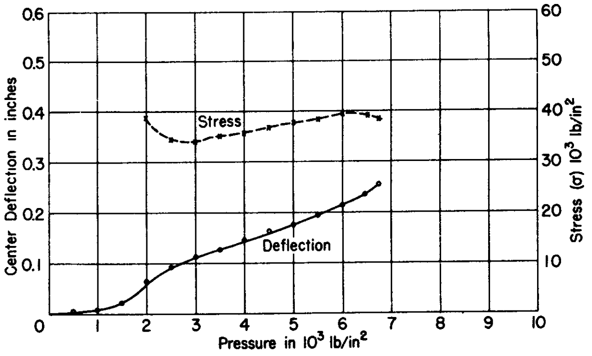

Figure 6b - Copper Diaphragm 0.0712 inch thick

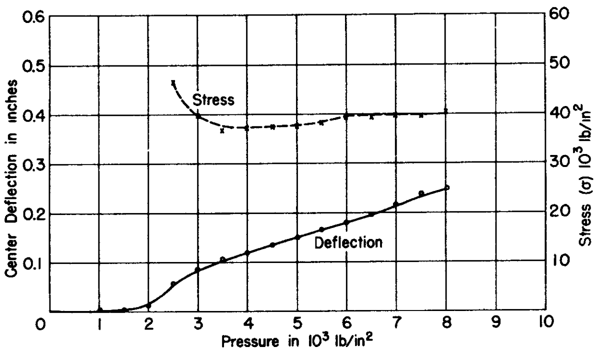

Figure 6c - Copper Diaphragm 0.0818 inch thick

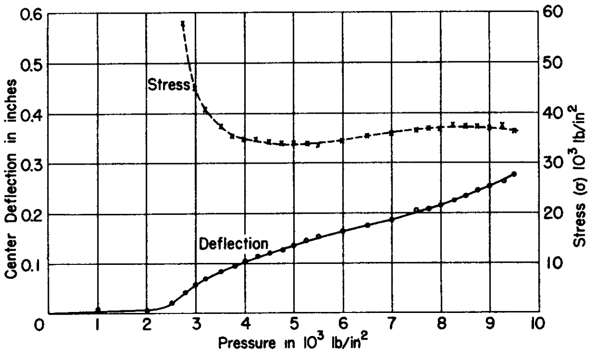

Figure 6d - Copper Diaphragm 0.1002 inch thick

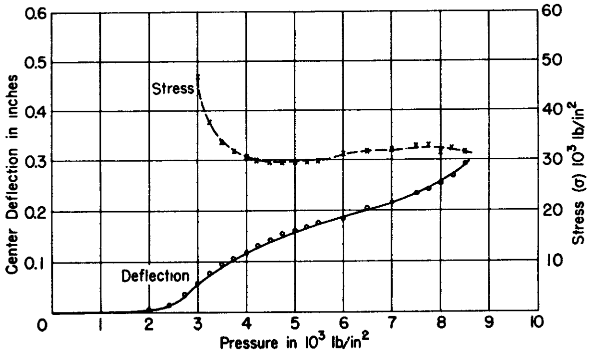

Figure 6e - Copper Diaphragm 0.1020 inch thick

Figure 6 - Curves of Deflection
and Plastic Stress for
Modugno Gage Disks

Starred Series

(Continued from Figure 5)

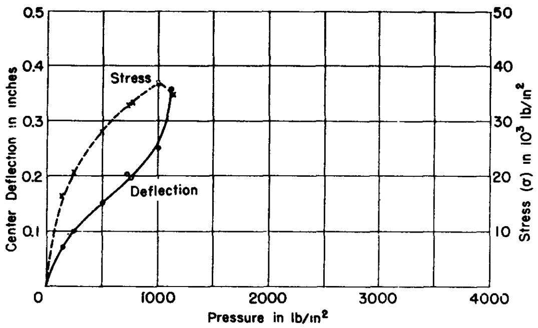

Figure 7a - Copper Diaphragm 0.0110 inch thick

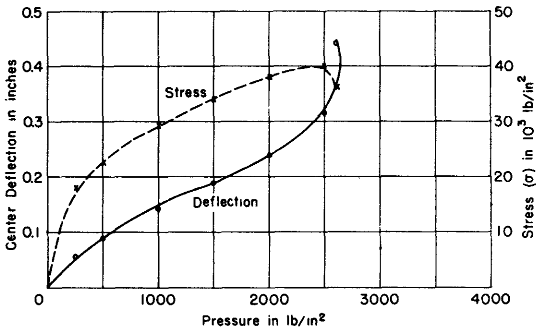

Figure 7b - Copper Diaphragm 0.0225 inch thick

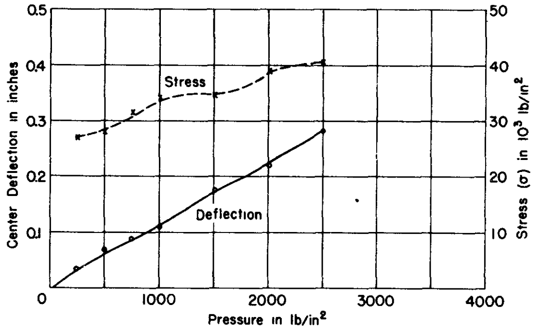

Figure 7c - Copper Diaphragm 0.0235 inch thick

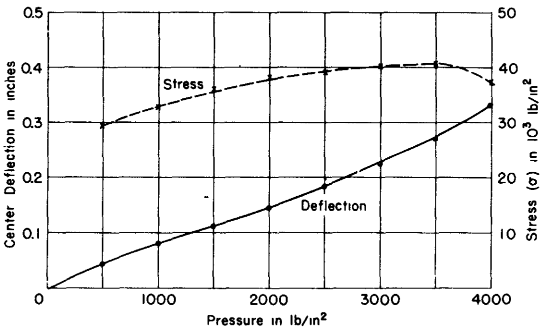

Figure 7d - Copper Diaphragm 0.0333 inch thick

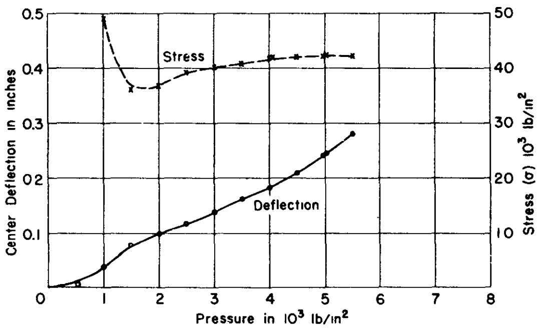

Figure 7e - Copper Diaphragm 0.0505 inch thick

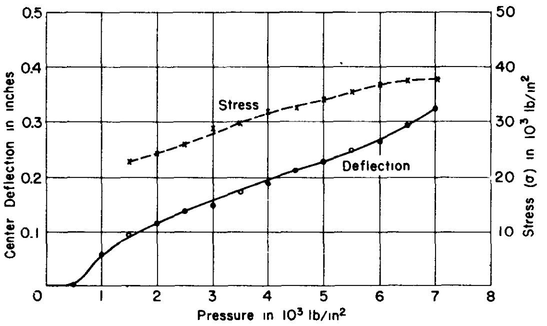

Figure 7f - Copper Diaphragm 0.0650 inch thick

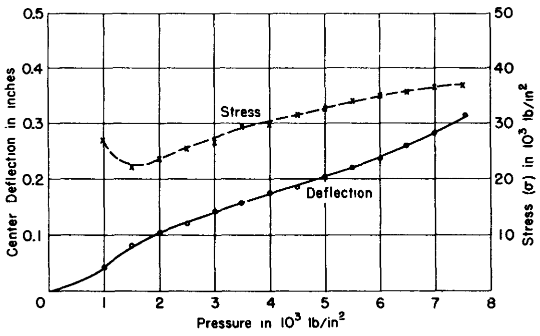

Figure 7g - Copper Diaphragm 0.0735 inch thick

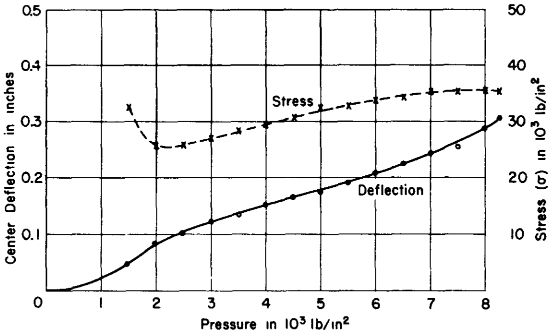

Figure 7h - Copper Diaphragm 0.0845 inch thick

Figure 7 - Curves of Deflection and Plastic Stress for Modugno Gage Disks
Unstarred Series

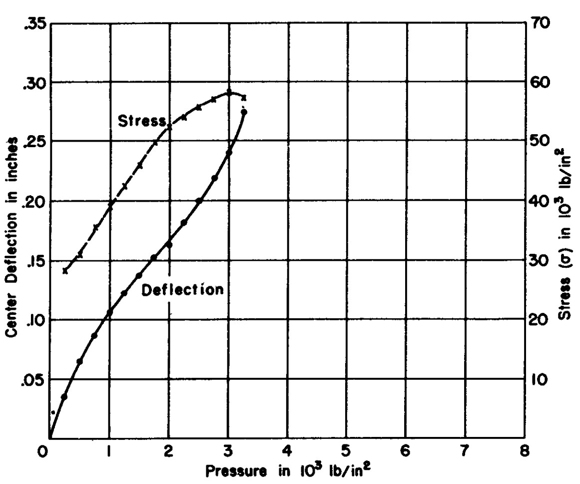

Figure 8a - Steel Diaphragm 0.022 inch thick

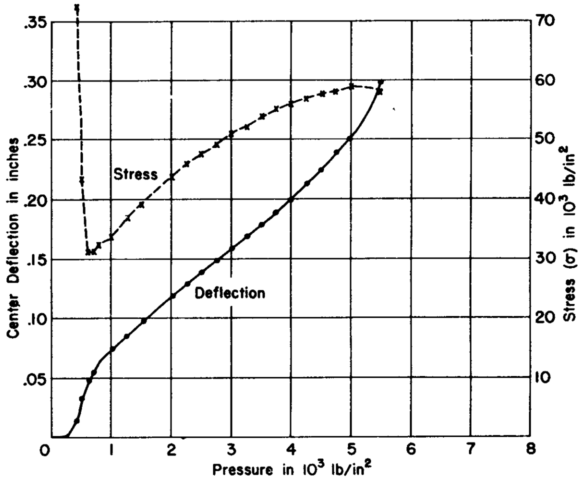

Figure 8b - Steel Diaphragm 0.0350 inch thick

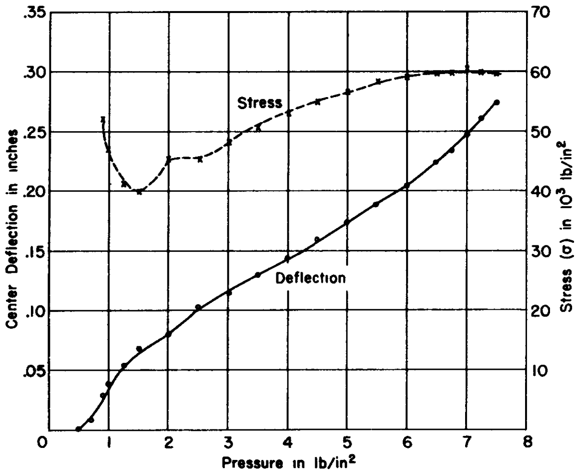

Figure 8c - Steel Diaphragm 0.0485 inch thick

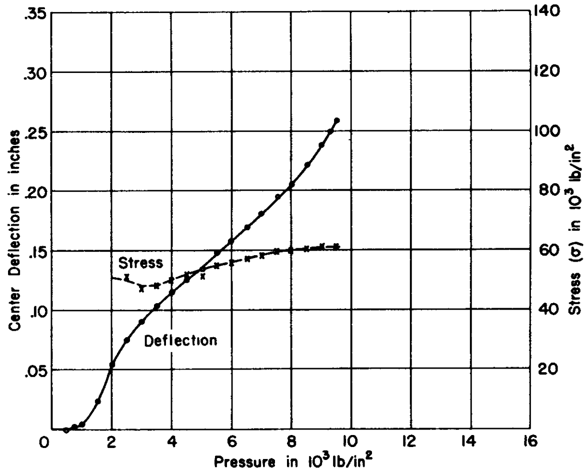

Figure 8d - Steel Diaphragm 0.0633 inch thick

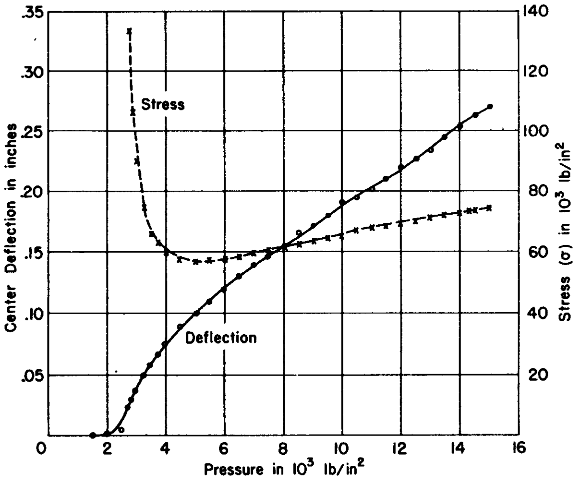

Figure 8e - Steel Diaphragm 0.0785 inch thick

Figure 8 - Curves of Deflection and Plastic Stress for Modugno Gage Disks of Steel

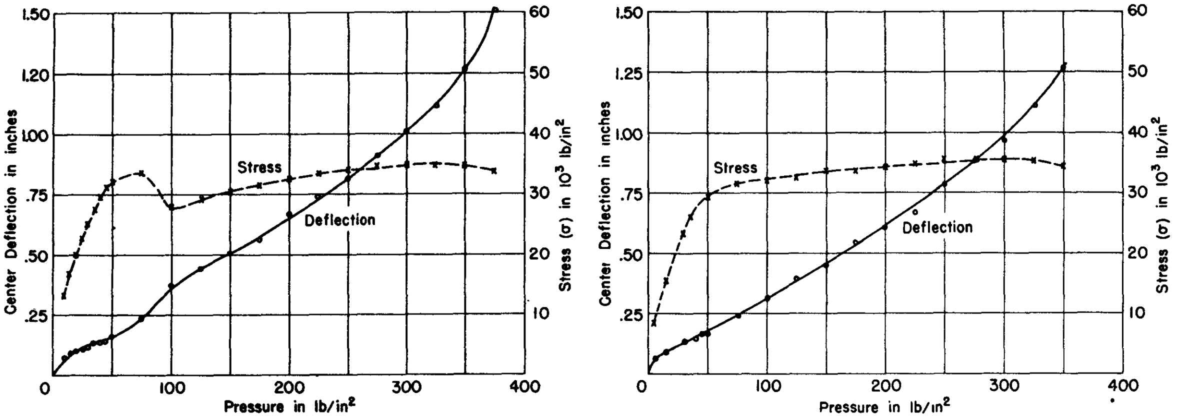

Figure 9a - Diaphragm 3, 1/64 inch thick

Figure 9b - Diaphragm 4, 1/64 inch thick

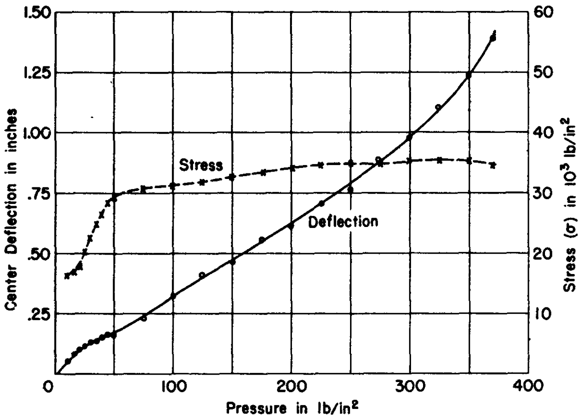

Figure 9c - Diaphragm 5, 1/64 inch thick

Figure 9 - Curves of Deflection and Plastic Stress for Copper Diaphragms 5 inches in Diameter

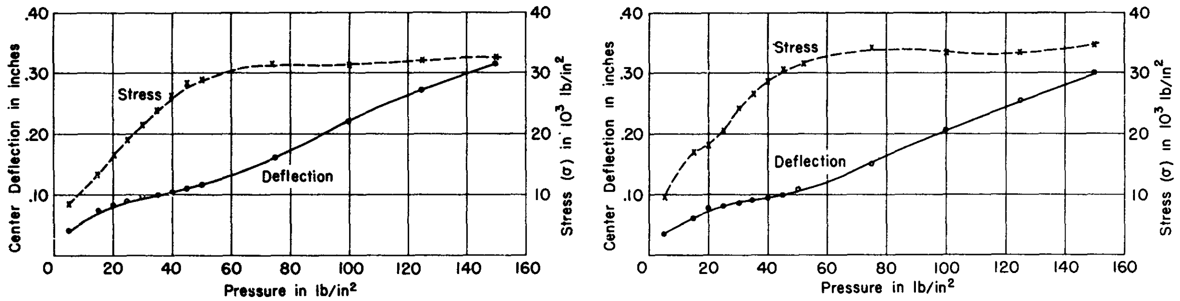

Figure 10a - Diaphragm 6, 0.023 inch thick

Figure 10b - Diaphragm 7, 0.023 inch thick

Figure 10 - Curves of Deflection and Plastic Stress for Furniture Steel Diaphragms 5 inches in Diameter

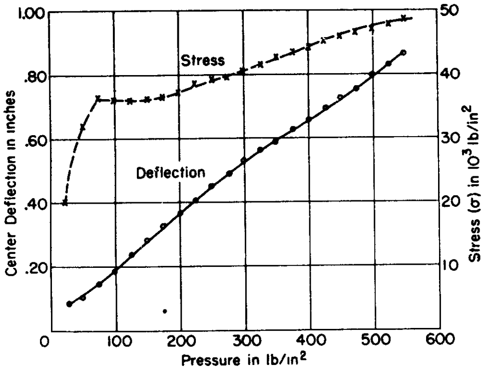

Figure 11a - Diaphragm 8, 0.023 inch thick, 29 Aug 42

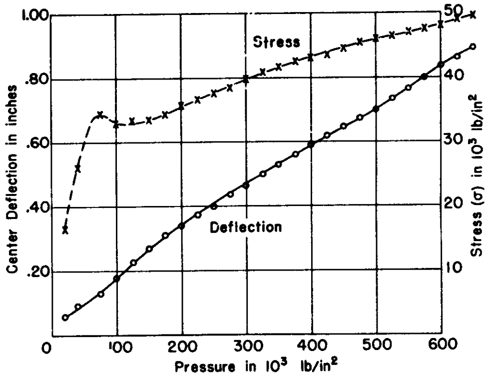

Figure 11b - Diaphragm 2, 0.026 inch thick, 20 Aug 42

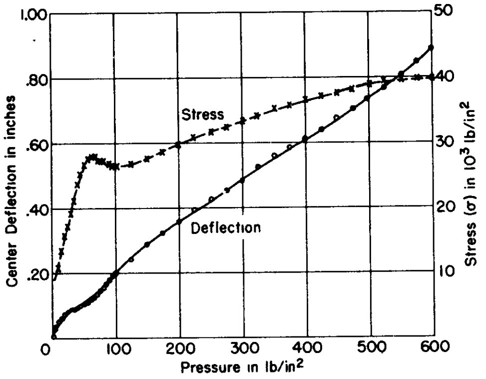

Figure 11c - Diaphragm 1, 0.030 inch thick, 14 Aug 42

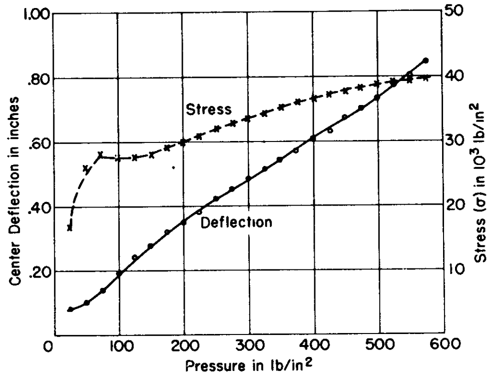

Figure 11d - Diaphragm 1, 0,030 inch thick, 20 Aug 42

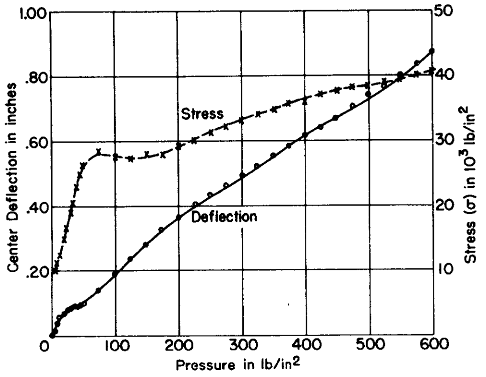

Figure 11e - Diaphragm 2, 0.030 inch thick, 14 Aug 42

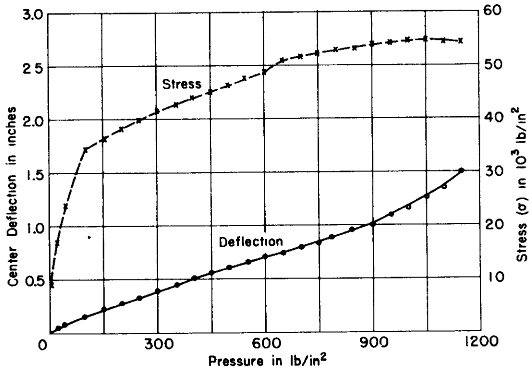

Figure 11f - Diaphragm 1, 0.030 inch thick, 24 Aug 42

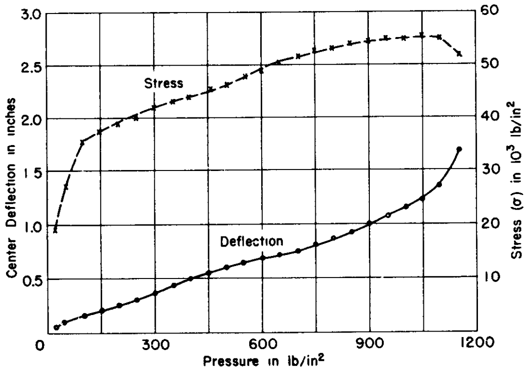

Figure 11g - Diaphragm 2, 0.030 inch thick, 24 Aug 42

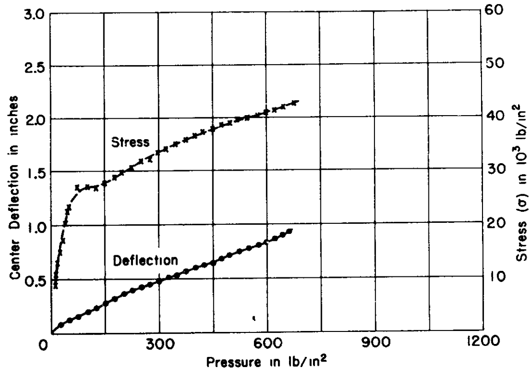

Figure 11h - Diaphragm 3, 0.030 inch thick, 20 Aug 42

Figure 11 - Curves of Deflection and Plastic Stress for Furniture Steel Diaphragms 5 inches in Diameter

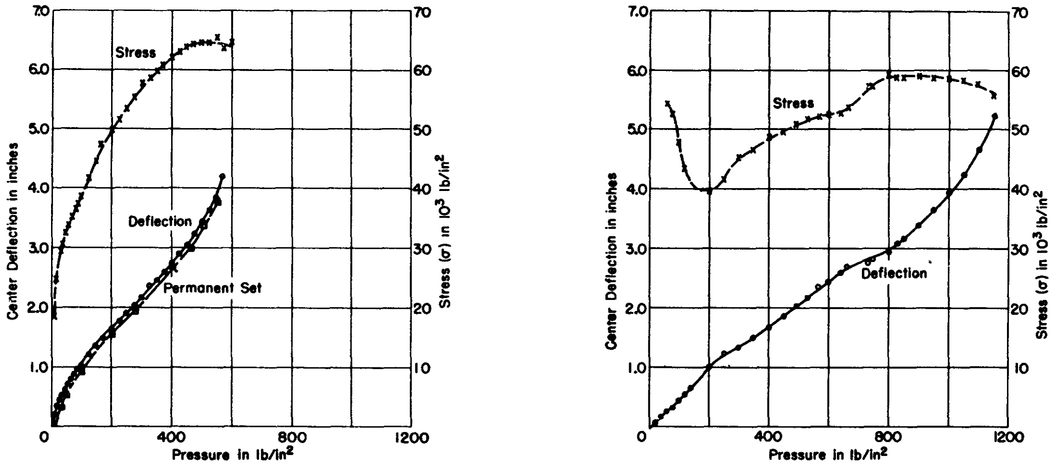

Figure 12a - Diaphragm 5, 0.063 inch thick

Figure 12b - Diaphragm 2W, 0.125 inch thick

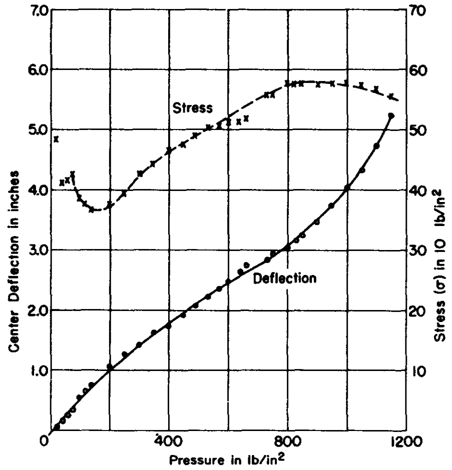

Figure 12c - Diaphragm 2E, 0.125 inch thick

Figure 12 - Curves of Deflection and Plastic Stress for Medium Steel Diaphragms 20 inches in Diameter

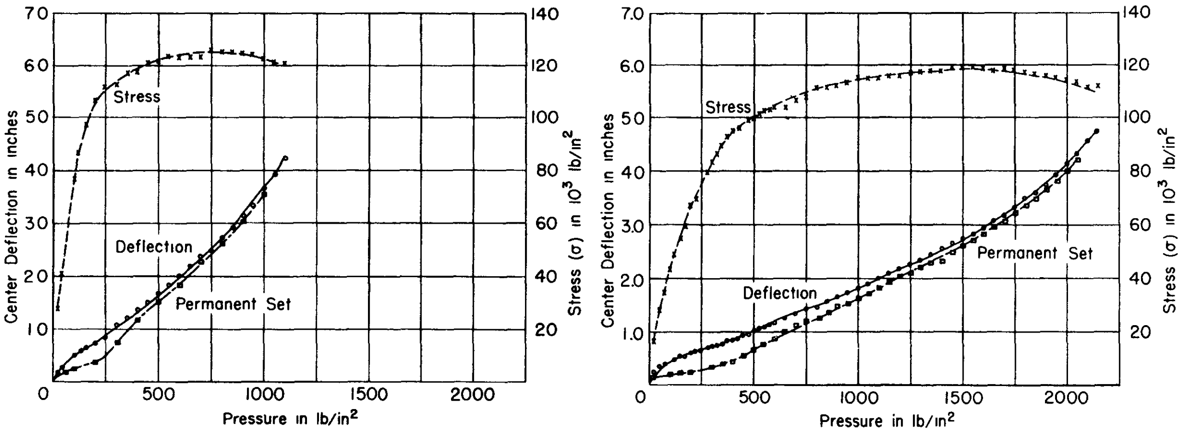

Figure 13a - Diaphragm 4, 0.063 inch thick

Figure 13b - Diaphragm 3, 0.125 inch thick

Figure 13 - Curves of Deflection and Plastic Stress for Special Treatment Steel Diaphragms 20 inches in Diameter

with the origin is allowed a certain tolerance.* The observed spots deviate somewhat from the lines so drawn. Casual examination is enough to show that there is in many cases a preliminary range of low pressures and small deflections in which deviations are systematic, followed by a higher range in which they are smaller and in which they fall more at random. Finally, when z is no longer small compared with a, the pressure deflection curve, even at constant $\sigma_p h_p$, will become concave toward the deflection axis, and will turn upward.

The random deviations may be attributed to experimental error, and especially to local variations in the metal itself. For purposes of obtaining a valid average, it is legitimate to make use of a mean line which will tell about all that can be expected from a different sample of the metal.

The systematic deviations may be attributed to:

(a) deferred establishment of fully plastic action, and persistence of variation in stress through the thickness of the diaphragm;

(b) deviations from flatness in the initial condition, so that the starting point is misjudged;

(c) finite thickness, which causes complications especially near the clamped boundary;

(d) effect of z^2 added to a^2 in the uppermost range.

The lines are straight, aside from item (d), if the systematic deviations can be ignored. This imposes two restrictions:

1. the deflections are valid only within certain limiting values. If these limits are known, this causes no special difficulty.

2. the datum for the zero pressure may not coincide with the actual position of the diaphragm at zero pressure as subject to direct measurement. This is much more difficult to allow for.

Test results on the diaphragms studied will now be discussed separately by groups.

COPPER DIAPHRAGMS FOR MODUGNO GAGES

Figure 5 on page 8 refers to copper Modugno disks up to 0.10 inch thick. All the deflection curves have a slight S-form; the upward turn at high deflection may be explained by the term z^2 in the denominator of Equation [1]. After allowance for this, the fraction $\frac{a^2}{4\sigma h} = \left(\frac{dz}{dp}\right)_{z=0}$ was evaluated

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* This has been done, but the lines are not reproduced in the figures since the reader may wish to make use of these graphs to apply another analysis.

by ignoring the departures from the theoretical line at small deflections and admitting an initial deflection z_0, found by extending a faired straight line to the zero pressure axis. σ was then found by dividing out the known value of a^2/h. The single parameter

$$\sigma = \frac{a^2}{4h\,\frac{dz}{dp}} \qquad [2]$$

which may be termed "nominal plastic stress," thus serves to characterize the curve. It offers an average constant value of σ, the insertion of which in Equation [1] will reasonably reproduce the curve of deflection on pressure in the fully plastic region.

The results so obtained are given in Table 2.

TABLE 2

Nominal Plastic Stress
Copper Modugno Disks, Starred Series

Figure	Original Thickness h inches	Initial Deflection z_0 inches	Nominal Plastic Stress σ kips per square inch
5a	0.0164	0.0	37.0
b	0.0214	0.02	34.3
c	0.0336	0.02	35.5
d	0.0462	0.01	34.8
e	0.0514	0.0	34.4
6a	0.0637	0.02	39.5
b	0.0712	- 0.01	32.6
c	0.0818	- 0.01	33.7
d	0.1002	- 0.02	24.0
e	0.1020	0.04	29.0

It seems hardly likely, at first consideration, that diaphragms would be used if the initial deflection was as great as the values indicated. The probability is, however, that the standard practice of crimping the edges of these disks actually throws them out of flat. It is believed, therefore, that the combined effects of the systematic errors at low pressure can be suitably compensated by a correction for the initial deflection, provided the gage is not used for final deflections less than, say, 0.10 inch. The nominal value for stress averaged from all diaphragms but the last two in Table 2 is

$$\bar{\sigma} = 35 \text{ kips per square inch}$$

as compared with 39 kips per square inch averaged from Figure 5, and 33.5 kips per square inch by tensile test.

Figure 7 contains the curves for an earlier series of copper Modugno disks, the results of which are summarized in Table 3. These results are similar, though more scattered, and the tensile yield strength values are in this case not available. No systematic variation of z_0 is apparent. Three thicker disks in which h/a exceeded 0.2 were omitted in view of divergent results.

TABLE 3

Nominal Plastic Stress

Copper Modugno Disks, Unstarred Series

Figure	Original Thickness h inches	Initial Deflection z_0 inches	Nominal Plastic Stress σ kips per square inch
7a	0.0110	0.03	35.4
b	0.0225	0.02	34.4
c	0.0235	0.0	33.2
d	0.0333	0.01	37.8
e	0.0505	0.01	38.8
f	0.0650	0.04	35.7
g	0.0735	0.05	37.5
h	0.0845	0.03	34.8

THE MODUGNO GAGE IN GENERAL

The summarized results indicated by the formula do not in all cases match closely with observations as plotted in Figures 5 and 6. However, it is doubtful whether any more refined theory would produce better consistency. It is believed that substitution in Equation [3], following, of gage deflections obtained in underwater service will yield fairly reliable field pressure values. In particular it is questioned whether much additional benefit is to be obtained by referring the readings for each thickness separately to a calibration curve for that thickness.

If a systematic variation of σ with thickness were discernible, the case would be better for using a separate set of constants, or even for using the original wholly empirical curve, separately for each thickness. Table 4 contains a comparison of the stress parameters obtained from the curve-fitting process with the maximum values read from the curves of Figures 5, 6, and 7. In neither of the two series is there any evidence of such a systematic variation. It is therefore concluded that a value for σ averaged from all thicknesses has a better chance of being right for a new diaphragm of any thickness

TABLE 4

Variation of Plastic Stress Value in Various Samples of Copper Used in Modugno Disks

Original Thickness h inches	Series	Observed Maximum Plastic Stress kips per square inch	Nominal Plastic Stress σ kips per square inch
0.0110	Unstarred		35.4
0.0164	Starred	41	37.0
0.0214	Starred	38	34.5
0.0225	Unstarred	40	34.4
0.0235	Unstarred	40	33.2
0.0333	Unstarred	41	37.8
0.0336	Starred	37	35.5
0.0462	Starred	39	34.8
0.0505	Unstarred	42	38.8
0.0514	Starred	40	34.4
0.0637	Starred	40	39.5
0.0650	Unstarred	38	35.7
0.0712	Starred	40	32.6
0.0735	Unstarred	38	37.5
0.0818	Starred	40	33.7
0.0845	Unstarred	36	34.8

cut from the same lot of material than a value based on one thickness only, even if the thickness is the same in the calibrated sample and the new sample. These considerations lead to the use of the formula

$$p = 4 \cdot 35 \cdot \frac{hz}{a^2 + z^2} \text{ kips per square inch} \qquad [3]$$

A graph similar to those of Figures 5 and 6 but based on Equation [3] is plotted in Figure 14 for reference and comparison, by substitution of the average stress, 35 kips per square inch for σ in Equation [1].

The family of curves in Figure 14 is plotted by use of constants in Equation [3] which were averaged from all

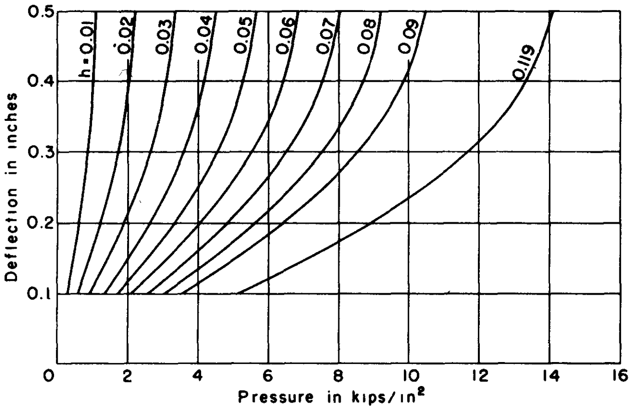

Figure 14 - Nominal Plastic Deflection of Copper Modugno Disks

thicknesses. Since the curves are inaccurate for values of z below 0.10 inch, that portion of the curves is blanked out. The maximum thickness for which validity of the curves is acceptable is considered to be $0.2a$ (= 0.119 inch).

It is suggested that the consistency of operation of the Modugno gage might be improved by using diaphragms pre-deflected to a known pressure value and considered to indicate excess over that value in terms of additional deflection observed.

STEEL DIAPHRAGMS FOR MODUGNO GAGES

The steel diaphragms for Modugno gages follow a pattern similar to that of the copper disks. A tendency seen also in the thicker copper appears here at a smaller thickness, namely, failure of the deflection curve to turn upward at the upper end. It is clear that as the thickness increases the assumptions involved in the simplified theory become less accurate. The reason why the deflection curve should show less upturn at large deflections as thickness increases is not clear, but until this anomaly is resolved it seems better to limit diaphragms in use to the smaller thicknesses for which the simple theory is valid.

In Table 5, which summarizes the data from Figure 8, the last item is considered to lie beyond the limit of suitable thickness. The first four items show a progressive variation of z_0 with h. The formula for these medium steel diaphragms, obtained as before, with additional terms to take care of a roughly systematic variation in initial deflection, is

$$p = 4 \cdot 71 \cdot h \cdot \frac{z - 0.047 + 0.25h}{a^2 + z^2} \text{ kips per square inch} \qquad [4]$$

Curves based on this equation are plotted for representative thicknesses in Figure 15; Equation [4] should not be used for thickness exceeding 0.07 inch or final deflection less than 0.10 inch.

TABLE 5

Nominal Plastic Stress

Medium Steel Modugno Disks

Figure	Original Thickness h inches	Initial Deflection z_0 inches	Nominal Plastic Stress σ kips per square inch
8a	0.022	0.042	72.3
b	0.035	0.038	68.1
c	0.0485	0.032	71.0
d	0.0633	0.031	71.8
e	0.0785	0.032	80.8

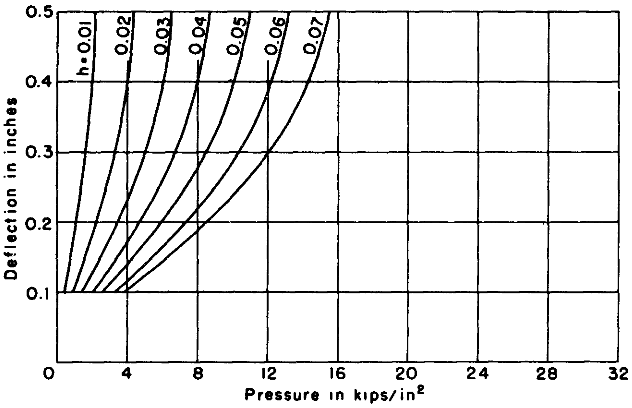

Figure 15 - Nominal Plastic Deflection of Steel Modugno Disks

TABLE 6

Nominal Plastic Stress in 5-inch Copper Disks 1/16 inch Thick

Figure	Initial Deflection z_0 inches	Nominal Plastic Stress σ kips per square inch
9a	0.10	38.9
b	0.04	36.7
c	0.08	39.6

5-INCH COPPER DIAPHRAGMS

Only three 5-inch copper diaphragms were tested, all of the same thickness. The data of Figure 9 are summarized in Table 6.

5-INCH DIAPHRAGMS OF FURNITURE STEEL

The series of ten 5-inch furniture-steel diaphragms included six of the same thickness, and this affords a good opportunity to estimate the dispersion in results to be expected from this material. Only very small thicknesses were used, so that no limitation to the simple theory arising from excessive thickness was found. The irregularities at small deflections extended to greater values of deflection. This may possibly be explained by supposing that plastic flow proceeded further before the effects of the stretcher-leveling* process are overcome. The values of z_0 are erratic, and no systematic variation in them is discernible.

In view of the extensive use of furniture steel in structural models, and especially in recent models loaded by underwater explosion, a quantitative estimate of the dispersion has been made, and the mean error of σ is found to be plus or minus 12 per cent. Figure 16 has been made up for the thicknesses of furniture steel in common use. The formula used, determined as previously, is

$$p = 4 \cdot 48 \cdot h \cdot \frac{z - 0.09 \text{ inch}}{a^2 + z^2} \text{ kips per square inch} \qquad [5]$$

The grossly approximate nature of the results as summarized here must, however, be emphasized and kept in mind if these curves are to be applied to

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[*] Stretcher-leveling is a process for flattening furniture steel, in which a long sheet is seized in corrugated grips at the ends and stretched until the yield point is exceeded throughout the plate.

TABLE 7

Nominal Plastic Stress in Furniture-Steel Diaphragms of 5-inch Diameter

Figure	Original Thickness h inches	Initial Deflection z_0 inches	Nominal Plastic Stress σ kips per square inch	$\Delta\sigma$ kips per square inch
10a	0.023	0.02	37.6	- 10.5
b	0.023	0.02	38.2	- 9.9
11a	0.023	0.06	49.3	+ 1.2
b	0.026	0.07	53.4	+ 5.3
c	0.030	0.12	45.9	- 2.2
d	0.030	0.12	45.2	- 2.9
e	0.030	0.10	55.7	+ 7.6
f	0.030	0.13	46.6	- 1.5
g	0.030	0.17	63.3	+ 15.2
h	0.030	0.07	46.2	- 1.9
Average		0.088	48.1	5.8

practical use. Their limitations can be judged from the dispersion data in Table 7.

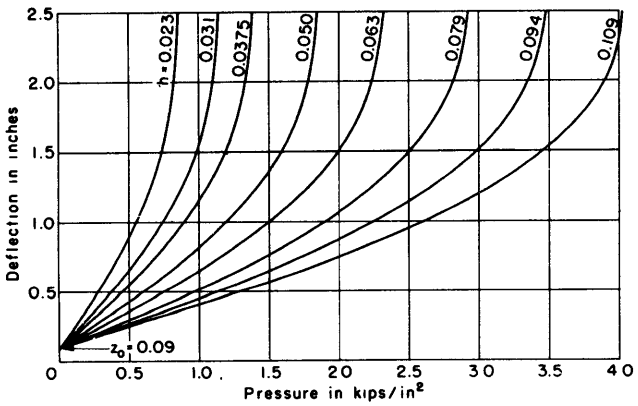

Figure 16 - Nominal Plastic Deflection of Furniture-Steel Diaphragms of 5-inch Diameter

20-INCH DIAPHRAGMS

A series of very carefully finished steel diaphragms of 20-inch diameter were made up for explosion test at the Norfolk Navy Yard in the assembly known as a "drum caisson." The five disks on which data are given in Figures 12 and 13 formed the controls for this series, which were subjected to static load only. The full results of these static tests are being reported separately (6). Their size, the care used in their finish, the extent of the data, and the precautions taken in the preparation and testing of these diaphragms make them the most dependable source of pertinent information available at the present writing. The irregularities which show in Diaphragm 2W make it appropriate to omit it; this leaves a series of 4 items for which the characteristics are given in Table 8.

TABLE 8

Characteristics and Nominal Plastic Stress of 20-inch Steel Diaphragms

Figure	Diaphragm Number	Original Thickness h inches	Material	Yield Point kips per square inch	Initial Deflection z_0 inches	Nominal Plastic Stress σ kips per square inch
12a	5	0.063	MS	31.2	0.6	78.8
c	2E	0.125	MS	31.2	0.7	78.1
13a	4	0.063	STS	105.0	0.00	120.1
b	3	0.125	STS	105.0	0.10	118.5

On the basis of this careful work, nominal charts of plastic deformation for a series of thicknesses of medium steel and special treatment steel have been made up and are presented in Figures 17 and 18. Dispersion and limiting thickness and deflection data for these cases are not available.

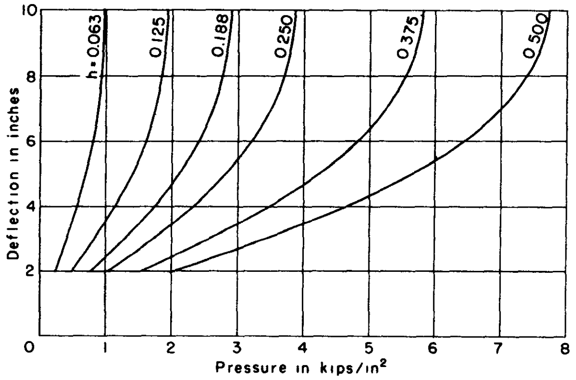

Figure 17 - Nominal Plastic Deflection of Medium-Steel Diaphragms of 20-inch Diameter

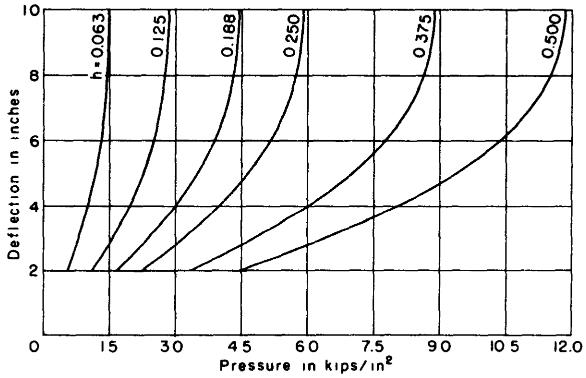

Figure 18 - Nominal Plastic Deflection of Special-Treatment-Steel Diaphragms of 20-inch Diameter

CONCLUSIONS

1. The simple nominal theory of Reference (1) is verified approximately for the static plastic deflection of copper and steel diaphragms at deflection values above a certain limit and thickness values below a certain limit.

2. The deflection, however, must in general be taken from an initial value other than zero, and sometimes varying as a linear function of the thickness.

3. The nominal stress for use in application of the simple theory is, for copper and special treatment steel, near but above the yield stress obtained in a standard tensile test; for medium steel it is considerably above the tensile yield stress. The nominal stress must for the present be found from the slope of the straight part of the deflection-pressure curve obtained in a diaphragm test, by the formula $\sigma = \frac{a^2/4h}{(dz/dp)}$.

REFERENCES

(1) "Protection Against Underwater Explosion - Plastic Deformation of a Circular Plate," by A.N. Gleyzal, TMB Report 490, September 1942.

(2) "The Modugno Gage, Its Construction, Use and Typical Results," Bureau of Ships Underwater Explosion Report 1942-3, October 1942.

(3) "Modugno Gages - Static Pressure Calibration Curves," Norfolk Plan C-41080.

(4) "Normal Pressure Tests of Circular Plates with Clamped Edges," by Albert E. McPherson, Walter Ramberg, and Samuel Levy, NACA Technical Note 848, June 1942, describing tests performed at the National Bureau of Standards.

(5) BuShips CONFIDENTIAL letter C-S81-3 (374) of 14 November 1941 to David Taylor Model Basin.

(6) "Plastic Strain and Deflection Tests on Clamped Circular Steel Plates 20 Inches in Diameter," by A.N. Gleyzal, TMB Report R-142, to be published.

For the convenience of the reader the following reports are suggested for supplementary reading.

(7) "Memorandum on the Plastic Deformation of Marine Structures by an Underwater Explosion," by John G. Kirkwood, NDRC Report, OSRD 793.

(8) "The Plastic Deformation of Marine Structures by an Underwater Explosion Wave II," by John G. Kirkwood, NDRC Report, OSRD 1115.