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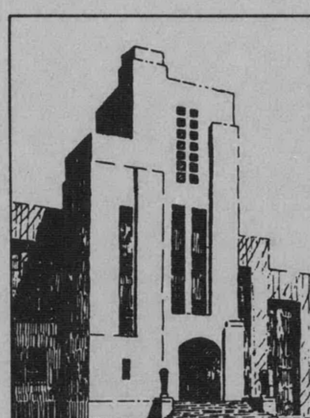
THE RESPONSE OF A BALL CRUSHER GAGE

by

George Chertock



April 1951

Report 751

NS 724-001

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ABSTRACT

Equations are derived for the response of a ball crusher gage to velocities or accelerations which are almost step functions. These equations show explicitly the effect of rise times, or decay times, on the motion; and also the effect of an initial set of the ball, or an initial stand-off of the piston. Formulas are also presented for the use of the gage to measure pressures.

I. INTRODUCTION

The ball crusher gage has been used extensively to measure simple parameters of shock motion.[1] In this gage, a copper ball is compressed between an inertial element and an anvil which is attached to the structure whose motion is being measured. There are several variations in the design of the gage depending upon the size of the copper ball, the size and shape of the inertial mass, and the number of directions in which the gage will record.

The great defect of the ball crusher gage is that the time pattern of the motion or pressure must be known before the gage readings can be accurately analyzed. One purpose of this report has been to demonstrate this fact quantitatively. Furthermore, it seems plausible that the gage should only be used where the acceleration to be measured is expected to be either an approximate step function of the time or an approximate impulsive function of the time during the response of the gage. Conversely, if it is desired to measure the magnitude of some motion whose approcimate time pattern is known, then a gage should be chosen with a response time which would make the expected motion approximately either a step function or an impulsive function.

In this report the two cases just mentioned are discussed and calibration formulas are derived which show explicitly the modifications due to small departures of the motion from the ideal which is assumed.

[1]References are listed on page 9.

II. EQUATION OF MOTION OF THE GAGE

We idealize the gage as shown in Figure 1. Suppose that in a time dt, the structure moves upward a distance dy, the ball is compressed by an amount dx, and the force exerted by the ball on the piston is F(x), x denoting the total compression of the ball. Then the displacement of the piston in this time is dy - dx, and the differential equation for its motion is

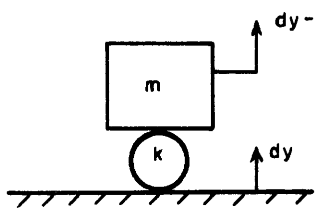

Figure 1 - Ball Crusher Gage

$$m \frac{d^2}{dt^2}(y - x) = F(x) \qquad [1]$$

Further analysis of the motion hinges on a knowledge of the force function F(x). This function is ordinarily known from a dynamic calibration of the deformation of the copper ball.[2] Such empirical calibrations show that, over a large range in compressions, the force function is simply proportional to the displacement, $F = kx$, where k is a constant dependent on the size and hardness of the copper balls.

Hence the differential equation becomes

$$\left.\begin{array}{ll} \ddot{x} + \omega^2 x = \ddot{y}(t) & \text{for } \dot{x} > 0 \\ \ddot{x} = 0 & \text{for } \dot{x} \leq 0 \end{array}\right\} \qquad [2]$$

$$\omega^2 = \frac{k}{m}$$

The second part of the equation expresses the fact that the copper ball retains the maximum deformation to which it has been compressed, i.e., that the compression is acquired plastically and not elastically. The first part of the equation is the same as that of a frictionless linear oscillator of mass m under an impressed force $+m\ddot{y}$. It is evident that the observed set compression of the copper ball after the shock must equal the maximum value of x in the equation, and is reached when $\dot{x}$ becomes zero.

III. RESPONSE TO AN ALMOST CONSTANT ACCELERATION

The first type of motion we consider is when the gage is used to measure an acceleration which jumps instantaneously to a peak value and then decays only slightly during the time the gage is responding. Such a motion can conveniently be represented for a sufficient length of time by an equation of the form

$$\ddot{y} = ae^{-\mu t} \quad \text{for } t \geq 0 \qquad [3]$$

$$\ddot{y} = 0 \qquad \text{for } t < 0$$

The solution of Equations [2] and [3] that makes $x = 0$ and $\dot{x} = 0$ at $t = 0$ is given by

$$x = \frac{a}{\mu^2+\omega^2}\left[e^{-\mu t} - \cos \omega t + \frac{\mu}{\omega}\sin \omega t\right]$$

The response time of the gage, T, is obtained by setting $\dot{x} = 0$ and is given by the first non-zero root of the transcendental equation

$$e^{-\mu T} = \frac{\omega}{\mu}\sin \omega T + \cos \omega T \qquad [4]$$

In terms of this response time the peak value of x is given by

$$x_m = \frac{a}{\mu \omega}\sin \omega T \qquad [5]$$

We solve Equation [4] for the case where $\frac{\mu}{\omega} \ll 1$ by assuming ωT to be expressed as a power series in $\frac{\mu}{\omega}$. Then by successive approximations, through terms of order $\left(\frac{\mu}{\omega}\right)^2$,

$$\omega T = \pi - 2\frac{\mu}{\omega} + \pi\left(\frac{\mu}{\omega}\right)^2 \qquad [6]$$

and substituting in Equation [5]

$$x_m = \frac{2a}{\omega^2}\left(1 - \frac{\pi}{2}\frac{\mu}{\omega}\right) \qquad [7]$$

If the acceleration did not decay during the response time of the gage, i.e., if $\frac{\mu}{\omega} = 0$, then this equation shows that the calibration formula relating an actual "step" acceleration, a_s, to the set deformation caused by it, x_m, is linear:

$$a_s = \frac{\omega^2 x_m}{2} \qquad [8]$$

In the more general case of a slow decay in acceleration from a peak of height a

$$a \cong a_s\left(1 + \frac{\pi}{2}\frac{\mu}{\omega}\right) \qquad [9]$$

where a_s is given by [8]. The term in μ represents the effect of the slow decay.

IV. RESPONSE TO AN ACCELERATION WITH FINITE RISE TIME

In practice the peak acceleration of a structure is not acquired instantaneously but is reached in some finite time. It is therefore important to estimate how this rise time will affect the response of the gage. As an example of an acceleration which rises gradually to its initial peak, let it be assumed that

$$\ddot{y} = a(1 - e^{-\mu t}) \quad t \geq 0 \qquad [10]$$

The solution of Equations [2] and [10] is given by

$$x = \frac{a}{\omega^2}\left[1 - \cos \omega t - \frac{\omega^2 e^{-\mu t}}{\mu^2+\omega^2} + \frac{\omega^2 \cos \omega t - \mu\omega \sin \omega t}{\mu^2 + \omega^2}\right] \qquad [11]$$

This equation is valid until $\dot{x}$ vanishes at a time T which is given by the first non-zero root of

$$\frac{\omega}{\mu} e^{-\mu t} = \frac{\omega}{\mu}\cos \omega T - \sin \omega T \qquad [12]$$

In terms of T, the peak value of x is given by

$$x_m = \frac{a}{\omega^2}\left(1 - \cos \omega T\right)$$

We solve Equation [12] for the case where $\frac{\omega}{\mu} \ll 1$ by assuming ωT to be expressed as a power series in $\frac{\omega}{\mu}$. Then, for small values of ωT,

$$\frac{\omega}{\mu} e^{-\frac{\mu}{\omega}\omega T}$$

is negligible and

$$\tan \omega T \simeq \frac{\omega}{\mu} \qquad [13]$$

ωT being in the third quadrant.

$$x_m = \frac{a}{\omega^2}\left(1 + \frac{\mu}{\sqrt{\mu^2+\omega^2}}\right) = \frac{a}{\omega^2}\left(2 - \frac{1}{2}\frac{\omega^2}{\mu^2}\right) \qquad [14]$$

Hence the peak acceleration a is given approximately by the equation

$$a = a_s\left[1 + \frac{1}{4}\left(\frac{\omega}{\mu}\right)^2\right] \qquad [15]$$

a_s being again given by [8]. Thus for small rise times, so that $\frac{\omega}{\mu} \ll 1$, the effect on the calibration formula is only of the second order in $\frac{\omega}{\mu}$.

Another extreme form of Equation [10] occurs when $\frac{\mu}{\omega} \ll 1$. In this case the acceleration increases so gradually that the forces on the inertial mass are always in quasi-static equilibrium (i.e., $\ddot{x}$ is negligible). The appropriate calibration formula for the peak acceleration then becomes, to a first approximation,

$$a = \omega^2 x_m$$

It is important to note, however, that for a case such as this the quasi-static value of the spring constant k of the copper ball must be used. It has been found experimentally that for quasi-static conditions the value of k is some 15 to 20 percent less than the effective value for dynamic deformation.

V. BALL CRUSHER GAGE AS PEAK VELOCITY GAGE

The use of the ball crusher gage to measure peak velocities is practically restricted to the cases where the peak velocity of the structure is acquired in a short time as compared with the response time of the gage, and the velocity is then maintained at almost constant value during the response time (i e., the motion of the structure must correspond almost to a step function of velocity, or almost to an impulsive acceleration). This type of motion can also be illustrated by Equation [3] but with the restriction that $\frac{\omega}{\mu} \ll 1$. For this case the response time T is again given by the appropriate root of Equation [4] and the final compression of the ball is

$$x_m = \frac{v_m}{\omega} \sin \omega T \qquad [16]$$

where v_m is the peak velocity of the structure.

Now in Equation [4], $e^{-\mu t}$ is negligibly small and, correct to the first order in $\frac{\omega}{\mu}$,

$$\tan \omega T = -\frac{\mu}{\omega}$$

$$\omega T = \frac{\pi}{2} + \frac{\omega}{\mu} \qquad [17]$$

and

$$x_m = \frac{v_m}{\omega} \cos \frac{\omega}{\mu} = \frac{v_m}{\omega} \left[1 - \frac{1}{2}\left(\frac{\omega}{\mu}\right)^2\right]$$

Hence

$$v_m = v_s \left[1 + \frac{1}{2}\left(\frac{\omega}{\mu}\right)^2\right] \qquad [18]$$

where $v_s = \omega x_m$ and is the equivalent step velocity for the same value of x_m.

The result of this special case makes it sufficiently certain that, if the peak velocity is not reached quite instantaneously, the effect on the calibration formula is only of the second order in $\frac{\omega}{\mu}$.

VI. EFFECT OF A NONLINEAR BALL CALIBRATION

In all of the previous derivations it is assumed that the force function F(x) is proportional to the compression, or, what is equivalent, that W(x), the plastic work done in compressing the ball (which is the quantity actually measured),[2] is proportional to the square of the compression. Occasionally the compression of the ball exceeds the limit for which this simple relation holds. For such cases the equations may be modified as follows:

We multiply both sides of Equation [1] by dx and integrate over the response time of the gage. Then

$$\int_0^{x_m} m\,\ddot{y}\,dx - \int_0^{x_m} d\left(\frac{m\dot{x}^2}{2}\right) = \int_0^{x_m} F\,dx = W(x_m)$$

Now the second term vanishes because $\dot{x} = 0$ when $x = 0$ and again when $x = x_m$. Thus

$$W(x_m) = \int_0^{x_m} m\,\ddot{y}\,dx \qquad [19]$$

Hence, in general, the copper ball deforms until the work of compression is equal to the work done on it by a force of magnitude $m\ddot{y}$.

For an acceleration which is a step function of the time, $\ddot{y}$ is constant and can be taken outside of the integral. Hence, if $\ddot{y} = a_s$

$$a_s = \frac{W(x_m)}{mx_m} \qquad [20]$$

If W(x) does not deviate too greatly from $kx^2/2$, so that $\omega = \sqrt{\frac{k}{m}}$ still has meaning, we would also expect that [9] will still remain valid or

$$a = a_s\left(1 + \frac{\pi\mu}{2\omega}\right), \quad \frac{\mu}{\omega} \ll 1 \qquad [9]$$

where W(x) is determined directly from the copper-ball calibration data.

A more general equation can also be derived if the acceleration changes impulsively and the velocity is a step function of the time. In that case the effect of the impulsive force $m\ddot{y}$ is to instantaneously give the inertial element a kinetic energy $mv_s^2/2$ with respect to the structure, which thereafter moves at constant velocity. This kinetic energy then becomes zero because the inertial element does plastic work on the copper ball, hence

$$W(x_m) = \frac{mv_s^2}{2}$$

and

$$v_s = \sqrt{\frac{2W(x_m)}{m}} \qquad [21]$$

VII. EFFECT OF INITIAL SET

Suppose the copper ball has an initial deformation before the motion of the structure begins. This initial deformation might be due to mishandling of the gage prior to attaching it to the structure, or it might be due to a previous shock motion of the structure. In any case, how does the presence of the initial deformation modify the subsequent deformation of the copper ball?

Suppose at any instant the ball has a set x and a zero rate of deformation $\dot{x}$. Then at this instant $\ddot{x}$ cannot be negative, since $\dot{x}$ can never become negative. Hence, if a is the acceleration of the structure, from [1]

$$ma = F + m\ddot{x} \geq F = kx$$

Therefore

$$a \geq \frac{k}{m} x = 2a_1 \qquad [22]$$

where a_1 is the magnitude of the step acceleration which could have produced the deformation x.

This result means that if a copper ball has already undergone a certain deformation in response to the motion of a structure, and if the rate of deformation has instantaneously vanished, then the deformation will not increase further in response to subsequent motion of the structure unless its acceleration exceeds twice the step acceleration equivalent to the deformation already present.

Even if the subsequent acceleration of the structure is large enough to deform the copper ball further, then the final deformation is in general <u>smaller</u> than it would have acquired if it did not have the initial set. For if x_i is the initial deformation, x_f the final deformation, and $\bar{a}$ a mean value of the acceleration of the structure averaged over the deformation, then, from [19],

$$m\bar{a}(x_f - x_i) = \frac{K}{2}(x_f - x_i)^2$$

$$\bar{a} = \frac{\omega^2}{2}(x_f + x_i) \qquad [23]$$

and

$$x_f = \frac{2a}{\omega^2} - x_i$$

VIII. EFFECT OF INITIAL STAND-OFF

The preceding analysis has been based on the assumption that the inertial mass is in contact with the ball at the start of the motion. If this condition does not hold, then the previously derived formulas will be incorrect, and the compression of the copper ball will be greater than would otherwise occur. This can be seen from the energy equation which is derived by integrating both sides of Equation [1] with respect to x over the range $x = 0$ to $x = x_m$.

$$\int_0^{x_m} m\ddot{y}\, dx - \int_0^{x_m} d\left(\frac{m\dot{x}^2}{2}\right) = \int_0^{x_m} F\, dx = W(x_m)$$

Now if, for example, the structure moves with a constant acceleration a, but the piston's initial stand-off from the ball is a distance s, then at the time when the piston first touches the ball the relative velocity between piston and structure will be $\sqrt{2as}$, and this must also be the value of $\dot{x}$ when $x = 0$. Hence

$$ma\, x_m + \frac{m}{2}(2\, as) = W(x_m) = \frac{kx_m^2}{2}$$

$$ma(x_m + s) = \frac{kx_m^2}{2}$$

$$a = \frac{\omega^2}{2}\,\frac{x_m^2}{x_m + s} \simeq \frac{W^2 x_m}{2}\left(1 - \frac{s}{x_m}\right), \text{ if } s \ll x$$

This result emphasizes the necessity for providing some mechanism in the gage for holding the inertial mass against the copper ball.

IX. BALL CRUSHER PRESSURE GAGE

The ball crusher gage has also been used extensively to measure shock-wave pressures. In such an application the inertial mass becomes a piston with one face exposed to the unknown pressure, while the base (or structure) remains fixed. The analysis is formally identical with the analysis already given except that $m\ddot{y}$ is replaced by PA where P(t) is the pressure on the face of the piston and A is the sectional area of the piston.* Hence the results may be summarized as follows.

For a step function of pressure of magnitude P_s,

$$P_s = \frac{k}{2A} x_m \text{ or } P_s = \frac{W(x_m)}{Ax_m} \qquad [25]$$

For a pressure of the form $Pe^{-\mu t}$, $\frac{\mu}{\omega} \ll 1$,

$$P = P_s\left(1 + \frac{\pi\mu}{2\omega}\right) \qquad [26]$$

where P_s is given by [25]. For a pressure which acts impulsively, so that the impulse $\int P\,dt$ is almost a step function during the response time of the gage, the total impulse per unit area is

$$I = \int P\,dt = \frac{m\omega x_m}{A} \text{ or } I = \sqrt{\frac{2W_m}{A}} \qquad [27]$$

Equation [26] is directly applicable to measurements of shock pressures from underwater explosions,[3] but does not seem to have been given in this simple form before.

REFERENCES

1. Blake, R.E., "Peak-Reading Accelerometers," NRL Report S-3276, Shock and Vibration Bulletin No. 8, March 1948.

2. Abkowitz, D.E., "High-Speed Compression Tests of 3/8-Inch Copper Balls," TMB CONFIDENTIAL Report C-147, November 1948.

3. Cole, R.H., "Underwater Explosions," Princeton University Press, 1948.

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*m must be increased by the mass of the entrained water for underwater use of this gage.

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