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EFFECT OF TRANSVERSE CURVATURE ON FRICTIONAL RESISTANCE

BY L. LANDWEBER

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EFFECT OF TRANSVERSE CURVATURE ON FRICTIONAL RESISTANCE

by

L. Landweber

ABSTRACT

A theoretical value for the effect of transverse curvature on frictional resistance is computed. An important consequence is the indication that an appreciable correction for transverse curvature is required in estimating the frictional resistance of small ship models.

INTRODUCTION

It is generally supposed that the frictional resistance of a flat plate is increased by the presence of any immersed edges of the plate parallel to a stream. This effect has been attributed to the continuation of the boundary layer beyond the edge of the plate, with the implication that since additional fluid has lost momentum within the additional boundary layer, the drag must be greater. As an extension of this argument Comstock and Hancock (1)* have reasoned that transverse curvature must also increase frictional resistance over that for a flat plate. Their argument appears to assume that, for the same Reynolds number (based on downstream distance from foremost point or leading edge), the curved surface and flat plate have the same thickness of boundary layer; and consequently, since the area of the section of the boundary layer would be slightly greater for the curved surface, it must have retarded more fluid and consequently have more drag. This argument involves several unsupported assumptions. Nevertheless the experiments of Kempf (2) on the drag of axially towed smooth cylinders indicate that frictional resistance increases with curvature.

Since 1942 it has been claimed that transverse curvature appreciably increases the frictional resistance of small ship models, the effect diminishing with increasing size (1). As a consequence, the Frictional Resistance Committee of the American Towing Tank Conference has considered a proposal for applying a curvature correction to flat-plate friction data when computing the frictional resistance of a small ship model. To evaluate this proposal, the effect of transverse curvature on frictional resistance has been calculated theoretically.

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*Numbers in parentheses indicate references listed on the last page of this report.

THEORY

To determine the effect of curvature it is proposed to compute the ratios of the shearing stress and drag of a cylinder (with axis parallel to the stream) to those on a flat plate, for the same Reynolds number. The procedure will consist in determining the resistance of a cylinder by extending Prandtl's method (3) for computing the resistance of a flat plate from laws derived from characteristics of turbulent flow in pipes. The method assumes the 1/7th-power law for the velocity distribution and Blasius' empirical formula for the shearing stress at the wall for a turbulent boundary layer. For a flat plate the method gives results in good agreement with experiment for Reynolds numbers between 2×10^5 and 2×10^7, thereby indicating that the assumed laws are independent of the curvature of the wall. In the present problem, since the resistance of the curved and flat plates will be computed upon the same basis, and we are concerned only with the ratio of the shearing stresses and drags, and not with their absolute values, the results should serve as a good approximation for even higher Reynolds numbers.

The calculations are based upon the following four equations: the first is the empirical Blasius formula for shearing stress at a wall in a turbulent boundary layer; the second represents the 1/7th-power law of velocity distribution; the third is the relation between shearing stress and drag for a circular cylinder; and the fourth is the momentum integral, which states that the drag of a length x of a cylinder is equal to the loss of momentum of the fluid at the distance x from the nose of the cylinder.

$$\tau = 0.0225 \rho U_0^{7/4} \left(\frac{\nu}{\delta}\right)^{1/4} \qquad [1]$$

$$\frac{U}{U_0} = \left(\frac{y}{\delta}\right)^{1/7} \qquad [2]$$

$$\tau = \frac{1}{2\pi r} \frac{dD}{dx} \qquad [3]$$

$$D = \rho \int_0^\delta U(U_0 - U) \cdot 2\pi\,(r + y)dy \qquad [4]$$

where τ is the shearing stress at the wall,

ρ is the mass density of the fluid,

ν is the kinematic viscosity,

δ is the boundary-layer thickness,

U_0 is the free-stream velocity relative to the body,

U is the velocity of the stream relative to the body at a distance y from the wall,

y is distance from the wall along a radius at a section of the cylinder,
r is the radius of the cylinder,
D is the drag of a length x of the cylinder, and
x is the distance along the cylinder, measured from the foremost point.

If the value for U in [2] is substituted into [4], the integral may be evaluated to give

$$D = \frac{7\pi}{36}\rho U_0^2 r\delta\left(1 + 0.30\frac{\delta}{r}\right) \qquad [5]$$

Also, from [1] and [3],

$$\frac{dD}{dx} = 0.045\pi\rho r U_0^{7/4}\left(\frac{\nu}{\delta}\right)^{1/4} \qquad [6]$$

Eliminating D between [5] and [6] gives the differential equation

$$\frac{7}{36}\frac{d\delta}{dx}\left(\delta^{1/4} + 0.60\,\frac{\delta^{5/4}}{r}\right) = 0.045\left(\frac{\nu}{U_0}\right)^{1/4}$$

or, integrating and simplifying,

$$\delta\left(1 + \frac{1}{3}\frac{\delta}{r}\right)^{0.8} = 0.371 x R_x^{-0.2} \qquad [7]$$

where $R_x = \frac{U_0 x}{\nu}$, a Reynolds number.

Let δ_0 be the thickness of the boundary layer for the flat plate $(r = \infty)$. Then, from [7]

$$\delta_0 = 0.371 x R_x^{-0.2} \qquad [8]$$

and hence

$$\frac{\delta_0}{\delta} = \left(1 + \frac{1}{3}\frac{\delta}{r}\right)^{0.8} \qquad [9]$$

An immediate consequence of [9] is that the boundary layer on a convex curved plate is thinner than on a flat plate at the same Reynolds number, in agreement with experiment; see Reference (1, p. 197). Let τ_0 be the shearing stress at the wall for a flat plate. Then, from [1],

$$\frac{\tau}{\tau_0} = \left(\frac{\delta_0}{\delta}\right)^{1/4} \qquad [10]$$

and hence, from [9],

$$\frac{\tau}{\tau_0} = \left(1 + \frac{1}{3}\frac{\delta}{r}\right)^{0.2} \qquad [11]$$

Let D_0 be the drag of a flat plate of length x, having the same developed area as the cylinder. Let A be the developed area; then $A = 2\pi r x$. Hence [5] may be written as

$$D = \frac{7}{72}\rho U_0^2 A \frac{\delta}{x}\left(1 + 0.30\frac{\delta}{r}\right) \qquad [12]$$

and then

$$D_o = \frac{7}{72}\rho U_0^2 A \frac{\delta_0}{x} \qquad [13]$$

Consequently

$$\frac{D}{D_o} = \frac{\delta}{\delta_o}\left(1 + 0.30\frac{\delta}{r}\right) \qquad [14]$$

and hence, from [9],

$$\frac{D}{D_o} = \frac{1 + 0.30\frac{\delta}{r}}{\left(1 + \frac{1}{3}\frac{\delta}{r}\right)^{0.8}} \qquad [15]$$

These results may be presented in a convenient parametric form by putting $\alpha = \frac{\delta}{r}$. Equations [7], [11], and [15] become

$$\frac{x}{r} R_x^{-0.2} = 2.70\alpha\left(1 + \frac{\alpha}{3}\right)^{0.8} \qquad [16]$$

$$\frac{\tau}{\tau_o} = \left(1 + \frac{\alpha}{3}\right)^{0.2} \qquad [17]$$

$$\frac{D}{D_0} = \frac{1 + 0.30\alpha}{\left(1 + \frac{\alpha}{3}\right)^{0.8}} \qquad [18]$$

An important special case is that where the thickness of the boundary layer is small compared with the radius, so that $\alpha/3 \ll 1$. We may then expand [16], [17], and [18] in powers of α and neglect second and higher-order terms, to obtain

$$\frac{x}{r} R_x^{-0.2} = 2.70\alpha \qquad [16a]$$

$$\frac{\tau}{\tau_o} = 1 + \frac{\alpha}{15} \qquad [17a]$$

$$\frac{D}{D_o} = 1 + \frac{\alpha}{30} \qquad [18a]$$

Explicit formulas for the shearing stress and drag ratios can now be derived by eliminating α by means of [16a]. Thus

$$\frac{\tau}{\tau_o} = 1 + 0.0248\frac{x}{r} R_x^{-0.2} \qquad [19]$$

and

$$\frac{D}{D_o} = 1 + 0.0124\frac{x}{r} R_x^{-0.2} \qquad [20]$$

Equations [19] and [20] are good approximations for $\frac{x}{r} R_x^{-0.2} < 1$, or $\alpha < 0.37$. For larger values of α, Equations [16], [17], and [18] should be used. Values of τ/τ_0 and D/D_0 against $\frac{x}{r} R_x^{-0.2}$, computed from Equations [16], [17], and [18], are listed in Table 1 and graphed in Figure 1.

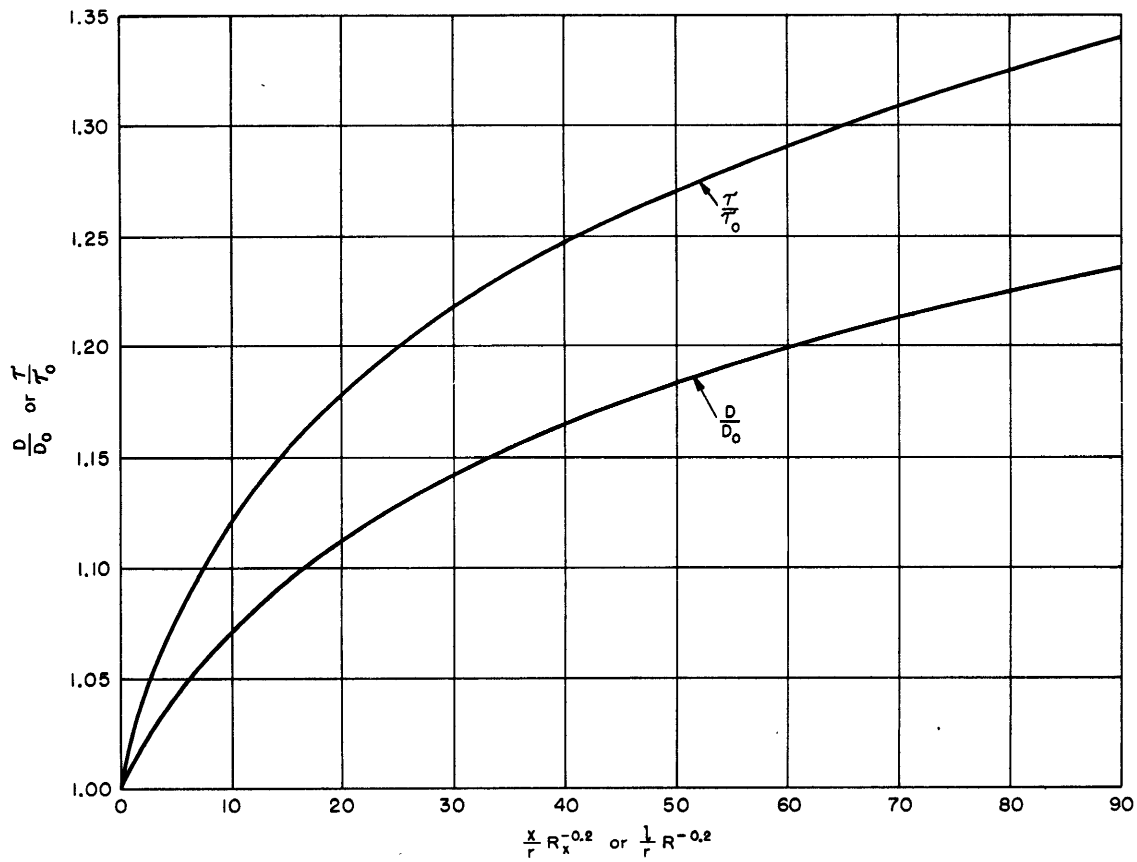

Figure 1 - Effect of Transverse Curvature on Frictional Resistance

The curves are computed by Blasius-Prandtl method for 1/7-th power law of velocity distribution in a turbulent boundary layer.

τ is shearing stress on wall,

D is frictional resistance,

x is distance from foremost point or edge of cylinder or plate,

r is radius of cylinder,

R_x is Reynolds number, $U_0 x/\nu$,

R is Reynolds number, $U_0 l/\nu$,

U_0 is stream velocity,

ν is kinematic viscosity, and

l is length of cylinder or plate.

Zero subscript denotes values for a flat plate.

TABLE 1

Cylinder-Flat Plate Ratios of Shearing Stress and Drag

α	$\frac{\tau}{\tau_0}$	$\frac{D}{D_0}$	$\frac{x}{r}R_x^{-0.2}$
0.1	1.005	1.002	0.277
0.3	1.019	1.010	0.895
0.6	1.038	1.020	1.873
0.9	1.052	1.030	3.000
1.5	1.081	1.047	5.60
2.0	1.107	1.062	8.13
3.0	1.150	1.091	14.10
6.0	1.245	1.162	39.0
10.0	1.337	1.234	87.5

APPLICATION TO A TOWED CYLINDER

Consider a smooth cylinder of radius r and length l towed longitudinally at a velocity U_0. It will be supposed that the nose and tail of the cylinder are faired, and that l/r is so great that end effects may be neglected.

If D is the drag of the cylinder and D_0 the drag of a flat plate of the same length and same developed area, Equations [16] and [18] are directly applicable with x replaced by l.

In a numerical problem, l, r, U_0, and ν will generally be given data from which $\frac{l}{r} R^{-0.2}$ can be computed, where $R = U_0 l/\nu$. The ratio D/D_0 is then given by Figure 1. D_0 may be calculated from one of the formulas or curves for the frictional resistance of a smooth, flat plate.

Example: Let $l = 500$, $r = 0.1$, $U_0 = 24$, $\nu = 1.2 \times 10^{-5}$

Then $R = 10^9$ and $\frac{l}{r} R^{-0.2} = 79.3$. Hence, from Figure 1, $D/D_0 = 1.224$. From the Schoenherr Mean Curve, at $R = 10^9$, the specific frictional resistance C_f is 0.00155. Then

$$D_0 = \frac{1}{2}\rho \cdot 2\pi r l U_0^2 C_f$$

Assume $\rho = 2$. Then $D_0 = 281$. Hence $D = 344$.

For Reynolds numbers as large as 10^9 the surface roughness may be critical and the drag may be greater than for an ideally smooth surface.

APPLICATION TO SHIP FORMS

On a complex geometrical surface, such as that of a ship hull, the curvature of the transverse sections varies from point to point. It will be assumed that the shearing stress at each point depends only upon the transverse radius of curvature r and the longitudinal position x at the point. The effect of longitudinal curvature and the increase in velocity due to the displacement flow are being neglected in these calculations.

Let l be the length of the ship (or model), A the wetted surface area, U_o the velocity. The radius $\bar{r}$ of an equivalent semisubmerged cylinder will be defined by the relation

$$\bar{r} = \frac{A}{\pi l} \qquad [21]$$

and it will be shown that the correction factor for curvature is given by Equation [20] with r and x replaced by $\bar{r}$ and l, i.e.

$$\frac{D}{D_o} = 1 + 0.0124 \frac{l}{\bar{r}} R^{-0.2}, \qquad [22]$$

or, substituting the value of $\bar{r}$ from [21],

$$\frac{D}{D_o} = 1 + 0.039 \frac{l^2}{A} R^{-0.2}. \qquad [23]$$

Since $R^{-0.2} \approx 13.9 C_f$, where C_f is the specific frictional resistance for a flat plate, [23] may also be written as

$$\frac{D}{D_o} = 1 + 0.54 \frac{l^2}{A} C_f \qquad [24]$$

The value of l^2/A may be expected to range from about 10 for a fine form to about 4 for a full form. C_f ranges from about 0.006 for a small model at low speed to about 0.0025 for a large model. Hence the curvature correction factor may be expected to range from about 1.032 for a small model at low speed to about 1.005 for a large model.

It remains to be proved that a semisubmerged cylinder having the same length and wetted surface area as the ship form, in accordance with [21], will have the same curvature correction. Let D be the frictional resistance of the ship form and D' that of the semisubmerged cylinder. It is to be proved that D = D'.

Since by definition the semisubmerged cylinder has the same length and wetted surface area as the ship form, the equivalent flat-plate resistance D_o is the same for both.

Let ds be an element of arc length along a transverse section, and θ the transverse inclination of the element. The friction force on the ship form is

$$D = \iint \tau \, dx \, ds \qquad [25]$$

But substituting for τ from [19], we get

$$D = \iint \tau_0 \left(1 + 0.0248 \frac{x}{r} R_x^{-0.2}\right) dx \, ds$$

or

$$D = D_0 + 0.0248 \iint \tau_0 \frac{x}{r} R_x^{-0.2} \, dx \, ds \qquad [26]$$

Similarly

$$D' = D_0 + 0.0248 \iint \tau_0 \frac{x}{\bar{r}} R_x^{-0.2} \, dx \, ds \qquad [27]$$

Now put $ds = r d\theta$ and suppose that the ship sides are vertical at the waterline, so that θ varies from $-\pi/2$ to $\pi/2$ along a transverse section. Then

$$\iint \tau_0 \frac{x}{r} R_x^{-0.2} \, dx \, ds = \int_0^x \int_{-\pi/2}^{\pi/2} \tau_0 x R_x^{-0.2} \, dx d\theta \qquad [28]$$

$$= \pi \int_0^x \tau_0 x R_x^{-0.2} \, dx.$$

Similarly, for a semisubmerged cylinder of radius $\bar{r}$,

$$\iint \tau_0 \frac{x}{\bar{r}} R_x^{-0.2} \, dx \, ds = \pi \int_0^x \tau_0 x R_x^{-0.2} \, dx$$

Hence the integrals in [26] and [27] are equal, so that $D = D'$, as we wished to prove.

REFERENCES

(1) Comstock, John P., and Hancock, C.H., "The Effect of Size of Towing Tank on Model Resistance," Trans. Soc. Naval Arch. Marine Eng., Vol. 50, pp. 149-197, 1942.

(2) Kempf, Gunther, "Über den Reibungswiderstand von Flächen verschiedener Form" (On the Frictional Resistance of Surfaces of Various Form), Proc. First Intern. Congr. Applied Mechanics, 1925.

(3) Durant, William Frederick, "Aerodynamic Theory," Vol. 3, pp. 146, 147.

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