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CALCULATION OF FLEXURAL CRITICAL FREQUENCIES OF SHIP HULLS BY PROHL'S METHOD

by

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ABSTRACT

This report extends Prohl's method for the calculation of critical speeds of flexible rotors to calculating flexural critical frequencies of ships. The method is simplified and set up so that the computations can be readily made with a punched card machine. The method is applied to a uniform bar as a check.

AUTHORIZATION

This study was undertaken under the general project established by Bureau of Ships letter S87-(8)(330) of 4 August 1943 for the development of techniques and instruments for observation of motions on ships.

INTRODUCTION

Most of the methods for obtaining the critical frequencies of ship hulls give only the frequency in the lowest mode of vibration. When attempts are made to apply them to higher modes either the accuracy is greatly reduced or the methods themselves are not applicable.

An accurate method for the calculation of critical frequencies in the higher modes would be very useful, provided the value of the method were not outweighed by the extent of the computations involved. However, it must be kept in mind that all such calculations when applied to ships involve in addition to the mass of the ship itself a quantity representing the virtual mass of the surrounding water. Although theories exist for the estimation of the virtual mass, References (1) and (2), experimental verification of the theories is still unsatisfactory. The method given in this report is based on a given mass-distribution curve for the ship which in this case must include the virtual mass of the water. The extent to which any method of calculating flexural frequencies of ships is limited by the uncertainty as to the virtual mass of the water must remain for the present a matter of individual judgment.

THEORY OF METHOD

The numerical calculation of critical speeds of flexible rotors as given by M. A. Prohl, Reference (3), may be extended to the calculation of flexural critical frequencies of ships. If the ship is assumed to be vibrating freely in one of its normal modes and at a circular frequency ω, the deflection at the instant of maximum displacement from the rest position considered as a function of distance from one end must satisfy the differential equation:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2y}{dx^2}\right) = m\omega^2 y$$

where

E is the modulus of elasticity, tons/ft.^{2},
I is the variable moment of inertia of cross-section of ship, ft.^{4},
x is the distance along the length of ship, ft.,
y is the deflection normal to longitudinal axis of ship, ft.,
m is the variable mass per unit length of ship at distance x from end, tons sec.^{2}/ft^2,
ω is the circular critical frequency of the vibration, radians/sec.
(2π times the frequency of flexural vibration in cycles per second)

There are four boundary conditions which must be satisfied for this fourth-order differential equation, and any frequency ω which will give a solution of the equation satisfying the four boundary conditions is a critical frequency.

In Reference (3), Prohl treats the case of a turbine-rotor shaft supported in two bearings and overhanging at one end. This imposes the condition of zero deflection at either of the bearings. Furthermore, by the assumption of simple support at the bearings Prohl obtains the condition of zero moment at the end that does not overhang. In the case of a ship the points of zero deflection are the nodal points whose location cannot be assumed to be known. The four known end conditions are that the moment and shear must be zero at each end, as no external forces or couples are assumed acting. If the ship is assumed to be vibrating with unit amplitude at one end and at a circular frequency ω, the shear and moment being taken as zero at that end, then in order for ω to represent a critical frequency it must have such a value that the shear and moment also become zero at the other end.

Prohl simulates the actual rotor by a lumped rotor in which the mass is concentrated at a large number of equally spaced points whereas the bending stiffness is concentrated in massless members connecting the lumped masses. This enables him to proceed step by step from one end of the shaft to the other.

In order to apply Prohl's method to a ship, the latter can be divided into twenty or more sections of equal length and the weight of each section can be lumped at the ends of the section, half at each end. The section ends are designated as stations and hence there are twenty-one stations for the twenty sections. Figure 1 shows the hull of a ship in its fundamental mode of vibration with the masses concentrated at twenty-one stations. Unit deflection is assumed at the left end of the ship, that is, Station 0, with other boundary conditions at thi end following the laws of dynamics for a free-free bar, that is the bending moment and shearing force must be zero. As the slope at the end cannot be assumed known, its value, designated as θ_0 , must be carried through the calculation as an unknown quantity. With the necessary boundary conditions known or assumed, the calculation proceeds for some chosen value of ω .

The following nomenclature is used in the calculation

- V is shearing force in tons
- M is bending moment in ton-ft.
- y is deflection normal to longitudinal axis in feet
- Δx is length of each section in feet

The computations involve also increments of these quantities.

By virtue of the fact that the ship is structurally continuous, the deflection curve for the ship will also be continuous; thus the value of deflection will be the same for the right end of Section 1 and the left end of Section 2, etc. For each station there is a process of four integrations in deducing the deflection at the station from the deflection at the pr ceding station; at each section are added increments of shear, bending moment, slope, and deflection until the calculation is completed for the length of the ship. The increments are desig nated by Δ's.

The following formulas as well as the tables to follow will help clarify the above discussion:

$$\Delta V_n = m_{n-1}\, y_{n-1}\, \omega^2$$

$$V_n = V_{n-1} + \Delta V_n$$

$$\Delta M_n = V_n\, \Delta x$$

$$M_n = M_{n-1} + \Delta M_n$$

$$\Delta \theta_n = \frac{\Delta x}{EI} M_n$$

$$\theta_n = \theta_{n-1} + \Delta \theta_n$$

$$\Delta y_n = \theta_n\, \Delta x$$

$$y_n = y_{n-1} + \Delta y_{n-1}$$

where n designates the number of the station from 0 to **20A**.

Since each term is of the form $a + b\theta_0$ **where** a and b real numbers and since the assumed conditions at the left end are that $y = 1$ and $\theta = \theta_0$ (an unknown value), y_1, expressed in the form $a + b\theta_0$ has the value $1 + 0 \cdot \theta_0$. A sample table for starting the calculations on a uniform bar is given on page 5. Since all computed quantities in this table will be of the form $a + b\theta_0$ and since the operations required to obtain one term from a preceding one are merely multiplications and additions, the "a" and "b" terms may be treated independently. Hence, it is simpler to make the computation by means of two separate tables, one for the "a" terms and one for the "b" terms. Thus, for Station 0 in the "a" table the value of y is 1.0 and the value of θ is 0 whereas in the "b" table the value of y is 0 and the value of θ is 1.0. The true value applicable to any of the spaces in the table on page 5 is

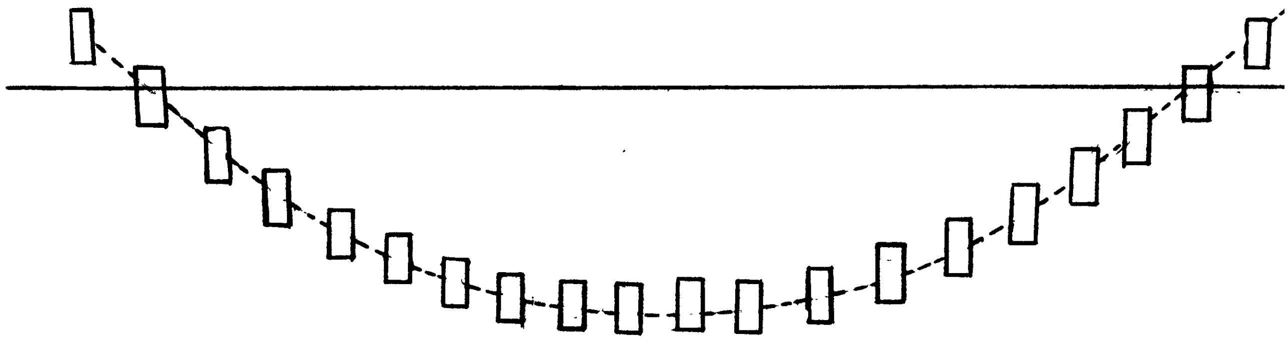

Figure 1 - Fundamental Mode of Vibration of the Hull of a Ship with Masses Concentrated at Twenty One Stations

Table 1 - Sample Table Showing the Initial Data for Calculating the Resonant Frequencies of a Uniform Bar

Station	m	y	ΔV	V	ΔM	M	$\frac{\Delta x}{EI} \times 10^{12}$	$\Delta\theta$	θ	Δ
0	38.8	$1+0\cdot\theta_0$		$0+0\cdot\theta_0$		$0+0\cdot\theta_0$			$0+1\cdot\theta_0$	0+
1	77.5						899			
2	77.5						899			
3	77.5						899			
4	77.5						899			
5	77.5						899			
6	77.5						899			
7	77.5						899			
8	77.5						899			
9	77.5						899			
10	77.5						899			
11	77.5						899			
12	77.5						899			
13	77.5						899			
14	77.5						899			
15	77.5						899			
16	77.5						899			
17	77.5						899			
18	77.5						899			
19	77.5						899			
20	38.8						899			
20A										

obtained by adding the corresponding values in the "a" and "b" tables and expressing the result in the form $a + b\theta_o$. Although dividing the ship into twenty segments with the station at the left end designated as 0 makes the station at the right end Station 20, the lumping of masses at the stations makes it necessary to consider an auxiliary Station 20A also. The values of shear and bending moment listed in the table on page 5 are the values which the section to the left exerts on the section to the right of the station in accordance with the ordinary conventions adopted in simple-beam theory. The shear load at Station 20A differs from that at 20 due to the inertia effect of the mass assumed concentrated at 20, that is, $V_{20A} = V_{20} + y_{20}\, m_{20}\, \omega^2$. The moment however, does not change abruptly, being a continuous quantity.

When the values for shear and bending moment for Station 20A have been obtained in the form $a + b\theta_o$ it is necessary to determine some relationship between the two values before the critical frequencies can be estimated. The residual shear and moment at the forward end of the ship are written in the form

$$V_{20A} = c + d\theta_o$$

$$M_{20A} = e + f\theta_o$$

where for Station 20A, c is the value of shear computed in the "a" table and d the value in the "b" table. Since the moment does not change at a point of concentrated mass M_{20A} and M_{20} are identical.

If the assumed frequency is a critical frequency then

$$V_{20A} = c + d\theta_o = 0$$

and

$$M_{20A} = e + f\theta_o = 0$$

Therefore $\theta_o = -c/d = -e/f$ and the natural frequencies occur when $c/d = e/f$. Hence, if c/d and e/f are plotted against the assumed frequencies, the critical frequencies will occur at the points of intersection of the curves.

It is simpler, however, to eliminate the plotting of the two curves by assuming either the shear or bending moment to

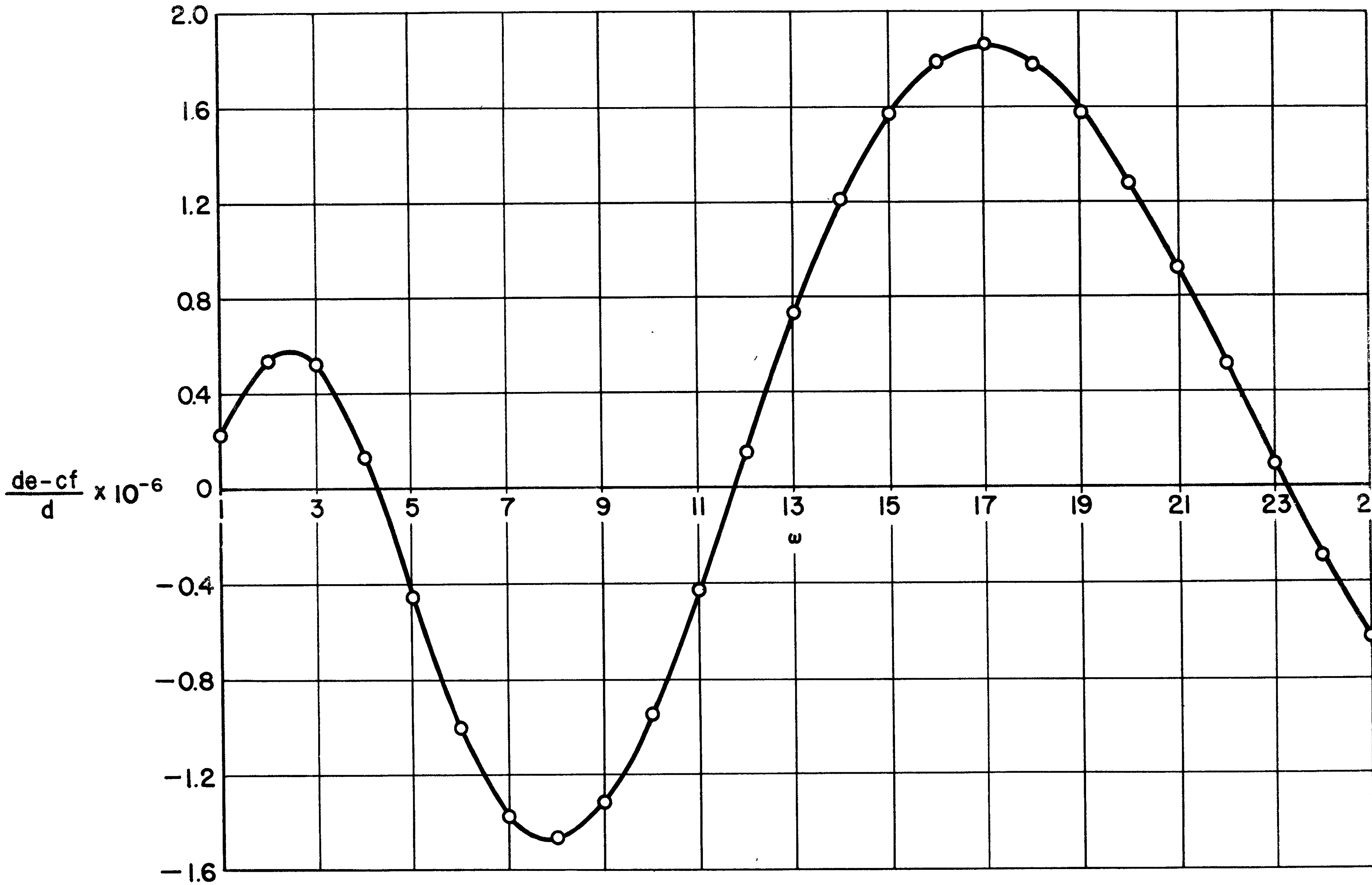

Figure 2 – Calculation of Natural Flexural Frequencies of Uniform Bar by Prohl's Method
(The criticals occur at the values of ω where $\frac{de-cf}{d}$ is equal to zero)

be zero:

$$V_{20A} = c + d\theta_0 = 0$$

$$M_{20A} = e + f\,\theta_0$$

Then M_{20} can be written in the form

$$M_{20A} = \frac{de - cf}{d},$$

which shows that M_{20A} is equal to zero only when $c/d = e/f$. Therefore $\frac{de - cf}{d}$ is plotted against the assumed frequencies and where the curve crosses the zero line is a critical frequency.

APPLICATION OF METHOD TO A UNIFORM BAR

In order to check the accuracy of the method, calculations were made for a uniform bar since in this case exact formulas are available for the natural frequencies. The initial data for making the calculation for the uniform bar are given in the table on page 5 where it will be observed that the "a" and "b" terms are included in the same table even though they can be treated separately. A typical punched-card machine calculation made from initial data for one of the assumed values of ω is given on page 8.

Similar calculations were made for frequencies from $\omega = 1$ through $\omega = 25$. The value for

$$M_{20A} = \frac{de - cf}{d}$$

was calculated for each ω assumed and the data were plotted in Figure 2. The critical values obtained from this curve are $\omega = 4.25$, $\omega = 11.74$, $\omega = 23.25$. These values check with the critical frequencies as computed by the well-known formulas for the uniform bar.

A punched-card type of calculating machine is believed to be the only practicable means of handling the calculations involved in this method. With the use of such a machine the time necessary for the calculations is reduced from several months to a few days. It appears from the results with the uniform bar calculation that the method is

ω	initial values Y	V	M	θ	station	m	ΔX	Y	V	M	θ
16	0	0	0	1	0	388	50				1000000000
16					1	775	50	500000000			1000000000
16					2	775	50	1022295200	992000000	49600000	1044590400
16					3	775	50	1634765352	302023368	200611684	1224940304
16					4	775	50	2478185049	626360814	513792091	1686839394
16					5	775	50	3803832147	1118032798	1072808455	2651294195
16					6	775	50	6032598898	1872713086	2009164968	4457533301
16					7	775	50	9854373853	3069580647	3543955292	7643549309
16					8	775	50	16398454820	5024688360	6056299472	13088162534
16					9	775	50	27525355069	8278141796	10195370370	22253800497
16					10	775	50	46322953260	13739178842	17064956491	37598196388
16					11	775	50	77944687371	22929646169	28529779576	63243468881
16					12	775	50	131019580166	38393878143	47786715648	106149785589
16					13	775	50	220018869896	64388156848	79920794072	177998579460
16					14	775	50	369824584830	108039900635	133940744390	298411308667
16					15	775	50	619382380060	181894046242	224587767511	500315711659
16					16	775	50	1038856782359	304179510446	376677522734	838948804597
16					17	775	50	1742335115568	510288696066	631821870767	1406956666417
16					18	775	50	2922196183865	855967982995	1059805862265	2359788136893
16					19	775	50	4901120688145	1435731705874	1777671715202	3957849008560
16					20	775	50	8220333261236	2408114050402	2981728740403	6638483146188
16					20a	388	50	8220333261236	3224623213246		
									(d)	(f)	
16	1	0	0	0	0	388	50	10000000			000000000
16					1	775	50	10823240	993280	496640	000446479
16					2	775	50	11348818	3021571	2007486	002251155
16					3	775	50	14561880	5273176	4644014	006426124
16					4	775	50	21696893	8162253	8725141	014270026
16					5	775	50	35555797	12466917	14958600	027717807
16					6	775	50	60525978	19521187	24719194	049940368
16					7	775	50	103693702	31529541	40483965	086335447
16					8	775	50	176768976	52102371	66535151	146150540
16					9	775	50	299344008	87173336	110121619	245150063
16					10	775	50	504358874	146563187	183403413	410029731
16					11	775	50	847243214	246627988	306717407	685768680
16					12	775	50	1421205583	414721042	514077928	1147224737
16					13	775	50	2382826660	696688230	862422043	1983242154
16					14	775	50	3994938319	1169441039	1447142563	3284283318
16					15	775	50	6698508338	1962036801	2428160964	5407140025
16					16	775	50	11233193635	3291020854	4073671391	9069370606
16					17	775	50	18839543763	5519686471	6833514627	15812700256
16					18	775	50	31598171043	9257451954	11462240604	25517254259
16					19	775	50	52998662887	15526529089	19225505149	42800983688
16					20	775	50	88893838286	26041463806	32246237052	71790350798
16					20a	388	50	88893838286	34871110975		
									(c)	(e)	

Table 2—Sample Punch Card Machine Calculation for a Uniform Bar for an Assumed Frequency ω = 16 (vertical lines denote decimal points)

reliable for flexural modes of free-free bars at least up to the 4-noded mode. The value of the method for use in ship design may be expected to increase as more experimental data are obtained as to the virtual mass of the surrounding water and as to the extent to which flexural vibration theory can be applied to ship hulls considered as built-up girders of varying section.

REFERENCES

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(3) "A General Method for Calculating Critical Speeds of Flexible Rotors," by M. A. Prohl, Journal of Applied Mechanics, vol. 12, No. 3, September 1945, pp. A-142 - A-148.