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THE DAVID W. TAYLOR MODEL BASIN

UNITED STATES NAVY

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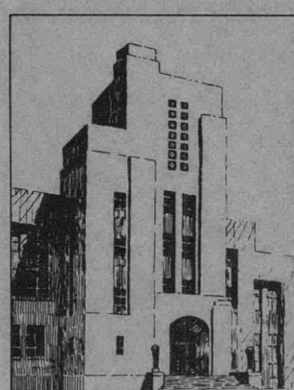
EXPLOSIVE LOAD ON UNDERWATER STRUCTURES AS MODIFIED BY BULK CAVITATION

BY PROF. E. H. KENNARD



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DAVID TAYLOR MODEL BASIN

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PERSONNEL

The development of this theoretical treatment, and the report describing it, are the work of Professor E.H. Kennard, of the David Taylor Model Basin staff. The computations required for the construction of the curves were made with the assistance of Mr. S. Pines.

NOTATION

c speed of sound in water
g acceleration due to gravity
L a convenient linear dimension
m mass per unit area of a plate
n distance from a surface along its normal
p pressure
p_b breaking-pressure, at which cavitation occurs
p_c cavity pressure or pressure in cavitated region
p_h hydrostatic pressure
p, p'' pressure in incident and reflected wave-trains, respectively
p_0 see Equation [12] on page 12
q see Equation [16] on page 13
v particle velocity
v_x, v_y, v_z cartesian components of v
v_b particle velocity just ahead of a breaking-front, reckoned as positive away from the front
v_{bn} component of v_b normal to the boundary
v_c particle velocity in a cavitated region, reckoned as positive toward a breaking-front but away from other boundaries
v_{cn} component of v_c normal to the boundary
v_{cx}, v_{cy}, v_{cz} cartesian components of v_c
V_b velocity of propagation of a breaking-front
V_c velocity of propagation of a closing-front
x, y, z cartesian coordinates
α see Equation [12]
η fraction of space free of water in a cavitated region
ρ density of water

EXPLOSIVE LOAD ON UNDERWATER STRUCTURES AS MODIFIED BY BULK* CAVITATION

ABSTRACT

Whereas cavitation is most commonly observed at the interface between water and a solid surface, there are indications that it may also occur in the midst of a mass of water. Such cavitation may modify the action of explosive pressure waves upon structures. This is possible wherever reflection of the wave gives rise to tension in the water. An extension of hydrodynamical theory to cover such cases is described in this report.

It is shown that cavitated regions should be formed through the propagation of breaking-fronts moving at supersonic velocity. The cavitation should usually take the form of small bubbles continuously distributed, rather than of large voids. Subsequently the cavitation will be destroyed as the boundary of the cavitated region contracts and acts as a closing-front. The relevant mathematical formulas are cited.

Similitude relations are discussed, and the theory is applied to a plane wave falling normally upon a plate, and to the explanation of the dome that is formed over large charges exploded in the sea.

INTRODUCTION

The study of the behavior of ship structures when loaded by an underwater explosion is a major project at the David W. Taylor Model Basin. Good progress has been made toward an understanding of the pressure field in open water, with all boundaries well removed, and to this extent the groundwork has been laid for defining *load*. Important gaps still exist in this line of information, however. The energy balance is still incomplete, so that it is not yet possible to say what fraction of the explosive energy is made available in the first cycle of pulsation of the gas globe. It is, therefore, still impossible to evaluate the effect of the displacement of the gas globe which may put its center at a point nearer the target at the end of the first cycle than at its beginning. Questions of this sort have led others also to conclude that sound fundamental data are still most necessary (1).**

Questions relating to the properties of the *target*, as distinguished from the load which the explosion puts on the target, are set aside for separate consideration. It may be assumed that in this report the load is treated

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* This term is rather new; it will be defined and discussed in the report.

** Numbers in parentheses indicate references on page 25 of this report.

in terms of a target of arbitrarily assumed properties, such as might serve to measure directly the pressure in the water.

The behavior of an acoustical wave incident on a solid or free boundary in water is rather fully understood; and the shock wave radiated from an explosion in water partakes in large degree of the nature of an acoustical wave. Even in such waves, however, the continuity of the medium is believed to be broken at times by tensile effects to which the water responds by cavitation. In the shock wave, where pressures are a whole order higher than in an acoustical wave, cavitation naturally has that much more significance. Among the problems which must be solved before interactions between field and target can be subjected to study by calculation, that of cavitation must rank high in importance.

HYDRODYNAMICAL THEORY OF CAVITATION IN BULK

In practical experience cavitation usually originates between water and a solid surface, such as a propeller blade. There are some indications, however, that it may also occur in the midst of a mass of water, as for example when explosive pressure waves are reflected from the surface of the sea. To determine the effect of such cavitation upon the motion of the water, a certain extension of hydrodynamical theory is required.

In the present report the necessary extension of the theory will be described, but the complete mathematical details will be published elsewhere (2). The theory is based upon certain simple assumptions, which are laid down without entering upon the complicated question as to the nature of the cavitation process itself. Two applications of the theory will be discussed, dealing respectively with the impact of a pressure wave upon a plate, page 10, and upon the surface of the sea, page 19.

The following assumptions will be made:

(a) cavitation occurs wherever the pressure in the water sinks to a fixed value p_b, called the *breaking-pressure*;

(b) upon the occurrence of cavitation, the pressure instantly becomes equal to a fixed value p_c, called the *cavity pressure,* which cannot be less than p_b, so that

$$p_b \leqq p_c \tag{1}$$

(c) when the pressure rises above p_c, the cavitation disappears instantly.

How far these assumptions correspond to the actual behavior of water is not yet known. The value to be assigned to p_b is discussed briefly on page 17. The cavitation will undoubtedly take the form of small bubbles scattered through the water. Such bubbles have often been observed, but in many cases they seem to contain air in addition to water vapor, and they do not always disappear when the pressure is raised. All such complications will be ignored here, however, in order to obtain a tractable analytical theory. The bubbles may be supposed to be so small that the resulting inhomogeneity of the water may be neglected; and p_c may be supposed to equal the vapor pressure of the water.

The discussion will be limited to motion that is irrotational or free from vortices, motion such as can be produced by the action of pressure upon frictionless liquid. Furthermore, all variations of pressure will be assumed to be small enough so that the usual theory of sound waves is applicable to the unbroken water; but no limit need be set upon the magnitude of its particle velocity.

BREAKING-FRONTS

Cavitation will begin, according to the assumptions just made, in a region where the pressure is falling, and at a point of minimum pressure, at the instant at which the pressure sinks to p_b. A cavity will form and this cavity, for reasons lying outside the assumptions of the analytical theory, will at once become subdivided into bubbles. Since, however, the pressure will be sinking in the neighboring water also, the same process will soon occur at neighboring points as well. Thus a cavitated region will form, surrounding the point of initiation. The boundary of this region will sweep out into the unbroken water as a breaking-front, Figure 1. Since the pressure gradient at the initial point of minimum pressure is zero, the velocity of advance of the breaking-front is seen to be infinite at first, just as, when a rounded bowl is lowered into water, the boundary of the wetted region moves out at first at infinite speed. Hence cavitation occurs almost simultaneously throughout a considerable volume, resulting in a fairly uniform distribution of bubbles; there is no reason to expect the immediate formation of a large cavity anywhere.

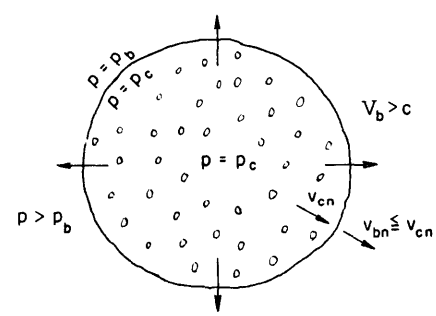

Figure 1 - An Expanding Breaking-Front, where $p = p_b$, Surrounding a Cavitated Region

The speed of propagation of the breaking-front relative to the water ahead of it, V_b, can be shown never to sink below the speed of sound, c. Usually V_b is greater than c. This means that no influence can be propagated past a breaking-front into the region ahead of it. The production of the cavitated region is thus a consequence solely of processes occurring in the unbroken water or on its boundaries, as a result of which the pressure in successive portions is lowered to the breaking-pressure; the front is merely a particular surface of constant pressure advancing in accordance with the ordinary equations of wave propagation. Its speed of advance is found to be (2)

$$V_b = \rho c^2 \frac{\dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_y}{\partial y} + \dfrac{\partial v_z}{\partial z}}{\dfrac{\partial p}{\partial n}} \qquad [2]$$

where $\partial p/\partial n$ denotes the normal pressure gradient or the rate of increase of the pressure along a normal to the front drawn into the unbroken water, and v_x, v_y, v_z are components of the particle velocity taken in the directions of cartesian axes. In order that the pressure may sink as the front approaches, the numerator in Equation [2] must be positive.

If p_b is less than p_c, there is a discontinuity of pressure at the breaking-front, so that the pressure is p_b ahead of it and p_c behind it. Thus, while the front is traversing an element of water, the element is kicked forward by the excess of pressure acting on its rear face. If v_b is the particle velocity just ahead of the front, and if v_{bn} is the component of this velocity in a direction perpendicular to the front or to the boundary of the cavitated region, taken positive toward the unbroken water, and if v_c and v_{cn} denote corresponding quantities in the cavitated region just behind the front, then the analysis (2) indicates that

$$v_{cn} = v_{bn} + \frac{p_c - p_b}{\rho V_b} \qquad [3]$$

Components of velocity parallel to the boundary are, however, left unaltered. Thus, if $p_b = p_c$, the particle velocity is left entirely unaltered by the passage of the breaking-front, but if p_b is less than p_c there is a discontinuity in its component perpendicular to the front.

THE CAVITATED REGION

Conditions within the region of cavitation must be comparatively simple. Since there is no pressure gradient, and the pressure is uniformly equal to p_c, the particle velocity must be constant in time, retaining the value at which it was left by the passage of the breaking-front.

If $p_b = p_c$, the particle velocity, being unaltered by the passage of the breaking-front, retains its expanding character. In this case, according to our assumptions, the fraction η of the space that is occupied by bubbles increases steadily from an initial value of zero. If p_b is less than p_c, however, a certain volume of space is freed at once by compression of the water as its pressure rises from p_b to p_c. The general formula for η at any point in the cavitated region at time t is (2)

$$\eta = \frac{p_c - p_b}{\rho c^2}\left[1 - \frac{c^2}{V_b^2}\right] + \int_{t_b}^{t}\left(\frac{\partial v_{cx}}{\partial x} + \frac{\partial v_{cy}}{\partial y} + \frac{\partial v_{cz}}{\partial z}\right) dt \qquad [4]$$

where t_b is the time at which cavitation occurred at this particular point and v_{cx}, v_{cy}, and v_{cz} are the components of the particle velocity v_c in the directions of the x, y, and z axes. Apparently, if p_b is less than p_c, η may either increase or decrease, or neither, after the breaking-front has passed.

THE CAVITATION BOUNDARY

When the boundary of the cavitated region, advancing as a breaking-front, arrives at a point beyond which V_b as given by Equation [2] would be less than the speed of sound, c, the analysis shows that it must halt abruptly. This may be regarded as happening either because the liquid ahead of the front is not expanding with sufficient rapidity, that is, the numerator in Equation [2] is too small, or because an excessive pressure gradient has been encountered, that is, the denominator is too large. The boundary may then do either of two things. Which it will do is found to depend in part upon the particle velocity in the neighboring cavitated region, but in larger degree upon conditions in the adjacent unbroken liquid.

One alternative is that the boundary may stand still as a *stationary boundary,* as shown in Figure 2, where any waves of pressure that may be incident upon it from the unbroken side are reflected as if from a free surface. This must occur whenever the incident waves are very weak.

The other alternative is that destruction of the cavitation may begin, that is, the boundary may recede toward the cavitated region, leaving the liquid unbroken again behind it. Such a boundary may be called a *closing-front.* Apparently it may be of either of two distinct types.

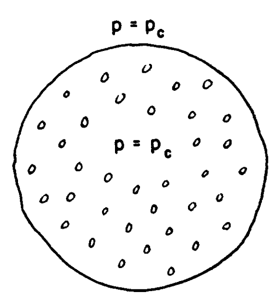

Figure 2 - A Stationary Boundary

CLOSING-FRONTS

Closing of the cavitation may result from a contracting motion in the cavitated region itself, when the distribution of the values of v_c at different points are such that the bubbles tend to decrease in size. This can happen, however, only if p_b is less than p_c; for, as already remarked, if $p_b = p_c$

the water retains the expanding motion which brought about the cavitation. If contraction of the bubbles occurs near a part of the boundary at which $\eta = 0$, this part of the boundary will advance into the cavitated region as a closing-front. A closing-front of this type may be called an *intrinsic* one; the analysis shows that it must advance at a speed exceeding the speed of sound, else it will at once change into the other type, to be described next.

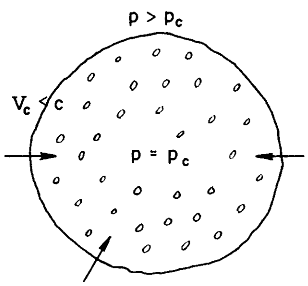

Figure 3 - A Forced Closing-Front

When recession of the boundary of the cavitated region is caused by conditions in the unbroken water, the boundary may be called a forced closing-front, Figure 3. Its motion is essentially an impact process, similar to that which occurs when a locomotive picks up the slack in a long string of cars. Layer after layer of the cavitated water is compressed impulsively from p_c to some higher pressure p, and its component of velocity normal to the boundary is likewise changed. It is assumed in the idealized theory, as already stated, that the cavitation bubbles close instantly as the closing-front passes over them. If, in reality, they contain a kernel of air or other foreign gas which requires time to redissolve in the liquid, the process will be modified.

It can be shown that a forced closing-front cannot move faster than sound, relatively to the unbroken liquid behind it, but exact equations covering its motion are difficult to formulate in the general case. The reason can be said to lie in diffraction of the waves that are incident on the boundary.

THE ONE-DIMENSIONAL CASE

The one-dimensional case, on the other hand, is easily treated in more detail. If the motion is confined to one dimension, use may be made of the familiar fact that any one-dimensional disturbance in unbroken liquid is equivalent to two superposed trains of plane waves traveling in opposite directions. One of these two trains will fall at normal incidence upon the plane boundary of the cavitated region, while the other will be leaving it continually as a reflected train of waves. Simple equations can then be written in terms of these trains.

Let p' denote the pressure in the incident wave train, and let v_c denote the particle velocity in the cavitated region, measured positively now toward the cavitated side of the boundary. Then the analysis (2) indicates that, if

$$p' \leqq \frac{1}{2}(p_c + \rho c v_c)$$

the boundary remains at rest, except, of course, as it may move slightly with the particle velocity of the water. The incident waves are reflected as if at a free surface at which the pressure is always p_c; see Figure 4. This case will occur, for example, whenever the incident waves are waves of tension but are not of sufficient strength to cause fresh cavitation.

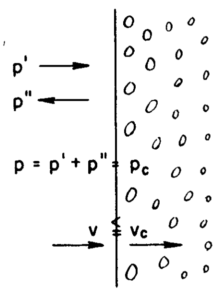

Figure 4 - A Plane Stationary Boundary

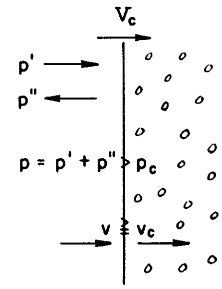

Figure 5 - A Plane Forced Closing-Front advancing toward the Right

If, on the other hand,

$$p' > \frac{1}{2}(p_c + \rho c v_c)$$

the boundary advances toward the cavitated region as a forced closing-front; see Figure 5. For the pressure p and the particle velocity v of the unbroken water just behind the front, the latter taken positive toward the side of cavitation, and for V_c, the speed of advance of the front relative to the cavitated water ahead of it, the following formulas are obtained (2)

$$p - p_c = \frac{(1-\eta)\rho c w^2}{2w + \eta(c - 2w)} \qquad [5]$$

$$v - v_c = w\,\frac{w + \eta(c-w)}{2w + \eta(c-2w)} \qquad [6]$$

$$V_c = c\,\frac{w}{w + \eta(c-w)} \qquad [7]$$

where

$$w = \frac{1}{\rho c}(2p' - p_c) - v_c \qquad [8]$$

ρ is the density of water and c the speed of sound in it, and η is the fraction of space that is occupied by bubbles.

According to Equation [7], $V_c = c$ if $\eta = 0$. The boundaries at which $\eta = 0$ constitute, however, a singular case which will usually be of momentary duration.

The most interesting example of such a boundary is a breaking-front which has just ceased advancing. Usually the advance ceases because V_b has sunk to c and would go below this value if the front advanced farther; then, by Equation [4], $\eta = 0$ at the front. Furthermore, by Equation [3], in which

$v_{cn} = - v_c$, $v_{bn} = - v_b$, and in view of the differences in the choice of the positive direction for velocity, as illustrated in Figures 1 and 4,

$$v_c = v_b - \frac{p_c - p_b}{\rho c}$$

Here p_b and v_b represent pressure and particle velocity just behind the front, so that $p_b = p' + p''$ where p'' is the pressure in the reflected wave; whereas by the usual acoustic equations $\rho c v_b = p' - p''$. It follows that

$$p' = \frac{1}{2}(p_c + \rho c v_c)$$

Comparison of this equation with inequalities previously written involving p' shows that the further behavior of the boundary will depend upon the subsequent course taken by the incident pressure p'. If p' increases as time goes on, the boundary will at once start back toward the cavitated region as a forced closing-front; whereas, if p' remains constant or decreases, the boundary will remain stationary, constituting a free surface.

FINITE GAPS

Cavitation in the midst of a mass of liquid must ordinarily consist of small bubbles which can be assumed, for analytical purposes, to be continuously distributed. There appear to be only two ways in which large spaces or gaps can be formed in a liquid by hydrodynamic action not involving the motion of solids.

Rotational motion may have the effect of lowering the pressure to the breaking-point, as in an eddy, and then forming a cavity. Such motion, however, is excluded in the present discussion.

If the motion is of the irrotational or potential type a gap can form only if $p_b < p_c$, where a wave of tension falls upon the boundary of a cavitated region already formed and causes the surface of the unbroken water to withdraw. Such a gap will presumably take the form of a layer of especially large bubbles between the broken and unbroken water.

When cavitation results from the impact of a wave of tension upon the interface between water and a solid, its character will depend upon the relative magnitudes of the breaking-pressure for a water-solid and for a water-water surface. If the breaking-pressure between solid and water is higher than that within the water itself, breaking will occur first at the solid, with the formation of a gap or cavity. Otherwise continuously distributed cavitation will form in the water, a layer of which will be left in contact with the solid. What the facts are in the case of explosive pressure waves impinging upon painted or corroded steel is not yet known.

The subsequent closing of a gap, provided it does not contain an appreciable amount of air or other foreign gas, will result in the usual water-hammer effect. If the gap closes against a rigid boundary moving at fixed velocity v'', and v' is the particle velocity of the advancing water, the pressure rises instantaneously from p_c to $p_c + \rho c(v' - v'')$. When a gap closes in the midst of the water, however, with a difference $v' - v''$ in the particle velocities on the two sides of the gap, the impact pressure is only $\frac{1}{2}\rho c(v' - v'')$; here the pressure at the gap rises instantaneously from p_c to $p_c + \frac{1}{2}\rho c(v' - v'')$. The action is, in fact, the same as if the two masses of water had impinged simultaneously and from opposite sides upon a thin solid sheet moving with the mean velocity of the water or a velocity $\frac{1}{2}(v' + v'')$.

CAVITATION AND DYNAMICAL SIMILARITY

Cavitation in the midst of a liquid differs in its effect upon relations of similarity from cavitation at the surface of a solid.

A glance at the differential equations of sound, or at some of the equations written in this report, shows that, in constructing a possible motion similar to a given one, but on a different scale, it is necessary to preserve unchanged at corresponding points the values of the two dimensionless quantities

$$\frac{p}{\rho v^2}, \ \frac{p}{\rho c^2},$$

where p is the pressure referred to any chosen datum or zero of pressure,
v is the particle velocity,
ρ is the density, and
c is the speed of sound in the liquid in question, here water.

In a given liquid, with fixed ρ and c, it follows that both p and the particle velocity must be preserved at corresponding points. The only transformation that is possible is thus the simple one, familiar in the discussion of underwater explosions, in which all linear dimensions and all times are changed in the same uniform ratio. The occurrence of cavitation at fixed values of p_b and p_c alters nothing in this conclusion so long as cavities of appreciable size do not form.

If large gaps occur, however, gravity may play a role in their neighborhood. Then, from such equations as $s = \frac{1}{2}gt^2$ and $p = \rho gh$, where s is the displacement in time t or h is the static head, it is evident that, for similarity to hold, an additional quantity must be preserved. This may be written in various forms, such as $gs\ t^2/s^2$ or

$$\frac{gL}{v^2}$$

where L is any convenient linear dimension. It is clear that L, like v^2, must be kept constant. Thus, if cavitation within the midst of a liquid is accompanied by the formation of cavities of considerable size, no transformation of similarity is possible at all.

The inclusion of effects of viscosity, on the other hand, requiring preservation of the quantity

$$\frac{\rho v L}{\nu}$$

where ν is the viscosity, is known to destroy the possibility of similarity, irrespective of whether cavitation occurs or not.

In experiments such as those on cavitating propellers, transformations of similarity can be made for two reasons. In the first place, the compressibility of the water can be neglected, as well as the viscosity effects, so that only two quantities need to be preserved in value, such as

$$\frac{p}{\rho v^2}, \quad \frac{gL}{v^2}$$

In the second place, only a single cavitation pressure is usually recognized, and this can be taken as the datum pressure which is held constant. The usual change of scale then becomes possible in which all linear dimensions and also the excess of pressure at each point over the cavitation pressure are changed in proportion to v^2. If, however, it became necessary to distinguish between two cavitation pressures, a breaking-pressure and a cavity pressure, then the fixed difference between these two would require all pressure differences to be fixed, and consequently similar motions on different linear scales could not occur.

APPLICATION: CAVITATION BEHIND* A PLATE

The simplest case to which the analytical theory of cavitation can be applied is that of plane waves of pressure falling at normal incidence upon a uniform plane sheet of solid material, where the sheet is so thin that elastic propagation through its thickness need not be considered; see Figure 6.

Various aspects of this case have been discussed in several reports (3) (4) (5) (6). If the pressure wave is of limited length and of sufficiently low intensity to make acoustic theory applicable, and if water can support the requisite tension, then it has been shown that the initial forward acceleration of the plate is followed by a phase during which it is brought to rest again by the action of tension in the water. The final displacement of the plate is equal to twice the total displacement of a particle

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* On the side acted on by the explosion.

of water due to the incident wave, or the same as the displacement of the water surface when the plate is absent.

The effect of cavitation, on the other hand, will vary somewhat, according to the point at which it occurs. There are two possibilities:

1. The plate may break loose from the water, or

2. cavitation may occur first in the water itself.

1. The plate may break loose from the water; see Figure 7.

The pressure at which this occurs may be either the cavity pressure p_c or some lower pressure p_b'. In either case, the surface of the liquid then becomes a free surface at which the pressure is constant and equal to p_c, and the remainder of the incident wave is reflected from this free surface. The plate, meantime, will continue moving forward until it is arrested by other forces. The pressure p_h, atmospheric or otherwise, acting on

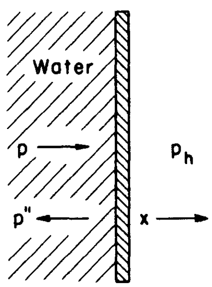

Figure 6 - Diagram representing Plane Waves of Pressure p in Water, falling upon a Large Thin Plate

This plate is backed by gas at the pressure p_h. A reflected wave of pressure p'' travels back into the water.

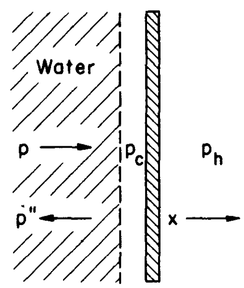

Figure 7 - Diagram illustrating the Case in which Cavitation occurs at a Thin Plate

Here the cavitation takes the form of a definite cavity in which the pressure is p_c.

the opposite face of the plate, may be assumed to exceed the pressure p_c in the cavity behind it; the difference, $p_h - p_c$, will suffice eventually to arrest the motion of the plate and to cause its return to contact with the water. There may also be other forces of elastic or plastic origin. It may happen, however, as suggested by Professor G.I. Taylor (5), that spray projected from the water surface will tend for a time to support the outward motion of the plate. When the returning plate strikes the water, an impact wave of pressure will be produced in the water as the plate comes exponentially to rest.

If the incident wave is of exponential form, explicit formulas are easily obtained. This case is discussed at length by Taylor (5), but a few details may be given here.

The equation of motion for the plate is

$$m \frac{d^2x}{dt^2} = p + p'' \qquad [9]$$

where p is the excess of pressure in the incident wave above the hydrostatic pressure p_h, which is assumed to be the same on both sides of the plate,

p'' is the excess of pressure over p_h in the reflected wave,

m is the mass of the plate per unit area,

x is the positional coordinate of the plate in a direction perpendicular to its surface, and

t is the time.

Elimination of p'' gives

$$m \frac{d^2x}{dt^2} + \rho c \frac{dx}{dt} = 2p \qquad [10]$$

where ρ is the density of water and c is the speed of sound in it. Compare here Equations [9] and [10] on page 24 of TMB Report 480 (4).

The solution of Equation [10] for $p = 0$ is

$$\frac{dx}{dt} = u_0 e^{-\frac{\rho c t}{m}} \qquad [11]$$

With a suitable choice of the constant u_0, this solution will represent the motion of the plate after returning to contact with the water if t represents the time measured from the instant of contact and u_0 is the velocity of the plate at that instant.

To represent the impact of the pressure wave, we set $p = 0$ for t less than 0 and, for t greater than 0,

$$p = p(t) = p_0 e^{-\alpha t} \qquad [12]$$

in terms of two constants p_0 and α. It is assumed that the displacement of the plate during the effective time of action of the wave is negligibly small. The solution of Equation [10] that represents the plate as starting from rest at $x = 0$ and $t = 0$ is then easily verified to be

$$\frac{dx}{dt} = \frac{2p_0}{\rho c - \alpha m}\left(e^{-\alpha t} - e^{-\frac{\rho c t}{m}}\right) \qquad [13]$$

see TMB Report 480, page 25.

The corresponding total pressure on the plate above hydrostatic is, from Equations [9] and [13],

$$p + p'' = m \frac{d^2x}{dt^2} = \frac{2p_0}{\rho c - \alpha m}\left(\rho c e^{-\frac{\rho c t}{m}} - \alpha m e^{-\alpha t}\right) \qquad [14]$$

If the simple assumption is now made that cavitation occurs at the surface of the plate as soon as the pressure sinks to a certain value p_b', the time at which it occurs can be found by putting $p + p'' = p_b' - p_h$ in Equation [14] and solving for t. The corresponding value of dx/dt as obtained from Equation [13] is then the velocity with which the plate leaves the water.

For the special case in which $p_b' = p_c = p_h$, this velocity is also the maximum velocity acquired by the plate and has the value

$$\frac{dx}{dt} = v_{\max} = \frac{2p_0}{\alpha m}\, q^{\frac{q}{1-q}} \qquad [15]$$

where

$$q = \frac{\rho c}{\alpha m} \qquad [16]$$

as given on page 7 of TMB Report 489 (6). The formula for $v_{\max}$ can also be written

$$v_{\max} = k\,\frac{p_0}{\rho c}, \quad k = 2q^{\frac{1}{1-q}} \qquad [17]$$

where k is a dimensionless number and $p_0/\rho c$ represents the particle velocity associated with the maximum pressure in the incident wave. A plot of k against q is shown in Figure 8.

If p_b' is less than p_h, the plate is slowed down somewhat by the action of the pressure p_h on its opposite face, assisted perhaps by tension in the water, so that it leaves the water with a velocity less than $v_{\max}$.

The initial velocities of diaphragms acted on by explosive pressure waves as measured at the David W. Taylor Model Basin have always been less than the calculated $v_{\max}$, but never less than half as great. Details will be reported elsewhere.

2. Cavitation may occur first in the water itself; see Figure 9.

Consideration of this case is new. If a fixed breaking-pressure p_b is assumed, the point at which cavitation starts may be found by examining the resultant pressure distribution in the water near the plate. The reflected pressure $p''(t)$ at the plate itself is, from Equations [12] and [14],

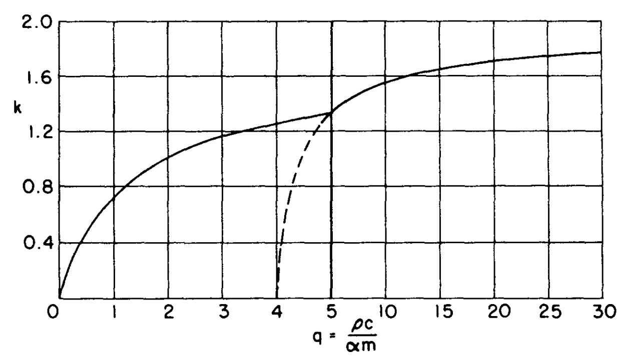

Figure 8 - Plot of the Coefficient k in Equation [17]

The broken curve continues the right-hand part of the curve backward on the same scale.

so long as cavitation does not occur,

$$p''(t) = \frac{p}{\rho c - \alpha m}\left[2\rho c e^{-\frac{\rho c t}{m}} - (\rho c + \alpha m)\, e^{-\alpha t}\right] \qquad [18]$$

Let x represent a distance from the plate, measured positively in the direction of propagation of the incident wave. Then the pressure at any point behind the plate, within the distance to which the reflected wave has traveled, will be

$$p = p\left(t - \frac{x}{c}\right) + p''\left(t + \frac{x}{c}\right) =$$

$$p_0\left[e^{-\alpha\left(t - \frac{x}{c}\right)} + \frac{2\rho c}{\rho c - \alpha m}\, e^{-\frac{\rho c}{m}\left(t + \frac{x}{c}\right)} - \frac{\rho c + \alpha m}{\rho c - \alpha m}\, e^{-\alpha\left(t + \frac{x}{c}\right)}\right] \qquad [19]$$

Cavitation will begin where p as given by this equation first sinks to the breaking-pressure p_b. From this point a breaking-front will advance toward the plate, perhaps all the way up to it, while another one travels back into the water. The latter front travels forever, in the present case, provided p_b is less than 0, but the cavitation behind it soon becomes negligible. This is because the incident wave soon becomes inappreciable, and the receding breaking-front soon becomes indistinguishable from that particular reflected wave at which $p'' = p_b - p_h$ and travels with this wave at the speed of sound. Equation [4] then gives $\eta = 0$ behind the front.

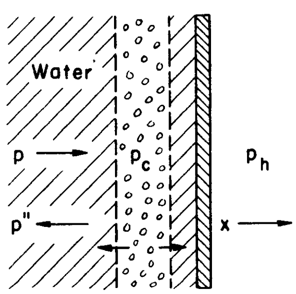

Figure 9 - Diagram illustrating the Case in which Bulk Cavitation occurs in the Water

The particle velocity v_n ahead of the front becomes that of the reflected wave or $v_{bn} = -(p_b - p_h)/\rho c$; hence by Equation [3], in which $V_b = c$ and the signs of v_{cn} and v_{bn} must be changed to allow for the difference in the direction chosen for a positive velocity, the particle velocity behind the front is $-(p_c - p_h)/\rho c$. Thus the part of the reflected wave from $2p_0$ to p_b travels on, with a discontinuity at its rear face, leaving the pressure uniformly equal to p_c and the velocity uniformly equal to $-(p_c - p_h)/\rho c$ behind it. Only a limited region of cavitation is formed near the plate.

The process by which the plate, after being returned by other forces such as air pressure, destroys the cavitation again, can be followed by numerical integration in any particular case that may arise.

EFFECT OF CAVITATION ON PRESSURE

The action of an exponential wave upon a plate is illustrated in Figures 10 and 11, which are drawn to represent very roughly the action of the shock wave from 300 pounds of TNT upon a plate of steel 1 inch thick, or from 1 ounce of TNT upon a plate 1/17 inch thick.

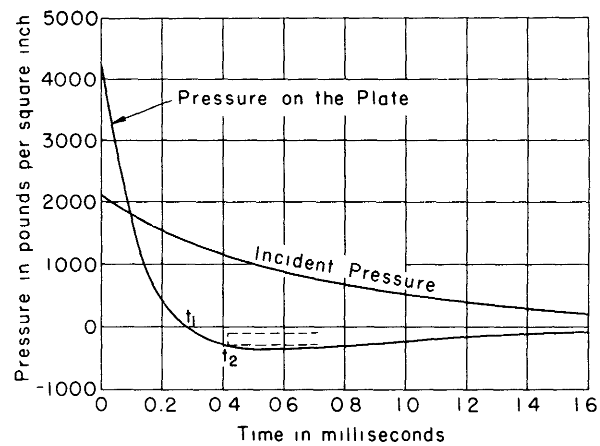

Figure 10 - Pressure on a Plate, in the Absence of Cavitation, plotted on a Time Base

Figure 10 shows the excess pressure on the plate itself, above hydrostatic pressure, plotted

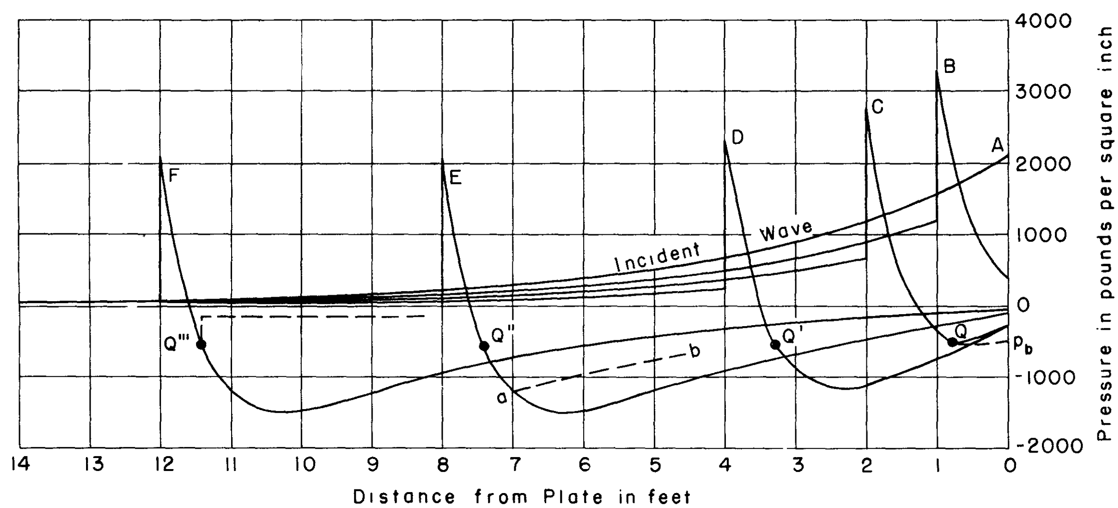

Figure 11 - Distributions of Pressure behind a Plate at Successive Instants of Time, in the Absence of Cavitation

The incident wave approaches from the left, hence distance from the plate is plotted in that direction.

on a basis of time. The time scale is labeled to correspond to 300 pounds of TNT; for 1 ounce the times would be 1/17 as great. One curve shows the incident pressure, or the pressure that would exist in the water at the location of the plate if the plate were absent, as given by Equation [12]. The other curve shows the actual pressure on the plate, as given by Equation [14]. This may be thought of as made up of the incident pressure p together with a component of pressure p'' due to a reflected wave that travels back into the water.

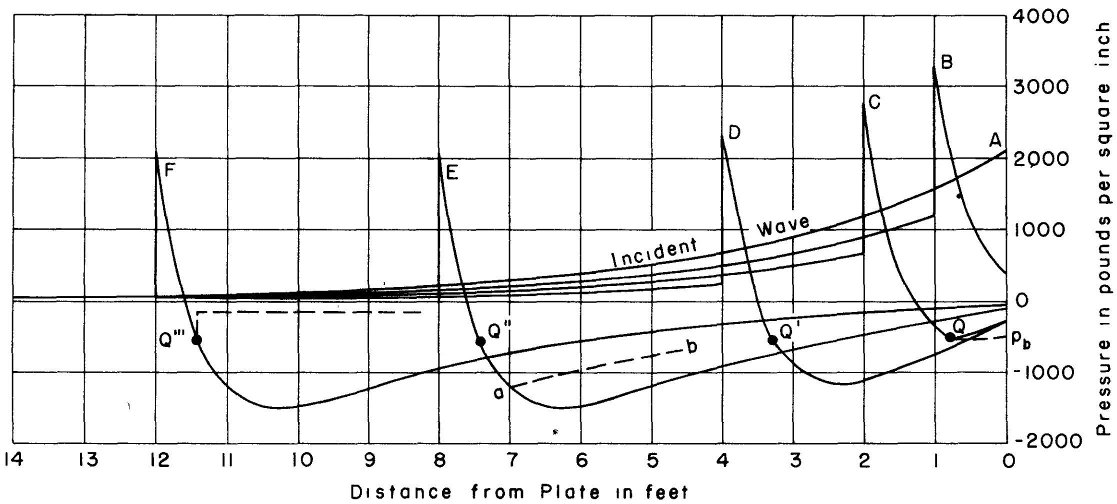

Figure 11 - Distributions of Pressure behind a Plate at Successive Instants of Time, in the Absence of Cavitation

Figure 11, on the other hand, shows the instantaneous distribution of pressure in the water adjacent to the plate, plotted against the distance from the plate. The distances shown in the figure correspond to 300 pounds of TNT; for 1 ounce they would be 1/17 as great. The curves are calculated by Equation [19] where $t + x/c$ is positive, and by Equation [12] elsewhere. Curve A shows the distribution of pressure at the instant at which the pressure wave first reaches the plate ($t = 0$). Curve B shows the distribution 0.2 millisecond later ($t = 0.0002$); at this time the reflected wave has advanced 1 foot from the plate. Curves C, D, E, F refer similarly to times about 0.4, 0.8, 1.6, 2.4 milliseconds after the arrival of the incident wave. Curve F serves also to represent the final form of the reflected wave; the incident wave has by this time completely disappeared.

These figures will be modified by the occurrence of cavitation in a way that depends upon the laws governing the cavitation.

Cavitation may occur *at the plate.* It may occur as soon as the pressure sinks to the hydrostatic pressure p_h; this will be at the instant marked t_1 in Figure 10. In this case the plate leaves the water with a velocity equal to $v_{\max}$ as given by Equation [17], and the curve for the pressure on the plate in Figure 10 coincides with the axis of zero pressure from the time t_1 onwards. An alternative possibility, however, is that cavitation may not begin until a lower pressure p_b' is reached, at a later time such as that marked t_2 in Figure 10. In this case the plate leaves the water at the time t_2 with a velocity less than $v_{\max}$. The pressure on the plate after t_2 will then be the constant cavity pressure p_c. If $p_c = p_b'$, the curve will extend horizontally from the point t_2, as shown by the lower of the broken lines, instead of continuing downward. If, on the other hand, p_c is greater than p_b', the pressure on the plate will rise suddenly to the value p_c at the instant t_2 and will then remain constant, as illustrated by the upper of the two broken lines in Figure 10.

The distributions of pressure in the water, as plotted in Figure 11, will be modified in ways to correspond. The part of the reflected wave that is reflected from the water surface after the occurrence of cavitation will be modified so as to contain higher pressures, since the pressure at the water surface is higher than it would have been if the water had continued in contact with the plate. In Figure 11, on each of the later curves there will be a point representing the instantaneous position of that part of the reflected wave which was reflected just as cavitation began; such a point is indicated by a on Curve E. The pressure to the right of this point contains a component that was reflected from the free water surface instead of from the plate and hence will lie somewhat higher than it would in the absence of cavitation, as is suggested in Figure 11 by the broken line ab.

As an alternative, cavitation might *begin in the water itself*. In such a case the analysis given in foregoing sections becomes applicable. Cavitation will start at a definite position as well as at a definite time. It might begin, for example, at Q in Figure 11; this point would then represent the position of that plane in the water, parallel to the plate, at which the pressure first sinks to the breaking-pressure p_b.

From this initial plane, a plane breaking-front will advance a short distance toward the plate, while another one will follow the reflected wave toward the left, moving a little more rapidly than this wave so as always to be in the position at which the total pressure equals p_b. Successive positions of the latter breaking-front are indicated in Figure 11 by Q′, Q″, Q‴. Behind this front, or on the right in the figure, lies the cavitated region, in which the pressure equals the cavity pressure p_c. The boundary of this region on the side toward the plate is not shown in Figure 11, since its position can only be inferred from a more detailed study of the motion of the water near the plate. The uniform pressure p_c behind the breaking-front, on the assumption that p_c is greater than p_b, is illustrated for a certain instant of time by the broken line behind Q‴. Thus, the part of Curve F to the left of Q‴, up to 12 feet from the plate, represents the part of the reflected wave that got past Q before cavitation began, diminished somewhat through being partially overtaken by the breaking-front which moves at first at supersonic velocity. The remainder of Curve F is replaced by the uniform pressure in the cavitated region or near the plate by an undetermined modified pressure.

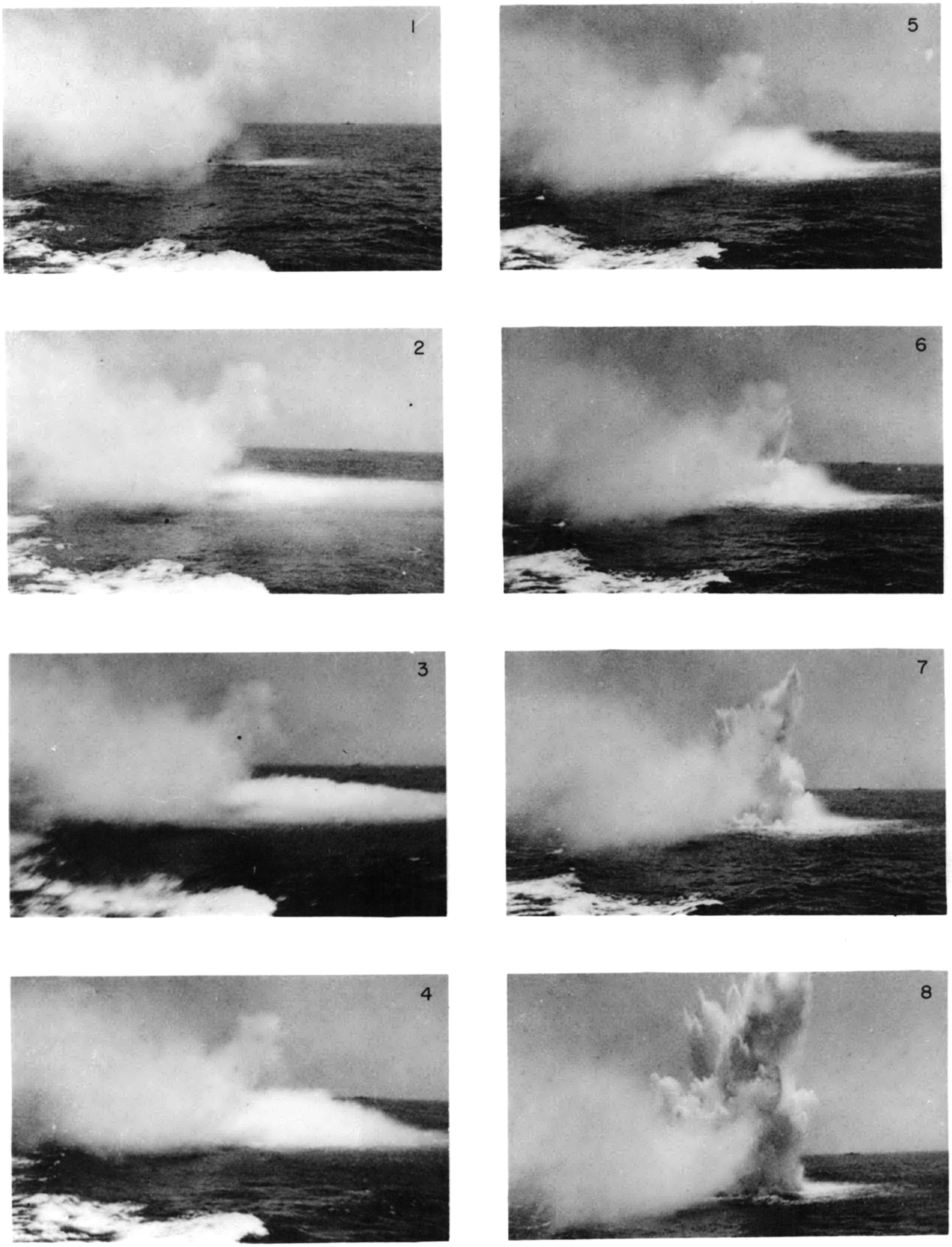

Figure 12 - Growth of a Dome and Plumes from an Underwater Explosion

The phenomenon is partially obscured by smoke from the ship from which the photographs were taken.

More complete figures are scarcely worth constructing until an actual known case presents itself for analysis.

CAVITATION UNDER THE SURFACE OF THE SEA

When a charge is exploded at a suitable depth in the ocean, a dome of white-appearing water is seen to rise somewhat above the surface, breaking after a moment into plumes of spray. The eight frames from a motion picture film, Figure 12, illustrate this phenomenon. The plumes are supposed to be associated with the escape of the explosion gases. The dome, however, has been ascribed to the occurrence of cavitation; a layer of water at the surface and just under it, after being kicked upward by the pressure wave, fails to be jerked to rest again by the action of a reflected wave of equal tension and continues rising until stopped by gravity and air pressure. This explanation will be considered briefly on the basis of the foregoing analysis.

It is necessary first to fix upon the value to be assumed for the breaking-pressure p_b. Hilliar (7) found that the dome was absent whenever, according to his measurements, the maximum pressure reaching the surface was under 0.3 ton or 670 pounds per square inch, and concluded that p_b was roughly of this magnitude. It will be assumed, therefore, for the moment, that p_b = -600 pounds per square inch.

To select a specific case for study, suppose that a charge of 300 pounds of TNT is detonated 50 feet below the surface. Then the pressure wave should be reflected from the surface as a wave of equal tension, diverging from the mirror image of the charge in the surface and decreasing in intensity as it progresses. Using Hilliar's data, it is easy to map out the lens-shaped volume within which the pressure would sink momentarily at least to -600 pounds per square inch if there were no cavitation. This volume is outlined roughly by the lower curve in Figure 13.

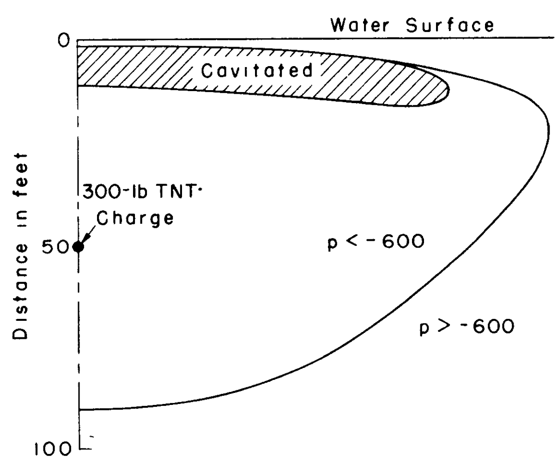

Figure 13 - Diagram of Region of Reduced Pressure following Reflection of a Pressure Wave from the Surface

Application of the criterion obtained from the analysis for the propagation of a breaking-front indicates, on the contrary, that cavitation would in reality be confined to a much smaller region, which is shaded in Figure 13. To locate this region, it is necessary to estimate the magnitudes of the incident and reflected waves as they become superposed upon each other at various points and at various times. The pressure in each wave is assumed to decrease in inverse proportion to the distance from its point of origin, real or assumed; and allowance must be made for the time of propagation. It is unnecessary to give details of the rather tedious calculations, which were carried out only roughly.

By trial, it is found that the total pressure should first reach the value of -600 pounds per square inch at a point situated directly over the charge and about 1 foot under the surface. Cavitation will begin at this point, according to the assumption made here, and from this point a closed breaking-front will sweep out, moving at supersonic velocity. The upper side of this front must obviously halt almost at once, for a tension of 600 pounds per square inch cannot occur close to the surface; but the lower side may descend to a considerable depth.

In Figure 14 are shown the estimated distributions of pressure along a vertical line through the charge at two different times, distinguished by the numbers 1 and 2. Heavy curves are drawn to represent the actual pressures; light curves above the axis represent the component pressures due to the incident wave, those below the axis the components due to the reflected wave.

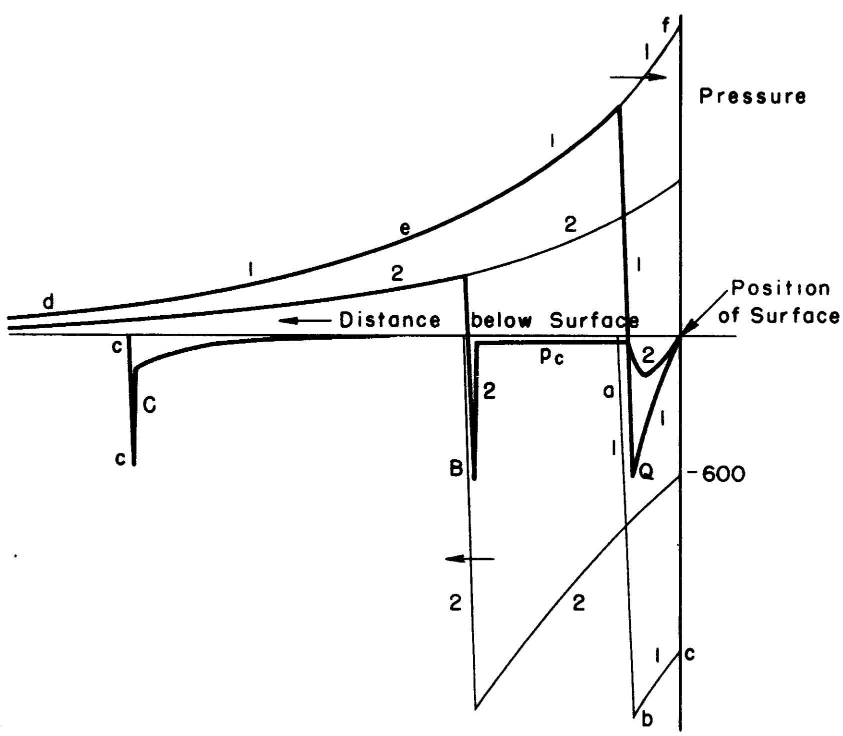

Figure 14 - Diagram illustrating the Distribution of Pressure below the Surface of the Sea, as explained in the Text

At the instant 1, a small part of the incident wave has already been converted at the surface into the reflected wave, shown by the light curve abc; the remainder of the incident wave is represented by the curve def. Together these two components make up the total pressure represented by the heavy curve 1111. Cavitation is just beginning at Q, where the pressure has sunk to -600 pounds per square inch.

From this time onward, the curve of total pressure is clipped off at -600 pounds per square inch by the breaking-front. Hence, at the time 2, for example, the curve has its minimum at -600 at B, and to the right of this point, or toward the surface, lies a cavitated region, in which the pressure has the small negative value p_c. Just under the surface, however, in unbroken water, larger negative pressures will probably occur. The distribution of pressure at this instant will thus be as shown by the heavy curve 222.

The breaking-front will finally cease advancing when V_b as given by Equation [2] becomes equal to c. In applying this criterion, it is more convenient to transform Equation [2] by substituting, from the theory of sound waves,

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = -\frac{1}{\rho c^2}\frac{\partial p}{\partial t}$$

The actual formula employed in making the rough estimate was Equation [46] in Reference (2). Using the author's provisional estimate of the later part of the pressure curve, as represented on page 15 of Reference (6), it was concluded in the manner just described that cavitation might ultimately extend throughout a volume such as that shaded in Figure 14, or to a horizontal radius of nearly 100 feet, but only to a maximum depth in the center of 10 feet.

After the boundary of the cavitated region has ceased advancing as a breaking-front, it will undoubtedly begin to recede as a closing-front. No attempt has been made to follow this process, however, since it seems to be possible to infer the gross features of the subsequent motion of the water from more general considerations.

The particle velocity just behind the front may be estimated from Equation [3]. Just above the top of the cavitated layer, v_{bn}, representing the resultant particle velocity due to incident and reflected waves, adds numerically to the last term in Equation [3] and gives a total upward particle velocity v_{cn} in the cavitated layer of about 49 feet per second. The simultaneous value at the surface is twice that in the incident wave or perhaps 34 feet per second. Where the descending part of the front halts, however, the positive direction for v_{bn} is downward, whereas the actual particle velocity is due almost entirely to the reflected wave and is upward. Thus

v_{bn} is here nearly equal to $p_b/\rho c$, so that v_{bn} just about cancels the last term in Equation [3] and v_{cn} is small. It may safely be inferred that the particle velocity in the cavitated region will grade from a small value at the bottom to about 49 feet per second at the top.

The whole cavitated layer, 9 feet thick, should therefore rise, carrying a thin uncavitated sheet on top of it. This "solid" sheet will increase in thickness as its lower boundary travels downward in the form of a forced closing-front. The numerical values cited indicate that the center of gravity of the upper 10 feet of water will start upward with a velocity of perhaps 25 feet per second; and there should be a downward acceleration of g due to gravity and of $(34/10)g$ due to air pressure on the top, or a total of 4.4 g. The center of gravity should rise, therefore, not over $s = v^2/2g = 25^2/8.8 \times 32 = 2.2$ feet, during a time $v/4g$ or 0.2 second. The surface of the water will rise higher but certainly not more than twice as high or, at the utmost, 5 feet.

Now this picture as inferred from the analysis appears not to agree too well with the facts. Hilliar's observations indicate that, in the case considered, the dome would certainly be less than 60 feet in radius but would rise in a second or so to a height of 15 or 20 feet. The analytical estimate would be changed considerably if a different breaking-pressure were assumed, or if more recent values for the incident pressure were employed, but a large disagreement with observation would remain. The large rise that is actually observed could be explained only by supposing that the disintegration of the water extends up to the surface and serves to admit atmospheric pressure to the interior. Cavitation up to the surface might result from the initial presence of air bubbles in the upper few feet of water, which would effectively raise p_b, perhaps up to p_c. The whiteness observed in all explosions of this kind does, in fact, extend to the very edge of the dome in the photographs; see Figures 12 and 15. It is not easy to believe, however, that air can mix sufficiently rapidly with the cavitated water to relieve the vacuum effectively.

The jaggedness of the edge of the dome, so clearly revealed by the photographs, suggests a modified hypothesis. Perhaps the general mass of water really does rise only a few feet, as the analysis suggests, and what is seen as a white dome of considerable height is only an umbrella of spray thrown up from the surface.

The origin of the spray itself is perhaps to be found in an instability of the surface under impulsive pressure. The pressure gradient is equivalent to a momentary increase of gravity by a factor of 100 to 1000, followed by a reversal to similar values. If there are any small waves on

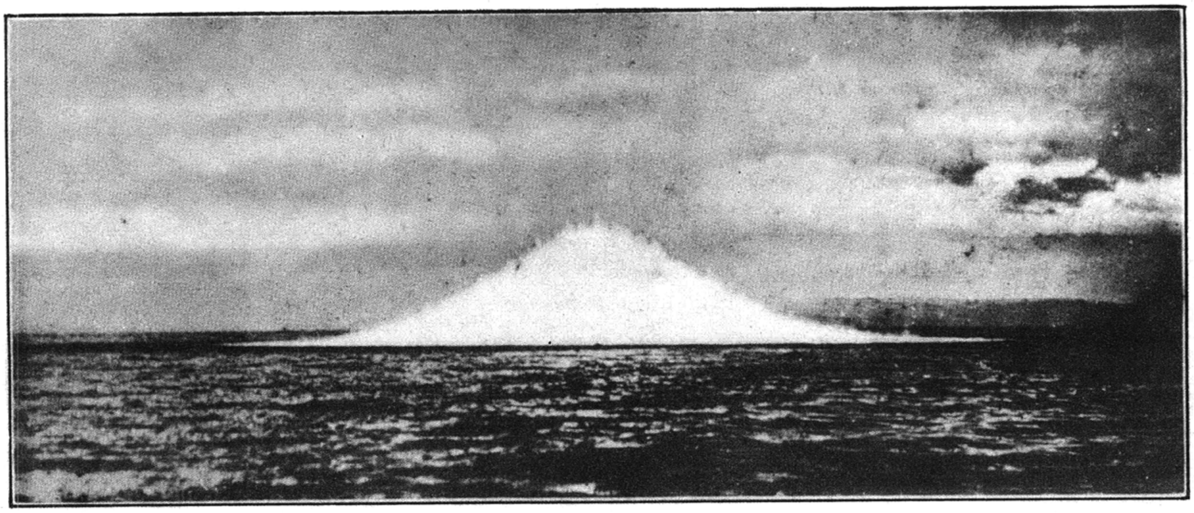

Figure 15 - The Dome, 53 feet high, raised by a Charge of 1900 pounds of Amatol, detonated 64 feet below the Surface

This photograph is from Hilliar, Reference (7), Figure 62.

the surface, the lesser mass of water under the troughs will be accelerated more violently than the greater mass under the crests, but the difference in the accelerations will be greater during the pressure phase than during the subsequent tension phase because the initial differential motion tends to smooth out the waves or even to reverse them. The initial troughs should thus tend to be thrown up as spray.

An indirect method of determining whether or not cavitation occurs under the surface is by studying the reflected wave of tension itself. In the absence of cavitation, this should be a reversed replica of the incident wave, reduced somewhat by the greater distance of travel. If, however, cavitation occurs, only the very short initial part of the tension wave as produced at the surface, containing the rapid drop to the breaking-pressure p_b, will continue traveling below the level at which the breaking-front halts. It is readily seen that the lower boundary of the cavitated region should stand still thereafter as a stationary boundary, as described on page 5. For, as noted on page 8, $2p' = p_c + \rho c v_c$ when the breaking-front halts, where p' is the positive pressure in the incident wave, and thereafter $2p' < p_c + \rho c v_c$ as p' decreases, so that the condition for a stationary boundary as stated on page 7 is met. The tail of the incident wave will be reflected from this boundary as a tension wave in which the pressure is $p'' = p_c - p'$. Thus the total reflected wave as it occurs below the region of cavitation will be qualitatively as sketched at C in Figure 14.

This conclusion is in general harmony with a series of piezoelectric observations reported in 1924 (8). Only relatively small tensions were found. Presumably an initial jab of high tension such as cc in Figure 14 would have little effect on the gage. The observed tensions would represent, therefore, merely the reflection of the tail of the incident wave from the bottom of the cavitated region, as is stated in the report.

The value of the breaking pressure may be inferred most easily from the minimum depth at which the reflected tension appears in full strength, indicating no cavitation. One of the observations mentioned points toward a relatively high value of p_b. A charge of 2 1/4 pounds of guncotton 60 feet below the surface gave a maximum pressure of 910 pounds per square inch on a gage placed 15 feet away and on the same level. Without cavitation, therefore, the maximum reflected tension should be about $910 \times 15/120 = 115$ pounds; but only 15 pounds was observed. Yet the maximum pressure at the surface would be only $910 \times 15/60 = 230$ pounds per square inch. If the gage was capable of measuring tensions effectively, the conclusion is justified that in this case the water must have cavitated at a tension scarcely exceeding 200 pounds.

It must be recognized, however, that cavitation at the gage might alter the conclusions materially. If cavitation over the gage occurs at higher pressures than it does in the water itself, then the tensions indicated by the gage set only a lower limit to the magnitude of the tension occurring in the water itself. The piezoelectric observations would be consistent with the assumption that no cavitation at all occurs in the midst of the sea.

A few remarks may be added concerning the similarity laws for surface phenomena. On page 9 it has been seen that the change to model scale, as it is commonly made in dealing with underwater explosions, is possible only so long as gravity effects can be neglected. In this change all linear dimensions and all times are changed in one and the same ratio; the pressures and velocities at corresponding points remain unchanged. It follows that the effects of air pressure upon surface phenomena will be relatively the same upon all scales. Insofar as these phenomena are influenced by gravity, however, similar motions on different scales are impossible. Similar motions would be possible only if the strength of gravity were changed in inverse ratio to the linear dimensions, so as to preserve the value of the quantity gL/v^2 or, since v^2 is unchanged, of gL itself; L is here any convenient linear dimension and v is the particle velocity. Small-scale phenomena thus correspond to large-scale ones occurring in a proportionately weaker gravitational field.

This conclusion is surprising, for it appears to mean that spray should be thrown to the same height by charges of all sizes. This would be in conflict with the suggestion that the dome over large charges may consist chiefly of spray, for a charge of an ounce throws spray to a height of a few feet at most. The explanation of the difference may possibly lie in an influence of surface tension upon spray formation. Since the pressure under a curved surface is $p = 2T/r$ in terms of the surface tension T and the radius r, the relative effect of surface tension, when the pressures are unchanged, will be the same only if T is changed in the ratio of the linear dimensions. Thus surface tension, being actually constant, will have a much larger effect upon small-scale than upon large-scale phenomena.

On the other hand, as we have seen, a dome of superficially solid water is limited chiefly by air pressure, hence it should follow the usual linear scale. The absence of a noticeable dome over small charges is thus consistent with the estimate of possible dome heights as made in the foregoing, and in turn constitutes evidence against the supposition that the dome over large charges consists largely of moderately disintegrated water.

It must be recognized, however, that other causes are possible for the difference in the surface phenomena on large and small scales. For one reason or another, cavitation might occur more easily in the salt water of the sea than in the fresh water in the laboratory. Or it might be that water can stand higher tension for the shorter times involved in the action of smaller charges. More evidence on these points is needed.

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