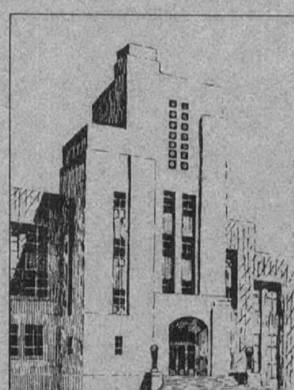
THE DAVID W. TAYLOR MODEL BASIN

UNITED STATES NAVY

INSTRUMENTATION FOR THE MEASUREMENT OF UNDERWATER EXPLOSION PRESSURES

BY M.A. GREENFIELD, Ph.D., AND M.M. SHAPIRO, Ph.D.



CONFIDENTIAL

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SEPTEMBER 1944 REPORT 523

NAVY DEPARTMENT

DAVID TAYLOR MODEL BASIN

WASHINGTON, D.C.

RESTRICTED

CONFIDENTIAL

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PERSONNEL

This report was prepared by Dr. M.A. Greenfield and Dr. M.M. Shapiro, who are responsible for the development of the pressure gages described in it. Dr. Avis Borden conducted the research on calibration techniques. Valuable assistance in all the experimental work was rendered by E.T. Habib, A.R. Cohen, and B. Stiller. All participants are members of the David Taylor Model Basin staff.

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INSTRUMENTATION FOR THE MEASUREMENT OF UNDERWATER EXPLOSION PRESSURES

ABSTRACT

The David Taylor Model Basin, in its efforts to provide instruments for the measurement of underwater explosion pressures, has developed three pressure gages, two of the piezoelectric and one of the resistance type. The construction and operation of the gages are described in detail.

The most important causes of distortion of a gage signal are analyzed, and the measures taken to overcome them are described. In particular the mechanical disturbance of a gage cable by an underwater explosion gives rise to a spurious signal which is large when ordinary coaxial cables are used. An investigation of this effect has resulted in the development of a coaxial cable which has many advantages over any other type that was investigated, and which permits accurate measurements of momentum.

Several techniques for calibrating the various gages are discussed. The best of these methods gives results which agree closely with those obtained by workers in other laboratories.

The gages were subjected to explosion tests with charges of about one ounce of tetryl, in which pressure-time records were obtained. Only the tourmaline gage has been tested with larger charges, of 100 grams or more. These tests yield records which exhibit a high degree of reproducibility.

INTRODUCTION

When intensive research into the fundamentals of underwater explosion phenomena was started in the United States less than four years ago, one of the first problems encountered was to find a satisfactory gage for measuring the high-intensity short-duration pressures in the water.

A number of gages had been developed by previous experimenters in this field, some of them mechanical and some electrical, but none of them were satisfactory for the new work being undertaken. Mechanical gages such as the Hilliar gage (1) (2)* were not fast enough to indicate the rate of rise and the peak pressures accurately, and the piezoelectric tourmaline crystal gage suggested by Sir J.J. Thomson (3) and used by Keys (4) was, like the Hilliar gage, so large that it was suitable only for work with large or service charges.

The rapid progress which had been made in the development of electronic instruments in the decade preceding 1940 suggested the use of some

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* Numbers in parentheses indicate references on page 47 of this report.

sort of electrical gage with oscillographic recording. Studies made with various types of electrical gages demonstrated that, at least for the small charges being used in the new series of experiments, the gage had likewise to be small in physical dimensions.

In view of the importance of the project and the necessity for devising an accurate and satisfactory gage before much real progress could be made on the research program, a number of different agencies began work on gages and carried it along simultaneously until all were satisfied that the desired result had been achieved. This is a case where duplication of effort, if it may be called by that name, was not only justifiable but necessary.

The David Taylor Model Basin was one of the agencies which engaged in this program of development. This report has been prepared to give an outline of the history of the project and a rather complete description of the results as they stand to the time of writing. All of the work in question has been done with electrical gages; two of the final designs are of the piezoelectric type and one is of the electrical resistance type.

For the convenience of the reader, the report is divided into five parts, giving first the general specifications for the gages, then in turn describing the pickup or sensitive elements and the cables and recording channels. The manner of calibrating the gages is described, and the report concludes with a description of the performance of the various designs during laboratory and field tests.

PART 1. GENERAL SPECIFICATIONS

The pressure of a shock wave resulting from the underwater detonation of a small charge of high explosive reaches its peak value in less than a microsecond (1×10^{-6} second); this peak value may be several thousand pounds per square inch. For explosive charges less than 1 pound, the duration* of the pressure wave is less than 70 microseconds. The characteristics of a typical wave are shown in Figure 1.

The pickup has to follow the pressure variations occurring in this time interval, and the period of its lowest mode of vibration must be small compared to the duration. For example, if this period is required to be less than 1/10 the duration, then the fundamental frequency of the pickup must exceed 1.5×10^5 cycles per second. This in turn means that none of the physical dimensions of the pickup may exceed about 1/2 inch.** The frequency

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* The duration is defined as the time required for the pressure to drop to $1/e$, or 36.8 per cent of its peak amplitude.

** For example, at the lowest mode a freely suspended steel disk 0.25 inch thick and 0.8 inch in diameter vibrates at a frequency of approximately 1.5×10^5 cycles per second.

should in fact be even higher than 1.5×10^5 cycles per second to prevent the pickup from overshooting the peak pressure.

The pickup must be free of hysteresis and have good mechanical properties. It must not be brittle or have a low tensile strength or be soluble in fresh or sea water. It must operate without permanent deformations and it must respond linearly to the pressure.

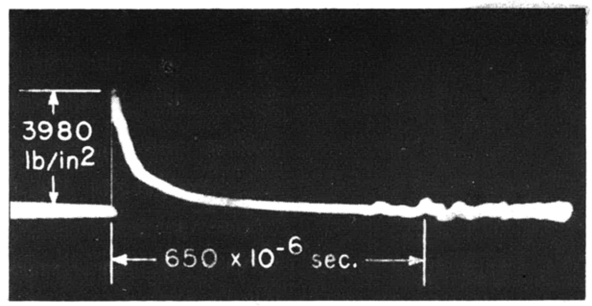

Figure 1 - Typical Pressure Record from an Underwater Explosion

This record was obtained by a tourmaline gage supplied by the Underwater Explosives Research Laboratory at Woods Hole; the charge was 150 grams of tetryl at 3 feet from the gage.

Finally, some way must be provided for transforming the mechanical variations of the pickup dimensions into electrical variations of voltage so that a record can be obtained with a cathode-ray oscillograph.

Both high- and low-impedance pickups were developed, to provide a check on the validity of the pressure measurements. The two types supplement each other, as will be discussed later in greater detail.

EARLIER PIEZOELECTRIC GAGES

Before the TMB gages and the manner in which they fulfill these requirements are described, the background of development should be filled in somewhat more completely than was done in the Introduction. As mentioned there, Sir J.J. Thomson was probably the first to suggest the use of piezoelectricity for the measurement of explosion pressures. An early form of piezoelectric manometer was devised by Keys, who used a dozen tourmaline crystals mounted in mosaic fashion inside a brass pressure vessel 6 inches in diameter. So large a voltage was developed by these crystals that the output could be impressed directly on the deflection plates of a cathode-ray oscillograph; no prior amplification was necessary.

A more recent and much smaller tourmaline crystal pressure gage was devised by Professor E.B. Wilson, Jr., and developed in collaboration with Dr. R.H. Cole of the Underwater Explosives Research Laboratory at the Woods Hole Oceanographic Institution, Woods Hole, Massachusetts (5). It consists of a single tourmaline crystal in the form of a thin plate, with two copper-plated faces or electrodes to which the cable leads are soldered. The pickup is insulated from the water by a thin protective coating, usually of Bostik cement. Typical overall dimensions of the crystal are about 0.6 inch by 0.3 inch by 0.08 inch. A shielded, rubber-insulated Belden cable connects the pickup to the amplifier.

Because the crystal is small, the output must be amplified before it is fed into a cathode-ray tube. However, this gage has the advantage that it can be used to study explosions of charges smaller than those required with the Keys type of gage.

PART 2. TMB PICKUP ELEMENTS

Two materials which satisfy all of the specified requirements for a high-impedance pickup are quartz and tourmaline crystals. Their piezoelectric properties are described in detail in References (6) to (14).

Quartz crystals have been used in the measurement of rapidly changing pressures in internal combustion engines (15). More recently the Naval Ordnance Laboratory has been developing a quartz indicator for the measurement of underwater explosion pressures (16). The two types of Taylor Model Basin piezoelectric gages use quartz and tourmaline, respectively.

THE TMB QUARTZ GAGE

When an "X-cut" plate of quartz* is subjected to a compressional force normal to its parallel faces, equal and opposite charges proportional to the force appear at these faces. Quartz is not sensitive to isotropic pressure and a quartz crystal forming the sensitive element of a pressure gage must be housed in a case so that the pressure will act only on the faces perpendicular to the electric axis. Quartz plates have been subjected to pressures up to 50,000 pounds per square inch in a testing machine, and the relationship between charge and pressure has been found to be linear over this full range (17).

The present TMB quartz gage has passed through several stages in its evolution. The earliest form had a large crystal and a plastic housing; a subsequent model used a double crystal and a brass housing; the gage described in this report is considerably smaller than the previous models and is of the single-crystal type. The techniques applied in the construction and assembly of a quartz gage at the Taylor Model Basin are described in the Appendix.

An idea of the progressive reduction in size, to obtain a high natural frequency of vibration for the quartz-mount system, is presented in Figure 2. In its present form the pickup element consists of a cylindrical quartz crystal 1/16 inch thick and 1/4 inch in diameter, housed in a brass cylindrical mount 7/32 inch high and 1/2 inch in diameter; see Figure 3. A brass tube, integral with this cylinder, contains the wire conductor attached

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* A plate of quartz is said to be "X-cut" if its plane is perpendicular to an electric axis of the crystal.

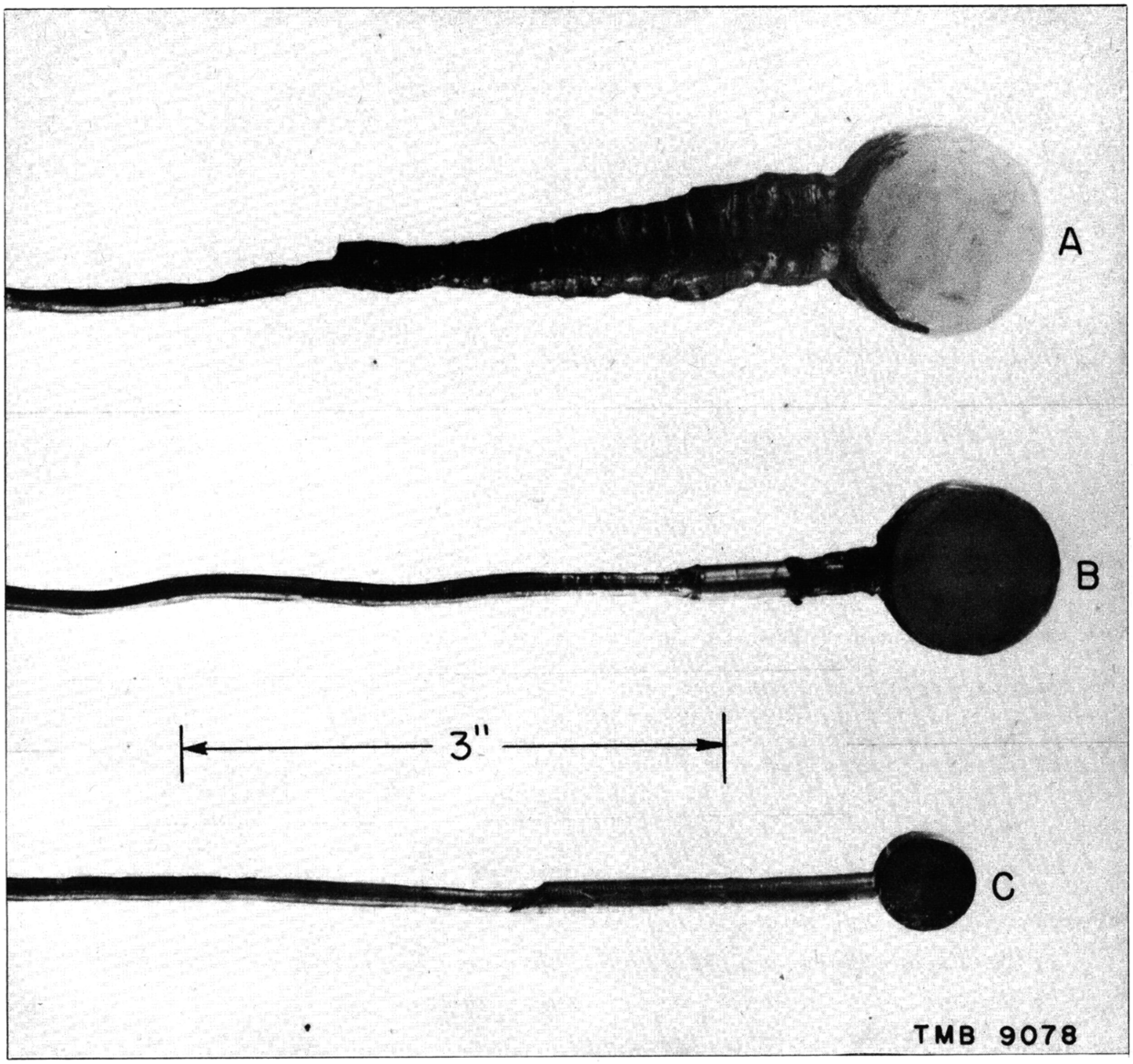

Figure 2 - Three Stages in the Evolution of the TMB Quartz Pressure Gage

A is an early model with plastic mount;
B is a double-crystal model with brass mount; and
C is a single-crystal model, shown in greater detail in Figure 3.
All three of the gages are reproduced here in approximately their natural size.

to one electrode on the quartz; the other side of the crystal is grounded to the mount. The brass tube is soldered to a long copper tube which serves as a cable.

The sensitivity of the pickup is determined by the area of the crystal and its total capacitance, the distributed cable capacitance, and any lumped capacitance that may be added. The output of the quartz crystal used is approximately

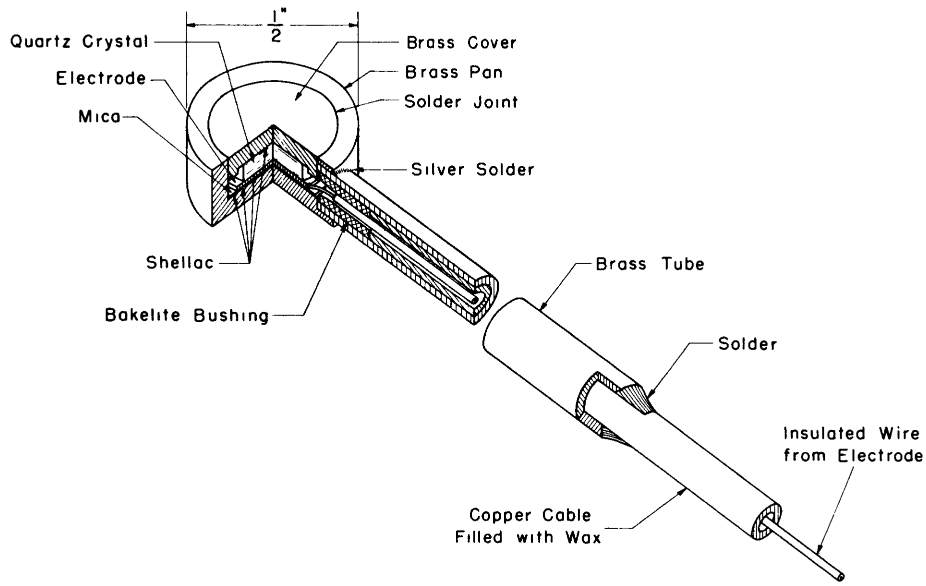

Figure 3 - Components of the TMB Quartz Piezoelectric Gage

The brass housing consists of three parts: a "pan" which contains the crystal; a cover; and a brass tube which is silver-soldered to the pan. A cylindrical, X-cut quartz crystal is cemented on one side to an electrode consisting of a thin copper disk, and on the other to the cover of the brass housing, through which it is grounded. The use of a copper disk for the electrode obviates the trouble of electroplating. Laterally the crystal is surrounded by an air gap which is required because quartz is insensitive to isotropic pressure. It is the need for this air gap which makes a housing necessary.

$$\frac{\Delta Q}{\Delta P} = KA = 5 \times 10^{-13} \; \frac{\text{coulombs}}{\text{pounds per square inch}}$$

where ΔQ is the charge produced, in coulombs,

ΔP is the pressure applied, in pounds per square inch,

K is the piezoelectric constant for quartz, about 1.03×10^{-11} coulombs per pound, and

A is the area of one side of the crystal, in square inches.

If a capacitance of 5000 micromicrofarads is used with a crystal 0.05 square inch in area, the voltage sensitivity is

$$\frac{KA}{C} = \frac{\Delta V}{\Delta P} = \frac{5 \times 10^{-13}}{5 \times 10^{-9}} \; \frac{\text{volt}}{\text{pounds per square inch}} = 10^{-4} \; \frac{\text{volt}}{\text{pounds per square inch}}$$

For a pressure of 4000 pounds per square inch the signal ΔV is 0.4 volt. The voltage variation across the capacitance is led through an amplifier into the input of a cathode-ray oscillograph. Then the record produced on the fluorescent screen can be photographed.

In any quartz gage intended for work under water, it is essential to have a watertight mount which will provide an air gap surrounding the lateral surface of the crystal. Rapid leakage of charge between the two electrodes must also be prevented. The leakage resistance of the TMB piezoelectric quartz gages is usually several thousand megohms.

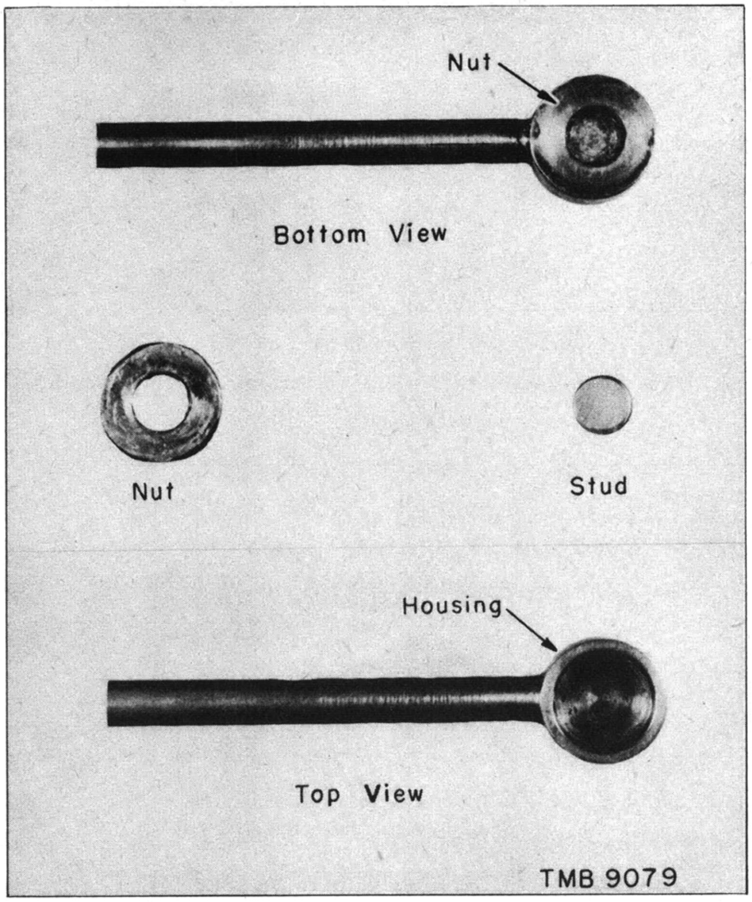

Figure 4 - Modification of Quartz Gage for Attachment to a Surface

A nut is soldered to the bottom of the housing, and a threaded stud is soldered to the surface at which the pressure is to be measured.

The brass housing of the quartz gage lends it great durability, and thus the gage will last for a very large number of explosions. This advantage is offset by the vibrations set up in the quartz mount when the gage is subjected to explosions. The observed frequency of the mount is of the order of 7×10^4 cycles per second, not high enough to prevent the introduction of an error in the determination of the peak pressure. A quartz gage of modified design which should possess a considerably higher natural frequency is under construction at the Taylor Model Basin.

The quartz mount can be readily adapted to the measurement of explosion pressures at the surface of an underwater structure. This is accomplished by silver-soldering a nut to the bottom of the mount, and attaching a threaded stud to the surface in question; see Figure 4.

THE TMB TOURMALINE GAGE

The TMB form of tourmaline pressure gage differs from the one developed by the Underwater Explosives Research Laboratory at Woods Hole chiefly

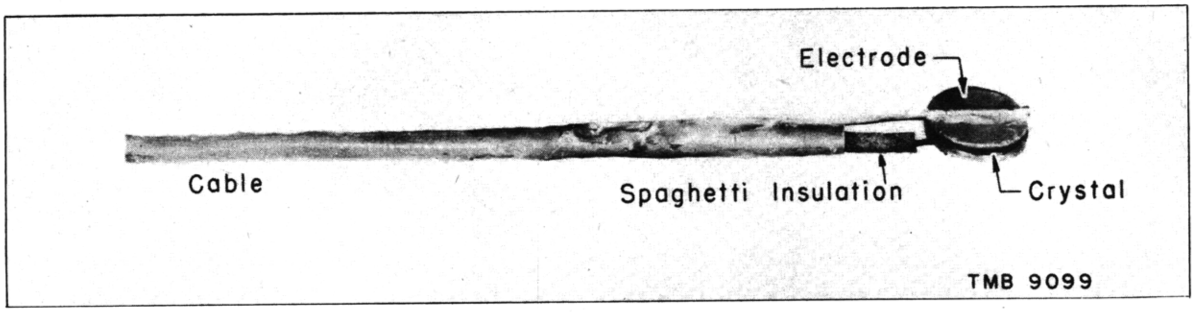

Figure 5a

This is a view of the gage element before the rubber coating is molded around the crystal.

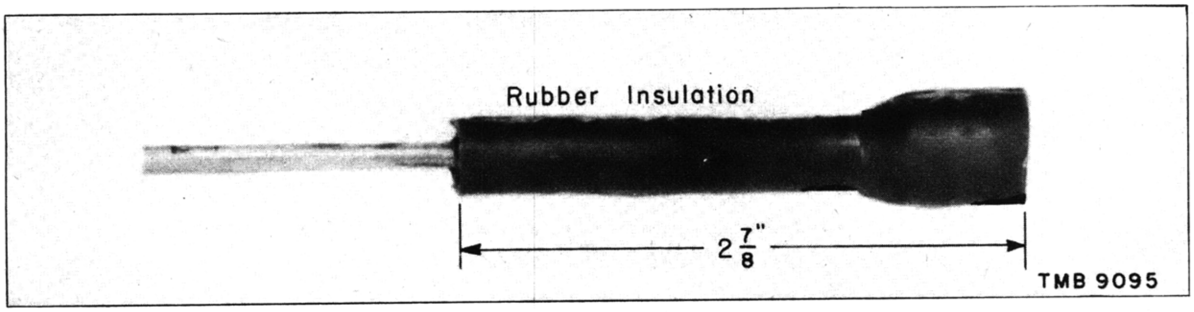

Figure 5b

This is a view of the completed gage.

Figure 5 - TMB Tourmaline Gage for Determining Underwater Explosion Pressures

in the type of insulation and cable employed. The construction of this gage and of a TMB copper cable have been described in detail elsewhere (18); they are shown here in the photographs, Figures 5a and 5b. Tourmaline is sensitive to isotropic pressure and therefore requires no mount. Thus the frequency of the lowest mode of vibration of the tourmaline gage is much greater than that of the quartz gage. The crystals employed have approximately the same area and sensitivity as the quartz crystals. The tourmaline crystals are insulated by molding a sheath of rubber with a low sulphur content about the crystal and that part of the cable to which it is attached.

A modified form of this gage has been used to obtain load-time records at a surface of an underwater structure exposed to explosive loading. The rubber insulation is not only molded about the crystal and the copper cable but also to a brass plate; see Figure 6. This plate is then bolted to the structure. The structure must be heavy enough to be unaffected in its deformation or its motion by the screws or by the weight of the brass plate. Pressure records obtained in this manner are compared with "open-water" records later in this report.

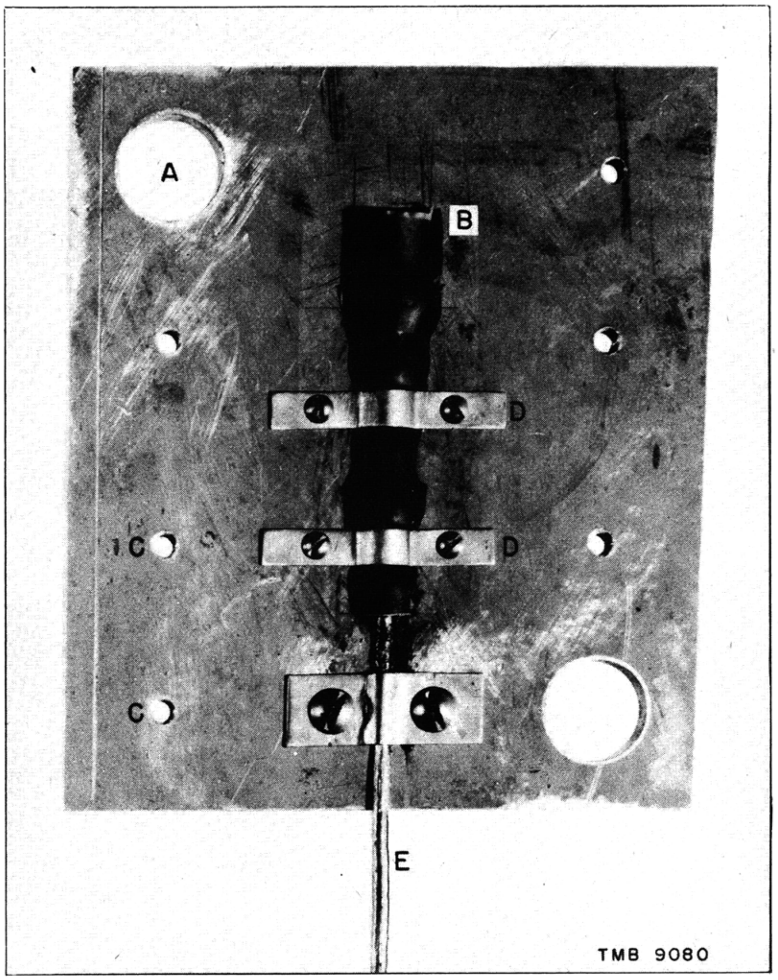

Figure 6 - Adaptation of Tourmaline Gage to Surface Pressure Measurements

A. Hole for alignment of brass plate with mold
B. Rubber insulation surrounding tourmaline crystal
C. Drilled hole to admit bolt for attaching brass plate to structure
D. Clamp to secure gage in cavity of brass plate
E. Wax-filled cable of copper tubing

In use, the entire assembly is bolted to the facing plate of the structure being tested.

PRECAUTIONS IN THE USE OF PIEZOELECTRIC GAGES

Inasmuch as piezoelectric gages are high-impedance sources, certain precautions must be taken in their use. If the cable leading from the pickup to the amplifier input is too long, then it acts as a transmission line with attendant frequency distortion of the signal. This is due to the fact that the high impedance of the gage does not match the low surge impedance of the cable; the latter is generally less than 100 ohms. This will be discussed in greater detail in the section headed "Cables and Recording Channels," pages 12 to 18.

The time constant of the gage is defined as the product of the total capacitance in parallel with the gage and the input resistance of the amplifier into which the gage output is sent. It is necessary that piezoelectric gages have a large time constant compared to the duration of the pressure under study. If the time constant is not sufficiently large, the signal will leak through the input resistance of the amplifier and will acquire a corresponding distortion; see pages 24 to 27, inclusive, for detailed calculations which provide practical criteria for the magnitude of the time constant needed.

Care must be exercised in the selection of a cable for piezoelectric pickup elements. It was found in the Taylor Model Basin tests that most cables, when subjected to an explosion, generate a signal of their own which has a low amplitude and long duration compared to the true signal. If the cable signal is not removed, measurements of momentum in the pressure field are particularly unreliable. A comparison of the signals produced by various cables is given in the section beginning on page 12. The cable finally selected (18) consists of an annealed copper tube with an outside diameter of 1/8 inch and a wall thickness of 1/32 inch. The central conductor is a Number 20 or Number 24 copper wire insulated with enamel and a double layer of glass fiber.* The space between the central conductor and the wall of the copper tube is filled with ceresin or paraffin. The wax prevents vibration of the central conductor inside the copper tube when the pressure pulse from an explosion strikes the cable.

THE TMB RESISTANCE GAGE

It was felt desirable to supplement the piezoelectric gages with a gage of the low-impedance type of approximately 100 ohms. Such a gage is particularly convenient if a very long line is required between the pickup element and the amplifier, or if a signal of long duration is to be studied. Since the impedance of the gage matches the surge impedance of the cable, there is no frequency distortion of the signal by the cable no matter how great its length.

The principle upon which the resistance gage operates is that its ohmic resistance changes when the pickup element is subjected to pressure. This change in resistance affects the current in a ballast circuit. The corresponding variation in voltage across part of the ballast resistance is recorded by the cathode-ray oscillograph. It is evident that insofar as the pickup element is concerned, there is no limitation on the maximum duration

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* This wire is manufactured by the Anaconda Wire and Cable Company under the trade name of DVE wire. "DVE" wire signifies double vitrotex enameled.

of signal that may be studied. The low impedance of the pickup also serves to eliminate cable signal.

An early form of a resistance gage, constructed at the Massachusetts Institute of Technology, consisted of a 250- or 500-ohm resistor* imbedded in a rubber sheath. The pressure changed the average distance between the carbon particles on the glass tube in the resistor, thus producing a change in resistance. This gage was found to be fairly sensitive when connected to a 1500-ohm ballast resistance in series with a 45-volt battery; the voltage sensitivity was then about $5 \times 10^{-5} \dfrac{\text{volts}}{\text{pounds per square inch}}$, of the same order of magnitude as the sensitivity of a piezoelectric pickup.

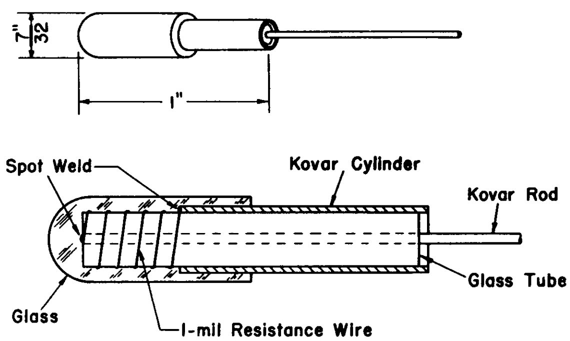

Figure 7 - Glass Resistance-Type Pickup

In the upper diagram the gage is shown in its natural size. The element is made of 705 glass as manufactured by the Corning Glass Company. Kovar is used for the metal parts because it has the same coefficient of expansion as glass over a large range of temperatures. The Kovar rod is slipped into a glass tube, and the tube is put into a Kovar cylinder. 1-mil Advance wire is then wrapped about the glass tube and the two ends of the wire are spot-welded to the Kovar wire and cylinder, respectively. A small glass tube is then slipped over the end of the assembly at which the Advance wire is spot-welded to the rod. The entire unit is heated with an oxygen-hydrogen torch until the glass parts and the Kovar are fused into an integral unit. After the element is annealed it is ready to be assembled on a cable.

However, these resistor gages showed high hysteresis. The calibrations of some of them varied by as much as 20 to 50 per cent after the element had been exposed to a number of explosions.

A gage was then developed at the Taylor Model Basin, which is elastic and which shows little hysteresis in the range of pressures encountered in underwater explosions. This gage consists of a glass element in which approximately 100 ohms of 1-mil Advance wire is imbedded; see Figure 7. The ends of this fine wire are spot-welded to a cylinder and a rod of an alloy which has equal thermal expansion with the glass, known by the trade name of Kovar. The overall dimensions of the glass element are about 1 inch by 7/32 inch. The Kovar cylinder and rod are set coaxially to prevent electrical leakage between them through the water. These elements can be calibrated in a static pressure chamber by measuring their resistance change with a Wheatstone bridge for a given applied pressure. They possess linear calibrations and show little hysteresis.

This resistance gage has the disadvantage of very low sensitivity compared to the piezoelectric gages. When a 100-ohm resistance element in

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* As manufactured by the International Resistance Company.

series with a 100-ohm ballast resistor and four 6-volt storage batteries is subjected to a pressure of 3000 pounds per square inch, the voltage across the element changes by 5 millivolts. Furthermore, the gages are fragile compared to the piezoelectric elements and they seldom withstand more than half a dozen explosions.

When long lines and signals of long duration are encountered, it is desirable to use the resistance element as a check on the piezoelectric gages. The resistance gage may also be useful in determining whether or not cable signal has been eliminated from a piezoelectric gage; this is particularly important in the case of explosions of large charges. Long lines are then used so that the operating personnel and the amplifiers will be at a safe distance. Thus successive parts of the cable are subjected to the pressure wave over a relatively long period of time; i.e., the time required for sound in water to traverse the cable length. If any signal resulted from this pressure on the cable, the tail of the true pressure curve would be obscured. The resistance element should show no cable signal under these circumstances. This is important particularly if the momentum as well as the peak pressure in the pressure pulse is being investigated.

At the time of writing, the resistance element is not sufficiently rugged to be used as a field gage.

The sensitivity of the glass elements to pressure changes varies from 1.4 to $1.8 \times 10^{-7} \frac{1}{\text{pounds per square inch}}$. It is interesting to note that the compressibilities of various types of glass range from about 1 to $2 \times 10^{-7} \frac{1}{\text{pounds per square inch}}$.

PART 3. CABLES AND RECORDING CHANNELS

TYPES OF DISTORTION OF A GAGE SIGNAL

It has been mentioned in Part 2 that a long cable will act as a transmission line. Thus a signal propagated by such a cable will at a given instant possess different phases at different points along the line. This may give rise to distortion of the gage signal. Moreover, there are other causes of signal distortion, such as mechanical disturbance of the cable, insufficiently large time constant of the circuit, and inadequate frequency response of the amplifiers. These will be discussed in turn.

DISTORTIONS DUE TO THE TRANSMISSION LINE

The usual equations (19) describing a transmission line are included here for the reader's convenience.

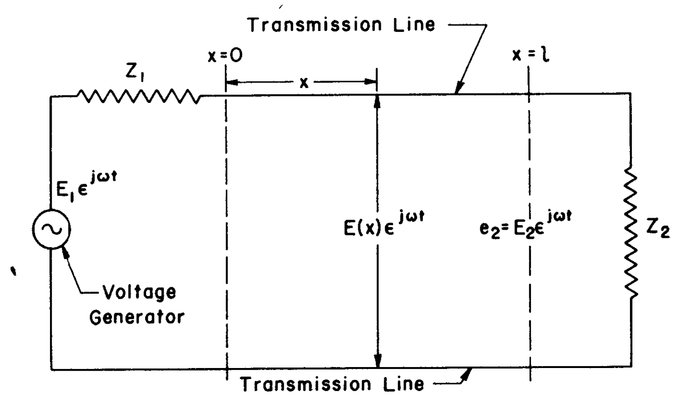

Figure 8 - Schematic Diagram of a Transmission Line Circuit

In Figure 8 $E_1\epsilon^{j\omega t}$ is the voltage produced by a generator,

Z_1 is the impedance of the generator,

x is a distance along the transmission line, varying from 0 to l,

$E(x)\epsilon^{j\omega t}$ is the voltage at x between the pair of conductors which constitute the transmission line,

Z is the combined impedance of the resistances, inductances, and capacitances as used in particular cases,

$e_2 = E_2\epsilon^{j\omega t}$ is $E(l)\epsilon^{j\omega t}$, the output voltage,

Z_2 is the terminating impedance, and

ω is the frequency of the impressed voltage, in radians per second.

These quantities are related by the equation

$$E(x) = \frac{Z_0 E_1\left[\epsilon^{\alpha[l-x]} + r_2\epsilon^{-\alpha[l-x]}\right]}{\left[Z_1 + Z_0\right]\left[\epsilon^{\alpha l} - r_1 r_2\epsilon^{-\alpha l}\right]} \qquad [1]$$

where $Z_0 = \sqrt{\dfrac{R + jL\omega}{G + jC\omega}}$ is the surge impedance of the cable, defined as that impedance which must terminate both ends of a dissipationless line to make it a distortionless line,

R is the resistance per unit length of the line, with both conductors in series,

G is the leakage conductance from one conductor to the other, per unit length,

L is the distributed inductance per unit length,

C is the distributed capacitance per unit length,

$\alpha = \sqrt{(R + jL\omega)(G + jC\omega)}$ is the propagation function of the line,

$$r_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0} \quad \text{and}$$

$$r_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0}$$

It is the quantity $E(l)$ which is equal to E_2 that is important. This represents the output voltage of the cable and therefore the input voltage for the amplifiers.

$$E_2 = \frac{E_1 Z_0 (1 + r_2)}{(Z_1 + Z_0)(\epsilon^{\alpha l} - r_1 r_2 \epsilon^{-\alpha l})} \qquad [2]$$

The assumption will now be made that the line has no dissipation; i.e., $R = G = 0$.* Since C is approximately 40×10^{-12} farad per foot, G is unimportant above 20 or 30 cycles per second. The approximation is not as good for R since this quantity is approximately 0.01 ohm per foot for Number 20 copper wire, while L is approximately 1.2×10^{-7} henry per foot. However, good agreement with experiment is obtained even when R is neglected. This approximation simplifies the equation considerably.

$$\alpha = j\omega\sqrt{LC}\,; \quad Z_0 = \sqrt{\frac{L}{C}} \qquad [3]$$

Z_0 is now a pure resistance.

$$E_2 = \frac{E_1 Z_0 (1 + r_2)}{\left(Z_1 + Z_0\right)\left(\epsilon^{\,j\omega l\sqrt{LC}} - r_1 r_2 \epsilon^{-j\omega l\sqrt{LC}}\right)} \qquad [4]$$

It is to be noted that even for a dissipationless line there will be frequency distortion; i.e., E_2 is still a function of the frequency $\omega/2\pi$. However, if the line were terminated at both ends by impedances equal to the surge impedance, i.e., if $Z_1 = Z_2 = Z_0$, then $r_1 = r_2 = 0$.

$$E_2 = \frac{E_1}{2}\,\epsilon^{-j\omega l\sqrt{LC}} \qquad [5]$$

and

$$e_2 = E_2\,\epsilon^{\,j\omega t} = \frac{E_1}{2}\,\epsilon^{\,j\omega(t - l\sqrt{LC})} \qquad [6]$$

The amplitude is now independent of the frequency, while the phase is altered only by having a constant subtracted from the time. Thus if the dissipationless line is terminated by its surge impedance, it becomes a

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* $G \leqq 10^{-11}$ mho per foot for the cables used.

distortionless transmission system. It may be noted that the phase of the original signal has been altered by subtracting a constant time, $l\sqrt{LC}$. Since $\sqrt{LC} = 1/v$, where v is the phase velocity, $l\sqrt{LC}$ represents a time lag which is independent of the frequency. It is the time required for a sinusoidal component of the signal, as in a Fourier analysis, to traverse the length of the cable. If a nonsinusoidal voltage is impressed on the line, all the harmonic components will be subjected to the same amplitude and phase changes during the transmission. Thus a replica of the impressed voltage will arrive at the receiving end. This addition of the harmonic components is possible since all the differential equations are linear.

The important question that remains is the following. What magnitude of error is introduced by having the terminations Z_1 and Z_2 not equal to Z? The following analysis will give criteria for the length of line and the size of terminating impedance that will keep the distortion below a specified amount.

Substituting the values of r_1, r_2, and $l\sqrt{LC} = lC\sqrt{L/C} = lCZ_0 = C_c Z_0$ into Equation [4], we obtain

$$E_2 = \frac{E_1 Z_0 Z_2}{Z_0(Z_1 + Z_2)\cos \omega Z_0 C_c + (Z_0^2 + Z_1 Z_2)\, j \sin \omega Z_0 C_c} \qquad [7]$$

$C_c = lC$ is the distributed capacitance of the whole cable.

In this problem the pressure pulse gives rise to the generator voltage $E_1 \epsilon^{j\omega t}$ of the piezoelectric gage. The generator has an internal impedance

$$Z_1 = \frac{1}{j\omega C_g}$$

where C_g is the capacitance of the piezoelectric pickup element. If q is the charge generated by the gage then

$$q = E_1 C_g$$

and therefore

$$E_2 = \frac{q Z_0 Z_2}{Z_0\left(C_g Z_2 + \frac{1}{j\omega}\right)\cos \omega Z_0 C_c + \left(Z_0^2 C_g + \frac{Z_2}{j\omega}\right) j \sin \omega Z_0 C_c} \qquad [8]$$

It is customary to terminate the cable with a capacitance to control the output voltage. Therefore let

$$Z_2 = \frac{1}{j\omega C_2}$$

where C_2 is the terminating capacitance. Then Equation [8] becomes

$$E_2 = \frac{q}{(C_g + C_2)\cos\omega Z_0 C_c + \left(1 - \omega^2 C_2 C_g Z_0^2\right)\left(\frac{\sin\omega Z_0 C_c}{\omega Z_0 C_c}\right) C_c} \qquad [9]$$

This result has been obtained by Lampson (20).

The order of magnitude of the various parameters may be compared. C_g is very small, probably less than 40×10^{-12} farad. C_2 usually varies from 10^{-9} farad to 10^{-7} farad. $C_c = lC$ where C is approximately 40×10^{-12} farad per foot and l ranges from 20 feet to 1000 feet. Z_0 is the surge impedance and ranges from 50 to 70 ohms for ordinary microphone cables. The range of frequencies $\omega/2\pi$ of interest in underwater explosion work extends up to several hundred kilocycles, say 3×10^5 cycles per second.

If the length of the cable l is small, then $\sin\omega Z_0 C_c/\omega Z_0 C_c$ and $\cos\omega Z_0 C_c$ may be replaced by unity. In that event

$$E_2 = \frac{q}{C_g + C_2 + C_c(1 - \omega^2 C_g Z_0^2 C_2)} \qquad [10]$$

If the terminating capacitance C_2 is not large, then

$$E_2 = \frac{q}{C_g + C_2 + C_c} \qquad [11]$$

This value for the output voltage of the line is valid for short lines and for terminating capacitances C_2 that are not too large.

A comparison of Equations [11] and [9] shows that the effect of a large value for C_2 or for l is to increase the value of E_2 over that which would be obtained from Equation [11].

The effect of finite lengths of line can be easily estimated. Suppose it is desired that the value of E_2 should vary by no more than 4 per cent for a range of frequencies up to 3×10^5 cycles per second; assume that C_c and C_2 are approximately equal and that C_g is negligible. Then Equation [9] can be rewritten

$$E_2 = \frac{q}{C_2\left(\cos\omega Z_0 C_c + \frac{\sin\omega Z_0 C_c}{\omega Z_0 C_c}\right)} \qquad [9a]$$

For small values of the argument $\omega Z_0 C_c$ we can write

$$\cos\omega Z_0 C_c = 1 - \frac{(\omega Z_0 C_c)^2}{2}$$

$$\frac{\sin\omega Z_0 C_c}{\omega Z_0 C_c} = 1 - \frac{(\omega Z_0 C_c)^2}{6}$$

Thus the prescribed maximum variation of 4 per cent in E_2 will occur if $\cos \omega Z_0 C_c$ is allowed to deviate from unity by 0.03, and $\sin \omega Z_0 C_c / \omega Z_0 C_c$ by 0.01. Then

$$1 - \frac{(\omega Z_0 C_c)^2}{6} = 0.99$$

and therefore

$$\omega Z_0 C_c = 0.245$$

Let $Z_0 = 55$ ohms and $C = 40 \times 10^{-12}$ farad per foot. Then

$$l = \frac{C_c}{C} = \frac{\omega Z_0 C_c}{\omega Z_0 C} = 59 \text{ feet}$$

This is the maximum length of line that may be used under the stated conditions.

Similarly the effect of C_2 in causing distortion may be estimated. Let the deviation of $1 - \omega^2 C_g Z_0^2 C_2$ from unity be 3 per cent for the range of frequencies up to 3×10^5 cycles per second. Let $C_g = 30 \times 10^{-12}$ farad, and $Z_0 = 55$ ohms. Then

$$\omega^2 C_g Z_0^2 C_2 < 0.03$$

Therefore

$$C_2 < 0.11 \times 10^{-6} \text{ farad}$$

A 0.1-microfarad terminating capacitor is thus the largest that could be employed under the given conditions.

This is not a serious limitation on the attenuation of signal voltage. Capacitors larger than 0.1 microfarad may be employed to obtain greater attenuation by using two capacitors in series on the end of the line. If one capacitor is kept below the limit of 0.1 microfarad, the other capacitor may be as large as needed to secure the desired attenuation. The signal would then be taken from the larger capacitor. The combination would be an equivalent capacitor whose capacitance is less than the smaller of the pair and hence less than 0.1 microfarad.

It is more difficult to compensate for the use of a very long line to avoid distortion of the signal. There are two methods of dealing with this problem. One method is to use a preamplifier on the end of a line; for example, in the case just discussed the line was less than 59 feet long. The preamplifier is so designed that its output impedance is low enough to match the surge impedance of the cable. A line of any length can then be used from the output of the preamplifier to the input of the next amplifier. Such a preamplifier has been designed and tested in conjunction with 500 feet of single conductor cable with a surge impedance of 72 ohms. The frequency-response curve of this preamplifier and 500 feet of cable is essentially flat between 100 cycles per second and one megacycle per second.

Another method consists in terminating the cable, at least approximately, in its surge impedance. This method has been described by Lampson (21). Other work has been done at the Taylor Model Basin on this problem (22).

DISTORTIONS DUE TO MECHANICAL DISTURBANCE OF THE CABLE

In Part 2 it was pointed out that the type of cable used with a gage is one of the factors which determine the degree of distortion of the signal obtained. In testing a piezoelectric gage at the Taylor Model Basin it was observed that mechanical disturbance of the cable modified the voltage signal perceptibly. It was evident that in using the gage to study the pressure-time variation near an underwater explosion, a spurious signal produced in the cable could introduce an appreciable error. Preliminary tests confirmed this conjecture. A series of tests was undertaken to compare the voltage signals from various types of shielded, single-conductor cables exposed without other pickup to an underwater explosion. An attempt was then made to improve one of the best of these, a coaxial copper-tube cable, so as to reduce its distortion of a gage signal. Finally, certain relevant properties of the modified cable were studied: The reproducibility of its voltage signal, and the effects of change in its orientation and configuration.

Each cable under investigation was prepared for immersion by insulating the wire at one end, both from the concentric shield and from the water, by rubber tape coated with Bostik cement. The other end was connected to the input of an amplifier. The cable was then taped to a plank and lowered diagonally into the water to a depth of 3 feet, as shown in Figure 9. Both cable and charge were mounted on suitable frames by means of which their depth and relative position could be accurately adjusted.

A Number 8 detonator was exploded, and the voltage output of the cable as a function of time was recorded with a Du Mont Type 208 cathode-ray oscillograph and an auxiliary camera. The time axis on the cathode-ray screen was swept out by a TMB sweep generator, and synchronized with the explosion by a trigger circuit. Voltage and time scales were calibrated with a General Radio Microvolter and an oscillator by impressing a sine wave of known amplitude and frequency on the screen of the cathode-ray tube. The photographic records were enlarged, and peak voltages were read off.

A detailed description of the cables, including the capacitance of each, and a tabulation of the results will be found in Tables 1 and 2, respectively. The peak voltages generated by the different cables range from

TABLE 1

Description of Cables Subjected to Underwater Explosions, and Peak Voltages Produced by These Cables

Serial Letter	Manufacturer's Designation	Central Wire B&S Gage	Insulation	Shield	Sheathing	Nominal Outside Diameter inches	Capacitance per foot μμf	Wall Thickness of Copper Tubes inches	Peak Voltage* millivolts
A	Belden 8421	20	Rubber	Copper Stranding	Rubber	0.260	44		136 ± 12
B	Belden 8401	26	Rubber	Copper - Phosphor-Bronze Stranding	Rubber	0.245	25		108 ± 10
C	Belden 8216	14	Rubber	Copper Stranding	Rubber	0.435	25.5		80 ± 9
D	Precision	20	Fiber-glass	Copper Tube	None	0.100	108	0.007	10 ± 2
E	Cable E	20	Fiber-glass	Copper Tube	None	0.125	57	0.034	13 ± 2
F	Precision	20	Fiber-glass and Celluloid	Copper Tube	None	0.218	107	0.013	40 ± 5
G	Belden 1637	20	Rubber	Copper Tube	None	0.256	161	0.012	21 ± 3
H	Belden 8863	20	Rubber, Cotton	Copper Stranding	None	0.225	33		26 ± 3

* The peak voltage was determined from at least four trials with each cable.

10 to 136 millivolts. Greater signals were produced by the rubber-sheathed cables than by the unsheathed ones, the former ranging from 80 to 136 millivolts, the latter from 10 to 40 millivolts. Typical durations of the signals were 2 or 3 milliseconds.

These large cable signals may give rise to important errors when used in conjunction with a crystal pickup in measurements on the initial part of the pressure pulse in an underwater explosion. Even larger relative errors will result, however, in pressure determinations during the ensuing period of several milliseconds characterized by distinctly lower pressures. The character of the tail of the curve corresponding to this later period may be completely masked by cable distortion. When measuring impulse or momentum the area under the pressure-time curve must be determined, and since the duration of the cable pulse is long compared to that of the true pressure pulse, a tail area of considerable size would make a large spurious contribution, say as large as 20 per cent, to the measured impulse.

The lowest signals, 10 and 13 millivolts, were produced by the cables designated as D and E respectively, see Table 1. The shielding in both consists of copper tubing rather than stranding; they have fiber-glass insulation, their outside diameter is small, 0.100 inch and 0.125 inch respectively, as compared to those of the other copper-tube cables tested, and

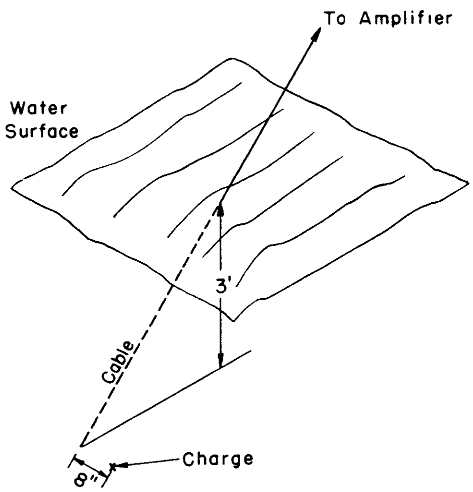


Figure 9a - Sidewise Orientation

The charge is offset from the cable, to produce lateral movement of the latter.

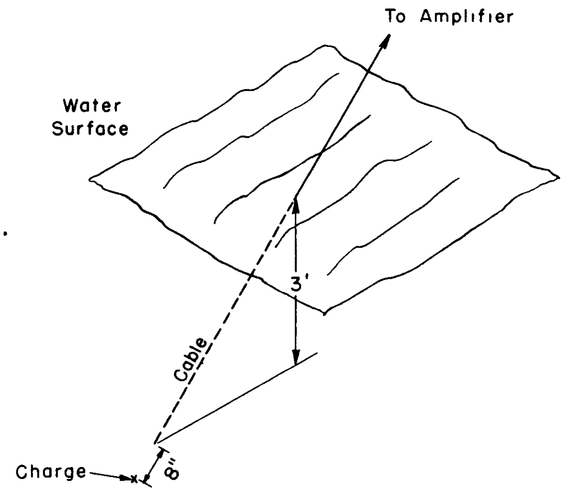


Figure 9b - Endwise Orientation

The charge is in line with the end of the cable.

Figure 9 - Orientations of Cable Relative to Charge in the Early Cable Tests

they are unsheathed. In Cable D, prepared by the Precision Tube Company, the copper tubing is placed over the insulation by a drawing operation. The resulting hardness of the tubing cannot be removed by annealing because this would damage the insulation. In Cable E, prepared at the Taylor Model Basin, Number 24 wire covered with fiber-glass insulation is inserted into tubing with a wall thickness of 0.034 inch. Because the tubing in the latter cable can be annealed *before* the wire is inserted, it has the advantage of flexibility. However, since the central wire is rather loose (this cable was made up without wax) it is capable of motion relative to the concentric tube. Hence the cable signal shows high-frequency oscillations due to the vibration of the wire; see Figure 10b.

To determine how much reduction in the peak voltage of Cable E could be effected by reducing the relative motion of the central wire, a test was made with a charge placed on an extension of the axis of the cable; see Figure 9b. The distance between the immersed end of the cable and the charge was kept fixed. The resulting peak voltage was 2 millivolts, compared to 13 millivolts when the charge was in the position shown in Figure 9a.

It appeared that of all the cables tested Cable E would be the most satisfactory for use in explosion gages, provided that it could be made insensitive to orientation. To accomplish this, it was decided to fill the space between the tube and the wire with wax (18). The modified cable, hereafter called the "TMB Cable," gave satisfactory results, as shown in Table 2,

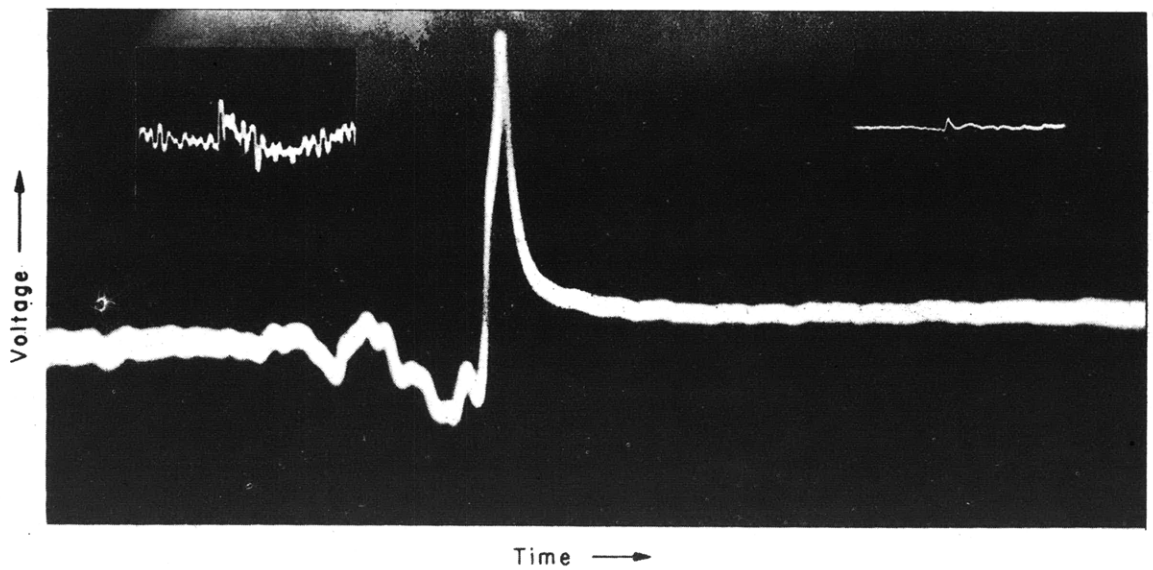


Figure 10a

Figure 10 - Voltage Signals Produced by Three Cables Subjected to the Underwater Explosion of a Detonator

Figure 10a shows a signal from a rubber-sheathed cable; Figure 10b, a signal from a coaxial copper-tube cable; and Figure 10c, a signal from a wax-filled TMB cable.

Figures 10b and 10c were originally taken at larger amplifications than those exhibited here, to permit accurate measurement. They have been reduced to show the three cable signals approximately to the same voltage scale. The apparently sharper focus in Figures 10b and 10c is due to this reduction. The time scales are different. The total time on each oscillogram is the same, 12 milliseconds.

TABLE 2

Comparison of Peak Voltages of TMB Wax-Filled Cable and Unmodified Cable

Cable	Peak Voltage, millivolts	
	Sidewise Orientation Figure 9a	Endwise Orientation Figure 9b
E	12.9	2.0
TMB	3.4	2.6

which gives the peak voltages obtained with the original and the modified cables for the positions shown in Figure 9. The capacitance of the TMB cable was 50 micromicrofarads per foot.

Figure 10 is a comparison of voltage-signal records from a rubber cable, from the unmodified copper-tube cable, and from the TMB cable. It should be noted that although the time scales are different in the three records, the voltage scales are the same.

The TMB cable possesses the following advantages:

a. Compared to the unmodified Cable E, it is relatively insensitive to orientation.

b. For sidewise orientation, Figure 9a, it gives a peak signal less than 1/3 as large as does the unmodified copper cable, and about 1/30 of that produced by commercial rubber-sheathed cables.

c. High-frequency voltage oscillations due to vibration of the wire in the unwaxed copper-tube cable are eliminated; compare Figures 10b and 10c.

d. It is very flexible, since the copper tubing remains in the annealed condition after the central wire is inserted.

e. The use of copper tubing as a shield makes it easy to lead the gage cable into a pressure chamber for calibration. With rubber cable it is difficult to seal the gage inside the pressure chamber so that it will hold high pressure without damaging the rubber insulation.

f. It has a relatively low dielectric absorption. Consequently, its capacitance shows a negligible change with frequency, as compared with variations as high as 15 per cent observed for rubber-sheathed cables between 150 cycles per second and 30,000 cycles per second.

Because of these characteristics the TMB cable appears to be better suited for use with underwater explosion-pressure gages than any of the others tested. It has been adopted for all such gages recently constructed at the Taylor Model Basin.

In addition, two other features of the cable were investigated; the reproducibility of the voltage signal and the effect of changing the configuration of the cable in the pressure field. In a series of five detonations under nominally identical conditions, the peak voltage signal from the cable alone was found to be reproducible to within a probable deviation of 4 per cent. To test the effect of departure from a linear configuration, the cable was bent into a circle with the charge at its center. There was little change in the resulting cable signal.

Further tests of the relative merits of the TMB cable and a rubber-sheathed cable were subsequently made at the Underwater Explosives Research

Laboratory at Woods Hole, in connection with the use of larger charges. These experiments differed from the earlier ones mainly in two respects: charges of 250 grams of tetryl were exploded instead of detonators, and each of the cables tested was part of a complete piezoelectric gage assembly. The crystal signal was delayed so as to separate it from the initial part of the cable signal. This was done by bending the cable into the shape of an isosceles triangle with the crystal at its vertex 30 inches from the base; see Figure 11. The charge was placed at the same distance on the other side of the base. Hence the pressure pulse struck the cable about half a millisecond before reaching the crystal.

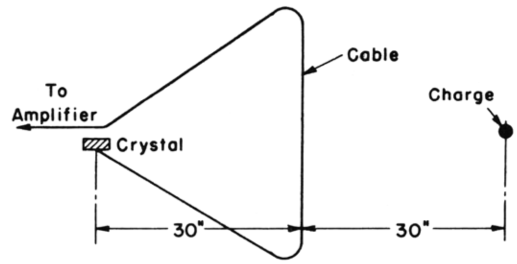


Figure 11 - Positions of Cable and Crystal Relative to Charge in the Cable

Cable signals obtained simultaneously from a Belden Series 8400 rubber-sheathed cable and from a TMB cable are shown in Figures 12 and 13. In these pictures the time sweep proceeds from left to right; thus the first disturbance at the left is that due to the cable. The steep signal which follows is produced by the crystal,* and the peculiar appearance of the subsequent portion of the curve was probably caused by the output of the crystal overloading the amplifier. Pairs of such records from ten explosions were obtained. These show a steady change in the condition of the rubber cable as

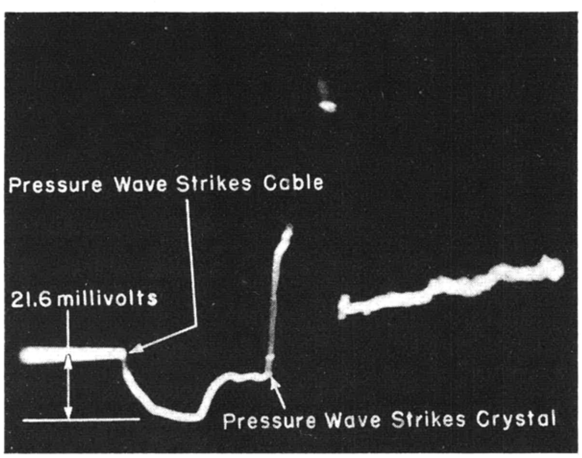


Figure 12 - Voltage Signal Produced by a Rubber Cable Subjected to an Underwater Explosion of 250 Grams of Tetryl

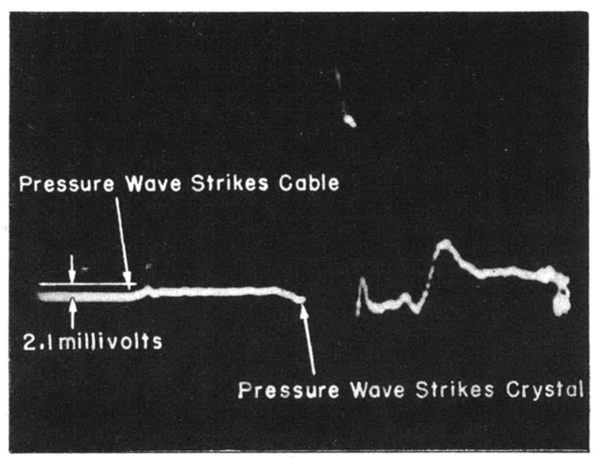


Figure 13 - Voltage Signal Produced by a TMB Cable Subjected to an Underwater Explosion of 250 Grams of Tetryl

* The amplifier gain required to produce a sizeable cable signal was much too high for the crystal signal.

it is subjected to additional explosions. The peak voltage in the cable signal decreased from 22 millivolts in the first explosion to 6 millivolts in the tenth, while that in the TMB cable fluctuated between 1 and 2 millivolts. This effect of "aging" on the rubber cable, due to repeated explosions, had been observed previously at the Underwater Explosives Research Laboratory. The investigators at that laboratory report that the change is reversible; if an "aged" cable is put aside for a week or two it reverts to its former state and again gives rise to large signals. Other rubber-sheathed cables showed the same features in even greater degree.

When large numbers of gages with copper cables were used simultaneously on a ship, the noise level became objectionably high, probably as a result of eddy currents through the copper tube which serves as a return lead. One way to overcome this difficulty may be to replace the present combination of single conductor and grounded copper tube with a double conductor enclosed in a copper tube, the latter acting only as a shield.

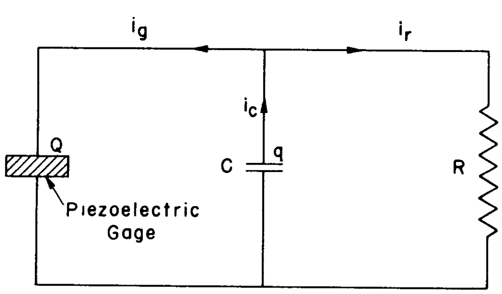
DISTORTION DUE TO INSUFFICIENT TIME CONSTANT

Another type of distortion of the gage signal occurs when the time constant of the circuit is too small compared to the duration of the signal. The following analysis provides practical criteria for the magnitude of the time constants needed to reduce this kind of distortion to a given percentage.

A piezoelectric gage circuit may be represented as in Figure 14.

To minimize distortion caused by leakage of the signal charge through the input resistance of the amplifier, the time constant of the circuit expressed by the product RC must be large compared to the duration of the signal.

The piezoelectric gage generates a charge which is proportional to the applied force. Since the peak pressure due to an underwater explosion occurs almost instantaneously, the peak charge is generated at the very start. As the pressure decreases, the crystal reabsorbs as much charge as it previously generated; this reverses the original current.



$Q(t)$ is the charge produced by the crystal; i.e., the true gage response which it is desired to measure,

$q(t)$ is the observed charge on the capacitor,

C is the combined distributed capacitance of the cable plus the terminating lumped capacitance,

R is the input resistance of the amplifier,

RC is the time constant of the circuit, and

i_g, i_r, and i_c are the instantaneous currents in the indicated branches of the circuit.

Figure 14 - Simplified Equivalent Circuit of a Piezoelectric Gage

Thus at first the charge is produced and thrown on the capacitor plates. Then as the pressure diminishes the capacitor discharges into the crystal and through the resistance R. It is apparent that if the resistance were infinite the voltage on the capacitor would follow the original pressure faithfully. However, the presence of a finite resistance causes a leakage through that as well as back into the crystal. Thus the voltage across the capacitor is always less than it would otherwise be. In fact, even if the original pressure were always positive, it would be possible, because of the leakage through R, for the capacitor to reverse its charge and indicate an apparent negative force. The distortion of the signal from a piezoelectric gage due to leakage is shown in Figure 15, in which V_0 is the observed voltage and V_t is the voltage to be expected for zero leakage.

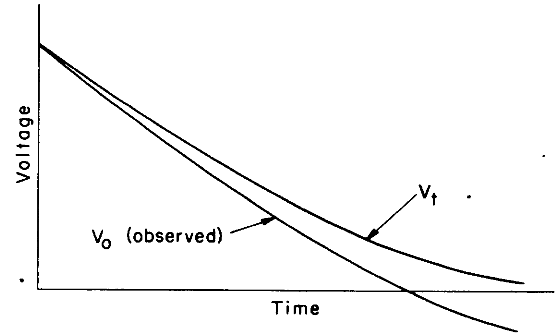


Figure 15 - Distortion of a Piezoelectric Gage Signal Due to RC Leakage

V_t is the value which would be obtained if the leakage were zero.

It is apparent that the relative error $(V_t - V_0)/V_t$ increases as time progresses. Thus if it is desired to study the late phases of the record, it is necessary to make the product RC quite large. This requirement is more severe than if the peak value alone were being investigated. An analysis of the distortion is given below.

Let $i_c = -dq/dt$ be the total current flowing out of the capacitor, $i_g = -dQ/dt$ be the current flowing into the crystal, and $i_r = q/RC$ be the current flowing through the resistance R. Then

$$i_c = i_g + i_r \qquad [12]$$

or

$$-\frac{dq}{dt} = \frac{q}{RC} - \frac{dQ}{dt}$$

and

$$\frac{dq}{dt} + \frac{q}{RC} = \frac{dQ}{dt}$$

$$q(t) = e^{-\int d\tau/RC}\left[\int \frac{dQ}{d\tau}\, e^{\int d\tau/RC}\, d\tau + c\right]$$

$$= e^{-t/RC}\left[Q(t)\, e^{t/RC} - \frac{1}{RC}\int_0^t Q(\tau)\, e^{\tau/RC}\, d\tau + C\right]$$

The initial conditions are that $q(0) = Q(0) = Q_0$, the initial charge produced. Therefore

$$q(t) = Q(t) - \frac{1}{RC} e^{-t/RC} \int_0^t Q(\tau) e^{\tau/RC} d\tau$$

where τ is the variable of integration. The relative error is

$$\frac{Q(t) - q(t)}{Q(t)} = \frac{\Delta Q}{Q} = \frac{1}{RC} \frac{\int_0^t Q(\tau) \epsilon^{\tau/RC} d\tau}{Q(t) e^{t/RC}} \qquad [13]$$

Thus far the expression for the charge produced by the crystal has been an arbitrary function. Now two special forms for $Q(t)$ will be considered.

$$\text{Case 1} \quad Q(t) = Q_0 \epsilon^{-t/t_0} \qquad [14]$$

$$\text{Case 2} \quad Q(t) = Q_0\left(1 - \frac{t}{t_0}\right) \qquad [15]$$

where $t_0 = RC$ is the time constant of the discharge.

For what time ought the relative error to be calculated? It is seen that the error increases with time. As a practical choice the comparison may be made for the time at which

$$\frac{t}{t_0} = 1 \text{ in Case 1}$$

and

$$\frac{t}{t_0} = \frac{2}{3} \text{ in Case 2}$$

In both cases $Q(t)$ is reduced to approximately 1/3 of its original value.

Table 3 is based on Equation [13]. It gives the relative error $\Delta Q/Q$ as a function of RC/t_0.

The conclusion to be drawn in each of the two cases is substantially the same. If the error is not to exceed 10 per cent, the time constant should be 10 to 15 times as

TABLE 3

Relative Error $\frac{\Delta Q}{Q}$ as a Function of $\frac{RC}{t_0}$ for Case 1 and Case 2

Case 1		Case 2	
$\frac{RC}{t_0}$	$\frac{\Delta Q}{Q}$	$\frac{RC}{t_0}$	$\frac{\Delta Q}{Q}$
1	1.00	1	0.92
2	0.65	2	0.55
3	0.47	3	0.39
4	0.37	4	0.31
5	0.31	5	0.25
6	0.26	8	0.13
7	0.23	10	0.11
10	0.16	16	0.07
15	0.11	.	.
20	0.084	.	.
.	.	.	.
.	.	.	.
.	.	.	.
∞	0	∞	0

large as the period of time which is of interest. Thus if the time of interest is 20 milliseconds, the time constant must exceed 250 milliseconds.

DISTORTIONS DUE TO FREQUENCY RESPONSE OF AMPLIFIERS

Thus far distortions associated with the cable and those due to inadequate time constant have been discussed. Another kind of signal distortion arises from inadequate frequency response of the amplifiers. There are several ways of describing this frequency response. The following are three methods in common use.

Let $E_1 \sin\omega t$ represent a standard sinusoidal signal going to the input of the amplifier. Then the output will have the form

$$E \sin(\omega t - \theta)$$

E is the amplitude of the output. If it is a function of the frequency ω, i.e., $E = E(\omega)$, then the amplifier has amplitude distortion. θ is the change in phase introduced by the amplifier. If θ is either a constant or a linear function of the frequency, i.e., $\theta = \theta(\omega) = \alpha\omega + \beta$, where α and β are constant, then there is no phase distortion. If θ is a nonlinear function of ω, then the amplifier has introduced phase distortion. In the case of the properly terminated cable treated in an earlier section, see Equation [5],

$E(\omega) = \frac{E_1}{2}$, which is independent of signal frequency ω;

$\theta(\omega) = jl\sqrt{LC}\,\omega$, which is a linear function of ω.

Thus the properly terminated cable is an example of a distortionless system. When $E(\omega)$ and $\theta(\omega)$ are determined experimentally, the frequency response is completely known.

A second method involves obtaining the response of the amplifier to a periodic square wave.

A third method is to obtain the response of the amplifier to a unit pulse; see Figure 16.

These modes of description are essentially equivalent, and any one may be derived from any other (23). Each may be useful for some specific purpose. Thus, for example, if it is desired to learn the effect of some parameter, such as the value of a resistance

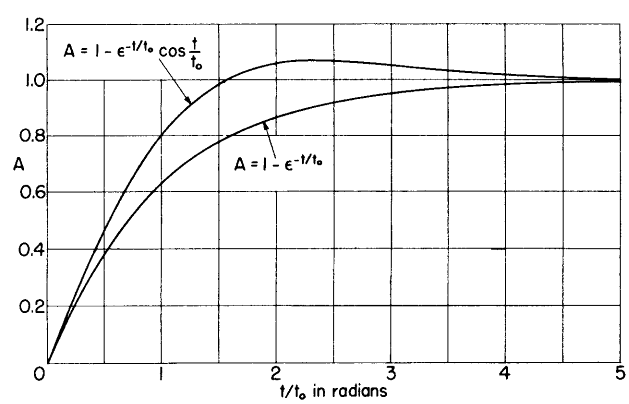


Figure 16 - Types of Response of Amplifiers to a Unit Pulse

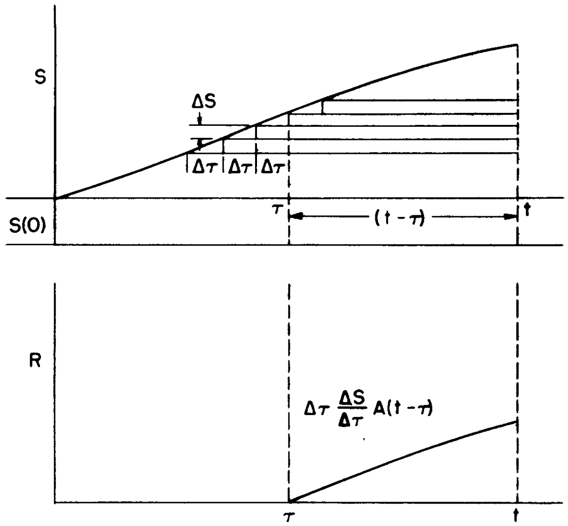


Figure 17 - Illustration of the Duhamel Integral

or capacitance, on the frequency response, then the response to a square wave may be "stopped" on the screen of a cathode-ray oscillograph, and the change in the response can be observed visually and continuously as the resistance, or capacitance, is altered. Thus the square-wave method is valuable to anyone building an amplifier. Once the amplifier is built it may be necessary to determine the response to a given input transient signal. It is difficult to derive mathematically the response to a transient signal in terms of a known response to a square wave (23). This question may, however, be answered very conveniently if the response to a unit pulse is known, and for studying distortion, the square wave of adequate duration is sufficiently equivalent to it. The following analysis (24) shows how the response of any amplifier to a given signal may be calculated provided the response to a unit pulse is known. Let

$$E(t) \text{ be } 1 \text{ for } t \geqq 0, \quad \text{be } 0 \text{ for } t < 0 \quad \Big\} \text{ the unit pulse,}$$

$A(t)$ be the response to the unit pulse, usually called the indicial admittance,

$S(t)$ be the input or excitation signal, and

$R(t)$ be the response of the amplifier to $S(t)$.

It will be assumed that the amplifier is a linear system, i.e., it is describable by linear differential equations. In that case the principle of superposition may be used. It is also necessary to assume that the coefficients of the differential equations of the system (amplifier) are constants. This means that the response is only a function of $(t - \tau)$ where τ is the time at which the unit pulse is applied, and t is the time at which the response is desired.

Consider the input signal S as represented in Figure 17. The signal is to be considered as the sum of many rectangular pulses. Each pulse starts at a time $\Delta\tau$ after the previous one. The height of the pulse is given by

$$\Delta S = \frac{\Delta S}{\Delta \tau} \Delta \tau$$

Thus a pulse of height ΔS is applied at the time τ, while at a time $\Delta \tau$ later another pulse is applied, and so on. It is desired to find the effect of all these pulses at the definite time t. The response of any one of them is pictured in the lower half of Figure 17, and is given by

$$\frac{\Delta S}{\Delta \tau} \Delta \tau \cdot A(t - \tau)$$

There is also the contribution of the pulse whose height is $S(0)$. The corresponding response is given by the product

$$S(0) \cdot A(t)$$

The total response is obtained by adding all the steps. This yields

$$R(t) = S(0)\, A(t) + \sum \frac{\Delta S}{\Delta \tau} A(t - \tau)\, \Delta \tau$$

In the limit as $\Delta \tau \to 0$, the summation is replaced by the following expression, known as the Duhamel integral

$$R(t) = S(0)\, A(t) + \int_0^t \frac{dS(\tau)}{d\tau} A(t - \tau)\, d\tau$$

This formula gives the response for a given input signal $S(t)$ provided the response to a unit pulse, i.e., $A(t)$, is known; see Figure 16 and the upper half of Figure 17.

A specific case will be worked out in detail to illustrate the method.

Assume for instance that the response to a unit pulse has been obtained experimentally and is representable by the empirical equation

$$A(t) = 1 - \epsilon^{-t/t_0}$$

t_0 is a measure of "the time of rise" of the amplifier.* Let the input signal be that due to an underwater explosion. This may be represented fairly well by the form**

$$S(t) = \epsilon^{-t/T}$$

where T is the duration of the signal. Thus

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* The response of actual amplifiers frequently approximates this form.

** For simplicity, $S(0)$ is chosen as unity, and $S(t)$ and $R(t)$ are dimensionless.

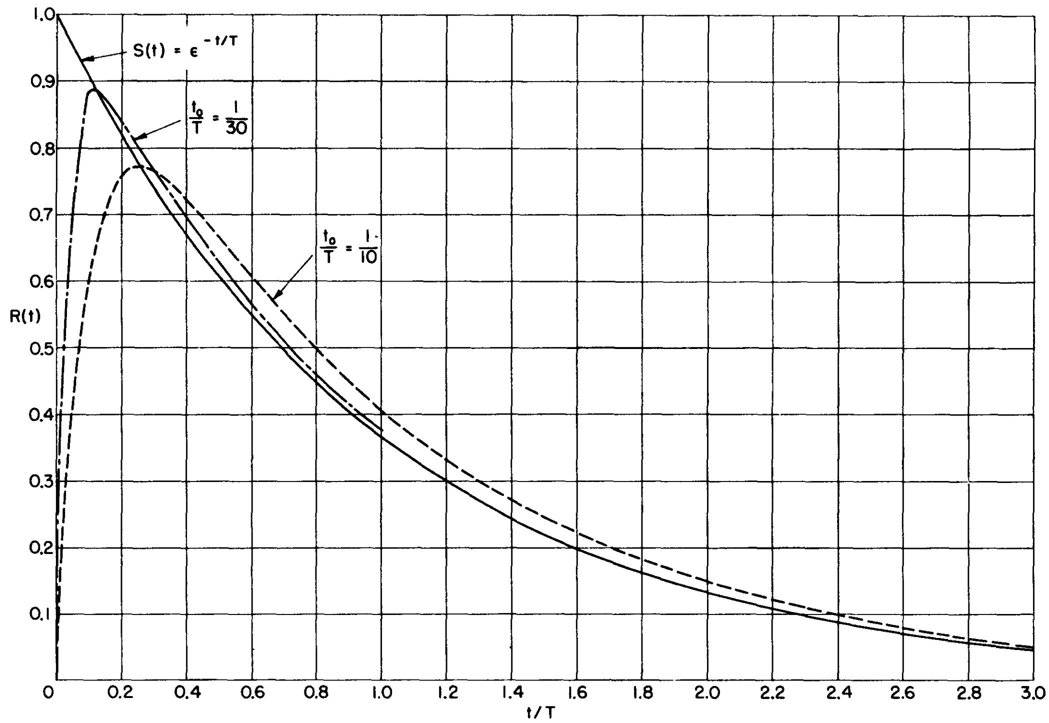

Figure 18 - Response of an Amplifier to an Input Signal of the Form $\epsilon^{-t/T}$

The amplifier in question is one which shows the response to a unit pulse depicted by the lower curve in Figure 16. The response is expressed by Equation [17] and is plotted for two values of t_0/T.

$$R(t) = 1 - \epsilon^{-t/t_0} - \int_0^t \frac{1}{T}\, \epsilon^{-\tau/T}\left(1 - \epsilon^{-(t-\tau)/t_0}\right) d\tau \qquad [16]$$

$$R(t) = \frac{T}{T - t_0}\left(\epsilon^{-t/T} - \epsilon^{-t/t_0}\right) \qquad [17]$$

See Reference (5).

The original signal $S(t)$ and response $R(t)$ are plotted for the indicated values of t_0/T in Figure 18. It is apparent that the response approaches the input signal more closely as t_0/T approaches zero. There is a considerable loss of peak signal even for small values of t_0/T; for example, a loss of 7.5 per cent for $t_0/T = 1/50$.

It is useful to obtain the peak response as a function of t_0/T. The t_0/T necessary for a given allowable loss of peak response for the assumed type of signal can then be specified. Differentiating Equation [17] yields

$$R'(t) = \frac{T}{T - t_0}\left[-\frac{1}{T}\,\epsilon^{-t/T} + \frac{1}{t_0}\,\epsilon^{-t/t_0}\right]$$

Setting this expression equal to zero, we obtain

$$\frac{1}{T}\,\epsilon^{-t_m/T} = \frac{1}{t_0}\,\epsilon^{-t_m/t_o}$$

where t_m is the time at which the maximum response occurs. The maximum response R_m at this time is given by

$$R_m = \frac{T}{T - t_0}\left(\epsilon^{-t_m/T} - \epsilon^{-t_m/t_o}\right) \qquad [18]$$

Let $X = t_0/T$. Then

$$R_m = \frac{1}{1 - X}\left(\epsilon^{-t_m/T} - \epsilon^{-t_m/t_o}\right)$$

but, from the preceding

$$\epsilon^{-t_m} = X^{t_o/(1-x)}$$

Eliminating t_m, we obtain

$$R_m = \frac{1}{1 - X}\left(X^{x/(1-x)} - X^{1/(1-x)}\right)$$

and

$$R_m = X^{x/(1-x)} \qquad [19]$$

This is the desired function giving the peak response to be expected from a given value of $X = t_0/T$. For values of X less than 1/20 we may use

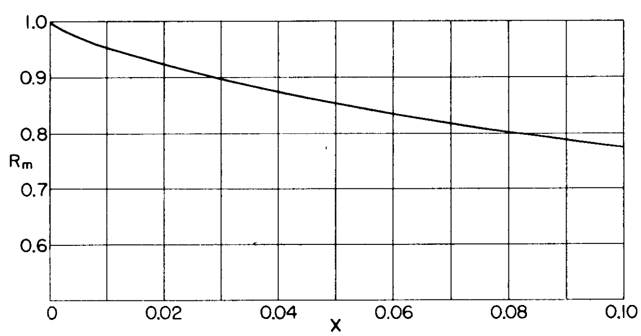

Figure 19 - Peak Response as a Function of $X = t_0/T$

The maxima of the family of curves in Figure 18 are plotted against the parameter t_0/T. This graph is a plot of Equation [19].

$R_m = X^X$ with an error which is less than 1 per cent. A plot of Equation [19] is given in Figure 19.

As a numerical example of the use of this result, suppose the amplifier is required to record 97 per cent of the peak pressure due to the underwater explosion of an ounce of tetryl. What time of rise should the amplifier have? The duration of the pressure from an ounce of tetryl is approximately 35 microseconds. Thus

$$T = 35$$

$$R_m = 0.97 = X^X$$

Then

$$X = \frac{1}{170} = \frac{t_0}{T}$$

Therefore

$$t_0 = \frac{35}{170} = 0.2 \text{ microsecond}$$

This imposes a severe requirement upon the amplifier, as can be seen from the following example. Suppose that the frequency response of an available amplifier falls off by 8 per cent at 300 kilocycles. It then has a time of rise equal to 3/4 microsecond, a time too large to give the desired accuracy for an explosion of an ounce of tetryl.

It may be asked, what is the smallest charge for which the cited amplifier will record 97 per cent of the peak pressure? The duration of the pressure varies as the cube root of the mass of the charge. Therefore, the desired value of weight W of the charge may be obtained if the required duration is known. In this case

$$R_m = 0.97$$

$$t_0 = 0.75$$

$$X = \frac{1}{170} = \frac{t_0}{T}$$

Therefore

$$T = 0.75 \times 170 = 127 \text{ microseconds}$$

Comparing this time with the duration of an explosion of one ounce of tetryl, we have

$$\frac{W}{1} = \left(\frac{127}{35}\right)^3 = 47.8 \text{ ounces of tetryl}$$

Therefore the required charge is 3 pounds or more of tetryl.

If valid measurements are to be made it is necessary to take such precautions as working with relatively large charges or using very high fidelity amplifiers and cables which are not too long.

THE TMB RECORDING CHANNEL

The Taylor Model Basin recording channel consists of a series of units as indicated by the block diagram in Figure 20. The amplifier was designed by members of the staff and responds to frequencies up to 300

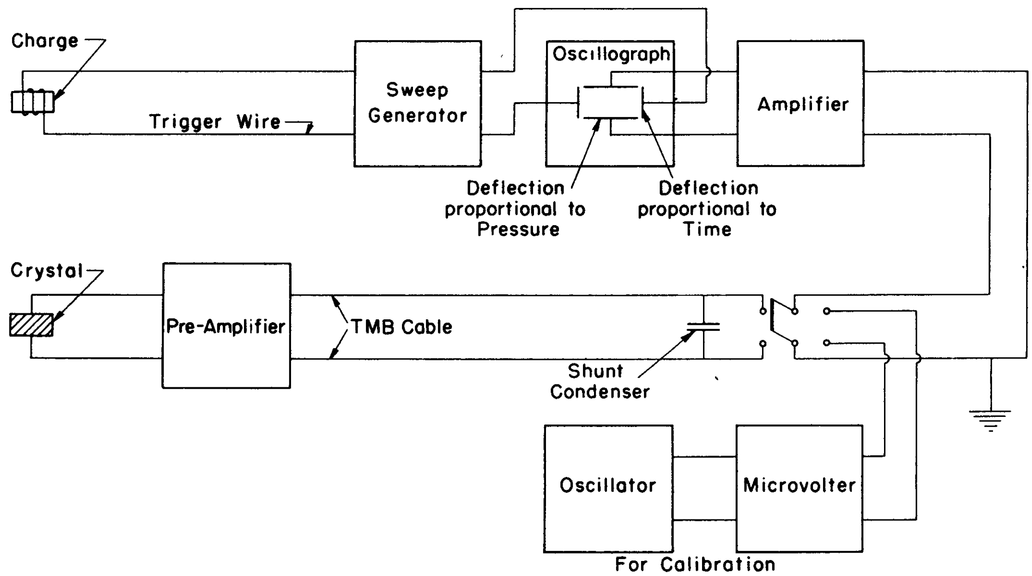

Figure 20 - Schematic Diagram of Channel used in Underwater Explosion Tests at the Taylor Model Basin

kilocycles. Its output goes directly to the vertical deflecting plates of a Du Mont Type 208 cathode-ray oscillograph. A high-vacuum type of sweep generator is used, which produces either single or continuous sweeps. A Hewlett-Packard oscillator and General Radio Microvolter are employed for time and voltage calibrations. A 35-mm Eastman Ektra camera using an f 1.9 lens records calibrations and explosion pressures. The sweep is initiated when a fine wire wrapped around the charge is broken by the explosion.

PART 4. CALIBRATION OF THE PRESSURE GAGES

The usefulness of a pressure gage for underwater explosions is manifestly limited by the accuracy and reliability of its calibration. This involves a precise determination of the electrical output of the gage when it is subjected to a known change in pressure. In this section the apparatus employed in applying a pressure change to the gage will be described first, then the circuit used in measuring its output by each of several different methods. Finally, the results obtained from the various calibrations will be compared with one another, with values of piezoelectric constants found in the literature, and with values obtained for the same crystals by two other laboratories.*

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* The Underwater Explosives Research Laboratory at the Oceanographic Institution, Woods Hole, Massachusetts, and the Exploration Laboratory of the Stanolind Oil and Gas Company, Tulsa, Oklahoma.

PRESSURE APPARATUS

Calibration pressures are developed inside a cylindrical steel pressure chamber* filled with a rust-inhibiting solution of potassium dichromate and sodium carbonate. Oil is not used, since upon repeated immersion in oil at high pressures the rubber insulation on the tourmaline gages would be damaged. The crystal end of the gage is sealed into the pressure vessel by neoprene washers sandwiched between split brass washers. The latter fit snugly around the copper-tube cable, and the neoprene washers are compressed by a retaining screw through which the cable emerges from the chamber. The pressure is raised to some predetermined value which is read on a Bourdon-tube gage. Three such gages with ranges up to 2000, 5000, and 10,000 pounds per square inch have been used. These have been calibrated twice at the National Bureau of Standards. They can be read to within 1/4 to 1/2 of one per cent at the usual test pressures.

In the calibration of the piezoelectric gages, a rapid change in pressure is required. Therefore, once the desired pressure has been attained, it is released by one of the following two methods. A release valve is suddenly opened, permitting a return to atmospheric pressure in approximately 0.05 second; or a thin brass diaphragm, sealed into the chamber on the opposite side from the gage, is punctured, releasing the pressure in 0.2 to 0.3 millisecond. The diaphragm must be thick enough to withstand a given pressure, but thin enough to be readily punctured with, say, a thrust of a screwdriver. In practice, the diaphragms are made of brass, 1.12 inch in diameter and 0.003 inch to 0.008 inch thick, depending upon the pressure to which they are to be subjected. It requires but a few minutes to remove a punctured diaphragm and replace it with a new one.

Unlike the transient character of a piezoelectric gage calibration, that of a glass resistance gage is static; i.e., the pressure remains at a predetermined value while the change in the gage resistance is measured.

CIRCUITS FOR MEASURING THE OUTPUT OF THE GAGES

For a piezoelectric gage the output to be measured is a quantity of charge, whereas for a resistance-type gage the output is a change in resistance. Accordingly, different circuits must be used in calibrating the two types.

CALIBRATION OF THE PIEZOELECTRIC GAGES

The charge ΔQ developed by a piezoelectric crystal under load is proportional to the change ΔF in the total force acting on the electrode

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* This apparatus was designed by Lt. Dennison Bancroft, USNR, of the David Taylor Model Basin staff.

surfaces, i.e., $\Delta Q = K\Delta F$, where K is the piezoelectric constant of the crystalline material along the electric axis. When a piezoelectric gage is subjected to hydrostatic pressure, it is convenient to write $\Delta Q = KA\,\Delta P$. The change in pressure ΔP acting over area A determines the total force transmitted to the electrode surface. From the constant KA of a particular gage, the change in pressure responsible for a given output of charge is inferred.

Since values of K are given in the literature, it may be inquired why calibration is necessary; why not simply multiply K by A? There are several reasons. The piezo constant is not the same for various specimens of tourmaline. It is difficult to know which of the values found in the literature is a correct value for the sample used in the gage.

In the case of the quartz gage there are two sources of possible error in computing KA rather than in determining it experimentally. In the first place, few quartz crystals are altogether free from "twinning" of right-handed and left-handed quartz. Twinning reduces the effective K of the crystal, since the two types of quartz show opposite polarities. Secondly, A is not known precisely. As pointed out in the Appendix, it is necessary that an air gap surround the quartz crystal laterally. The brass cover which transmits the external pressure to the crystal is supported partly by the crystal and partly by the cylindrical wall of the housing; see Figure 3 on page 6. Moreover, a portion of the brass cover lies over the air gap. Thus the effective area A subjected to pressure differs from the electrode surface of the crystal alone. Though it is possible to estimate approximately how much of the area over the air gap is supported by the crystal, it is not easy to compute this exactly. However, if the gage is calibrated, it is not necessary to know A, since the product KA is obtained directly.

For these reasons it is desirable to determine experimentally the constant KA of each gage. Now for any particular method of calibration, results of high internal consistency are not incompatible with the occurrence of a systematic error. To ensure reliable KA values, therefore, it is important to compare the results obtained by a given method of calibration with those obtained by another method, and, if possible, by an independent observer. At the Taylor Model Basin, in addition to the alternative methods of applying a pressure change which have already been described, experiments have been made with several different circuits for measuring the charge. One of these has proved especially successful, yielding results which not only show internal consistency but which agree closely with those obtained for the same crystals in other laboratories. Experiments have also been performed by two other methods of calibration. One of these used a Compton quadrant electrometer; the other utilized a "microcoulometer" or high-impedance current amplifier and a moving-coil galvanometer. These latter two methods have the disadvantage of requiring exceedingly high gage impedance, at least 30,000 megohms, for successful operation. The oscillographic method, on the other hand, requires an impedance of only about 10 megohms. The various calibration techniques will now be described.

Oscillographic Recording

In this method of calibration the pressure is released by a bursting diaphragm, and the output is recorded on the screen of a cathode-ray tube. The gage is shunted by a mica capacitor and connected into the amplifier of a Du Mont Type 208 cathode-ray oscillograph. A horizontal time sweep is provided by a TMB sweep generator and synchronized with the pressure release by a simple trigger circuit. The output of the crystal as it is subjected to a pressure change appears as a voltage-time curve on the screen of the cathode-ray tube; see Figure 21. It is photographed with an Ektra camera, and the film records are measured with a micrometer microscope. The time required to release the pressure by puncturing a diaphragm, about 0.2 to 0.3 millisecond, makes the process especially amenable to oscillographic recording. The event is fast enough so that the required *RC* is not inconveniently large; on the other hand, it is slow enough so that the demands of flat frequency response on the amplifier and adequate beam intensity on the cathode-ray tube are not excessive. The slow descent of the portion of the curve to the right of the peak gives an indication of the large *RC*, 40 milliseconds, which was used. This time constant is so large compared to the time required for the release of the pressure that the leakage error in the measurement of peak pressure, as in Figure 15 on page 25, is less than 1 per cent. For time calibration a sine wave of known frequency is fed into the amplifier from a Hewlett-Packard oscillator. The amplitude is calibrated by putting a step voltage from a potentiometer into the amplifier of the Du Mont oscillograph.

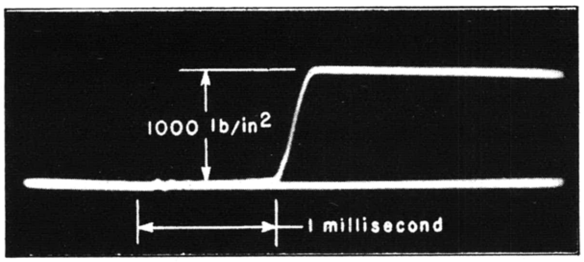

Figure 21 - Calibration Record Obtained with the Direct Oscillographic Method

The pressure inside a calibration chamber is released by puncturing a thin brass diaphragm. The time scale runs from left to right.

Table 4 gives the results of a typical set of calibrations on five tourmaline crystals. Each *KA* value cited in Column 3 is the result of the number of determinations or "trials" indicated in Column 2. Thus the *KA* value

TABLE 4

Piezoelectric Calibration Constants of Typical Tourmaline Gages

1	2	3	4	5	6	7	8
Gage	Number of Trials	$KA \times 10^{13}$ coulombs per pounds per square inch		Per Cent Difference in *KA*	Area square inches	$K \times 10^{11}$ coulombs per pound	
		TMB	Stanolind			TMB	Stanolind
275	36	9.16 ± 0.08	9.21	0.5	0.0865	1.058	1.064
282	30	9.02 ± 0.06	8.87	1.7	0.084	1.074	1.058
284	17	9.10 ± 0.11	9.02	0.9	0.085	1.070	1.061
285	28	7.85 ± 0.18	7.89	0.5	0.074	1.061	1.066
XT-503	41	11.82 ± 0.11	12.06	2.0	0.112	1.053	1.074

for Gage 275 is based on the measurement of 36 oscillograms like the one in Figure 21. The deviation given in Column 3 is in every instance the standard deviation. It will be seen that the *KA* determinations for a given gage are internally consistent to within one or two per cent.

It is interesting to compare these *KA* values with those obtained for these same crystals by the Exploration Laboratory of the Stanolind Company,* from whom the plated crystals are procured. Their figures are listed in Column 4, and the percentage differences between these values and those obtained at the Taylor Model Basin are given in Column 5. The TMB calibrations differ from those of Stanolind by 2 per cent or less. It should be emphasized that the two sets of calibrations were performed by independent observers using different techniques. One of these gages, XT-503, was previously calibrated by the Underwater Explosives Research Laboratory at Woods Hole; the value obtained there is in agreement with Stanolind's within 1 per cent.

From the value of *KA* and the measured area of *A* of the crystals listed in Column 6, the piezoelectric constant *K* of the tourmaline used in these gages is computed. The values of *K* deduced from the Taylor Model Basin determinations and from those of the Stanolind Laboratory are listed in Columns 7 and 8 respectively. These figures may be compared with those computed from piezoelectric moduli given in the International Critical Tables (10). Using the moduli of Riecke and Voigt found in these tables, the piezoelectric constant of tourmaline along its electrical axis when the crystal is subjected

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* Dr. Daniel Silverman of the Stanolind Company carried out these calibrations with a microcoulometer.

to hydrostatic pressure is computed to be 1.078×10^{-11} coulombs per pound. If the moduli determined by Röntgen are used, a value 1.087×10^{-11} coulombs per pound is obtained. The agreement with either of these values is thus quite satisfactory.

Quadrant Electrometer

An alternative method of calibrating the piezoelectric gage utilizes the Compton quadrant electrometer.* This instrument possesses much higher sensitivity than that required to measure the typical output of a TMB piezoelectric gage. By shunting the gage with a total capacitance of about 0.01 microfarad, as illustrated in Figure 22, a potential difference of about 0.1 volt was impressed on the quadrants. Although the instrument is capable of many refined sensitivity adjustments, it was found possible to get the desired range of sensitivities merely by varying the needle potential between 45 and 65 volts. Because of the high capacitance in parallel with the gage, the instrument was usually stable and free from excessive drift. However, for satisfactory operation the impedance of the electrometer system had to be kept high, of the order of 50,000 megohms. Gages of lower impedance could not be calibrated with the electrometer. To prevent moisture from collecting on the insulation, a small quantity of calcium chloride was kept inside the casing of the electrometer as well as inside the box which shielded the switches and capacitors.

Gages were calibrated in the following manner: After the needle potential was adjusted for the desired sensitivity, the "floating" quadrants were grounded by momentarily closing switch K_1; see Figure 22. Then, with K_2

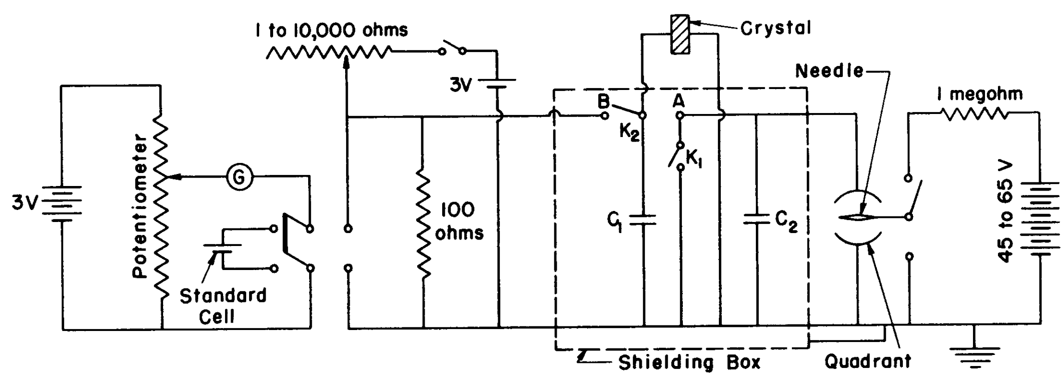

Figure 22 - Electrometer Calibration Circuit with Potentiometer

The deflections of a Compton quadrant electrometer are calibrated by putting a known voltage on the needle with a Leeds and Northrup Type K potentiometer.

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* The electrometer was obtained on loan through the courtesy of Dr. L.E. Curtis of the Radioactivity Section, National Bureau of Standards.

in position A, the pressure on the crystal was released. The corresponding ballistic deflection was read on a scale about 1 meter from the electrometer. This was repeated for various pressures.

Voltage calibration of the scale was carried out as follows: First K_1 was momentarily closed. Then K_2 was thrown to position B, charging up a mica capacitor C_1 to a potential difference which was precisely determined with a Leeds and Northrup Type K potentiometer. By reversing K_2 the known voltage was impressed upon C_2 and the quadrants. Deflections of the needle were observed for a suitable range of voltages.

Voltages ΔV were plotted against corresponding pressure changes ΔP and a "best line" was drawn. Usually the experimental points were so nearly collinear that the slope $\Delta V/\Delta P$ could be determined without recourse to the method of least squares. The piezoelectric constant KA of the gage is given by $C(\Delta V/\Delta P)$, where C is the total capacitance in parallel with the gage. C was measured with a Schering capacitance-bridge circuit manufactured by the General Radio Company.

The results obtained with this method of calibration are described at the end of the next section.

Microcoulometer

A third calibration technique makes use of an impedance coupler, Figure 23, with a large time constant, together with a voltmeter consisting of a moving-coil galvanometer* in series with a suitable resistance. This circuit has been called a microcoulometer. It has an input impedance of 30,000 megohms and an input capacitance of 10^{-8} farad. Hence its time constant, about 200 seconds, is long in comparison with the time of deflection of the galvanometer coil, which is about 2 seconds. By the switch K_1, Figure 24, either the piezoelectric crystal or a potentiometer may be

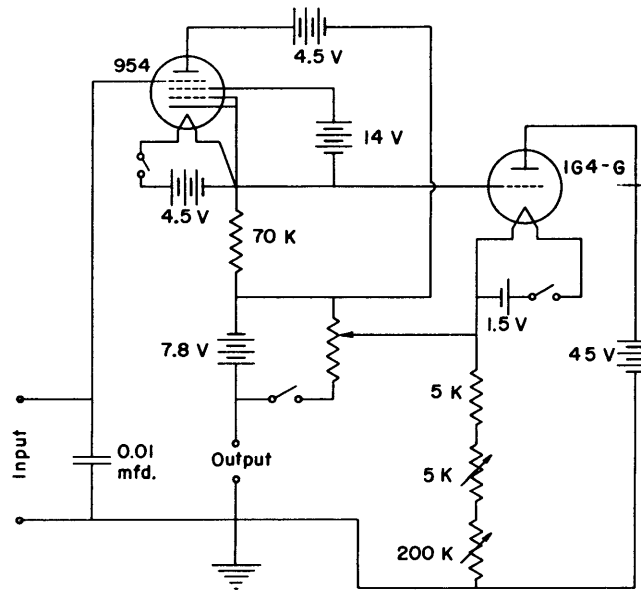

Figure 23 - Piezoelectric Gage Impedance Coupler with Large Time Constant, Termed a Microcoulometer

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* The galvanometer employed is manufactured by the General Electric Company under Serial Number 32C.

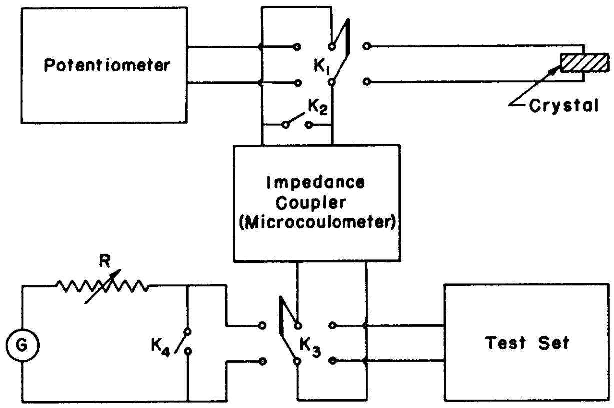

Figure 24 - Calibration Circuit Employing the Microcoulometer

The circuit in Figure 23 is represented by the box labeled "Impedance Coupler." The "input" and "output" of Figure 23 correspond to connections K_1 and K_3, respectively. The galvanometer G together with the series resistance *R* constitute the voltmeter.

connected to the input of the impedance coupler. With K_3 the output can be fed either into the galvanometer or into a Simpson test set.

The following calibration procedure is used: First the gage is momentarily shorted by the switch K_2. Then the variable resistor in the impedance coupler is adjusted until the output potential is zero. For preliminary adjustment of the output, the test set is a convenient indicator. In the interest of uniformly high accuracy, it is desirable to get nearly full-scale deflections of the galvanometer for each release of pressure. This is achieved by varying the series resistance *R*, which controls the galvanometer's voltage sensitivity.

To translate galvanometer deflections into volts, the potentiometer is employed in much the same manner as that described for the electrometer. Several voltages are applied to the microcoulometer for each value of *R* previously used, and the corresponding galvanometer deflections are noted. With these data it is possible to plot voltage against pressure change and as before, *KA* is determined from *C* and the slope $\Delta V/\Delta P$ of the curve.

The values of the calibration constant *KA* obtained with the electrometer and the microcoulometer are between 7 and 13 per cent lower than those determined with the oscillographic recording technique. The oscillographic values are judged to be correct within the experimental error, inasmuch as they have been independently substantiated by workers in two other laboratories. Moreover, they lead to a value of *K* which agree closely with that found by both Voigt and Röntgen. Both the electrometer and microcoulometer methods offer considerable difficulty because of the high input impedance which must be maintained for successful operation; the former is especially troublesome as it requires an impedance of 50,000 megohms. The discrepancy of 7 to 13 per cent has recently been accounted for (25).

Program for Further Piezoelectric Calibration Studies

Several important questions concerning piezoelectric calibrations remain to be answered. Some observers have found an indication of a difference between the *KA* value determined with a "bare" crystal, i.e., before the insulation is coated on, and the value obtained after the gage has been coated with rubber. This question is being investigated.*

It is also planned to study the effect, if any, on KA when the gage is subjected to a series of explosions. In this connection it should be observed that the oscillographic calibration technique has a great advantage over the other methods, in that it can be applied to a gage after repeated explosions. It is found, for example, that the impedance of a gage after several explosions usually drops well below the 30,000 megohms required for calibration with the microcoulometer. Hence recalibration by this method is impossible. On the other hand, the oscillographic method does not demand an impedance of more than about 10 megohms, and the gage impedance seldom drops below several hundred megohms. When it does, the gage becomes useless in any event, for the RC of the gage circuit is then too small.

CALIBRATION OF THE GLASS RESISTANCE GAGE

The glass resistance gage is calibrated with a Wheatstone bridge. The gage is sealed into the pressure chamber in the same way as are the piezoelectric gages, except that a brass washer soldered to the cable above the glass element facilitates the closure.

A series of pressure increments ΔP up to 6000 pounds per square inch are then applied to the gage, and the change ΔR_g in its resistance is measured. Displacements on the galvanometer scale, which are proportional to the resistance changes ΔR_g, are plotted against ΔP. From the slope of the resulting line the calibration constant of the gage $1/R_g\ (\Delta R_g/\Delta P)$ is obtained. Values of this constant are of the order of $1.5 \times 10^{-7}\ \frac{1}{\text{pounds per square inch}}$. Since R_g is about 100 ohms, ΔR_g is of the order of 0.015 ohm for a change in pressure of 1000 pounds per square inch.

PART 5. GAGE PERFORMANCE IN TESTS

UNDERWATER EXPLOSION TESTING

It was thought desirable to test the piezoelectric gages under a variety of circumstances, using larger charges than could be fired at the Taylor Model Basin at the time this work was done. Tests were therefore conducted by Taylor Model Basin personnel at the Underwater Explosives Research Laboratory, Woods Hole. Considerable work on explosion pressures had been done by investigators at that laboratory with tourmaline gages manufactured by the Stanolind Company, and some of these gages were available for direct comparison with TMB tourmaline gages. Therefore it was decided to concentrate

* Dr. A. Borden reports that preliminary experiments indicate a difference of less than 1 per cent between the bare and the coated crystals.

initially on testing the performance of the tourmaline gage rather than that of the other gages. The recording facilities at Woods Hole were generously made available. The overall frequency response of the cable and amplifier was good to 300 kilocycles per second.

PRESSURES IN OPEN WATER

A welded ring 9 feet in diameter of 1 1/2-inch steel pipe was used for the mounting of charges and gages; see Figure 25. The charges, which ranged from 50 to 250 grams of loose tetryl, were placed in the center of the steel ring. The gages were mounted in the plane of the ring 36 inches from the charge. The charge and the gages were secured by vertical and horizontal wires. The wires in turn were attached to vertical steel rods passed through drilled holes in the 9-foot ring.

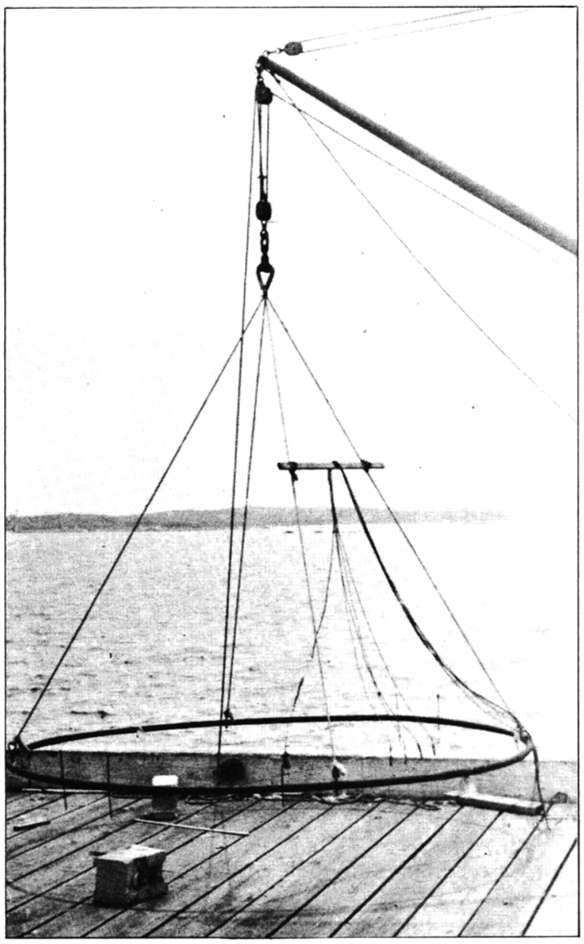

Figure 25 - Welded Steel Ring Used for Mounting Charges and Pressure Gages

This ring was used by TMB personnel in tests at the Underwater Explosives Research Laboratory at Woods Hole.

Three gages and recording channels were ordinarily employed; these gave at least two good records for each explosion. Two of the gages were usually the TMB copper-cable tourmaline type, whereas the third was a similar tourmaline gage mounted on a Belden 8400 rubber-sheathed cable; the latter gage was supplied by the Underwater Explosives Research Laboratory. The tetryl was detonated by firing a Number 8 detonator cap placed in the center of the charge.

Figure 26 shows photographic reproductions of some typical records. Table 5 summarizes some of the results obtained. Peak pressures recorded by the two types of gages generally agreed within 3 per cent. The average deviation of the pressures recorded by a single gage was also about 3 per cent.

After the consistent operation of the tourmaline gages when used with charges of this size had been established, it was decided to test their

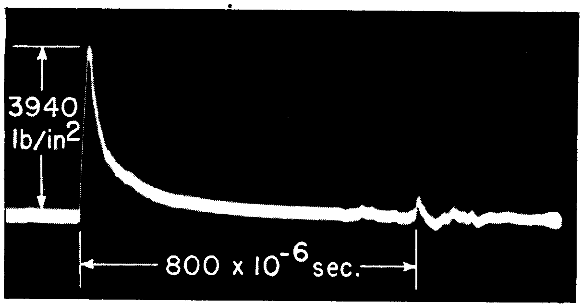

Figure 26a - Record Obtained with a TMB Tourmaline Pressure Gage

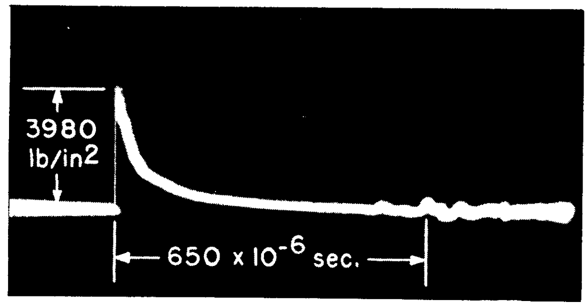

Figure 26b - Record Obtained with a Gage Supplied by the Underwater Explosives Research Laboratory

Figure 26 - Pressure-Time Oscillograms Obtained with Two Tourmaline Gages in Open Water

150 grams of tetryl 3 feet from the gage was used in each of the two explosions.

TABLE 5

Peak Pressure and Duration of Explosions as Registered by Tourmaline Piezoelectric Gages

The distance from the gages to the charge was 36 inches, plus or minus 1 inch.

Tetryl grams	Gage		Number of Records	Peak Pressure pounds per square inch	Duration microseconds
100	XT-C2	TMB*	9	3140 ± 110	55.4 ± 6.2
100	S-300	UERL**	8	3010 ± 120	52.3 ± 4.8
150	XT-C2	TMB	7	3840 ± 140	63.3 ± 2.2
150	XT-C4	TMB	11	3840 ± 50	58.9 ± 2.0
150	XT-B13	TMB	6	3740 ± 125	52.2 ± 1.7
150	S-300	UERL	14	3880 ± 65	54.5 ± 2.3

* "TMB" refers to a Taylor Model Basin gage.

** "UERL" refers to a gage belonging to the Underwater Explosives Research Laboratory.

performance when used with explosions of 1 ounce or less of tetryl.* In addition a comparison of the records obtained with tourmaline gages with those of the quartz and glass gages has been undertaken. These investigations are still in progress, and the results will be given in a subsequent report. Experience in these tests has shown clearly that the task of obtaining valid pressure-time records of explosions is greatly complicated when small charges are employed. For reasons set forth elsewhere in this report, the shorter

* The upper limit of 1 ounce was imposed by the fact that these tests were to be performed at the Taylor Model Basin, where facilities for larger explosions were then not available.

duration of such explosions imposes requirements upon the pickup, amplifier, sweep generator, and cathode-ray tube which are much more stringent than for explosions of larger charges.

PRESSURES AT A SURFACE OF AN UNDERWATER STRUCTURE

A modified form of the tourmaline gage has been used to obtain pressure-time records at a surface. The rubber insulation is molded about the crystal and also in a cavity sunk in a brass plate. The latter is bolted to the surface of an underwater structure; see Figure 6 on page 9. Thus far only preliminary comparisons have been made between this surface type of gage and the usual open-water type.

The brass plate was bolted to the center of a piece of plywood 2 feet square by 1.5 inch thick. The plywood was supported only by a rope so that it was free to move in the water. A 22-gram charge of tetryl was placed 20 inches from the surface gage so that the line joining the gage and the charge was perpendicular to the plane of the plywood. An open-water type of tourmaline gage, also placed 20 inches from the charge, served as a comparison pickup.

Figures 27a and 27b show a typical pair of comparison records. Two interesting differences always appear in these comparisons. The surface gage shows a higher peak pressure than is observed in open water; the ratio of these peak pressures is somewhat less than 2. The time histories of the two records are strikingly different. In Figure 27b the pressure indicated by the surface gage has dropped to zero in a time interval during which the pressure indicated by the open-water gage has decayed only to approximately 1/3 of its peak value.

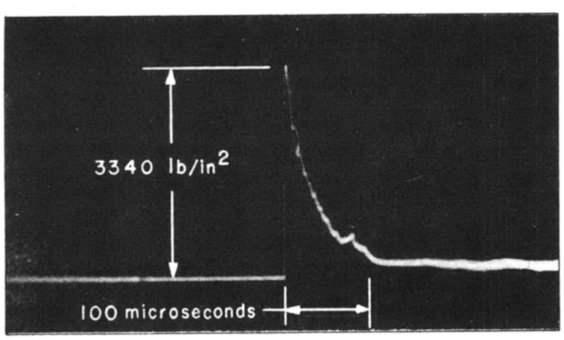

Figure 27a - Explosion Pressures in Open Water

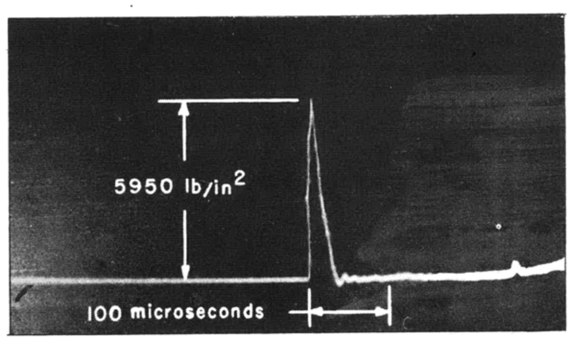

Figure 27b - Explosion Pressures on a Surface of an Underwater Structure

Figure 27 - Comparison of Explosion Pressures on a Surface of an Underwater Structure and in Open Water

The record in Figure 27a was obtained with the usual type of open-water gage; that in Figure 27b with a surface-type of tourmaline gage sunk into a brass plate. The return to zero pressure is much more rapid on the surface than in open water.

The approximate doubling of the peak pressure is to be expected because of the reflection of the pressure wave that takes place at the surface of the brass plate. Professor Kennard of the Taylor Model Basin staff has offered the following explanation for the rapid fall of the pressure to zero which is indicated by the surface gage. The pressure wave impinges upon the brass plate and causes it to move initially as if it were backed by air, because of the small acoustic impedance of the plywood. The brass plate undergoes very large accelerations and after a certain time t_s cavitation occurs in the water. If we assume that cavitation sets in at zero pressure, and that the form of the shock wave is given by $e^{-\alpha t}$, then the time t_s at which the record should indicate zero pressure is (26)*

$$t_s = \frac{1}{\alpha} \frac{1}{x-1} \ln x$$

where

$$x = \frac{\rho c}{\alpha m}$$

ρc is the specific acoustic impedance of the water; m is the mass per unit area of the brass plate.

α was determined from the open-water record given by Gage 280 in Figure 27a. It is approximately 28,300 seconds^{-1} if t is in seconds; m is 0.075 pound per square inch. This gives

$$t_s = 35.3 \text{ microseconds}$$

Direct measurement on the curve in Figure 27b gives 35 microseconds for the time required to reach zero pressure.

USE OF THE TOURMALINE GAGE IN STUDYING A SLOWER PRESSURE CHANGE

The duration of an explosion of 100 grams of tetryl is about 50 microseconds. Recently the TMB tourmaline gage has been used to study pressure changes which are a thousand times as slow as this, i.e., about 50 to 100 milliseconds in duration.

The phenomenon under investigation was the load on an experimental 6-inch, 47-caliber, high-angle turret due to the recoil of the guns when they were fired. The shock of recoil is absorbed by a hydraulic brake or buffer. A 1/3-scale model of the turret at the Philadelphia Navy Yard was used for this test. The load due to recoil was simulated by the impact of a 2000-pound car rolling down an inclined track, and striking the piston of the buffer (28) (29).

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* See also Reference (27), which deals with the explosive load on underwater structures as modified by bulk cavitation.

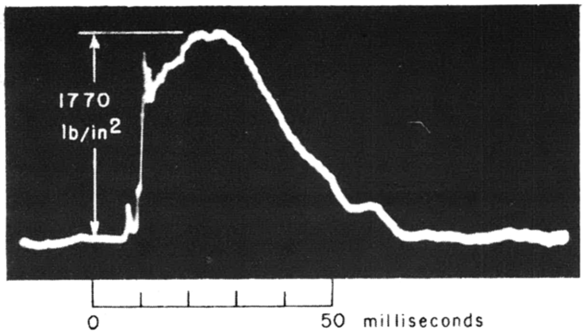

Figure 28 - Oscillogram Showing Pressure as a Function of Time Inside a Hydraulic Brake or Buffer

This record was obtained with a TMB tourmaline gage. The duration of this phenomenon is about 1000 times that of typical explosions described in this report.

From a measurement of the peak pressure inside the buffer the peak load can be deduced. This measurement has usually beem made with a Crosby high-pressure engine indicator. It was desired to calibrate this instrument against a tourmaline gage.

The gage was sealed into the buffer by a special adapter. An RC constant of 1 second was provided by shunting the gage with a 0.01-microfarad capacitor and using a cathode follower with an input impedance of 100 megohms. A trigger circuit was designed in which a capacitor discharge, actuated by mechanical contact of two wires, set off the sweep. Contact was made at a predetermined time before impact when a bolt projecting from the moving car pushed one wire against the other. To record the pressure-time curve, a Du Mont Type 208 cathode-ray oscillograph was used together with the auxiliary equipment usually employed in recording transient pressures at the Taylor Model Basin.

The car was released from various heights up to 20 feet, and a pressure-time curve was obtained for each impact. Figure 28 shows a typical record for a 15-foot drop. The initial, sharp pulse is due to a shock-pressure wave transmitted through the buffer. This is followed by the main-peak portion of the curve which shows the variation of pressure inside the cylinder as the piston moves through it.

In an effort to check the validity of the pressure-time curve, the change in momentum mv of the car due to impact was compared with the total impulse $\int F dt$ deduced from the area under the pressure-time curve.* m is the mass of the car; v its velocity at the initial moment of impact; $F = Pa$, where P is the pressure recorded by the gage at time t, and a is the area of the piston. The integration, performed with a planimeter, is extended over a time from the initial moment of impact until the pressure returns to zero. The impulse measured in this way agreed with the calculated change in momentum to within 5 per cent.

Thus, besides its usefulness in explosion research, the TMB tourmaline gage appears to be equally suitable for the measurement of much slower pressure changes.

* The recoil velocity of the car is negligible.

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APPENDIX

CONSTRUCTION OF THE QUARTZ PIEZOELECTRIC GAGE

Figure 3 is a view of the quartz gage with a section cut away to show the various components. The principal parts of the gage will first be described, then the details of its construction.

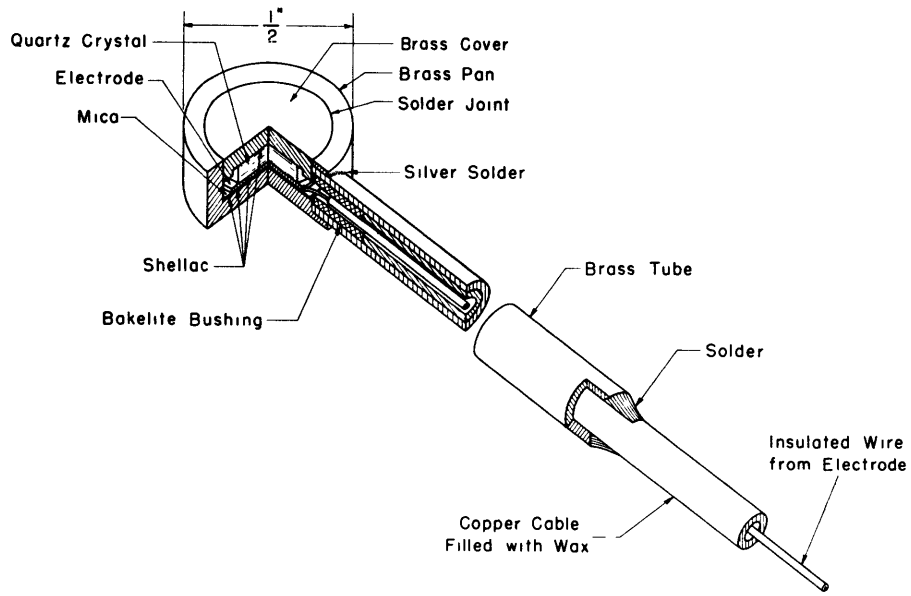

Figure 3 - Components of the TMB Quartz Piezoelectric Gage

The brass housing consists of three parts: a "pan" which contains the crystal; a cover; and a brass tube which is silver-soldered to the pan. A cylindrical, X-cut quartz crystal is cemented on one side to an electrode consisting of a thin copper disk, and on the other to the cover of the brass housing, through which it is grounded. The use of a copper disk for the electrode obviates the trouble of electroplating. Laterally the crystal is surrounded by an air gap which is required because quartz is insensitive to isotropic pressure. It is the need for this air gap which makes a housing necessary.

DESCRIPTION OF THE PRINCIPAL PARTS

The copper electrode is insulated from the brass pan by a thin sheet of mica. It is connected to the central lead wire, which is covered with enamel and a double layer of glass fiber. The space between this insulated wire and the concentric copper tube is filled with ceresin wax. Near its entrance into the pan the wire is insulated from the brass tube by a bakelite bushing. The crystal is so oriented that a compressional force produces a negative charge on the surface next to the copper electrode. When the gage is in use, the electrode is connected to the grid of an amplifier tube; thus the grid is charged negatively during compression. The brass housing is sealed watertight by soft-soldering the cover to the pan.

ASSEMBLY OF THE GAGE

Before assembly the various components of the gage are prepared as follows: an annealed copper-tube single-conductor cable is filled with wax, using a technique developed at the Taylor Model Basin (18). The outer surface at one end of this cable, as well as the inner surface of the brass tube to which it will be soldered, are tinned. The lateral surface of the cover is also tinned to prepare it for soldering to the pan. Pure orange shellac crystals are heated until they just melt; then thin coats of shellac are applied to the inside surface of the cover and pan, as well as to both surfaces

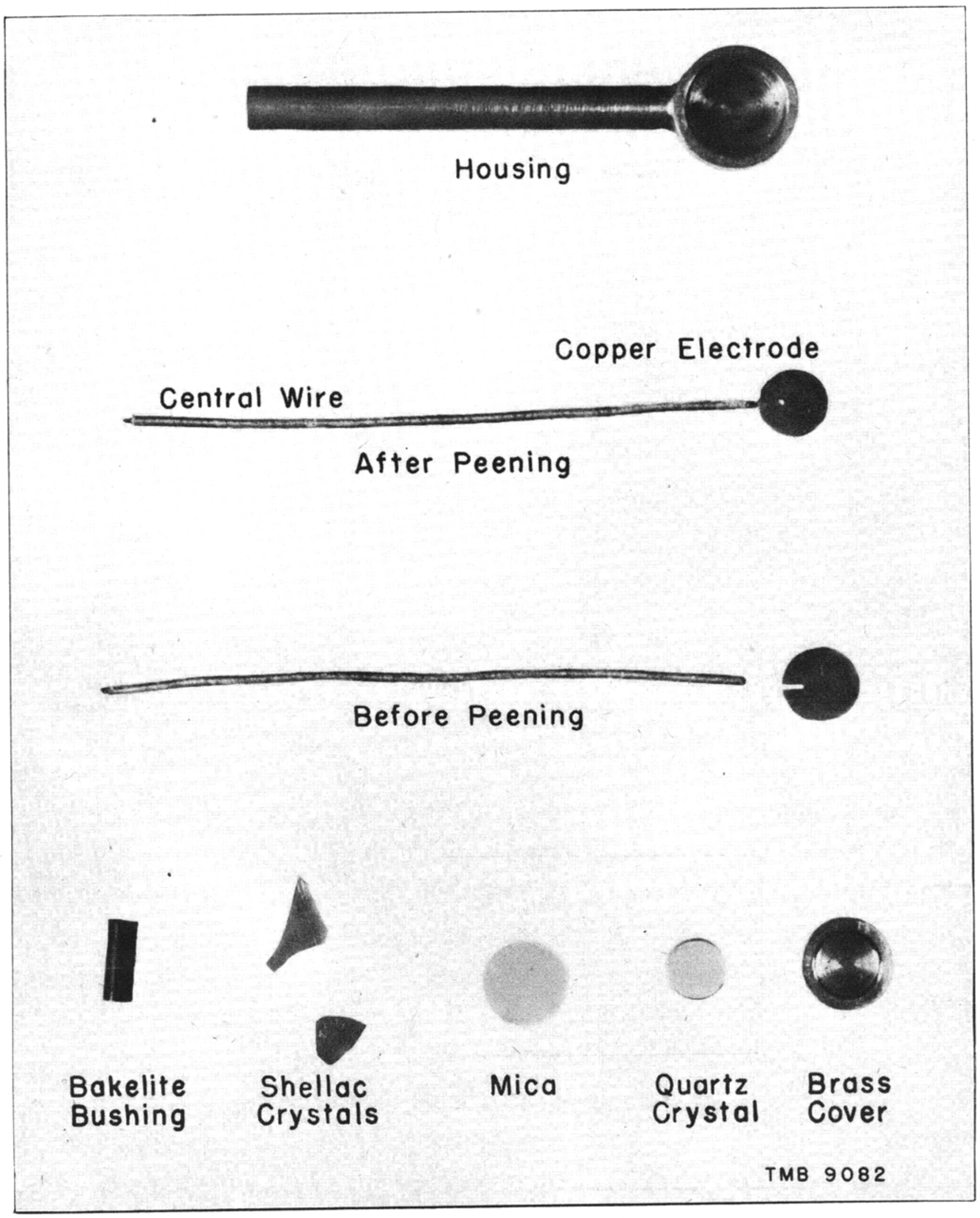

Figure 29 - Details of Assembly of the Quartz Gage

of the quartz crystal and mica.* Before the shellac has dried, the mica is cemented to the pan; similarly, the crystal is attached centrally to the cover. The slight depression in the cover serves a dual purpose: It helps in aligning the crystal, and later when the gage is finally assembled, the wall of the depression prevents the crystal from slipping sidewise off the electrode; this ensures that the full area of the crystal is utilized.

Before the wire is connected to the electrode, the copper cable is pushed into the brass tube farther than it will be in the completed gage, so that the wire emerging from it protrudes beyond the pan. A bakelite bushing is then slipped over this wire. The end of the wire is bared and attached to the electrode at the slit with a peening hammer, see Figure 29. A tight friction joint results; the short length of wire in the slit becomes virtually a part of the electrode. Then, while the bakelite bushing is inserted into the brass tube, the cable is pulled back until the copper electrode lies concentrically over the mica. The cover with the attached crystal is forced into the pan with a small C-clamp, and presure is maintained by this clamp while the cover is soft-soldered to the pan. During the soldering the shellac films remelt; thus, upon cooling, the various components are bound firmly together. Finally, the copper cable is soldered to the brass tube. The C-clamp is removed only after the metal has cooled. In both soldering operations great care is taken to keep the surfaces clean, so that the joints will be impervious to water under explosion pressures. If all the work has been done properly, the gage is now complete and ready for testing.

STATIC TESTING OF THE GAGE

At this stage the impedance of the gage is measured. For calibration with the microcoulometer described elsewhere in this report, the impedance should be at least 30,000 megohms. Next, to make sure that the gage is watertight, it is sealed in a pressure chamber and subjected to hydrostatic pressures up to 3000 pounds per square inch. Once more the impedance is measured. If the latter has dropped below its pre-immersion value, then the soldered joints may be leaky. If, on the other hand, the impedance is still high, the gage is ready for calibration.

SURFACE-PRESSURE MODIFICATION OF THE QUARTZ GAGE

The housing of the quartz gage has been successfully adapted to the measurement of explosion pressures on the surface of an underwater structure.

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* When the crystal is subjected to a change in pressure the opposite charges developed on its two faces are transmitted by induction to the central electrode and the grounded cover, respectively, through the layers of shellac.

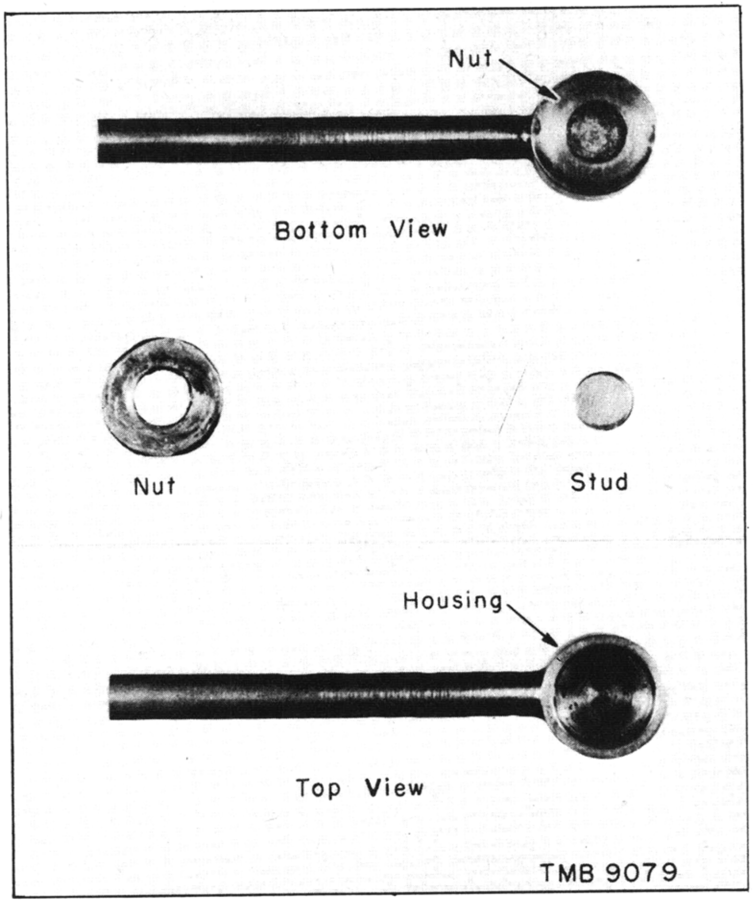

Figure 4 - Modification of Quartz Gage for Attachment to a Surface

A nut is soldered to the bottom of the housing, and a threaded stud is soldered to the surface at which the pressure is to be measured.

A brass nut whose outside diameter is the same as that of the pan, 1/2 inch, and whose inside diameter is 1/4 inch, is silver-soldered to the bottom of the housing. Since this nut is only 0.08 inch thick, it does not add excessively to the size or mass of the gage. A brass stud 1/4 inch in diameter and 0.075 inch thick is threaded to fit the nut. This stud is soldered to the surface under investigation, and the gage is then screwed down securely over the stud. Figure 4 shows the brass mount before and after adaptation to the surface type of gage, together with the nut and stud used in the conversion.

This type of "surface gage" has been designed so that it can be firmly attached not only to a rigid surface, but also to the surface of a diaphragm subjected to explosive loading. In the latter case the brass-plate type of tourmaline gage illustrated in Figure 6, on page 9, is inapplicable, as it is much too massive.