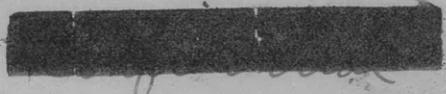


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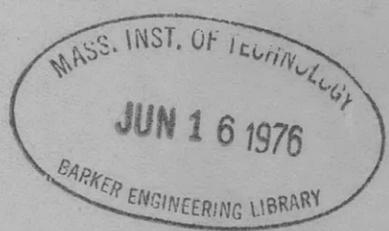
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UNITED STATES EXPERIMENTAL MODEL BASIN

THE ULTIMATE AND CRITICAL COMPRESSIVE STRENGTH
OF Tee STIFFENERS

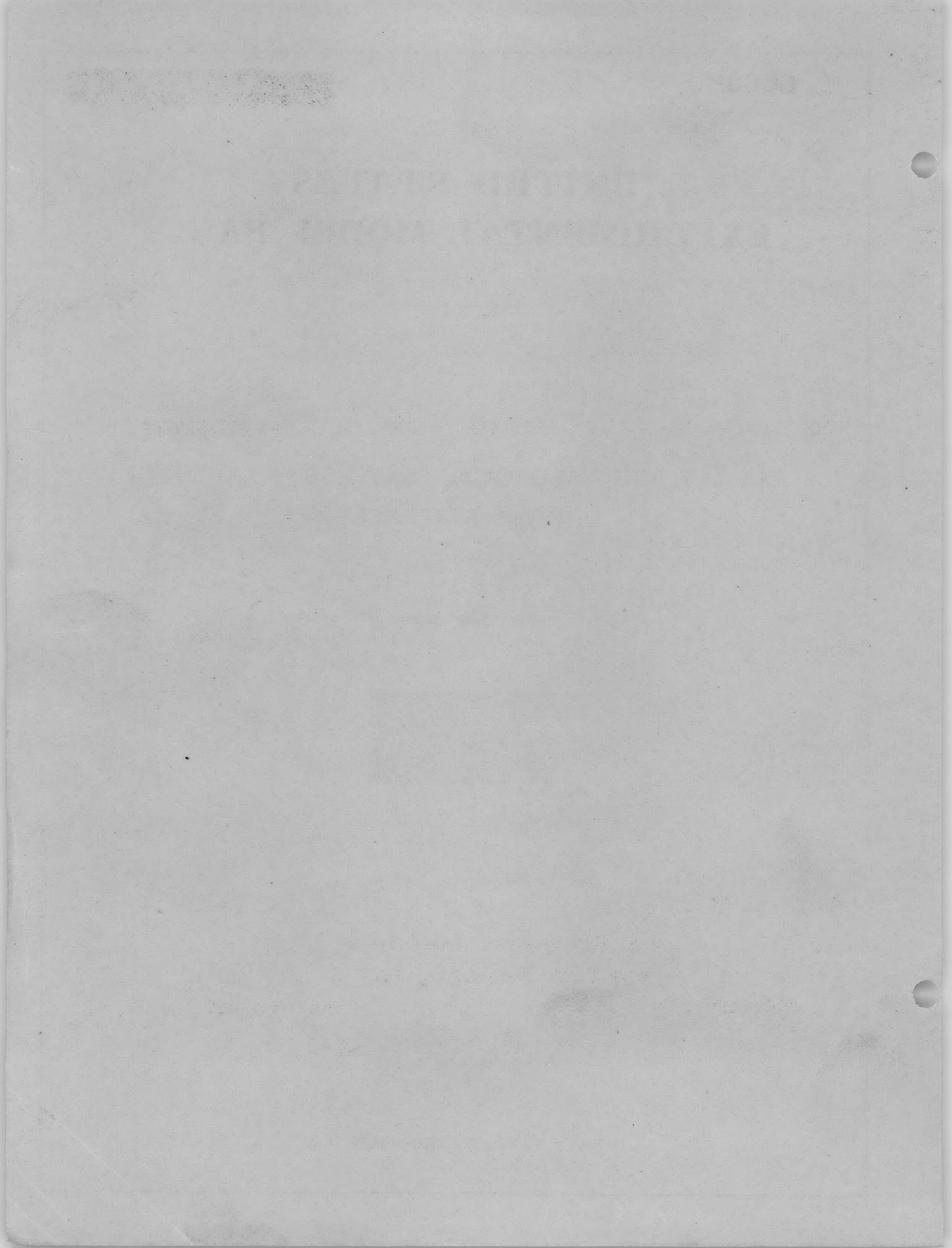
BY JOHN VASTA



FEBRUARY, 1938

REPORT NO. 445✓

NAVY YARD, WASHINGTON, D.C.



THE ULTIMATE AND CRITICAL COMPRESSIVE STRENGTH OF Tee STIFFENERS

By J. Vasta

Summary

Experimental work on the ultimate and critical compressive strength of Tee stiffeners of uniform thickness simply supported at the toe, is summarized in Fig. 4 and Fig. 5 of this report. Figure 4 specifies the minimum required flange width to develop the maximum compressive strength of the stiffener, while Fig. 5 gives the critical strength in terms of the ultimate strength. The application and extension of these results to non-uniform Tee sections is discussed and illustrated.

Introduction

In a report on "Stiffeners and Flanges", of January, 1935, designated as "Progress Report No. 5" (of a series of reports on "Strength of Hull Plating in Compression") the results of a preliminary investigation on the strength of Tee stiffeners were presented. These results were summarized in Fig. 10 of that report. The experimental work has since been extended to cover a range not previously investigated.

The present experiments were systematically planned and better controlled to bring forth the relationships among all of the variables involved. Because of this, and improvements in the method of analysis, the results submitted here are more accurate than the previous ones. Therefore, Fig. 4 of the present report should supersede the previous Fig. 10.

One of the major functions of longitudinal stiffeners is to support the plate adequately against compressive forces induced when the ship bends as a girder. If the stiffener is unstable or if it lacks rigidity, then it will fail to render adequate support to the plating which, in turn, will collapse at a lower stress than it can normally develop if properly supported.

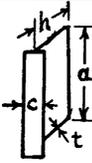
Experimentally, the adequacy of a stiffener can be determined only by two separate investigations. (1) The first phase considers the stability, i.e., the ability of the stiffener to retain its shape and hence its ability to sustain high compressive stresses by itself. This phase has been investigated and is being reported now. The problem here was to test Tee stiffeners not attached to plating but supported by cylindrical columns at the toe. The analysis resolved itself into finding the proper dimensions of the standing flange that enabled the Tee section to develop the maximum possible compressive strength. (2) The second phase of the investigation will consider the rigidity or stiffness, i.e. the required moment of inertia of the stiffener which will cause it to support the plate adequately. The method here is to attach stiffeners, which are known in advance to be

stable, to a plate and test the combination as a unit. In this manner, the plate will be elastically supported at the edges with stiffeners of known stability and performance, and the interaction between the two as affecting the strength of the plating can be determined. Such an investigation is now in progress.

Characteristics of Test Models

About 120 models, constructed of furniture steel, were tested. The dimensions of the section, and the physical characteristics of the material are given in Table I.

TABLE I

Length/Depth of Web Plate a/h	Range of Flange Width/Thickness Ratio (c/t) for				Thick- ness t in.	Tensile Yield Stress lbs/sq. in.
	$\frac{h}{t} = 52$	$\frac{h}{t} = 41$	$\frac{h}{t} = 31$	$\frac{h}{t} = 22$		
 4	6 to 30	6 to 24	6 to 22	6 to 16	0.070	39600
8	8 to 30	6 to 24	6 to 23	6 to 17	0.070	38200
12	6 to 28	6 to 24	6 to 22	6 to 16	0.050	37200
16	6 to 30	6 to 24	6 to 23	6 to 17	0.050	34000) 37200)

The webs and flanges were of the same thickness, being cut from the same sheet, and were assembled by continuous solder fillet. The loaded ends of the models were machined parallel in order to obtain good load distribution.

For any one constant value of length-depth ratio (a/h) of the web plate, two quantities were varied independently; the depth-thickness ratio (h/t) of the web and the width-thickness ratio (c/t) of the flange. In this way several optimum combinations were experimentally determined. An optimum stiffener is one which for a constant length and web depth has the smallest flange width that permits it to develop the maximum compressive strength.

Method of Testing

All models were supported at the toe by a pair of cylindrical columns clamped together with finger tight clamps as shown in the photograph, Fig. 10. These columns hold the toe of the stiffener straight, and since they are shorter than the models, they take no compressive load. This type of support has been described in previous reports*, and is intended to give the nearest experimental approach to simple support. The deflection of one edge of the flange at its mid-length was recorded by means of a dial micrometer.

*A description of the method of support of lateral edges of flat plates loaded in edge compression is given in Progress Reports 1, 2, and 5.

Modes of Failure

As the model is compressed it undergoes certain deformations. At first, the deformation is small and imperceptible, but as the load increases it becomes appreciable and noticeable. At very high loads, the model takes a definite characteristic configuration which is directly related to its dimensions. When failure occurs, however, this configuration, in most cases, changes and the model assumes a shape not wholly related to the one exhibited earlier. It is this initial mode of failure, and not the final one, that is of importance. In general, three distinct types of initial failure were noted.

(1) With models having an $a/h \leq 8$ and insufficient flange width the web plate and flange bent together as a unit, characteristic of a slender column failure.

(2) With models having an $a/h \leq 8$ and adequate width (optimum sections) the web plate buckled locally, and the flange wrinkled in line with this buckle. The rest of the model remained essentially straight.

(3) With models having an $a/h \geq 12$, and all flange widths tested, failure occurred by a combined bending and twisting.

Analysis of Test Data - Results

The ultimate unit stresses (P/A) of the models have been plotted against the ratio of flange width to thickness (c/t) for a constant (h/t) and (a/h). Figure 1 shows a typical set of curves for a Tee stiffener of constant length-depth ratio of approximately $a/h = 8$. Similar experimental charts have been prepared to cover the ranges of $a/h = 4, 12, \text{ and } 16$.

From curves faired through the experimental points, as in Fig. 1, values of "strength ratios" have been obtained. These are summarized in Table III, and are shown graphically in Fig. 2. The "strength ratio" is obtained by dividing the ultimate or collapsing stress of any particular model by the maximum stress value defined by the horizontal portion of the faired curves. Thus, from Fig. 1, the strength ratio for a Tee stiffener having $h/t = 41$, $a/h = 8.2$, and $c/t = 10$ is 0.925, or 34800 lb. per sq. in. divided by 37600 lb. per sq. in., the maximum stress developed by the combination under consideration.

The data are expressed in a more usable form in Fig. 3. This figure, which represents an auxiliary step for arriving at Fig. 4, has been derived by plotting for any given (h/t) and (a/h) the minimum value of (c/t) that yields a "strength ratio" of unity. With Fig. 3 as a basis, Fig. 4 has been constructed showing contour curves of flange width. This figure summarizes the experimental work, and presents a clear picture of the relationship of the variables involved.

In order to make possible, for those cases where strength requirements are of secondary importance, the selection of a flange width other than that dictated for maximum strength, Fig. 5 has been prepared from Table III. The curves in this figure give the value of (c/t) which will develop any desired fraction of the

maximum compressive stress. Thus, for a web plate-thickness ratio of 52, and $a/h = 12$, the strength ratio has a value of 0.81 for $c/t = 12$, and increases to 0.95 for $c/t = 16$. It is to be noted here that the maximum spread in the "strength ratio" is experienced with the deeper webs ($h/t = 52$).

Critical Stress

The critical stress of the models was obtained by plotting compressive load against the edge deflection of the flange measured at its mid-length. This deflection was recorded by a dial micrometer reading to 1/1000 in. The deviation of the plotted curve from the straight line was taken as the critical load.

The critical stress for the optimum Tee stiffeners is given in Table II. This value is plotted as a fraction of the ultimate stress against the length-depth ratio of the web, (see Fig. 6). The curve shows that for short stiffeners ($a/h = 4$) the critical stress is coincident with the ultimate stress. This means that an adequate stiffener having $a/h = 4$ will keep its section undistorted up to the maximum load. With increasing length ($a/h > 4$) there is a uniform rate of decrease in the ratio of critical to ultimate stress until at $a/h = 16$, the critical stress has dropped to approximately 75 per cent of the ultimate. The significance of this must be noted. It means that stiffeners with high a/h ratios and with adequate flange width specified by Fig. 4, though developing the maximum compressive strength, begin to bend and twist at a load lower than the ultimate. A moderate increase in flange width above the minimum required will not materially increase the critical stress. Moreover, above this critical value the deformations become progressively large, and the section distorts visibly. The curve shows further that within the range studied the ratio of critical to ultimate stress is approximately independent of the web plate depths, and of the yield stress of the material. It must be pointed out that values of critical stresses were obtained with Tee sections having machined flat ends, a condition that is probably very similar to the connections of longitudinal stiffeners to web frames of the ship. This makes the results directly comparable.

Discussion of Results

The maximum stress value defined by the horizontal portion of the curves of the type shown in Fig. 1 is approximately equal to the collapsing stress developed by the control plate* and also to the predicted value from Fig. 12A of the supplement to Progress Report No. 2. The maximum discrepancy in the three stresses is well within a range of ± 5 per cent which can very well be accounted

*The control plate strength was established by testing 2 flat plates having the same (h/t) as the web of the Tee stiffener and supported at the lateral edges by cylindrical columns. This type of support has been described in previous reports.

for by variations in thickness and yield stress of the material. The maximum strength of a Tee stiffener is found to be that of the control plate. When the web h/t is less than 40 the maximum strength of a stiffener is the compressive yield strength of the material.

An inspection of Fig. 4 shows that for high values of a/h a correspondingly large value of c/t is required in order to develop the maximum compressive strength of the stiffener. A Tee section may readily grow out of reasonable proportion; i.e., in some instances the flange becomes unduly wide: Hence practical considerations limit the selection. The upper limit here should be defined by a stiffener having a flange width not exceeding 30 thicknesses. If the unsupported flange width $c/2$ is larger than $15t$, the flange itself will become unstable, wrinkling along the free edge, and will cause the stiffener to fail at lower ultimate stress. The limit of $15t$ as a safe and conservative value was indicated in Progress Report No. 6, "The Buckling or Critical Strength of Flat Bar Stiffeners," of May 1937.

The curves of Fig. 5 make possible the selection of Tee stiffeners for practically every strength requirement. The design of Tee sections which do not develop the maximum compressive strength, however, should be discouraged because of excessive deformation and distortion. Stiffeners that develop only a fraction of their maximum strength (insufficient flange width) have generally a much lower critical strength than the optimum section. As the width of the flange becomes adequate the critical stress of the stiffener increases until it approaches that of the optimum section. The range beyond this optimum section has not been sufficiently investigated to warrant definite statements. It is to be noted, however, that as the flange increases in width it introduces a problem of its own; i.e. the possible wrinkling of the free edges. Since the major function of the stiffener is to support the plating adequately, it becomes important that its critical stress be kept as high as possible, - preferably equal to or above the ultimate compressive strength of the plating. Only then, is there any assurance that the stiffener is effectively supporting the plating.

It was stated in Progress Report No. 5, and is equally true in this case, that these results represent the performance of Tee stiffeners not attached to plating, but simply supported at the toe by cylindrical columns held together by finger-tight clamps. These tests give evidence that the flange of an optimum Tee yields a type of edge support which closely approximates the edge support obtained with cylindrical columns; i.e. simple support. Probably the toe of the Tee likewise receives simple support when attached to the plating, but this will be known more definitely after the current investigation is completed. The elastic interaction existing between the stiffener and plating loaded together in compression is being studied experimentally.

Non-Uniform Tee Section

For purpose of analysis, a non-uniform section is defined as one whose average flange thickness is greater than the web thickness. This report so far has considered only Tees of uniform section, having the same flange and web thickness. Non-uniform sections are commonly used, however, and it is possible to extend these results to such sections. To do so it is necessary to determine a quantity which will be designated as the "equivalent c/t " of the section, and which is defined by the relation

$$\frac{I_{F(Y-Y)}}{t^4} = \frac{1}{12} \left(\frac{c}{t} \right)^3$$

where $I_{F(Y-Y)}$ is the moment of inertia of the flange of the non-uniform section taken about the web axis (Y-Y), and t is the web thickness. This relation has been plotted graphically in Fig. 8 from which the "equivalent c/t " can be readily determined.

If the "equivalent c/t " of the section satisfies the requirements of Fig. 4 then the original non-uniform Tee section can be considered satisfactory. This criterion is based on the fact that given two sections, as in Fig. 7, which have the same web dimensions and same flange width then the polar moment of inertia about the toe (I_p), and the flange moment of inertia about the Y-Y axis $I_{F(Y-Y)}$ * of the non-uniform stiffener A will always be greater than the corresponding quantities for the optimum uniform stiffener B. Since the moment of inertia of the flange $I_{F(Y-Y)}$ is a measure of the resistance of the stiffener to failure by "laying over," and the polar moment of inertia I_p is a measure of the resistance of the stiffener to failure by twist, the non-uniform Tee of Fig. 7 is considered as good or better than the optimum uniform section with which it is compared.

As a first check on Tee stiffeners cut from rolled non-uniform sections, the designer should determine whether the flange width divided by the web thickness (c/t) meets the requirements of Fig. 4. If so, the stiffener is satisfactory and has some margin of strength. If, however, the c/t ratio falls short of the value required in Fig. 4 the designer should determine the "equivalent c/t " of the section with the aid of Fig. 8 and check this new ratio for adequacy.

*For practical purposes the quantity I_p taken about the toe of the Tee stiffener is approximately equal to the moment of inertia of the section about the X-X axis. (See Fig. 7). For the range covered in this investigation the approximation leads to an error not greater than 1.5 per cent. The same can be said for the quantity $I_{F(Y-Y)}$ which very nearly equals the moment of inertia of the whole section about the Y-Y axis. These close approximations make it possible to select values of I_p and $I_{F(Y-Y)}$ directly from a structural handbook when the Tees are cut from rolled I beams.

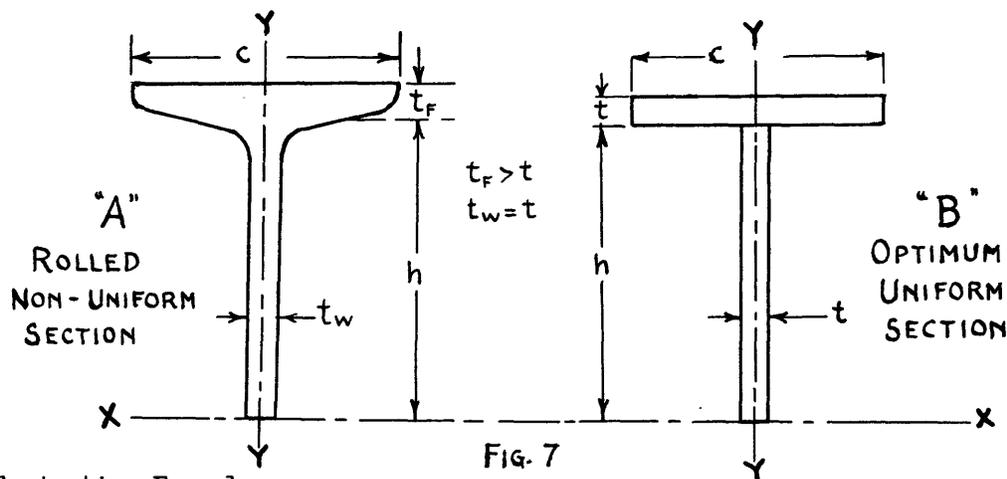


FIG. 7

Illustrative Example

The longitudinal stiffener at the bottom of the shell plating at frame 97½ of DD No. 409-420 is made up of a 6" x 3.06" x 5.87 lb. Tee cut from a J and L junior I-beam. The belt frame spacing is 7 feet. It is desired to check this stiffener for adequacy in compressive strength by the above criterion.

Given	$a = 7 \text{ ft.} = 84 \text{ in.}$	$I_P = 36.1 \text{ in.}^4 \text{ approx.}$
	$h = 5.75 \text{ in.}$	$I_{F(Y-Y)} = 0.49 \text{ in.}^4 \text{ approx.}$
	$t_w = 0.175 \text{ in.}$	
	$t_F = 0.25 \text{ (average)}$	
	$c = 3.06 \text{ in.}$	

From the given quantities the following ratios are established:

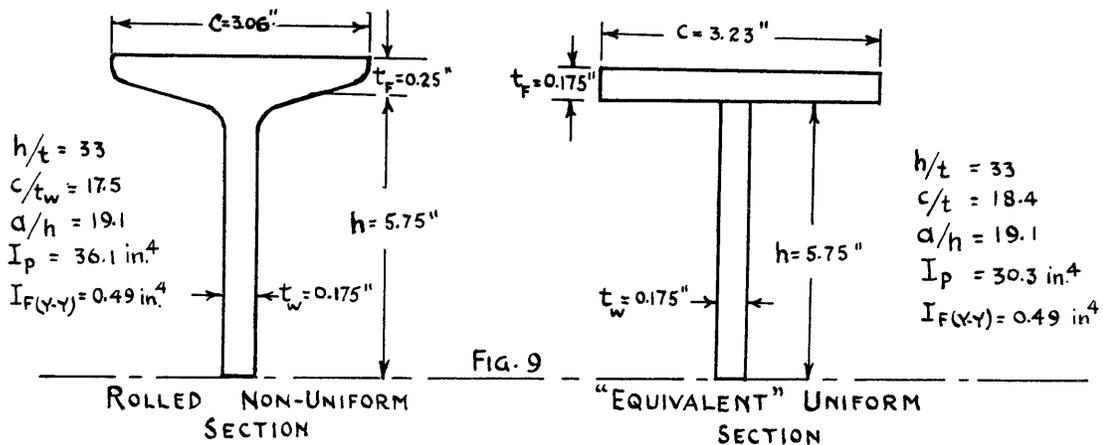
$$a/h = 14.6, \quad c/t_w = 17.5, \quad h/t = 33.$$

Reference to Fig. 4 of this report shows that a stiffener of such proportions would require a c/t ratio of only 16 in order to develop its maximum compressive strength. Actually the given ratio c/t_w is 17.5, and the given section is more than adequate to fulfill its function. The further fact that this is a rolled non-uniform section with the flange thickness greater than that of the web gives the Tee a moderate margin of safety over the corresponding uniform section.

If it were desired now to use the given stiffener between transverse belt frames 110 inches apart the question would arise whether or not the Tee would still be adequate in strength. With $a = 110 \text{ in.}$, the new a/h ratio becomes 19.1. Reference to Fig. 4 shows that the minimum required c/t for this new a/h ratio should be about 18. Since the given ratio is 17.5 there may be some doubt as to the suitability of the section. However, from the given quantities $I_{F(Y-Y)}$ and t_w , the "equivalent c/t " of the section is derived with the aid of Fig. 8, and a new ratio of $c/t = 18.4$ is established. Consequently the given stiffener is satisfactory as far as ultimate compressive strength is concerned. It must be pointed out however that its critical strength has dropped approximately 10 per cent.

The comparison between the rolled non-uniform section and the "equivalent" uniform section is summarized in Fig. 9. It will be seen here that the "equivalent"

Tee, which can now be considered as the optimum section, will require a polar moment of inertia $I_p = 30.3 \text{ in.}^4$ as contrasted with the given $I_p = 36.1 \text{ in.}^4$. This differential will always be on the safe side since from the geometry of the figure the I_p of the rolled non-uniform section will always be greater than the I_p of the equivalent uniform section for a constant $I_{F(Y-Y)}$. In other words, if the quantity $I_{F(Y-Y)}$ is held constant in the equivalent uniform section, the value I_p is satisfied automatically.



Conclusions

(1) The maximum compressive strength of a Tee stiffener of uniform thickness when simply supported at the toe is secured when it has the minimum required flange width indicated in Fig. 4. It is then an "optimum stiffener".

(2) The maximum compressive strength of a Tee stiffener of non-uniform section when simply supported at the toe is secured when its "equivalent c/t ", flange width-thickness ratio, determined by Fig. 8 meets the requirements of Fig. 4. A simple approximation regarding the adequacy of this non-uniform Tee section can be made from Fig. 4 by considering the web thickness (and not the flange thickness) in the ratio c/t . This approximation is always on the safe side, and should be considered before proceeding to determine the "equivalent c/t ".

(3) The ratio of the critical stress to the maximum compressive stress of Tee stiffeners when simply supported at the toe and defined by Fig. 4 decreases from unity for short lengths ($a/h = 4$) to 0.75 for $a/h = 16$. Stiffeners with inadequate flange width have lower critical stress than the optimum section.

(4) Optimum stiffeners having a depth of web ($h/t \geq 40$) will develop an ultimate average stress that is equivalent to the compressive strength of a flat plate of the same dimensions as the web of the stiffener and simply supported at the edges by cylindrical columns. With a smaller web depth ($h/t < 40$) the

stiffener will develop the full compressive yield stress of the material.

(5) Optimum stiffeners with deep webs ($h/t > 40$) and moderate lengths ($a/h \leq 8$) remain essentially straight at failure, and are said to be stable. The required flange width in this case insures them against "laying over".

(6) Optimum stiffeners with deep webs ($h/t > 40$) and great lengths ($a/h > 12$) will "lay over", and fail by combined bending and twisting accompanied by local web buckling.

(7) All rolled non-uniform sections when compared with optimum uniform sections with the same web dimensions have values of polar moment of inertia I_p , and flange moment of inertia $I_{F(Y-Y)}$ that are equal to or greater than the minimum required. This makes them safe and adequate stiffeners.

TABLE II

CRITICAL STRESS OF OPTIMUM UNIFORM Tee SECTIONS

Model No.	Critical Load P_{cr}	Critical Stress $P_{cr} = P_{cr}/A$	Ultimate Stress $P_U = P/A$	Nomianl Value of h/t a/h		Ratio P_{cr}/P_U
5601	5000	38200	38900	20	4	0.98
5611	7500	38500	38400	30	4	1.00
5621	10500	38600	39700	40	4	0.97
5632	12000	37500	38400	50	4	0.98
5802	5000	35200	40000	20	8	0.88
5807	6500	33500	39400	30	8	0.85
5815	9000	34100	37300	40	8	0.91
5824	11500	32700	34500	50	8	0.95
5903	2500	29500	39000	20	12	0.76
5909	3500	29500	36600	30	12	0.81
5915	4000	28800	34300	40	12	0.84
5922	4600	26400	32200	50	12	0.82
5927	2200	27300	39400	20	16	0.69
5933	2500	23800	34300	30	16	0.69
5940	3500	25300	33200	40	16	0.76
5948	4000	22200	27300	50	16	0.76

TABLE III

"STRENGTH RATIOS" OF Tee STIFFENERS - VALUES TAKEN FROM EXPERIMENTAL PLOTS -

$\frac{c}{t}$	a/h=4	$\frac{h}{t} = 52$			$\frac{h}{t} = 41$				$\frac{h}{t} = 31$				$\frac{h}{t} = 22$			
		8	12	16	4	8	12	16	4	8	12	16	4	8	12	16
6	.574				.891	.744	.850	.880	.972		.844	.920	1.000	1.000	.930	.970
7	.706				.922	.798			.982						.967	
8	.836	.580	.680	.535	.948	.857	.907	.906	.990		.930	.946			.988	.986
9		.664			.974				1.000+	.975			1.000	1.000	.998	
10	.954	.751	.748	.674	.990	.925	.952	.924	1.000	.990	.966	.966			1.000+	.996
11		.840			1.000+	.952	.966									1.000+
12	.993	.884	.810	.764		.970	.984	.946		1.000+	.980	.987				1.000
13												.992				
14	1.000+	.980	.880	.848		1.000+	.996	.968		1.000	.990	.996				
16		1.000+	.946	.913		1.000	1.000+	.986			1.000+	1.000+				
18	1.000		.990	.962				.994								
20			1.000+	.987				1.000+								
22		1.000		1.000+												
24			1.000	1.000												
Max.	38400	34600	32400	28800	38800	37600	34200	33000	38500	39700	36800	34500	39000	39700	39000	39000
Avg. Stress																

+These values have been used to plot curves in Fig. 3.

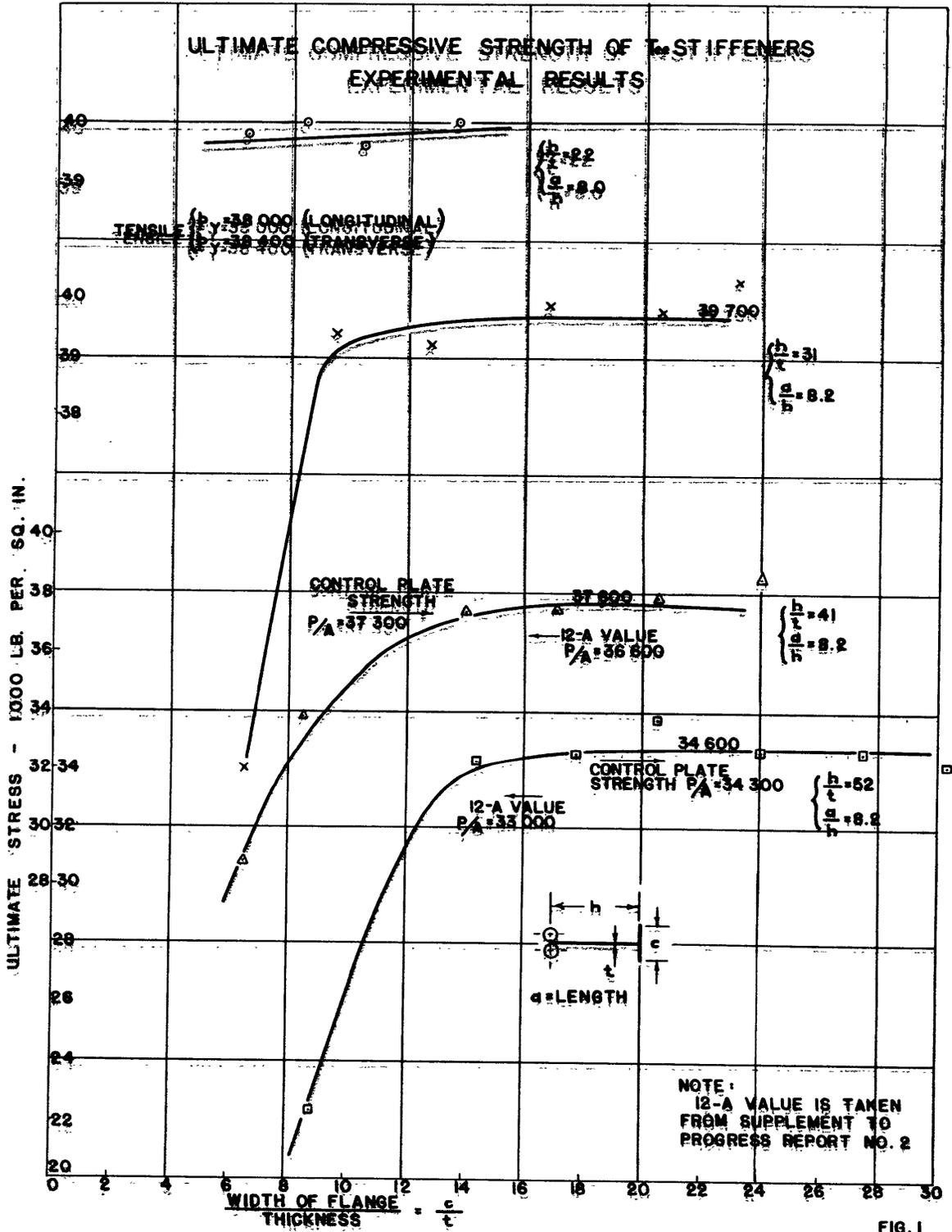


FIG. 1

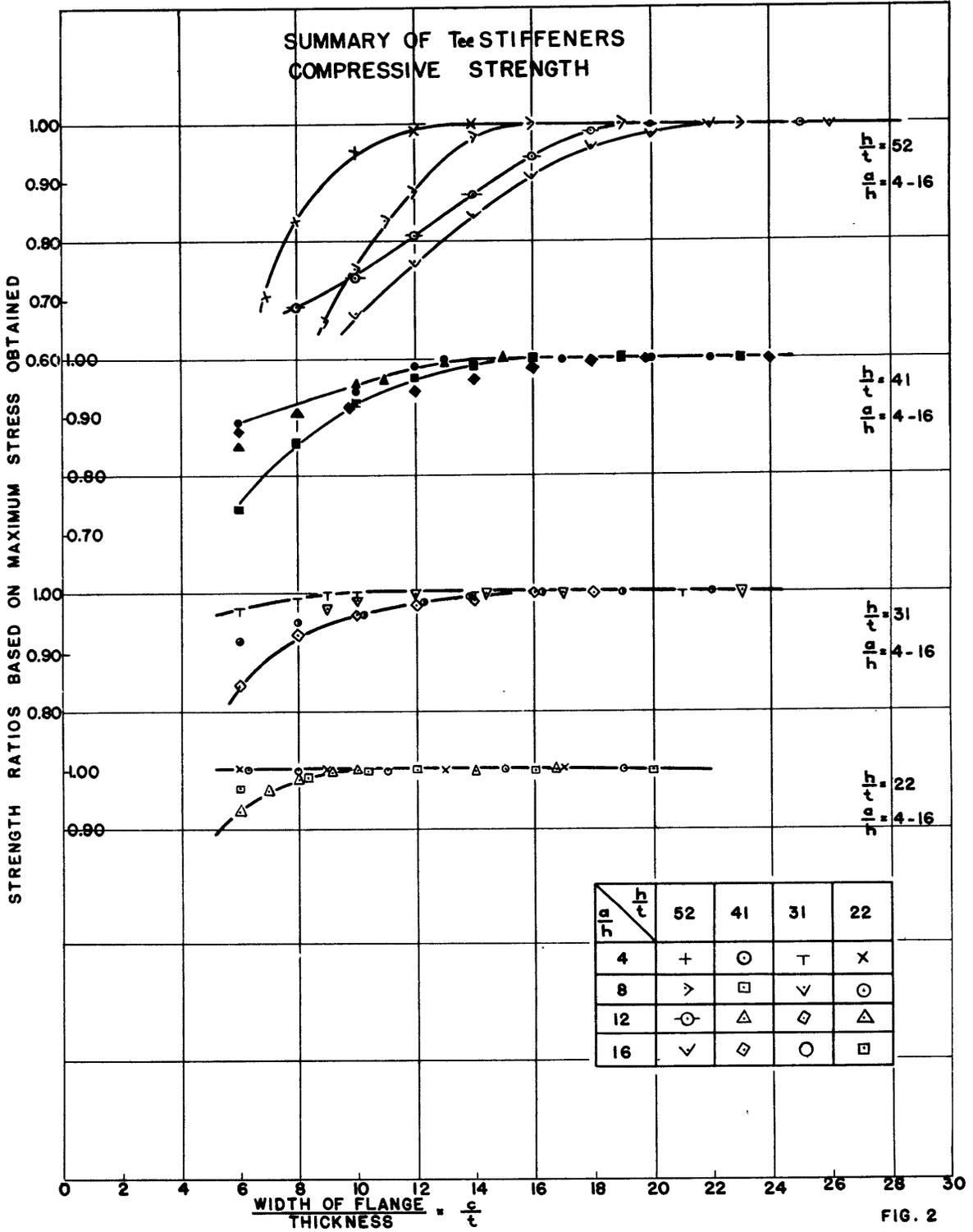


FIG. 2

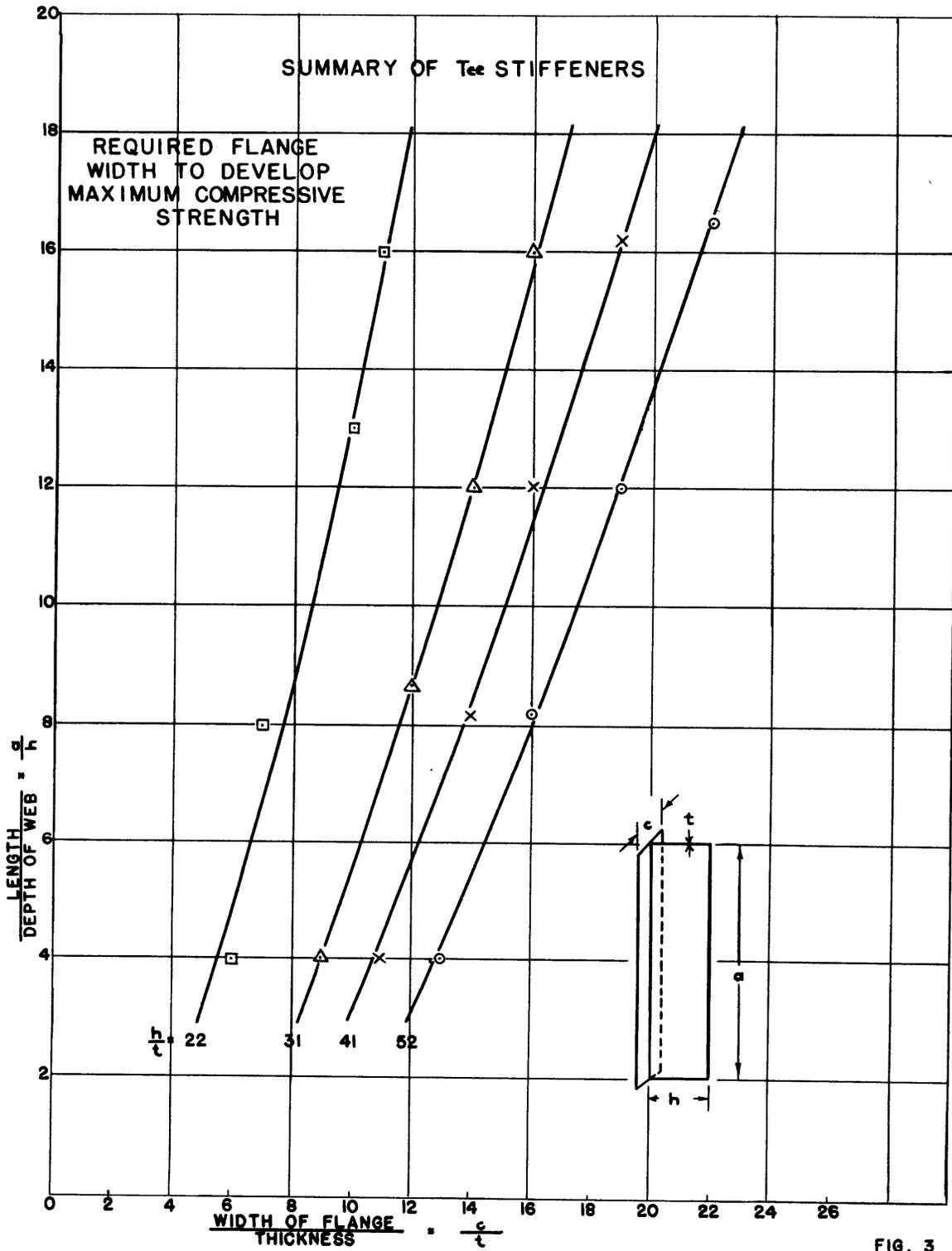


FIG. 3

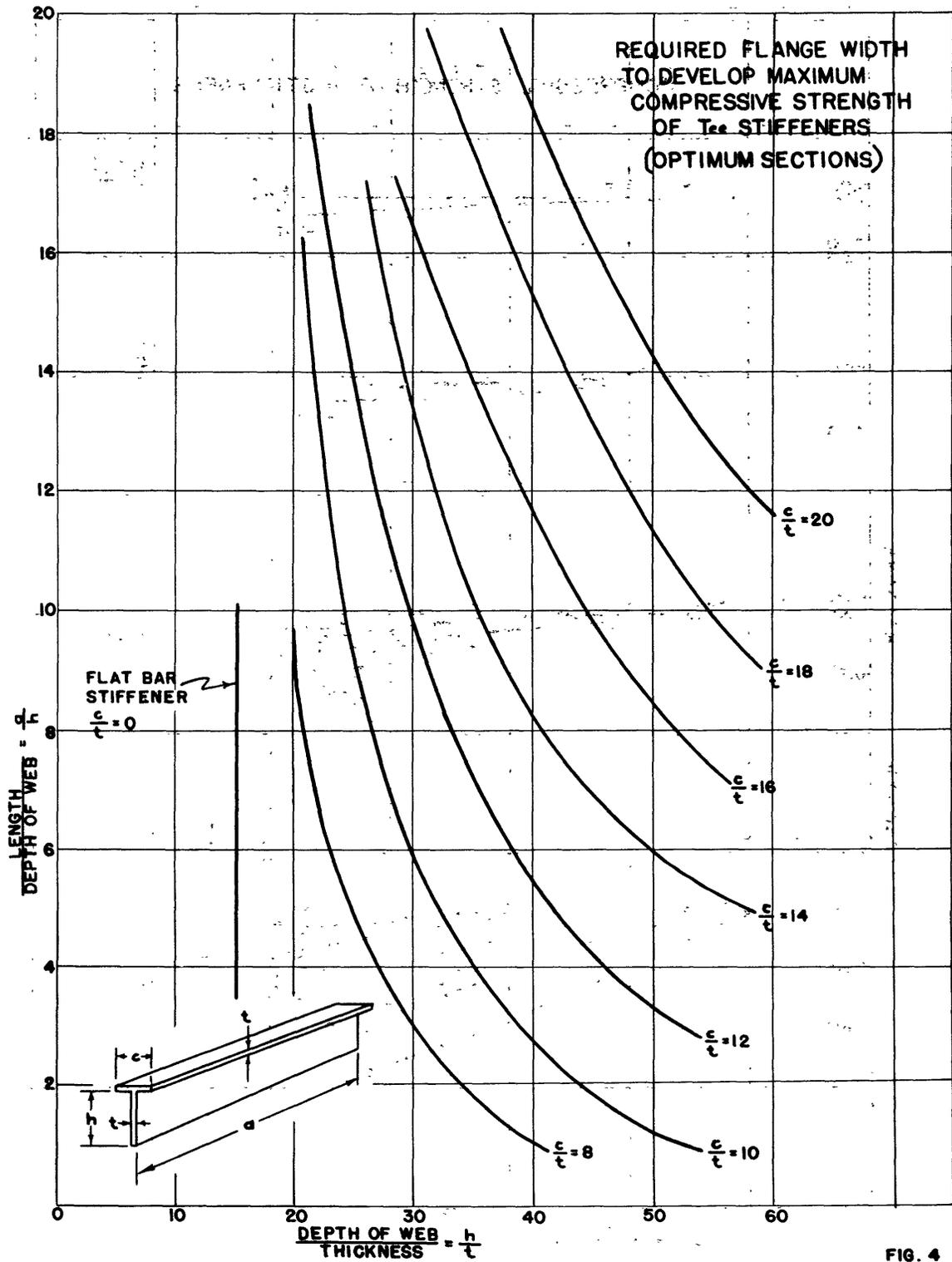


FIG. 4

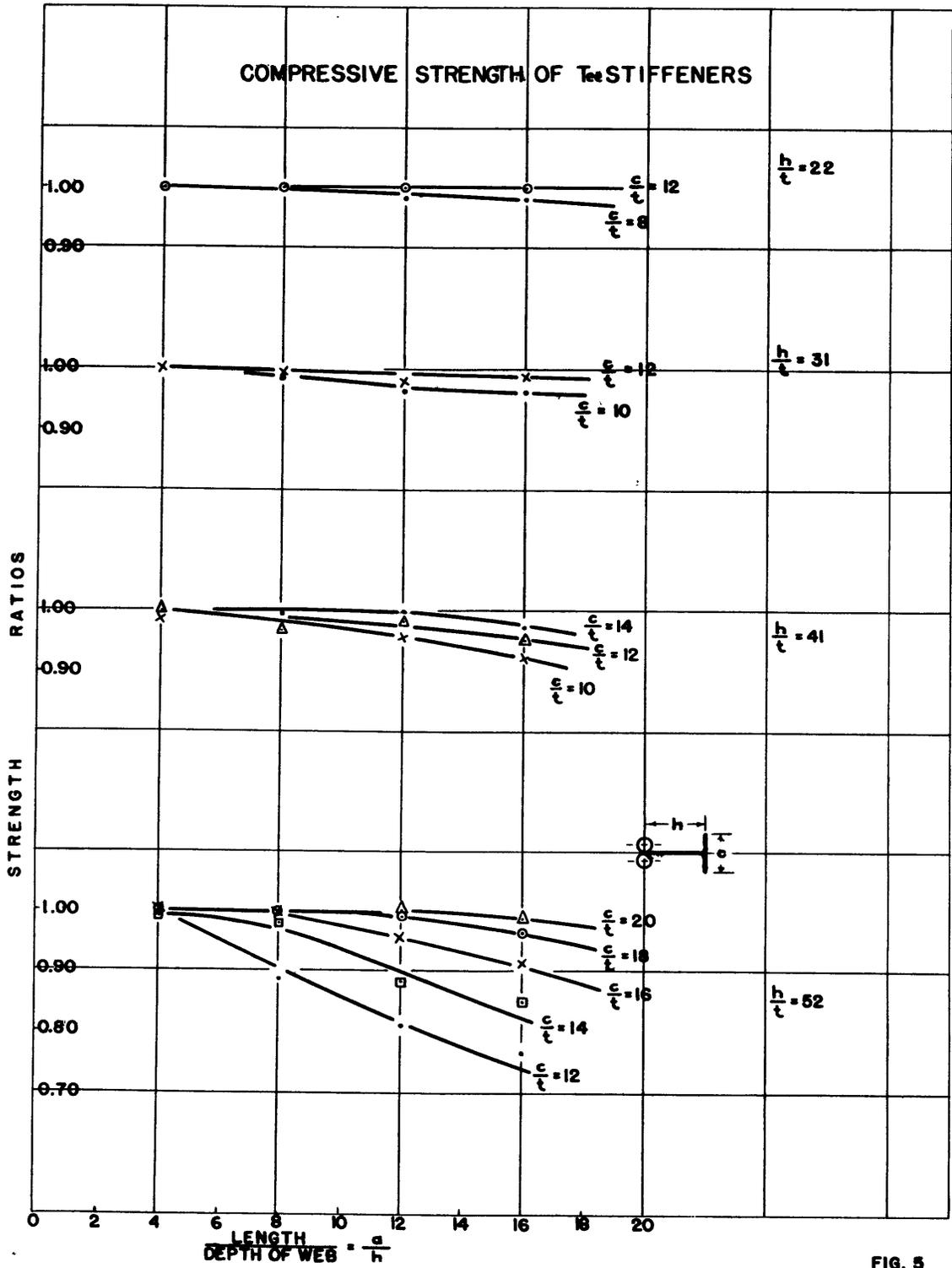


FIG. 5

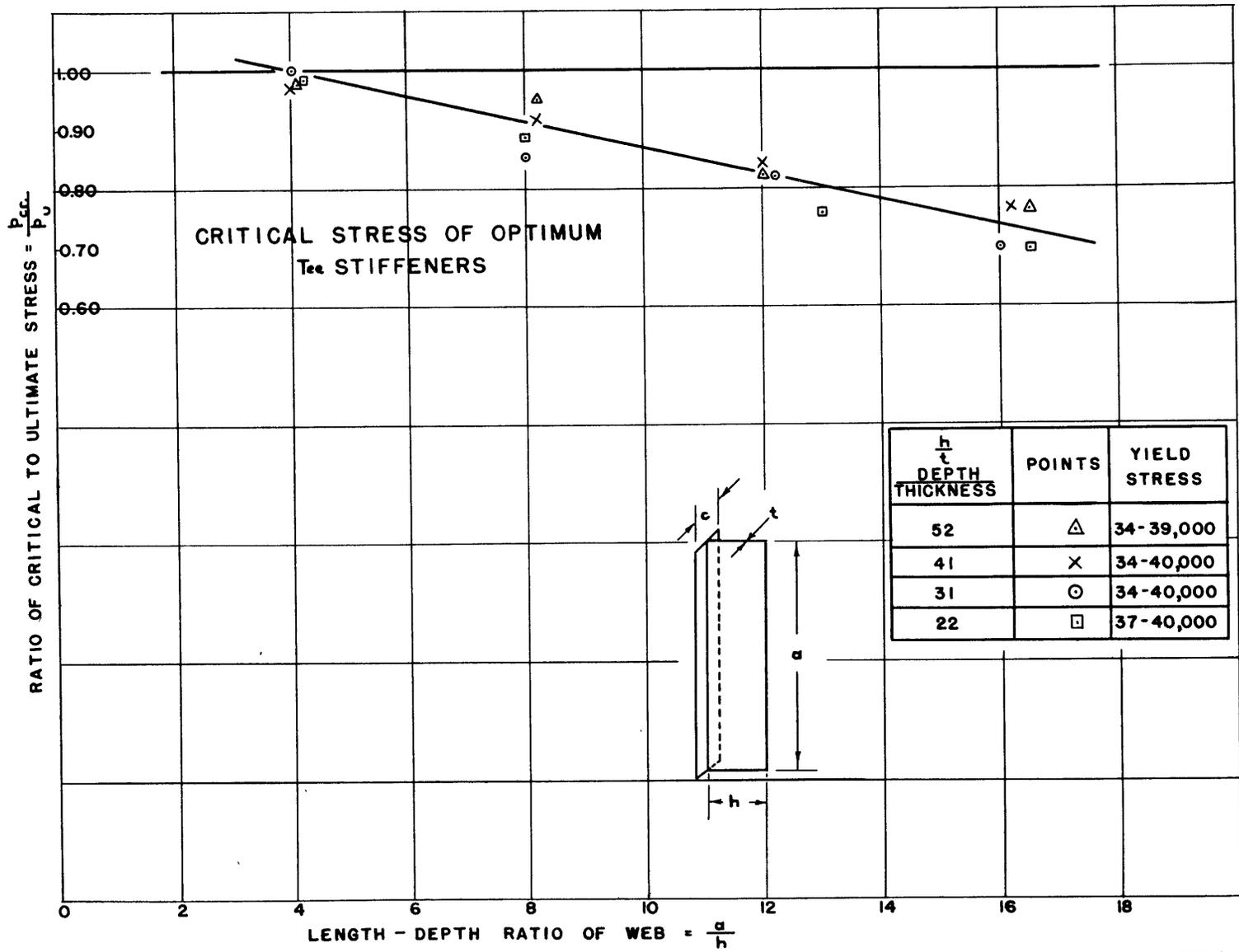


FIG. 6

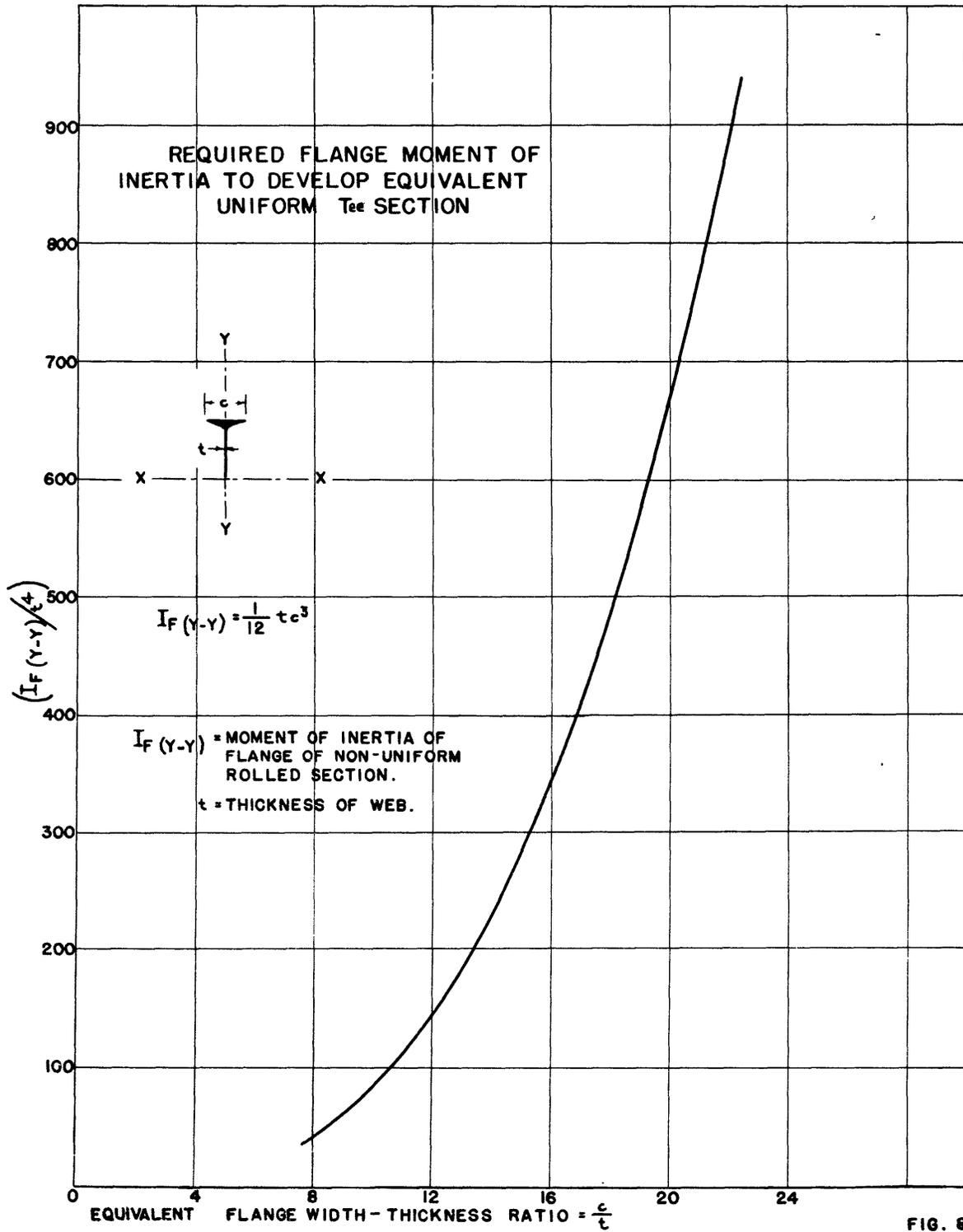


FIG. 8

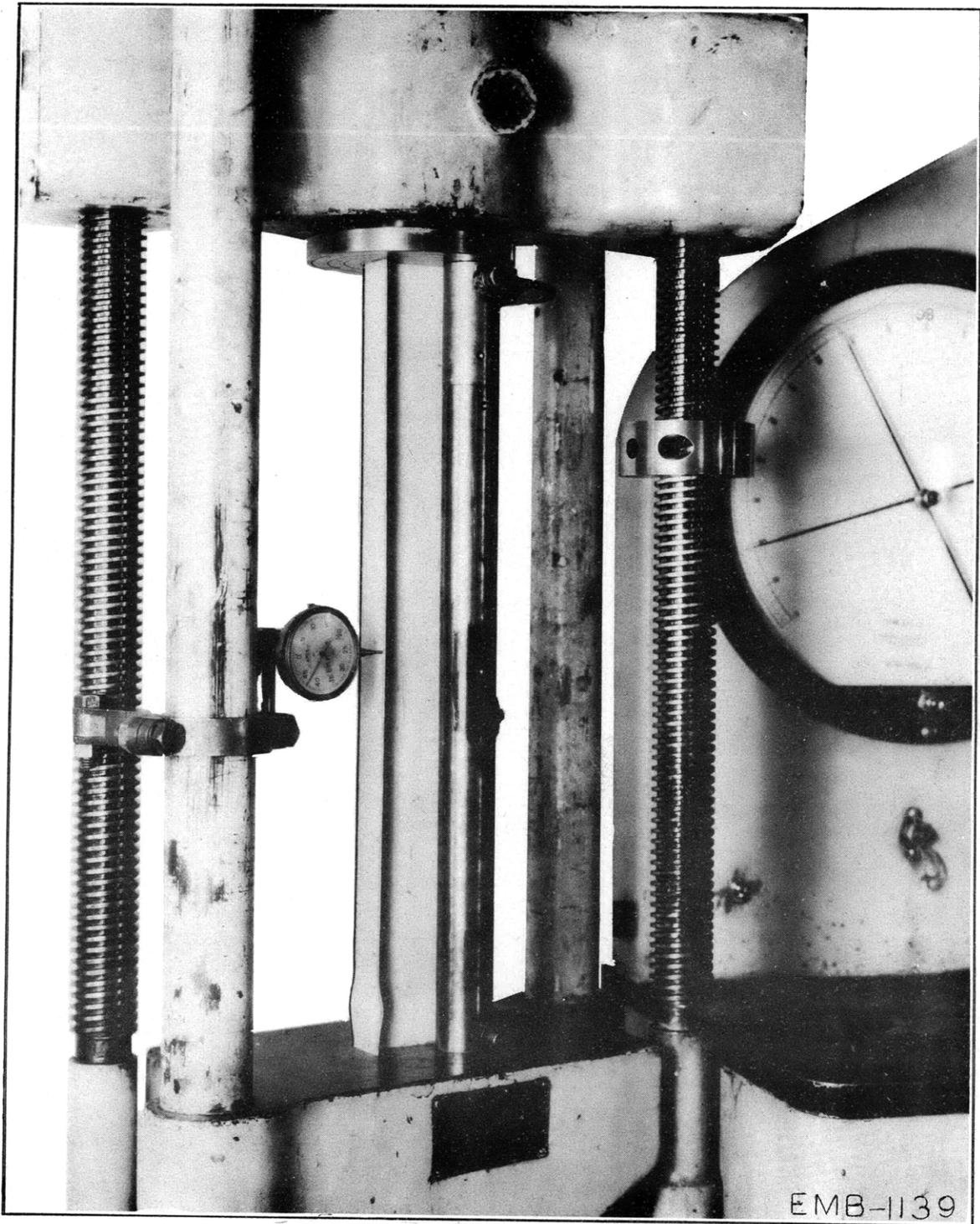


FIGURE 10

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