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UNITED STATES EXPERIMENTAL MODEL BASIN

NAVY YARD, WASHINGTON, D.C.

THE STRENGTH OF SHIP PLATING UNDER EDGE COMPRESSION

BY J. M. FRANKLAND

EXPERIMENTAL MODEL BASIN
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A Review of Published and Unpublished
Information to 1940.

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SUMMARY

A review is given of the experimental work on the strength of plating in compression, stiffened and unstiffened, on the strength of stiffeners, and on the strength of plating with lightening holes. The data are compared with the results obtained by other investigators in this field. An analysis is then made to draw conclusions useful to the designer. Unsolved problems are pointed out and recommendations made for further study. An appendix contains an illustrative application of this information in design.

INTRODUCTION

Since the end of 1932 the U.S. Experimental Model Basin has been conducting tests to determine the strength of longitudinally stiffened plating under compressive loading in the plane of the plating. From time to time the major results of these tests have been discussed in progress reports, (1) to (7).* During the course of the testing a considerable development of technique and an appreciable advance in understanding of the phenomena involved has taken place. At the same time independent investigations, particularly in the aeronautical field, have contributed materially to the subject from the theoretical as well as from the experimental point of view. It appears appropriate at this time to review the work that has been done in a more systematic and critical light than was possible at the time of writing the progress reports and to indicate how the earlier conclusions have been modified in the light of subsequent experimental work.

THE EXPERIMENTAL MODEL BASIN INVESTIGATIONS

A. Scope of the Tests.

The experimental work at the Model Basin has been devoted principally to the following phases of the problem:

(a) The strength of rectangular panels of flat plating supported either by edge constraints carrying no load or by longitudinal stiffeners. References (1), (2), (16), and (17) are concerned with this phase.

(b) The strength of rectangular flat plates with lightening holes, reinforced and unreinforced at the edge of the hole. References (4) (5), and (21) report the results of these tests.

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* Numbers in parentheses designate references at end of report.

(c) The strength of stiffeners, including the design of stiffeners to avoid local failure and the size of stiffener required to furnish adequate support to the plate. References (3), (6), and (7) describe the related experimental work. Reference (8) gives a theory developed at the Experimental Model Basin on which the basis of the design of stiffener sections is presented. This theory is corroborated by the tests. Reference (9), a translation of Timoshenko's analysis of the problem of finding the smallest stiffener adequate to support the plating, supplements this work.

The foregoing investigations were concerned only with flat rectangular plates supported longitudinally, that is, in the direction of loading. No tests of curved plates have been made.

A few preliminary tests on the effect of normal hydrostatic pressure on the compressive strength of plating have indicated a negligible effect of hydrostatic loading. This point, however, cannot be regarded as settled by these tests, since certain difficult problems of experimental technique remain to be solved and the specimens that have been tested do not cover an adequate range of conditions.

B. The Strength of Medium Steel Plating in Compression.

The most important of the characteristics which determine the strength of plating is the ratio of the panel width (or spacing of longitudinal stiffeners) to the plate thickness, denoted as width-thickness ratio, or b/t . At a b/t of 100, med ium steel plating (supported by stiffeners not farther apart than the transverse frames) will buckle into approximately square lobes at a stress of about 11 kips per square inch. This type of buckling has been described by Bryan and by Timoshen (10), and is discussed in Experimental Model Basin Progress Reports (1) and (2). I the stiffeners are strong enough, however, the combination will continue to take lo up to an average stress in the plating of about 19 kips per square inch. At a load corresponding to the latter condition, the specimen continues to shorten at constan load and failure follows. Such a condition is shown in Figure 1.

Buckling of the plating between stiffeners is not, therefore, an immediat cause of failure of the stiffened structure. What does cause the failure is the hi stress in the stiffener and in the plating at the stiffener. The plating between stiffeners shirks its share of the load because of the buckles, and throws it on th plating near the stiffeners and on the stiffeners. This results in peaks of stress in the plating in way of the stiffeners, and in valleys of stress in the unsupporte parts of the plate where the buckles are pronounced. The stress distributions of Figure 2 illustrate this condition for successive stages of loading of plating wit high values of b/t , say 100 or more.

The stress at which buckling of the plate occurs decreases rapidly as the values of b/t increase, and at $b/t = 200$ has fallen to less than 3 kips per square inch. When b/t is reduced below 100, the buckling stress rises and approaches the compressive strength of the plating.

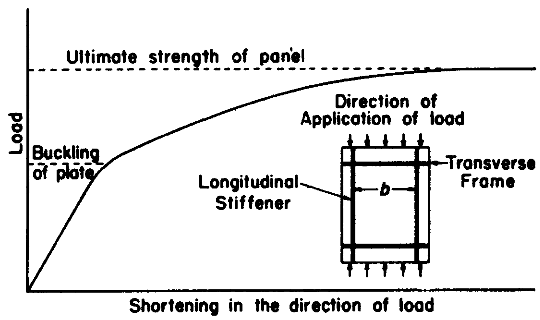

Figure 1 – Behavior of a Flat Plate Under Edge Compression

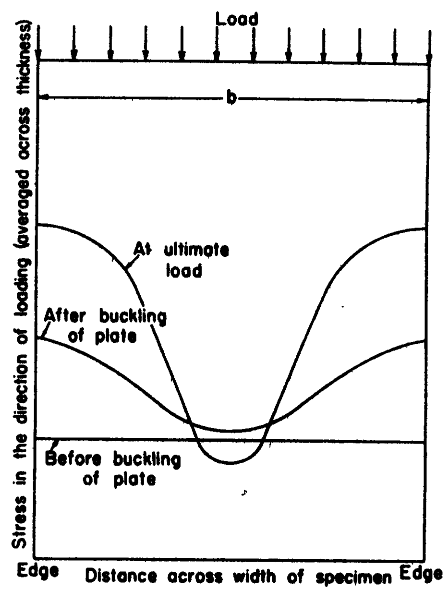

Figure 2 – Stress-Distribution Across Width of a Wide Plate Under Edge Compression

It is evident from Figure 2 that the shirking of load by the plating between stiffeners becomes more pronounced as the load is increased beyond the point at which the plate buckles. A structure in which the plate buckles at a low stress is clearly inefficient. In certain types of structures, for example aircraft, it may be necessary for reasons of fabrication and minimum allowable thicknesses to accept an uneconomically wide spacing of stiffeners. The naval architect, however, has ample sections at his disposal, and such poor economy is unnecessary.

Accordingly, we shall here deal with the lower range of b/t (ratios less than 100 for medium steel). In this range, plasticity of the material is an important factor in the strength of stiffened plating, just as in the short-column range of column strength [Reference (10), pages 156 to 165]. The effect can be noted in the observed buckling stresses, which agree well with Bryan's theoretical value, Equation [10] of Progress Report 1, for values up to some seventy per cent of the yield point of medium steel. This may be seen from the summary of test data, Figure 3. At higher stresses, the Bryan formula gives values higher than those observed. The curve of plate buckling in Figure 3 is based on the Bryan formula for the solid portion of the curve below $F = 0.5$, and for the dashed part of the curve on the type of modification suggested by Bleich [Reference (18), pages 216 to 223]. Insufficient data exist to establish definitely the dashed portion of the plate-buckling

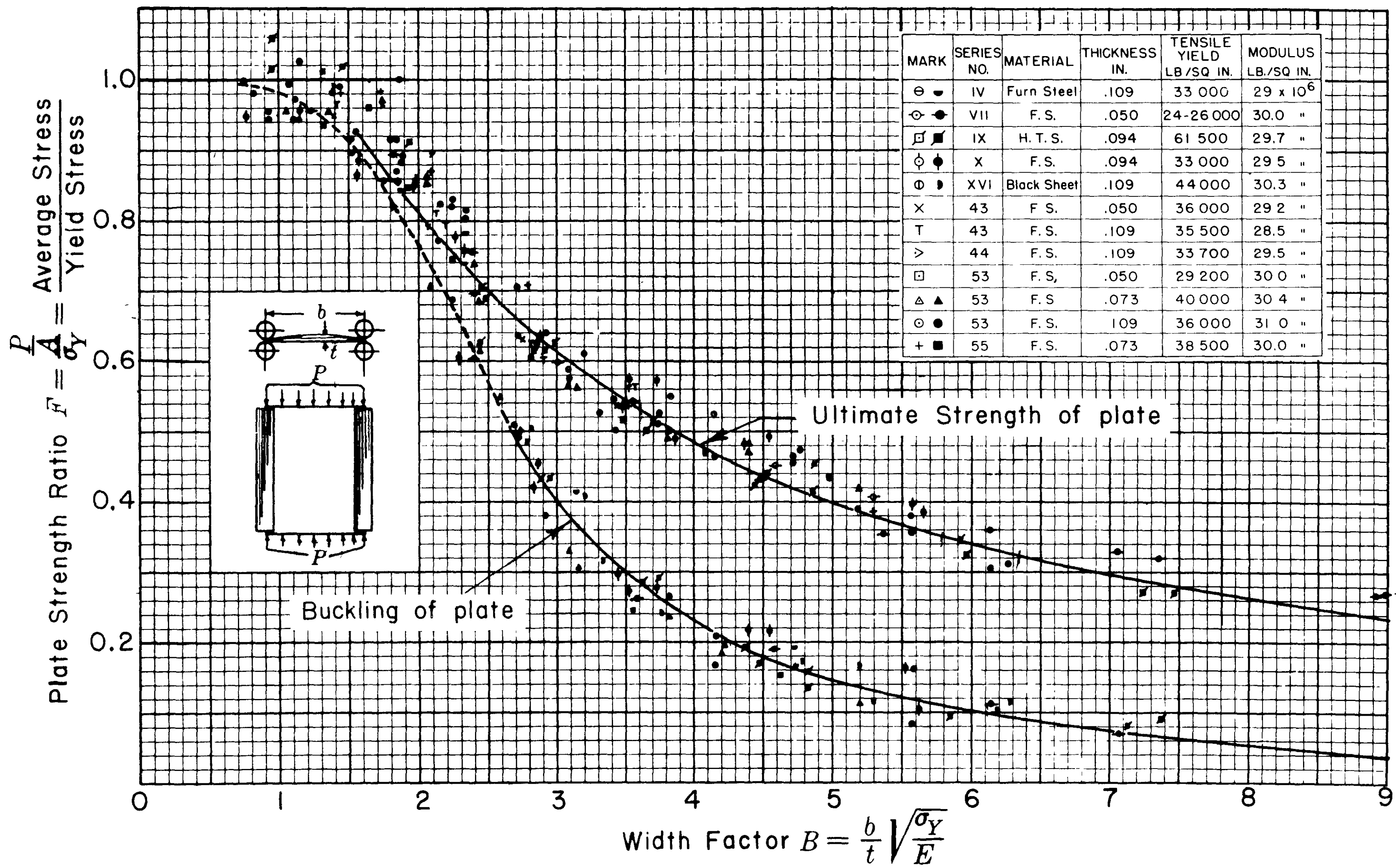

Figure 3 – Ultimate and Critical Compressive Strength of Flat Plates – Summary of Experimental Work

curve of Figure 3 by experiment, but enough information exists to justify reliance on this part of the curve as a good approximation.

Stiffened plating, even after buckling, continues to carry an increasing load until the structure collapses from high stresses at the stiffeners. Tests have been made of the ultimate strength of plating with edge supports simulating very rigid stiffeners. The strengths developed in terms of average stress are shown in the points along the upper curve of Figure 3. The data shown in this figure were obtained from tests of plates simply supported on the unloaded edges by heavy cylinders, as shown diagrammatically in the small sketch and in the photograph, Figure 4. These cylinders were shorter than the test specimens, and themselves carried none of the compressive load. In this respect the conditions of test differ from those of service in which stiffeners are welded or otherwise firmly secured to the plating.

The data in Figure 3 have been reduced to non-dimensional terms by the use of the following variables:

$$F = \frac{\sigma}{\sigma_Y} \qquad [1]$$

$$B = \frac{b}{t}\sqrt{\frac{\sigma_Y}{E}} \qquad [2]$$

where σ is the average stress in the plate at buckling or at the ultimate compressive load, σ_Y is the yield strength, b the stiffener spacing, t the thickness of the plating, and E is Young's modulus. F is then a non-dimensional strength factor and B is a non-dimensional width factor analogous to b/t . Ultimate compressive strength and buckling strength may then be represented by single curves which are unique for a specific kind of material.

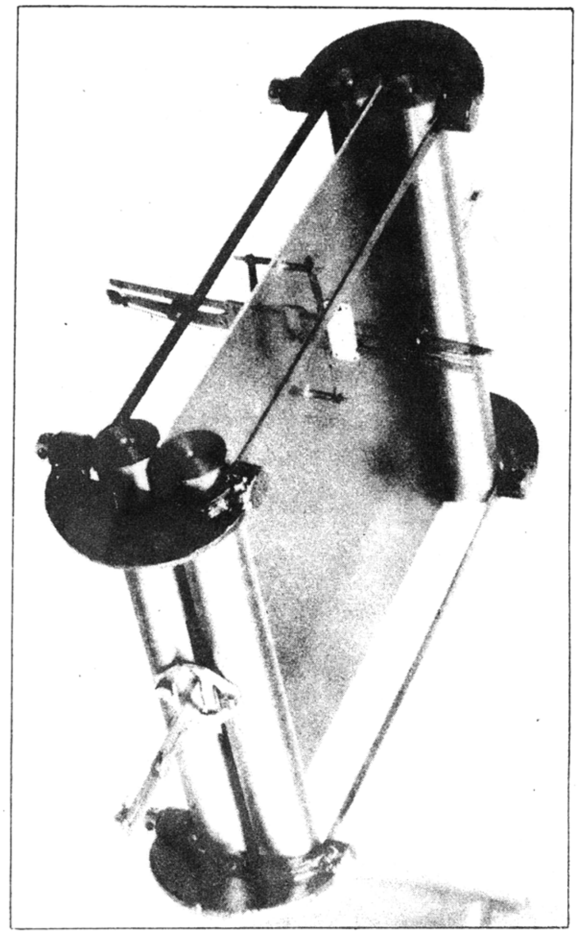

Figure 4 – Test Assembly of Plate and Edge Supports

The use of these non-dimensional variables should serve also to eliminate the scattering of data caused by variations in strength properties from one sample to the next. They will do so exactly if the compressive properties are known and the stress–strain curves of the material are affine, that is to say, speaking somewhat loosely, if the stress–strain curves are of the same shape*. This is approximately true for medium steel and for high tensile steel.

Figure 3, based upon an unpublished report by Vasta, formerly of the Experimental Model Basin Staff, shows the success of the non-dimensional method of interpreting these data. The curve representing the compressive strength data of Figure 3 has the following equation:

$$F = \frac{2.25}{B} - \frac{1.25}{B^2} \qquad [3]$$

and fits the data with reasonable accuracy. The consistency of the data at the high-stress end of the curve is not very satisfactory. The scatter is due largely to inaccuracy in estimating the compressive yield strength from the tensile stress-strain curve.

Compressive failure of the specimens occurs when the edge stresses in the plate reach a value high enough so that yielding occurs along the edges. For values of B less than 2, buckling and compression failure are practically simultaneous, as shown in Figure 3, and the plate can carry no appreciable load above that at which it buckles. From this a conclusion important in design may be drawn: for values of B less than 2, (b/t in medium steel less than about 60), the stiffener required to support the plate up to the limit of its compressive strength need only be strong enough to carry in itself, without buckling, a stress equal to the average stress in the plate at failure.

Such strength curves as those in Figure 3 are specific for a type of material. The ones shown are considered to be representative of medium steel. Strength curves for plating of other materials, such as aluminum alloy, differ somewhat from those of Figure 3 in the range of B values between 0 and 4. The curves are somewhat conservative for high-tensile steel, as indicated by the points for Series IX at low values of B, but this difference is not large.

For design purposes the ultimate strength curves in Figure 5 have been drawn for steels of various yield strengths. In the process, the ultimate strength curve of Figure 3, because of its assumed non-dimensional characteristics, has been

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* More precisely, two curves or figures are said to be affine when one can be transformed into the other by a linear transformation of the type

$$\begin{aligned} X &= c_1 x + c_2 y + c_3 z + c_4 \\ Y &= c_1' x + c_2' y + c_3' z + c_4' \\ Z &= c_1'' x + c_2'' y + c_3'' z + c_4'' \end{aligned}$$

where the coefficients c_1, c_2, ... are constants. In its most general form, such a transformation is equivalent geometrically to a rigid body displacement accompanied by a shear and a change in scale along each axis. For example, all parallelograms are affine, and all ellipsoids likewise.

extrapolated to predict the behavior of steels having higher yield strengths than any actually tested. This procedure is probably somewhat conservative at the high yield strengths.

Figure 5 represents the best information to date and supersedes the similar curves of Progress Report 2, (2).

The objection has been raised to the type of test reported in Figure 3 that the test conditions impose no lateral restraint against transverse contraction in the plane of the plate. If not thus restrained, a buckled rectangular panel will contract transversely across the crest of the buckle. If laterally adjacent panels of plating are present, there exists a lateral restraint opposing this contraction and constraining the panel to remain rectangular. In such a case the depth of the buckle is reduced and an increase in effectiveness of plating under compression can be expected. The corresponding increase in strength would be appreciable only when the buckling is marked, say for B greater than 2.5.

Some tests to investigate this point have been carried out at the Experimental Model Basin with specimens stiffened with several longitudinals, but at such low values of B that no differences were observed with change in the number of panels. In the well-supported plate representative of efficient ship construction, where B is less than 2, this effect should be of no importance. It is probable, however, that the strength curve of Figure 3 is conservative for this reason for the range of B greater than 2.5.

C. The Strength of Stiffeners and Stiffened Plating

In the experimental work described so far, the compressive load was carried by the plate alone, and the support offered was effective only in a direction normal to the plate. In actual ship assemblies the supporting members are themselves subject to axial compressive load.

Final collapse in compression in stiffened plating is caused by high stresses in the stiffener and the adjacent plating. To obtain high strength in stiffened plating it is clearly necessary, then, to have stiffeners which furnish adequate support to the plate and which can themselves carry high direct stresses without premature collapse. Although stiffeners and plating receive separate consideration and are separately varied in the tests, it is their mutual action in the assembly which produces the desired results. It is necessary to bear this point in mind in planning tests such as these even more than has been the case in the past.

The strength of longitudinal stiffeners supporting plating under compression is dependent on the following factors:

(a) Adequate rigidity of the stiffener against general instability, either by column failure or by twisting failure (tripping, laying over).

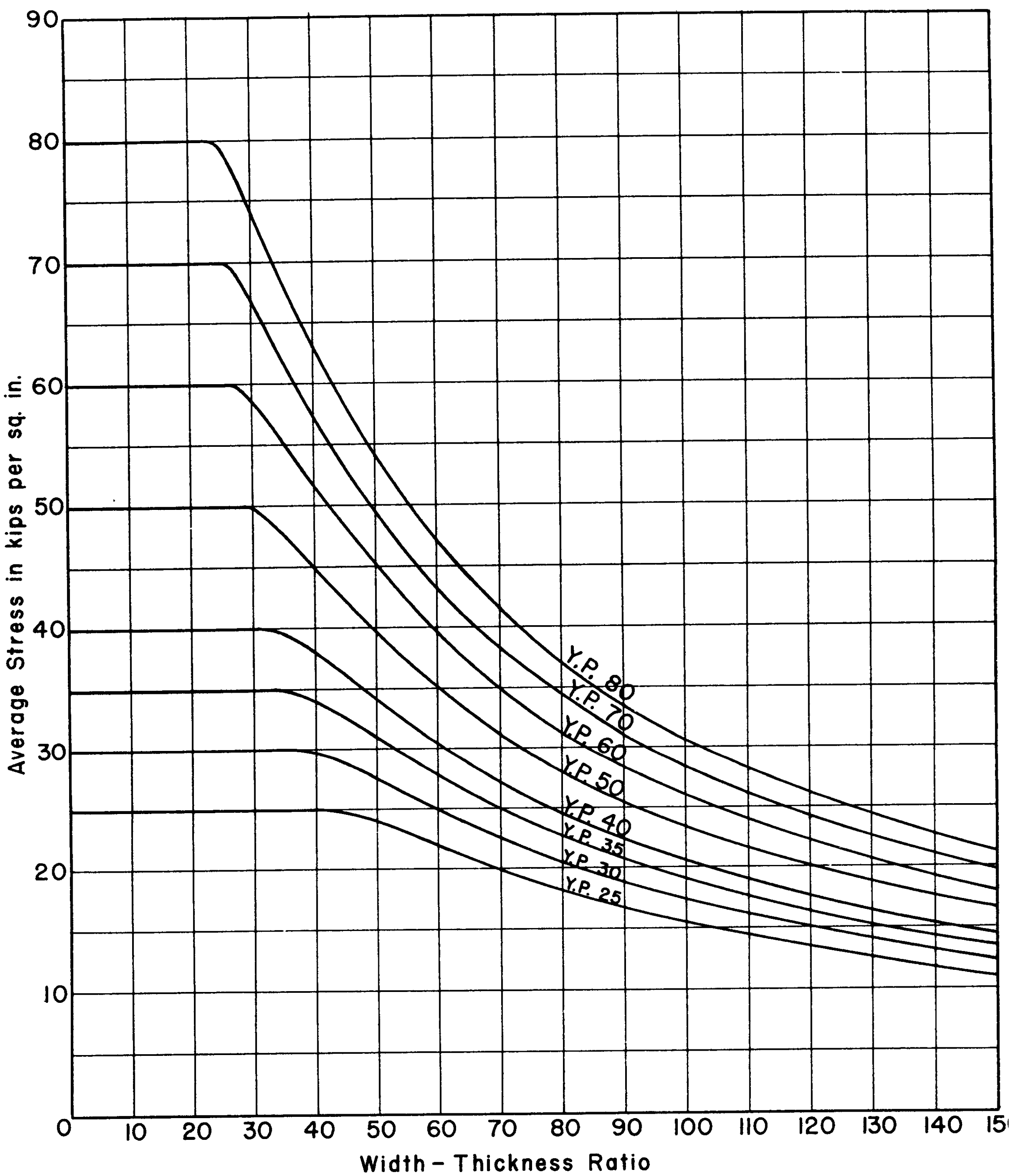

Figure 5 - The Ultimate Strength of Steel Plating Under Edge Compression. The various curves are for steels having the yield strengths indicated, in kips per square inch.

(b) Choice of suitable proportions of web and flange to obtain a sturdy section, that is, one which will fail by general instability prior to local crippling.

These features have previously been considered in References (3), (6), (7), (8), (9), (15), and (16).

The following points indicate the scope of the Experimental Model Basin work on this subject:

1. Investigation of the strength of channel-section columns. This gives the results of simple flat-end tests and tests of flat-end specimens supported by flexure plates at the third points. This type of section in which the middle bay only is the test section, was intended to reduce the end fixation in the column in order to approach pin-end conditions. The data include results on sections with inadequate flanges or with unusually thin webs. For these specimens the failure was not a simple column failure, but was complicated by local effects. The eccentricity of load application was great enough to cause failure in the upper end of most of the three-bay specimens. For these reasons the strengths observed are considered to be low, although the "lower limit of the probable ultimate strength" chosen for practical design of columns (20) was still lower. Even so, the tests point to column strengths greater than those on which the Bureau of Construction and Repair column tables are based.

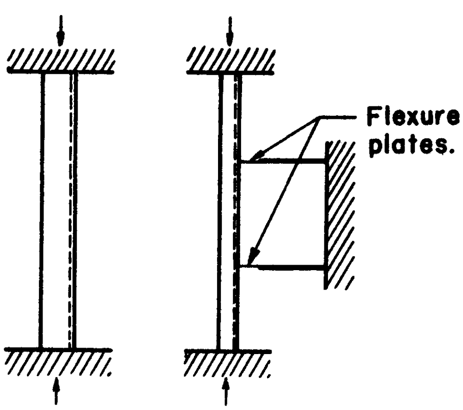

Figure 6 - Column Tests

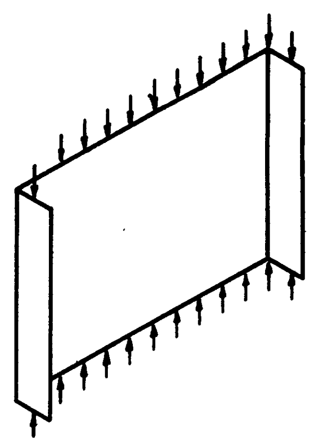

Figure 7 - Plates Stiffened with Flat Bars

2. An extensive series of tests was carried out on plates stiffened with flat bar stiffeners; these have been summarized (16), not formally reported. The investigation was directed to finding the proportions of stiffener which would develop an ultimate strength in compression of a

stiffened plate as great as that found in tests with cylindrical standards, as shown in Figure 4. The tests indicated that a depth of 15 to 20 bar thicknesses gave the optimum stiffener. A shallower stiffener does not permit the development of the maximum radius of gyration, while a deeper stiffener fails prematurely by waving of the outstanding edge. Attention is drawn to the requirement in the Bureau column tables that outstanding or free flanges shall be no wider than 10 thicknesses. These tests indicate that wider flanges would be acceptable.

Multiple-bay specimens including up to five bar stiffeners were tested in this investigation to determine the effect on the strength of lateral restraint in the plane of the plating. As all the specimens had a width-thickness ratio of 60, there was little development of plate buckling prior to failure. At this width-thickness ratio, the specimens with single bays of plate failed at the same stress as those with multiple bays, a result to be expected at values of b/t no greater than this.

3. The problem of the minimum rigidity of stiffener required to restrict the buckling of the plate in a stiffened panel to the plating between stiffeners is discussed in Reference (9). This is a translation of a theoretical analysis by S. Timoshenko. The assumption is made that all stresses lie entirely within the elastic range, and the conclusions are accordingly open to question at stresses above 75 per cent of the yield strength. However, some tests at the Experimental Model Basin with stiffeners of low slenderness ratio were not in conflict with this theory at high failure stresses. Further testing is needed to establish the applicability of this theory to highly stressed structures.

4. The optimum cross-sectional shape and dimensions for stiffeners have been extensively studied, with results reported in References (6), (7), (15), and (16). A theoretical analysis of this problem has been carried out by Windenburg (8), leading to practical criteria in good accord with the experimental data that have been obtained. Figure 8 is reproduced from this report and summarizes the important conclusions for tee stiffeners in which the web and flange are of the same thickness. The curves give the flange proportions required to support a given web or to prevent laying over (twisting failure) of the stiffener. For depth-thickness ratios of the web less than $c/t = 70$, the flange proportions on the chart are dictated by twisting failure. With deeper webs, buckling of the web occurs before twisting failure develops. The curves for flange widths of $f/t = 8$ and $f/t = 10$ are broken off at the lower end at the point where these narrow flanges fail to develop the full strength of the web. With wide flanges above $f/t = 30$, failure

is precipitated by local buckling of the flange. In the case of webs deeper than c/t = 50, local failure of the web will occur below the stress for column failure at a slenderness ratio of 40 or less.

Attention is drawn to the fact that Figure 8 applies to medium steel only. For high-tensile steel it must be modified, and definite results can be given only after additional tests have been made.

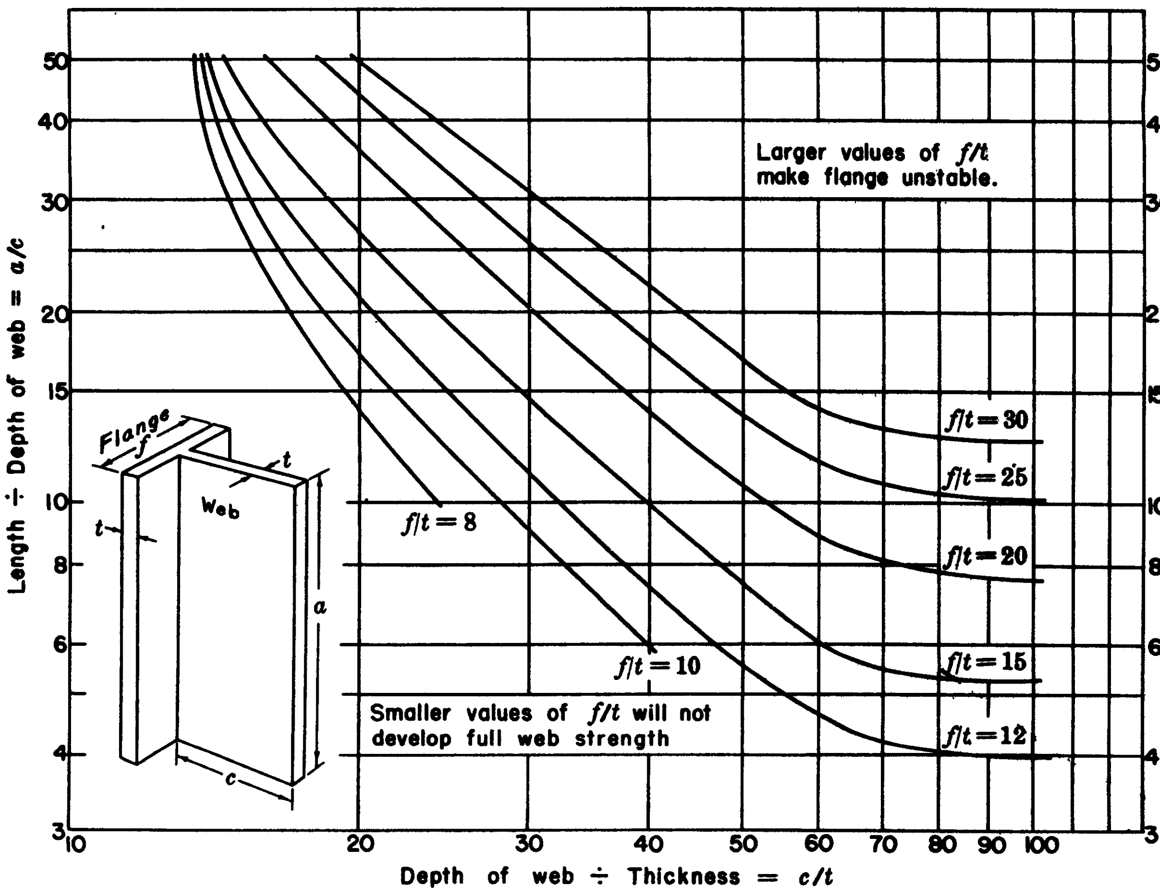

Figure 8 - Flange Proportions Required for a Tee Stiffener in Medium Steel.

5. Experimental Model Basin Report 452 gave the results of compression tests of four structural assemblies representative of ship-bottom construction. Premature failures in the connections occurred in two specimens. The other two specimens failed at loads in excellent agreement with those predicted from previous tests of stiffeners and plating. Wrinkling between increments of intermittent welding observed in these tests points to the inadvisability of discontinuous welding for attachment of members heavily stressed in compression. It

was observed that the channel-section stiffeners showed a tendency to lay over, which was more pronounced than in the case of the tee-section stiffeners. The effect of assymmetry in stiffeners would always be expected to favor such action, but other tests in addition to the short series here mentioned should be made to clarify details.

The design of stiffeners for plating in compression is closely related to the design of columns. Uncertainty in knowledge of column strength is reflected to an equal degree in the stiffener problem. The work described here is insufficient to dispose finally of the question. The choice of stiffener sections by the designer has been in a large measure covered by the work on the optimum proportions of cross section (8), but the choice of the proper depth of stiffener for a given length is still not free from uncertainty.

D. Lightening Holes in Plating Under Compression.

The effect of lightening holes on strength and the criteria for stiffening the edges of the holes have been investigated experimentally for plates loaded in pure compression (4), (5), and (20). If there is no stiffening on the edge of the hole, the minimum section A–A through the hole in Figure 9 will behave like two flat-bars, in that the net section will carry the yield strength of the material at failure if the width of the remaining plate at each side of the hole does not exceed 15 plate thicknesses. Should the widths of the remaining pieces of plate exceed 15 to 20 plate thicknesses, failure will occur at the minimum section at an average stress on the net section less than the yield strength, and this stress will depend somewhat on the shape of hole. Circular holes are somewhat stronger than elongated holes in this range because the length of unsupported edge is less.

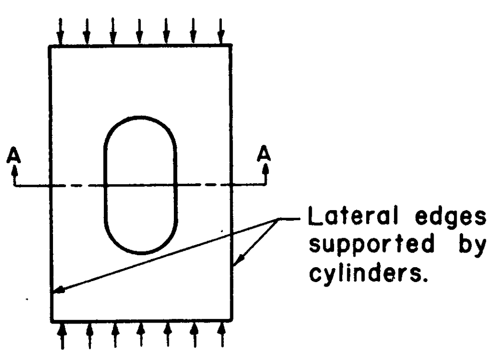

Figure 9 – Plate with Lightening Hole

There are thus two reasons for stiffening the edges of lightening holes:

(a) To make up for the reduced section of the plate at the hole,

(b) To support the plating at the edge of the hole in cases where the remaining pieces of plating at the net section have width-thickness ratios greater than 15.

Experimental data (5) and (21) on the effect of using different sizes of flanges to stiffen lightening holes are in accord with the theoretical curves of Figure 8.

Attention is drawn to the fact that the preceding conclusions refer only to members in pure compression, and that the presence of appreciable shear in addition to compression may call for modifications in these criteria.

OTHER WORK IN THIS FIELD

The state of knowledge among naval architects with regard to compressive strength of plating was summarized by Rossell in 1935 (14). This paper gives an excellent historical survey and serves as a good introduction to the subject for those concerned with questions of design.

Experimental and theoretical work of importance is being actively carried on in the aeronautical field, in Germany and the United States in particular. References (11), (12), and (13) survey the current activity. However, the interest in aeronautical circles is centered largely on a range of width factor B greater than used in current ship design. An even more important point of difference is that the stiffener stresses at failure are relatively lower (i.e., smaller fractions of the yield strength) in aeronautical construction.

The latest data in the aeronautical field (11) and (13), confirm Marguerre's formula for the efficiency of plating:

$$\frac{\text{Average stress in plating}}{\text{Stiffener stress}} \equiv \text{Efficiency} = \sqrt[3]{\frac{\sigma_{crit}}{\sigma_s}} \qquad [4]$$

in which σ_{crit} is the buckling stress of the plate and
σ_s is the stress in the stiffener at the load for which the efficiency is to be calculated.

This formula appears to hold quite satisfactorily for σ_s less than 85 per cent of the yield strength and for B greater than 2.5. This result is at variance with the Experimental Model Basin data shown in Figure 3, since the Marguerre formula gives values which are about 20 per cent high. The reason for this discrepancy is believed to lie in the following differences in the features of the tests:

(a) Higher stresses at the stiffeners or at the supported edges of the sheet in the Experimental Model Basin tests.

(b) Presence of lateral restraint of the plate panels (furnished by adjacent panels of plating) in the aeronautical tests.

(c) Emphasis in the aeronautical tests on higher values of B.

(d) Possible specific effects of the material (strong aluminum alloy compared with steel). This, however, must be a minor point, since the curve of Figure 8 of Reference (2), giving the strength of duralumin plating, is likewise appreciably lower than the aeronautical tests would indicate.

A particular type of major instability of open-section compression members has recently been made the subject of a new treatment by H. Wagner. This is the twisting failure exhibited by short members with sections of low torsional rigidity, particularly channels and angles, and even tee stiffeners at low slenderness ratios. Failures of zees by twisting are described in detail in Reference (13). In many cases laying-over of sections is due to a failure of this type. The subject has received some attention at the Experimental Model Basin, having been taken into account in the theoretical study of Reference (8), but there is need for more investigation of this type of failure.

An important contribution that the aeronautical researches have made lies in the light thrown on the subject by the development of theoretical analysis. Comparisons of some of these theories with detailed experimental observations are reported in Reference (13). The theory appears to furnish a reasonably good basis for planning definitive test programs with much saving in number of specimens and testing time.

It is highly desirable that the existing gaps and discrepancies between the results of Experimental Model Basin research and those of aeronautical research be removed in order that the results of the aeronautical research may be used as a check on the Model Basin studies.

APPLICATION OF EXPERIMENTAL RESULTS IN DESIGN

The use in actual design of the experimental and theoretical data described above depends in large measure upon the general approach of the designer in reducing the design problem to a form susceptible of analysis by conventional methods. The approach adopted here is indicated briefly in the next paragraphs, following which will be given the conclusions valid in design which can be derived from the existing information.

Compressive loading raises questions of detail design which do not occur when attention is given wholly to tensile action. A sufficient total area of section is naturally the first concern. Beyond that point it is necessary to ensure that the local construction be sturdy, that is, that local failures will not occur below loads which the major structure should be capable of carrying. This involves distribution of the material over the section, and the use of plates and shapes of optimum properties on a given sectional area. The results are then checked by calculation of the strength after the section has been designed.

It is desirable that the calculations should be reducible to the conventional girder analysis of ship stresses. This may be done in several ways. For example, from a given design cross section one may derive effective values of area by reducing the areas of actual components in proportion to their efficiencies, and may then apply the simple beam theory to the reduced section. On the other hand, the designer may use the actual cross section and express the strength characteristics in terms of an average or nominal stress at failure. In other words, one may deal either with a reduced area at maximum stress, or with the full area at a reduced stress value. In aeronautical practice the first approach is almost universal, since the second method in some cases leads to appreciable errors. For the well-stiffened structures characteristic of present longitudinal construction in ships it is believed, however, that the simpler second method is quite satisfactory. This approach has accordingly been adopted as the basis for interpreting the Experimental Model Basin test results for use in design.

The case of individual shapes used to stiffen plating should be distinguished from compartmented construction such as that formed by the longitudinals in double bottoms. In the latter case, the longitudinals and the inner and outer bottoms are all to be considered as plating when applying the results of the Experimental Model Basin investigations. Angle bars connecting these plate components are so well supported that their strength in compression may be taken as the compressive yield point of the material. Plate elements in the compartmented areas should be subdivided by longitudinal stiffeners of adequate section so that the plating will be able to carry a high average stress when the boundary bars reach the yield strength.

To do this calls for sufficient stiffening to achieve a width factor B of less than about 2 in the plating in order to develop a plate strength of 80 per cent or more of the yield strength. The criterion is properly expressed in terms of B rather than b/t, since the appropriate b/t values will vary with the yield strength of the steel used. For medium steel with a yield strength of 34 kips per square inch the stiffener spacing should not exceed 58t if the ultimate strength of the plate is to be maintained above 80 per cent of the yield strength, as is shown in the design chart of Figure 5. On high tensile steel plating the spacing of stiffeners must be closer if the metal is to be loaded successfully to a proportionately higher average stress; with a yield strength of 50 kips per square inch, it should not exceed 49t if the same efficiency of 80 per cent is to be achieved.

The average stress to be allowed should be chosen on the basis of the strength curves of Figure 5. This is done by reducing the assigned maximum allowable stress by the ratio of the curve value of plate strength at the given stiffener spacing to the specification yield strength. The spacing of stiffeners required to develop given average strength at failure for various values of the yield strength can be taken from the curves of Figure 5. The basis for these is the non-dimensional equation

$$F = \frac{2.25}{B} - \frac{1.25}{B^2}$$

derived from the tests on plate panels (see Figure 3).

Stiffener sections are to be so chosen that, in combination with the plate, they will exhibit a column strength sufficient to develop the plate strength corresponding to the stiffener spacing. The principal question here is the determination of the appropriate radius of gyration of the equivalent column. Timoshenko (9) suggested the use of the radius of gyration of the stiffener section alone, taken about an axis lying in the adjacent surface of the plating. It is believed that this estimate of radius of gyration errs on the high side in many cases. It is recommended, for B less than 2, that the radius of gyration be calculated for the combined section of stiffener plus the associated plate extending halfway to the adjacent stiffeners and neglecting any curvature in the plating. The axis about which the radius of gyration should be calculated is the centroidal axis of the total section.

The stiffener should be considered as pin-ended at the transverse members since between successive frames the stiffener may be deflected inward and outward alternately. The slenderness ratio is calculated on the basis of the spacing of these frames. To develop the strength of plating shown in the design chart of Figure 5, this slenderness ratio should not exceed 40. Should stiffeners with slenderness ratios above the limit recommended be necessary in some locations, they will not develop the plate strength shown in Figure 5. In such cases, the allowed strength of plating taken from the curves should be reduced in the ratio of the column strength at the higher slenderness ratio to the strength of a column at a slenderness ratio of 40 by whatever standards of column strength are in actual use by the designer.

The sections used as stiffeners should meet the requirements of Reference (8) with regard to crippling strength, ability to perform as a sturdy column, and strength under conditions of twisting failure. A guide to the design of tee sections for axial loading is given in Figure 10, in which data from Figure 8 have been supplemented by curves calculated for a flange thickness t twice as great as the web thickness h. The figure shows that flanges of the same area in these two cases are about equally effective in stabilizing the web, except in the case of very narrow flanges. The width-thickness ratio f/t of the flange should not exceed 30, beyond which flange buckling may be expected. Narrow flanges will stabilize only shallow webs, as is shown by the curves for $f/t = 7.5$, 8, and 10 (in the case of $t = h$). These curves are accordingly cut off at the point where the web reaches the maximum depth which the flange is capable of supporting. The other curves are stopped at a web depth c/t of 50, since deeper webs will buckle prior to general instability of the stiffener. Should deeper webs be necessary, a longitudinal stiffener should be provided along the web.

Figure 10 applies only to the case of sections with flanges of uniform thickness. The results can be extended to rolled sections with tapered flanges by the use of an effective width-thickness ratio f/t for the flange equal to 0.9 of the width divided by the mean thickness.

Reinforcement of the edges of access holes by means of symmetrically placed flanges can be designed by the same methods as those recommended here for tee stiffeners without allowance for benefit of curvature. The minimum section should be chosen to meet the requirements of Figure 10. Holes for lightening the webs of compression members should be avoided from the point of view of strength wherever this is possible, as more weight can be saved by accurate design of intact webs.

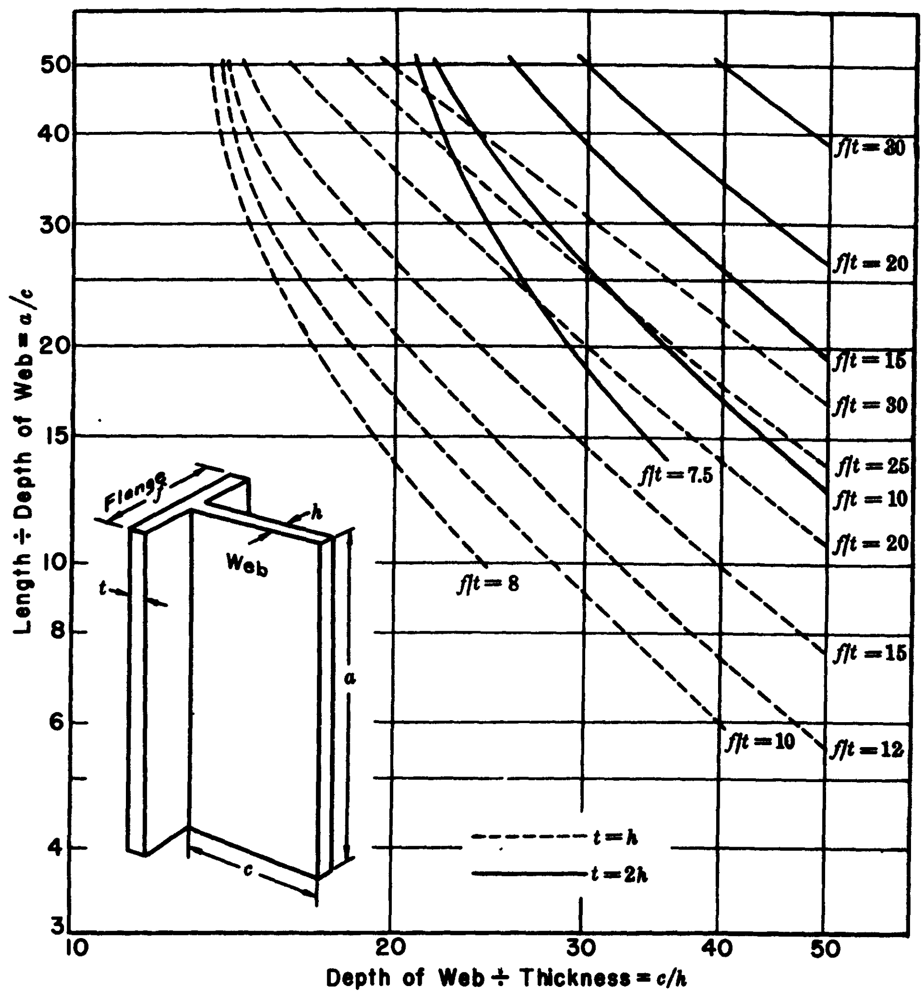

Figure 10 – Design Chart for Tee Stiffeners

It should be emphasized that the foregoing requirements are for members under loading which is primarily axial. Stiffeners whose design is controlled by transverse loading conditions have somewhat different optimum proportions, and the section modulus is of prime importance in the latter case.

Bulkheads will usually be designed to carry both types of load, either separately or together, and it may be advisable to undertake some revision of existing tabular design data in the light of these requirements.

The foregoing recommendations for the design of stiffener sections refer only to steels with yield point below 40 kips per square inch. In high-tensile steels, the optimum sections should be somewhat more compact; specific criteria have not, however, been worked out for stiffeners.

A greater tendency to twisting failure in the case of channels has been noted in the tests of Reference (19). Tee sections appear to be superior to channels or other unsymmetrical shapes in this regard.

Attention is drawn to the fact that the Experimental Model Basin results to date apply only to the case of flat plating in longitudinally stiffened structures, and that no allowance has been made for possible weakening effects due to the presence of shear or hydrostatic pressure in addition to compression. With respect to the latter point, it is noted that the twisting of unsymmetrical sections, such as channels or zees, may result in a reduction in axial strength when transverse loading is present.

When the designer has chosen his detailed disposition of material in accordance with the foregoing recommendations, he is then in position to verify the strength of the ship section. The allowable maximum fiber stress that has been chosen must be modified by multiplying by the strength factor F at the chosen b/t, that is, by the ratio of plate strength to yield strength. This furnishes an allowable average fiber stress which must not be exceeded. Comparison of this with the quotient of maximum bending moment by section modulus affords the final check on the safety of the design.

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE STUDY

The data obtained to date present an adequate basis for the design of stiffened flat plating in compression to an accuracy at least as good as that of the current column-design tables. It is believed, however, that substantial improvements can still be made in the accuracy of these calculations, which should be reflected in decreased structural weight.

The technique of testing and the understanding of the behavior of stiffened plating and compression testing in general has progressed to such a point that significant results may be achieved more rapidly, more accurately, and at lower expense than formerly, and the time appears ripe for a new advance. This can be achieved by development of theory as a guide to experimental research and by careful correlation of the compressive properties of the material with the compressive properties of the structure. Determination of stress-strain curves in compression on samples from all actual models tested is essential, and this is already in hand for current work.

Future work should include:

(a) A determination of column strength for ship steels;

(b) An investigation of twisting failure of shapes in compression;

(c) A brief resurvey of the fundamental details of the investigations on stiffened plates in compression with particular attention to low width-thickness ratios;

(d) A study of the strength of curved plating in compression;

(e) An investigation of the effect of shear on the compressive strength of plating;

(f) An investigation of the combined effect of hydrostatic pressure and edge compression on stiffened plating.

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BIBLIOGRAPHY

(1) "The Strength of Hull Plating Under Compression," by R. J. Sweeney, EMB Progress Report 1. (Confidential)

(2) "The Strength of Hull Plating Under Compression," EMB Progress Report 2, with supplement dated 31 October 1933. (Confidential)

(3) "The Stiffening of Hull Plating to Withstand Compression," by R. J. Sweeney, EMB Technical Memorandum. (Confidential)

(4) "The Strength of Hull Plating Under Compression," by John Vasta, EMB Progress Report 3. (Confidential)

(5) "The Strength of Hull Plating Under Compression," by John Vasta, EMB Progress Report 4. (Confidential)

(6) "The Strength of Hull Plating Under Compression - Stiffeners and Flanges," by John Vasta, EMB Progress Report 5, January 1935. (Confidential)

(7) "The Strength of Hull Plating Under Compression - Buckling or Critical Strength of Flat Bar Stiffeners," EMB Progress Report 6, May 1937. (Confidential)

(8) "The Elastic Stability of Tee Stiffeners," by Dwight F. Windenburg, EMB Report 457, November 1938.

(9) "Über die Stabilität versteifter Platten," (On the Stability of Stiffened Plates), by S. Timoshenko, EMB Report 402, (Translation 8), July 1935.

(10) "Theory of Elastic Stability," by S. Timoshenko. McGraw-Hill, 1936.

(11) "Die mittragende Breite der gedrückten Platte," (The Effective Width of a Plate Under Compression), by K. Marguerre, Luftfahrtforschung, Vol. 14, No. 3, 20 March 1937, pp. 121-128.

(12) "Experimental Studies of the Effective Width of Buckled Sheets," by R. Lahde and H. Wagner, NACA Technical Memorandum 814, 1936.

(13) "Experimental Study of Deformation and Effective Width in Axially Loaded Sheet-Stringer Panels," by Walter Ramberg, Albert E. McPherson, and Sam Levy, NACA Technical Memorandum 684, February 1939.

(14) "Strength of Plating in Compression," by Commander H. E. Rossell, (CC),U.S.N.(Ret.), Trans. S.N.A.M.E. Vol. 43, 1935, pp. 121-138.

(15) "The Ultimate and Critical Compressive Strength of Tee Stiffeners," by John Vasta, EMB Report 445, February 1938. (Confidential)

(16) Memorandum by John Vasta on "Strength of Plates Supported by Stiffeners," (unpublished).

(17) Memorandum by John Vasta on "Strength of Unstiffened Flat Plates," (unpublished).

(18) "Theorie und Berechnung der eisernen Brücken," (Theory and Calculation of Steel Bridges), by Friedrich Bleich, J. Springer, Berlin, 1924. (English translation will appear in the near future as EMB Translation 30).

(19) "Compression Tests of Ship-Structure Assemblies," by John Vasta, EMB Report 452, June 1938. (Confidential)

(20) General Specifications, Appendix 20, Column Tables, Edition of 1910, reprinted 1932.

(21) Tensile and Compressive Tests of Longitudinals of CL51 to 54 with and without Manholes. August 1939. EMB Test Report.

APPENDIX

EXAMPLE ILLUSTRATING THE RECOMMENDED DESIGN PRACTICES

It is required to design a midship section of approximately rectangular shape with a beam of 32 feet and a depth of 12 feet. The section must be capable of carrying a bending moment of 17,000 foot-tons in both hog and sag at an allowable extreme fiber stress of 16 kips per square inch in medium steel. Longitudinal framing is to be used with transverse web frames 8 feet apart. Concentrated loads on the deck are to be carried by two fore and aft high tensile steel box girders placed 4 feet on either side of the center line. The deck is supported by stanchions at each transverse frame, the stanchions landing on deck longitudinals in the bottom. Openings are required in the deck with clear diameters of 42 inches.

The questions of design involved will be taken in the same order as that in which a designer would encounter them in practice. Thus, it is the usual experience that the design of certain features of the section is determined by certain detail requirements not otherwise affecting the general features of the design. In the problem considered here it will be necessary to dispose first of certain details in the design of the heavy longitudinal members before proceeding with the design of the section as a whole.

The design of the longitudinal girders in the bottom is controlled primarily by the concentrated loads they carry. The bending and shear requirements call for a light web in 0.2-inch plate with a depth of 19.6 inches, giving a web-depth ratio of $c/h = 98$. The high value of this ratio indicates that the web is too thin for an efficient member carrying axial load, so supplementary stiffening is required to prevent web buckling.

As shown in Figure 11, this necessary support is provided by a longitudinal tee stiffener at mid-height of the web. This gives the web plate an acceptable width-thickness ratio of 49. Vertical stiffeners for the webs of the longitudinal girders are provided on 4-foot spacing and are bracketed into the longitudinal web stiffeners. The effective column length of the longitudinal stiffeners is consequently 48 inches.

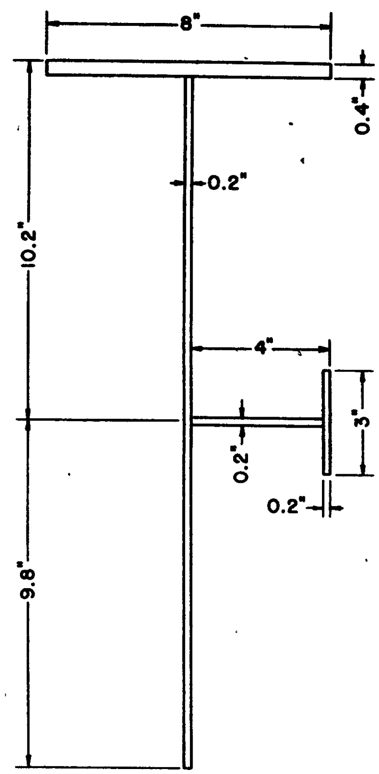

Figure 11 - Longitudinal Girder

For practical reasons of fabrication, a stiffener depth of 4 inches and a plate thickness of 0.2 inch was chosen. The size of flange required is found from the curves of Figure 10 for $t = h$. In this particular case

a/c = 12.6, and c/h = 19. The minimum permissible width ratio of the flange is found to be f/t = 8. For reasons of convenience, a flange 3 inches wide is chosen with f/t = 15, which is more than sufficient to stabilize the stiffener, provided the value for radius of gyration is satisfactory.

As the radius of gyration of a tee stiffener with associated plate is usually between 30 and 40 per cent of the depth, which in this case is 4 inches, this size of stiffener will probably show a slenderness ratio of less than 40 for the combination.

To verify the actual slenderness ratio, the following formula is convenient:

$$I = I_s + \left(y_s + \frac{1}{2}t\right)^2 \frac{A_p A_s}{A_p + A_s} + \frac{1}{12}t^2 A_p \qquad [5]$$

where I is the moment of inertia of a combined section of plate and stiffener, (this is taken about the gravity axis parallel to the plate)

I_s is the moment of inertia of the stiffener alone about a parallel axis through its center of gravity,

y_s is the distance of the center of gravity of the stiffener section from the faying surface,

t is the thickness of plating,

A_p is the cross-sectional area of associated plating,

A_s is the cross-sectional area of stiffener.

For the stiffener considered, $A_s = 1.36$ in²., y_s = 2.78 in., and I_s = 2.26 in⁴. The associated web plate is to be taken as extending half way to the flange on the one side and half way to the shell plating on the other side. From this one obtains t = 0.2 in. and $A_p = 0.2 \times 9.8 = 1.96$ in².

Substituting these values in [5], the moment of inertia of the plate-stiffener combination is found to be I = 8.92 in⁴. From these data, the radius of gyration is

$$\sqrt{\frac{8.92}{1.36 + 1.96}} = 1.64 \text{ in.}$$

and the slenderness ratio is 48/1.64 = 29. This figure is acceptable, since it is smaller than the slenderness ratio of 40 required to develop a high failing stress in the stiffener.

The web of the longitudinal girder has now been satisfactorily stabilized by the addition of the longitudinal stiffener. It then becomes necessary to make a similar calculation for the longitudinal girder as a whole. Since the radius of gyration of the member and the associated shell plating is of the order of one-third the depth, it is evident that the girder has a slenderness ratio well below the upper limit of 40. It remains to check the flange dimensions.

The flange area is determined by the bending moments produced by concentrated loads. The required area is 3.2 in². A width-thickness ratio f/t less than 30 should be chosen to prevent buckling of the flange itself, but it must be wide enough to stabilize the girder under the axial loads it carries: that is, an f/t must be chosen large enough to support the web from buckling and to provide sufficient stiffness against twisting failure.

To check the first point, the web between the flange and the web stiffener is considered, with $c/h = 49$. The length-depth ratio of this portion of the web is $a/c = 9.8$. Trying a flange 0.4 inch by 8 inches with $f/t = 20$ and $t = 2h$, it will be found from the curves of Figure 10 that this flange is more than sufficient to support the web.

✓The check on twisting failure requires going back to the formulas of (8), since Figure 10 does not apply to webs of such large depth-thickness ratios. The web stiffener is to be neglected, as it can play but small part in a twisting failure. From Equation [24] of reference (8), with a fixity coefficient γ of 2 as recommended there, the critical stress for twisting failure is calculated to be 231 kips per square inch. This is, of course, only a fictitious stress, as the calculation assumes an unrestricted elastic range. It should be interpreted in this case as meaning merely that the member is stable up to the yield point with regard to twisting.

The preceding investigation assures longitudinal girders in the bottom that can be relied upon to carry their full share of the longitudinal bending load in the bottom.

The remaining portion of the bottom consists of shell plating with suitable stiffeners. The depth required for these stiffeners can be estimated to be about three times the minimum required radius of gyration, which in this case is the frame spacing divided by 40, or 2.4 inches; a stiffener 8 inches deep should provide a satisfactory margin. Choose a welded tee stiffener of plate thickness 0.2-inch in both web and flange. Figure 10 gives the information required to determine the flange width. The depth-thickness ratio of the web is $c/h = 39$, the length-depth ratio of the web is $a/c = 12.3$. A width-thickness ratio f/t of 20 for the flange will be found from the curves for $t = h$ to be sufficient to stabilize the stiffener. The section so determined is shown in Figure 12.

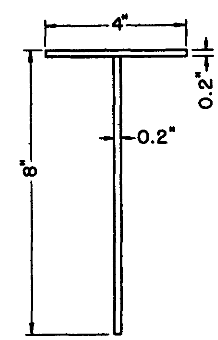

Figure 12 - Shell Stiffener

✓The deck carries two fore and aft box girders in 0.375-inch high-tensile plate. Concentrated loads from rails determine the design of these sections, which are 18 inches high and 12 inches wide. Reinforcing of the deck beneath the girders is required, necessitating the use of a fore and aft plate in medium steel which is 0.375 inch by 24 inches in section.

Now that these preliminary questions have been disposed of, attention can be turned to the design of the midship section as a whole. A preliminary choice of $b/t = 50$ for shell stiffener spacing will be made. This may be subject to slight revision when locating the stiffeners in the space available.

If the plating is assumed to have a specified yield strength of 34 kips per square inch, a width-thickness ratio of 50 corresponds to a width factor of

$$B = 50\sqrt{\frac{34}{29,000}} = 1.71$$

and, from Figure 3, to a strength ratio or efficiency of

$$F = 0.888$$

The average stress at compressive failure in such stiffened plating is the product of F and the yield strength, or 30.2 kips per square inch. Alternatively, this strength can be determined from Figure 5 by interpolation between the curves for yield points of 30 and 35 kips per square inch.

The allowable average stress is then obtained by multiplying the allowable maximum stress of 16 kips per square inch by the strength ratio. One obtains

$$0.888 \times 16 = 14.2 \text{ kips per square inch}$$

This is to be considered as an average for both plating and longitudinals.

Since the bending moments in hog and sag are equal, the most economical section is obtained with equal areas in deck and bottom, the neutral axis being at mid-height of the section. At an allowable average stress in deck and bottom of 14.2 kips per square inch and with this location of the neutral axis, the deep longitudinals and the box girders will carry a bending moment of about 2400 foot-tons.

The shell and its local stiffening must carry the remaining bending moment of 14,600 foot-tons. This requires a section modulus for the stiffened shell alone of

$$\frac{14,600 \times 2.24}{14.2} = 2303 \text{ in.}^2\text{-ft.}$$

Neglecting the effect of the side plating in carrying bending moment, this section modulus calls for an average thickness of stiffened plating in deck and bottom of 0.499 inch. This figure includes the distributed area of the stiffeners in addition to the plating.

The stiffener, Figure 12, has a cross-sectional area of 2.36 square inches. For 0.375-inch plate and a stiffener spacing of 50 x 0.375 = 18.75 inches, the average thickness is 0.501 inch which agrees satisfactorily with the estimated requirement. Shell plating 0.375 inch thick is accordingly chosen for a preliminary layout.

A sketch of the cross section so determined is shown in Figure 13. To accommodate uniformly spaced stiffeners between the longitudinals, the stiffener spacing is reduced to 18 inches, which results in a width-thickness ratio of 48.

A check of the slenderness ratio of the stiffened plating is required. By use of Equation [5], it will be found that the radius of gyration of the plate-stiffener combination is 2.73 inches and the slenderness ratio is 48/2.73 = 35.2, which is satisfactorily below the desirable upper limit of 40.

The only other detail requiring attention before a final stress analysis of the section is the reinforcement of the deck openings. An edge reinforcement is chosen consisting of a tee flange 6 inches wide made from 0.375-inch plate. The deck plate adjacent to the opening can be considered as the web of a tee stiffener with $c/h = 48$. The width-thickness ratio of the reinforcing flange is $f/t = 16$, and the flange and web thicknesses are equal. With these proportions, the curves of Figure 10 indicate that the flange chosen will stabilize the deck plating for a length of 8 times the web depth, or 12 feet. As the deck opening has a length less the frame spacing of 8 feet, the flange chosen is of ample size.

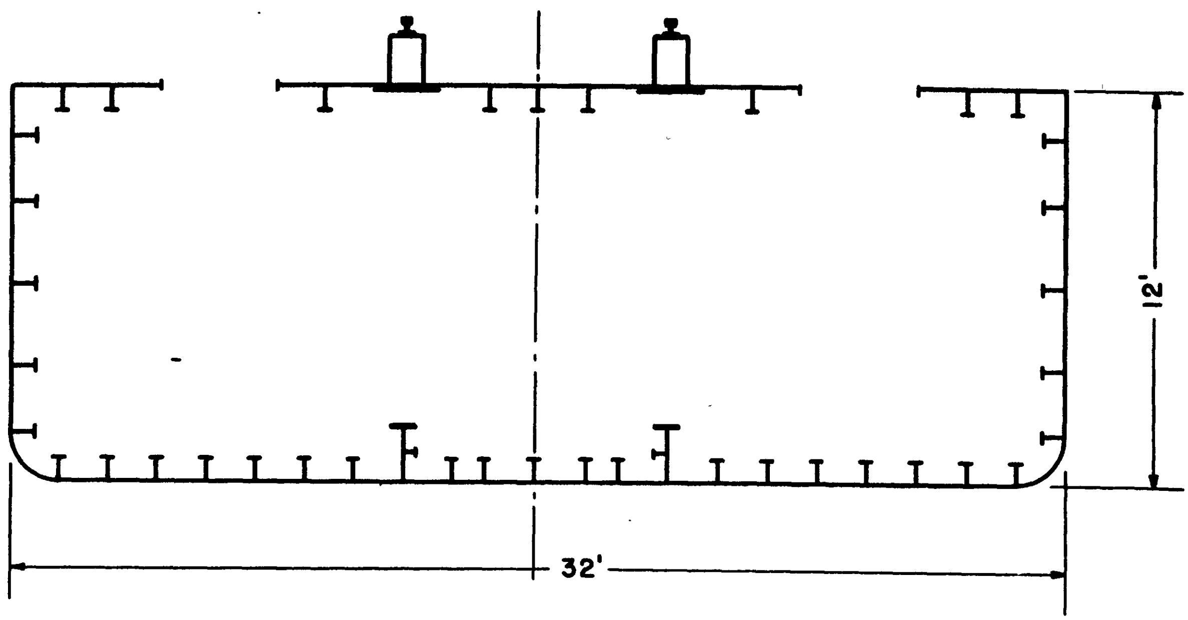

Figure 13 - Proposed Midship Section

Calculation of the properties of the midship section so chosen give a neutral axis 6.07 feet above the inside of the bottom plating and a least section modulus of 2663 in^2. - ft.

Since the stiffener spacing has been reduced from $b/t = 50$ to $b/t = 48$, a slight increase in the average allowable stress is permitted. It will be found that

$$B = 48\sqrt{\frac{35}{29,000}} = 1.67$$

$$F = 0.900$$

The average allowable stress is 0.9 x 16 = 14.4 kips per square inch.

Under the maximum applied bending moment of 17,000 foot-tons, the extreme fiber stress is

$$\frac{17,000 \times 2.24}{2663} = 14.3 \text{ kips per square inch}$$

The tentative scantlings are therefore satisfactory and show an economical use of material.

Many of the calculations made in this example would be dispensed with by the designer in practice, as the information would undoubtedly be available on suitable data sheets for approved combinations of plate and stiffener. The preparation of such data sheets is best done by those immediately concerned with the detailed problems and routine of design.

It should be realized that the preceding remarks do not pretend to be a discussion of design conditions, but only to discuss the application of the criteria proposed in earlier sections. In fact, the design conditions have been simplified somewhat artificially in order to point the illustration more sharply. In particular, no attention has been paid to the effect of transverse hydrostatic loading on the strength of stiffened plating in the bottom. This loading may possibly be more important than the bending load in the design of stiffeners and plating.

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