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# EFFECT OF ENTRAINED WATER ON THE MASS MOMENT OF INERTIA OF SHIP PROPELLERS



U.S. Experimental Model Basin,
Navy Yard, Washington, D. C.

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## Summary

Several series of model propellers varying in pitch and in blade width were set in torsional vibration on vertical shafts. The resonant frequency was measured with the propeller in air and in water. A value was assigned to the increase in moment of inertia in water on the assumption that the change in resonant frequency was due wholly to this cause. The effect was found to increase with pitch and with blade width and also to be dependent on both amplitude and frequency in water. From the data obtained no simple mathematical expression could be found which would be universally applicable. With the apparatus in its present state it is not possible to study the effects of amplitude and frequency separately.


## General

A ship's propeller and its shaft, combined with turbine, reduction gear, or engine, form a system capable of torsional vibration of various natural frequencies depending on the relative masses of the members attached to the shaft.

The effect of the propeller on the natural frequency depends only on its mass moment of inertia providing that the damping forces acting on it are negligible. When immersed the moment of inertia of the propeller may be considered to increase, due to the entrained water, which also damps the vibration. As far as the problem of determining the natural frequency of vibration is concerned we may include the damping effect with the inertia effect. However, the two are quite distinct, for damping causes dissipation of energy and decrease of amplitude as well as lowering of natural frequency whereas an increase in mass alone may cause a greater amplitude.

## Apparatus and Method of Tests

The first attempt at a solution of the problem was to suspend the propeller from a steel wire as a torsion pendulum and find the free periods in air and in water. This was not possible, however, because the motion became aperiodic in water due to the excessive damping.

Next the apparatus shown on drawing No. A-986 and illustrated in Figs. 2 and 3 was made.

This consists of a vertical shaft mounted in a ball bearing and having the propeller keyed to the lower end. On the upper end is attached a disk which is set oscillating

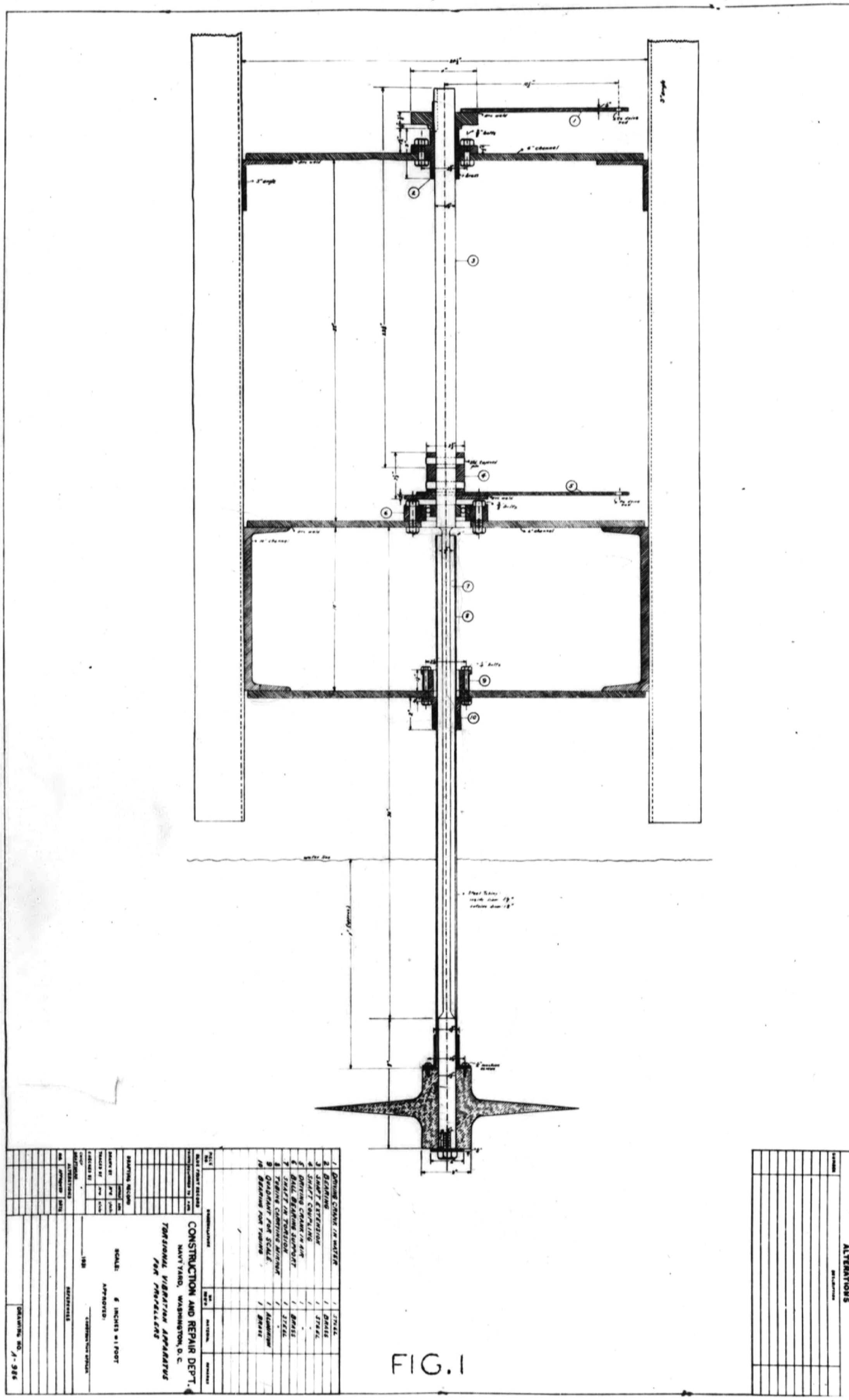
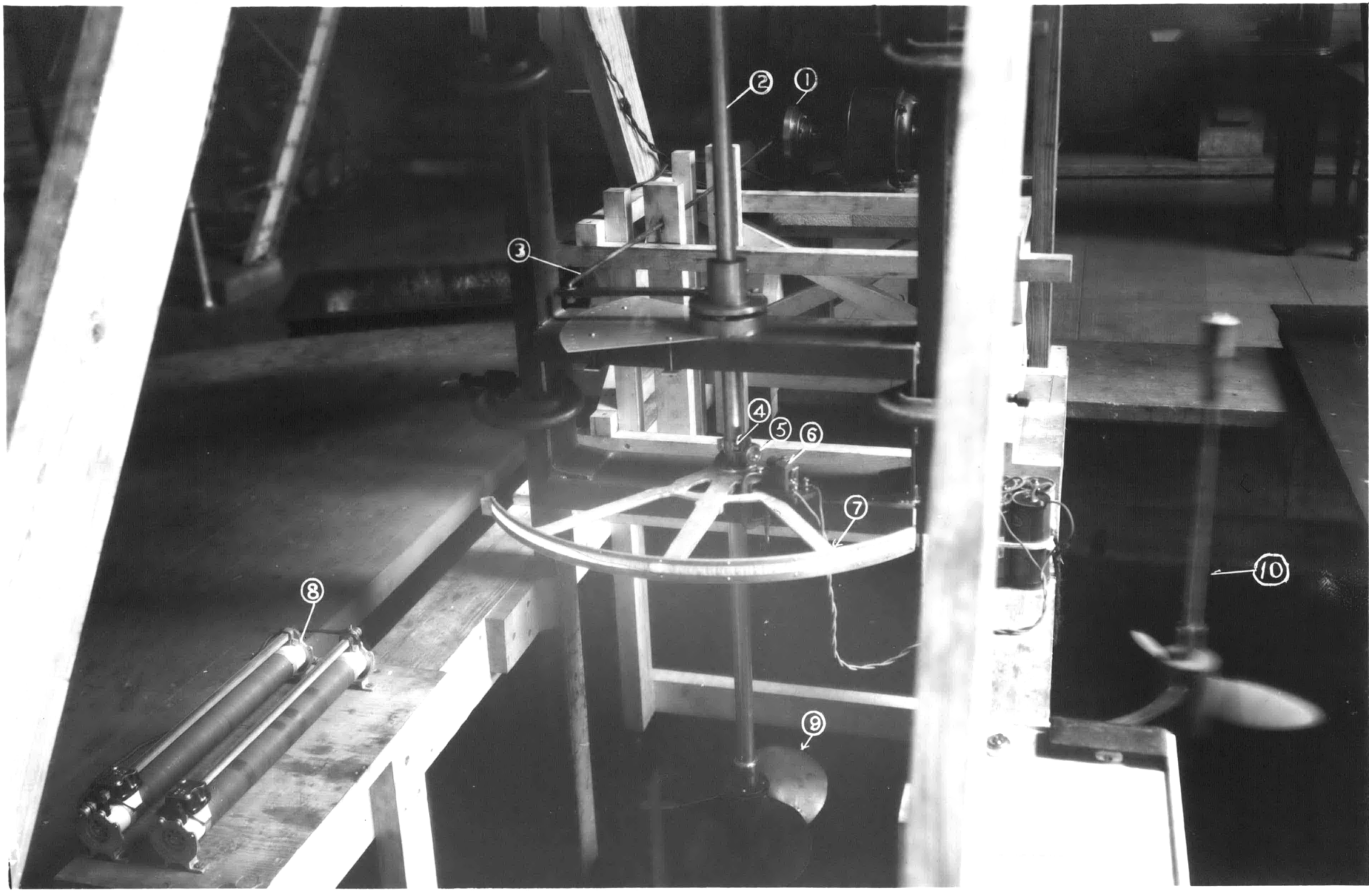
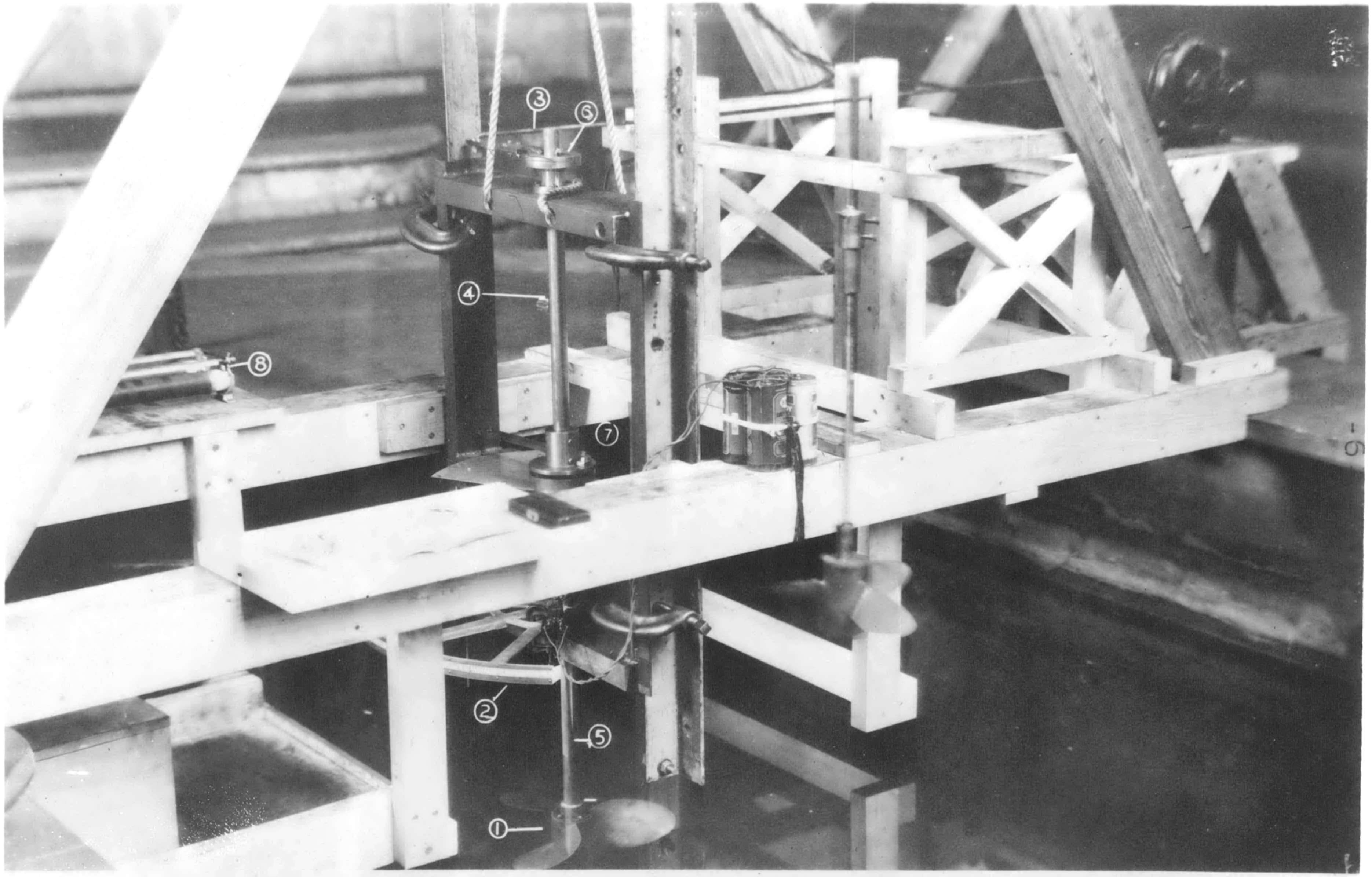

1 DRIVING CRANK IN WATER — 1 — STEEL
2 BEARING — 1 — BRASS
3 SHAFT EXTENSION — 1 — STEEL
4 SHAFT COUPLING — 1
5 DRIVING CRANK IN AIR — 1
6 BALL BEARING SUPPORT — 1 — BRASS
7 SHAFT IN TORSION — 1 — STEEL
8 TUBING CARRYING MIRROR — 1
9 QUADRANT FOR SCALE — 1 — ALUMINUM
10 BEARING FOR TUBING — 1 — BRASS
CONSTRUCTION AND REPAIR DEPT.
NAVY YARD, WASHINGTON, D.C.
TORSIONAL VIBRATION APPARATUS
FOR PROPELLERS
SCALE: 6 INCHES = 1 FOOT
APPROVED:
DRAWING NO. A-986
ALTERATIONS

FIG.1




Torsional Vibration Apparatus for Propellers.

1. Eccentric 3. Drive rod 5. Lens 7. Scale 9. Propeller
2. Shaft extension 4. Mirror 6. Lamp holder 8. Control rheostats




Torsional Vibration Apparatus for Propellers.

1. Propeller 3. Drive rod 5. Tube surrounding shaft in torsior
2. Scale for recording 4. Shaft extension 6. Driving disc when in water

by a rod attached to an eccentric on the shaft of a motor. The amplitude at the top end is fixed but the frequency can be varied by changing the resistance in series with the armature of the motor. The portion of the shaft between the bearing and the propellers is turned down so as to bring the natural frequency of the propeller on the shaft within suitable limits. A steel tube 1/16" thick surrounds the shaft and follows the motion of the propeller. A small mirror fastened to this tubing throws a beam of light on to a scale, thereby furnishing a measure of the amplitude of the propeller. An extension is added to the shaft so that when the propeller is lowered into water the drive rod may be attached to the upper driving arm without changing the level of the motor. The diameter of the shaft extension is so much greater than the turned down portion of the shaft that the effective length of shaft in torsion is not appreciably altered.

The amplitude of the propeller varies with the frequency and resonance is found by varying the speed of the motor until maximum travel of the beam of light is indicated on the scale. While this corresponds to a shaft with infinite mass at one end as explained subsequently (see page 10), and hence does not represent exactly a ship's propeller shaft, it does not have the disadvantage of a shifting node and furnishes a simple formula for computing the mass moment of inertia of the propeller.

## General Theory

The expression for the torque required to twist one end of a cylindrical shaft thru an angle $\theta$, the other end being held fixed, is:

$$T = \frac{E_s J \theta}{l}$$

where $T$ = torque in lb.

$E_s$ = shear modulus of elasticity in $lb/in^2$ (approximately $11.9 \times 10^6$ for steel)

$J$ = polar moment of inertia of shaft in $inches^4$ ($J = \frac{\pi D^4}{32}$ for a circle)

$\theta$ = angle of torsion in radians

$l$ = length of shaft in inches

If a propeller is keyed to a shaft held rigidly at the opposite end and turned thru an angle $\theta$, it will be subject to a restoring torque $T = \frac{E_s J \theta}{l}$ and, since this torque is proportional to the angular displacement, if the propeller is released, it will perform simple harmonic oscillations. The period of a simple harmonic motion varies as the square root of the ratio of the mass to the restoring force per unit displacement from the mean position; or, in the case of angular motion, to the square root of the ratio of the moment of inertia to the restoring torque per radian. Therefore, if a mass of moment of inertia I is subject to a restoring torque k per radian of twist it will perform angular simple harmonic motion of period:

$$T = 2\pi\sqrt{\frac{I}{k}}$$

In this case $k = \dfrac{\text{torque}}{\text{angular displacement}} = \dfrac{\dfrac{E_s J \theta}{l}}{\theta} = \dfrac{E_s J}{l}$

Hence if the moment of inertia of the shaft is negligible in comparison with that of the propeller, the free period of the propeller when the shaft is held rigidly at the end is:

$$T = 2\pi\sqrt{\frac{I\,l}{E_s\,J}}$$

For greater accuracy we must add 1/3 the moment of inertia of the shaft to that of the propeller. In terms of the dimensions of the shaft this formula becomes (using inch pound units):

$$T = 2\pi\sqrt{\frac{32\,I\,l}{E_s \times \pi \times d^4} \times \frac{1}{12 \times 32.2}}$$

where

$T$ = period in seconds

$I$ = moment of inertia of propeller in lb x $in^2$

$l$ = length of shaft in inches

$d$ = diameter of shaft in inches

$E_s$ = shear modulus in $lb/in^2$

This same formula applies when the shaft is reduced to the dimensions of a wire, resulting in a torsion pendulum. Since for a given wire all these terms are constant, the period will vary as the square root of the moment of inertia of the suspended mass. Hence, if the period of a body of known moment of inertia is found, the moment of inertia of another body can be found at once by measuring its free period of oscillation on the same wire.

If, instead of being held rigidly at one end, the shaft is mounted in bearings with a mass at each end, and these are turned in opposite directions and released, each mass will perform simple harmonic oscillations, but there will be a node in the shaft whose position depends on the ratio of the moments of inertia of the two masses. The position of the node is given by the equation:

$$\frac{l_1}{l_2} = \frac{I_2}{I_1}$$

where $l_1$ represents the distance of the node from the end to which the mass having moment of inertia $I_1$ is fixed.

The two masses have the same free period which is given by the expression:

$$T = 2\pi\sqrt{\frac{32\, I_1 I_2 \ell}{E_s \pi d^4 (I_1 + I_2)} \times \frac{1}{12 \times 32.2}}$$

Thus, if instead of a rigid fastening, we had a mass of the same moment of inertia as the propeller on the other end, the effective length of the shaft would be reduced by one half and the natural period reduced in the ratio $\frac{1}{2}$.

If a propeller is keyed to one end of a shaft and the other end is set in simple harmonic oscillation with a fixed amplitude, the amplitude of the propeller will vary with the frequency. At very low frequencies the amplitudes at both ends of the shaft will be the same and there will be no torsion, but as the frequency increases the inertia of the propeller causes its motion to lag and the amplitude increases up to a certain point after which it begins to decrease. At infinite frequency the propeller will come to rest even though the amplitude at the crank end of the shaft remains constant.

It can be shown that the maximum amplitude occurs at the same frequency as that at which the propeller would oscillate freely if the mass of the crank end were infinite.

Let $\theta_1$ = the instantaneous angular displacement of the crank end of the shaft, and $\theta$ the instantaneous angular displacement of the propeller.

The propeller is subject to a torque $k(\theta_1 - \theta)$, where k is the torque required to twist the propeller thru one radian when the crank end is held fixed. If I is the moment of inertia of the propeller, we have the equation of motion:

$$I \frac{d^2 \theta}{d t^2} = k(\theta_1 - \theta)$$

Since we are impressing on the crank end a simple harmonic motion of constant amplitude and varying frequency, we can express $\theta$ in the form:

$$\theta_1 = A \operatorname{Sin} wt$$

where $w = 2\pi \times$ frequency

Therefore:

$$\frac{d^2 \theta}{dt^2} = -\frac{k}{I}\theta + \frac{k}{I} A \operatorname{Sin} wt$$

The complete solution of this differential equation is:

$$\theta = \Phi \operatorname{Sin}\left(\sqrt{\frac{k}{I}}\,t + \beta\right) + \frac{\frac{K}{I} A}{\frac{k}{I} - w^2} \operatorname{Sin} wt$$

$\Phi$ and $\beta$ being arbitrary constants.

When $w^2 = \frac{k}{I}$, the amplitude becomes infinite, and hence this is the condition for resonance; but we have already shown that the free period for infinite mass at the crank end is:

$$T = 2\pi\sqrt{\frac{I}{k}}$$

or the frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$$

Since

$$w = 2\pi f$$

$$w^2 = 4\pi^2 f^2$$

Hence at resonance, in the case of the forced oscillation:

$$4\pi^2 f^2 = \frac{k}{I}$$

or

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$$

and the frequencies are the same in both cases.

So far we have neglected the effect of damping. Damping affects both amplitude and period. The case of simple harmonic motion with a damping force proportional to the velocity can be solved. In the case of a propeller vibrating in water the law of damping is not known. For simplicity the change in period has been ascribed wholly to increase in mass moment of inertia.

Conduct of Tests

Measurements were made on six series of three-bladed brass propellers, all of 16" diameter and varying in pitch ratio (p/d) from .60 to 2.00. Each series consisted of five propellers of the same pitch but of different blade widths - the mean width ratios varying from .15 to .35 by steps of .05.

The mass moment of inertia of each propeller in air was measured by keying it to a short piece of shafting suspended from a steel wire about 12 feet in length and measuring the period of oscillation. A steel cylinder whose moment of inertia was computed was used as a standard of reference. Next the propellers were keyed to the shaft of the vibration apparatus which in the first test was turned down to a diameter of .500" over a length of 30".

The resonant frequency was measured in air and in water, and the increase in moment of inertia in water computed by the formula:

$$\sqrt{\text{moment of inertia}} \times \text{resonant frequency} = \text{shaft constant.}$$

A correction was made for the additional mass moment of inertia of the tubing and other fittings attached to the propeller.

To see whether the effect varied with frequency and amplitude three shafts were made 20 inches in length and of diameters .045", .050", and .057", respectively. This permitted testing the same propeller at different resonant frequencies. Two of the series of propellers were tested on these three shafts and the results obtained were compared with those on the 30" shaft.

Results

The data obtained in the test with the 30 inch shaft (.500" in diameter) are given below:

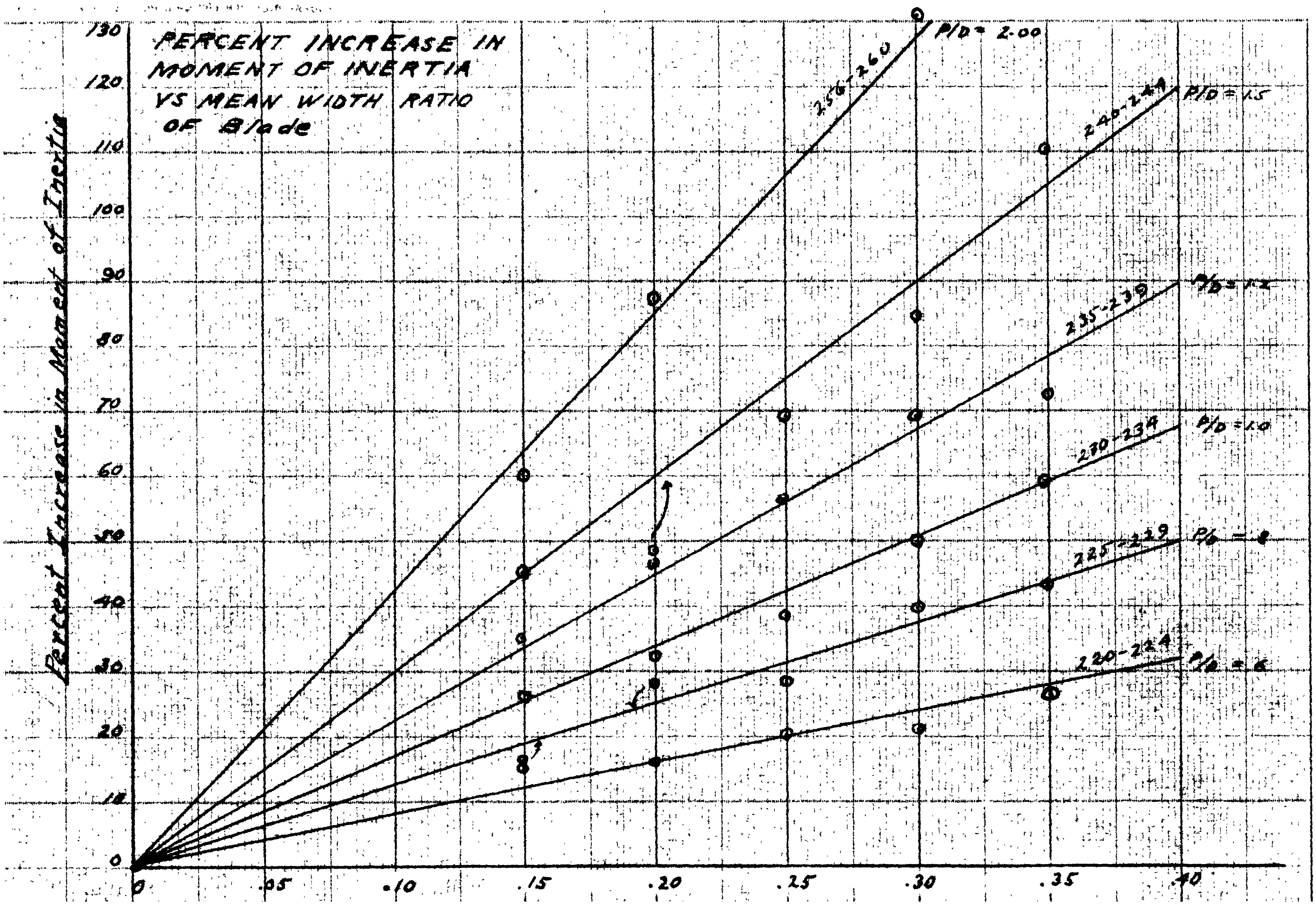
| E.M.B. Propeller No. | Pitch Ratio | Mean Width Ratio | Resonant frequency in air | Resonant frequency in water | Moment of inertia in air (lb x in²) | Moment of inertia in water (lb x in²) | % Increase |
|---|---|---|---|---|---|---|---|
| 220 | .60 | .15 | 1335 | 1245 | 54.2 | 62.6 | 15.5 |
| 221 | .60 | .20 | 1290 | 1200 | 60.2 | 69.9 | 16.1 |
| 222 | .60 | .25 | 1131 | 1044 | 79.4 | 95.7 | 20.5 |
| 223 | .60 | .30 | 1059 | 960 | 91.4 | 110.7 | 21.1 |
| 224 | .60 | .35 | 1059 | 939 | 91.4 | 115.8 | 26.7 |
| 225 | .80 | .15 | 1320 | 1224 | 58.6 | 68.4 | 16.7 |
| 226 | .80 | .20 | 1236 | 1095 | 63.6 | 81.6 | 28.3 |
| 227 | .80 | .25 | 1119 | 990 | 80.1 | 102.7 | 28.2 |
| 228 | .80 | .30 | 1125 | 954 | 79.4 | 111.0 | 39.8 |
| 229 | .80 | .35 | 1074 | 900 | 88.4 | 127.1 | 43.7 |
| 230 | 1.00 | .15 | 1380 | 1230 | 52.6 | 66.6 | 26.7 |
| 231 | 1.00 | .20 | 1230 | 1071 | 65.0 | 86.2 | 32.6 |
| 232 | 1.00 | .25 | 1116 | 957 | 79.5 | 110.4 | 38.8 |
| 233 | 1.00 | .30 | 1083 | 888 | 88.0 | 132.0 | 50.0 |
| 234 | 1.00 | .35 | 1038 | 825 | 95.8 | 152.7 | 59.3 |
| 235 | 1.20 | .15 | 1383 | 1194 | 52.6 | 71.2 | 35.3 |
| 236 | 1.20 | .20 | 1302 | 1080 | 59.6 | 87.5 | 46.8 |
| 237 | 1.20 | .25 | 1200 | 957 | 69.4 | 108.8 | 56.8 |
| 238 | 1.20 | .30 | 1113 | 858 | 81.4 | 138.0 | 69.6 |
| 239 | 1.20 | .35 | 1020 | 780 | 96.3 | 166.1 | 72.8 |
| 240 | 1.50 | .15 | 1320 | 1140 | 52.6 | 76.7 | 45.7 |
| 241 | 1.50 | .20 | 1200 | 984 | 70.1 | 103.9 | 48.3 |
| 242 | 1.50 | .25 | 1200 | 921 | 70.1 | 118.7 | 69.3 |
| 243 | 1.50 | .30 | 1149 | 846 | 76.2 | 140.9 | 85.0 |
| 244 | 1.50 | .35 | 1044 | 720 | 87.0 | 183.5 | 111.0 |
| 256 | 2.00 | .15 | 1350 | 1074 | 55.4 | 89.0 | 60.7 |
| 257 | 2.00 | .20 | 1260 | 927 | 64.2 | 120.4 | 87.7 |
| 258 | 2.00 | .25 | -- | -- | -- | -- | -- |
| 259 | 2.00 | .30 | 1089 | 723 | 83.9 | 194.4 | 131.8 |
| 260 | 2.00 | .35 | 1044 | 675 | 95.4 | 230.9 | 142.0 |

These results are shown graphically on Plate I. For a given pitch ratio the moment of inertia in water seems to increase directly with the blade width ratio, the slope of the curve depending on the pitch ratio. The relation between slope of curve and pitch ratio appears also to be a linear one as shown on Plate II.

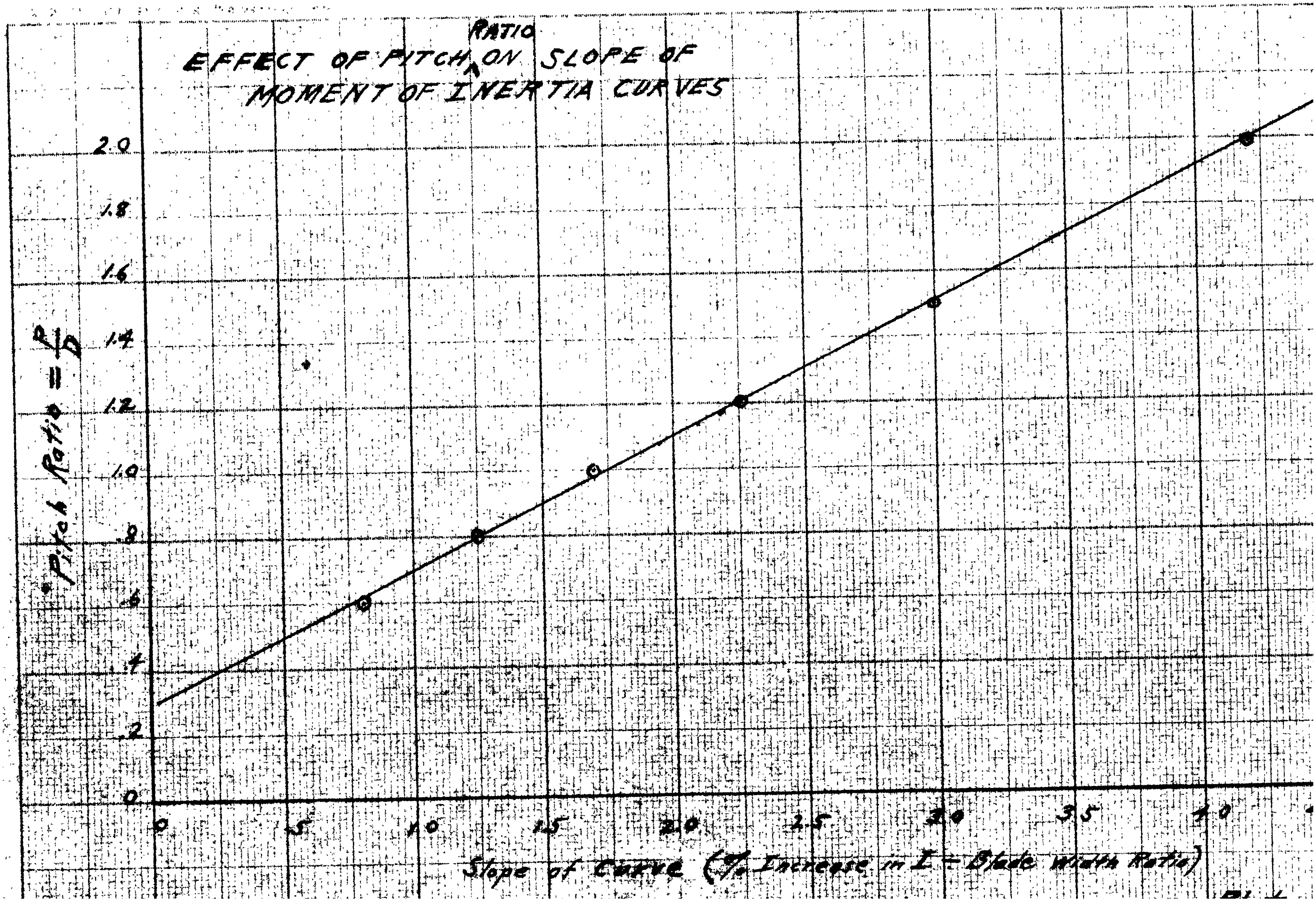
Most of the irregularities in these curves were found to be due to non-uniformity in the hubs. Where a propeller had a large amount of lead added for balancing the percentage increase was usually found to be lower than the value given by the straight line.

From the results it appeared that one simple formula would suffice to account for all cases, namely: % increase in moment of inertia = constant x blade width ratio x (constant + pitch ratio).

Further experiments, however, showed that the problem was much more complicated. The effect increases with frequency and amplitude. This is to be expected on the theory that the damping is the predominating factor, for the maximum velocity in simple harmonic motion varies both as the amplitude and the frequency and fluid resistance usually increases about as the square of the velocity.



PERCENT INCREASE IN
MOMENT OF INERTIA
VS MEAN WIDTH RATIO
OF Blade
Percent Increase in Moment of Inertia
256-260
P/D = 2.00
240-244
P/D = 1.5
235-239
P/D = 1.2
230-234
P/D = 1.0
225-229
P/D = .8
220-224
P/D = .6
130
120
110
100
90
80
70
60
50
40
30
20
10
0
0
.05
.10
.15
.20
.25
.30
.35
.40



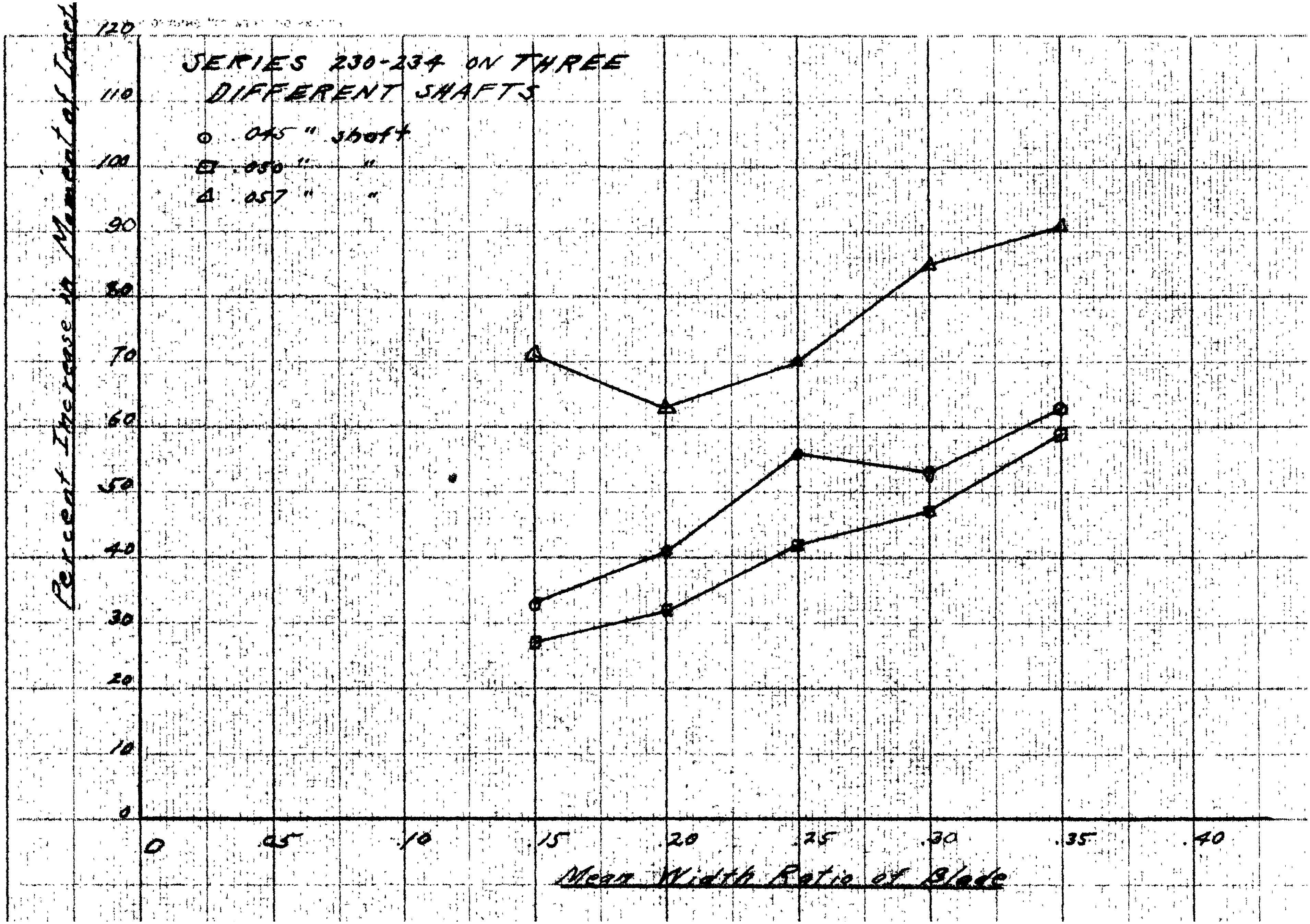
EFFECT OF PITCH RATIO ON SLOPE OF
MOMENT OF INERTIA CURVES
Pitch Ratio = P/D
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
0
.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Slope of Curve (% Increase in I ÷ Blade Width Ratio)

The apparatus in its present form does not permit studying the effects of amplitude and frequency separately, for the amplitude of the propeller at resonance cannot be accurately controlled.

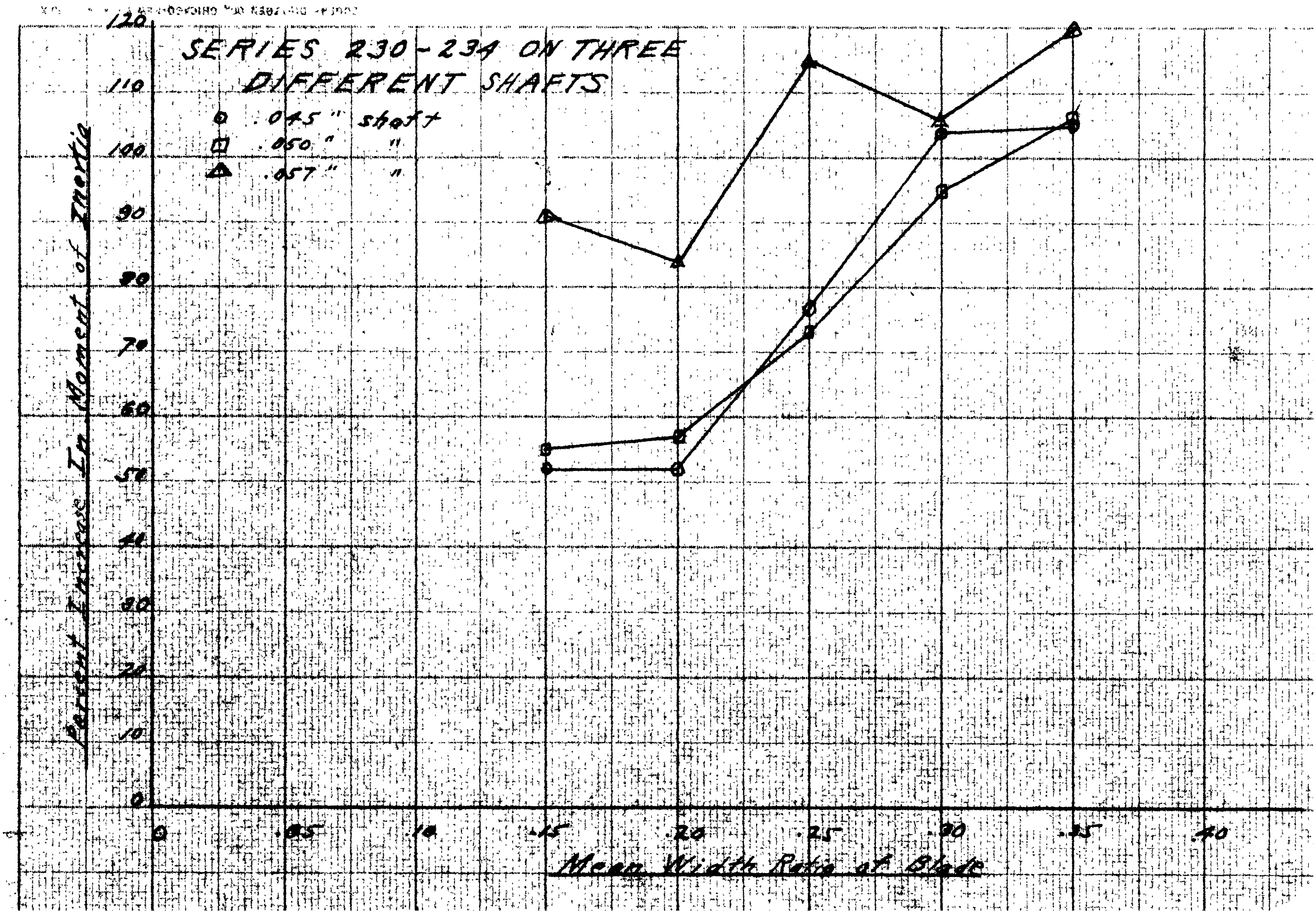
The data for two of the series of propellers on three different shafts giving different resonant frequencies and different amplitudes are shown below.

Plates 3 and 4 illustrate these data graphically. The curves of Plate 3 illustrate how the effect of amplitude obscures the effect of frequency. The middle curve was obtained from the shaft giving the lowest resonant frequencies but, since the amplitude was greater due to the greater flexibility of the shaft, a greater effect was observed than with the next larger shaft.

| E.M.B. Propeller No. | Pitch Ratio $\frac{P}{D}$ | Mean Width Ratio | Resonant frequency in air | Resonant frequency in water | Moment of inertia in air (lb x in$^2$) | Moment of inertia in water (lb x in$^2$) | % Increase in mom of ine |
|---|---|---|---|---|---|---|---|
| | | | | ".045 SHAFT | | | |
| 230 | .60 | .15 | 1110 | 993 | 52.6 | 70.0 | 33 |
| 231 | .60 | .20 | 1020 | 870 | 65.0 | 91.7 | 41 |
| 232 | .60 | .25 | 927 | 750 | 79.5 | 124.2 | 56 |
| 233 | .60 | .30 | 897 | 720 | 88.0 | 135.0 | 53 |
| 234 | .60 | .35 | 870 | 669 | 95.8 | 156.0 | 63 |
| 240 | 1.50 | .15 | 1110 | 930 | 52.6 | 80.0 | 52 |
| 241 | 1.50 | .20 | 1005 | 810 | 70.1 | 106.2 | 52 |
| 242 | 1.50 | .25 | 1005 | 750 | 70.1 | 124.2 | 77 |
| 243 | 1.50 | .30 | 960 | 675 | 76.2 | 153.9 | 104 |
| 244 | 1.50 | .35 | 900 | 630 | 87.0 | 176.8 | 105 |
| | | | | ".050 SHAFT | | | |
| 230 | .60 | .15 | 1383 | 1224 | 52.6 | 66.6 | 27 |
| 231 | .60 | .20 | 1233 | 1065 | 65.0 | 88.6 | 32 |
| 232 | .60 | .25 | 1116 | 945 | 79.5 | 113.00 | 42 |
| 233 | .60 | .30 | 1080 | 885 | 88.0 | 129.4 | 47 |
| 234 | .60 | .35 | 1035 | 815 | 95.8 | 153.8 | 59 |
| 240 | 1.50 | .15 | 1335 | 1110 | 52.6 | 81.6 | 55 |
| 241 | 1.50 | .20 | 1206 | 960 | 70.1 | 109.7 | 57 |
| 242 | 1.50 | .25 | 1200 | 900 | 70.1 | 124.9 | 73 |
| 243 | 1.50 | .30 | 1140 | 816 | 76.2 | 152.7 | 95 |
| 244 | 1.50 | .35 | 1080 | 753 | 87.0 | 179.5 | 106 |
| | | | | ".057 SHAFT | | | |
| 230 | .60 | .15 | -- | 1410 | 52.6 | 89.9 | 71 |
| 231 | .60 | .20 | 1605 | 1305 | 65.0 | 105.7 | 62 |
| 232 | .60 | .25 | 1515 | 1155 | 79.5 | 135.4 | 70 |
| 233 | .60 | .30 | 1395 | 1056 | 88.0 | 162.4 | 85 |
| 234 | .60 | .35 | 1326 | 996 | 95.8 | 182.9 | 91 |
| 240 | 1.50 | .15 | 1815 | 1335 | 52.6 | 100.7 | 91 |
| 241 | 1.50 | .20 | 1620 | 1185 | 70.1 | 128.7 | 84 |
| 242 | 1.50 | .25 | 1635 | 1095 | 70.1 | 150.9 | 115 |
| 243 | 1.50 | .30 | 1545 | 1074 | 76.2 | 156.9 | 106 |
| 244 | 1.50 | .35 | 1470 | 975 | 87.0 | 190.8 | 120 |



SERIES 230-234 ON THREE DIFFERENT SHAFTS
○ .045" shaft
□ .050" "
△ .057" "
Percent Increase in Moment of Inertia
120
110
100
90
80
70
60
50
40
30
20
10
0
0
.05
.10
.15
.20
.25
.30
.35
.40
Mean Width Ratio of Blade


SERIES 230-234 ON THREE DIFFERENT SHAFTS
.045" shaft
.050" "
.057" "
Percent Increase In Moment of Inertia
Mean Width Ratio of Blade
0
10
20
30
40
50
60
70
80
90
100
110
120
.05
.10
.15
.20
.25
.30
.35
.40

## Conclusion

In spite of the inadequacy of the experiments so far the following conclusions may definitely be drawn:

(a) For a fixed amplitude and frequency the effect varies directly with the blade width ratio, and with the pitch ratio.

(b) For a given propeller the effect increases with frequency and amplitude.

(c) In order to assign a value to the per cent increase in moment of inertia in any given case the amplitude and frequency of the propeller must be approximately known.

## Suggestions for Future Course of Experiments

Work has been started in adapting a vibration generator to the apparatus now in use. This machine will be mounted on the upper end of the shaft and will be capable of setting up torsional oscillations up to a speed of 3600 r.p.m. This will give the condition of a shaft without rigid connections subject to periodic forces as in the case of a ship's propeller shaft but will not permit controlling the amplitude even as closely as in the apparatus now in use.

In order to extend the curves to cover blade widths now in use brass propellers of 16 inch diameter, having mean width ratios of .40, .45, .50, and .55, should be available.

It has been proposed to check some of the data obtained with models against full scale propellers. This may be accomplished, for example, on the USS HAMILTON when the thrustmeters are to be installed by mounting the vibration generator on an arm attached to the shaft. It may also be possible to attach the vibration generator to the shaft at the reduction gear. Such a test in dry dock and in water would furnish a basis for comparison with model experiments. A brass model of the HAMILTON's propeller would be needed for this comparison.

[illegible] excellent [illegible] [illegible] [illegible], because [illegible] nos. 10, 15, 20, and 25, should be available.

It [illegible] to [illegible] of the [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. It [illegible] [illegible] [illegible] today [illegible] [illegible] [illegible] [illegible]