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A DISCUSSION OF SYNCHRONISM OF SHIPS WITH WAVES

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NAVY YARD, WASHINGTON, D. C.

JUNE, 1931 REPORT NO. 301.

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1. Theoretically a ship with no damping can, by controlling the starting conditions, be made to roll in the period of a non-synchronous wave train. Ordinarily such a ship would roll in a period somewhere between her own natural one and that of the wave train.

2. Actually there is no such thing as a ship with no damping. An actual ship in a uniform wave train will eventually roll in the period of the waves regardless of starting condition. Under such forced oscillations, the maximum inclination of the ship will occur at mid height of the waves when the period is well removed from the natural one of the ship. As the wave and ship periods approach synchronism, the point of maximum inclination approaches the crests and troughs of the waves. Large angles of roll will be generated only in conditions close to synchronism. As a ship rolls to large angles, the period is lengthened, hence she will be thrown out of exact synchronism. When a ship is in a uniform wave train of synchronous period, each passing crest and trough will add an increment to the angle of roll of the ship. This increment is constant and depends on the wave slope in magnitude being $\pi/2 \times \phi$ where ϕ is the maximum effective wave slope. The ship will continue to build up amplitude of roll until the increment added per wave is equal to the decrement per roll occasioned by the damping.

Since the position of equilibrium of the ship is normal to the effective wave slope, the maximum inclination from vertical which the ship will attain is the angle at which the increment is equal to the decrement plus the angle of effective wave slope.

3. A wave can be considered to be made up of a series of trochoidal surfaces which flatten out as the depth below the surface increases. If a ship were of very shallow draft its position of equilibrium would be normal to the surface of the wave. As the draft increases the ship is acted on by a number of different slopes and assumes a position of equilibrium at an angle somewhat less than the normal to the surface. The slope of the theoretical layer of the trochoidal wave to which this equilibrium position is normal is called the effective wave slope. The maximum angle of slope is very nearly at mid height. In speaking of wave slope the maximum is generally meant.

4. Any ship will roll heavily if in a synchronous wave train of height = L/20 or greater. The amplitude of roll of a ship in a synchronous wave train depends on

(a) The effective wave slope.

(b) The damping.

5. To encounter very heavy rolling we must have waves that synchronize with the ship's period, steep waves, and small damping of the ship.

The period of the ship depends on the GM and the radius of gyration. The period of the wave is tied up with its length. The slope of the wave depends on length and height. The damping of the ship depends on its form and appendages.

6. Assume a ship whose period is about 17.3 sec.

For a wave $L = \left(\frac{P}{.442}\right)^2$

The length of a synchronous wave is 1550 feet. It is not likely that a wave of this length would ever be encountered.

7. Suppose this ship to be in a uniform series of waves 800 feet in length period 12.6 sec. and height 31 feet. On a course 30° to the wave crest the ship, for synchronism, would have to steam at about 21 knots. The maximum wave slope would be $\pi/2 \times 2\frac{H}{L} = \pi \times 31/800 = .116 = 6^\circ.65$. As the ship is at an angle of 30° to the crest this maximum slope would not act over the whole length at one time. The projected length of the ship in the direction of travel of the wave is 550 sin 30° = 225 feet. This is a little over half the half length of the wave. At any point on the wave profile the slope is very nearly $\pi/2 \;\; H/L \sin \pi/T_1 \, t$ where H is the height, L the length from trough to crest, T_1 the period of the wave from trough to crest, and t the elapsed time from the trough. Then the mean slope when the ship is spanning the center height of the wave is

$$\frac{2}{T_1} \;\; \pi/2 . H/L \int_{T_1/4}^{3T_1/4} \sin \pi/T_1 . t \, dt =$$

$$\frac{2}{T_1} \;\; \pi/2 . \frac{H}{L} . \;\; \frac{T}{\pi} \;\; \left[-\cos \pi/T_1 \, t \right]_{T_1/4}^{3T_1/4} =$$

$$\frac{2}{T_1} . \; \pi/2 . \frac{H}{L} . \frac{T}{\pi} \;\; . \; .1414 = .1095 = 6^\circ.27$$

8. For conditions as above the increment of roll would be $9^{\circ}.8$. This increment however is based on the surface slope of the wave and as explained previously the effective slope acting on the ship is less than the surface slope. If the effective wave slope is that of the layer ten feet below the surface the slope will be about $4^{\circ}.18$. The increment will be $6^{\circ}.57$. The ship fitted with bilge keels will have damping such that the decrement per roll will equal the above increment before any great angle of inclination is reached, probably from 15° to 20° inclination from the vertical.

9. The speed of a ship on any course relative to the wave crests to give synchronism with the waves can be computed by the formula

$$V_s \sin \alpha = V_w \left(1 \pm \frac{P_w}{P_s} \right)$$

V_s = speed of ship P_s = period of ship (complete oscillation)

V_w = speed of wave P_w = period of wave.

10. The example worked out above assumes a uniform wave train and in actual operation at sea such waves would be rarely encountered. All assumptions made are against the ship, that is, on the side of heavy rolling.

11. Now let us take a ship of the scout cruiser class W = 11,500 tons, GM = 5.55 ft. B = 64 feet, L = 570 feet. The period of a complete oscillation is about 12.2 sec. The length of a synchronous wave for this ship is about 760 feet and height

30 feet. The maximum effective slope of this wave is 4°.75. The increment per roll for the ship when broadside to the waves would be about 7°.5. The maximum inclination from vertical would be about 25° with the ship dead in the water, less when underway.

12. The tables I and II made out for the two ships, one with period 17 sec., the other 12 sec. show the courses and speeds necessary for synchronism with various wave series.

13. The tables indicate that either ship can synchronize with waves of 200' length when steaming at speeds greater than the wave speed only when on headings of 60° or greater angle to the wave crest. On these waves at these headings the ship would span two crests and the effective wave slope would be very small when integrated over the length of the ship. Little rolling would be expected. At speeds less than the speed of the wave the 12 sec. ship can effect synchronism on all courses greater than 15° relative to the crest; the 17 sec. ship on courses greater than 22°. However on courses greater than 30° the ships would span two crests. In waves of 400' length and over the ships could synchronize with the waves only by steaming slower than the waves. The 12 sec. ship could synchronize on all courses greater than 13° relative to the crests, the 17 sec. ship on courses greater than 23°. On courses greater than 45° the ships would span two crests. In 600 foot waves the 12 sec. ship could synchronize on headings greater than 6°; the 17 sec. ship on courses greater than 22°. In 800' waves the 12 sec. ship could synchronize on all courses greater than 2° by steaming slowly into the sea. The period of

an 800 foot wave is about 12.4 sec. (nearly that of the ship). The 17 sec. ship could synchronize on headings greater than 18° relative to the crests. On the 600 and 800 foot waves, headings greater than 45° relative to the crests would give small effective wave slopes. The slopes of waves of length less than 400 feet would probably be small.

14. The data of the tables are plotted on curves I and II. These curves show clearly the large area over which synchronism could be effected by the 12 sec. ship and the small area in which the 17 sec. ship might synchronize with any probable sea waves. It must be borne in mind that it is in situations of synchronism that the waves have their greatest effect on the ship in inducing roll. The amount of roll induced depends on the damping characteristics of the ship. It is apparent however that of two ships with identical damping characteristics the one least likely to synchronize with probable sea waves will be the better sea boat.

15. Waves below 200' in length would have little effect on the ships here considered. Waves over 800' in length are of very rare occurrence. For these reasons the present investigation was bounded by 200' and 800' waves. An investigation of the behavior of ships after synchronism with the waves has been effected is underway.

PERIOD 12 sec.

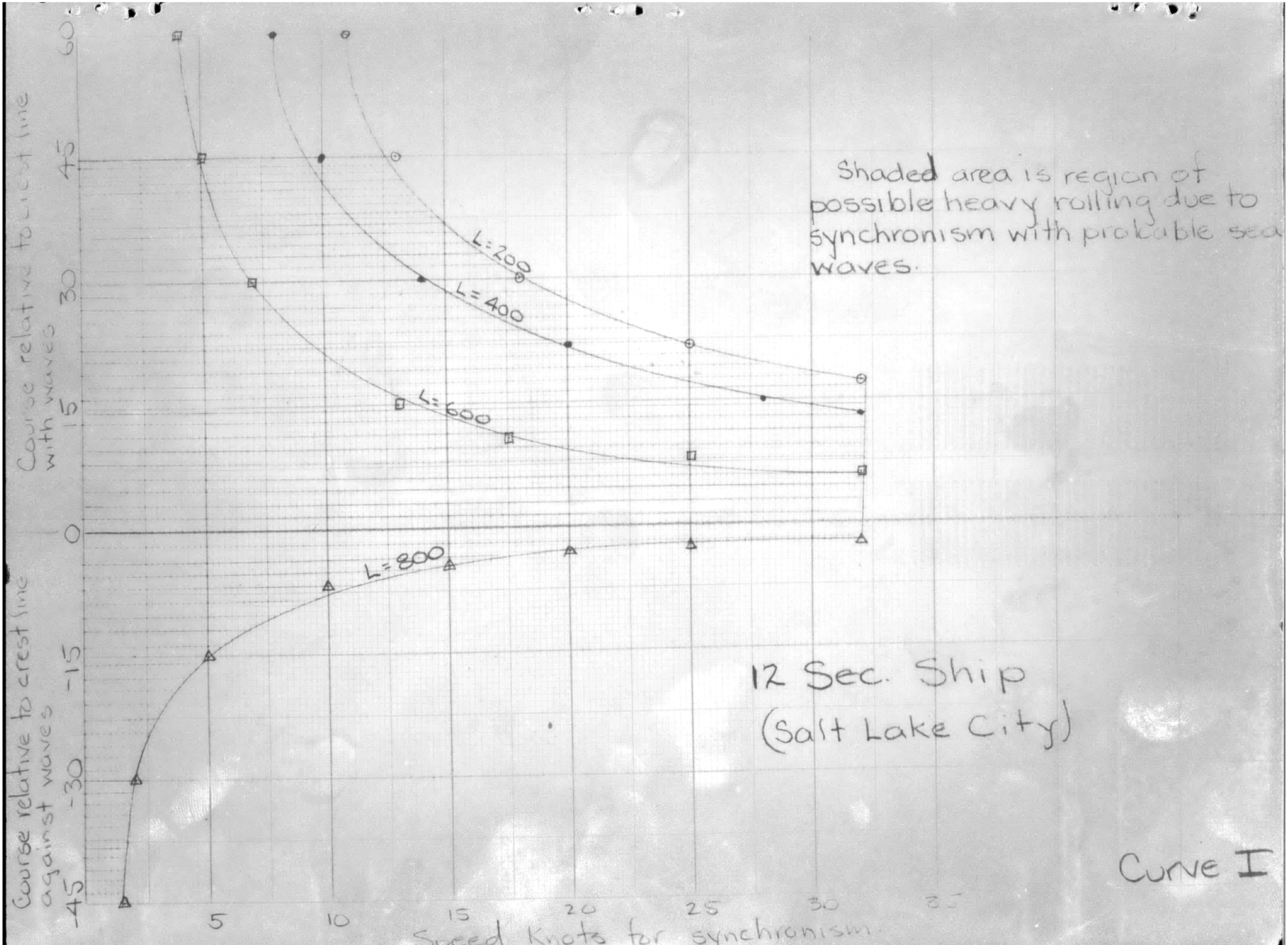
W.L.	Course relative to crest	Ships speed for synchr.	
	0	∞	
	15	111	35
200	30	58	18
	45	41	13
	60	33	11
	0	∞	
	15	180	28
400	30	93	14
	45	66	10
	60	53	8
	0	∞	
	15	240	13
600	30	125	7
	45	88	5
	60	72	4
	0	Nearly synchronous	
	15	300	-5
800	30	150	-2
	45	110	-1.5
	60	90	-1

TABLE I

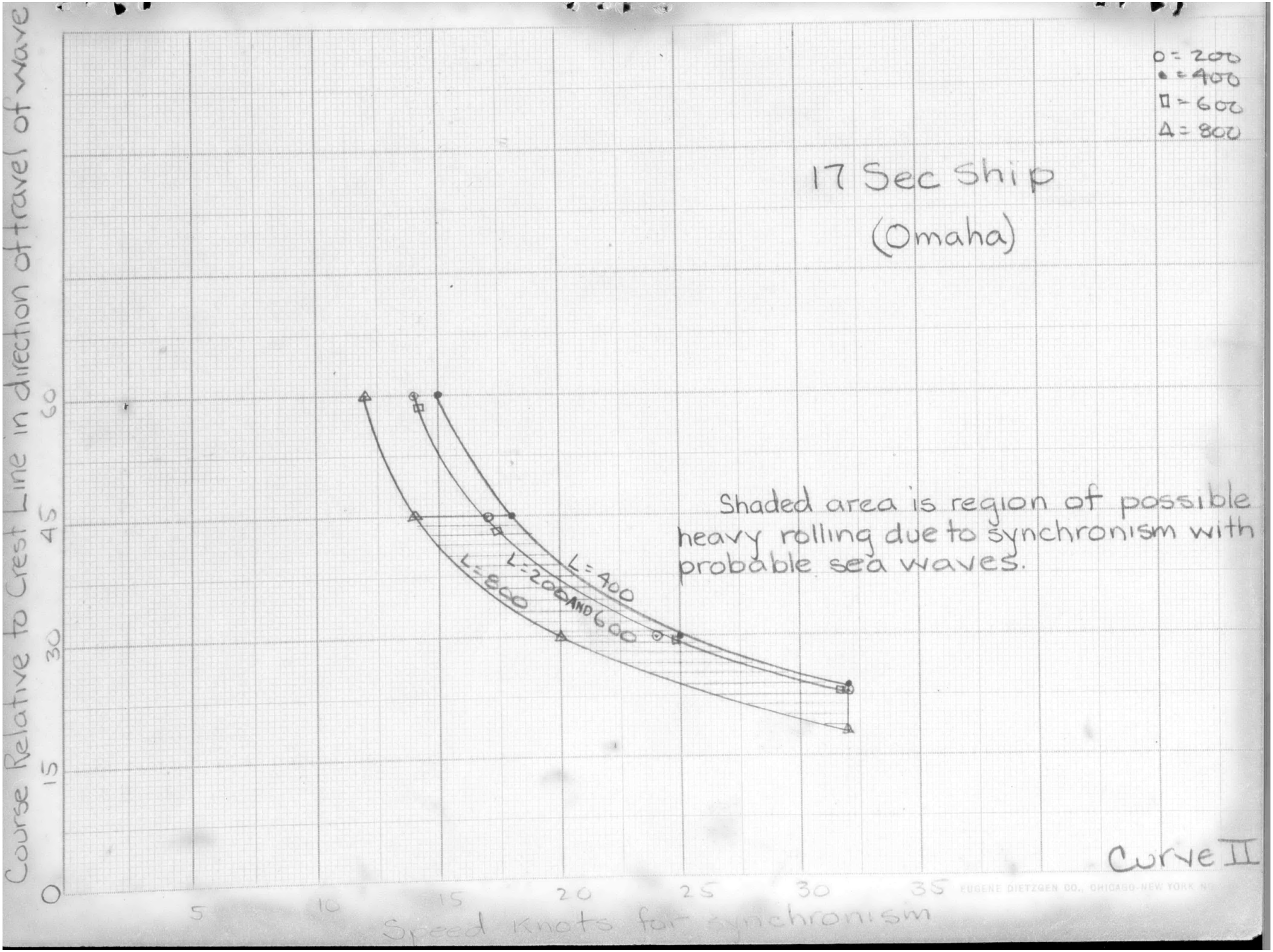
PERIOD 17 sec.

Wave length	Course relative to crest	Ships speed for synch.		
200	0°	∞		
	15	100 kts	or	47 kts
	30	52 "	"	24 "
	45	37 "	"	17 "
	60	30 "	"	14 "
400	0	∞		
	15	160	or	49
	30	83		25
	45	58		18
	60	48		15
600	0	∞		
	15	208		46
	30	107		24
	45	76		17
	60	62		14
800	0	∞		
	15	253		39
	30	131		20
	45	93		14
	60	76		12

TABLE II



Shaded area is region of possible heavy rolling due to synchronism with probable sea waves.
L=200
L=400
L=600
L=800
12 Sec. Ship
(Salt Lake City)
Curve I
Course relative to crest line with waves
Course relative to crest line against waves
Speed Knots for synchronism
60
45
30
15
0
-15
-30
-45
5
10
15
20
25
30
35



o = 200
• = 400
□ = 600
Δ = 800
17 Sec Ship
(Omaha)
Shaded area is region of possible heavy rolling due to synchronism with probable sea waves.
L = 400
L = 200 AND 600
L = 800
Course Relative to Crest Line in direction of travel of wave
0
15
30
45
60
5
10
15
20
25
30
35
Speed Knots for synchronism
Curve II
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