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UNITED STATES

EXPERIMENTAL MODEL BASIN

NAVY YARD, WASHINGTON, D.C.

THE INFLUENCE OF STIFFENING RINGS ON THE STRENGTH OF THIN CYLINDRICAL SHELLS UNDER EXTERNAL PRESSURE

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OUTLINE

THE INFLUENCE OF STIFFENING RINGS ON THE STRENGTH OF THIN CYLINDRICAL SHELLS UNDER EXTERNAL PRESSURE

by Charles Trilling

This report considers the strength of the frames or circumferential stiffening rings of thin circular cylindrical shells subjected to external pressure, and the influence of the size of the stiffeners upon the strength of the shell.

A survey is made of the theoretical work pertaining to this problem.

Theoretical formulas are compared with the results of tests conducted at the U. S. Experimental Model Basin on a series of model pressure vessels with frames of varying size.

Changes in the present design of frames of a submarine pressure hull are suggested in the report. The changes proposed would result in an appreciable reduction in the weight of the frames. This weight could be either saved or distributed to better advantage.

The conclusions reached are based both upon theoretical considerations and upon the results of the model experiments.

GENERAL THEORY

Shell Strength

A thin cylindrical shell under external pressure, like a column, may collapse either "by yield" or "by instability." Collapse by yield is caused by stresses in the shell reaching the yield point of the material. Collapse by instability is caused by buckling of the shell at stresses which may be considerably below the yield point. In either case the effective length of the vessel (Ref 1 and 2 of the list of references at the end of the report) may be taken as the length of unsupported cylinder between adjacent transverse or circumferential supports, whether these supports are the solid ends of the vessel, whether they are bulkheads or partitions, or whether they are circumferential stiffening rings or frames, provided the frames possess adequate rigidity. Ignoring, for the time being, what constitutes adequate rigidity, and assuming that we are dealing only with frames of sufficient strength to insure that the distance between them determines the effective length of the vessel, we shall investigate the influence of the size of the frame upon the strength of the shell. In the case of vessels which collapse by instability at low stresses, this influence, as far as is known, is nil. In the case of vessels which fail by yield, this influence has been evaluated in the rather well known von Sanden-Gunther formulas, (92) and (92a), (Ref 3, p 220), (Ref 5, p 48). These formulas are:

Formula (92)

$$p_y = \frac{2\frac{t}{D}s_y}{\frac{1}{2} + \sqrt{\frac{3}{1-\mu^2}}\,K\,\frac{1-\frac{1}{2}\mu - B}{1+\beta}} = \frac{2\frac{t}{D}s_y}{\frac{1}{2} + 1.81\,K\,\frac{0.85-B}{1+\beta}} \qquad (1)$$

Formula (92a)

$$p_y = \frac{2\frac{t}{D}s_y}{1 + H\,\frac{1-\frac{1}{2}\mu - B}{1+\beta}} = \frac{2\frac{t}{D}s_y}{1 + H\,\frac{0.85-B}{1+\beta}} \qquad (2)$$

where p_y = external collapsing pressure of shell.
s_y = yield point of the material.
μ = Poisson's ratio, assumed = 0.3.
D = diameter to the neutral axis of the shell.
t = thickness of shell.
L = effective length of vessel.
b = flange width of frame in contact with shell.
A = cross sectional area of frame.

$$B = \frac{bt}{A + bt}$$

$$\theta = 10\sqrt[4]{12(1-\mu^2)}\;\frac{L/D}{\sqrt{100\ t/D}} = \frac{18.2\ L/D}{\sqrt{100\ t/D}}$$

$$\beta = 2\,\frac{L}{b}\,\frac{NB}{\theta} = \frac{2Nt}{\frac{\theta}{L}(A + bt)} = \frac{11N}{\sqrt{100\ t/D}}\;\frac{t^2}{A + bt}$$

$$N = \frac{\cosh\theta - \cos\theta}{\sinh\theta + \sin\theta}$$

$$K = \frac{\sinh\theta - \sin\theta}{\sinh\theta + \sin\theta}$$

$$H = -2\,\frac{\left[1 + \sqrt{\frac{3\mu^2}{1-\mu^2}}\right]\sinh\frac{\theta}{2}\cos\frac{\theta}{2} + \left[1 - \sqrt{\frac{3\mu^2}{1-\mu^2}}\right]\cosh\frac{\theta}{2}\sin\frac{\theta}{2}}{\sinh\theta + \sin\theta}$$

$$= -\,\frac{3\sinh\frac{\theta}{2}\cos\frac{\theta}{2} + \cosh\frac{\theta}{2}\sin\frac{\theta}{2}}{\sinh\theta + \sin\theta} \qquad \text{(approximately)}$$

Formula (92a) as originally published (Ref 3, 5) was slightly different from Eq (2) given above. The original equation contained an error in that the coefficients of the transcendental terms in the numerator of the function H were interchanged, and the sign connecting these terms was minus instead of plus.

For convenience graphs of the functions N, K, and H, plotted against the argument θ, are shown in the Appendix as Figs 4, 5, and 6.

It was mentioned that these formulas imply that the vessel collapses by stresses in the shell reaching the yield point of the material. In formula (92) it is the longitudinal stress (bending plus axial stress) at the frame that reaches the yield point, and in formula (92a) it is the tangential or circumferential stress midway between frames. That formula is considered determinative which gives the lower value of the collapsing pressure. Usually the greater stress is longitudinal and consequently formula (92) is used extensively while formula (92a) is hardly ever used.

According to formula (92) and, except for very short thick tubes (in which case H is negative), according to formula (92a) also, the stronger the frame the weaker the vessel. This surprising result is explained by the fact that a light frame is less of a so-called "hard spot" than a heavy frame, that is the light frame contracts more under external pressure thereby allowing a more uniform contraction of the shell and lower stresses (particularly the longitudinal bending stresses) in the shell.

<u>The simple "hoop stress formula"</u> for the tangential stress in a ring of rectangular cross section

$$p_y = 2 \frac{t}{D} s_y \tag{3}$$

gives results which do not differ greatly from those given either by formula (92) (for frames of ordinary dimensions) or by formula (92a). Strictly speaking, outside diameter, not diameter to the neutral axis, should be used in Eq (3).

<u>A discussion of the application of yield formulas to thin cylindrical shells</u> will now be given to bring out an important point to which no attention has been given in the past. Thin-walled cylinders (with thicknesses say from 0.001 D to 0.01 D) collapse not by yield but by instability with the accompanying characteristic lobe formation in contrast to so-called "thick cylinders," in which a true yield failure may occur. What is generally regarded as collapse by yield of a thin cylinder is really a collapse by instability in the intermediate region (region of reduced modulus) at stresses near the yield point. In such a case, however, yield formulas can still be applied. This is best made clear by a detailed consideration of the case.

In a previous report (Ref 7) the collapse of pressure vessels by instability or buckling of the walls has been fully treated. It is not necessary in this discussion to quote formulas giving the pressure which causes unstable collapse; it is sufficient merely to call attention to the fact that in all such instability formulas the collapsing pressure is directly proportional to the modulus of elasticity of the material. These instability formulas should continue to be valid for stresses above the proportional limit provided the correct value of the effective modulus (Ref 8, 9) is used.

Consider now a relatively short thick pressure vessel, one which would commonly be supposed to collapse by yield. The maximum tangential stress, s, due to any external pressure, p, is given, let it be assumed for simplicity, by the simple hoop stress relation

$$p = 2 \frac{t}{D} s \qquad (3a)$$

As long as s is less than the proportional limit, s_e, (practically the same as the elastic limit) then for a vessel of the type considered, p is far below the pressure required by instability formulas for an unstable collapse. If, however, p be increased sufficiently so that s exceeds s_e, the effective modulus decreases rapidly, and finally at a critical stress s_c, where

$$s_e < s_c < s_y$$

it becomes so small that, as shown by instability formulas, an unstable collapse occurs. The collapsing pressure is, by Eq (3a)

$$p_c = 2 \frac{t}{D} s_c \qquad (3b)$$

Thus it is seen that collapse is not by yield for the yield point of the material (for which the effective modulus is zero) is not reached. However, in the vessel considered, s_c differs but little from s_y and hence comparison of Eq (3) and (3b) shows that p_c differs but little from p_y. For a relatively short thick pressure vessel, then, a yield formula predicts very closely the pressure at which collapse by instability occurs.

The use of yield formulas in the intermediate region of stress is thus justified. However, a different mode of reasoning now guides the decision as to which yield formula to use. Since the collapse of a relatively short thick cylinder is by instability and is due to tangential stresses approaching the yield point, tangential stresses only need be considered. The longitudinal bending stresses at the frames may be higher but they are of little consequence for at most they merely cause the shell to bend over the frame and relieve the stress. Convincing evidence of this is afforded by a photograph in a previous report (Ref 1, Fig 3), (Ref 10, p 44). Ridges are shown in the shell where it bends over internal stiffening rings, indicating that longitudinal bending stresses had reached the yield point without causing collapse of the shell.

Since, then, tangential stress, not longitudinal stress, is determinative in causing collapse of thin-walled cylinders, formula (92a), not (92), is the correct yield formula to use. The experimental results given later further substantiate this conclusion.

Frame Strength

The preceding discussion of shell strength was based on the assumption that only frames of "adequate strength" were involved. What constitutes adequate frame strength will now be investigated. The frame is first regarded simply as a ring under uniform external pressure. Such a ring may collapse either by instability or by yield.

Collapse by instability will be considered first. The generally accepted formula for the unstable collapse of a simple circular ring subjected to a uniform external pressure is that due to Lévy* (Ref 11)(Ref 12, p 247)

$$q_f = \frac{3\,E\,I}{R_f^3} = \frac{24\,E\,I}{D_f^3} \tag{4}$$

where q_f = collapsing load per unit circumferential length, or collapsing pressure on unit axial length of the ring.

E = modulus of elasticity of the material.

I = moment of inertia of the ring cross section about the neutral axis parallel to the axis of the ring.

$D_f = 2R_f$ = diameter to the neutral axis of the ring.

In order to apply Lévy's formula to the frame of a pressure vessel, a relation between the load q on the frame per unit circumferential length and the external pressure p on the shell must be established.

Standard practice follows the recommendation of von Sanden and Günther (Ref 4, p 508)(Ref 6, p 13). The policy is to make the frames 10 per cent stronger than is necessary to meet the requirement that the frames are to hold up even after the shell has collapsed by bulging between them. The shell is assumed to offer no support after buckling, but to transfer all of its load to the frames. Thus the frames are designed for the load

$$q = 1.1\ p\ L' = 1.1\ p\ (L + b) \tag{5}$$

where L and b are defined in connection with Eq (1) and (2), and the quantity L' is the frame spacing, i.e. the center to center distance between adjacent frames.

* Lévy's formula, Eq (4), is better known as Föppl's formula. Von Sanden and Günther (Ref 3, No 8, p 167) attribute it to Boussinesq (Ref 13) who, however, is responsible most likely for only a preliminary equation (Ref 12, Eq (223), p 230).

Eq (5) and (4) determine what value of I is required for the frame. Although in standard practice the support of the shell is ignored in computing the load on the frame, it is recognized that a portion of the shell material acts with the frame and contributes to its moment of inertia. Just how much of the shell is effective is not definitely established, but it is customary to use a strip of width equal to the flange width b in contact with the shell; this is believed to be a conservative allowance (Ref 1), (Ref 4, p 507), (Ref 6, p 8). Hence the I in Eq (4) must be replaced by I_b which denotes the moment of inertia of the compound section consisting of the frame section and an area bt of the shell. With this modification, combining Eq (4) and (5) gives the standard practice design formula

$$p_f = \frac{1}{1.1}\,\frac{24\,E\,I_b}{D_f^3 L'} \tag{6}$$

where p_f is the greatest external pressure on the shell the frame is considered capable of supporting. If the predetermined collapsing pressure of the shell be denoted by p_s, a frame – that is an I_b – is selected so as to satisfy the relation

$$p_f \geqq p_s \tag{7}$$

Von Sanden and Günther (Ref 3, Eq (88), p 221)(Ref 5, Eq (88), p 50) developed, but never used, the following relation between q and p, which shall be referred to as Formula (88).

$$F = \frac{q}{p} = \frac{1\,\cancel{\beta}}{1+\beta}\left[(1 - \tfrac{1}{2}\mu)\,\beta\,\left(\frac{A}{t} + b\right) + b\right] \tag{8}$$

The symbols used are described in connection with Eq (1) and (2). Here, however, q is the load on a compound frame structure consisting of the frame and the contiguous strip of the shell of width b. This load is composed of the external pressure on the strip and the shear forces exerted by the shell on each side of the strip (Ref 3, No 9, p 196)(Ref 5, p 30). Eq (8) with $\mu = 0.3$, can be put in the form

$$q = p\,F \tag{8a}$$

$$F = b\left[\frac{1 + \frac{0.85\beta}{B}}{1 + \beta}\right] \tag{8b}$$

Substituting Eq (8a) in Lévy's formula, Eq (4), and, replacing I by I_b and p by p_f, gives the following frame design formula

$$p_f = \frac{24\,E\,I_b}{D_f^3\,F} \tag{9}$$

Eq (9) should be compared with Eq (6).

Tokugawa (Ref 14) developed the following frame design formula

$$p_f = \beta \frac{2\,E}{1-\mu^2}\left(\frac{t}{D}\right)^3 + \gamma \frac{24\,E\,I}{D^3 L'} \tag{10}$$

where

$$\beta = 1 + 3\left(\frac{2\delta}{t} - 1\right)^2$$

$$\gamma = 1 + \frac{t^2}{4\,i_o^2}\left[\left(1 + \frac{2\nu}{t}\right) - \left(\frac{2\delta}{t} - 1\right)\right]^2$$

$$\delta - \frac{t}{2} = \frac{A\,(\nu + \frac{t}{2})}{A + L't}$$

or $$\left(\frac{2\delta}{t} - 1\right) = \frac{1 + \frac{2\nu}{t}}{1 + \frac{L't}{A}} = \left[\frac{A}{bd}\right]\frac{d}{t}\,\frac{1 + \frac{2\nu}{t}}{\frac{L'}{b} + \left[\frac{A}{bd}\right]\frac{d}{t}}$$

and where (see diagram, Fig. 1)

I = moment of inertia of the frame section alone.

i_o = radius of gyration of the frame section alone.

ν = distance from inner surface of the shell to the neutral axis of frame section alone.

δ = distance from outer surface of the shell to the neutral axis of the compound section.

d = depth of frame = 2ν in most cases.

The other symbols have been previously defined, particularly in connection with Eq (1) and (2).

This formula gives the pressure on the shell which causes what Tokugawa terms (Ref 14, pp 219, 253) "bodily collapse of shell and frame." Tokugawa's original formula (Ref 14, Eq (10), p 254) is actually more general than Eq (10). It involves not only the frame spacing L' but also the overall length of the vessel. Eq (10) is for the special case of a tube of infinite overall length.

Eq (10)(with the $1/(1-\mu^2)$ factor omitted) is equivalent to Lévy's formula applied to a compound frame structure consisting of the frame and a strip of the shell of width equal to the frame spacing L'. This is readily seen by writing Eq (10) in the form

$$p_f = \frac{24\,E}{D^3 L'}\left\{\frac{1}{1-\mu^2}\left[\frac{1}{12}L't^3 + L't\left(\delta - \frac{t}{2}\right)^2\right] + \left[I + A(t - \delta + \nu)^2\right]\right\} \tag{10a}$$

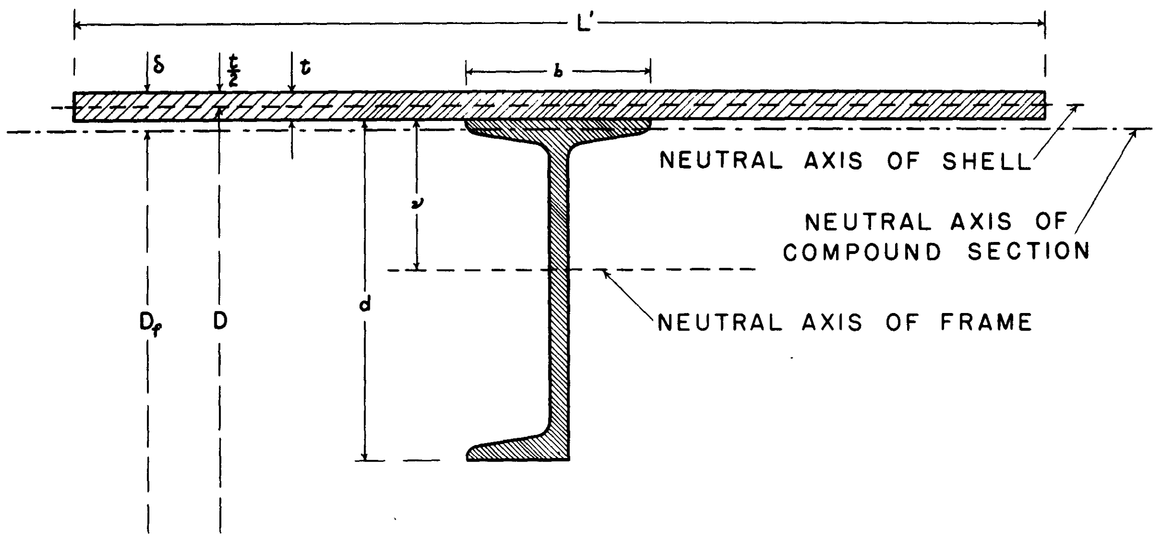

FIG. 1. DIAGRAM TO ILLUSTRATE VARIABLES IN TOKUGAWA FORMULA

The expression in the first pair of brackets is seen to be the moment of inertia of the shell with respect to the neutral axis of the compound section; the expression in the second pair of brackets is the moment of inertia of the frame with respect to this axis. It is to be noted, however, that D and not D_f is used. The use of Lévy's formula for such a section is justified only when the frames are so light that both shell and frame collapse as a unit.

The "plate factor" $1/(1 - \mu^2)$ which appears in Eq (10) is due to the slight difference in behavior between a wide thin plate and a narrow strip under bending (Ref 12, pp 52, 260), and likewise between a long thin tube and a ring under external pressure. For $A = 0$ ($I = 0$, $\beta = 1$) Eq (10) reduces to the generally accepted Bresse-Bryan formula (Ref 1, 7) for a thin cylindrical shell of infinite length, which verifies the correctness of inserting the $1/(1 - \mu^2)$ factor.

Eq (9) and (10) which treat the problem of frame design by considering the vessel as a whole and by taking into account the resistance of the shell in computing the frame strength warrant more discussion.

A frame cannot fail before the shell fails. For this reason, the viewpoint of standard practice that the frames should form a basic framework to support the whole structure against collapse is perhaps not the most desirable. A frame might better be regarded as an auxiliary of the plate, whose function is to stiffen the plate and develop its maximum compressive strength. From this viewpoint, not the strength of the stiffener. but only its influence upon shell strength seems to be of interest. However, it will be shown that the strength of the stiffener can be used as a criterion for adequate stiffening action.

Starting with a tube with extremely light frames (i.e. practically an unstiffened tube), let us consider the effect of gradually increasing the size of the frames in such a way as to always increase their cross sectional moment of inertia. The strength of the tube is greatly influenced at first, and it increases rapidly with increased size of frame. Then for a certain "critical size" of frame, this increase in shell strength ceases abruptly (as is shown later in Fig 2), and further increase in the size of the frame may have even a slight weakening effect. This critical size of frame is the dividing point between frames which do and frames which do not adequately support the shell. Adequate support consists of maintaining the circularity of the cylinder at the frames (as required by both instability and yield formulas for the shell) which results in making the length between frames the effective length L of the vessel. The fundamental assumption that when a stiffening ring becomes unstable it is incapable of maintaining circularity and of giving adequate support to the shell is a reasonable assumption and is the basis of Eq (9) and (10).

Eq (9) and (10), then, involve two methods of determining the pressure on the shell which causes the frame to become unstable. Eq (9) is the more direct method. Lévy's formula is applied directly to the frame (really to the compound frame structure that includes the contiguous shell), using the load transmitted by the shell to the frame. In Eq (10) the frame and shell are considered as a unit and the pressure for which the combined structure becomes unstable is determined. Although both equations seem theoretically sound, it will be shown later that the agreement between them is not very good. No satisfactory explanation for this is seen.

Collapse of the frame by yield will now be briefly examined. The maximum tangential stress in a ring of rectangular cross section under compression occurs on the inner surface. The failing pressure of the ring due to this stress reaching the yield point is given in the following formula of Lamé (Ref 15), (Ref 12, Eq (244), p 252), (also Ref 16, p 56)

$$\frac{q_f}{b} = \frac{R_2^2 - R_1^2}{2\,R_2^2}\, s_y \tag{11}$$

where q_f is defined in connection with Eq (4) and

b = width of ring, or flange width.

q_f/b = external failing pressure of the ring.

R_1 = inside radius.

R_2 = outside radius.

s_y = yield point of the material.

For frames of ordinary dimensions Eq (11) is but little different from the simple hoop stress formula, Eq (3), which can be written

$$\frac{q_f}{b} = \frac{R_2 - R_1}{R_2} s_y \tag{12}$$

Although Eq (11) and (12) apply only to rings of rectangular cross section, the collapsing load on a ring of any cross section can be roughly approximated by the following modification of Eq (12)(cf. Eq (22) and (32), p 166, Ref 3; pp 11, 12, Ref 5)

$$\frac{q_f}{b} = \frac{A}{bd} \frac{R_2 - R_1}{R_2} s_y \tag{12a}$$

Eq (12) and (12a) assume uniform stress distribution, a fair approximation to the actual state of stress, the error depending on the size and shape of the ring.

Prior to collapse of the shell the maximum stress in the frame is always less than the maximum stress in the shell. This can be readily checked by Lamé's formula with the von Sanden – Günther load, Eq (8). Hence collapse of the frame by yield need be considered only when it is desired, as in standard practice, that the frames hold up even after the shell has collapsed.

Graphical Representation of Theoretical Formulas

The various theoretical formulas presented for both shell strength and frame strength are represented graphically in Fig. 2 on the same set of axes. The abscissa is d/t, the ratio of depth of frame to thickness of shell, and the ordinate is the variable ψ, called the "pressure factor," and defined by the relation

$$\psi = \frac{p}{2 \frac{t}{D} s_y} \tag{13}$$

where p is the pressure on the shell that causes collapse of either shell or frame depending on the formula plotted.

Experimental models to be described later had the nominal dimensions

$$\begin{aligned} L/D &= 0.140 \\ t/D &= 0.003 \end{aligned} \tag{14}$$

and these values were used in plotting the formulas to allow comparison with experimental results. The small circles and crosses in Fig. 2, representing experimental points, will be discussed later.

The three formulas for the strength of the shell, the von Sanden–Günther formulas (92) and (92a), Eq (1) and (2), and the simple hoop stress formula Eq (3) are represented in Fig. 2 by the curves A, B, and C respectively. These formulas

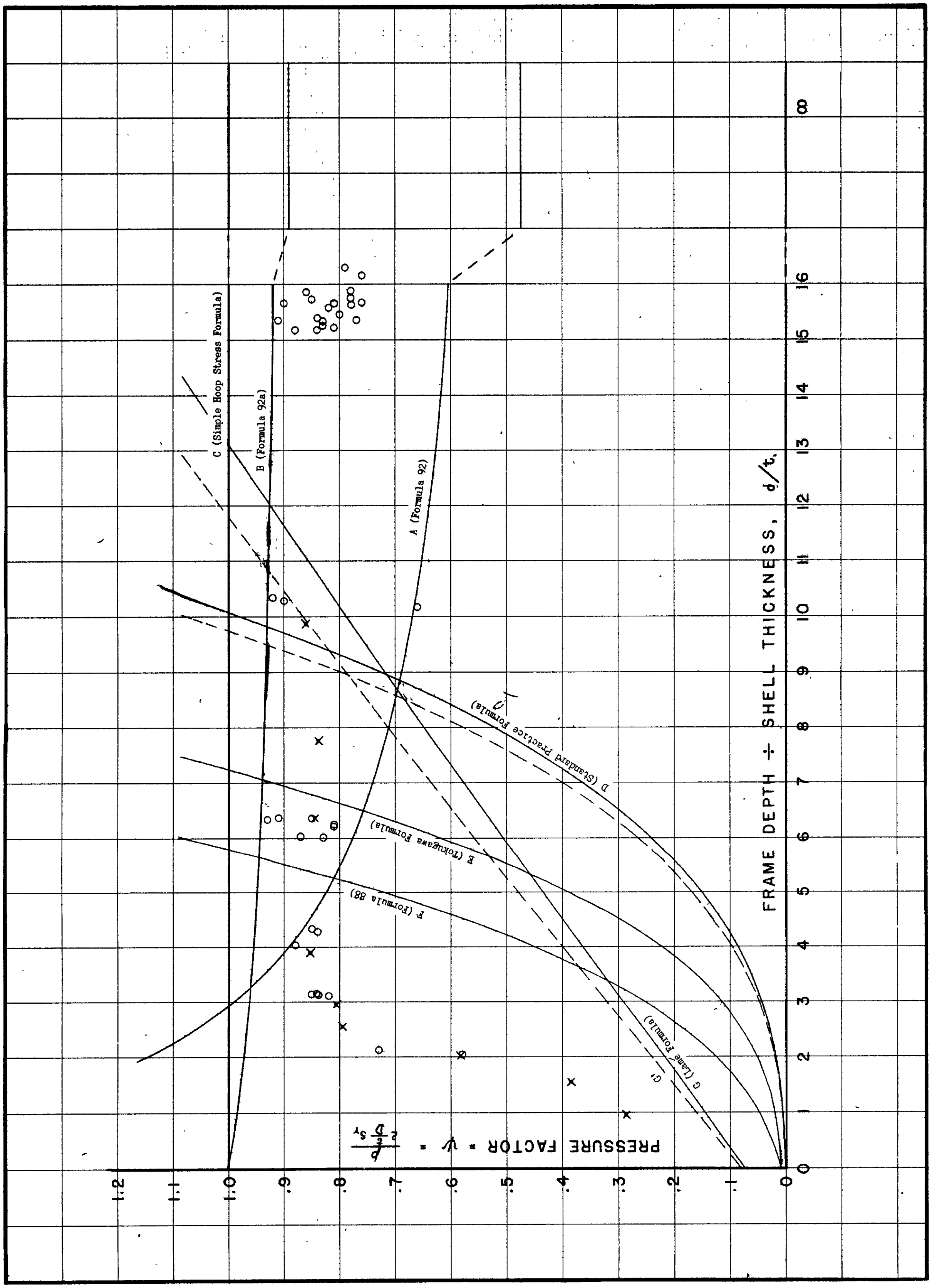

FIG 2. GRAPHICAL REPRESENTATION OF THEORETICAL FORMULAS AND EXPERIMENTAL DATA.

show how the shell strength varies with the size of the frame, but they do not consider whether the frames are strong enough to support the shell up to its predicted collapsing pressure, that is, they imply adequate frame strength. For that reason these curves show no drop in the region of small values of d/t.

Fig. 2 does not show a curve for collapse of the shell by instability. The curve representing the U.S. Experimental Model Basin instability formula (Ref 7, Eq (9)), assuming the dimensions of Eq (14) and the following values of the physical properties of the material

$$
\begin{aligned}
E &= 30{,}000{,}000 \text{ lb. per sq. in.} \\
s_y &= 30{,}000 \text{ lb. per sq. in.} \\
\mu &= 0.3
\end{aligned}
\qquad (15)
$$

would be the horizontal line

$$\psi = 1.85 \qquad (16)$$

which is far above curves A and B representing collapse by yield. Hence models with the dimensions of Eq (14) fail at stresses near the yield point, and unless the correct value of the effect modulus is used instead of E, instability formulas can not be used to compute collapsing pressures of these models.

Various theoretical formulas for the strength of the frame are plotted. Wherever necessary the values of Eq (14) and (15) were assumed.

Curves D and D' both represent Eq (6), the design formula of standard practice, the 1/1.1 factor being omitted in curve D'.

Curve E represents Eq (10), Tokugawa's formula. This formula is practically equivalent to Lévy's formula applied to the compound structure composed of the frame and a strip of the shell of width L'.

Curve F represents Eq (9), the design formula based on determining the load on the compound structure composed of the frame and a strip of the shell of width b by the von Sanden-Günther formula (88), Eq (8), and then, using this load, applying Lévy's formula to this frame structure.

Curves G and G' both represent Lamé's formula, Eq (11), with the standard practice load, Eq (5), on the frame, the 1.1 factor being omitted in curve G'. These curves, like curves D and D', imply that the frame is to hold up even after the shell collapses.

All these curves for the frame strength start at or near the origin and rise rapidly, soon intersecting the curves for the shell strength, A and B. The complete theoretical curve, then, consists of two intersecting curves, the proper frame strength curve on the left (small values of d/t) and the proper shell strength curve on the right (large values of d/t). Their point of intersection determines the theoretical value of the "critical d/t," corresponding to the "critical strength" of frame previously mentioned. It is clearly seen that as d/t is decreased from infinity, theoretically the strength of the vessel first increases somewhat and then, at the critical d/t, falls off rapidly.

MODEL EXPERIMENTS

Twenty model pressure vessels with frames of varying size were built and tested. The method of constructing and testing model pressure vessels at the U. S. Experimental Model Basin has been previously described in detail (Ref 10, 17). The twenty models mentioned form a group which has received the designation "Type VI." The characteristics of these models are shown in the drawing, Fig 3, and in the photographs, Fig 7, 8, 9 in the appendix. It is seen that each model has two pairs of frames; a heavy pair which is the same for all models, and a light pair of constant width and varying depth. The t/D and L/D ratios are about the same as those for a submarine pressure hull of the type of the USS CACHALOT.

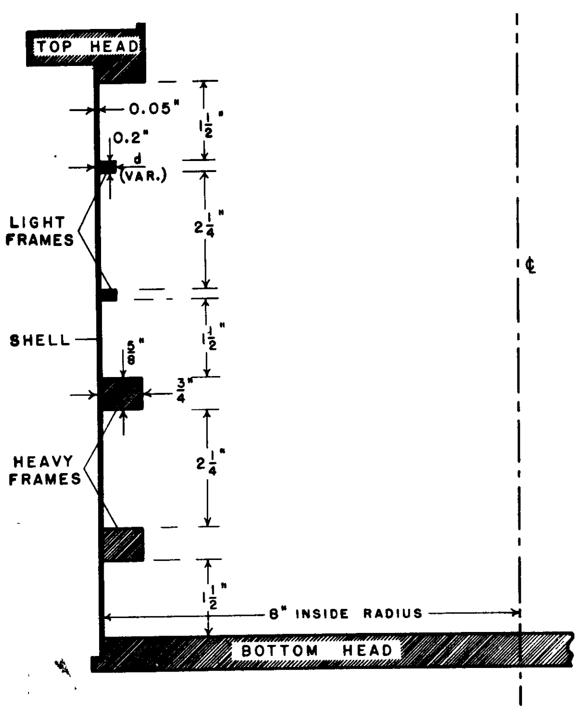

FIG 3. MODEL PRESSURE VESSELS, TYPE VI. LONGITUDINAL CROSS SECTION.

The heavy frames were used as a control. It was feared that the results obtained with the light frames might not be comparable because variations in the out-of-roundness of the models would cause differences in collapsing pressure greater than the differences to be measured. It was hoped that the effect of irregularities in the models would be indicated and a basis of comparison obtained from the collapsing pressures of the shell between the heavy frames.

In testing a type VI model the usual "polar diagrams" (Ref 1, Fig 4) (Ref 10) were obtained. In order to continue the test after collapse of the shell between one pair of frames, this space was reinforced by a heavy segmented reinforcing ring placed midway between the frames.

The control (heavy frames) proved to be more of a hindrance than a help. This was due chiefly to the fact that in order to obtain the ultimate collapsing pressure of the shell between one pair of frames it was necessary to damage the model considerably. It was then difficult and often impossible to continue the test and determine the collapsing pressure of the shell between the other pair of frames—particularly when the shell between the light frames failed first.

The experimental results obtained from tests of the twenty type VI models are listed in Table I. These results are plotted as small circles in Fig 2, and can be compared with the predictions of the theoretical formulas. The use of ψ instead of p as the ordinate in Fig 2 has the advantage of automatically correcting for variations in the shell thickness and in the yield point of the material.

Control data were not used to correct the experimental results for the light frames in any way. Variations in out-of-roundness of the different models of the series seemed to have small influence, and the control results are far from being uniform and accurate enough to evaluate this influence. Thus the use of a control not only was a hindrance but also it did not fulfill its purpose. While the observed collapsing pressures of the shell between the heavy frames are not of value as a control, they are, however, of value in themselves. They furnish twenty additional points to the diagram, Fig 2, in a region otherwise unexplored.

Tokugawa's experiments (Ref 14, p 256) on small brass models are similar to those performed at the Model Basin. The models, machined from thick cold-drawn brass tubes, were 100 mm (3.9 in) in diameter and 0.5 mm (0.02 in) nominal thickness, giving a t/D ratio of 0.005. The frames, which formed an integral part of the shell, were 3 mm (1/8 in) wide and of varying depth. The models were divided into groups, each with a different frame spacing. Only models of Tokugawa's series J_5 will be considered, for these models had an L/D ratio of 0.12 which corresponds closely to the type VI model L/D ratio of 0.14. However none of Tokugawa's models are strictly comparable with the Model Basin models because of the large difference in the t/D ratio and in the material properties.

Tokugawa's experimental results are represented by crosses in Fig 2 assuming the arbitrary value

$$s_y = 4000 \text{ Kg/cm}^2 = 57000 \text{ lb/in}^2$$

for the yield point of the brass. It is considered significant that the experimental curves determined by the two sets of data had the same general shape and showed a similar rapid decrease in collapsing strength at practically the same value of d/t, but otherwise the agreement in ψ values between the two sets of points is of no importance since it depends on the arbitrarily chosen value of s_y.

Comparison of Theory with Experiment.

The experimental results shown in Fig 2 support the conclusion (previously stated on page 5) that formula (92a), Eq (2), and not (92), Eq (1), is the correct yield formula. Considering only the region on the right in that figure, we note that the large decrease in shell strength with increasing frame strength predicted by formula (92) is not borne out by the experimental points. Beyond a certain relatively small value of d/t the frame strength is found to have no appreciable influence on the shell strength. Even formula (92a) for the extreme case $d/t = \infty$

TABLE I
EXPERIMENTAL RESULTS

For all models (see Fig 3, page 13) the diameter D = 16.05 in., and the unsupported length of shell between both the light frames and the heavy frames is L = 2.25 in.; thus L/D = 0.140. The light frames are of constant flange width, b = 0.20 in. and of varying depth, d. The heavy frames are 3/4 x 5/8 in.

LIGHT FRAMES

Model No.	d (in.)	t (in.)	d/t	100 t/D	$10^{-3}s_y$ (lb. per	$p_{expt.}$ sq. in.)	$\psi_{expt.}$	Frame collapse with shell
79	0.50	0.0483	10.35	0.301	27	150+*	0.92+	
82	.50	.0486	10.29	.303	28	152+	.90+	
76	.50	.0493	10.14	.307	42	170+	.66+	
88	.30	.0471	6.37	.293	27	135	.85	
75	.30	.0472	6.36	.294	39	210	.91	
81	.30	.0474	6.33	.295	28	154	.93	
84	.30	.0481	6.24	.300	29	141	.81	
87	.30	.0482	6.22	.300	29	141	.81	
86	.30	.0483	6.21	.301	29	141	.81	
80	.30	.0498	6.02	.310	28	152	.87	
85	.30	.0499	6.01	.311	29	150+	.83+	
92	.20	.0463	4.32	.288	32	156	.85	Yes
91	.20	.0466	4.29	.290	32	156	.84	Yes
83	.20	.0495	4.04	.308	29	157	.88	
96	.15	.0478	3.14	.298	28	142	.85	Yes
94	.15	.0479	3.13	.298	28	140	.84	Yes
89	.15	.0481	3.12	.300	27	136+	.84+	
90	.15	.0482	3.11	.300	27	133	.82	Yes
93	.10	.0471	2.12	.293	28	120	.73	Yes
95	.10	.0490	2.04	.305	34	120	.58	Yes

HEAVY FRAMES

Model No.	t (in.)	d/t	100 t/D	$10^{-3}s_y$ (lb. per	$p_{expt.}$ sq. in.)	$\psi_{expt.}$
79	0.0489	15.34	0.305	27	150	0.91
82	.0489	15.34	.305	28	142+	.83+
76	.0493	15.21	.307	42	210	.81
88	.0476	15.76	.297	27	125	.78
75	.0472	15.89	.294	39	180+	.78+
81	.0479	15.66	.298	28	150+	.90+
84	.0478	15.69	.298	29	131+	.76+
87	.0485	15.46	.302	29	141	.80
86	.0487	15.40	.303	29	148	.84
80	.0494	15.18	.308	28	152	.86
85	.0494	15.18	.308	29	150	.84
92	.0460	16.30	.287	32	146	.79
91	.0464	16.16	.289	32	141+	.76+
83	.0491	15.27	.306	29	147+	.83+
96	.0477	15.72	.297	28	142	.85
94	.0480	15.63	.299	28	130+	.78+
89	.0479	15.66	.298	27	131	.81
90	.0481	15.59	.300	27	133	.82
93	.0473	15.86	.295	28	143	.86
95	.0488	15.37	.304	34	160	.77

* The plus sign indicates that the ultimate collapsing pressure was likely slightly higher than the value recorded. (See remarks on page 13 concerning the experimental difficulties arising from the use of a control.)

(bulkhead for frame) gives a value of ψ (or of p) 11 per cent lower than the simple hoop stress formula value. The average difference between these two formulas, however, is much less. It is well to note that while the experimental points verify the general shape of the curve representing formula (92a), they all fall below it. This can probably be attributed to defects and irregularities in the models.

The various formulas for the frame strength determine several different theoretical values of the critical d/t. These values are all higher than the one determined by the experimental points in Fig 2. All the frame formulas, then, seem to be conservative, while the design formula of standard practice (represented by curve D on the extreme right) seems ultra-conservative. It would appear advantageous, therefore, to turn from standard practice a more liberal design. However, it must be considered that these experimental data are for frames of rectangular cross section only and that it is not certain they can be applied to all types of frames. Consequently the curve determined by the experimental points in Fig 2 should not be used as a basis of design; rather a more conservative curve, such as the theoretical curve E or F, should be chosen. It is suggested that Tokugawa's formula, Eq (10), (represented by curve E in Fig 2) be used in the design of frames. The less conservative Eq (9)(represented by curve F) is believed to be safe and can probably be recommended for use when more experimental data on stiffening rings of structural shapes are available to check its safety.

As far as the theoretical basis for Eq (9) and for Eq (10) goes, there is little to choose between these two formulas. Eq (10) by Tokugawa was selected merely because it is more conservative. Moreover the Tokugawa formula is somewhat simpler, and is particularly attractive in the case of T stiffeners or angle stiffeners welded to the shell at their toes. For such frames uncertainty arises over what value to use for b in formula (88) which is involved in Eq (9), and this difficulty is avoided by using Eq (10), in which b does not appear.

Standard practice has been declared ultra-conservative and more liberal methods of design have been suggested. It should be emphasized that these conclusions were based on the premise that the sole function of the frame is to give adequate support to the shell prior to collapse. The conclusions are no longer tenable if the further requirement is imposed that the frame be strong enough to hold up even after the shell has collapsed. Standard practice meets this additional requirement. At first this might seem open to question because standard practice ignores the following two possible sources of weakness in the frame which are now introduced.

First, there is the possibility of eccentric loading of the frame by the collapsed shell. This may have a large weakening influence, especially in the case where the frame would normally collapse by instability. Secondly, there is the possibility that after collapse of the shell the stresses in the frame may exceed the yield point (or at least the proportional limit) of the material. Comparison of curves D and G in Fig 2 shows that when standard practice was used in conjunction with the incorrect formula (92) collapse by yield may have seemed impossible, but that when the more correct formula (92a) is used this possibility should not be ignored.

Standard practice, in compensation for its unwarranted disregard of these two possibilities, calls for a 10 per cent margin of additional strength as mentioned on page 5. Moreover, the collapsed shell probably contributes much more to the effective moment of inertia of the frame than the small allowance of a strip of width b. These factors appear to outweigh the effects neglected by standard practice, for experience indicates that frames designed by this method are capable of holding up after shell collapse. This is confirmed by the experimental results shown in Table I. Only those frames with depth d equal to or less than 0.2 in. (d/t = about 4.2) collapsed with the shell, and frames of these dimensions are represented far to the left of the standard practice curve in Fig 2. (A collapsed frame is pictured in Fig 9 in the appendix.) Thus standard practice, notwithstanding its unjustifiable neglect of certain weakening factors, appears to be moderately conservative even with the demand that the frame hold up after the shell has collapsed.

The advantage of the present design of submarine frames, based on standard practice, lies in the consideration that these frames avoid the possibility of a sudden, complete collapse of a submerged vessel; lack of this assurance is regarded as detrimental to the morale of the submarine personnel. On the other hand a vessel with lighter frames and heavier shell than one designed by standard practice is a stronger vessel, whose larger factor of safety for a given depth should more than compensate for its inability to avoid complete destruction after collapse has begun. The larger factor of safety has also the added military advantage that the vessel is better prepared in an emergency to submerge to depths greater than the design depth. The stronger vessel, obtained by distributing the material of the vessel designed by standard practice to better advantage, is considered superior. In the design of a submarine pressure hull the ultimate goal should be the strongest vessel for a given weight of material.

CONCLUSION

The little used von Sanden-Günther formula (92a) is the correct yield formula; the widely used von Sanden-Günther formula (92) should not be used. Tangential stress, not longitudinal stress, is determinative in causing collapse of a thin-walled pressure vessel.

The present method of frame design of a submarine pressure hull, based on "standard practice," produces an excessively heavy frame. Lightening this frame will not weaken the vessel, and if the material saved is distributed to better advantage, a stronger vessel can be obtained.

Other methods of frame design, described in this report, are considered preferable to standard practice.

APPENDIX

In this appendix various theoretical formulas will be put in forms that are convenient for plotting. It is often of value to express the physical quantities involved in the formulas as non-dimensional ratios. For example, useful expressions of Eq (4) are

$$\frac{q_f/b}{2 \times 10^{-6} E} = \left[\frac{12\ I}{b\ d^3}\right]\left[\frac{100\ d}{D_f}\right]^3 = \left[\frac{12\ I}{b\ t^3}\right]\left[\frac{100\ t}{D_f}\right]^3 = \frac{L'}{b}\left[\frac{12\ I}{L' t^3}\right]\left[\frac{100\ t}{D_f}\right]^3 \tag{4a}$$

To find p_f by Eq (6) and (9) it is found expedient first to compute q_f/b by Eq (4) or (4a), replacing I by I_b, and then determine p_f by dividing q_f/b by 1.1 L'/b and F/b in turn.

For an inside rectangular frame of depth d and width b, Eq (4a) with I replaced by I_b becomes

$$\frac{q_f/b}{2 \times 10^{-6} E} = \left[\frac{1 + \frac{d}{t}}{\frac{D}{100\ t} - \frac{d}{100\ t}}\right]^3 \tag{4b}$$

Lamé's formula, Eq (11), can be conveniently written

$$\frac{q_f}{b} = C_L \quad 2(1 + \frac{d}{t})\ \frac{t}{D}\ s_y \tag{11a}$$

where

$$C_L = \frac{1 - \frac{d}{t}\ \frac{t}{D}}{(1 + \frac{t}{D})^2} = 1 - (2 + \frac{d}{t})\ \frac{t}{D} + (3 + 2\ \frac{d}{t})\ (\frac{t}{D})^2 - \ldots \tag{11b}$$

The Lamé correction factor C_L is nearly equal to unity. With $C_L = 1$, Eq (11a) is merely the simple hoop stress formula, Eq (12), (with the approximation $D = 2\ R_2$) applied to the compound section composed of a rectangular frame section and an area b t of the shell.

Combining Eq (11a) with Eq (5) and Eq (13) we get for the standard practice frame load

$$\psi_f = \frac{C_L}{1.1}\ \frac{(1 + \frac{d}{t})}{L'/b} \tag{11c}$$

In Figs 4, 5, and 6 following are the graphs of the functions N, K, H mentioned on page 3.

In Figs 7, 8, and 9 following are photographs of the models described on page 13.

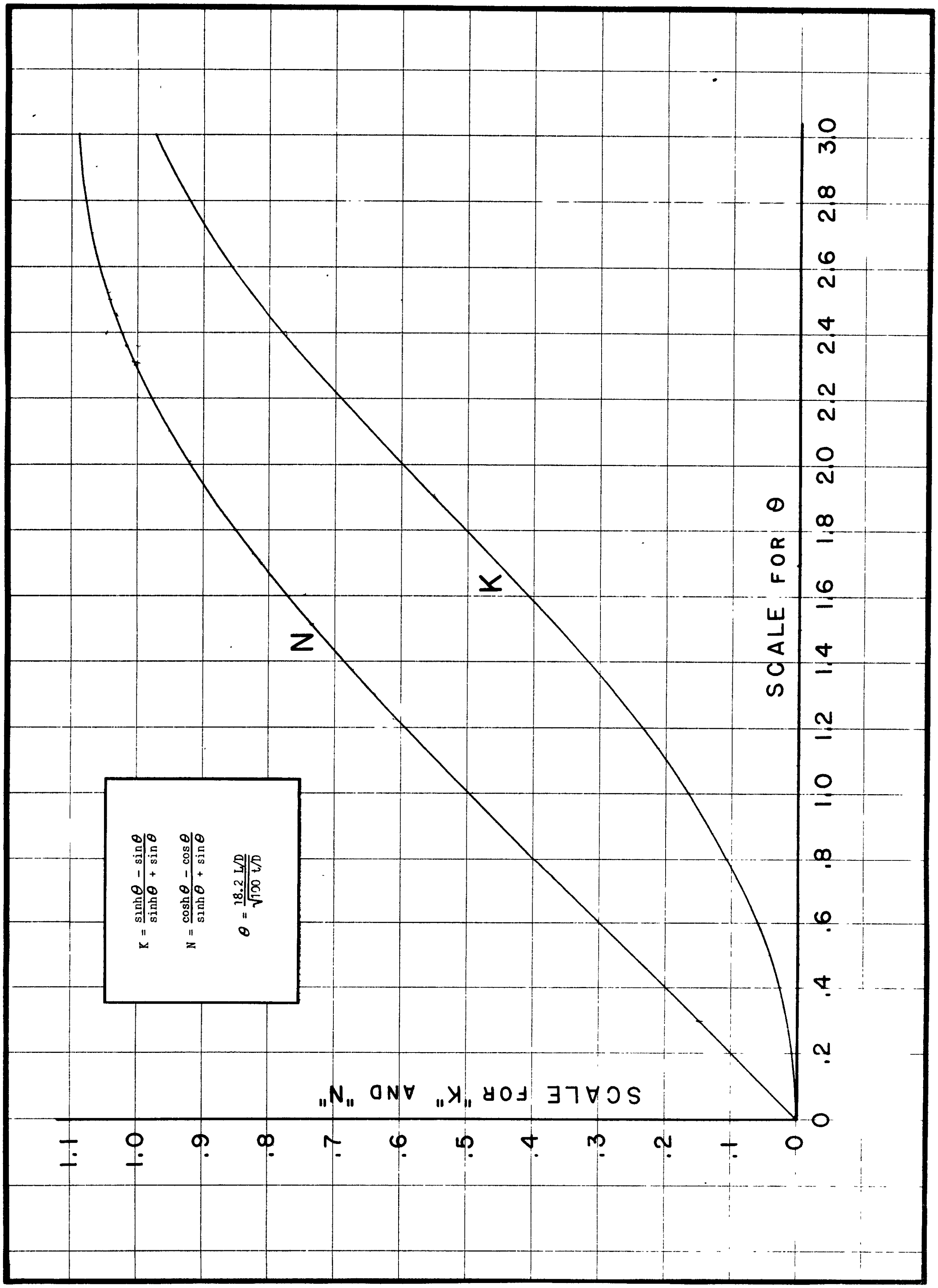

FIG 4. AUXILIARY FUNCTIONS, K AND N, FOR USE IN THE VON SANDEN–GÜNTHER FORMULAS (92) AND (92a).

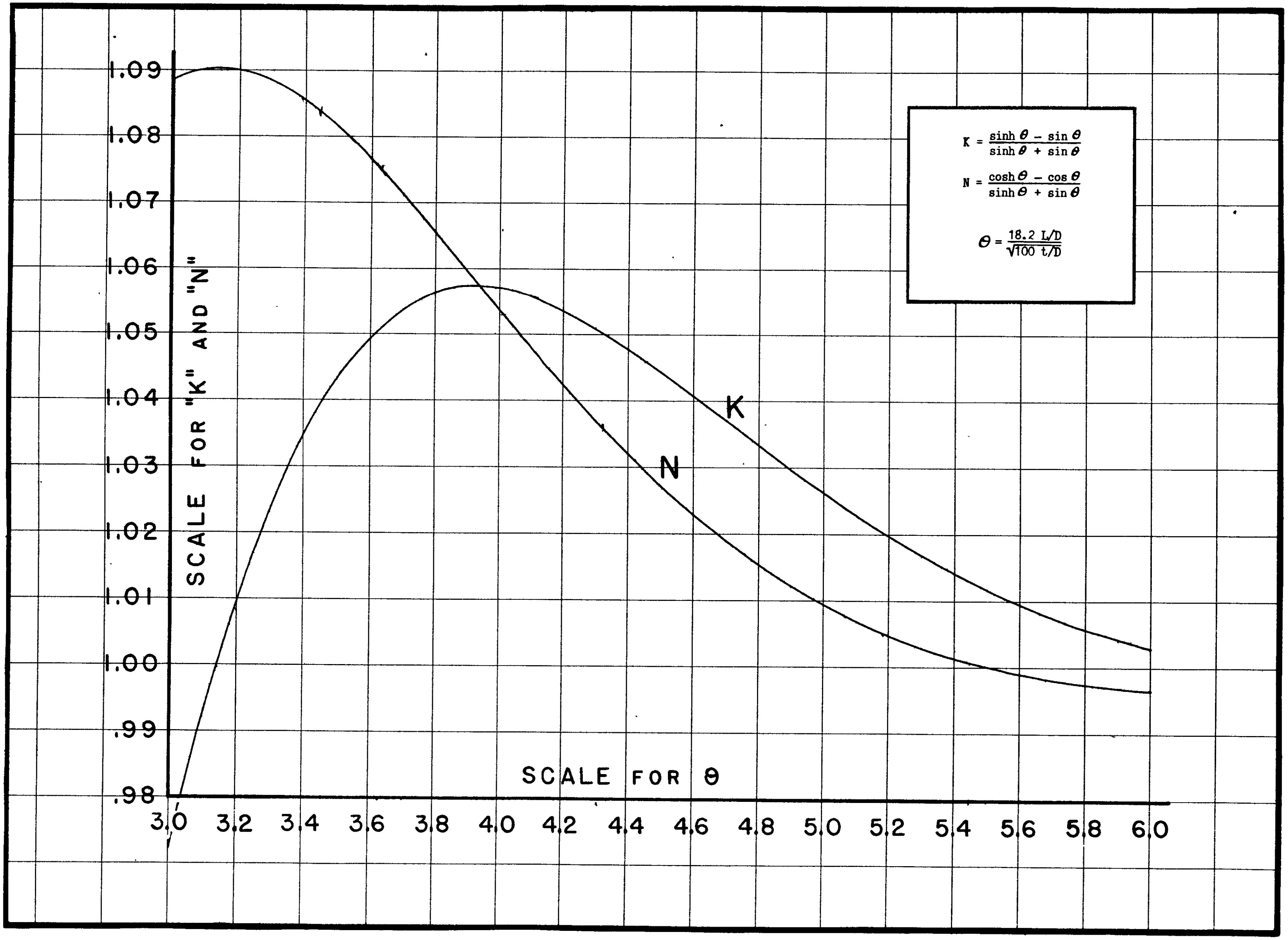

FIG 5. AUXILIARY FUNCTIONS, K AND N, FOR USE IN THE VON SANDEN–GÜNTHER FORMULAS (92) AND (92a).

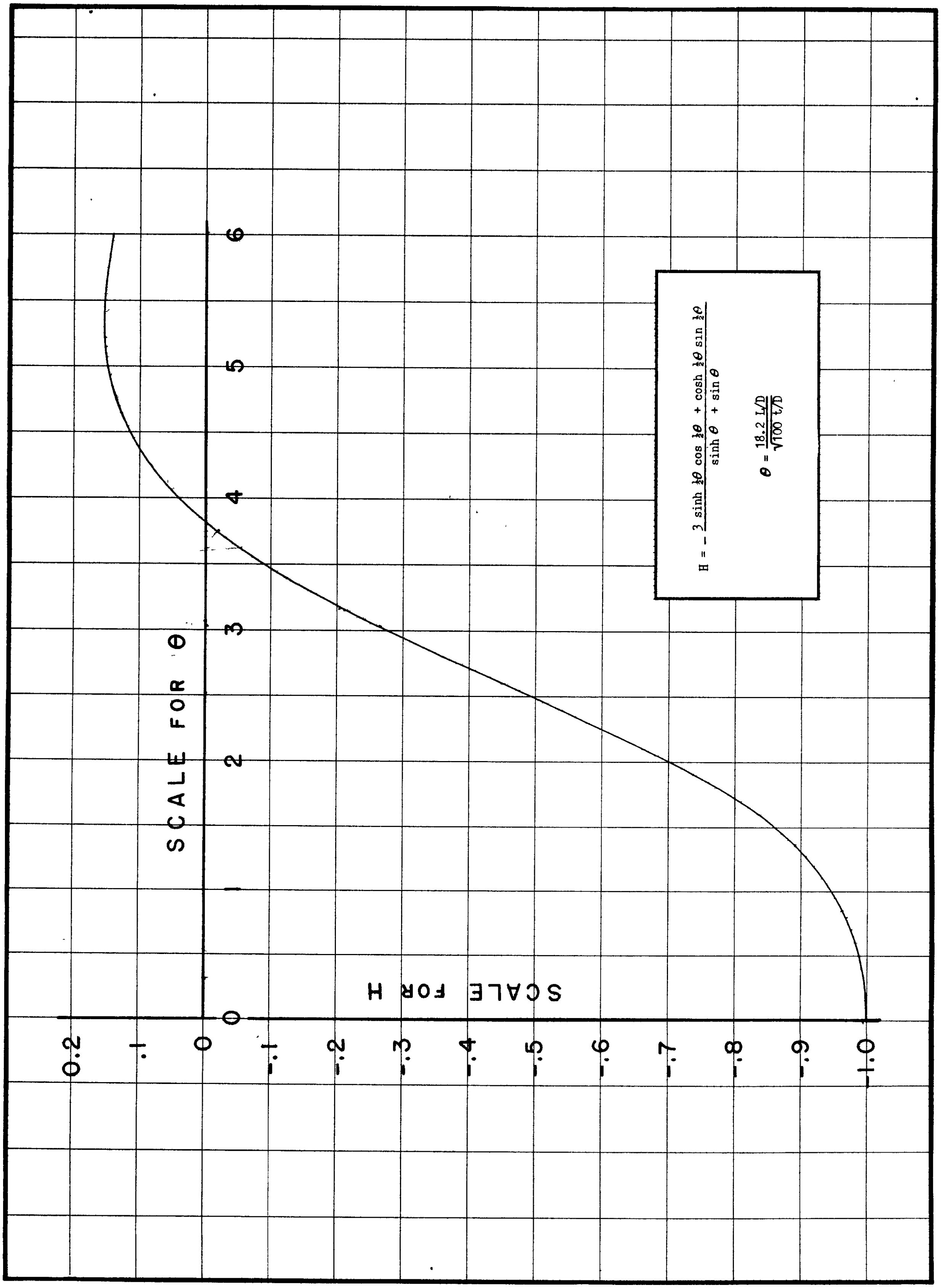

FIG 6. AUXILIARY FUNCTION, H, FOR USE IN THE VON SANDEN–GÜNTHER FORMULA (92a).

FIG. 7

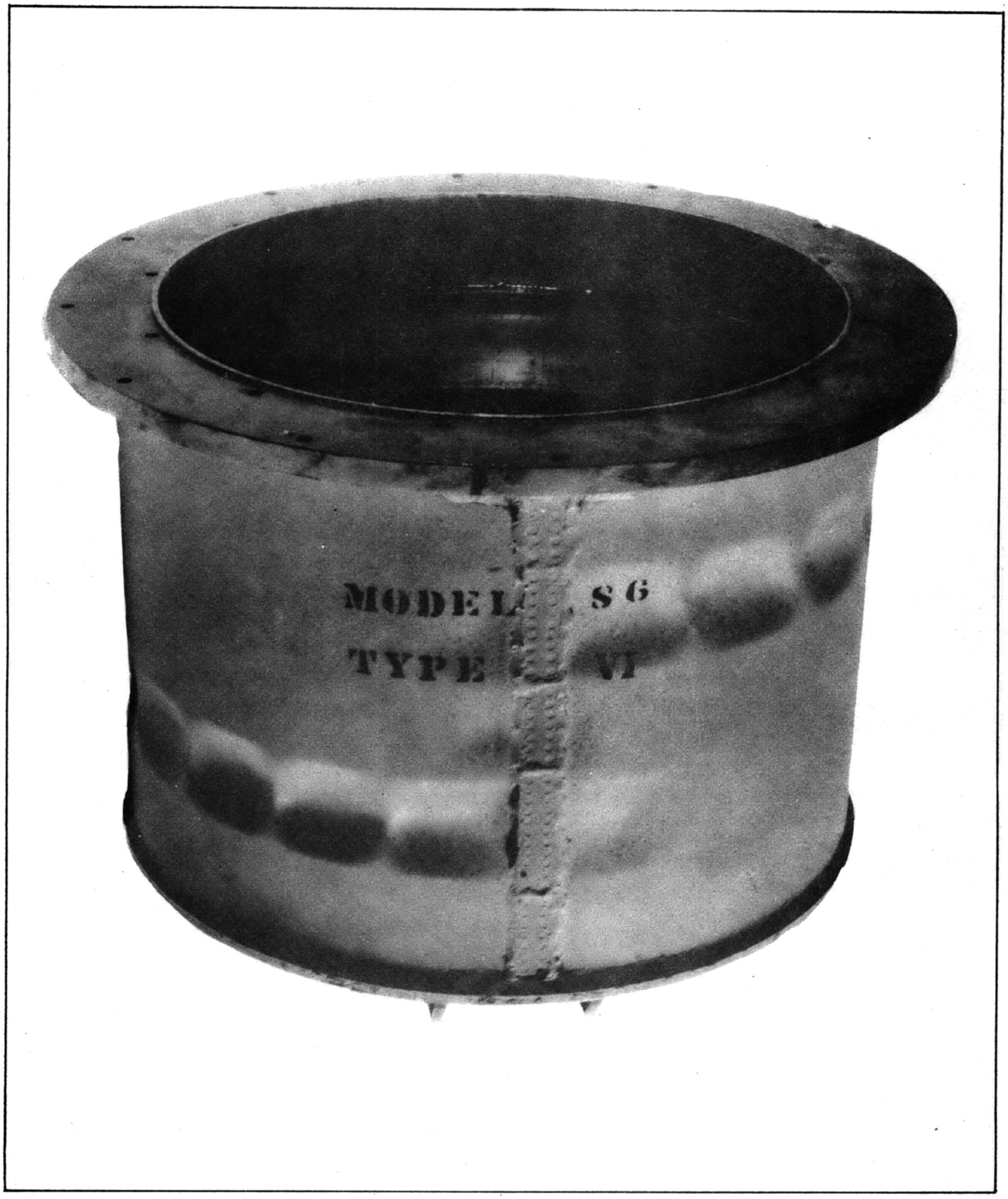

FIG. 8

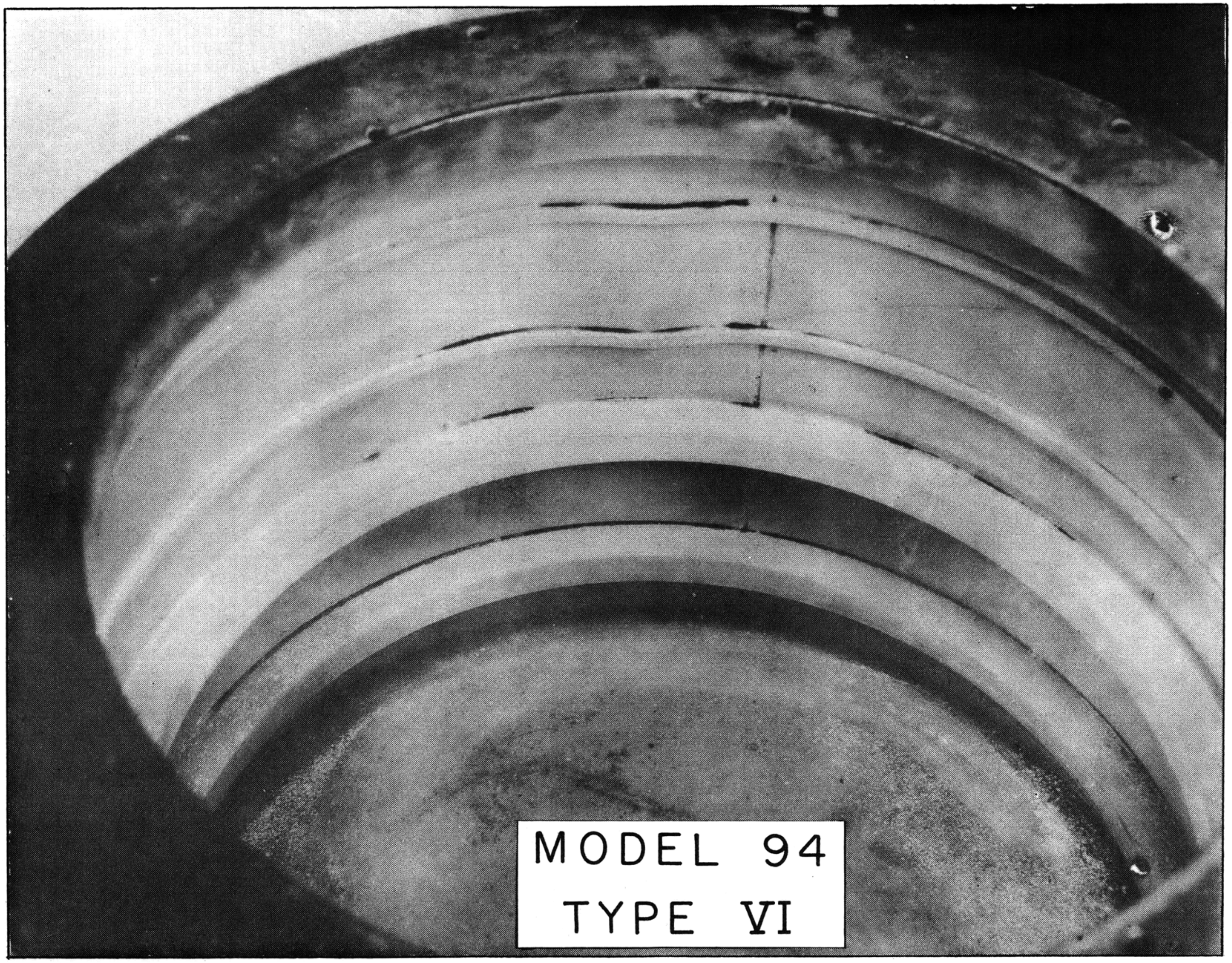

FIG. 9

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