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PORT OF A STUDY OF THE APPARENT LOSS OF METACENTRIC HEIGHT OF A DESTROYER WHEN UNDERWAY

U. S. EXPERIMENTAL MODEL BASIN

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MARCH 10, 1931. REPORT NO. 287

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I.

INTRODUCTORY

1. In accordance with request contained in a memorandum from the Chief Constructor without file number, dated Nov. 21, 1922, an investigation has been made of the apparent change in metacentric height of a destroyer when underway. It has been reported by the Board of Inspection and Survey that destroyers, when underway, have shown an apparent loss in metacentric height, it having been observed on several trials that these vessels take a more pronounced heel when making turns than would be expected from their stability characteristics, and that they have a tendency to hold this inclination after straightening out on the new course. It is obvious that a change in metacentric height itself when underway can occur only from a change in the height of the center of buoyancy, or a change in the moment of inertia of the water surface in the running condition as compared with the water plane when in a motionless condition, and it would appear that such changes should be relatively small as compared with the effects actually observed. Apparent changes in the stability when underway would, therefore,

arise principally from causes other than a change in the metacentric height of the vessel. One explanation of the phenomena reported by the Board of Inspection and Survey is the possibility that unsymmetrical dynamic water pressures exist upon the bilge keels, docking keels, or rudder when the ship is in an inclined position. Such dissymmetry of pressure would produce a heeling moment, and if this moment should be in such a direction as to hold the ship in the inclined position, the effect might be erroneously interpreted as due to a reduction in metacentric height.

II

DESCRIPTION OF APPARATUS AND PROCEDURE OF CONDUCTING TEST

2. Model #2344, representing a destroyer of the PRUITT class, was selected for these experiments. The model was ballasted to a displacement of 251.8 lbs., corresponding to a displacement in the ship of 1250 tons. The ballast was so adjusted as to bring the model to a metacentric height of .65 inches, corresponding to a metacentric height of 14.48 inches on the ship. A frame, fitted on the model as shown in the accompanying photographs, Plates I and II, was arranged to carry two movable weights and a plumb line and scale. The heavier of these weights was used as an inclining weight and was fitted with a peg for transverse positions to each side

of the center line, so chosen as to produce heels of 5°, 10°, 15°, and 20°. The smaller weight, carried on a guide bar, was provided to measure the change in heeling moment when underway. The model was towed by a line attached at the bow near the water line. Guide wires fitted at the bow and stern, to hold the model on its course, were attached to the model practically at the water line, in such a manner as to produce only a negligible heeling moment.

3. The procedure of conducting the test consisted in inclining the vessel to one of the angles just mentioned, with the small weight initially on the center line. Then when underway at a definite speed the amount of shift in the small weight necessary to restore the vessel to the predetermined inclination was noted. The moment of the shift of this small weight could then be regarded as equal to the heeling moment of the water force on the hull with the vessel underway. These experiments were conducted with the vessel heeled both to port and starboard, at the various angles mentioned above, and at speeds corresponding to speeds of the destroyer between 18 and 40 knots.

4. After completing the observations for the condition with all appendages in place, the bilge keels were removed and all runs repeated. Finally the rudder was removed and a set of observations made in this last condition with both bilge keels and rudder omitted.

III

METHOD OF WORKING UP AND INTERPRETING DATA

5. In working up the heeling observations, the results from the port and starboard runs for each speed and angle were averaged. The data from the two sides were found, in general, to be consistent. Curves of inclining moment, as determined by the shift of the small weight at the various angles of heel, were plotted against speed of the model. Final curves representing the corresponding effect upon the ship were prepared, but in the latter case the change in heeling moment, as observed from the experiments, was expressed in terms of equivalent loss of metacentric height. These latter curves, having speed of the ship in knots as abscissae and loss of metacentric height when underway as a percentage of metacentric height at rest for ordinates, appear on Plate III.

6. It is quite possible that any influence that will produce an apparent change in the stability characteristics of the vessel when underway as compared with its stability when at rest will also have an effect upon the rolling characteristics of the vessel. If, for instance, the vessel when inclined is subjected to unsymmetrical dynamic pressures upon the bilge keels which act in such a direction as to oppose her return to an upright position, it might be expected that

such forces would cause an apparent decrease in the damping characteristics of the vessel when rolling. With this point in mind, but having in mind also the contrasting possibilities that; firstly, the bilge keels would show a greater effect in extinguishing rolling when underway due to continually moving into undistrubed water as compared with rolling at rest; and secondly, the increased lateral frictional resistance due to the increase in lateral velocity of the water with respect to the ship when underway as compared with the lateral velocity without forward motion (See Appendix), the heeling observations for the several conditions of the model as regards appendages were paralleled by rolling observations. A roll and pitch recorder was installed in the model to obtain data upon rolling characteristics. The model was inclined by hand to an angle of about ten degrees and released, and the record was continued until the rolling so produced had entirely died out. This procedure was carried out with the model at rest and also when proceeding at various uniform speeds up to a speed of about eight knots.

7. In analyzing the data obtained by the roll and pitch recorder, certain assumptions were made regarding the nature of the oscillations. As a simplification of the analysis it was assumed that the damping force which produces the decay of the oscillation is proportional to the angular velocity of roll. If we let "y" represent the angular displacement at

any instant and assuming the damping force proportional to the velocity, and also assuming a linear proportionality of restoring force to the angle of displacement, we may set up the following equation for damped vibration:

$$I \frac{d^2y}{dt^2} + R \frac{dy}{dt} + \frac{y}{C} = 0 \qquad (a)$$

where I is the moment of inertia of the vessel about the longitudinal axis of rolling, C is the proportionality factor representing the restoring moment resulting from an angular displacement, and R is the proportionality factor representing the frictional drag whose effect is to reduce the amplitude of oscillation.

8. The general solution of this differential equation is:

$$y = A e^{-kt} \cdot \text{Sin}\ (wt-\theta) \qquad (b)$$

where "A" and "θ" are undetermined constants of integration and where "k" is a constant involving frictional resistance and moment of inertia of the vessel but which for convenience may be described in the following discussion as the damping coefficient. "A" is the initial angle of roll and "w" is a factor involving the vessel's natural period of oscillation with damped rolling. It is apparent that the exponential curve $y = Ae^{-kt}$ is the envelope containing the actual curve representing the vessel's oscillation where the effect of the Sine curve in equation (b) is taken into consideration. The ordinates of the extreme angles of roll for succeeding swings fall on the exponential curve $y = Ae^{-kt}$. The curves obtained representing the decay of oscillation in accordance with this equation are shown on Plates IV a, b, c.

9. Proceeding on the assumption that the rolling of the vessel would follow the simplified law stated above, the data from the roll and pitch recorder were analyzed by making plots on semilogarithmic paper. Plate V shows such a plot. It will be observed that in the case of the record obtained with the vessel standing still, all of the spots fall very well on a straight line, indicating that under such conditions the oscillations follow the assumed law within the limits of error of the observations. In the case of the data obtained when under way and to a more pronounced extent as the speed was increased, there is evidence of some departure from this law. Assuming, however, that the law represents the data on the average, the straight lines obtained on this plot, Plate V, yield by their slope the value of the exponent "k" which has been described as the damping coefficient.

10. After working up all of the rolling observations by this procedure, the curves shown on Plate VI were prepared where there appear for the model the curves of damping coefficient as ordinates plotted against speed of model as abscissae with an auxiliary line to permit conveniently the conversion of model speed to ship speed.

IV

DISCUSSION OF DATA AND CONCLUSIONS

11. An examination of the curves on Plate III shows that with an increase of speed of the vessel, there is a marked change in the stability characteristics, this change, when expressed in terms of equivalent loss in metacentric height amounting, at 35 knots, to approximately 33% at a 5 degree inclination, and 20% at a 20 degree inclination when all appendages are complete. When the bilge keels and rudder were removed, the equivalent loss in metacentric height at the same speed was reduced to approximately 23% with a 5 degree inclination and 18% at the 20 degree inclination. When the bilge keels were removed, leaving the rudder in place, however, the effect was somewhat different, the removal of the bilge keels reducing the apparent loss in metacentric height at the smaller inclination as compared with the condition where all appendages were in place but increasing the apparent loss at the larger inclination. From this fact, it would be concluded that at the smaller angles of inclination the bilge keels and rudder act together in their effect toward assisting the angular inclination, but that at the larger inclination the bilge keels and rudder act in opposition.

12. From the curves of damping coefficients, Plate VI, it appears that at speeds up to about 24 knots for the ship, and considering only the conditions where the model is equipped with or without bilge keels but in either case with

the rudder fitted, the damping coefficient is greater when the bilge keels are fitted but that at speeds in excess of the speed just mentioned, the damping coefficient is less when the bilge keels are fitted. Thus at a speed of about 35 knots the damping coefficient with bilge keels is approximately only one-half as great as without bilge keels. The somewhat unexpected condition appears to exist that at speeds above 24 knots the bilge keels appear to assist rolling rather than to restrain it.

13. From the fact that the model shows an apparent loss in metacentric height at the various inclinations even when the bilge keels and rudder are removed, it is apparent that the remaining appendages, consisting of docking keels, shafts, and struts, are acted upon by dynamic forces, or else that there is an actual loss in metacentric height when underway, resulting from a change in the position of the center of buoyancy, change in trim, and change in the moment of inertia of the water surface.

14. The specific resistance curve and also the curves of trim by bow and stern are shown on Plate VII. It will be noted that for the larger angles of inclination (greater than 10 degrees) the speed at which the trim is zero, and this is approximately the same as the speed at which the resistance curve begins its sharp rise, is also the speed at which the effect of the bilge keels on the ships' stability begins to change from an upsetting to a righting moment.

This indicates that, besides the effect on stability due to increased velocity of the water past the ship's surface, the trim of the vessel and the wave pattern set up about it are also primary items in determining the action of the bilge keels.

15. The ships' loss of righting moment with speed when it is inclined at various angles and when with and without appendages is shown by the curves on Plate VIII.

16. The curves on Plate IX show, for change of angle of inclination for four different speeds, the net righting moments due to the keels. Thus for all speeds through the angles of inclination from three to ten degrees the action of the bilge keels is to reduce the stability or effective righting moment of the ship. At approximately ten degrees the effect of the keels, as far as righting moment is concenned and regardless of speed, appears to approach zero. This indicates that for this position the action of either bilge keel is such as to neutralize or balance that of the other. For greater angles of inclination and speeds less than 27 knots, their combined action is again negative. For speeds above 27 knots, however, they develope a positive righting moment that increases rapidly both with speed and angle of inclination.

17. These experiments do not indicate the upper limits of this righting action. It appears logical, though, that

for extreme angles of inclination the action may again revert to an upsetting moment. The particular design of the keels will obviously be the determining factor here as it undoubtedly is also for all of the preceding cases noted.

18. In general all of the results of these experiments show that the bilge keels of the type attached to destroyers of the PRUITT class do affect the stability of the ship and that the effect may be very considerable at high speeds and when rolling thru large angles. Also, depending upon both the speed of the ship and the amplitude of its angle of roll, the keels have a variable effect on damping the ships' oscillations.

19. Whether or not it is possible to design bilge keels so that their combined action on stability and rolling is desirable under all conditions remains to be answered by further experiments. It seems likely that the best form of bilge keels in any particular case will be greatly determined by the form of the hull to which it is to be attached, and the speeds at which the vessel is to be operated.

APPENDIX

The following is a consideration, suggested by A. W. Johns*, of how forward motion of a ship increases the frictional damping resistance on rolling when assuming that the frictional resistance varies as the square of the velocity.

Let V = Speed of ship.

lV = Longitudinal velocity of water past the ship's bottom when latter is underway.

η = Mean radius from ship's center of oscillation to its wetted surface.

ΔS = A small area of the ship's bottom.

θ = Angle of roll at any instant, and

f = Friction coefficient determined by nature of ship's bottom.

then $\eta \frac{d\theta}{dt}$ is the lateral velocity of the water past the ship's bottom when it is rolling at rest.

Combining rolling and forward motion the resultant velocity past the ship's bottom is given by

$$V_r = \sqrt{\left(\eta \frac{d\theta}{dt}\right)^2 + l^2 V^2}$$

The frictional force on the small area ΔS will then be

$$\Delta R_f = F \cdot \Delta S \left[\left(\eta \frac{d\theta}{dt}\right)^2 + l^2 V^2\right]$$

The lateral component of this is

$$\Delta R_L = \Delta R_f \text{ Sine } \alpha$$

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where α is the angle between the line of flow and the direction of forward motion of the ship, or

$$\Delta R_L = f \, . \, \Delta S \left[\left(\eta \frac{d\theta}{dt} \right)^2 + l^2 v^2 \right] \, . \, \frac{\eta \frac{d\theta}{dt}}{\left[\left(\eta \frac{d\theta}{dt} \right)^2 + l^2 v^2 \right]^{\frac{1}{2}}}$$

$$= f \, . \, \Delta S \, . \, \eta \frac{d\theta}{dt} \, . \, \left[\left(\eta \frac{d\theta}{dt} \right)^2 + l^2 v^2 \right]^{\frac{1}{2}}$$

From this equation it is readily seen that the damping effect due to frictional resistance should increase with increase of forward motion.

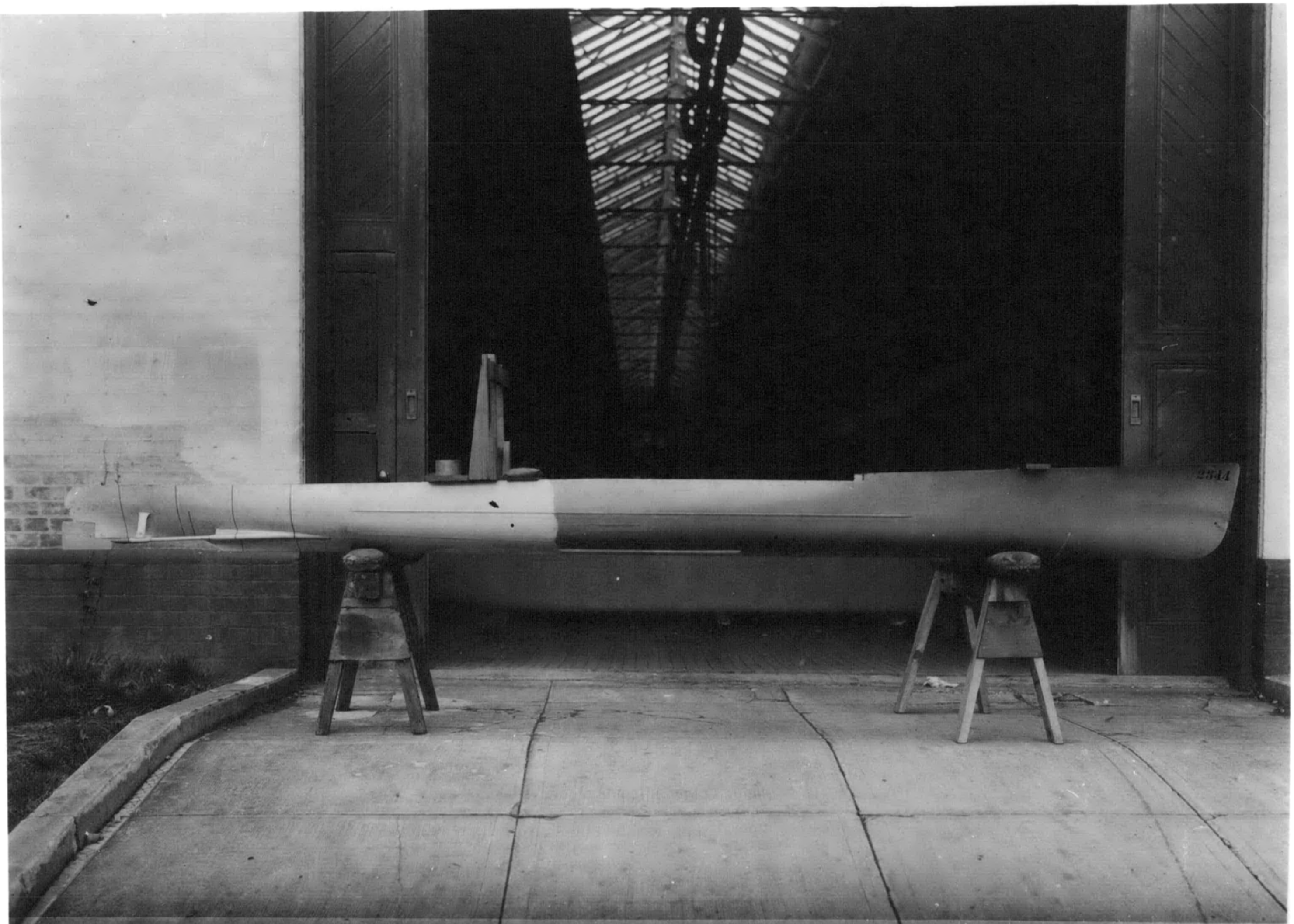


PLATE I DESTROYER MODEL PREPARED FOR STABILITY TESTS

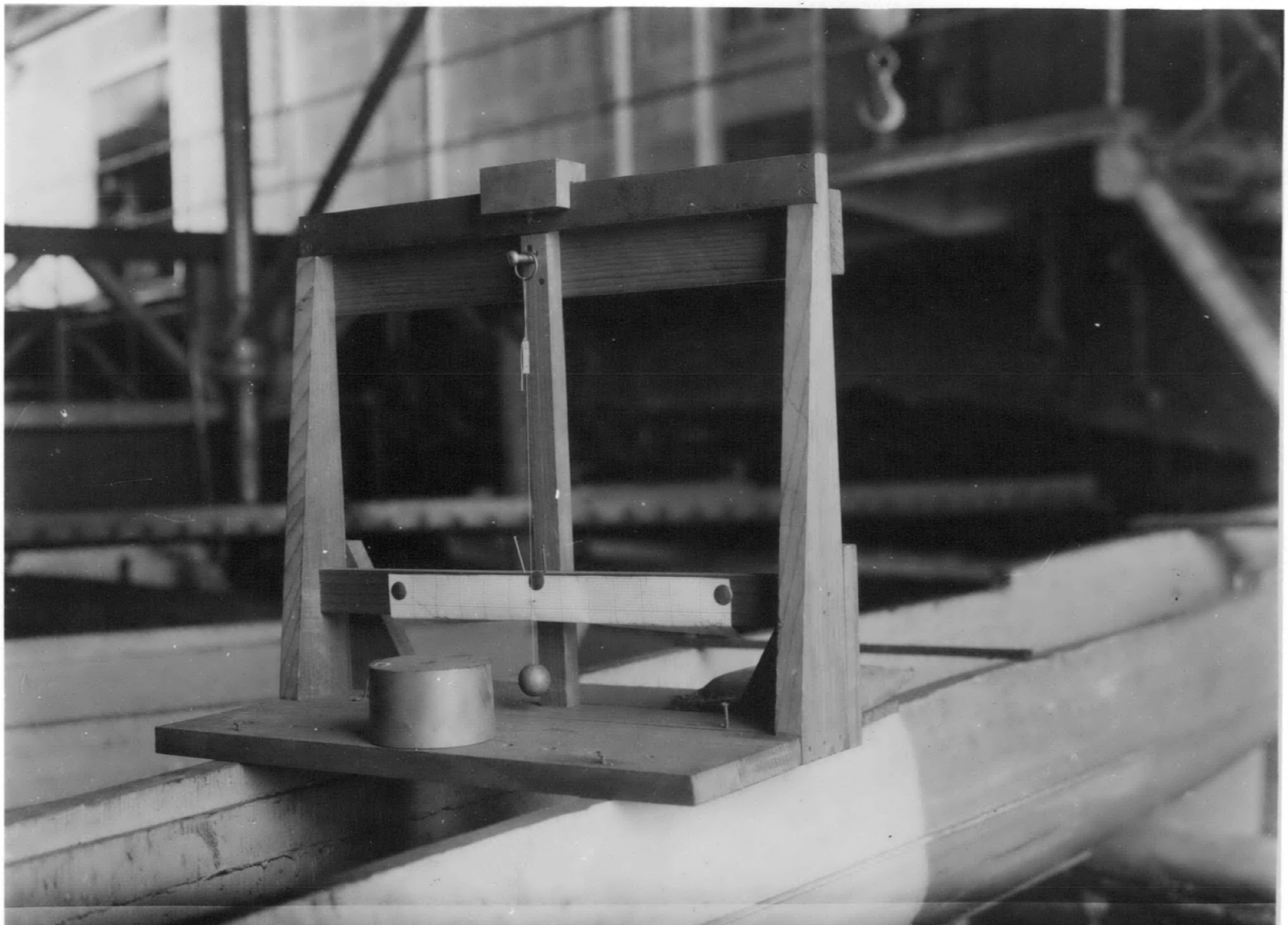


PLATE II PENDULUM & WEIGHTS ON DESTROYER MODEL

CURVES OF PER CENT DECREASE IN METACENTRIC HEIGHT
MODEL #2344 DESTROYER 'PRUITT'
CONVERTED INTO VALUES FOR SHIP

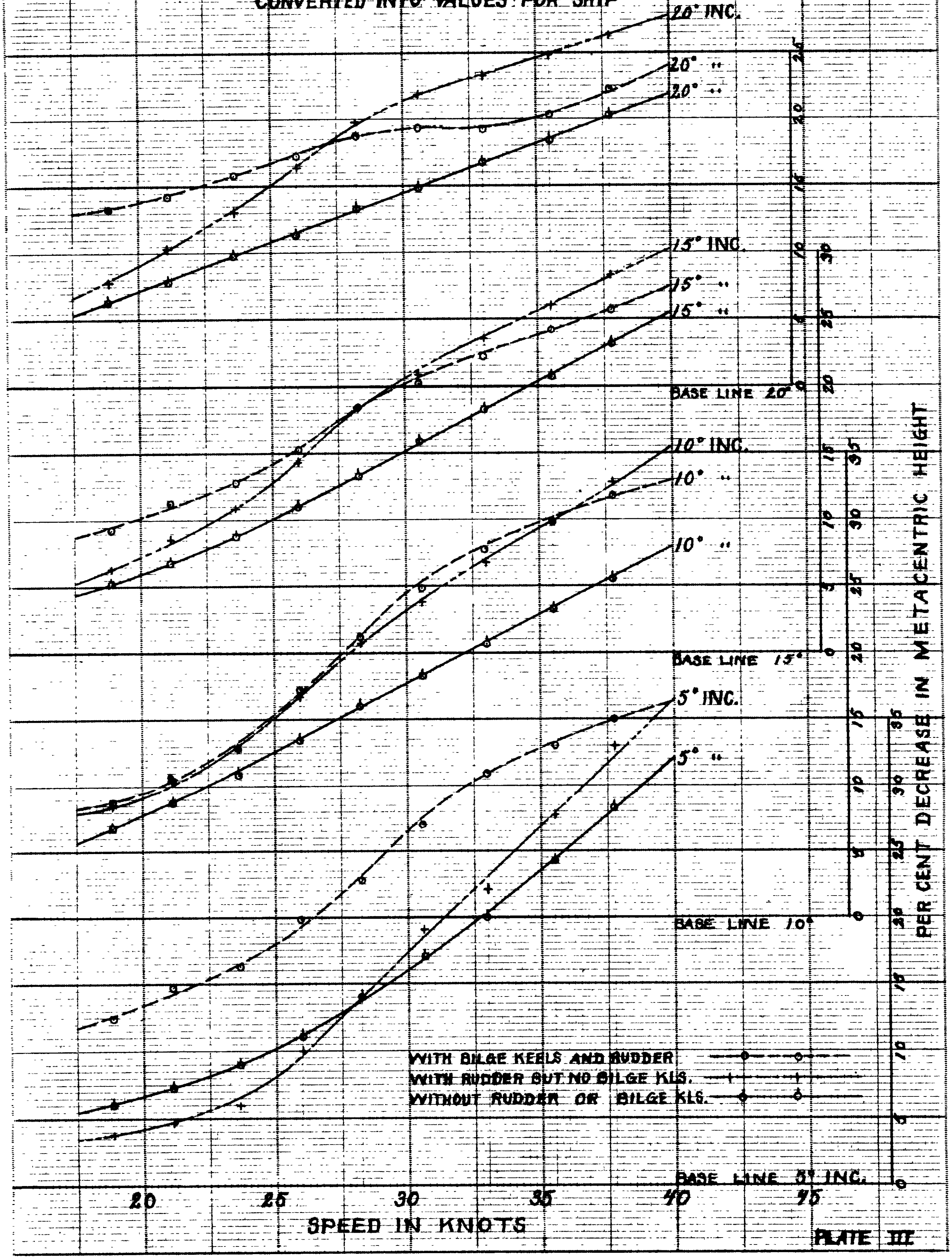
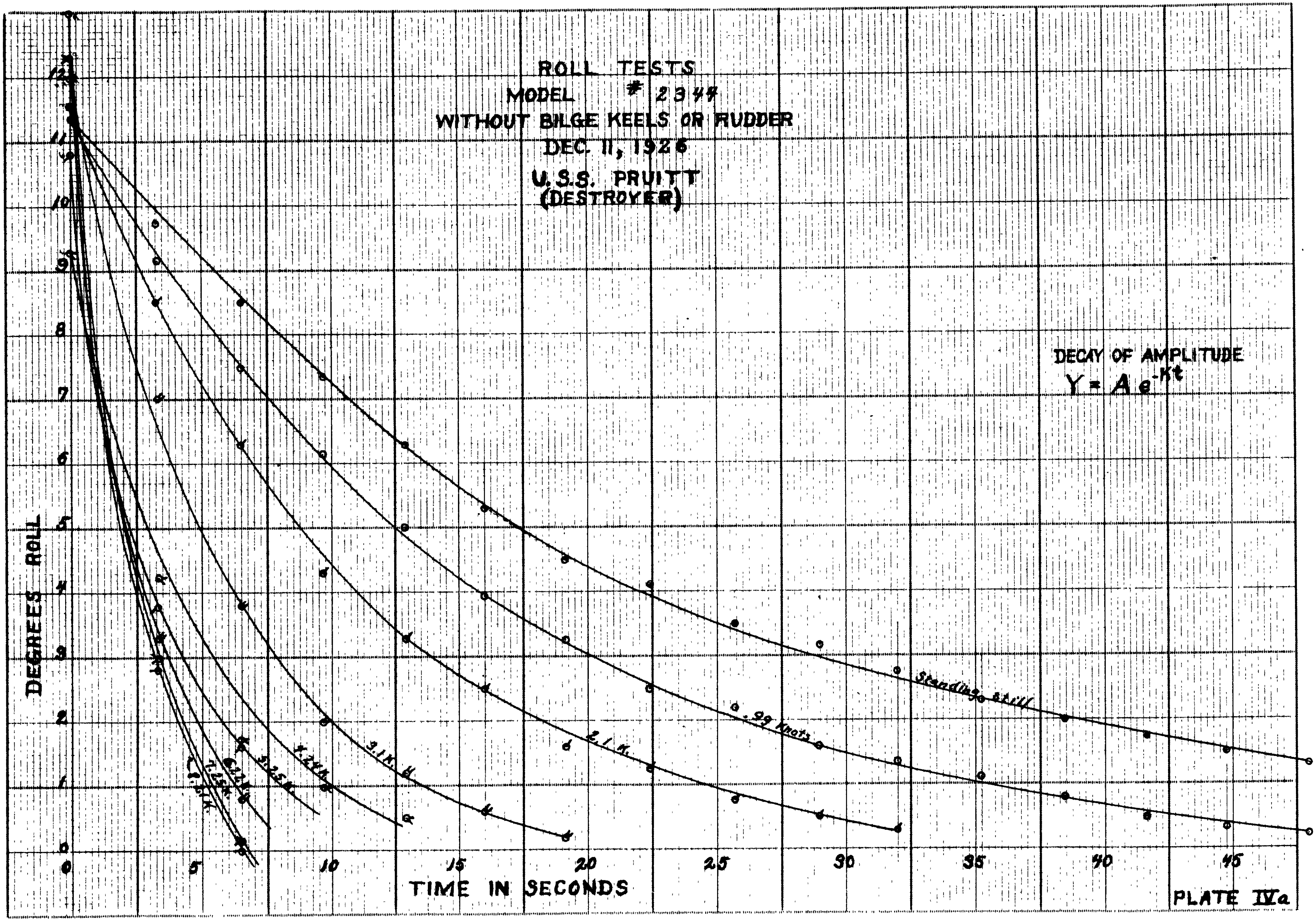
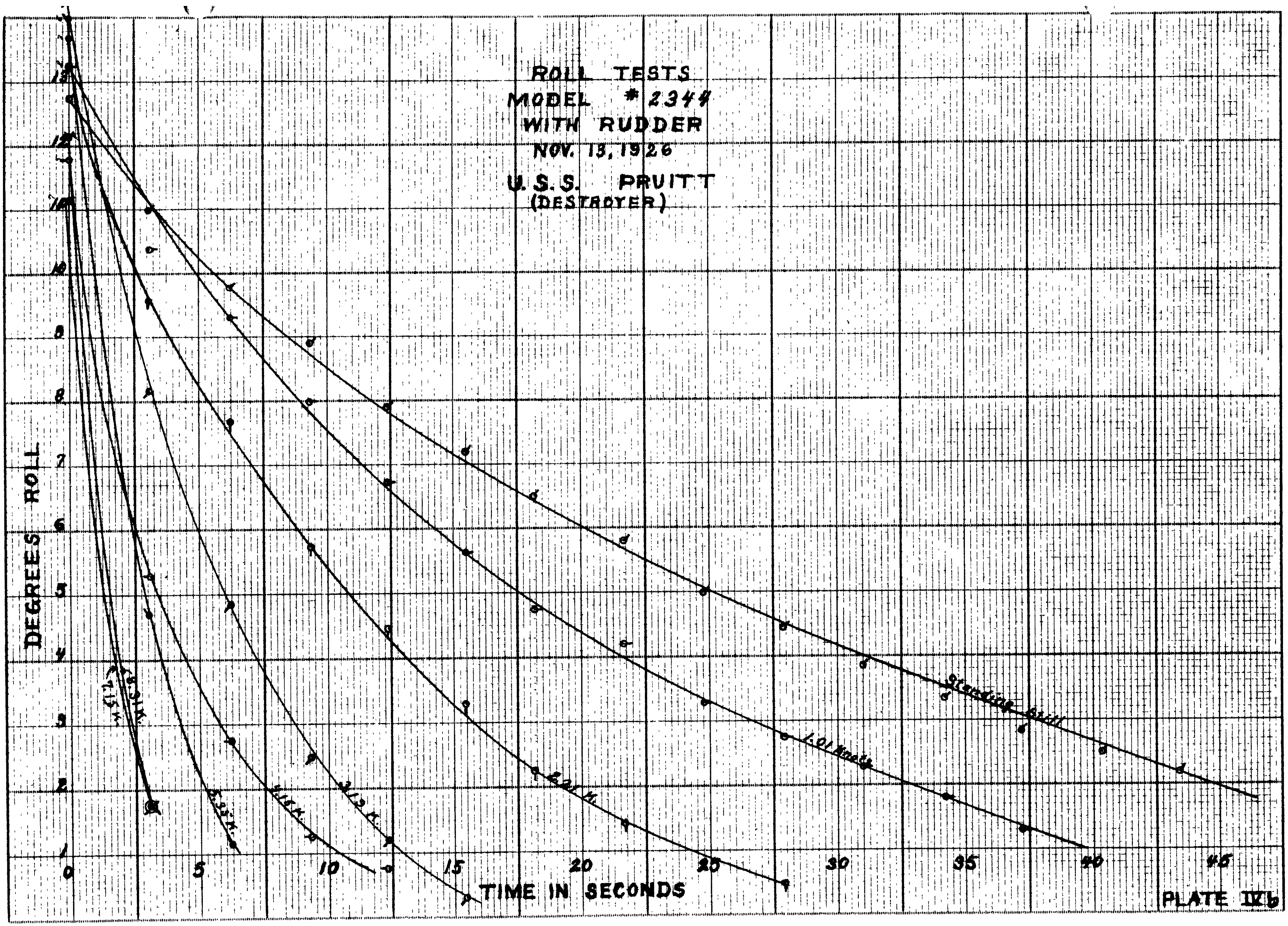


PLATE III



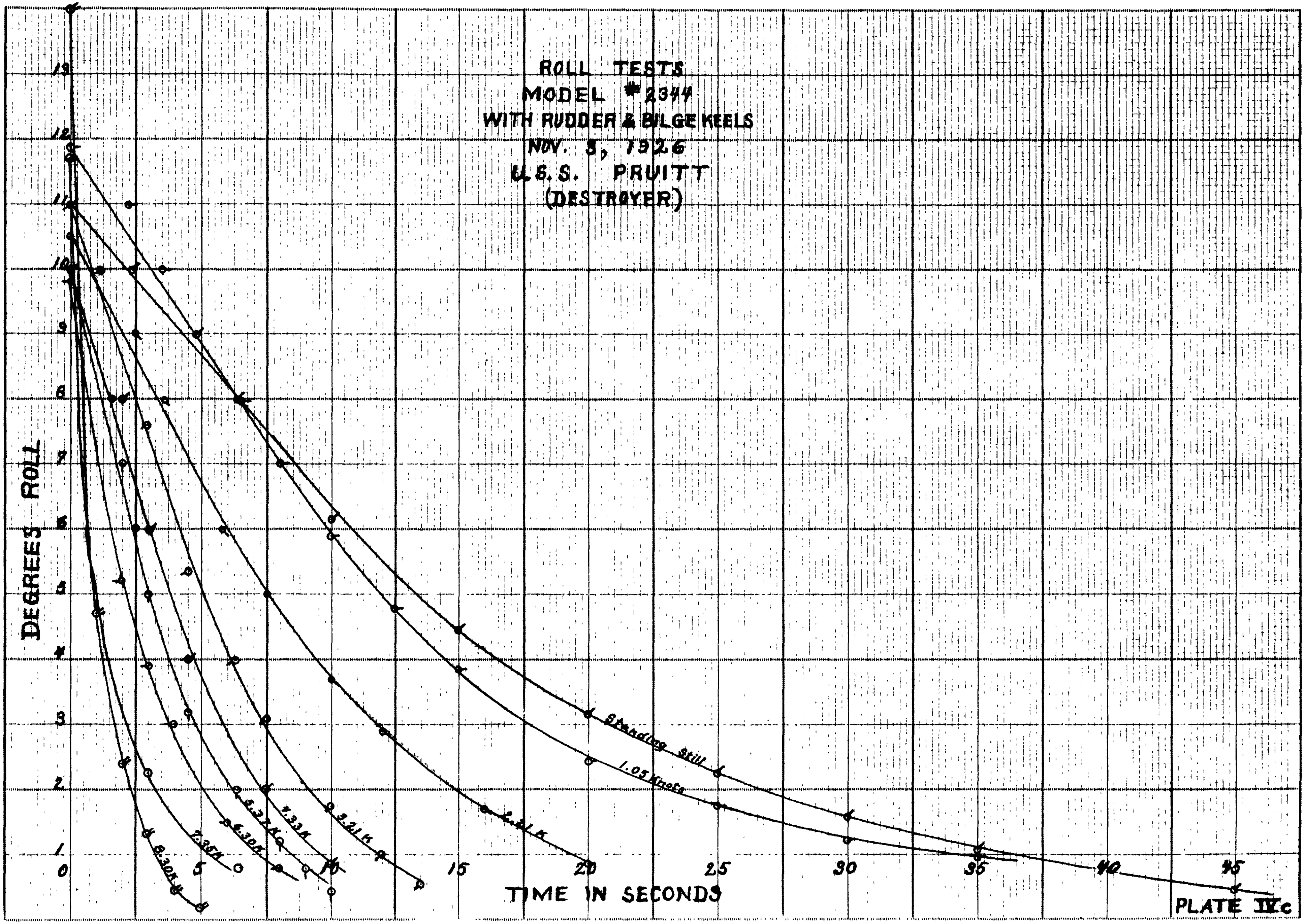
ROLL TESTS
MODEL # 2344
WITHOUT BILGE KEELS OR RUDDER
DEC. 11, 1926
U.S.S. PRUITT
(DESTROYER)

PLATE IVa



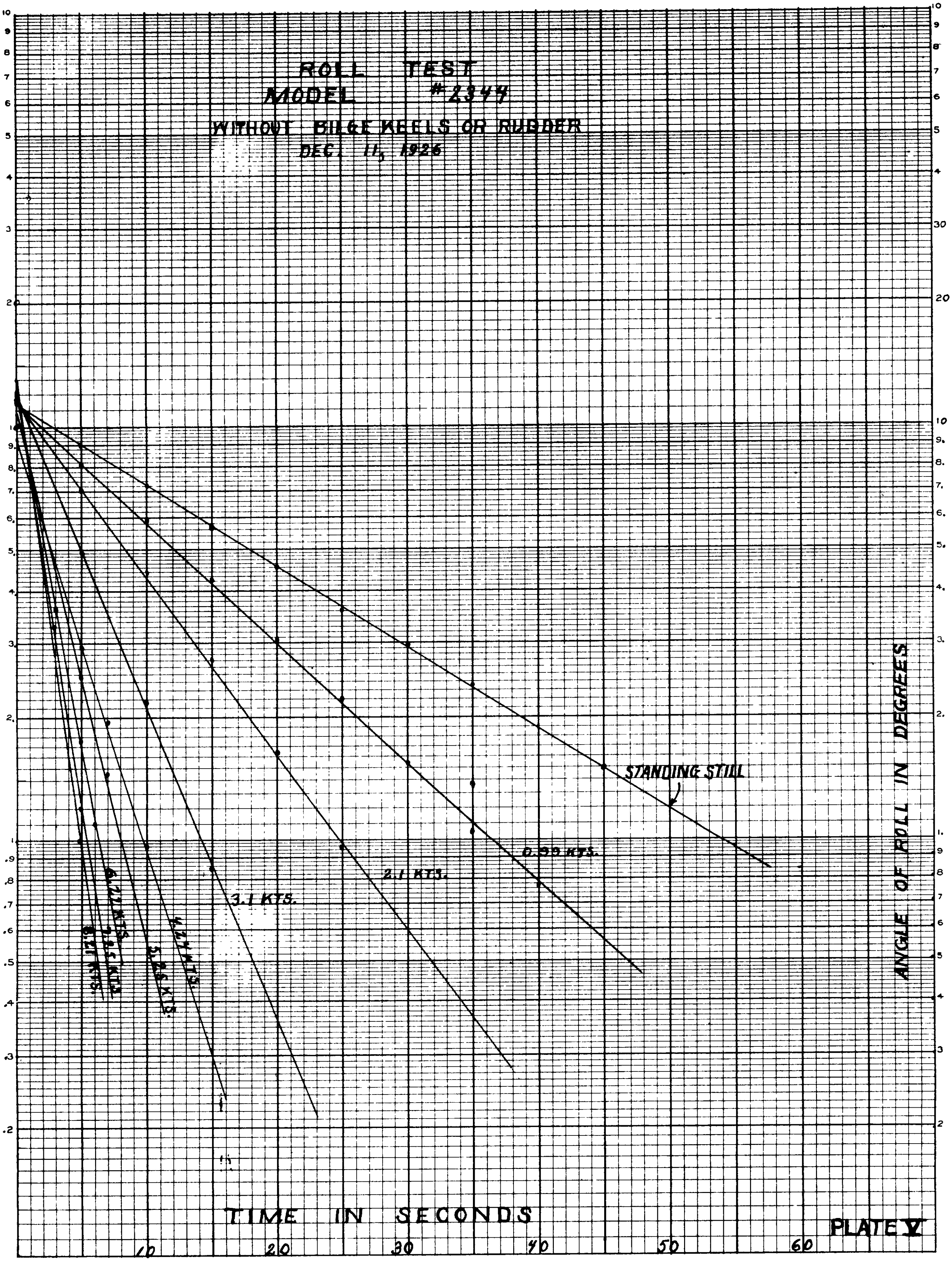
ROLL TESTS
MODEL #2344
WITH RUDDER
NOV. 13, 1926
U.S.S. PRUITT
(DESTROYER)
DEGREES ROLL
TIME IN SECONDS
0
5
10
15
20
25
30
35
40
45
1
2
3
4
5
6
7
8
9
10
11
12
13
7.15 K.
6.31 K.
5.35 K.
4.16 K.
3.13 K.
2.21 K.
1.01 Knots
Standing Still

PLATE IVb



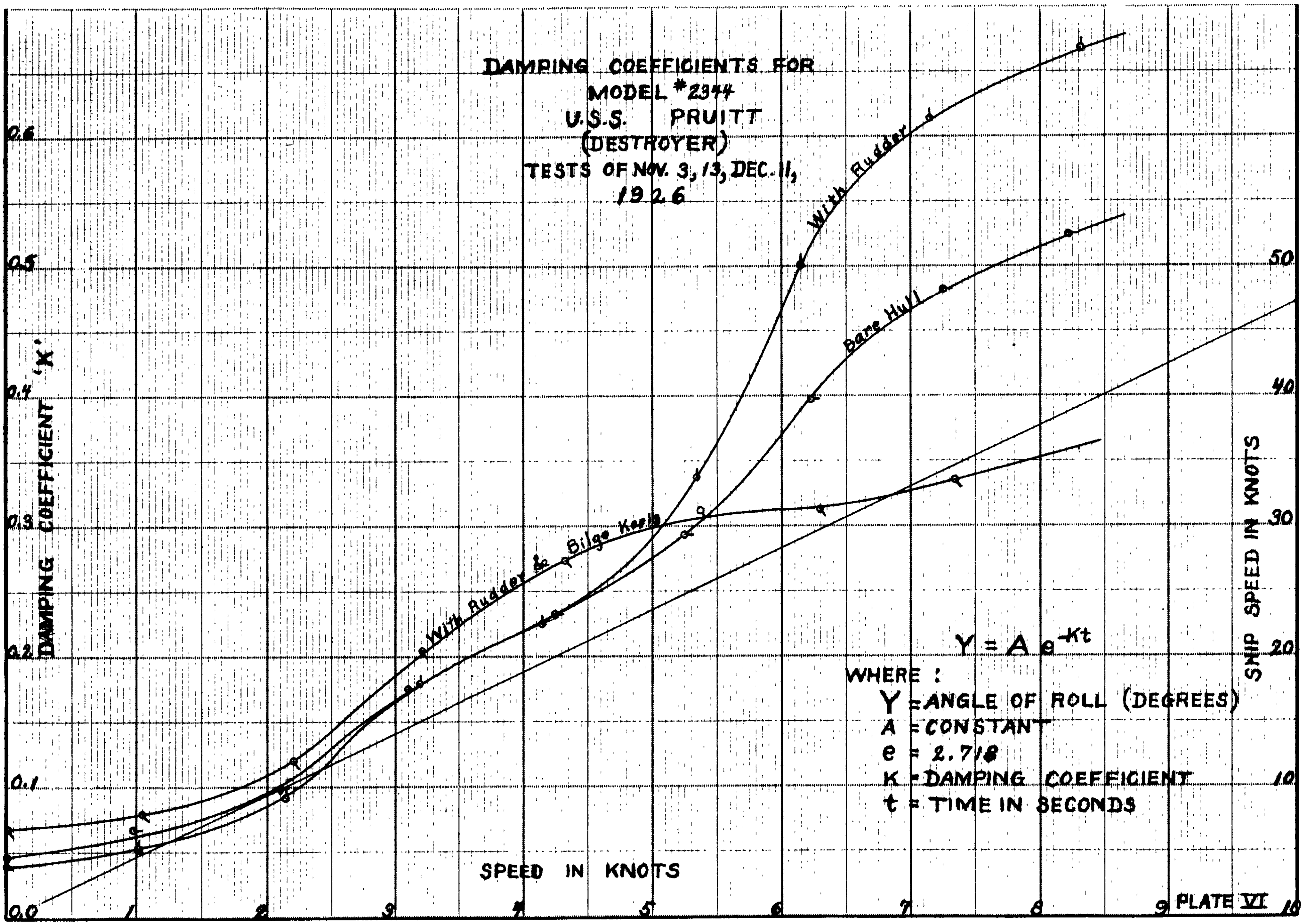
ROLL TESTS
MODEL #2344
WITH RUDDER & BILGE KEELS
NOV. 3, 1926
U.S.S. PRUITT
(DESTROYER)

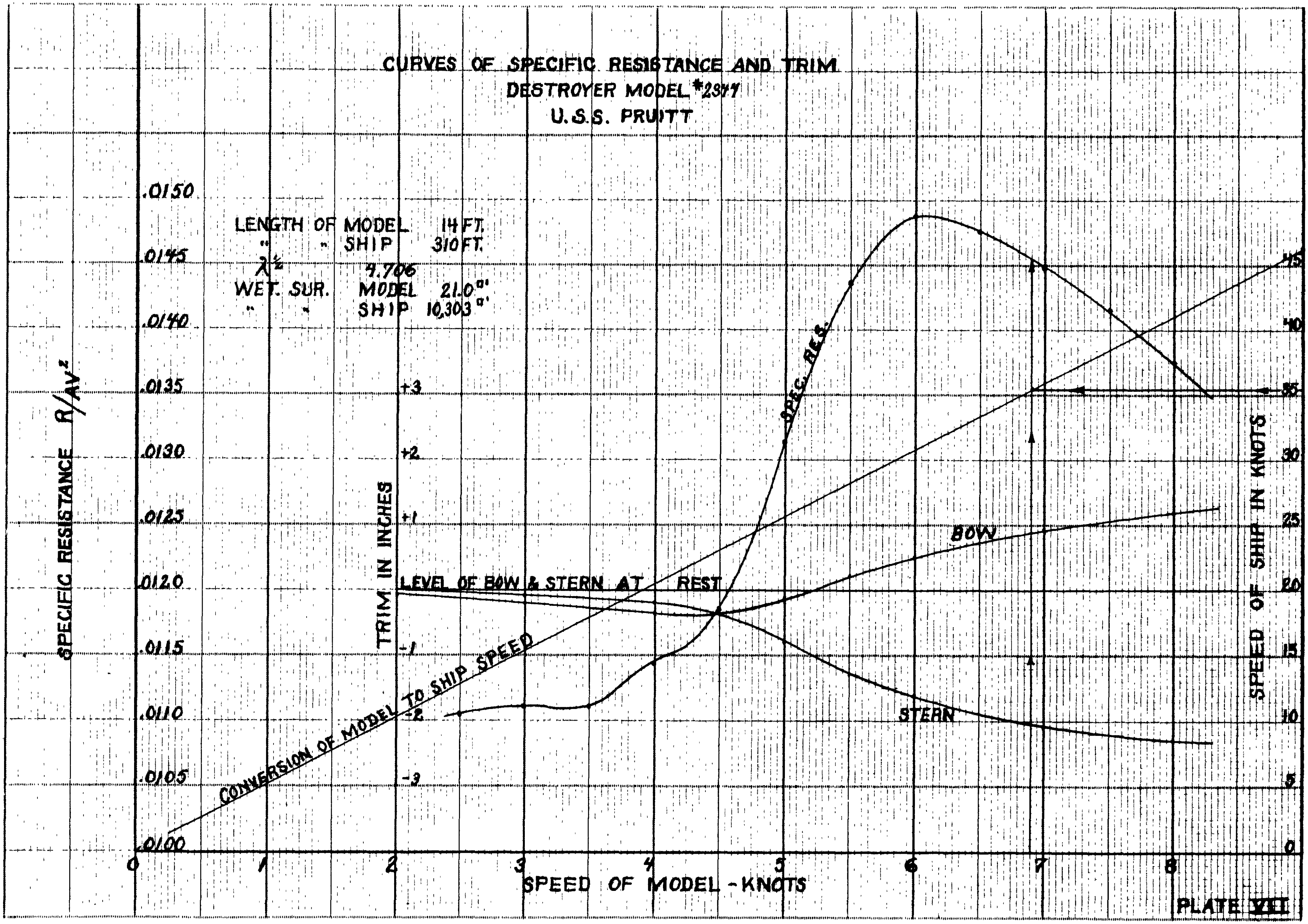
PLATE IVc



ROLL TEST
MODEL #2344
WITHOUT BILGE KEELS OR RUDDER
DEC. 11, 1926

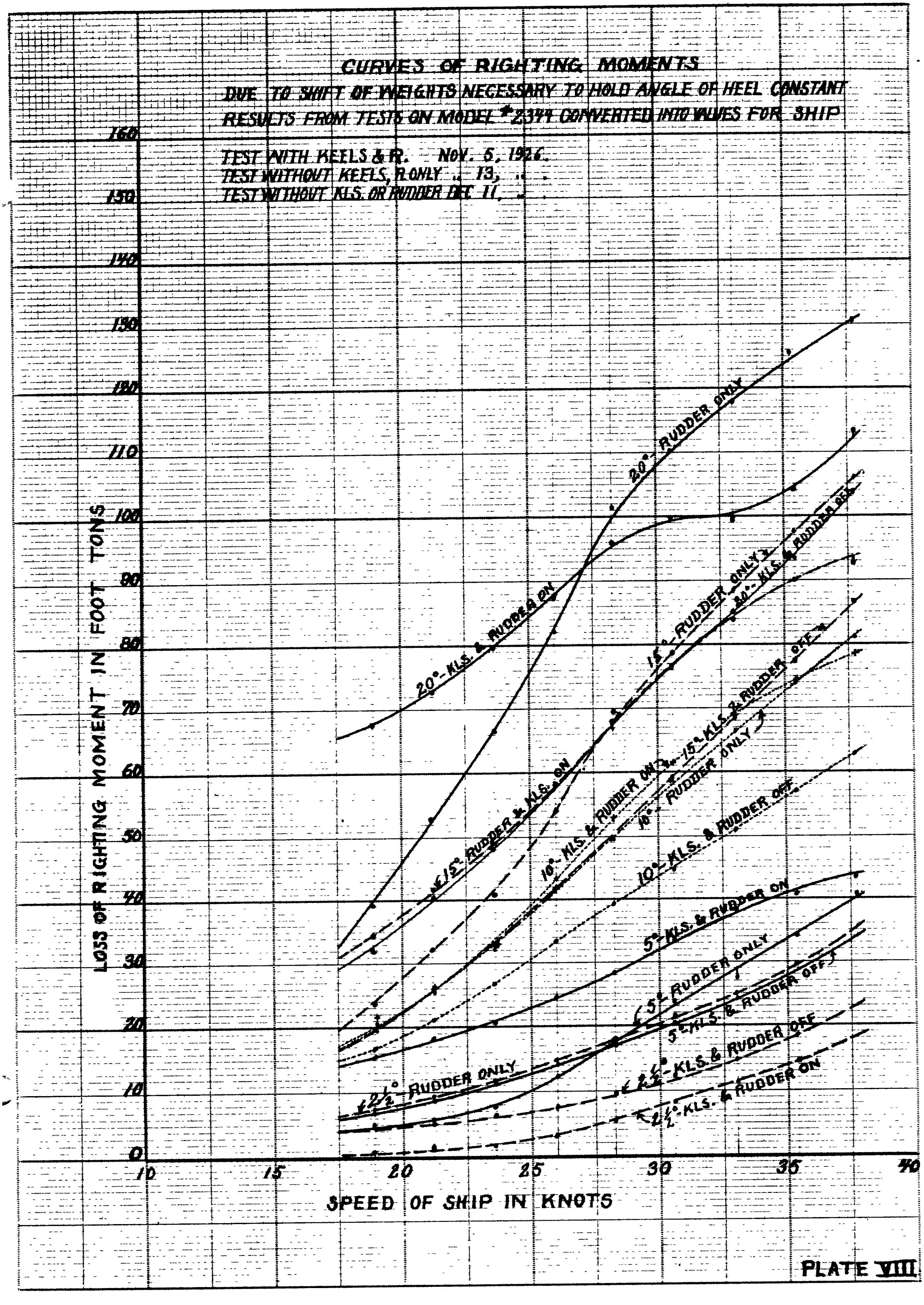
PLATE V

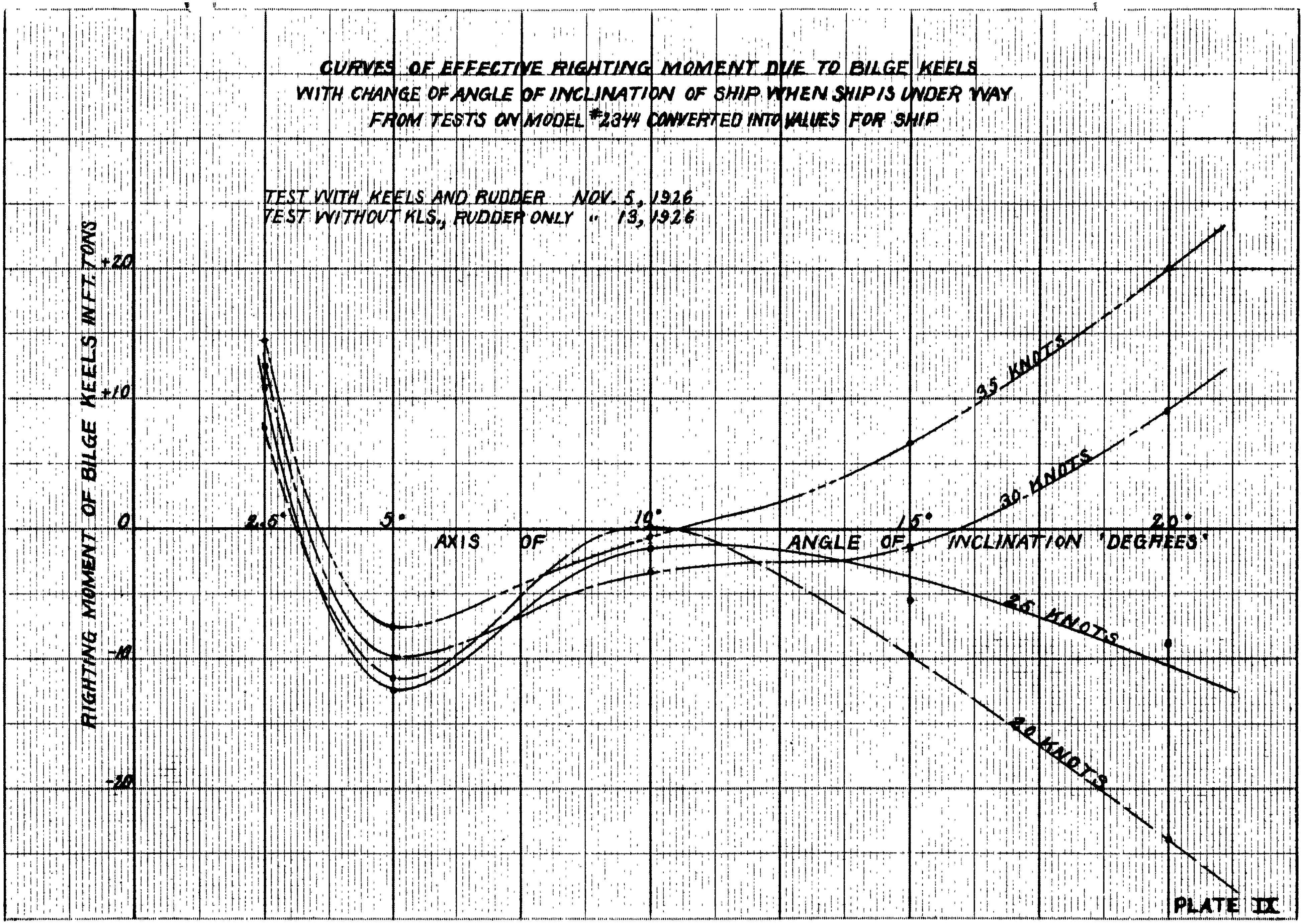

DAMPING COEFFICIENTS FOR
MODEL #2344
U.S.S. PRUITT
(DESTROYER)
TESTS OF NOV. 3, 13, DEC. 11,
1926
DAMPING COEFFICIENT 'K'
SHIP SPEED IN KNOTS
SPEED IN KNOTS
With Rudder
Bare Hull
With Rudder & Bilge Keels
Y = A e^-Kt
WHERE :
Y = ANGLE OF ROLL (DEGREES)
A = CONSTANT
e = 2.718
K = DAMPING COEFFICIENT
t = TIME IN SECONDS
0.0
0.1
0.2
0.3
0.4
0.5
0.6
1
2
3
4
5
6
7
8
9
10
20
30
40
50
PLATE VI



CURVES OF SPECIFIC RESISTANCE AND TRIM
DESTROYER MODEL #2377
U.S.S. PRUITT

PLATE VII





CURVES OF EFFECTIVE RIGHTING MOMENT DUE TO BILGE KEELS
WITH CHANGE OF ANGLE OF INCLINATION OF SHIP WHEN SHIP IS UNDER WAY
FROM TESTS ON MODEL #2344 CONVERTED INTO VALUES FOR SHIP

TEST WITH KEELS AND RUDDER NOV. 5, 1926
TEST WITHOUT KLS., RUDDER ONLY " 13, 1926

PLATE IX