PROJECT WHIRLWIND

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FOREWORD

Project Whirlwind

Project Whirlwind at the Massachusetts Institute of Technology Digital Computer Laboratory is sponsored by the Office of Naval Research under Contract N5ori60. The objectives of the Project are (1) the application of an electronic digital computer of large capacity and very high speed (Whirlwind I) to problems in mathematics, science, engineering, simulation, and control, and (2) the study and development of component reliability in Whirlwind I.

The Whirlwind I Computer

Whirlwind I is of the high-speed electronic digital type, in which quantities are represented as discrete numbers, and complex problems are solved by the repeated use of fundamental arithmetic and logical (i.e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the magnetic-core memory, in which binary digits are stored as one of two directions of magnetic flux within ferro-magnetic cores.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access magnetic-core memory has a capacity of 32,768 binary digits. Present speed of the computer is 40,000 single-address operations per second, equivalent to about 20,000 multiplications per second. This speed is higher than general scientific computation demands at the present state of the art, but is needed for control and simulation studies.

Reports

Quarterly summary reports are issued to maintain a supply of up-to-date information on the status of the Project. Detailed information on technical aspects of the Whirlwind program may be found in the formal R-series reports and the informal M-series memorandums that are issued to cover the work as it progresses. A list of selected publications, issued during the period covered by this Summary, appears in the Appendix.

1. QUARTERLY REVIEW AND ABSTRACT

Computer time allocated to the Scientific and Engineering Computation (S&EC) Group was divided among 48 problems. Previously developed computer logging facilities, now tested, are adequate; henceforth, WWI will process the logging tapes for the biweekly and quarterly reports of S&EC computer time. And finally -- a statistical study of tape preparation was completed.

The systems group, eliminating system weaknesses, replaced power-distribution panels with more reliable plug-in-type relays and revised the central d-c power-supply system for better regulation on transients.

The magnetic-tape system, now maintained by the systems group, has been given increased flexibility by the addition of a second delayed-printout typewriter.

To facilitate maintenance and to increase the reliability of terminal equipment, marginal-checking facilities have been improved, the file of test programs has been expanded, and checking and monitoring equipment has been added.

Pending engineering analysis, varying vacuum-tube heater voltage in conjunction with standard d-c marginal checking appears valuable in locating certain classes of tube faults.

Continued investigation indicates that previous results in the problem of transient changes in oxide cathodes are faulty; revised results are now being prepared.

Three seminars were held, one on advanced programming techniques, two on computing-machine methods. A number of papers were presented by staff members, the largest number at the Ann Arbor meeting of the ACM (at which there was also a showing of the recently completed Digital Computer Laboratory movie, "Making Electrons Count").

2. MATHEMATICS, CODING, AND APPLICATIONS

2.1 Introduction

During the period covered by this report 48 problems made use of the computer time allotted to the Scientific and Engineering Computation (S&EC) Group. Progress reports as submitted by the various programmers are presented in numerical order in Section 2.2. Of these, twenty (162, 167, 169, 170, 173 through 184, 186 through 188, and 192) represent new problems that are being described for the first time. Nine problems (113, 119, 142, 143, 144, 149, 161, 176, and 187) have been completed. During this period, ten theses (one bachelor's, seven master's, two doctorate) were written incorporating WWI results.

Tables 2-I and 2-II have been set up to provide the reader with a convenient reference to various interesting aspects of these problems. Table 2-I lists the problems according to their fields of application. The column labeled "% of WWI Time" has been included to indicate each problem's share of the WWI time allotted to the S&EC Group. The eight problems marked with asterisks represent work being performed by members of the MIT Project on Machine Methods of Computation, another phase of the work under contract N5ori60, which is reported more extensively in a separate report ("Machine Methods of Computation and Numerical Analysis," Quarterly Progress Report No. 12, Project DIC 6915, MIT, June 15, 1954).

The mathematical problems and procedures represented by the various current problems are tabulated separately in Table 2-II, because different problems in the same field frequently involve different mathematical methods, while problems in different fields sometimes are solved by identical methods.

The revised version of the comprehensive system of service routines (called CS II) has been in routine daily use during this report period. A flexible read-in procedure has been developed and is described under Problem 100. The logging facilities that were described in Summary Report No. 37 have been used in parallel with manually kept time records and have proved satisfactory. It is planned to have the computer process the logging tapes to produce the biweekly and quarterly reports on S&EC use of WWI time.

During the past six months, an attempt was made to compile some relevant statistics on the time expended in the preparation of punched paper tapes for WWI. The procedure was simply to have the typists indicate the type of tape (viz., an original complete tape, a typed modification, or a manual modification) being prepared, the approximate number of registers in the tape, and the approximate time expended. The conclusions reached are presented in Section 2.3 of this report.

Field	Description	Problem Number	% WWI Time	Source
Aeronautical Engineering	Construction and testing of a delta-wing flutter model	166.C	1.33	MIT
	Determination of the downwash at the tail of an airplane due to the lift response of the wing to a sharp-edged gust	168.C	0.26	MIT
	Low-aspect ratio flutter	177.D	0.04	MIT
	Transient temperature of a box-type beam	179.C	0.35	MIT
	Blast response of aircraft	183.D	0.67	MIT
Chemical Engineering	Transient effects in distillation	*167.D	2.35	MIT
	Effect of gravity on relative water production in oil reservoirs	188.C	0.76	Atlantic Refining Co.
Chemistry	Optical properties of thin metal films	101.C	0.91	MIT
Civil Engineering	Analysis of reinforced concrete walls	*113.C	0.93	MIT
	Study of shock waves: vibration problems in solid bodies	*142.D	2.14	MIT
	Response of cohesive and cohesionless soils to transient loading	*161.C	0.32	MIT
Dynamic Analysis & Control Lab	Response of a fuel-flow controller	187.C	0.20	MIT
Electrical Engineering	Switching-circuit design	*169.B	0.17	MIT
	Connector provision in automatic telephone exchanges	176.B	1.60	MIT
	Crosscorrelation of blast furnace input-output data	180.B	1.22	MIT
Geology & Geophysics	Geophysical data analysis	106.C	3.15	MIT
	Interpretation of earth-surface resistivity measurements	*123.C	0.57	MIT
	Frequency and phase spectrum analysis of seismograms	192.D	1.05	MIT
Instrumentation Lab	An interpretive program to accept mathematical symbols	108.C	0.13	MIT
	System of nonlinear differential equations	109.C	0.18	MIT
	Trajectory study against an evading target	178.D	0.29	MIT
Insulation Research	Crystal structures	182.C	0.36	MIT
Lincoln Lab	Digital methods of detecting signal from noise	149.C	0.25	MIT
	Transmission in a rectangular waveguide containing a single ferrite slab	163.C	0.53	MIT
	Tracking response characteristics of the human operator	186.C	0.05	MIT
Mathematics	Spherical wave propagation	*119.C	0.40	MIT
Mechanical Engineering	Turbine design (aerothermopressor)	120.D	1.28	MIT
Meteorology	Synoptic climatology	155.D	2.27	MIT
Physics	Vibrational frequency spectrum of a copper crystal	143.D	0.63	MIT
	Self-consistent molecular orbitals	144.C	0.23	MIT
	Energy bands in crystals	147.C	3.10	MIT
	Evaluation of the reflection coefficient in a semi-infinite open rectangular waveguide	156.A	0.04	MIT
	Use of water storage in a hydroelectric system to minimize the expected operating cost	159.D	1.15	MIT
	Determination of phase shifts from experimental cross-sections	162.C	0.11	MIT
	Inverse and inverse square root of a real symmetric matrix	170.C	0.01	MIT
	Overlap integrals of molecular and crystal physics	*172.B	2.66	MIT
	Tight-binding calculations in crystals	174.C	0.32	MIT
	Impurity levels in crystals	175.C	0.41	MIT
	Perturbed coulomb wave functions	181.C	0.08	MIT
	Scattering electrons from hydrogen	184.D	0.18	MIT
Servomechanisms Lab	Autocorrelation and Fourier transform calculation	107.C	0.71	MIT
	Subroutine for the numerically controlled milling machine	132.C	0.39	MIT
	Data-reduction program: polynomial fitting	126.C	0.40	MIT
	Improved power-spectra calculations	171.C	1.30	MIT
Miscellaneous	Comprehensive system of service routines	100	61.15	MIT
	Special problems (staff training, etc.)	131	1.64	MIT
	Library of subroutines	141	0.40	MIT
	Course 6.537: digital computer application practice	173	1.33	MIT

Table 2-I. Current Problems Arranged According to Field of Application
(* MIT Project on Machine Methods of Computation)

Mathematical Problem	Procedure	Problem Number
1. Matrix algebra and equations		
System of fifteen linear equations	Relaxation	*113.C
Evaluation of elements and roots of a third-order secular determinant	Standard analytical solution	141.D
Matrix multiplication, addition, diagonalization	Basic	144.C
Solution of a matrix equation	Iteration using Crout's method and matrix algebra	166.C
Inverse and square root of inverse	Diagonalization	170.C
Eigenvalues of a matrix	Diagonalization	174.C
Inversion	Diagonalization	175.C
Systems of up to 200 nonlinear equations	iteration	176.B
Matrix equation	Crout's method	177.D
Matrix equation	Diagonalization	182.C
2. Transcendental equations		
Set of two transcendental equations	Iteration	101.C
Curve fitting	Iteration using least squares	162.C
Two simultaneous transcendental equations	Iteration	163.C
3. Ordinary differential equations		
General system	Gill's modified fourth-order Runge-Kutta	108.C
Fourteen simultaneous nonlinear	Fourth-order Runge-Kutta	109.C
Seven nonlinear first order	Step-by-step Euler method	120.D
Second order	Finite difference approximation	*142.D
Second-order linear with variable coefficients	Gauss-Jackson forward integration formula	147.C
Second-order nonlinear	Fourth-order Runge-Kutta	*161.C
Set of nonlinear first-order equations	Second-order Runge-Kutta	*167.D
Wave equation	Manning-Millman method	181.C
System of five first-order nonlinear	Gill's method	187.C
4. Partial differential equations		
Nonlinear hyperbolic	Difference equations written along the characteristics and solved simultaneously by iteration	*119.C
Second-order parabolic	Explicit finite difference approximation	179.C
Elliptic boundary value problem with free interface	Iteration (successive overrelaxation)	188.C
5. Integration		
Auto-, crosscorrelation and Fourier transform	Simpson's rule	107.C
Integral evaluation	Trapezoidal rule	*123.C
Complex integral evaluation	Trapezoidal rule	156.A
Numerical integration	Trapezoidal rule	159.D
Numerical integration	Simpson's rule	168.C
Power spectra and confidence limits	Simpson's rule	171.C
Overlap integrals	Evaluation of analytic forms	*172.B
Fourier transform of the crosscorrelation of two functions	Simpson's rule	180.B
Two-dimensional integrals	Simpson's rule	184.D
Calculation of transfer functions	Simpson's rule	186.C
6. Statistics		
Multiple time series	Prediction by linear operators	106.C
Detecting signal from noise	Weighted counting	149.C
Calculation of the coefficients of a multiple regression system	Inner products	155.D
Phase and frequency spectra	Simpson's rule and Tukey's method of smoothing	192.D
7. Data-reduction program	Polynomial fitting, etc.	126.C
8. Network minimization	Boolean algebra	*169

Table 2-II. Current Problems Arranged According to the Mathematics Involved (* MIT Project on Machine Methods of Computation)

2.2 Problems Being Solved

100 COMPREHENSIVE SYSTEM OF SERVICE ROUTINES

The comprehensive system of service routines has been developed by the Scientific and Engineering Computation (S&EC) Group to simplify the process of coding. The system in use since the fall of 1952, described in Summary Reports 32-35, is now called CS I. A new system, called CS II, which is based on CS I, was described in Summary Reports 36 and 37. CS II has been thoroughly tested during the past quarter and is now being used on a routine basis.

Since the reader will find references in some of the reports below to the number system used in CS II, the following brief description is included here for the reader's convenience.

(m, n) numbers shall mean numbers which are of the form $z = x \cdot 2^y$ where x is an m-binary-digit number and y is an n-binary-digit number. For example, (24, 6) signifies a two-register floating-point system dealing with numbers of 24 significant binary digits (roughly 7 decimal digits) with magnitudes between 2^{63} and 2^{-64}.

Arithmetic involving these (m, n) numbers is carried out by means of (m, n) interpretive subroutines. These subroutines enable the programmer to write coded programs using (m, n) numbers as easily as, or even more easily than, he might write programs in the single-length fixed-point (15, 0) number system which is built into Whirlwind I.

The Comprehensive System of Utility Programs

CS I and CS II form part of a library of utility programs which are used daily by the S&EC Group. This library includes other conversion systems, post-mortem programs, input programs, and equipment programs. The major part of the library has been semipermanently recorded on either magnetic tape or the magnetic drum. A program called the utility control program, which arranges for the automatic selection of utility programs from the library, has been semipermanently recorded on the magnetic drum. The over-all system (the control program plus the library) is called the comprehensive system of utility programs.

The Utility Control Program

Normal operation of the computer is initiated by pressing the read-in button. This transfers computer control to a short program in test storage which records the contents of magnetic-core memory on drum group 0 and reads the utility control program from the drum into core memory. Computer control is then transferred to the utility control program.

The utility control program selects the utility program required, and the program is executed. After the program is executed the utility control program normally restores the contents of core memory (by reading the contents of drum group 0 into core memory) and stops the computer.

Utility-Program Selection

The utility program required for a read-in can be selected in the following ways:

1. By sensing the initial characters on a punched paper tape,
2. By pressing a button on the control panel (the examine-selector-panel button) and inserting a number into a set of octal pushbuttons (the selector panel) on the control panel.

The first method is called a normal read-in procedure, and the second is called an automatic read-in procedure.

Normal Read-In Procedure

In a normal read-in procedure (selected by not pushing the examine-selector-panel button) the utility control program assumes that a punched paper tape will be read in using the photoelectric tape reader. The utility program required is then selected by sensing the first two Flexowriter characters on the paper tape. The following cases arise on this basis:

1. fb tapes: the control program selects the binary-input program which reads in a binary tape.
2. fc tapes: the control program selects CS II which reads in the Flexo tape. Any output from conversion (tape title, floating-address table, conversion post-mortem) is recorded for delayed printing. No binary tape is produced.
3. fp tapes: the control program selects the post-mortem program which reads in the Flexo request tape and performs the requested post-mortem.
4. fs tapes: the control program selects the summer session computer (a special-purpose conversion system used for instructional purposes) which reads in the Flexo tape.

Whenever a paper tape is read in the utility control program displays and photographs the tape title and records both the tape title and the time of read-in on the direct punch. Thus two logs are produced for any operating period on the computer.

Manual Read-In Procedure

In a manual read-in procedure (selected by pushing the examine-selector-panel button) the control program selects the utility program required in accordance with the number put in the selector panel. The following simplified table gives the contents of the selector panel and the corresponding utility program.

Contents	Utility Program
1	Read in an fc tape and produce a binary tape. Record conversation results (title, floating-address table, binary tape, conversion post-mortem) for delayed printing and punching.
2	Record a Flexo stop character for delayed printing.
4	Display and photograph the contents of core memory.
5	Perform a programmed-arithmetic post-mortem, and record the results for delayed printing.
6	Perform a programmed-arithmetic post-mortem and record the re-
9	sults on the direct printer.
7	Record the title of the tape last read in for delayed printing.
8	Read in an fc tape and do not produce a binary tape. Record conversion results on the direct printer and punch.
15	Record the title of the tape last read in on the direct printer.
16, 17	Select CS I, convert the Flexo tape in the photoelectric tape reader, and record the binary tape for delayed punching.
24	Read in the magnetic-drum checking program.
32	Read in the magnetic-tape checking program.
40	Read in the scope-calibration program.
48	Read in the photoelectric-tape-reader checking program.
49	Terminate the paper-tape log.
62	Initiate the paper-tape log.
64	Perform a normal read-in procedure using the mechanical tape reader for input instead of the photoelectric tape reader.

101 OPTICAL PROPERTIES OF THIN METAL FILMS

The present stage of research in electronic digital computation applied to the optical and electrical properties of thin metal films was completed by A. Loeb of the MIT Chemistry Department during the past month. The final production runs have taken place, and the next month will be devoted to the analysis and publication of the results. While there is a possibility that some short runs may be desirable, this seems to be the appropriate time for a final summary.

Six modes of operation have been programmed, developed, tested, and used to deal with the functional relationships between the variables of a physical system consisting of a thin metal film on a nonabsorbing, thick backing. (The expressions thin and thick are used relative to the wavelength of incident radiation.) These variables are: film thickness, index of refraction of film, absorption coefficient of film, electrical conductivity of film, dielectric constant of film, index of refraction of backing, wavelength of incident radiation, and reflectance and transmission of incident radiation. The six modes are outlined below. It should be emphasized that each mode is fully automatic and requires no human interference.

A For a given set of optical constants (index of refraction and absorption coefficients) the reflectance and transmission are calculated.

B_1 and B_2 Approximate relations have been derived which are simple, and quite valid in the infrared. Modes B_1 and B_2 have been used to test their reliability. These relations enable one to calculate the conductivity, dielectric constant, index of refraction, and absorption coefficient of a film in terms of observed reflectance and transmission and the other physical variables of the system. For each set of reflectance and transmission there are two sets of optical and electrical constants. Of these only one is physically significant, but only physical insight can discriminate between them. Since WWI does not possess this physical insight, provisions have been made to compute both.

The modes B_1 and B_2 use the approximate equations to calculate the optical and electrical constants. These constants are then used according to mode A to compute a reflectance and transmission exactly. The discrepancy between the observed and calculated reflectance and transmission is an indication of the validity of the approximate expressions. The difference between modes B_1 and B_2 is that the former approximately computes and checks one set of optical constants, while the latter computes and checks the other set.

C_1 and C_2 The same approximations are used as in modes B_1 and B_2, and again a reflectance and a transmission are calculated exactly from the optical constants computed by the approximate equations. This time, however, instead of printing out the discrepancies between the observed and the calculated values, WWI uses these discrepancies to make a better approximation. Successive approximations are then made by WWI until the discrepancies between the calculated and observed reflectance and transmission are less than the expected experimental errors. Modes C_1 and C_2 correspond to the two sets of optical and electrical constants.

D When first estimates for the optical constants are already available, these may be read into WWI along with the other variables. WWI then calculates the reflectance and transmission corresponding to the first estimates, and, if there is a discrepancy between the observed and calculated values, successive approximations are made as in modes C_1 and C_2 until the discrepancies are less than the expected experimental errors.

The six modes described above were used for three purposes. In the first place, data published by others have been evaluated, and in many cases the approximate evaluation made by the authors has been improved upon. Secondly, large numbers of data obtained in our own laboratory on gold films with visible radiation ($4000 \text{ A} < \lambda < 6600 \text{ A}$) and on the aluminum-oxide films with infrared radiation ($1.5\mu < \lambda < 15\mu$) were evaluated. Thirdly, the validity of approximate equations that have been derived for use when no computer is

available was checked on WWI. The computer has been intimately integrated into the laboratory program. Since the density of thin metal films is not easily determined, the rapid calculations made it possible to repeat the calculations assuming different densities. Also, a quality check on some of our measurements was obtained because an underestimate of the reflectance, which is likely to occur with transparent films, was spotted by WWI. This led to the discovery of a lower limit of our film systems, which could be derived from the newly tested approximate equations.

The analysis of the results obtained from WWI has enabled us to calculate the relaxation time of electrons in metal blacks and metal brights. Also it enabled us to explain the abnormally high d-c resistance of some films, for the variation with wavelength of the optically evaluated conductivity of such films indicates that imperfections in the films act as condensers. The optical constants calculated from results obtained with radiation incident on the film differ in some cases from those calculated with radiation incident on the backing. This discrepancy has been blamed on the condition of the surface of the backing; improved cleaning methods for the backing in our own laboratory have decreased the discrepancies.

During the next year no new programs will be developed; whatever calculations will be performed will use one of the modes C_1, C_2, and D. It is hoped that in the future this research will be continued to encompass systems containing more than two films, each of which may be absorbing. To do so it would be most practical to develop a system analogous to the CS which can deal directly with complex numbers. This would then be used to solve Maxwell's equations directly on WWI for a system of various media separated by a series of parallel walls, each medium being characterized by its thickness and two optical or electrical constants.

The following publications include results of calculations made on WWI:

"Conductance and Relaxation Time of Electrons in Gold Blacks from Transmission and Reflection Measurements in the Far Infrared," Louis Harris and Arthur L. Loeb, J. Opt. Soc. Am. 43, 1114-1118.

"The Preparation and Infrared Properties of Aluminum Oxide Films," Louis Harris. Submitted to J. Opt. Soc. Am.

"The Evaluation of Optical and Electrical Constants of Thin Metal Films from Reflectance and Transmission Measurements by Electronic Digital Methods," Louis Harris and Arthur L. Loeb. To be submitted to J. Opt. Soc. Am.

"Electronic Digital Computation of Optical and Electrical Constants of Thin Films" (tentative title), Louis Harris, Harry H. Denman, and Arthur L. Loeb. To be submitted to J. S. I. A. M.

106 MIT SEISMIC PROJECT

Recently a new approach to picking seismic reflections was attempted by W. Walsh of the Geophysical Analysis Group. In this method, amplitude spectra of successive overlapping intervals (traveling spectra) of a seismogram trace were calculated and plotted in the fashion of a contour map; i.e., the time associated with each interval was plotted along one axis (the abscissa), the frequency along another (the ordinate), and the amplitude along a third axis perpendicular to the aforementioned two in the manner of contours. Both computation and plotting were accomplished automatically on Whirlwind.

The plotting of amplitude for frequency and interval time was achieved by plotting a number of spots (proportional to amplitude) in an area of predetermined magnitude on the oscilloscope screen. While each amplitude was being so displayed, a photograph of it was taken to afford the permanent record of the contour plot.

The results of such an analysis in a particular case, where reflection times were known, were quite encouraging. Not only were the times of reflections put into evidence, but also the frequency contributions of both signal and noise were displayed in such a fashion as to make possible the construction of mathematical and electronic filters. It is hoped that future analysis of this sort will prove just as valuable in those cases where reflections are not in visible evidence on the seismogram.

In addition, a generalized correlation program which will use the standard Problem 106 data tapes and compute auto- or crosscorrelations from either the Wiener or Tukey formulas over any sections of the seismic records on the data tapes has been started by H. Briscoe. The correlation is done in (24, 6) arithmetic although the standard data tapes are in (15, 0).

107 (a) AUTOCORRELATION AND (b) FOURIER TRANSFORM-INTEGRAL EVALUATION

Routines for calculating autocorrelation functions and Fourier sine or cosine transforms were developed as part of this problem by D. T. Ross of the MIT Servomechanisms Laboratory. The development work was completed in the second quarter of 1953 and described in Summary Report No. 34.

During the past quarter these routines were used by A. Fleischer of the MIT Meteorology Department, F. Raichlen of the MIT Hydrodynamics Laboratory, and Dr. J. T. Farrar of the Evans Memorial Hospital. The reports included below were submitted by the groups in question.

1. The Spectrum of Sunspot Variations

This is another aspect of the general problem of the effect on the atmosphere of solar variations. The sunspot numbers are an index of this variability that have the longest

history; an analysis of their statistics is therefore most meaningful. The spectrum of the sunspots is the simplest statistic related to predictability. Its possible application to meteorology is clear.

The spectrum of the mean monthly sunspot numbers was completed. This function was desired for its possible applications in predictions where solar effects on the atmosphere would be significant, e.g., radio reception and meteorological conditions. The work is now completed, and a report is being prepared by A. Fleischer of the MIT Meteorology Department.

2. Investigation of Turbulent Flow

This project is being carried out by F. Raichlen of the MIT Hydrodynamics Laboratory. It deals mainly with the investigation of turbulent flow by means of a Pitot-tube pressure-cell combination. The velocity fluctuations which are recorded by means of an oscilloscope motion-picture-camera arrangement are of a random nature.

From these records 1800 points with time spacings of 0.001182 second were then sent to the Digital Computer Laboratory, and an autocorrelation curve was obtained to determine the relative importance of the signal distortion caused by the resonant frequency of the pressure cell. This was found to be negligible in comparison to the true signal.

A number of runs were then made at different levels in a high-velocity open-channel flow. From these records 1800 points were taken at 0.00151-second intervals and submitted to the Digital Computer Laboratory for correlation. The autocorrelation curves obtained showed in addition to the 180 cps component, caused by the natural frequency of the gage, a low-frequency (3.5 cps) component which is believed to be inherent to the flow. Mean-intensity spectra were then obtained from these autocorrelation curves for the frequency ranges 0 to 25 cps and 0 to 250 cps. From the 0 to 250 cps mean-intensity spectrum the fact that the distortion of the signal caused by resonance was negligible was again confirmed.

The program used for obtaining the mean-intensity spectrum consisted of convolving the autocorrelation function with the unit step function.

$$W(f) = \int_0^{\tau_0} R(\tau) \cos \omega\tau d\tau \quad \int_0^{\tau_0} \cos \omega\tau d\tau$$

$$W(f) = \left[\int_0^{\tau_0} R(\tau) \cos \omega\tau d\tau\right] \left[\frac{\sin \omega\tau_0}{\omega}\right]$$

Wherever there is a periodic component the integral $\int_0^{\tau_0} R(\tau) \cos \omega\tau d\tau$ will give a spike, and $\frac{\sin \omega\tau_0}{\omega}$ will give peaks and troughs on either side of the spike. For a maximum delay time of $t_0 = 0.45$ second these peaks and troughs were subtracted from the results obtained from the Digital Computer Laboratory. Mean-intensity spectra showing 90 per cent of the power to be below 25 cps and periodicities occurring at 3.6 and 10 cps were obtained. This type of spectrum agrees with theoretical results, the periodicities thought to be in the flow itself, not in the measuring instrument.

3. Analysis of Intestinal-Motility Records

During the past year, it has been demonstrated that rabbit intestinal-motility records may be satisfactorily analyzed by the generation of their autocorrelation functions. Human intestinal-motility records have also yielded good autocorrelations, computed by Whirlwind. However, the autocorrelation functions of the complex human records are still too complex to permit accurate quantitation of the power of the component functions making up the total motility record.

During the past three months, further analysis of the four human records referred to in Summary Report No. 37 has been undertaken. In each case, Whirlwind performed Fourier transformation of the autocorrelations, using the program developed by D. T. Ross of the MIT Servomechanisms Laboratory. In all the transforms, considerable power was seen in the low-frequency ranges (0-3 cpm). In order to characterize these frequencies more accurately, several runs were obtained with different frequency tapes.

In two of the human records, a significant spike on the frequency spectra is seen in the range of 10.2-11.4 cpm. This corresponds to the estimated frequency of one of the known intestinal waveforms.

The transform on the subject was further subjected to a "smoothing-out" process which eliminates the large oscillations seen on the frequency spectra obtained by the currently used Fourier programs. This step appears to make analysis of the frequency spectra more accurate as it obviates the necessity of manually drawing a curve.

During the next three months, it is planned to continue using Whirlwind. The work may be divided into two parts: (1) The analysis of intestinal motor activity in rabbits before and after total body x-irradiation. This will involve autocorrelation and Fourier transformation of approximately 120 4-min motility records. These studies have been performed under a contract with the U.S. Atomic Energy Commission. (2) Further evaluation of the use of autocorrelation and Fourier transformation in the analysis of human intestinal motor activity. One serious deterrent to the use of these analytic methods for human records is the inadequacies of the recording system. The records analyzed have been obtained with a balloon-water kymographic system. Since the recording balloon may move "downstream" as a result of propulsive motility, different segments of the intestine are involved, and a stationary time series is not achieved. It is hoped that we can overcome these recording deficiencies.

This work is being carried out by J. T. Farrar, M.D., of the Massachusetts Memorial Hospitals.

108 AN INTERPRETIVE PROGRAM

This program will accept as input algebraic equations, differential equations, etc., expressed on Flexowriter punched paper tape in conventional mathematical notation (within limits imposed by the Flexowriter) and automatically provide the desired solution.

For information concerning the use of this program, a programmer's manual (E-364) is available at the Instrumentation Laboratory Library.

The work done on this program during this quarter was that of combining all the tapes used in developing the program into one master tape and then making tests on this combined tape.

109 AN AIRPLANE PURSUIT-COURSE PROGRAM

An airplane pursuit-course program has been written which restrains the airplane and target to the same horizontal plane. This program computes prediction times for a two-gyro gunsight and a three-gyro gunsight. A slant-airplane pursuit-course program has also been written which computes prediction times for a two-gyro gunsight and a three-gyro gunsight and correction-time ratios for the latter gunsight. Each of these programs includes the effects of airplane dynamics and projectile ballistics.

A test problem has been run using these two programs with comparable initial conditions for each. The results of these runs are being analyzed and will be included in the final report on this problem.

It is anticipated that a number of runs will be made using the slant-plane pursuit-course program with the airplane and target initially in a slant plane. From the results of these runs it will be possible to evaluate previous methods used to calibrate two-gyro and three-gyro gunsights.

This study is being carried out by M. H. Hellman of the MIT Instrumentation Laboratory.

113 STRESS ANALYSIS OF A THROATLESS PRESS

The results of the Whirlwind calculations for the stress analysis of the throatless press have been compared with an experimental photoelastic analysis. The two solutions are in fairly close agreement. One noticeable difference in the results is the stress values at reentrant corners of the structure. These values are higher in the photoelastic analysis. If a finer network of bars was used to approximate the plate, more accurate values would be obtained at the points of high stress concentration. The indeterminacy of the structure increases, however, and the computation time required for a solution also increases as the network of bars is made finer.

Figure 2-1 is a typical stress plot at one section through the roof of the press. The two analyses are in close agreement except at the lower edge where the photo-elastic shear curve has a higher stress than might normally be expected. This section is at the edge of the loaded area of the experimental model, and the high shear stress can be attributed to this loading condition. There was no corresponding load concentration in the Whirlwind analysis. Ref.: Sydney, S. H., "Analysis of a Massive Closed-Ring Frame by the Lattice Analogy Method," Master's Thesis, Dept. of Civil Engineering, MIT, June 1954.

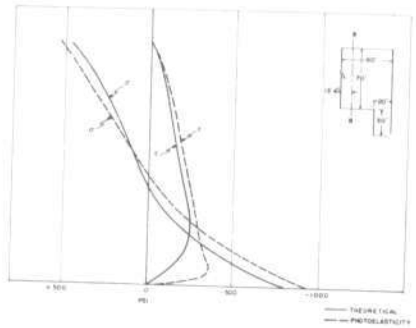

Fig. 2-1

Typical Stress Plot at Section B-B Through the Roof of the Press

119 SPHERICAL WAVE PROPAGATION

The problem which has been investigated is that of the flow which results when a compressed sphere of air, initially at rest in the atmosphere, is suddenly released. General solutions of the equations governing the flow have been found for small (i.e., infinitesimal) amplitude waves, but in the case of finite amplitude waves the equations have defied general analytical solution because of their nonlinearity.

The conditions of this problem are believed to simulate, in simplified fashion, the conditions of an explosion. A complete solution has been obtained for the case (Case A) corresponding to a quite mild explosion in that the compressed sphere considered has a maximum density at its center of three times atmospheric density, and the density falls off exponentially from there. In Case B the maximum density is six times atmospheric density at the center and again falls off exponentially from there.

In both cases the results indicate that in its early stages, as would be expected, the motion is that of a single outward-moving spherical wave whose amplitude falls off with increasing distance from the center. This opposes the tendency (as in the plane-wave case) for the wave to become a shock wave. Thus any shock which might form would be very weak. Actually the data in Case A indicates that no discernible shock forms as this initial wave moves outward, and it also seems very unlikely that this initial wave forms a shock in Case B.

After a short time, in both cases, the center of the sphere becomes overexpanded (i.e., its density becomes less than atmospheric density). At this point a recoil wave forms

traveling inwards. Thus the center of the sphere becomes recompressed. From this point on the two solutions differ. In Case A the center is recompressed to a maximum density only slightly above atmospheric. Therefore, the second outward-going wave that forms is very weak, so weak in fact that no discernible second inward wave forms. By this time conditions are close to atmospheric everywhere, and what motion is left dies out rapidly. These results differ not only with those of Case B but also with the results of J. J. Unwin* who considered precisely the problem of Case A. His first recoil wave was of such strength that the center of the sphere was recompressed to a density substantially above that of the original compression. This was naturally followed by a second strong outward wave followed by a second recompression. Unwin was forced to stop his calculations before reaching the maximum of the second recompression because of numerical difficulties in his numerical process, which is different in various respects from the one used here. Physical checks applied to the results indicate that the solution obtained here is very accurate and more accurate than that obtained by Unwin.

In Case B the first recompression is markedly stronger than that in Case A as indicated by the velocities with which the particles move toward the center. It even appears, in contrast with Case A, that a shock forms near the center of the sphere. However, before the calculations could be carried to the shock, two major numerical difficulties were encountered. The first is that near the shock the extrapolation method used to estimate the indeterminate quantity $2u/r$ on the t-axis becomes very inaccurate and causes serious numerical errors. The second difficulty is caused by the inherent numerical error in the difference process being used to solve the problem. The process used is second order with a general per-step error of $0(h^3)$. However, for small r the error tends to be of the order of h^2 because of the r in the denominator of the quantity $2u/r$.

Before any further work can be done on the problem it will be necessary to determine, first, the relative significance of the two errors and, second, some method of reducing the effect of the errors.

A complete discussion of this problem and the two sources of error may be found in a final report written by A. Ralston for the MIT Committee on Machine Methods of Computation.

* Unwin, J.J., "The Production of Waves by the Sudden Release of a Spherical Distribution of Compressed Air in the Atmosphere," Proc. Roy. Soc. (London), Ser. A, Vol. 178

120 THERMODYNAMIC AND DYNAMIC EFFECTS OF WATER INJECTION INTO GAS STREAMS OF HIGH TEMPERATURE AND HIGH VELOCITY

This problem is connected with the development of a potential gas-turbine component, called an "aerothermopressor," in which a net rise in stagnation pressure of a hot gas stream is brought about by the evaporation of liquid (water) injected into a high-velocity region of the flow. The so-called "stagnation pressure" of a compressible gas stream is analogous to the quantity "total head" in the more familiar problems of hydraulics. The function of the aerothermopressor in the gas-turbine cycle is to reduce both specific fuel and specific air consumption of the gas-turbine power plant, and it is analogous to the role of the condenser in a steam power plant. Further description of this device may be found in earlier reports, beginning with Summary Report No. 32, Fourth Quarter 1952.

During the past quarter, considerable effort was directed toward revision and improvement of existing calculation procedures. Difficulties arising from instability and truncation error have led to a complete re-evaluation of the numerical analysis currently in use. This study is still in progress.

At the same time, computations of aerothermopressor performance for various Mach number variations have continued, and a new calculation procedure has been written and successfully tested which to a great extent allows Whirlwind itself to determine the Mach number variation (and, hence, geometrical shape of duct) for best performance. This calculation procedure shows great promise for providing a routine method for aerothermopressor design contained on a single WWI punched tape.

Construction of the large aerothermopressor test facility at the MIT Gas Turbine Laboratory has been completed, and the equipment is now being tested. Experimental data which will be obtained from this device together with the results of the WWI theoretical analysis will in the future be used hand in hand to obtain a better understanding of the complicated physical processes which occur in the aerothermopressor.

The aerothermopressor development program is under the sponsorship of the Office of Naval Research and the guidance of Prof. Ascher H. Shapiro of the Department of Mechanical Engineering. The Whirlwind numerical analysis is being carried out by Dr. Bruce D. Gavril.

123 EARTH RESISTIVITY INTERPRETATION

The program for the Newton analysis of the Slichter kernel (described in Summary Report No. 37) has been successfully tested on Whirlwind by K. Vozoff of the MIT Geophysics Department. It has thus far been applied to four different kernels, three theoretical and one practical, with various degrees of success. More data must be run before the usefulness of the method is fully established.

One important variation was necessary. It was found that the full values of the calculated parameter changes could not be applied because of the nonlinearity of the kernel function. The resultant parameters would oscillate wildly. At the suggestion of T. R. Madden, a parameter-change multiplier was introduced, so that a fraction of the calculated change could be used each time. To expedite convergence, this fraction should not be made too small. It is expected that the maximum allowable value of the parameter will approach unity as the true solution is neared. In the cases tested, 0.05 has been the largest allowable fraction at the beginning of a calculation, but 0.4 has been used later in the calculations.

Of the four kernels tried, two runs were stopped after two iterations. This was caused by a provision for automatic stopping when the sum of the squared errors increases. The error increase resulted from parameter oscillation. These two kernels have not yet been retested. The third case, a theoretical kernel for the case of thicknesses $1/1/\infty$ and resistivities 1/10/0.1 was run through 25 iterations. The parameters were approaching asymptotes of about the right values. The sums of squares of errors decreased by 81 per cent in the 25 iterations.

The only kernel which it is believed was iterated a sufficient number of times had thicknesses $1/1/\infty$ and resistivities 1/0.1/1. This case was iterated 91 times, increasing the parameter-change multiplier every 20 iterations, from 0.05 to 0.4. Automatic cutoff was taken out of the program. After 40 steps, the correct solutions were obtained, but the calculations were continued and the errors continued to decrease. After 50 steps, the thickness and resistivity of the second layer went negative, but the error increased slightly. Beyond this point, the first-layer thickness oscillated about a value 3 per cent too large, the second-layer thickness and resistivity oscillated about zero, and the error continued to decrease very slowly. At the end of the calculation the error was 0.2 per cent of the original error, whereas at the correct solution (50 iterations before the end) it had already decreased to 0.5 per cent of the original error. There is no question of the validity of this solution, and the question of how accurate a solution to expect arises. It would not, however, seem proper to cease calculation when the error reaches some predetermined value, because all solutions beyond this point are equally valid.

The particular case considered here is a difficult one in that the derivatives and the parameters do not each have a range (in λ) of predominance. It would be expected that an increase in the accuracy of the kernel data would help. However, the data used here were accurate to five places, two more than could reasonably be expected from field data. This then implies a fundamental restriction on the whole approach of studying kernels. Significantly, this restriction is of the sort required for the use of the Pekeris method of interpretation (see Summary Reports No. 9, 10, and 11). It may be that the potentials themselves are more amenable to relaxation-type analyses than are the kernels. This would not be expected since the potential must be obtained from the kernel by an integration of the form

$$r \int k(\lambda)\ j_o(\lambda r)\, d\lambda$$

Thus even less separation of ranges of dominance is anticipated. This would then imply a general restriction on analysis of resistivity data. As stated previously, however, more data should be run before definite conclusions are drawn. This will be done.

126 A DATA-REDUCTION PROGRAM

Problem 126 is a very large data-reduction program for use in the Servomechanisms Laboratory. The over-all problem is composed of many component sections which will be developed separately and then combined at a later date. Thus far, efforts have been focused on the development of utility-type programs. These programs, which have been described in previous quarterly reports, include a fully automatic program to fit polynomials to arbitrary empirical functions; a mistake diagnosis routine, an automatic interruptive checking routine; a general purpose Lagrange interpolation program; and a flexible and fairly elaborate post mortem routine. The programs now being used have been written by Douglas T. Ross and Richard J. Turyn, Servomechanisms Laboratory staff members, with the assistance of Miss Dorothy A. Hamilton.

Testing of the first complete version of the data-reduction program was begun during this quarter. An initial test showed that the first sections operate correctly, but an unassigned constant prevented a complete run. Succeeding tests are to be run using the mistake diagnosis routine to extract approximately 30 intermediate results for accuracy checks with values previously computed using desk calculators. These tests have been delayed because of technical difficulties. Hand calculations require over half a day per solution, but it is hoped that a complete run of the program, 250-300 solutions, can be made in a matter of minutes.

131 SPECIAL PROBLEMS (STAFF TRAINING, DEMONSTRATIONS, ETC.)

A request was received from the Boston Museum of Science to factor four numbers (each containing eleven to thirteen digits) into prime factors, the largest of which was not to exceed 101. In case the given numbers did not contain such factors, it was requested that they be varied over the range $\pm$ 15,000 while testing for the desired factors.

A program for carrying out this factorization was developed and described by Dr. H. Denman of the S&EC Group under Problem 94 in Summary Report No. 32. This program was reconstructed by J. Ackley of the S&EC Group during the past quarterly period. No trouble was encountered with three of the numbers, but for the fourth, no numbers could

be found in the interval 3,662,563,604,200 $\pm$ 15,000 whose largest prime factor did not exceed 101. For this case factors were given for certain numbers outside the interval. Also for one number in the interval prime factors not exceeding 173 were given.

These results will be used in the design of the gear chains in the planetarium of the Boston Museum of Science.

132 SUBROUTINES FOR THE NUMERICALLY CONTROLLED MILLING MACHINE

During this quarter, four new or revised subroutines were added by J. Runyon of the Servomechanisms Laboratory to the library of routines for computations for the MIT numerically controlled milling machine. Among these are two routines for use on jobs requiring interpolation. One of these is for selecting a set of points for the interpolation by means of the Lagrangian formula.

When it became necessary to check several very long milling-machine tapes, a routine for reading these tapes into the computer was brought up to date. The Ferranti reader facilitates the process since no feedout is required where the tape must stop. The revised routine has been incorporated in a program for printing out in decimal form the numbers on the tape or for finding and printing out cumulative totals of milling-machine orders. A program for preparing tape for series-16 wing templates has been run several times, but the major portion of it has not performed satisfactorily. Error diagnosis is continuing.

Six test pieces were cut in wood from tapes prepared on WWI in the preceding quarter. These are cones with sinusoidal cross sections. The pieces illustrate two different methods of cut spacing and three degrees of approximation to the surface. Since the results of this machining were satisfactory, a program for preparing tape for a series-16 conic wing section has been obtained by modifying the sinusoidal-cone program. It will be run sometime in the future.

141 S&EC SUBROUTINE STUDY

A list of all available and tested library subroutines, including the storage requirements for each, has been added to the CS programming manual to inform programmers of the subroutines currently available.

A new subroutine for calculating the principal value of $\tan^{-1}X$ has been written, tested, and added to the subroutine library under the title, FU 7: $\tan^{-1}X$, (30-j, j), j=1, 2, ... 15. This closed subroutine, written in CS code, is entered with the value of X in the MRA (where $-\infty < X < \infty$, within the limits of the number system used). The subroutine makes use of two approximations; the value of X determines which is used.

$$\text{For } 0 \leq |X| \leq 0.00405, \quad \tan^{-1} X \simeq X$$

$$\text{For } 0.00405 < |X| < \infty, \quad \tan^{-1} X \simeq \frac{\pi}{4} + \sum_{0}^{7} C_{2i+1} \left(\frac{X-1}{X+1}\right)^{2i+1}$$

This second approximate expression is given in the Rand Corporation Approximations in Numerical Analysis. The maximum absolute error for $\tan^{-1}X$ is about 1×10^{-7}, with the average error much less (about 3×10^{-8}). The maximum relative error is about 10^{-5}, and again the average is much less.

142 A STUDY OF SHOCK WAVES

Problem 142, temporarily terminated in December, was reopened by S. Sydney of the MIT Civil Engineering Department in the early spring after the results of the previous calculations had been analyzed. (See Summary Report No. 36.)

It was found that the shock wave had travelled through the two-dimensional solid, had reflected off the rear face, and had gone about halfway back towards the front face at the conclusion of the fall analysis. Further calculations were made to propagate the wave to the front face and then back again to the rear face. On the basis of the results already analyzed, maximum stress intensities are expected to occur when the wave is reflected off the front face.

The program was revised in order to obtain stress intensities directly rather than displacement and velocity values of the individual mass nodal points. These values were then plotted directly without any intermediate hand computations.

References: Bart, R., "Propagation of Shock Waves in Two Dimensional Solids," Doctoral Thesis in Civil Engineering, MIT June, 1954.

143 THE VIBRATIONAL FREQUENCY SPECTRUM OF A COPPER CRYSTAL

The secular determinant described in Summary Report No. 36 has been solved for 3417 wave vectors q and the resulting frequencies sorted into a histogram of 100 divisions by Whirlwind I. The results appear in Figs. 2-2 and 2-3 as a smooth curve drawn through the points of the histogram. The solid curve of Fig. 2-2 is the composite spectrum of the three possible thermal waves propagating through the copper lattice (one "longitudinal," two "transverse" vibrations for each wave-vector). Figure 2-3 presents the decomposition of Fig. 2-2 into longitudinal and transverse spectra as indicated. The following values of the force constants were determined from X-ray temperature diffuse scattering experiments and were employed in the present calculation:

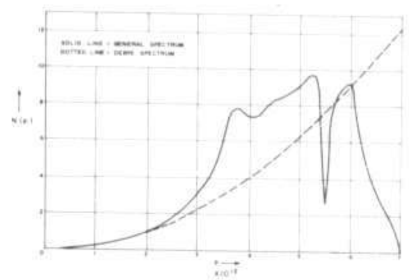

Fig. 2-2
Composite Spectra

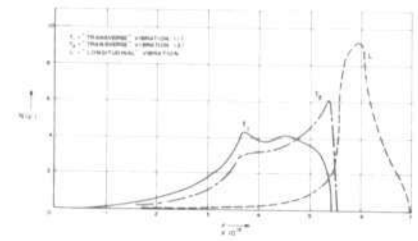

Fig. 2-3
Decomposition of Vibrational Spectrum into Branches

$$\alpha_1' = \alpha_1 + 2\alpha 3 \quad , \quad \beta_1' = \beta_1 + 2\beta 3$$

$$\alpha_1'' = \alpha_1' + 2\beta_1' \qquad \gamma_1' = \gamma_1 + 2\gamma 3$$

$\alpha_1 = 0.48 \times 10^4$ dy/cm
$\beta_1 = 0.87 \times 10^4$ dy/cm
$\gamma_1 = 1.25 \times 10^4$ dy/cm
$\beta_2 = 0.35 \times 10^4$ dy/cm
$\alpha_2 = -\ 0.072 \times 10^4$ dy/cm

$\alpha_3 = 0.09 \times 10^4$ dy/cm
$\beta_3 = -\ 0.022 \times 10^4$ dy/cm
$\gamma_3 = -\ 0.015 \times 10^4$ dy/cm
$\delta_3 = 0.06 \times 10^4$ dy/cm

The dotted line of Fig. 2-2 is the Debye spectrum adjusted to the cut-off frequency of the secular determinant which is 7.01×10^{12} cps. It is well known that the simple Debye spectrum does produce a specific heat in close agreement with experiment at medium low temperatures. The reason is obvious from Fig. 2-2 wherein it will be noted that the two curves coincide over about 1/3 of the spectrum. This is the most important region at low temperatures. Also, it will be noticed that the high-frequency end of the spectrum is sharply peaked due entirely to "longitudinal" waves. This result also occurs in the simple lattice models where exact frequency spectra can be easily calculated by analytic means and is of interest when considering electron conduction phenomena wherein the scattering of electrons by lattice waves is caused primarily by the longitudinal waves. This part of the spectrum can be approximated by a delta function or a rectangle in such cases.

The general value of this spectrum calculation has been to show that the exact model does not introduce any essentially new features over the simpler models which have been employed frequently and with success in the past. At first glance, this outcome appears disappointing. Nonetheless it is worthwhile by way of developing programming methods applicable to cases where new results might be obtainable and of indicating valid approximations for cases where exact calculation is not practical.

A similar spectrum calculation will be performed for the body-centered cubic lattice of α-iron and will employ the general atomic-force constants determined by H. Curien of the College de France. This result is expected to appear in the next quarterly report.

These studies are being carried out by E. H. Jacobsen of the MIT Physics Department.

144 SELF-CONSISTENT MOLECULAR ORBITALS

The routine described in previous Summary Reports has been rewritten by Dr. A. Meckler of the MIT Solid State and Molecular Theory Group in a style more suitable for production runs. The main program exists on a permanent tape and is fed in to a fixed position in storage. This position is determined by the largest problem to be handled by the routine. The data is fed into fixed addresses, after which control is transferred to the main program, the calculation is performed, results are displayed, and control is transferred back to the read-in program so as to bring in the next set of independent data, if any.

As yet, no actual molecule has been run. A study of the ammonia molecule will be begun in co-operation with Dr. H. Kaplan of the Solid State and Molecular Theory Group and will be described under Problem 201 in future reports.

147 ENERGY BANDS IN CRYSTALS

The formation and solution of the secular equations which form the final step in this problem have been programmed by Dr. D. Howarth of the MIT Solid State and Molecular Theory Group following the lines laid down in Summary Report No. 37. A large amount of production work has been carried out on metallic copper, and the results are considerably better than were expected. The success of the method depends upon the rate of convergence of the secular problem, and inclusion of only eight terms in the expansion has produced convergence to four significant fugures, making this possibly the most accurate method yet devised for handling energy bands in solids. The convergence is so good that an extension of the work is in progress to investigate energies at points of lower symmetry in momentum-space than have been possible to compute by other methods.

A program has been devised to print and plot the resulting wave functions as well as the eigenvalues, which are of some physical interest and will be of practical use for other problems. The routine to solve secular equations involving nondiagonal overlap matrices

has been used to solve 52 such equations, of order up to twenty, for Dr. F. Herman of RCA, Princeton; satisfactory results were obtained in all cases.

149 DIGITAL METHODS OF DETECTING SIGNAL FROM NOISE

During this quarter the study of digital methods of detecting signal from noise has been essentially completed by Dr. G. P. Dinneen of the Lincoln Laboratory.

Programs have been written which successfully generate a sequence of zeros and ones which simulate both signal and noise. The noise and signal regions, which differ only in the density of ones, have been so generated that the necessary records for a large number of trials may be kept. Simple statistical tests have been made to test the accuracy of this model.

Quite a large number of detectors of three different classes have been programmed and tested. The three classes are the sequential-observer detector described in Lincoln Laboratory Technical Report No. 20. the success-run observer based on the theory of success runs; and the density detector which consists of a moving slot inside of which the density of ones is measured. Data has been obtained on the operation of these detectors for several different signal-to-noise ratios and parameter settings. A report is now being prepared to present the results of this study.

Reference: William Feller, An Introduction to Probability Theory and Its Applications, John Wiley and Sons, Inc. 1950.

155 SYNOPTIC CLIMATOLOGY

The objectives of the synoptic climatology project under Prof. T. F. Malone of the MIT Meteorology Department have been stated in Summary Report No. 37. The five basic questions to be answered will be restated here along with the progress made toward their solution during the past quarterly period.

Questions:

1. How large a sample must be taken to insure stability of the prediction operators?
2. Over how large an area must the circulation pattern be considered?
3. How far back in time must the autocorrelation and crosscorrelation be extended before the point of diminishing returns on information is reached?
4. How much independence really exists between the several levels in the atmosphere?
5. How precisely must the circulation pattern be specified?

Progress:

1. The question of sample size seems to have been settled for the time being with the conclusion being that the six-year sample chosen is sufficient for obtaining stable operators. This is based on a preliminary comparison with a sample of twenty years.

2. For pressure prediction, results show that the grid size chosen originally (65°-125° W, 30°-60° N) appears to be superior to a grid less than half its size or one more than double its size. The lack of equal specification precision for all grid sizes used may have led to an erroneous conclusion. More work is planned to substantiate or disprove the preliminary conclusions reached thus far.

3. This is one question which was not investigated at all during the past quarter.

4. A major portion of our work during the next quarter will be spent dealing with the problem of upper levels and their contribution toward prediction independent of surface information. A four-dimensional model (x, y, z, t) is being put together to determine the upper-level contribution.

5. A full-scale operation has recently been completed which enables a 24-hour climatological prediction of surface pressure over a grid from 65°-125° W to 30°-60° N. In a preliminary analysis of the errors encountered, the question of specification appears to be quite important. Errors seem to originate because of coarse specification of the patterns used for predicting.

The following is a summary of work completed by Don G. Friedman. This work is described in detail in an Sc. D. thesis submitted to the Meteorology Department of MIT.

An investigation relating to Question 4 was terminated during the second quarter. This study involved an investigation of the relative amounts of information available in the large-scale circulation patterns both near the surface and aloft for a classification of temperature and precipitation data. This approach to the classification of climatic data related a mathematical specification (based on an ensemble of Tschebyscheff orthogonal polynomials) of the circulation over a region bounded by 30° - 60°N latitude and 65° - 125°W longitude to the surface weather at a point in the region by the use of a linear operator. The linear operator was obtained by a multiple linear regression analysis.

The usefulness of this objective method was shown by the comparison of a classification of temperature and precipitation data for January 1948-1952 at a number of stations with a classification of the same data based on previously accepted methods. The method was used to obtain a classification of the temperature and precipitation data based on the sea-level pressure patterns and the 700-mb height patterns, respectively. The comparison indicated the practicability of using the upper-air flow pattern for the classification of weather data was obtained by the comparison of results based on 700-mb height, 700-mb standardized height, and 700-mb cumulative probability of height patterns. Circulation patterns

based on the 700-mb standardized height appear to be superior to the 700-mb height patterns for use as objective specifiers of temperature and precipitation.

The problem on Whirlwind involved computing fourteen coefficients of Tschebyscheff orthogonal polynomials fit to:

1. sea-level pressure,
2. 700-mb height,
3. 700-mb standardized height,
4. 700-mb cumulative-probability-of-height

for each day of five Januarys (1948-1952). Ninety-one sampling points were taken to represent the circulation over the region for each representation. Sums of cross-products of the various combinations of the orthogonal polynomial coefficients and the weather data were needed to supply the initial data for the multiple linear regression analysis. These cross-products were calculated on Whirlwind. Very little difficulty was encountered in the solution of these problems.

156 THE EVALUATION OF THE REFLECTION COEFFICIENT IN A SEMI-INFINITE OPEN RECTANGULAR WAVEGUIDE

The evaluation of the reflection coefficient in a semi-infinite open rectangular waveguide is obtained approximately by using Fourier transform techniques on the integral equations of the Wiener-Hopf type. The integrals are to be evaluated by the trapezoidal rule.

After making suitable checks, it was decided that the results from the test run for $0 \leq a < \pi^2$ were correct for the interval used in the trapezoidal rule but that this was not sufficiently accurate.

To obtain the additional accuracy desired, the intervals used in the trapezoidal rule were taken smaller around the singularity of the integrand. Corrections were made in the program to introduce these changes, and, so far, a successful run has not been obtained.

Once a successful run is completed on the above, the final section of the problem will remain to be done, i.e., the section for $-30 \leq a \leq 0$.

A successful run was made for $200 \leq a \leq 300$ in steps of 5.

This work has been carried out by A. Balser of Columbia University. Work was delayed during this quarter because of Mrs. Balser's absence from Cambridge.

159 WATER USE IN A HYDROELECTRIC SYSTEM

The following programs have been written and appear to be working satisfactorily:

1. A data program takes 40 years of weekly flows of the Columbia River at Grand Coulee, processes them, and stores them on the drum.

2. Another program takes these flows and deduces from them conditional-probability distributions of river flow and stores them on the drum.

3. An output program is available which punches out the flows and probability distributions in binary form so that they may be put back on the drum directly without recalculation for each run.

4. Another output program prints out the non-zero parts of the conditional-probability matrices in a convenient layout.

5. An optimization program computes tables which determine operation of a given one-dam hydro system for a given criterion of good performance. This program was written entirely in (15, 0) so as to be fast.

6. Another program takes the calculated tables, uses them for operating the system for the historical flows, and prints out the results. Thus it can be seen how the calculated optimum operation reacts to real flows.

It is planned to explore various criteria for good performance and various models of dams.

This study is being carried out by J. D. C. Little of the MIT Operations Research Group. The results will be included in a doctorate thesis to be submitted to the MIT Physics Department.

161 RESPONSE OF MASS-PLASTIC SPRING SYSTEM TO TRANSIENT LOADING

This problem involves the motion of a building foundation bearing on soil when it is subjected to a blast loading. The behavior of the soil-foundation system is approximated by the action of a single degree of freedom mass-spring system with nonlinear characteristics. The properties of this system have been developed from established static-footing action concepts and some assumptions regarding the manner in which the foundation soil acts under transient-loading conditions.

The physical properties of actual soils, bearing capacity, density, internal-friction angle, strain rate, etc., have been determined to a large extent by empirical means. This information has been used to predict the behavior of these soils under the shock-loading conditions by the equations developed for this analysis. This analysis has been divided into two sections in order to treat the cohesionless soils and cohesive soils separately. Cohesive soils exhibit a strain-rate effect which is not present in cohesionless soils. Under a constant load, a building will settle if the soil possesses this strain-rate effect. Different equations were developed for these two types of soil, and separate analyses have been made to evaluate the effect of this characteristic feature of cohesive soils.

The results of these analyses have been included by S. Sydney in a report to the MIT Civil Engineering as a set of load-settlement curves for various sizes of footings.

A designer, in using these results, would first establish the maximum allowable transient settlement for the building and then enter these charts to obtain the required footing size under the specified blast-loading condition.

162 DETERMINATION OF PHASE SHIFTS FROM EXPERIMENTAL CROSS SECTIONS

This problem, undertaken by Dr. F. J. Eppling of the MIT Laboratory for Nuclear Science, is concerned with the analysis of a nuclear-scattering experiment, the elastic scattering of protons by O^{16}, over a range of bombarding energies from about 0.5 Mev to 4.6 Mev. Cross sections were measured at eight scattering angles from 168.0° to 90.4° (in the center of the mass system). From the experiment a series of eight curves is obtained in which the absolute differential cross section, d /d , is plotted as a function of the bombarding energy of the incident protons at each of the eight scattering angles. From these curves it is then possible to obtain d /d as a function of scattering angle at any given bombarding energy.

For a reaction such as the one above the theoretical expression for d /d is given as follows:

$$\frac{d\sigma}{d\omega} = \lambda\!\!\!^{-2} \Bigg| \frac{-\eta}{2} \csc^2 \frac{\theta}{2}\, e^{i\eta \ell \eta \csc^2 \frac{\theta}{2}} + \sum_{\ell=o}^{\infty} (\ell+1) P_\ell e^{i(\alpha_\ell + \delta_\ell^+)} \sin \delta_\ell^+ + \sum_{\ell=1}^{\infty} \ell P_\ell e^{i(\alpha_\ell + \delta_\ell^-)} \sin \delta_\ell^- \Bigg|^2 + \lambda\!\!\!^{-2} \sin^2\theta \Bigg| \sum_{\ell=1}^{\infty} P_\ell' \, e^{i\alpha_\ell} \left\{ e^{i\delta_\ell^-} \sin \delta_\ell^- - e^{i\delta_\ell^+} \sin \delta_\ell^+ \right\} \Bigg|^2$$

where:

$\lambda\!\!\!^{-} = \frac{\hbar}{\mu v}$,

$\eta = \frac{zZ\epsilon^2}{\hbar v}$,

θ = the scattering angle in the center of mass system,

$P_\ell(\cos\theta)$ = Legendre polynomial or order ℓ,

$P_\ell'(\cos\theta) = \frac{d}{d(\cos\theta)} P_\ell(\cos\theta)$,

$\alpha_\ell = 2 \sum_{x=1}^{\ell} \tan^{-1} \frac{n}{x}$; and $\alpha_o = 0$.

$\delta_{\ell}^{\pm} = \beta_{\ell}^{\pm} - \phi_{\ell}$ where $\beta_{\ell}^{\pm}$ is the resonant phase shift and ϕ_{ℓ} is usually called the potential-scattering phase shift or "hard sphere" phase shift. $\delta_{\ell}^{\pm}$ is the non-coulomb phase shift of the partial wave of orbital angular ℓ and total angular momentum $J = \ell \mp 1/2$; and where

$k = 1/\lambda\!\!\!^{-}$

μ = the reduced mass of the system,

v = the velocity of relative motion,

z, Z = the atomic numbers of the colliding particles,

ϵ = the electronic charge

$\hbar$ = Plank's constant divided by 2π, and

$\ell\hbar$ = the orbital angular momentum of the incident protons.

Using this expression for $d\sigma/d\omega$ it is desired to extract the δ_{ℓ}'s from $\ell = 0$ through $\ell = 3$, i.e., the S-wave, the P-wave, the D-wave, and the F-wave phase shifts, explicitly from the experimental angular distributions and to determine their limits compatible with experimental errors, which are estimated to be about 1 to 2 percent over most of the energy range covered by the experiment. If possible, it is desired to establish the minimum number and type of phase shifts required to fit any given angular distribution. The criterion of fit being used is to find the phase shifts which make the sum of the squares of the percentage errors in the cross section a minimum.

From these phase shifts it will be possible to determine the properties of the excited states of F^{17} within the region of excitation energies which the experiment covers and, in addition, to determine, within the limits of the experimental errors, whether or not a number of more subtle factors, such as the ground state of F^{17}, a low-lying bound state, and possible effects of core excitation, are influencing the scattering.

A preliminary program has been planned to obtain plots of the sum of the squares of the percentage errors in the cross section as a function of each phase shift over a range of -180° to $+180^{\circ}$. This is be ng done in order to study the behavior of the minimum. So far, two tapes have been successfully run, one giving the errors as a function of δ_0, the S-wave phase shift, and the other giving it as a function of δ_1, the P-wave phase shift.

163 FERRITE PHASE SHIFTERS IN RECTANGULAR WAVEGUIDE

The solution of the electromagnetic boundary-value problem dealing with a ferrite slab in a rectangular waveguide is being studied by K. J. Button of the Lincoln Laboratory at MIT. Four complete sets of solutions of the two simultaneous transcendental equations for the complex propagation constant, $\Gamma = \alpha + j\beta$, for both directions of propagation in the ferrite loaded X-band waveguide have been provided by Whirlwind I. Each set of solutions consisted of a continuous determination of the attenuation constant and phase constant for both directions of propagation as a function of the position of the ferrite in the waveguide for a constant value of the magnetic field of 2100 oersteds. A different value of magnetic field intensity was used for each set of solutions.

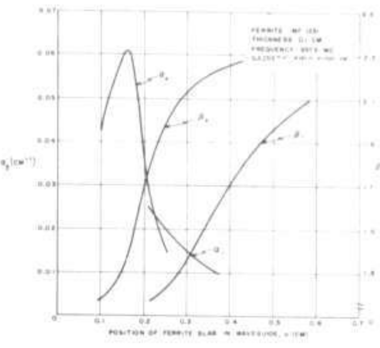

Fig. 2-4

The Attenuation Constant and Phase Constant for the Positive and Negative Directions of Propagation as a Function of Ferrite Slab Position

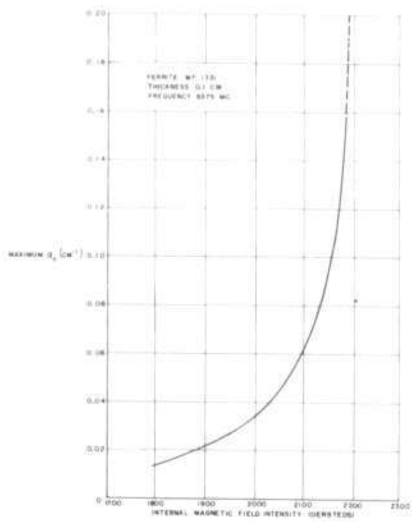

Fig. 2-5

Maximum Value of Attenuation Constant for Forward Propagation as a Function of Internal Magnetic Field Intensity

The dependence of the propagation constants upon the magnetic field intensity within the ferrite material has been determined in a region below ferromagnetic resonance. As the internal field was increased in an effort to look further into the resonance region, the peak value of the α_+ curve (see Fig. 2-4) increased sharply as expected. This indicated a strong absorption of the signal power by the ferrite material near the ferromagnetic-resonance condition. A plot of the maximum value of the attenuation constant for forward propagation is shown as a function of field intensity in Fig. 2-5.

At 2200 oersteds, the CS program was found to be incapable of providing values of α_+ larger than 0.2. Above this value the individual solutions of the two transcendental equations did not converge to a simultaneous solution. This indicates that the behavior of one or more of the transcendental factors as parameters are changed was not predicted properly. Several methods of analysis and approximation have been attempted without success. It has been established that an analysis involving extensive hand computation will be required in order to study the behavior of the rapidly changing transcendental factors which contribute to the difficulty in this restricted region.

Therefore, Digital Computer Laboratory time will not be requested during the next several months. Machine computation will not be resumed until the present theoretical findings have been investigated experimentally and the reasons for program failure have been determined by hand computation. Since the hand computation of a single point on curves similar to those shown in Fig. 2-4 normally requires two weeks of full-time effort, additional personnel must be assigned to assist the programmer. An effort is now being made to find suitable assistance.

166 CONSTRUCTION AND TESTING OF A DELTA-WING FLUTTER MODEL

The problem of designing, constructing, and testing a delta-wing flutter model to simulate a given set of flexibility influence coefficients was described in Summary Report No. 37 by M. M. Chen and S. I. Gravitz of the MIT Aeronautical Engineering Department.

With the completion of the building-block routines covering the necessary matrix algebra for the program, work was begun on the task of evolving and testing the complete program for the analysis and design of the lattice network. Eventually the basic program was debugged, and useful machine time was obtained to yield meaningful results. These results have been described by Gravitz in an S. M. thesis.

The basic program as it now stands is divided into various units, each one on a different tape, to facilitate modifications to the analysis and to reduce the total amount of tape read-in time. The units and their functions are as follows:

1. The first unit contains the input data for the physical system. Specifically the given influence coefficients, $[C]$, and the first trial bending and torsional stiffness, $\{X^{(1)}\}$, are placed in secondary storage for use in the computations to follow.

2. The geometry of the lattice network, the $[a]$'s and the $[\Delta a]$'s is similarly stored for future use of the second unit.

3. The third unit is a long computation tape composed of several previously tested tapes spliced together to perform the calculations corresponding to the errors in the influence coefficients, $[\Delta c]$. At this point in the program it is known whether the trial stiffness estimates are valid.

4. If the influence-coefficient differences are not negligible, it is necessary to compute corrections to the initial trial-stiffness values. This phase of the program does this for the i^{th} trial and prepares the $(i + 1)^{th}$ stiffness estimate:

$$\{X^{(i)}\} + \{\Delta X^{(i)}\} = \{X^{(i+1)}\}$$

When the basic program was completed it was possible to utilize it to learn more about the physical aspects of the lattice-network design problem. On the basis of the runs that were made, the following conclusions were made evident:

1. Using simplifying assumptions, having perhaps questionable physical validity, it is possible to quickly and easily arrive at first trial stiffnesses which yield influence coefficients which match the given coefficients to well within 100 per cent on the average.

2. When it is necessary to eliminate an unknown stiffness, it is more satisfactory, mathematically, to assume a reasonable member cross section, thus defining the ratio of torsional to bending stiffness, rather than to equate two unknown stiffnesses. Although both types of assumption are reasonable from the physical point of view, the latter type leads to ill conditioning of the final matrix used to evaluate the X's.

The agenda for the immediate future will include constructing and testing a three-bay lattice structure of the physical dimensions incorporated in the present program. By co-ordinating the testing between the computer and structures laboratory phases, it is expected that a satisfactory conclusion to the problem can be obtained.

2.3 Tape-Preparation Statistics

During the past six months time records have been kept on all punched paper tapes prepared for Whirlwind I. These records have distinguished among three types of tapes because of the difference in procedures involved.

1. Original complete tapes are prepared directly on standard Flexowriter equipment from a coded manuscript submitted by the programmers.

In order to catch undetected mistakes the tape is verified by having a second person type the manuscript independently on a verifier. As the tape is typed, the first tape is compared line by line with the newly punched one. Disagreements in tapes will cause the verifier keyboard to lock while the typist determines which tape is correct.

Library subroutines can be inserted anywhere in the program by reproducing the subroutine tape onto the program tape. A comparator is used to detect errors in the duplication by comparing the original tape with the copy. If any difference is detected the comparator will stop.

When the checking has been completed the tape is fed back through the Flexowriter, producing two typed copies of the program. One copy is filed, and the other is returned with the manuscript to the programmer.

If the programmer desires to have his tape in the more compact binary (556) form, he need only request this. The standard Flexowriter tape is then converted by the computer to the binary form. However, it is not necessary to carry out this conversion since the Flexowrtier tape can be read directly into the computer.

2. Typed modifications are prepared when the programmer desires to change 11 or more registers in a binary tape or when he desires to make a change in a Flexowriter tape. The modifications for the binary tape are typed, verified, converted, and then attached by splicing to the main program. The modification to the Flexowriter tape is accomplished by duplicating the main tape, inserting or substituting the changes where desired, and finally checking the duplicated portions of the tape by means of the comparator.

3. Manual modifications are changes that are punched directly in binary (556) form and attached to the converted tape.

The following conclusions were reached:

a. An original tape requires 0.18 minute per register for preparation and is approximately 216 registers long.

b. A typed modification requires 1.06 minutes per register and is approximately 30 registers long.

c. The average length of a manual modification is 5 registers and requires 1.64 minutes per register.

d. It has been observed that the minutes per register required for the preparation of an original tape vary inversely with the number of registers in the tape.

e. The controlling factor in a typed modification of a Flexowriter tape is the length of the tape in which the changes are to be made rather than the number of changes.

f. Manual modifications consume more tape-preparation time per register. However, once done, no further time is needed for conversion, and valuable computer time is conserved.

167 TRANSIENT EFFECTS IN DISTILLATION

Work has continued under the direction of J. F. O'Donnell of the MIT Chemical Engineering Department on three basic problems which have been mentioned in previous reports. These problems all involve transient effects of holdup in binary distillation. They are: product takeoff in batch distillation, equilibration in batch distillation, and transients in continuous distillation.

For each case two types of programs have been written for Whirlwind. The first program finds the set of steady-state conditions (initial or final) for a specific set of parameters. This involves solving a set of simultaneous nonlinear algebraic equations and usually takes about one minute of machine time. The second program determines the variation of the compositions in the system as a function of time for a set of parameters during a transient period. These programs require the solution of sets of simultaneous nonlinear differential equations and usually have taken from five to twenty minutes of Whirlwind time each.

A decision was made to solve the differential equations by a Runge-Kutta method. When some trials were made using a fourth-order subroutine, they showed that the maximum permissible interval size was limited by instability rather than by convergence problems. Consequently, solutions were tried using a second-order routine and were found to give adequate results. This method was used henceforth.

The above-described programs are working satisfactorily and have been used to obtain considerable data. Much valuable information has been gained by studying the data, and the study is continuing. Sufficient data is now on hand for some phases, and further data will be taken where they are necessary.

In most cases qualitative relationships have already been established. Now the emphasis is on obtaining quantitative relationships for use by designers. For the case of product takeoff in batch distillation this has been difficult. At present a supplementary program is being written for Whirlwind which will calculate the product composition as a function of time for any set of parameters when the holdup in the system is taken to be zero. This is a limiting case which the present program will not solve. Comparisons of results with holdup to results without holdup, other conditions being the same, appear promising. Again a Runge-Kutta integration will be done of the nonlinear differential equation.

For the case of equilibration in batch distillation much of the work thus far has been done by Myers and has been described in detail in his thesis.[1]

1. Myers, H. J., "Equilibration Time in a Batch Distillation System," S. M. Thesis, Department of Chemical Engineering, MIT (1954).

For the case of transients in continuous distillation much of the work thus far has been done by Polk and Smith and has been described in their thesis.[1] Jordan is doing a Master's thesis this summer in a continuation of the latter work.

Work will continue on the over-all project. Final results will be included in a Ph. D. thesis to be submitted by J. F. O'Donnell to the MIT Chemical Engineering Department.

168 INDICIAL DOWNWASH BEHIND A TWO-DIMENSIONAL WING

The downwash at the tail of an airplane caused by the lift response of the wing to a sharp-edged gust is being investigated by N. P. Hobbs of the MIT Aeronautical Engineering Department. The integrals to be evaluated were described in Summary Report No. 37.

During the last three months, the several integrals required for the determination of the indicial downwash have been evaluated. The indicial-downwash solution, i.e., the downwash corresponding to the Wagner indicial-lift case, has been completed for four downstream observation points. The remaining problem is to utilize the indicial-downwash solutions to obtain by numerical integration the downwash for immersion speeds of the airfoil other than infinity.

169 UTILIZING A GENERAL-PURPOSE DIGITAL COMPUTER IN SWITCHING-CIRCUIT DESIGN

This problem will be covered in detail in an S.M. thesis to be submitted by E. C. Hoy to the MIT Electrical Engineering Department before September 1954.

Although the underlying motivating question of the thesis concerns the applicability of a general-purpose digital computer to switching-circuit design, the specific problem is obtaining the (or a) minimal[2] sum-of-products form of a prescribed switching function. The solution to this problem will aid in constructing the most economical realizations of two-terminal networks. As two-terminal networks are not only among the most frequently used switching circuits but also form the building blocks for the more complex networks, the solution to the aforementioned specific problem will have a wide field of application.

Input and output routines have been written which permit the specification of the desired function in conventional, standard notation and the printout of the solution in a conventional switching-circuit symbology.

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1. Polk, J. G., and Smith, A. T., "Behavior of Continuous Distillation Columns Under Transient Operating Conditions," S. M. Thesis, Department of Chemical Engineering, MIT (1954).

2. i.e., one with the fewest possible variable appearances or one for which no equivalent can be found with fewer variable appearances.

The reduction of the function is performed by the following rules (note: all operations and variables are Boolean; let x be a "single variable"[1] and A and B be the products of any number of single variables):

1. $AX + AX' = A$
2. $XB + X'AB = XB + AB$
3. $A + AB = A$ (this includes $A + A = A$)
4.[2] $X\underbrace{(\text{all or any part of A})}_{f_1\{A\}} + X'\underbrace{(\text{all or any part of A})}_{F_2\{A\}} + A = X f_1\{A\}) + X' (f_2\{A\})$

Note that an appearance of X indicates a lack of generality of the rule involved. Whether or not this is a major shortcoming remains to be seen. Spatial and temporal sequence applications of the rules have been found to be important; these points must be investigated further. The present system of applying the first three rules in order (1, 2, and 3) to each pair of terms and then applying rule 4 has led to satisfactory solutions in trial runs but is not always the best answer to the problem. It does not lead to the (or a) minimum solution for each arbitrary function. Several ideas for obtaining solutions that can be proved to be minimal in every case have been tried, and a fresh approach is being sought. If a more suitable solution is found, it will be programmed.

170 INVERSE AND INVERSE SQUARE ROOT OF A REAL SYMMETRIC MATRIX

A routine has been written by Dr. A. Meckler of the Solid State and Molecular Theory Group for the library of subroutines which calculates the inverse or the square root of the inverse of a real symmetric matrix. The routine has as its core the matrix-diagonalization routine developed under Problem 134. The given matrix is first diagonalized, the eigenvalues remaining in fast storage, the transformation matrix on a drum group. The reciprocals or the square roots of the reciprocals are then taken, and this diagonal matrix is then undiagonalized by the transposed use of the original transforming matrix.

The parameters to be planted in the routine are the order of the matrix, the starting address of the matrix, the criterion for diagonalization, and a drum group. This preparation is exactly like that necessary for straight diagonalization. The order of the matrix is again restricted to be less than or equal to 32 because of drum-group capacity.

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1. i.e., one of the relay-operation variables that form the group of independent variables for the E. C. Hoy prescribed switching function.

2. "All or any part of A" is to mean a product of any number of the single variables composing A (the product of no variable to be construed as "1"), e.g., if A is xyz, a product of single variables, a part of A would by xy; this explanation is necessary because when considering the origins of xyz and xy, xy actually includes xyz.

171 IMPROVED POWER SPECTRUM ESTIMATES

A major problem which arises in the calculation of power spectra for frequency analysis of an empirical function is the step of Fourier transformation of the autocorrelation function to give the power spectrum. (See Problem 107.) The use of a finite rather than infinite integral in the definition of the Fourier transform introduces a large amount of spurious ripple (Gibbs's phenomenon) which is superimposed upon the correct answer and makes reasonable interpretation of results extremely difficult. Problem 171 is concerned with the realization, as a WWI computer program, of techniques for minimizing this difficulty, derived by D. T. Ross in partial fulfillment of the requirements for a Master's Degree in the Department of Electrical Engineering. The program which has been written calculates a series of spectra, each closer to the correct spectrum than the previous one. The program then plots calibrated graphs of these functions on the scope along with a graph of the given function and a "Measure of Indecision" curve which shows, as a function of frequency, the relative confidence which can be placed in the series of spectra.

Using an auxiliary program which generates test functions with known spectra, ten complete runs were made to obtain five, ten, or fifteen spectra in each case. One run used a sample autocorrelation function of a certain type of radar noise previously obtained in Problem 107. Results were good, and much was learned about the capabilities and limitations of the method. The thesis report is being published as Servomechanisms Laboratory Report No. 7138-R-5, "Improved Computational Techniques for Fourier Transformation," by Douglas T. Ross, June 25, 1954. Problem 171 will be reopened sometime in the future in order to test some further techniques for improved frequency resolution as proposed in the thesis report.

172 OVERLAP INTEGRALS OF MOLECULAR AND CRYSTAL PHYSICS

In order to evaluate certain types of two-center integrals arising in molecular and solid-state physics, it is convenient to express the wave functions and potentials as linear combinations of analytic Slater atomic orbitals. When this has been done, the resultant integrations can be evaluated by means of complicated but systematic formulas. In order to mechanize this procedure, several programs have been written for the Whirlwind I computer by F. J. Corbató of the MIT Physics Department. In addition, it was found possible to devise a program which fits linear combinations of Slater atomic orbitals to tabulated Hartree-Foch wave functions.

The calculational programs consist of a "basic" subroutine for evaluating overlap integrals, several "mixer" routines which combine overlap integrals into related integrals, and "driver" routines which direct and control the calculations according to the specifications of the various wave functions and potentials involved. These programs have been written, tested, and combined along with an automatic "assembly" routine into one main tape. In operation, a toggle-switch setting allows any of six types of integrals to be obtained from the one tape.

At present these programs are being used to make calculations involved in the author's thesis in which the energy bands of graphite are being investigated by means of the tight-binding method.

173 COURSE 6.537 -- DIGITAL COMPUTER APPLICATION PRACTICE

Twelve students enrolled in the second semester Electrical Engineering course 6.537, entitled "Digital Computer Applications Practice," given by Professor C. W. Adams. The purpose of this course was to study the advanced preparation of coded programs for automatic electronic digital computers, particularly for Whirlwind I.

Each student programmed, prepared on punched tape, and executed on Whirlwind one problem of his own choosing, making use of CS II. The problems undertaken included: the simultaneous solution of a set of linear algebraic equations by the Crout method, the computation of a double integral of a probability equation, the solution of a delay-line spurious-noise problem, the formation of the Fourier components for discrete data taken over a period, the computation of the time of occurrence and the location of a quake by a least-square fit of empirical distance time curve, the finding of the roots of a polynomial by Newton's method, the solution of a set of linear equations by the Gauss-Jordan reduction method, the solution of a matrix equation by the Crout method, and the programming of an arc tangent subroutine.

174 TIGHT-BINDING CALCULATIONS IN CRYSTALS

The unperturbed wave function for a crystal is approximated by a linear combination of atomic functions. A Hamiltonian matrix is set up between these functions; the eigenvalues of this matrix are the energy levels of the electrons. The diagonalization of the Hamiltonian would be done on WWI.

A program has been written which computes the Hamiltonian matrix in the machine for the calculation of the energy levels of the electrons in nickel. Each element in the Hamiltonian matrix is a trigonometric polynomial of low order. After calculating the matrix the matrix is then diagonalized. This program has been tested, and the complete calculation of the energy levels of nickel is completed within this approximation of tight binding. The results are now being studied by Dr. G. Koster of the MIT Solid State and Molecular Theory Group.

175 IMPURITY LEVELS IN CRYSTALS

The calculation of impurity levels in crystals involves the solution of difference equations which are solved by a Green's function method. This involves finding the inverse of matrices (H-E) where H is the unperturbed Hamiltonian and E is the energy of the impurity level. The inversion is being done on Whirlwind I.

A program has been written by Dr. G. Koster of the MIT Solid State and Molecular Theory Group which inverts the matrix indicated above. The inversion is done for a succession of E values for a given matrix H. The program has been tested and is now being used to calculate the impurity energy levels in chromium. This involves the inversion of 5 by 5 matrices. The machine carries out the inversion of 160 matrices in roughly 25 minutes.

176 CONNECTOR PROVISION IN AUTOMATIC TELEPHONE EXCHANGES

The general traffic problem consists of determining the relationship between A, the amount of traffic offered to a number of channels, N, the number of channels available, and B, the probability of an item of traffic failing to secure a channel because of the limited number of channels available.

The traffic considered may be telephone calls, customers at a store, automobiles, visitors at an exhibition, telegrams to be sent over a communications system, etc. The corresponding channels might be switching circuits, shop attendants, lanes in a highway, turnstiles at an entrance, or bandwidths of the system.

Many solutions to the problem have been obtained under varying sets of assumptions, but none is satisfactory for the case of connectors in step-by-step automatic telephone exchanges. All solutions available tend to overestimate requirements by amounts usually varying between 5 and 20 per cent. The most accurate traffic tables available are based on the binomial formula and were first applied in 1905 by E. C. Molina.

Because of the rapid development in electronics, and hence in toll service, most of the recent work on telephone-traffic theories has been devoted to the problem of lost calls delayed. However, the connector circuit is still a vital link in automatic telephone connection, and, inexpensive as it may be by comparison with the toll circuit, its importance lies in the large number of connector groups in operation. (There are approximately a quarter of a million groups in use in the United States alone, with each circuit costing approximately $100.)

It is believed that one of the main reasons for the overestimation of connector circuits, as indicated by existing traffic tables, is that none of the theories on which they are based makes allowance for sources also being available as sinks. So far as can be ascer-

tained, no endeavor has been made to allow for this interaction of sources and sinks. Consideration of the interaction of sources and sinks produces the following equations.

$$k P_k = \frac{p}{1 - p} P_{k-1} \sum_{w=0}^{S-k} (S - w - k + 1) Q_w \qquad (1a)$$

(k = 1, 2, 3. . . . , S)

$$\sum_{w=0}^{S} P_w = 1 \qquad (1b)$$

where P_w is the probability that exactly w circuits will be simultaneously engaged as sinks,

S is the number of terminals which may serve either as a source or sink,

p is the probability of any given terminal being in use as a scurce at any given instant (assumed constant for all terminals),

and Q_w is the probability that exactly w circuits will be simultaneously engaged as sources (assumed equal to P_w).

From solutions to the above equations it is then required to find the realtionship between A = pS and N for a given B in the following equation.

$$B = \frac{1}{S(1 - p)} \sum_{k=N}^{S-1} \sum_{w=0}^{S-k-1} (S - k - w) Q_w P_k \qquad (2)$$

A simple iterative procedure in which each P_k was calculated in turn as a function of the best available approximation of all the other P_k was first utilized. Although the method converged for small values of p, it diverged for large values of p.

A second procedure in which Equations (1a) and (1b) were reduced to a set of linear simultaneous equations was then devised. These equations were then solved by conventional means, and iteration of the process was found to converge over all values of p required.

A study has been completed using this second method over the following ranges of values:

Equation (1) p = 0.02 to 0.20
S = 25, 50, 100, 200.
Equation (2) N = Sp, Sp + 1, Sp + 2, . . . until $B \leq 0.001$.

Allowance for a typical distribution of calling rates produces the following equations:

$$P_k = \frac{1}{k} \cdot \frac{p}{1-p} \cdot P_{k-1} \sum_{w=0}^{S-1} (S - w - k + 1)\, Q_w \left(1 - \frac{p}{1-p} \cdot \frac{w+k-1}{S-1}\right) \tag{3a}$$

$$\sum_{k=0}^{S} P_k = 1 \tag{3b}$$

$$B(N, S, p) = \frac{1}{S(1-p)} \sum_{k=N}^{S-1} \sum_{w=0}^{S-k-1} (S - k - w) Q_w P_k \left(1 - \frac{p}{1-p} \cdot \frac{k+w}{S-1}\right) \tag{4}$$

These equations have been solved for $S = 25$ and $p = 0.02(0.01)\,0.20$. Results differ so little from those given by the original set of equations that it is clear that the effect of the distribution of calling rates may be safely neglected.

All of the results for this problem have been included in an S. M. thesis submitted by B. Marrows to the MIT Electrical Engineering Department.

177 LOW ASPECT RATIO FLUTTER

This problem involves the determination of the lowest speed at which self-excited oscillations of an elastic structure (aircraft wing) in an airstream can occur. This requires a means of specifying the pressure distribution on an arbitrary planform for arbitrary motions of the surface, followed by solution of the equations of motion to determine the velocity-frequency pairs (double eigenvalues) for which the oscillation can occur.

The former problem has been reduced analytically to an expression involving the integral of the unknown pressures. This cannot be inverted analytically but has been reduced to a matrix equation by numerical integration. The resulting matrix equation must be inverted.

The flutter problem, also initially an integral equation, is similarly reduced to a matrix equation. Since it is an eigenvalue problem in two parameters and not explicit in both, the solution is necessarily by trial and error.

As with any numerical-integration scheme, the accuracy is improved by refining the interval; hence the matrix equations are necessarily of high order.

The coding of the problem for solution of simultaneous equations for arbitrary rank up to and including 26 has been completed and tested. As a result it has been possible to determine the pressure distribution on a flat plate of aspect ratio unity at steady angle of attack. The results indicate that both the theoretical approach and machine solution are satisfactory.

Several similar examples are now in preparation. It is intended to extend the available rank of the present program, now limited by high-speed storage space, then to program the complex eigenvalue problem for the machine.

This study is being conducted by H. M. Voss of the MIT Aero-Elastic and Structures Research Laboratory.

178 TRAJECTORY STUDY AGAINST AN EVADING TARGET

This problem has been concerned with a trajectory study of a missile seeking an evading target. It was desired to determine the minimum passing distance of these two objects for a variety of missile velocities, target velocities, and evasive courses. For each case it was assumed that the target evaded by flying a circular course instead of a predicted linear one. The procedure was to compute the distances between the missile and the target with reference to a certain coordinate system for some initial guess at the independent variable time. The rate of change of the distance was calculated. When this rate came sufficiently close to zero the corresponding value of the distance was considered to be the minimum passing distance.

The precedure was coded by C. Block of the MIT Instrumentation Laboratory. About 2000 runs were made and the results have been presented in tabular and graphic forms in Instrumentation Laboratory Report T-50. This report has been submitted as an M. S. thesis by R. Hansen and R. B. Doane to the Electrical Engineering Department.

179 TRANSIENT TEMPERATURE OF A BOX-TYPE BEAM

The transient temperature response of a box beam due to a time varying heat-flux input on one flange has been calculated by L. A. Schmit of the MIT Aero-Elastic and Structures Research Laboratory making use of an uncoupled finite-difference procedure. The system was treated as two dimensional (of unit length). Reradiation and convection losses were neglected, and the thermal properties of the structure were assumed constant. The calculated results are being compared with experimental test data.

As a result of this study two factors affecting the temperature distribution in this type problem come ot light.

1. The absorptivity of the irradiated surface as measured in the laboratory prior to field test may be considerably (as much as 30 per cent) different from the absorptivity of the exposed surface at the time of the field test. The full explanation of this substantial difference is not known, but a partial explanation is thought to lie in the accumulation on the exposed surface of highly reflective sand and dust between the time of laboratory measurements of absorptivity and the time of exposure in the field test. This difficulty is overcome by calculating the effective absorptivity from calorimeter and temperature data obtained during the field test. The final calculations (Tape 179-86-8) are based on a calculated

effective absorptivity.

2. The detail of the web-flange joint, particularly the use of steel screws in an aluminum structure, can be shown to produce large increases in joint contact pressure with increasing mean-joint temperature. As a result of this effect much higher values of the contact admittance parameter than were originally thought possible are used in the final calculations (Tape 179-86-8).

The final correlation obtained is not entirely satisfactory. However, there are indications that the major remaining cause of discrepancy is caused by reflected radiation input on surfaces other than the exposed flange. This difficulty can be overcome by adequately shielding all surfaces except the exposed one in future tests.

180 CROSSCORRELATION OF BLAST-FURNACE INPUT-OUTPUT DATA

This problem was undertaken by R. G. Mills of the MIT Electrical Engineering Department in connection with his S. B. thesis.

The original objective of this problem was to compute the Fourier transform of the crosscorrelation function relating blast-furnace input and output information for the purpose of evaluating a proposed technique for analyzing complex production processes. Two collateral objectives have arisen in the course of carrying out this computation; a sine-cosine subroutine for the summer session computer was needed, and a combined program, which would carry out all phases of the complex computation involved in correlating a function (or functions) and transforming the resulting correlation function, was also desired.

The subroutine was written and tested; it has been made available to the library. The magnitude of the argument is reduced to less than 2π before the solution of the series is carried out, thus reducing the error.

The portion of the project relating to the thesis was successfully carried out, and analysis of the resulting information showed that the method employed was a valid and useful one. In the interest of simplified programming, since time was short, the correlation program was originally written using the summer session computer. The desired results were obtained with a minimum of programming difficulty, but the computer and tape-preparation time required for correlating functions having more than about 120 values was prohibitive because of (1) the inherent slowness of the summer session computer, and (2) the lack of storage capacity of SS (300 registers), which necessitated breaking the longer functions up into several parts, computing each separately, and correctly combining the results.

The Fourier transformation was carried out using a program in CS II originally written by D. R. Ross and modified by him for the purposes of this problem. The combined use of the SS correlation program and the CS II transform program gave results which

were entirely satisfactory for the purposes of the thesis, but, because of the several stages of intermediate hand computation and tape preparation which were required, such a system is valueless for production work and does not make efficient use of computer time. Required output time, in particular, is excessive.

In order to render the program more generally useful, the correlation portion has been rewritten in CS II and combined with the transform portion in such a way that tape preparation is greatly simplified, and computer and output time is minimized. The program handles functions having up to 250 values; it may be used whenever it is desired to cross-correlate two functions and evaluate the cosine and sine transforms of the resulting function or to autocorrelate a single function and transform the result (since only the cosine transform is required in this case, the program may be stopped after this has been obtained, thus reducing the required time further).

Output is obtained in the form of oscilloscope plots of (1) the even part of the correlation function, (2) the measure of indecision of the cosine transform, and (3) & (4) the cosine transform of the even part of the correlation function <u>before</u> (3) and <u>after</u> (4) a smoothing process which is written into the transform program. When the sine transform is desired four additional frames are plotted, similar to the above, except (1) shows the <u>odd</u> part, and (2), (3), and (4) involve the sine transform of the <u>odd</u> part of the function. The scope and camera are the only output devices used by the program, which considerably simplifies operation, especially when it is run with other programs which make use of the delayed-output equipment (magnetic tape).

181 PERTURBED COULOMB WAVE FUNCTIONS

Nuclear scattering in general remains the most direct way to get information about nuclear forces. Of particular interest is the comparison between neutron-deuteron scattering and proton-deuteron scattering, because it may be assumed that the forces involved in the two cases differ only by the addition of an electrostatic interaction between the charged proton and deuteron. The validity of this assumption requires that the specifically nuclear force between two protons be the same as that between two neutrons, and its proof or disproof is of basic interest.

An analysis of the nuclear-deuteron scattering data requires knowledge of the scattering that would result with only an electrostatic interaction between proton and deuteron. In the case of two point charges, this is given by the so-called Coulomb wave functions, tabulated in some detail by the U.S. Bureau of Standards. For proton-deuteron scattering, the charge distribution of the deuteron cannot be considered at a point, and the corresponding functions must come directly from the linear second-order differential equation by a numerical solution.

With these functions at hand, one can use the neutron-deuteron scattering data to predict proton-deuteron scattering. Sufficient data of both kinds have been taken in this laboratory to make a significant comparison.

A test of this so-called charge-symmetry of nuclear forces may be obtained by comparing p-D and n-D scattering. To do this, one compares the experimental p-D phase shift, δ, with that calculated from

$$\frac{\phi'(b)}{\phi(b)} = \frac{G'(b) + \cot\delta\, F'(b)}{G(b) + \cot\delta\, F(b)}$$

where G(r) and F(r) are the regular and irregular solutions of the p-D wave equation

$$\frac{d^2\phi}{dr^2} + \left[k^2 - \frac{1 - e^{-\frac{2r}{a}}}{Rr}\right]\phi = 0$$

k = wave no.
R = protonic bhor radius
a = scattering length

The numerical solution of this differential equation has been carried out on WWI by the following procedure for R. Zimmerman of the MIT Nuclear Science Laboratory.

The solution is started by a power series.

$$\phi = \sum_{n=o}^{\infty} bnr^n$$

Regular Sol. $\begin{cases} b_o = 0 \\ b_1 = 1 \end{cases}$

Irregular Sol. $\begin{cases} b_o = 1 \\ b_1 = 0 \end{cases}$

$$b_n = \frac{-k^2 b_{n-2}}{n(n-1)} - \frac{1}{R_{n(n-1)}}\left[(-\frac{2}{a})b_{n-2} + (-\frac{2}{a})^2 \frac{bn-3}{2!} + \cdots + (-\frac{2}{a})^{m-2}\frac{1}{(n-2)!} + (-\frac{2}{a})^{m-1}\frac{b_o}{(n-1)!}\right]$$

The function is then generated by the Manning-Millman method.

$$\phi\, n+1 = \frac{2\phi_n - \phi_{n-1} + \frac{h^2}{12}\left[10\phi''_n + \phi''_{n-1}\right]}{1 + \frac{h^2}{12} f_{n+1}(x)}$$

$$\phi'' = f(x)\,\phi$$

$$f(x) = \left[k^2 - \frac{1 - e^{-\frac{2r}{a}}}{Rr}\right]$$

The regular and irregular solutions have been done successfully for $0 \leq r \leq 3$ at intervals of 0.05, for a = 0.216, $R_1 = 2.147$ and $R_2 = 0.447$, and $k_1^2 = 0.2158$, $k_2^2 = 0.4316$, $k_3^2 = 0.6474$, $k_4^2 = 0.8632$, $k_5^2 = 1.0790$, $k_6^2 = 1.6185$, $k_7^2 = 2.1580$. This concludes the problem.

182 CRYSTAL STRUCTURES

In solving a crystal structure, the corrections to the atomic parameters are obtained by a least-squares method as a set of n simultaneous linear equations in n unknowns; n may be 30 or more. It is required to solve this system.

This problem is a continuation of Problem 105 to make use of routines developed by Dr. A. Meckler under Problem 134.

This problem involves the inversion of a 30th-order matrix with an eigenvalue ratio of about 1000 : 1. The only WWI routine currently available for the inversion of a matrix of this order proved to converge too slowly to be feasible. This routine is being rewritten in a form which will operate at least five times as rapidly as the former version. Consideration is being given to the development of other routines for handling matrices too large to be stored in rapid-access memory.

183 BLAST RESPONSE OF AIRCRAFT STRUCTURES

The present program is the first phase of a study by H. Lin of the MIT Aero-Elastic and Structures Research Laboratory attempting to establish a lethal criterion for airplane structures upon encountering a blast gust. For the purpose of examining some of the simplifying assumptions necessarily made in actual aircraft structures and of investigating the feasibility of a step-by-step solution in solving a nonlinear dynamic problem, this first phase treats the simple case of a cantilever, uniform, weightless beam subject to a triangular pulse at its tip mass. The structural characteristics of the beam are assumed to be such that its moment-curvature curve is bilinear. The resulting equation is a second-order nonlinear ordinary differential equation.

All quantities in the governing nonlinear equation have been nondimensionalized. The parameters of each case to be computed by WWI are:

K = buckled slope of the moment-curvature curve,
ϵ_y = yield curvature of the beam, and
τ_f = duration of triangular pulse.

The ultimate quantities desired from the present program are:

ϵ_a = maximum curvature at the root (the fixed end),
R' = ratio of maximum curvature to yield curvature, and
$Z_a \Delta\tau$ = time to reach maximum root curvature.

For a given beam of known ductility factor R defined by

$$R = \frac{\epsilon_c}{\epsilon_y},$$

where ϵ_c = failure curvature of the beam, the calculated value of R' will show that the beam will fail if

$$R \geq R.$$

A parameteric variation of K, ϵ_y, and τ_f will result in a plot of a family of curves from which a lethal criterion for this beam may be obtained under the aforementioned forcing function.

The numerical procedure employed in the step-by-step solution can be outlined as follows:

1. The velocity of the tip mass at any instant is extrapolated from the accelerations at previous three instants.

2. The displacement of the tip mass is then integrated from the velocity obtained above by Simpson's rule.

3. The net tip force necessary to produce this tip displacement is then solved by the Newton-Raphson iteration method from the cubic structural equation.

4. The acceleration of the tip mass at this instant is then determined from the equilibrium equation.

5. The root curvature at this instant is also calculated.

6. The cycle is repeated until a maximum value of root curvature is obtained.

7. To start the recurrent procedure outlined above, the necessary quantities at the initial three instants are determined by Legendre polynomials as described in the National Advisory Council on Aeronautics Report, NACA TN 2060.

8. When the beam is in its plastic range, the curvature is defined in terms of a special function, ψ, obtainable as the largest real positive root of a cubic equation. This root is found by the Newton-Raphson iteration procedure.

The formulation of the simplified case was programmed. Progress on this problem has been such that production runs have begun. Results are being consistently obtained to four significant figures.

184 SCATTERING OF ELECTRONS FROM HYDROGEN

This problem originated as a suggestion by Professor P. M. Morse that recently developed mathematical variational techniques, together with machine methods of computation, could be used to solve certain heretofore incompletely treated problems in atomic scattering of electrons from hydrogen, taking into account excitation and exchange, falls into that category. Part of this problem, which forms M. C. Newstein's Ph. D. thesis to be submitted to the MIT Physics Department, has been to construct a stationary expression for the scattering amplitude and to study the multidimensional integrals that are involved. These have been reduced to two-dimensional integrals which may be solved to the required

accuracy on WWI for certain trial functions. If the calculation is successful, it is expected that the method will prove important in the calculation of the cross sections for scattering from more complicated atomic and nuclear systems.

An expression for the differential amplitude for the scattering of electrons from hydrogen atoms has been devised and has the property of being stationary with respect to variations in the wave function. The integrals in this expression have been reduced, for a given trial wave function, to a two-dimensional form. These two-dimensional integrals are to be evaluated by Simpson's rule for various values of angle of scattering and incident electron energy.

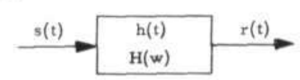
Programs have been written and tested for various forms of the integrand. A program to evaluate a two-dimensional integral, for a general integrand, has been written and tested for various mesh sizes.

186 TRACKING RESPONSE CHARACTERISTICS OF THE HUMAN OPERATOR

In many control systems (fire control, missile control, aircraft flight controls, etc.) the human operator is an important component of the system. To design such systems for optimum performance we must know the characteristics for all important components including the human operator.

The object of this investigation by J. I. Elkind of the Lincoln Laboratory at MIT is to measure the system characteristics of the human operator. To do this we determine the best linear representation for the human operator as well as try to measure his nonlinear characteristics. It is known that the human operator's system characteristics will change when some of the parameters of the control system of which he is a part are varied. His system characteristics tend to vary with time and also have been shown to be a function of the display, input signals, and output devices used. It is planned to explore these variations so as to be able to predict how the human operator will behave in a particular situation. Such information should be of considerable aid in the design of nonmachine systems.

In the experimental part of this study s randomly moving target is displayed, and the subject is asked to move a control to track the target movements to the best of his ability. In the simplified case we can represent the human operator by a linear filter with impulse response h(t). This simulation is shown in the figure below. The input stimulus is s(t); the operator's response is r(t).



Several relations are immediately apparent:

$$r(t) = \int_{-\infty}^{\infty} h(\sigma)s(t - \sigma)d\sigma$$

$$\phi_{sr}(\tau) = \int_{-\infty}^{\infty} h(\sigma)\,\phi_{ss}(\tau - \sigma)d\sigma$$

$$\Phi_{sr}(w) = H(w)\,\Phi_{ss}(w)$$

where $\phi_{sr}(\tau)$ is the crosscorrelation between stimulus and response, $\phi_{ss}(\tau)$ is the autocorrelation function of the stimulus, and $\Phi_{ss}(w)$ and $\Phi_{sr}(w)$ are the corresponding power-density functions. WWI will be used to obtain an approximate solution for h(t) from the convolution integrals and will also be used to Fourier transform the correlation functions in order to obtain the power-density functions.

To date one set of correlation functions has been Fourier transformed, and a transfer function H(w) has been computed for one test on one subject. However, the results have not been completely analyzed.

187 RESPONSE OF A FUEL-FLOW CONTROLLER

A system of first-order nonlinear ordinary differential equations describing the response of a fule-flow controller was solved on WWI making use of CS II by M. Merwin of the MIT Dynamic Analysis and Control Laboratory. The solution did not prove to be physically realistic so a second problem was then solved using the same equations with different constants. This second solution proved to be satisfactory. The Gill method, a variation of the fourth-order Runge-Kutta technique, was used.

The system was the following:

$$\dot{y} = c_1 y + c_2$$

$$\ddot{x}_3 = - c_3\bar{k}v - c_4\dot{x}_3 - c_5 x_3 - c_6\omega_2 + c_7$$

$$\dot{x}_4 = c_8 x_3 \begin{cases} \sqrt{\omega_1 - \omega_2} \text{ if } x_3 > 0 \\ \sqrt{\omega_2} \text{ if } x_3 < 0 \end{cases}$$

$$\dot{z} = c_9\left[- z + c_{10}\right]$$

where

$\bar{k} = f(z)$

$\omega_1 =$ const.

ω_2 is a quadratic function of x_4

188 EFFECT OF GRAVITY ON RELATIVE WATER PRODUCTION IN OIL RESERVOIRS

In a large number of oil reservoirs water, which underlies the oil, is produced along with the oil. The relative amount of water production depends for a given geometrical configuration on the effect of gravitational forces relative to the imposed pressure forces. In order to enhance the effect of gravity, the common practice is to plug the bottom part of the well bore so that fluid is produced from near the top of the formation; in some instances the imposed pressure forces are held to a minimum.

This problem has been undertaken in an effort to gain quantitative information concerning the effect on relative water production of partially penetrating wells and reduced pressure forces.

The interesting feature of this problem is the demarcation surface separating the oil and the water. This is a free surface in the sense that its location is not known beforehand but must be determined in the course of the computation.

The pressure p in either the oil or water zones obeys the following equation:

$$\frac{1}{r}\,\frac{\partial}{\partial r}\left\{r\,\frac{\partial p}{\partial r}\right\} + \frac{\partial^2 p}{\partial z^2} = 0 \qquad (1)$$

where r is the radial distance from the center of the well bore and z is the vertical depth below the top of the reservoir.

Since no fluid moves across the upper and lower boundaries of the reservoir nor across the cylindrical surface where the well bore is plugged, simple boundary conditions in terms of the normal derivative of p are obtained at these boundaries. At the well bore and the outer radius of the formation the pressure is a predetermined function of depth. At the interface between the oil and the water the pressure in the water zone must equal the pressure in the oil zone; furthermore, since no fluid moves across the interface, the normal derivatives of water and oil pressures are simply related.

The proposed method of solution is to assume a fixed interface position and solve Equation (1) for the water and oil zones independently. The interface position and the pressure difference between the well bore and the outer radial boundary are then adjusted so that water and oil pressures at the interface are equal in the least-squares sense. The procedure is then repeated with the new interface position and new boundary pressures.

In the actual calculation the procedure has been simplified by making a linear transformation on pressure; a logarithmic transformation has been made on r to increase accuracy near the well bore.

Preliminary results have indicated that the logarithmic transformation greatly decreases the rate of convergence of the numerical solution of Equation (1); furthermore, it appears that Equation (1) must be solved with a great amount of precision before the proposed adjustment of the interface is stable.

For this reason an effort will be made to improve the procedure for adjusting the interface making use of physical concepts and future results.

This study is being carried out by L. Kern of the Atlantic Refining Company.

192 SIMPSON'S RULE AND TUKEY'S METHOD OF SMOOTHING

This problem, being carried out by W. Walsh of the MIT Geology and Geophysics Department, consists of two sections.

In the first section it is desired to determine the contribution of various specified frequencies to the reflected and refracted energy recorded on prospecting records. It is hoped that this knowledge coupled with information of the noise spectrum will give rise to signal-to-noise ratios which will assist in the design of electronic and mathematical filters to be used in putting these reflections and refractions into a better relief in time.

In the second section -- as in the first -- it is desired to ascertain the spectrum (both phase and frequency) of the various direct, reflected, and refracted body waves recorded at a number of seismic stations situated on the continent of North America. It is hoped that if such information were displayed in time a great assistance would be rendered seismologists in the determination and "picking" of these phases. Furthermore, it is intended to apply similar analysis to the surface wave and "coda" section of earthquake records in the hope of determining the magnitude and presence of dispersion, which knowledge would in turn be instrumental in the determination of the type of continental and suboceanic layering.

In addition to the above, a frequency analysis of microseisms and the so-called T-Phase will be made. The intent here is to obtain knowledge which should be most instrumental in deciding the nature and origin of these phenomena.

During the period covered by this report, computation was confined to the calculation of traveling spectra of the various component seismograms of a T-Phase whose origin occurred on the north coast of the Dominican Republic. These spectra were plotted according to a density-plot routine. The results clearly show that this phase has dispersive characteristics peculiar to Rayleigh waves traveling at a ground water interface. Additional computation on other phases originating at different places must transpire, however, before definite conclusions can be made as to the nature of this phase.

In the following weeks it is intended to investigate by spectral analysis microseisms recorded at Weston, Mass., in the hope that additional proofs may be made concerning the

nature and origin of these waves. Thus far, analysis has shown them to be Rayleigh waves of high frequency generated by standing ocean waves.

For the past few weeks amplitude spectra of two earthquake seismograms were computed and plotted according to a density-plot routine as previously described. Both analyses displayed visually many of the reflected and refracted body waves as well as dispersive characteristics in the surface waves which have been predicted from theory.

In addition to this, several power spectra of microseism trains were computed. These computations afforded additional evidence for the origin and nature of microseisms and also indicated a resonance phenomenon existing when the origin of same has moved to deeper regions of the ocean.

In the future a similar analysis will be made on those waves which arrive first from the source of an earthquake.

3. OPERATION OF WHIRLWIND I

The Whirlwind Computer System reliability figure remained essentially the same as for the last 2 quarters. Reports by users show that for 90.8 per cent of the 1026 hours of assigned applications time the computer operated satisfactorily. Fig. 3-1 lists the division of computer time among the various major activities.

The systems group has continued its work of eliminating system weaknesses. Two major projects completed were replacement of power-distribution panels in order to substitute more reliable, plug-in-type relays and revision of the central d-c power-supply system to give better regulation on transients. The latter work has been in progress for about a year.

The magnetic-tape units are now maintained by the systems group. More flexibility has been provided in this tape system by the addition of a second delayed-printout typewriter.

The work on terminal equipment has included refinements on existing units to increase reliability and permit more effective maintenance. To facilitate maintenance and to increase the reliability of terminal equipment, progress has been made in improving the marginal-checking facilities, expanding the file of test programs, and adding checking and monitoring equipment.

Activity	Hours Per Week													
	April					May				June				Total Hours
	2-8	9-15	16-22	23-29	30-6	7-13	14-20	21-27	28-3	4-10	11-17	18-24	25-1	
Marginal Checking	5	4	4	4	4	4	4	4	4	4	4	4	4	53
Installation	8	8	0	8	8	9	7	7	0	8	8	8	8	87
Maintenance	23	27	15	22	25	15	22	24	5	25	21	19	24	267
Terminal Equipment Testing	29	31	31	28	23	30	28	32	29	32	32	41	33	399
Technician Instruction	0	0	2	2	2	2	2	0	0	2	2	4	2	20
Scientific and Engineering Computation	50	46	40	62	57	43	41	40	48	40	44	58	46	615
Other Applications	33	31	29	20	29	36	43	41	39	36	34	12	28	411
Total Hours	148	147	121	146	148	139	147	148	125	147	145	146	145	1852

Fig. 3-1. Allocation of Computer Time

3.1 Systems Engineering

3.1.1 System Performance

The computer performance record for this quarter was essentially the same as for the previous quarter. During the 1026 hours assigned to computer applications, operators' reports showed an average reliability figure of 90.8 per cent. Approximately 22 hours (or 25 per cent) of the time lost were traced to sudden tube failures. In one particular instance, an intermittent short in a buffer amplifier of clock-pulse control which provides synchronizing pulses for the input-output devices resulted in the loss of 12 hours. As a consequence, additional checking facilities have been subjoined to the input-output-control marginal-checking test program to provide a more comprehensive test for such failures.

3.1.2 Replacement of Power-Distribution Panels

Within the past quarter the entire complement of "P" Row power-distribution panels has been replaced with units of new design. The new panels contain plug-in-type relays that should be more reliable and should reduce maintenance time considerably. After installation was started it was discovered that approximately 5 per cent of the new relays were defective The wires connecting the relay internally with the octal base plug were improperly soldered. A thorough inspection of all relays of this type, therefore, was carried out to select satisfactory units. The manufacturer has been notified and has assured us that he will modify his soldering process and institute an adequate final inspection to eliminate the defects.

3.1.3 Magnetic-Tape Facilities

The magnetic-tape units functioned with improved reliability. A second delayed-printout system has been installed and debugged. All of this equipment can be combined in a very flexible manner. Three of the 5 magnetic-tape units can be used interchangeably with either printout system, and both systems can operate simultaneously.

A design change in the head circuits of the read-record switch and reading amplifiers decreased the voltage level on the head windings from 150 to 0 volts The result diminishes the probability of insulation breakdown in the read-record heads.

3.1.4 Power-Supply Modifications

All of the original WWI d-c power supplies have now been replaced or modified to improve their operation. The replacement program was started about a year ago as a result of a Master's thesis research by J. J. Gano on power-supply and regulator design. Some of the improvements over the old supplies are faster response to the transients of line voltage and load current, slower commutation to lengthen thyratron life, and more stable voltage reference.

The speed of response of each supply has been increased severalfold by eliminating all time delays except the output filter and applying servomechanism techniques to the closed loop. A narrow-band compensating circuit is synthesized to replace the long-time-delay anti-hunt network. With these changes, output-voltage excursions are reduced to approximately 20 per cent of their former value.

3.2 Input-Output System

3.2.1 Auxiliary-Drum System

The auxiliary-drum system, as originally received from the vendor, used one writing amplifier for each digit of a computer word. Each writer was shared by the 12 heads used for corresponding digits in the 12 groups of registers on the drum. Switching among the heads was accomplished by relays. This system had several drawbacks:

1. The relays were not covered, making them susceptible to dust and dirt. Space limitations made it impossible to install covered relays without a large amount of bay re-wiring.
2. The relay-operate time increased with use. After several months' operation it was necessary to increase the time delay counted by the computer to allow for the relay-switching operation. It is not known whether or not this slowing down is a continuous phenomenon. If corrective action had been postponed it was felt that considerable expense and trouble might have resulted.
3. The relays were very difficult to adjust, partially because of the compact nature of the chassis on which they were mounted.
4. The relay operation was slow compared to electronic switching.

For these reasons it was decided to install electronic group switching and remove the relays.

There are several possible methods which might have been chosen to accomplish the switching function electronically. The system used in the buffer drum (providing a separate writer for each head and selecting the writer by means of gates) was decided upon since it would be the most expeditious and would minimize the number of different circuit types used in the WWI drums.

The change to electronic write-group selection will cost the addition of about 200 circuits to the auxiliary-drum control. This increases by about 75 per cent the number of tubes used in the auxiliary system. However, most of the increase is in repetitive circuits and involves plug-in chassis which have proved very reliable in operation.

The disadvantages of increasing the electronics in the system are offset by reduced maintenance time and increased operating speed.

The time required for all the bay wiring necessary to install the new writers was comparable to the maintenance time already spent on present relay chassis (even though no attempt had been made to keep them operating at their original speed); a negligible amount of time was spent in maintaining the writers themselves. The number previously used in both drum systems is about equal to the number to be added.

With electronic group switching it is estimated that the group-switching operation will take about 128 microseconds as compared to 32 milliseconds which was allowed for the relays to operate.

At present all the bay wiring for the changeover has been completed (Fig. 3-2). One digit at a time is being removed from the relays and connected to the electronic write-group-selection circuits. Thus far, 8 digits have been converted. The additional 8 digits await the delivery of more writers from the shops. It is estimated that the writers will be received and the entire auxiliary-drum system converted to electronic write-group selection early in July 1954.

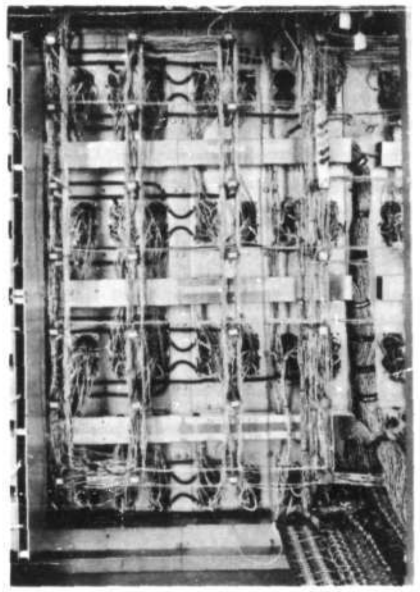


Fig. 3-2

Bay Wiring for Electronic Write-Group Selection in the WWI Auxiliary-Drum System. This bay has a capacity of 32 plug-in-type chassis, of which 27 are used for the new write switching. These 27 chassis replace 6 chassis used for relay switching.

3.2.2 Buffer-Drum System

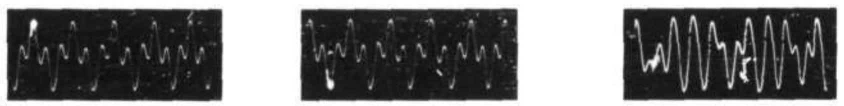
A parity check has been added to the auxiliary-storage section of the buffer drum and has been of considerable assistance in detecting marginal operation of the buffer drum and in helping to localize system troubles.

Plans are presently being made to add 2 groups to the auxiliary-storage section of the buffer drum by utilizing presently unused tracks.

3.2.3 Drum-Storage Monitors

The magnetic drums used in WWI are installed in a room on the floor below the computer and the test-control room. In order to facilitate trouble shooting, a monitor scope has been installed in the test-control room on which may be viewed the signal output from any of the tracks on the auxiliary drum and on the auxiliary-storage section of the buffer drum. A coincidence detector is provided so that any address can be selected to synchronize the scope and to intensify the waveform corresponding to the selected address. This permits immediate reading of the output signal at the selected address and detection of many types of system malfunctions. It is especially valuable for an over-all view of the track waveform which permits detection of writing between the slots, the most common cause of drum parity alarms.

Fig. 3-3 shows track waveforms from the drum-monitor system illustrating the readout from 2 normal tracks and the readout from a track on which there is writing between the slots. The intensified spot is used to locate a particular address and to indicate whether it contains a ONE (Fig. 3-3a) or a ZERO (Fig. 3-3b). Distortion of the waveform (Fig. 3-3c) indicates writing between the slots.



(a) (b) (c)

Fig. 3-3. Typical Waveforms of Magnetic-Drum Signals on the Drum-Monitor Scope

3.2.4 Ferranti Photoelectric Tape Reader

A high-speed Ferranti photoelectric tape reader was acquired for use with the WWI computer. The reader (Fig. 3-4), using prototype amplifier and control circuits, has been in service for the last 4 months. It has given satisfactory performance during most of this period except for a short time when the output-signal amplitudes were reduced to a marginal level by an accumulation of dust on the envelope of the incandescent exciter lamp. Now that this dust accumulation is recognized as a trouble source, routine cleaning

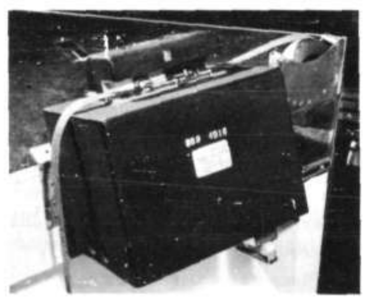


Fig. 3-4

Ferranti Photoelectric-Tape-Reader Installation in the Test-Control Room

has prevented any further occurrence of the same trouble.

Initially, difficulty with the reader was expected from 2 apparently weak points in its design. The first of these was the brake-clutch mechanism. A high degree of performance is required of this device, since it must, if the reader is to operate properly, stop the paper tape from a speed of 20 inches per second in only 0.04 inch of tape travel. One instance of trouble occurred when mechanical adjustment loosened and caused the tape-drive to bind. By using lockwashers at those points which loosened, further trouble from this cause has been prevented. Some concern is still felt that wear may adversely affect future performance of the mechanism.

The second expected source of trouble was the exciter-lamp filament, the object actually focused onto the cathodes of the photoelectric tubes by the optical systems in the reader. Any motion of this filament causes its image to shift with respect to the photoelectric tube. (In fact, a shift of a sixteenth of an inch in the filament moves the image entirely off the cathode surface of the photoelectric tube.) The filament of the exciter lamp has proven sufficiently rugged to prevent this trouble.

It is now planned to install a second reader in the WWI system. The control for these 2 readers will be shared; the computer operator determines which reader is to be used by manually selecting one or the other. Both readers are kept on a standby basis with filament and plate voltages applied, allowing either to be selected for immediate use. Effectively, this provides a replacement reader that can quickly be put into service if one reader should fail. A further advantage is that the changeover can be made by the operator instead of a technician or an engineer. The design of the 2-reader system is complete, and the second unit should be installed during the next 3 months.

4. CIRCUITS AND COMPONENTS

4.1 Vacuum Tubes

4.1.1 Vacuum-Tube Life

During the second quarter of 1954 the WWI computer operated for 1760 hours. Because there were few holidays during this period, the number of hours is slightly greater than during the past two quarters.

Vacuum-tube life has been calculated for five different tube types as outlined in Summary Report No. 36. A summary of this information together with previous data is shown below.

	FAILURE RATE, PER CENT PER 1000 HOURS			
Tube Type	1952	1953	First Quarter 1954	Second Quarter 1954
7AD7/SR1407/6145	2.00	3.3	1.75	1.6
7AK7	0.26	0.43	0.5	0.6
6080/6080WA/6AS7G		6.6*	1.1	1.5
5965		0.2*	0.4	0.3
6BL7GT		0.7*	0.3	0.5

* Last quarter 1953 only

The figures show little change from the first quarter. The large percentile fluctuation in the last three types can be expected, because the samples are not too large so the number of failures is small. When the average number of failures over a quarter is only 4, Poisson statistics indicate that the standard deviation should be 2. Hence, the expected range is rather large. The records of the 5965 and 6BL7GT continue to be very good, however.

A series of special tests was run on the WWI computer during this past quarter. In these tests the heater voltage was changed by 5 per cent, both above and below the rated value; then standard marginal checking was attempted. For heater voltage above the rated value by 5 per cent no great trouble was expected and little was found. Several tubes with grid emission were detected, however.

When computer operation was attempted with heater voltages 5 per cent below the usual values, a great deal of difficulty was encountered. There were three main classes of circuits which caused trouble: flip-flops using old 7AD7 tubes, pulse amplifiers using

old 6Y6G tubes, and cathode followers using 5687 tubes. Engineering analysis of these tubes is not yet complete so that no firm data can be given on the condition of the troublesome tubes. Most of them were original tubes, which means that the 7AD7 and 6Y6G tubes had had about 25,000 hours of operation. The 5687 tubes were used in newer circuits; they had had only about 12,000 hours of operation. None of these tubes appears on the failure record for this quarter, since processing of the test information has been delayed by engineering tests.

First indications are that variation of heater voltage in conjunction with standard d-c marginal checking may be a valuable tool for locating certain classes of faults. However, the engineering analysis must be completed before conclusions can be drawn.

4.1.2 Vacuum-Tube Research

Continued analysis has shown that the analytical results presented in the last two Summary Reports are faulty. A requirement for electrical neutrality within the oxide layer was neglected. When this requirement is included in the analysis, the limiting value for iR_oq_o/kT (q_o assumed much less than the charge on 1 electron) is 2 rather than $2\sqrt{3}$. Experimentally, it appears that q_o, the average charge on a donor, is nearly that of 1 electron. Under such conditions, electron diffusion as well as electron mobility becomes important in the conduction process; then, the limiting value of iR_oq_o/kT is 4 rather than 2 as in the case when q_o is much less than 1 electron charge. The thesis study is now being written up, and the complete analysis, which is rather extensive, will be available in the thesis within the next several months. Thus, none of the details of the analysis will be included in this report.

This study of diffusion and electrolysis in oxide cathodes was conducted with the object of determining the reason for the "droop" frequently seen in the output of direct-coupled amplifiers when driven by a step function lasting for several seconds. In many cases this droop can be traced to the cathodes of various tubes in the amplifiers. As an original hypothesis it was assumed that the droop was caused by changes in the coating resistance of the oxide cathodes, following Nergaard's[1] depletion layer theory. The experimental results have shown fairly well that changes in the coating resistance are probably not the cause of the droop. In particular, it has been shown that the emission of an oxide cathode is strongly coupled to the direct current flowing through the cathode. When the direct current is sufficient to cause a voltage drop across the cathode of about 0.2 volt, the emission is frequently about equal to the direct current. This situation may correspond to a reduction in the zero-field emission of the cathode for direct current to as little as

1. Nergaard, L. S., RCA Rev. 13, 464 (Dec. 1952)

			Reason for failure, number failed				
Type	Total in Service	Hours of Operation	Change in Characteristics	Shorts, Opens	Breakage	Burn-out	Gassy
C3J	18	0-1000	1				
OA2	76	no hours kept		1			
OC3	6	3000-4000	1				
OD3/VR150	25	0-1000			1		
2D21	203	0-1000	2				
		1000-2000	5				
		4000-5000	1				
		7000-8000	1				
		no clock hours	2				
3E29/829B	141	no hours kept	1				
3E29		23000-24000	1				
		24000-25000		1			
5U4G	19	5000-6000				1	
		9000-10000		1			
		14000-15000					1
5Y3GT	6	11000-12000				1	
6AC7	113	3000-4000		1			
		10000-11000					2
6AG7	81	0-1000		1			
6AK5/5654	11	2000-3000	1				
6AK5							
6AL5	6	7000-8000	1				
6AN5	102	0-1000		1	1		
		2000-3000			1		
		3000-4000			1		
		6000-7000			1		
6AS7G/6080/6080WA	718						
6AS7G		21000-22000		1			
6080		0-1000		2			
		1000-2000	2	1			1
		2000-3000		2			1
		3000-4000		1			
		4000-5000		2			
6080WA		1000-2000					1
		2000-3000			1		
		4000-5000	2			1	
6AU6/6136	244						
6AU6		0-1000		5			
		1000-2000	2	6	1		1
		5000-6000		2			
6136		0-1000		1	1		
		1000-2000			1		
		2000-3000		2			
6BL7GT	492	0-1000		2			
		1000-2000		1			
		2000-3000		1			
6J5	8	4000-5000		1			
		6000-7000			1		
6L6G/5881	55						
6L6G		4000-5000	1				
		11000-12000	1				
5881		2000-3000	1				

Fig. 4-1. WWI Tube Failures

			Reason for failure, number failed				
Type	Total in Service	Hours of Operation	Change in Characteristics	Shorts, Opens	Breakage	Burn-out	Gassy
6SH7	29	2000-3000	1				
6SN7	361	0-1000		1			
		7000-8000	3	1			
		11000-12000	1				
		23000-24000	3				
		24000-25000		1			
6V6GT	30	0-1000		1			
		2000-3000		1			
		23000-24000	3		1		
6X5GT	25	6000-7000		1			
		9000-10000		1			
6145/7AD7/SR-1407	3118	10000-11000		1			
6145		no hours kept		1			
		0-1000	2	16		1	
		1000-2000	1	6			1
		2000-3000	2	7			
		3000-4000	1	9			
		4000-5000		3			
		5000-6000	1	4			
7AD7		7000-8000		2			1
		18000-19000		2			
		19000-20000		1			
		20000-21000	2	2			
		23000-24000		2			
		24000-25000	8	8	1		
		25000-26000		2			
7AK7	2592	0-1000	1	1		2	
		1000-2000	1	3			
		2000-3000	6	6			
		3000-4000	1				
		7000-8000		1			
		11000-12000		1			
		23000-24000	1	1			
		24000-25000			1		
12AU7/5963	402						
5963		1000-2000	1	2			
		2000-3000	8	1			
		7000-8000	3				
		8000-9000	2			1	
		10000-11000	1	1			
715C	124	no clock hours	1				
5651	36	4000-5000			1		
5687	105	0-1000		3			
		1000-2000		3			
		2000-3000	1	2			
		3000-4000	1	1			
		5000-6000	1				
		9000-10000	1				
		10000-11000		2			
5855	6	3000-4000	1				
5965	627	3000-4000		1			
		4000-5000		1			
		5000-6000		1			
6072	42	8000-9000	1				
		10000-11000	1				
6293	8	0-1000			1		
8008	30	4000-5000	1				

April 1 - June 30, 1954

1/5 the zero-field emission observed for low-duty-factor pulses. For cathodes which operate with heavy direct-current loadings so that the steady current is only an order of magnitude or so below the zero-field pulse emission with no direct current flowing, such a reduction in emission may well cause a large change in vacuum-tube characteristics.

The manner in which emission changes with direct current is illustrated in Fig. 4-1.

This study indicates that vacuum tubes designed along the following lines should probably have a low tendency to introduce droop in direct-coupled amplifiers used for step functions lasting several seconds or so.

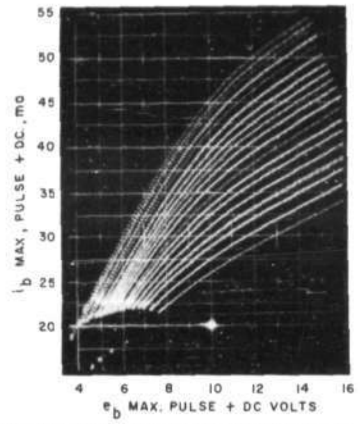
1. The current loading (ma/cm^2) of the cathode should be as low as practicable.

2. The cathode should have a zero-field emission several orders of magnitude greater than the normal cathode loading.

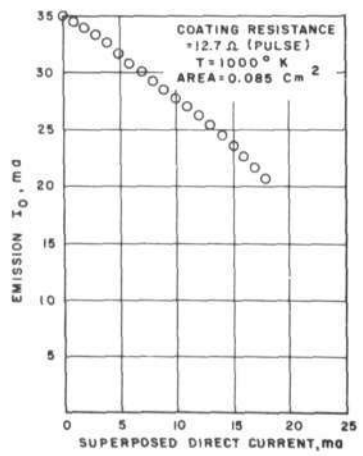
3. The cathode coating should be relatively thin and should be processed for good stability on life.

4. The cathode core material should be one not subject to trouble from interface impedance.

5. Consideration of possible dimensional changes with loading of the screen grid indicates that the screen-grid material should have good strength, a high thermal conductivity, and a low coefficient of thermal expansion. If the side rods are rigid and firmly fitted into the micas, the ratio of major to minor diameters must be



VOLTAGE-CURRENT CURVES FOR 1ma DC STEPS FROM ZERO TO 18ma DC



INFLECTION-POINT PULSE EMISSION OF RT412 AS A FUNCTION OF DIRECT-CURRENT FLOW

Fig. 4-2
Variation of Cathode Emission with Direct Current

as small as possible. For a given minor diameter, the amount of thermal distortion is very nearly proportional to the square of this ratio.

These considerations indicate that tubes which can be expected to have very little drift for steps of several seconds probably will not have exceptional performances. Engineering compromises in practical design should allow tubes adequate for most purposes, however.

4.2 Component Replacements

Fig. 4-3 lists the replacements of components other than tubes during the second quarter of 1954.

Component	Type	Total in Service	No. of Failures	Hours of Operation	Comments
Capacitors	0.1- f 500-v mica	2513	1	22000-23000	Leakage
	10- f 500-v mica	14	1	22000-23000	Leakage
	0.02- f ceramic disc	10	1	0	Shorted
	4.0- f 450-v electrolytic decoupling	105	1	4000-5000	Low capacity
Circuit Breaker	Heineman, 0411H, 5-amp 125-v DC	20	1	0	Short
Potentiometer	25-watt 1000-ohm W/W ± 10%	160	1	0	Open
Crystal Diodes	D-358/1N38A/1N6b	4090			
	D-358		1	25000-26000	Low R_b
	1N38A		2	no clock hours	Short
			1	1000-2000	Low R_b
			1	7000-8000	Low R_b
			1	19000-20000	Low R_b
	1N68		1	0-1000	Open
	D-357/1N34A	13883			
	D-357		1	16000-17000	Low R_b
			1	17000-18000	Low R_b
	1N34A		6	1000-2000	5 low R_b; 1 drift
			1	2000-3000	Low R_b
			1	3000-4000	Low R_b
			1	5000-6000	Low R_b
			1	6000-7000	Low R_b
			3	7000-8000	2 low R_b; 1 short
			1	8000-9000	Low R_b
	1N92	71	1	no clock hours	Drift
Rectifier	Selenium 10-ma DCL No. 134-1	834	2	0-1000	Short; intermittent
Relay	Clare Relay No. 40EC	36	1	1000-2000	Stack short
Switches	DPDT toggle	27	1	0	Intermittent
	SPST toggle	606	2	24000-25000	Intermittent
Transformers	Pulse 1:1 193-6	1046	1	4000-5000	Open secondary
	UTC J-14	6	1	10000-11000	Sensitive to vibration

Fig. 4-3. WWI Component Failures April 1 - June 30, 1954

5. ACADEMIC PROGRAM

5.1 Advanced Seminars on Computing

5.1.1 Seminar on Advanced Programming Techniques

These seminars provide an opportunity for the exchange of information on programming, components, logical design, and general developments in the computer field. The program was concluded during this quarter with the following talk:

9 April 1954	Recent Developments in the Digital Computer Field Including Current Trends and New Computers	C. W. Adams

5.1.2 Seminar on Computing Machine Methods

The seminars on Computing Machine Methods are arranged jointly by representatives of the MIT Committee on Machine Methods of Computation and the MIT Digital Computer Laboratory. The series was concluded during this quarter with the following two talks:

6 April 1954	An Electronic Analog Device for Delay Line Synthesis	Prof. James B. Reswick
20 April 1954	The Solution of Linear Programming Problems by the Simplex Method of Computation	Hrand Saxenian

5.2 MIT Courses

Twelve students enrolled in Electrical Engineering subject 6.537. Each student programmed, prepared on punched tape, debugged, and executed on Whirlwind I a problem of his own choice. The problems have been completed and are described under problem 173, above.

A series of three lectures was given on the subject of digital computers by J. D. Porter as part of Prof. T. Malone's course 119.33, Applied Climatology. A total of 23 students and staff members attended the lectures.

5.3 Visitors

On 27 April 1954, nine CBS-TV employees spent the day filming the computer for a TV film on MIT research projects to be shown on a program called "The Search."

Tours of the WWI installation include computer and Flexowriter demonstrations and an informal discussion of the major computer components. During the past quarter the following tours were conducted:

6 April	20 representatives of industrial concerns and 15 MIT representatives
26 April	20 members of the Society of Women Engineers
28 April	14 students from Harvard University
3 May	26 representatives of MIT and Harvard
6 June	5 MIT and 1 Wentworth Institute personnel
22 June	64 members of the summer course on Operations Research
28 June	19 businessmen attending the summer course on Control Problems

5.4 Making Electrons Count

The MIT Digital Computer Laboratory has recently completed a 23-minute, single-reel, 16-mm motion picture in color with narrative and background music. This film, entitled "Making Electrons Count," shows something of the location and work of the Laboratory, pictures a number of different applications of digital computers, and then tells the story of a typical user learning to use MIT's Whirlwind I computer to solve his problem. The history of computers and the various component parts of Whirlwind I also are shown briefly. The film has little or no technical detail and emphasizes the use, rather than the construction, of computers. It attempts to describe the use of a computer honestly, without underplaying the unpleasant details, but in a fairly light-hearted vein.

Any organization wishing to borrow this film should complete application DCL-2 obtainable at the Digital Computer Laboratory.

The film was shown on 23 June at the ACM meeting in Ann Arbor, Michigan.

6. APPENDIX

6.1 Reports and Publications

Project Whirlwind technical reports and memorandums are routinely distributed to only a restricted group known to have a particular interest in the Project, and to ASTIA (Armed Services Technical Information Agency) Document Service Center, Knott Building, Dayton, Ohio. Regular requests for copies of individual reports should be made to ASTIA; emergency requests, to John B. Bennett, Lincoln Laboratory, P. O. Box 73, Lexington 73, Mass. Att: Code DCL-6.1.

The following reports and memorandums were among those issued during the second quarter of 1954.

No.	Title	No. of Pages	Date	Author
SR-36	Summary Report No. 36, Fourth Quarter 1953	66		
R-221	Whirlwind I Operation Logic	109	5-1-54	M. Mann R. Rathbone J. Bennett
M-2728	Increased Facilities for Visual Display in the WWI Input and Output	18	3-17-54	G. A. Young
M-2729	Paper Tape Units and Printers in the WWI Input-Output System	19	3-15-54	G. A. Young
M-2834	Proposed Memory Address Selection System	3	5-26-54	D. Shansky
M-2868	Operation of the Magnetic Tape Printout Systems	2	6-16-54	A. Roberts J. Cahill R. Davis A. Favret C. Grandy
M-2870	Progress Report, May 17 through June 13, 1954	22	6-13-54	S & EC Group

6.2 Professional Society Papers

Title	Author	
Digital Computers and Their Applications	J. D. Porter	Boston Section, Society of Women Engineers, 26 April 1954
The MIT System of Automatic Coding: Comprehensive, Summer Session, and Algebraic	C. W. Adams	Symposium on Automatic Programming for Digital Computers, Washington, D. C., 13 May 1954
Ferrites for Digital Computers	F. E. Vinal	MIT Course 3.72, 18 May 1954
Transistor Switching Circuitry for High-Speed Digital Computers	E. W. Cohler	Association for Computing Machinery, Ann Arbor, 23 - 25 June 1954
The MIT Comprehensive System of Service Routines	D. Combelic	Association for Computing Machinery, Ann Arbor, 23 - 25 June 1954
The Programmed Synthesis of Digital Computers within Digital Computers	F. E. Heart	Association for Computing Machinery, Ann Arbor, 23 - 25 June 1954
Algebraic Synthesis of Logical Feedback Nets	R. C. Jeffrey	Association for Computing Machinery, Ann Arbor, 23 - 25 June 1954
Development of the 64 x 64 Core Memory	W. N. Papian	Association for Computing Machinery, Ann Arbor, 23 - 25 June 1954